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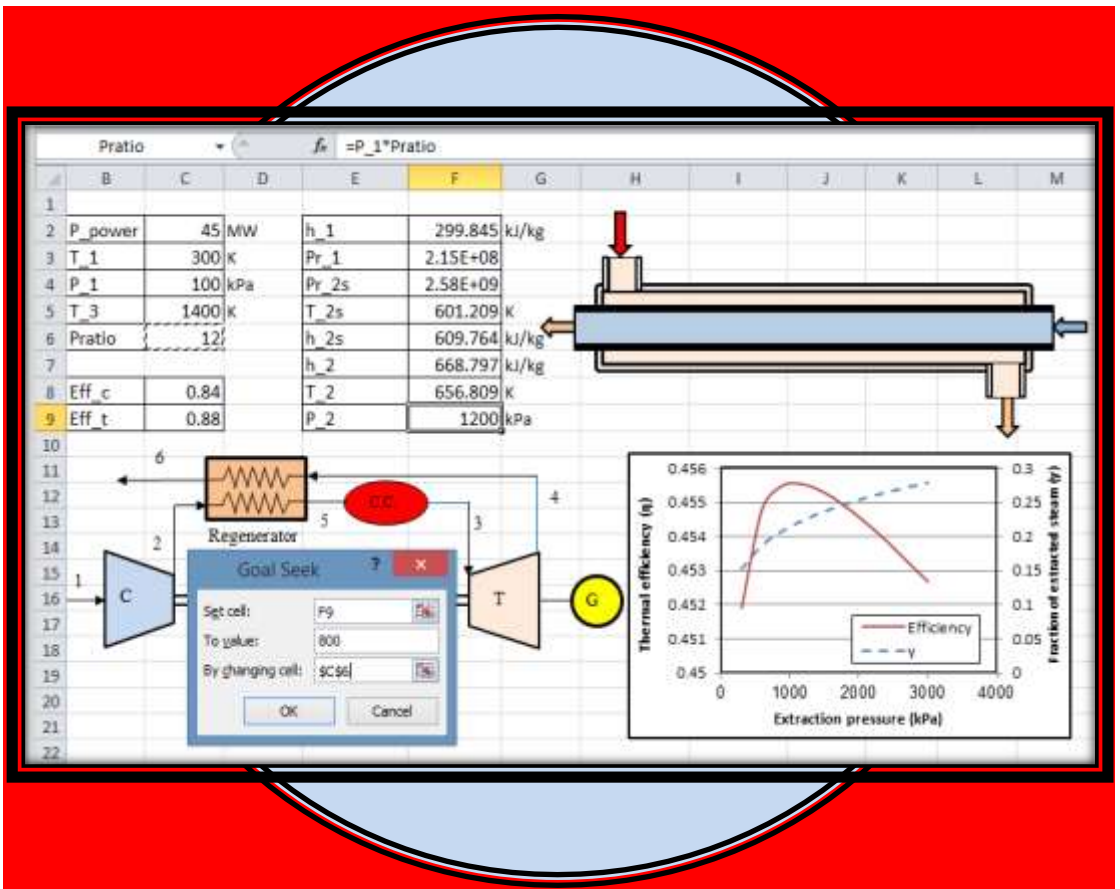
# Computer-Aided Thermofluid Analyses using Excel



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# Computer-Aided Thermofluid Analyses using Excel

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*This book is dedicated to the memory of my parents*

*Amna and El-Awad*

*May Allah bless their souls in Heaven*

**August 4, 2019**

**For the Thermax add-in visit:**

https://www.researchgate.net/publication/334959438_Thermax

## Preface

This book shows how Microsoft Excel can be used as an effective educational platform for thermofluid analyses. Excel provides a rich library of built-in functions and powerful tools for data visualisation and analysis. The Solver add-in that comes with it enables the students to perform multi-variable optimisation analyses of thermofluid systems. Building on these capabilities, the Excel-based platform for thermofluid analyses presented in this book also utilises the Visual Basic for Applications (VBA) programming language that comes with Microsoft Office. The book shows how VBA can be used for developing additional user-defined functions (UDFs) for supplementing Excel's built-in functions and introduces an Excel add-in, called Thermax, that provides UDFs for determining the thermodynamic fluid properties. The fluids covered by Thermax include twelve ideal gases, saturated water and superheated steam mixtures, six refrigerants, humid air for psychrometric analyses, and air at standard atmospheric pressure. Property add-ins developed by other academic and research institutions or individuals can be used to extend the range of thermofluid analyses that can be performed with the Excel-based platform.

Compared to the software that is dedicated to thermofluid analyses, the main advantage of Excel as an educational tool is its wide availability on computers and mobile phones. The introductory courses in computer applications usually taught to junior engineering students make them familiar with the basic functions of the spreadsheet. The Excel-based modelling platform described in this book provides the needed transparency and flexibility that allow the students to build white-box models from basic principles. An important advantage of this platform is that it minimises the need for the students to be skilful in numerical methods and computer-programming in order to apply computer-based methods in thermofluid analyses. This allows the students to pay more attention to the application of thermofluid principles pertinent to their analyses.

The principles underlying thermofluid analyses are usually taught to engineering students in three separate courses: thermodynamics, fluid mechanics, and heat transfer. Rather than dealing with thermofluid analyses in this segregated manner, most chapters of the book adopt a unified learning-by-example approach that best suits the students who have already studied the three basic courses. Most of the examples considered in the book are based on relevant examples given in popular textbooks so that the solutions obtained with Excel can be verified. Exercises are given at the end of each chapter to help students sharpen their skills related to that chapter. The last three chapters of the book differ in that they deal with selected topics related to fluid mechanics, heat transfer, and thermodynamics. The extended exercises given at the end of these three chapters involve more challenging assignments that suit mini projects. This arrangement makes the book also useful for those students interested in only one thermofluid subject.

Although the book is primarily written for mechanical engineering students, it may be equally useful other engineering specialisations. Enough material is covered in the book for a stand-alone course at the intermediate level on computer-aided analyses of thermofluid and energy systems. Selected topics or examples can also be used to supplement standard courses on thermodynamics, fluid dynamics, and heat transfer. It is also hoped that the book can be a useful reference for practicing engineers in the area of thermofluid and energy systems.

# CONTENTS

## Nomenclature

| | |
|---|---|
| $A$ | Area |
| $C$ | Friction coefficient in Hazen-Williams equation, Eq. (1.27) |
| $C_f$ | Friction coefficient, defined by Eq. (4.34) |
| $c_p$ | Specific heat at constant pressure, kJ/kg·°C |
| $c_v$ | Specific heat at constant volume, kJ/kg·°C |
| $D$ | Diameter |
| $f$ | Friction factor |
| $F$ | Correction factor, defined by Eq. (D.4) |
| $F_D$ | Drag force, defined by Eq. (4.34) |
| $g$ | Acceleration of gravity |
| $h$ | Average heat-transfer coefficient |
| $h$ | Enthalpy, kJ/kg |
| $h_f$ | Major friction in a pipe system |
| $k$ | Thermal conductivity, W/m·°C |
| $k$ | Isentropic exponent, dimensionless ($=c_p/c_v$) |
| $K$ | Minor losses friction coefficient in a pipe system, defined by Eq. (1.28) |
| $L$ | Length |
| $m$ | Mass |
| $\dot{m}$ | Mass rate of flow |
| M | Molecular weight |
| $P$ | Pressure, usually kPa |
| $P_r$ | Reduced pressure |
| $P_r$ | Relative pressure (for an ideal gas), defined by Eq. (4.4) |
| $q$ | Heat-transfer per kg of the working fluid, usually kJ |
| $Q$ | Heat, usually kJ |
| $Q$ | Volume flow rate, defined by Eq. (1.19) |
| $\dot{Q}$ | Rate of heat transfer, W or kW |
| $r$ | Radius or radial distance |
| $R$ | Gas constant, kJ/kg.K |
| $R_{th}$ | Thermal resistance, usually °C/W |
| $R_u$ | Universal gas constant kJ/kmol.K |
| $s$ | Entropy |
| $T$ | Temperature |
| $T_c$ | Critical temperature |
| $T_r$ | Reduced temperature |
| $u$ | Internal energy |
| $U$ | Overall heat-transfer coefficient of a heat-exchanger |
| $v$ | Specific volume, m$^3$/kg |
| $v_r$ | Relative volume (for an ideal gas), defined by Eq. (4.8) |
| $V$ | Velocity, usually m/s |
| $w$ | Work-done per kg of the working fluid, usually kJ |

$\dot{W}$      Power, W or kW

$x,y,z$      Space coordinates in cartesian system

**Greek Characters**

$\alpha$      Amortisation rate, defined by Eq. (6.24)

$\delta$      Thickness (e.g. of insulation)

$\Delta$      Difference (e.g. temperature)

$\varepsilon$      Roughness of surface material

$\varepsilon$      Heat-exchanger effectiveness

$\varepsilon_f$      Flow exergy

$\eta$      Efficiency

$v$      Kinematic viscosity, $m^2/s$

$\rho$      Density, $kg/m^3$

$\tau$      Time, annual operating hours of a system

**Dimensionless Groups**

Nu      Nusselt number

Pr      Prandtl number

Re      Reynolds number

**Subscripts**

$f$      Saturated liquid condition

$fg$      Difference in property between saturated liquid and saturated vapour

$g$      Saturated vapour condition

$lm$      Log-mean

$s$      Saturation temperature or pressure

$s$      Evaluated at the surface

$\infty$      Evaluation at free-stream ambient conditions

# 1

# Introduction

Cars, refrigerators, and air-conditioners that have become indispensible belongings for the individuals and families around the globe require large amounts of energy that mainly come from burning fossil fuels in power-generation plants. Apart from being non-renewable sources of energy, large-scale combustion of fossil fuels is the main cause for global warming and its devastating effects are felt at different parts of the world. Therefore, proper design and operation of these and other energy-conversion devices is becoming increasingly important. The principles underlying the designs of these systems are usually taught to engineering students in three *thermofluid* subjects; which are thermodynamics, fluid mechanics, and heat transfer. This chapter reviews these principles and their application for typical analyses in each subject. Due to the variation of the working fluids properties with temperature and pressure, the equations involved in these analyses are usually nonlinear and difficult to solve without introducing many simplifying assumptions that reduce their accuracy. In this respect, the chapter highlights the advantages of computer-aided methods and describes the Excel-based modelling platform for thermofluid analysis used in this book.

## 1.1.    A review of thermofluid subjects

The two principles that form the framework for thermofluid analyses are the conservation of mass (the continuity equation) and the conservation of energy (the first-law of thermodynamics). These principles take different mathematical forms depending on the nature of the flow (steady or unsteady), type of fluid (compressible or incompressible), and whether the system is open or closed. Auxiliary relationships are needed in order to quantify the various parameters involved in the relevant equations such as pressure-variations, friction losses, and rates of heat-transfer. The following sections review the main concepts of thermofluid analyses introduced in the three subjects of thermodynamics, fluid dynamics, and heat-transfer and illustrate the application of these principles by considering relevant cases.

## 1.1.1.    Thermodynamics

The principles of *engineering thermodynamics* allow us to determine the amount of energy transfer between the system and its surroundings in the form of work or heat and to determine the effectiveness of energy utilization in the system. Thermodynamics has four basic laws, the most important of which are the first law and the second law of thermodynamics. In addition to these two basic laws, thermodynamic analyses use many relationships that describe the behaviour of the particular system being considered (closed or open) or the particular fluid involved (a liquid, a liquid-vapour mixture, a gas, or a gaseous mixture). To illustrate the application of thermodynamic laws and relationships in a typical analysis, consider the air-compression system shown in Figure 1.1 that has two stages of compression separated by an intercooler. Air enters the system at a temperature $T_1$ and pressure $P_1$. The first-stage compressor, $C_1$, compresses the air adiabatically to state 2, after which it enters the intercooler where its temperature is reduced to $T_3$. The second-stage compressor, $C_2$, then increases the air pressure to $P_4$ and the temperature to $T_4$.
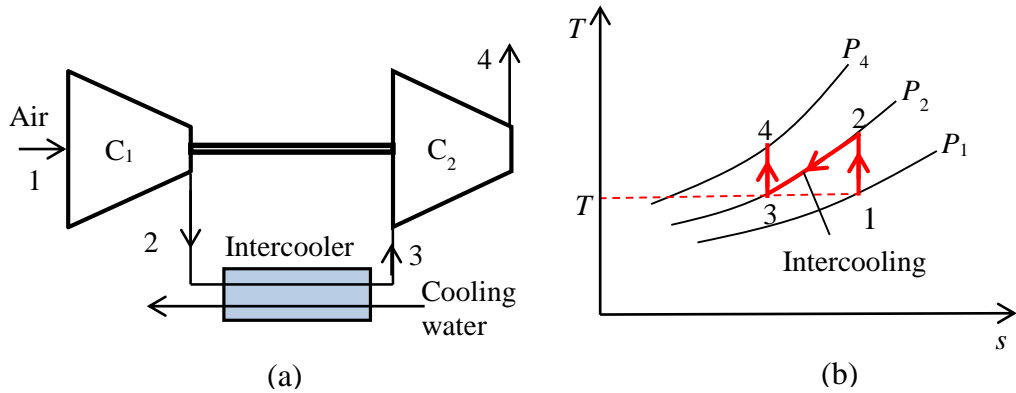
Figure 1.1. Schematic and *T-s* diagrams of a two-stage air compressor with inter-stage intercooling

The total compression work depends on how the total pressure ratio is divided between the two stages and there is a certain value of the intermediate pressure ($P_i$) that minimises the total work. The principles of thermodynamics help us to determine this optimum intermediate pressure as shown below.

Treating the two compressor stages as steady-flow processes, and neglecting changes in kinetic and potential energy, the first-law of thermodynamics leads to [1]:

$$q - w = \left(h_{out} - h_{in}\right) \tag{1.1}$$

Where $q$ and $w$ are the amounts of heat transfer and work transfer per unit mass flow of air, respectively, and ($h_o - h_i$) is the resulting enthalpy change in the stage. Equation (1.1) adopts the sign convention that heat into the system is positive, while work into the system is negative. Assuming the compression processes in both stages to be adiabatic ($q$=0) and reversible means that the processes are isentropic as shown in Figure 1.1.b. Using an average specific heat for air at constant pressure ($c_p$), the compression work per unit mass flow of air in stage 1 ($w_1$) and in stage 2 ($w_2$) can be determined from Equation (1.1) as follows:

$$w_1 = -\left(h_2 - h_1\right) = -c_p\left(T_2 - T_1\right) \tag{1.2}$$

$$w_2 = -\left(h_4 - h_3\right) = -c_p\left(T_4 - T_3\right) \tag{1.3}$$

Therefore, the total compression work in both stages ($w_{total}$) is given by:

$$w_{total} = w_1 + w_2 = -c_p\left[\left(T_2 - T_1\right) + \left(T_4 - T_3\right)\right] \tag{1.4}$$

Assuming perfect intercooling, i.e. $T_3 = T_1$, and rearranging Equation (1.4):

$$w_{total} = c_p T_1 \left[ \left( 1 - \frac{T_2}{T_1} \right) + \left( 1 - \frac{T_4}{T_3} \right) \right] = c_p T_1 \left[ 2 - \left( \frac{T_2}{T_1} \right) - \left( \frac{T_4}{T_3} \right) \right] \qquad (1.5)$$

Since we already assumed the two compression processes to be isentropic and the specific heats $c_p$ and $c_v$ for air to be constant, the temperature ratios in Equation (1.5) can be converted into pressure ratios by using the following relationships:

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \qquad (1.6)$$

$$\frac{T_4}{T_3} = \left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}} \qquad (1.7)$$

Where $k$ is the ratio ($c_p/c_v$); $c_v$ is the specific heat for air at constant volume. Appendix A gives properties of atmospheric air at different temperatures. Making another assumption that there is no pressure loss in the intercooler, then $P_3 = P_2 = P_i$. Substituting from Equations (1.6) and (1.7) into Equation (1.5), we get:

$$w_{total} = c_p T_1 \left[ 2 - \left( \frac{P_i}{P_1} \right)^{\frac{k-1}{k}} - \left( \frac{P_4}{p_i} \right)^{\frac{k-1}{k}} \right] \qquad (1.8)$$

To see how the total compression work varies with the intermediate pressure $P_i$, let us consider the specific case in which $T_1 = 300K$, $P_1 = 100$ kPa, and $P_4 = 900$ kPa. Using Equation (1.8), the total compression work in the system was calculated for different values of $P_i$ and the result is shown in Figure 1.2. The figure shows that the value of $P_i$ at which the total compression work is minimal is around 300 kPa. Increasing or decreasing $P_i$ from this value will increase the compression work.
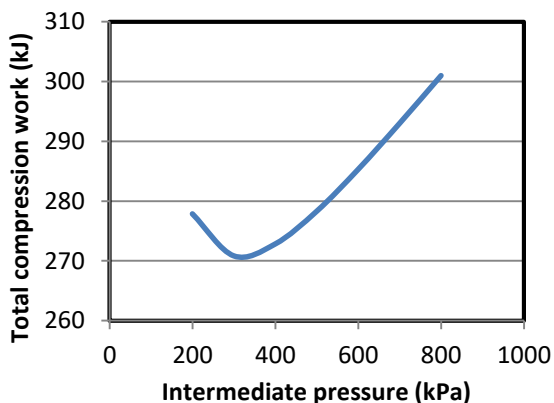


Figure 1.2. Variation of the total compression work with the intermediate pressure

The principles of thermodynamics are useful for performance analyses and optimisation of various types of power-generation and refrigeration systems. For example, consider the regenerative steam-turbine power plant shown in Figure 1.3. This plant consists of a boiler house for producing superheated steam, a high-pressure steam turbine (HPT), a low-pressure steam turbine (LPT), a condenser, an open feed-water heater (FWH) and two feed-water pumps. A fraction of the steam ($y$) is extracted after the HPT for preheating the feed-water before going back to the boiler house.
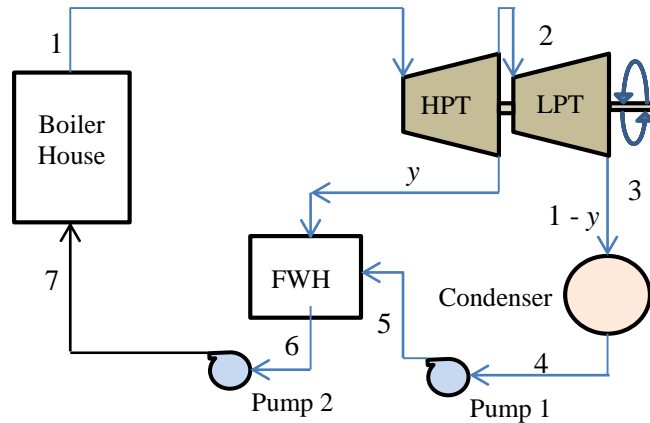


Figure 1.3. Schematic diagram of a regenerative steam-turbine power plant

The extracted steam reduces the work output from plant, but it also reduces the amount of heat added in the boiler and its net effect is to increase the thermal efficiency of the plant. There is also a certain extraction pressure for the steam at which the plant's thermal efficiency attains a maximum value. As shown below, the principles of thermo-dynamics can also be used to determine this optimum steam-extraction pressure.

The total specific work output from the two turbines ($w_{out}$) and the total work input to the two pumps ($w_{in}$) are given by:

$$w_{out} = w_{HPT} + w_{LPT} \tag{1.9}$$

$$w_{in} = w_{P1} + w_{P2} \tag{1.10}$$

Where $w_{HPT}$ and $w_{LPT}$ are the specific work output from the high-pressure turbine and the low-pressure turbine, respectively, and $w_{P1}$ and $w_{P2}$ are the specific work input in pump 1 and pump 2, respectively. Assuming the two turbines and the two pumps to be adiabatic and neglecting the changes in kinetic and potential energies, the work output or input for each device can be determined from the enthalpy difference across the device. Per each kg of steam generated in the boiler, these are given by:

$$w_{HPT} = (h_1 - h_2)$$ (1.11)

$$w_{LPT} = (1 - y)(h_2 - h_3)$$ (1.12)

$$w_{P1} = (1 - y)(h_5 - h_4)$$ (1.13)

$$w_{P2} = (h_7 - h_6)$$ (1.14)

Mass and energy balance over the open feed-water heater gives:

$$yh_2 + (1 - y)h_5 = 1 \times h_6$$ (1.15)

The net specific work output from the plant ($w_{net}$) is then given by:

$$w_{net} = w_{out} - w_{in}$$ (1.16)

The specific heat input to the boiler ($q_{in}$) can also be determined from the relevant enthalpy change as follows:

$$q_{in} = (h_1 - h_7)$$ (1.17)

Therefore, the thermal efficiency of the plant ($\eta$) can be calculated from:

$$\eta = w_{net} / q_{in}$$ (1.18)

Both $w_{net}$ and $\eta$ depend on the fraction of steam extracted for regeneration ($y$); which in turn depends on the extraction pressure ($P_2$). Figure 1.4 shows the variation of $y$ and $\eta$ with $P_2$ for an ideal cycle in which $P_1$ = 15 MPa, $T_1$ = 600°C, and $P_4$ = 10 kPa. The figure shows that the cycle's efficiency attains a maximum value of 45.55% when $P_2$ is in the range of 1000 kPa.

It should be mentioned that the working fluid in the above power plant changes phase from subcooled liquid water to superheated steam in the boiler, to saturated mixture of water and steam in the low-pressure turbine, and returns to subcooled water in the condenser. Therefore, appropriate property relationships, tables, or charts are needed in order to determine the working fluid properties at different states. The principles of thermodynamics are also applied in the analyses of air-conditioning and combustion processes as well as the analyses of processes or systems involving chemical reactions. For such analyses, thermodynamics provides the basic relationships needed to quantify the effects of fluid mixing and chemical reactions on the properties of working fluid and on the transfer of energy and effluents to or from the thermofluid system.
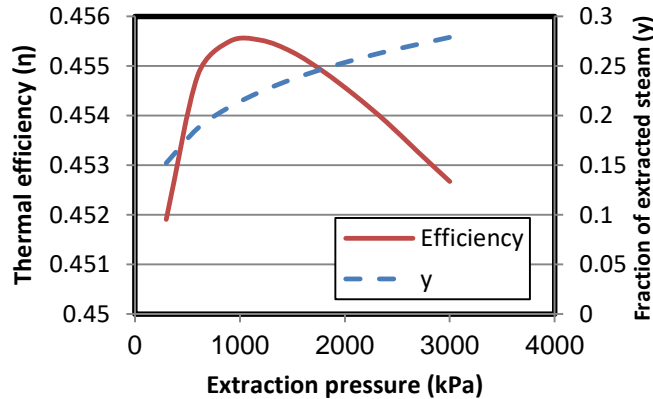
Figure 1.4. The effect of intermediate pressure ($P_2$) on the fraction of extracted steam
($y$) and thermal efficiency ($\eta$) of a regenerative steam-turbine power plant

### 1.1.2. Fluid dynamics

Fluid-transporting systems have many components such as pipes, pumps, compressors, control valves, flow-measuring devices, etc. The principles of *fluid dynamics* help us to estimate the power needed for overcoming friction and pressure losses in these components and to determine their suitable types and sizes. To illustrate the application of these principles, consider the simple pump-pipe system shown in Figure 1.5 that conveys a liquid between two non-pressurised tanks. Suppose that we want to determine the pump power needed to deliver a liquid of a specific weight $\gamma$ between points $A$ and $B$ at a rate of $Q$ if the pipe's length is $L$ and diameter $D$.
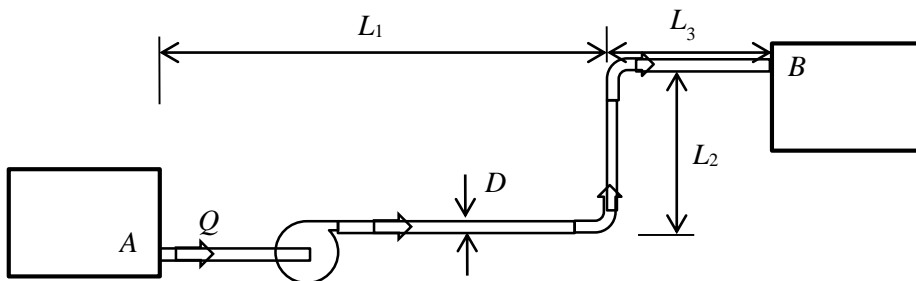


Figure 1.5. Schematic diagram of a simple pump-pipe system

The power needed for the pump ($\dot{W}$), in W, can be determined from the following power equation:

$$\dot{W} = \gamma \times Q \times h_p / \eta \tag{1.19}$$

Where $\gamma$ is the specific weight of the transported liquid (N/m³), $Q$ is the volume flow rate of the liquid (m³/s), $h_p$ is the pump head (m) needed to circulate the fluid through the pipe, and $\eta$ is the combined efficiency of the pump and the electric motor. For

steady flow of an incompressible fluid, $h_p$ can be determined from the following energy equation:

$$h_p = h_{f,total} + (z_B - z_A) + \frac{V_B^2 - V_A^2}{2g} \tag{1.20}$$

Where $h_{f,total}$ is the total head loss through the system due to friction (m), $z_A$ and $z_B$ are the elevations (m) at points $A$ and $B$, respectively, and $V_A$ and $V_B$ are the corresponding fluid velocities (m/s).

The total friction head loss $h_{f,total}$ consists of two parts: the *major friction loss* ($h_f$), which is the part lost in the pipe itself, and the *minor friction head loss* ($h_{f,minor}$), which is the part lost in other components of the system, i.e., elbows and tees, valves, etc. The major friction loss can be determined from the following Darcy-Weisbach equation [2]:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{1.21}$$

Where $f$ is the Dracy friction factor (dimensionless), $V$ the fluid velocity (m/s), $L$ the total length of the pipe (m), and $D$ the internal diameter of the pipe (m). The value of the friction factor, which depends on the roughness of the pipe surface and on whether the flow is laminar or turbulent, can be obtained from a Moody diagram [2] or calculated from a relevant formula. For laminar flows, $f$ can be calculated from:

$$f = 64/\mathrm{Re} \qquad \mathrm{Re} < 2300 \tag{1.22}$$

Where Re is the Reynolds number defined as:

$$\mathrm{Re} = VD/\nu \tag{1.23}$$

Where $\nu$ is the kinematic viscosity of the flowing fluid (m²/s). For a turbulent flow in smooth tubes, $f$ can be determined from the first Petukhov equation [2]:

$$f = (0.790 \ln \mathrm{Re} - 1.64)^{-2} \qquad 10^4 < \mathrm{Re} < 10^6 \tag{1.24}$$

For a turbulent flow in rough pipes, $f$ can be obtained from the following Swamee-Jain equation:

$$f = 0.25 \Bigg/ \left[ \log_{10} \left( \frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}} \right) \right]^2 \qquad \mathrm{Re} > 4000 \tag{1.25}$$

Chemical engineers usually determine the pipe friction by using the Chezy-Manning equation instead of the Darcy-Weisbach equation. According to this equation:

$$h_f = 2f \frac{L}{D} \frac{V^2}{g}$$
(1.26)

Where $f$ is the Fanning friction factor. Comparison with Equation (1.21) reveals that the value of the Fanning friction factor used in Equation (1.26) is 4 times the corresponding value of the Darcy friction factor. Civil engineers determine the friction head loss in water-transporting pipes by using the following Hazen-Williams equation:

$$h_f = \frac{10.67 L Q^{1.852}}{C^{1.852} D^{4.8704}}$$
(1.27)

Where $C$ is a coefficient that depends on the roughness of the pipe. Unlike Equations (1.21) and (1.26), Equation (1.27) is applicable for both laminar and turbulent flows.

The minor friction loss, $h_{f,minor}$, can be determined from the following equation:

$$h_{f,\mathrm{min}or} = \sum_1^n K \frac{V^2}{2g}$$
(1.28)

Where $n$ is the total number of components in the system and $K$ is a coefficient the value of which for each component can be found in relevant tables.

The equations described above can be used to determine the required pump power for specified values of the pipe length, pipe diameter, flow rate, fluid viscosity, and pipe material. The equations can also be used to determine the minimum diameter of the pipe, or the maximum flow rate of the fluid to be delivered, such that the friction loss in the system or the needed pump power does not exceed a specified limit. By also taking into consideration the initial cost of the pump-pipe system and the cost of electrical energy needed by the pump, fluid-dynamics equations can be used to determine the pipe diameter that gives the best economic compromise between the initial cost and the operating cost of the system over its life-time.

The principles of fluid dynamics also enable us to select the appropriate type and size of the pump for a given pump-pipe system. This is achieved with the help of characteristic curves usually provided by the manufacturers such as that shown in Figure 1.6 for a centrifugal pump. In many practical situations a single pump or a single compressor may not be adequate to meet the required fluid flow rate or delivery pressure and more than one pump or compressor have to be used. In this situation, the principles of fluid dynamics allow us to decide when to arrange the pumps/compressor in parallel or in

series. Figures 1.7 and 1.8 show the characteristic curves of two different arrangements of centrifugal pumps.
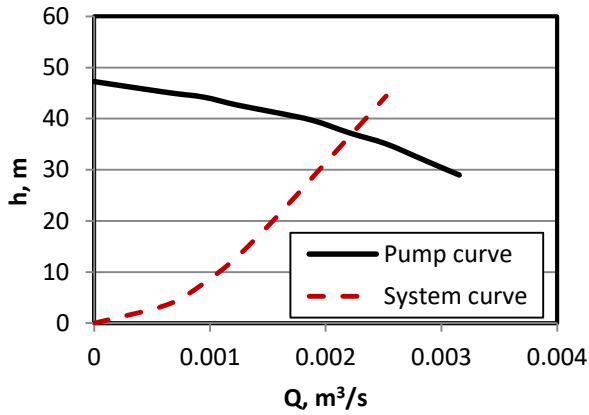


Figure 1.6. Characteristic curve for a centrifugal pump
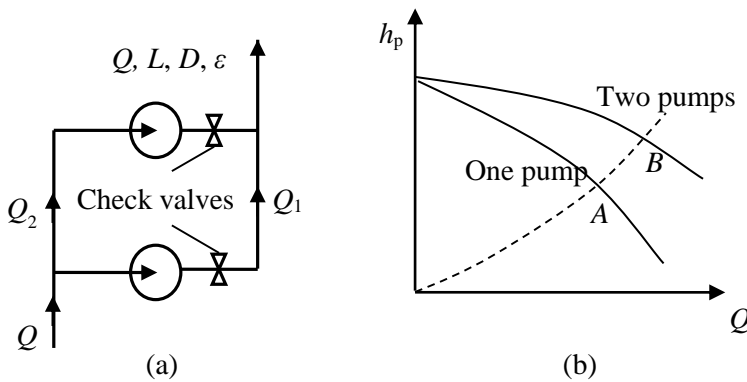


Figure 1.7. Two centrifugal pumps connected in parallel (adapted from Burmeister [3])
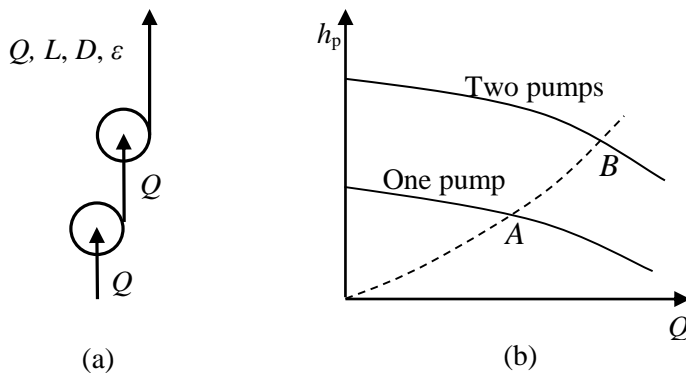


Figure 1.8. Two centrifugal pumps connected in series (adapted from Burmeister [3])

### 1.1.3.  Heat transfer

The principles of *heat transfer* are required for design analyses of thermofluid devices like boilers, condensers, and heat exchangers used to transfer thermal energy between the system's components or between the system and its surroundings. The subject also describes the methods that can be used to minimise or maximise the rate of heat-transfer such as thermal insulation, extended surfaces (fins), etc. To illustrate the use of heat-transfer concepts in thermofluid analyses, consider Figure 1.9 that shows a pipe of internal radius $r_1$ and external radius $r_2$. The fluid inside the pipe is at a temperature $T_i$ while the temperature of the surrounding air is $T_\infty$. The temperature difference between the pipe and the surroundings will cause heat gain or heat loss to/from the pipe and, in order to reduce this heat gain or heat loss, the pipe has to be surrounded by an insulation material. The principles of *heat transfer* help us to determine the required thickness of insulation ($\delta = r_3 - r_2$) that keeps the rate of heat-transfer within a specified limit.
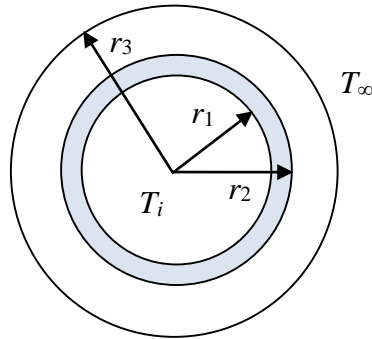


Figure 1.9. Schematic for an insulated metal pipe

The rate of heat transfer ($\dot{Q}$) to/from the pipe can be calculated from [4]:

$$\dot{Q} = (T_i - T_\infty)/R_{th} \tag{1.29}$$

Where $R_{th}$ is the combined thermal resistance to heat-transfer by conduction, convection, and radiation, which is given by [4]:

$$R_{th} = \frac{1}{h_i A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{1}{h_o A_3} \tag{1.30}$$

Where $h_i$ and $A_1$ are the heat-transfer coefficient and surface area inside the pipe, respectively, $h_o$ and $A_3$ are the heat-transfer coefficient and surface area outside the insulated pipe, respectively, $L$ is the length of the pipe, and $k_1$ and $k_2$ are the thermal conductivities of the pipe and the insulation, respectively. To simplify the analysis, it is usually assumed that $h_o$ takes into account the heat-transfer by both convection and

radiation to/from the insulation surface. The thickness of the metal pipe is usually small compared to its diameter, while its thermal conductivity is much higher than that of the insulation material. Therefore, the analysis can be simplified further by neglecting the second term that gives the thermal resistance due to conduction through the pipe.

The above analytical model can be used to determine the thickness of insulation required for reducing the rate of heat transfer to a specified limit or for controlling the surface temperature to a limit that is dictated by safety or other practical considerations. Although the thicker the insulation, the lower will be the rate heat transfer, the cost of insulation increases with its thickness and there is a certain thickness beyond which adding more insulation will be uneconomical. This *economical thickness* of insulation can be determined by extending the above heat-transfer model so that the cost of insulation and that of the saved energy can be calculated and compared.

Figure 1.10 shows a typical arrangement in which circular fins are attached to the surface of a pipe so as to boost the rate of heat-transfer between the fluid being transported with the pipe and the surrounding gas, usually air. As shown in Appendix C, the principles of *heat transfer* can be used to develop the required mathematical equations that describe the variation of temperature and rate of heat transfer over the surface of the fin. These equations can then be used to evaluate the effectiveness and efficiency of the fin.
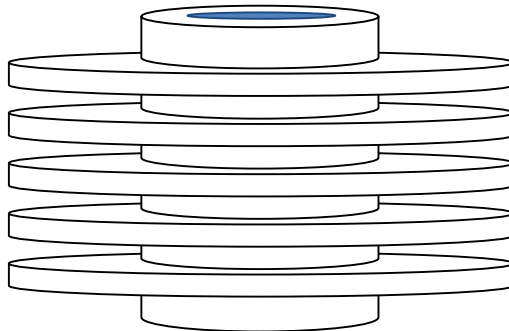


Figure 1.10. Circular fins attached to a pipe

Another important application of these principles in thermofluid analyses is that related to heat-exchangers. A heat-exchanger is a device used for transferring heat between two fluids through a separating surface usually a pipe or a tube. Figures 1.11 and 1.12 show a shell-and-tube heat-exchanger and a cross-flow heat-exchanger, respectively. These two types of heat-exchangers are commonly used in industries, power-plants, and vehicles. Heat-exchanger analyses either aim at determining the required size (i.e. surface area) for a specified heat-transfer duty or determining the exit temperatures of the two streams from a specified heat-exchanger type and size. Appendix D describes the log-mean temperature difference (LMTD) method and the effectiveness-NTU method used for these two types of analyses.
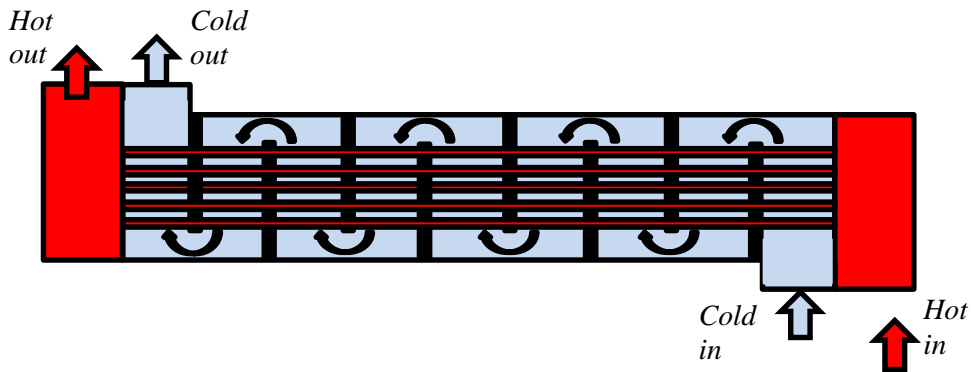
Figure 1.11. A parallel-flow shell-and-tube exchanger
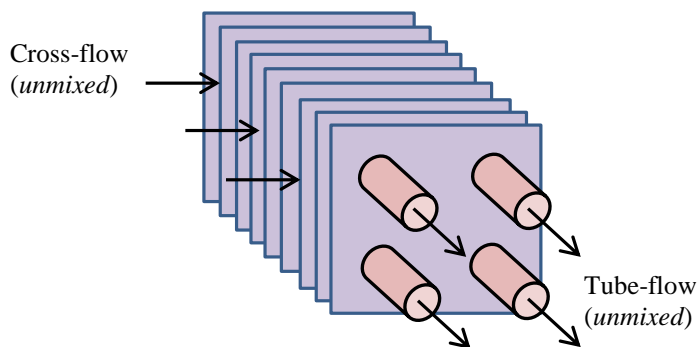


Figure 1.12. A cross-flow exchanger with both streams unmixed

Three independent physical laws are used to quantify the *rate* of heat transfer between the system and its surroundings by conduction (Fourier's Law), convection (Newton's law of cooling), and radiation (Stefan-Boltzmann law). While the physical properties that determine the rates of heat transfer by conduction and radiation, i.e. the thermal conductivity ($k$) and surface emissivity ($\varepsilon$), respectively, are substance-specific, the convection heat-transfer coefficient ($h$) depends on both the fluid and the flow. Numerous analytical relationships and empirical formulae are used for determining $h$ depending on whether the flow is forced or natural. For forced flows, the formulae also depend on whether the flow is internal or external to the system being considered. These formulae usually give the Nusselt number ($Nu$) from which the heat-transfer coefficient can be calculated. For example, the following Dittus-Boelter equation is used for determining $Nu$ inside a fluid-transporting pipe due to forced convection:

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n} \tag{1.31}$$

Where *Re* is the Reynolds number, *Pr* the Prandtl number, and *n* is a constant that takes the value of 0.4 when the pipe is being heated and 0.3 when it is being cooled. The heat transfer coefficient is related to the Nusselt number as follows:

$$h = \frac{k}{D} Nu \tag{1.32}$$

Where *D* is the pipe diameter and *k* is the thermal conductivity of the transported fluid. Many other analytical or empirical formulae are used for determining the Nusselt number for forced or natural flows over single tubes, bank of tubes, plates, etc. [4,5].

## 1.2.   Advantages of computer-aided thermofluid analyses

Apart from saving the time and eliminating possible human errors, computer-aided methods of analysis offer a number of advantages over traditional methods that use property tables and charts. An important advantage of these methods with respect to thermofluid analyses is their ability to give more realistic results by avoiding unnecessary simplification of the models and by using more accurate estimations of fluid properties. Moreover, they offer reliable techniques for iterative solutions and optimisation analyses and for the analyses of complex thermofluid systems. In what follows, these advantages are illustrated by means of relevant examples.

### 1.2.1.   Avoiding excessive simplification of the model

In many situations, traditional analytical methods adopt excessive simplifications of the analytical models; which makes their results grossly deviate from the behaviour of real systems. A good example of this situation is given by the models of internal-combustion (IC) engines. Traditional air-standard models of IC engines, such as the Otto cycle and the Diesel cycle, involve many simplifications such as neglecting heat-transfer and friction losses, treating the combustion process as heat-addition from an external source, and using constant specific heats. These assumptions enable the engine processes to be represented by simple closed-form relations for calculating the amount of heat added to the engine and net work from the engine [6]. However, air-standard models usually overestimate the engine's output and thermal efficiency. By comparison, computer-aided models of IC engines such as those described by Ferguson [7] take into consideration the geometrical as well as the thermodynamic characteristics of the engines. These models, which closely mimic the behaviour of actual IC engines, can be used to investigate the effect of important design and operation factors such the ignition or injection timing on the engine performance or the effect of engine speed on the specific fuel consumption. However, the formulation of these models leads to a set of ordinary differential equations that need to be solved simultaneously by using a numerical solver such as the Newton-Raphson method.

### 1.2.2.   Accurate representation of fluid properties and processes

The behaviour of real gasses and vapours is frequently modelled by using the following ideal-gas law:

$$P\tilde{v} = R_u T \tag{1.33}$$

Where $P$ is the absolute pressure of the gas, $\tilde{v}$ is the molar specific volume, $R_u$ is the universal gas constant, and $T$ is the absolute temperature of the gas. The ideal gas law can be used with reasonable accuracy for determining the specific volume of a superheated vapour, but when the temperature approaches the saturation line, the value of the specific volume determined by the ideal-gas law departs significantly from the actual volume. More accurate estimates can be obtained by using the Soave-Redlich-Kwong (SRK) equation of state [1]:

$$P = \frac{R_u T}{\tilde{v} - b} - \frac{a\alpha}{\tilde{v}(\tilde{v} + b)} \tag{1.34}$$

Where the constants $a$, $b$ and $\alpha$ are fluid-dependent. Figure 1.13 shows the deviations from the tabulated values by those obtained from the ideal-gas law and the SRK equation of state for refrigerant R134a at 0.2 MPa.
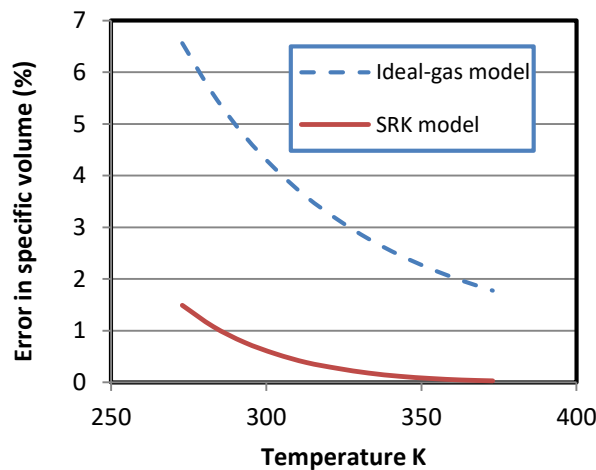


Figure 1.13. Errors in the specific volume of R134a by the ideal-gas law and the SRK equation of state

The figure shows that the error of the ideal-gas law is more that 2% even at high temperatures and increases as the temperature approaches the saturation value, but the accuracy of the SRK equation remained higher than 99% even close to the saturation line. However, since the SRK equation is implicit in $\tilde{v}$, it cannot be used directly to determine the specific volume, but has to be solved iteratively. A number of standard iterative procedures (e.g. Newton-Raphson method) can be used to solve the equation, but they are not convenient for hand calculations.

There are many similar situations in thermofluid analyses where a nonlinear equation like the SRK equation gives advantage to computer-aided numerical methods by enabling more realistic and accurate estimations. Another important implicit equation for thermofluid analyses is the Colebrook-White equation (usually referred to as the Colebrook equation) that determines the friction factor (*f*) in turbulent pipe flows:

$$\sqrt{\frac{1}{f}} = -2.0 \log_{10}\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{1.35}$$

Where Re is the Reynolds number, $D$ is the pipe's diameter, and $\varepsilon$ is the roughness of the pipe material. Since the equation involves the friction factor $f$ on both sides, it needs to be solved iteratively. This is why traditional methods prefer to use explicit relationships, such as the Swamee-Jain formula given by Equation (1.25), even though the Colebrook equation is more accurate.

### 1.2.3.  Dealing with iterative solutions and optimisation analyses

Thermofluid analyses involving iterative solutions and optimisation analyses are two common types that suit computer-aided methods more than manual methods even for simple systems. A good example is found in pipe-flow analyses. Pipe flow problems that require the friction head loss to be determined from Equation (1.21) when both the diameter and flow rate are known can be solved in a straightforward manner. However, in design analyses of pump-pipe systems we may need to find the flow rate in a given pipe that gives a specified head loss or to find a suitable pipe diameter for specified head loss, flow rate, and pipe length. In these two cases, the friction factor $f$ cannot be determined in advance because it depends on the Reynolds number. Therefore, these two types of pipe-flow problems, referred to as type-2 and type-3 problems, need to be solved by iteration. It is much easier to carry out the iterative process to the required level of accuracy by using a computer-aided method than by doing it manually. There are many other types of thermofluid problems that also require iterative solutions such as the determination of the unknown fluids' exit temperatures from a given heat exchanger or the determination of the adiabatic flame temperature by first-law analysis of the combustion process.

Optimisation analyses are needed for determining the best design for a thermofluid system such as the optimum intermediate pressure for an air-compression system, the optimum steam-extraction pressure for a regenerative Rankine cycle, and the best thickness of insulation for a pipe. While simple optimisation analyses that involve a single design parameter can be performed by means of calculus techniques and graphic tools, optimisation analyses of complex systems that involve multiple variables require the use of computer-aided techniques.

### 1.2.4.    Analyses of complex systems

The physical complexity of some thermofluid systems makes their analyses only possible with the help of computer-aided methods. An example of such systems is the looped pipe network shown in Figure 1.14. This network is fed by two water tanks, A and B, and consists of four pipe loops and four consumption points. Suppose that the pipe diameters and lengths are given, the flow rates from the supply tanks are specified, and it is required to determine the flows in the different pipes and the discharges at the consumption points. Although the solution is mainly based on the two well-known principles of the conservation of mass and the conservation of energy, it is difficult to solve the problem by using manual analytical methods especially when a minimum pressure level is to be met at the discharge points. Therefore, a computer-aided method, such as the Hardy-Cross method, has to be used [8, 9].
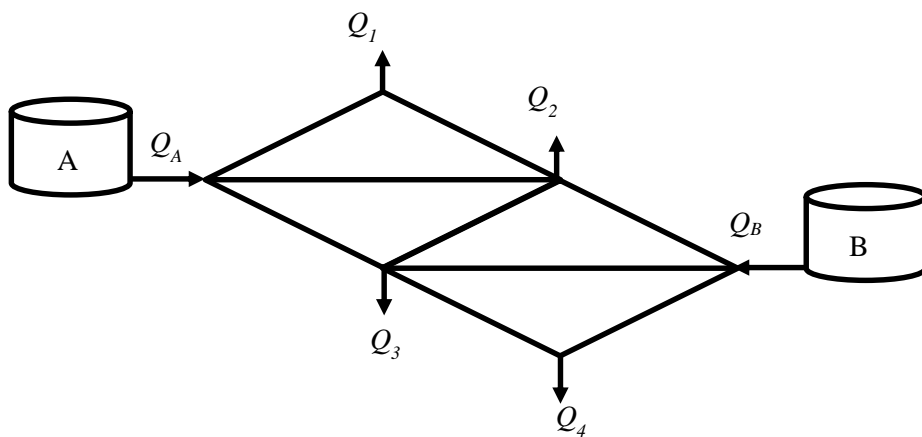


Figure 1.14. A looped pipe network supplied by two tanks

Another type of thermofluid analyses for which computer-aided numerical methods are necessary are the analyses of multi-dimensional fluid-flow and heat transfer in complex geometries. This type of analyses involves coupled and nonlinear partial differential equations that have to be solved by using computational fluid dynamics (CFD) methods such as the finite-volume method or the finite-difference method. Many commercial CFD applications are available nowadays that offer great flexibility and user-friendliness.

### 1.3.    An Excel-based modelling platform for thermofluid analyses

Microsoft Excel is a spread-sheet application developed mainly for statistical analyses and presentations of tabulated data. Considering the simplicity of its user-interface and the flexibility of its graphing tools, Excel has been used in some engineering textbooks for dealing with simple computer-based operations like matrix inversion and matrix multiplications [4,5]. However, Excel is equipped with other features that make it a capable modelling platform for a wide range of engineering analyses [10-12]. In addition to its "What-if" analyses tools that include the Goal Seek command and the

Solver add-in, the "Developer" ribbon in Excel provides a programming language called Visual Basic for Applications (VBA) that can also be used for developing customised user-defined functions (UDFs) needed for engineering analyses when such functions are not provided by Excel. The Developer ribbon also allows the use of macros to remove the tedium of parametric studies and repetitive calculations.

The main limitation of Excel as modelling platform for thermofluid analyses is the lack of built-in functions for fluid properties. This problem could be solved by developing add-ins for this purpose. In this respect, the Mechanical Engineering Department at the University of Alabama developed a set of add-ins for various thermofluid analyses [13-15]. Their *Thermotable* add-in for fluid properties deals with ideal gases, water and superheated steam, and four refrigerants R134a, R22, R410A, and R407C. They also developed other add-ins for psychrometric and compressible flow analyses. Goodwin [16] developed an educational Excel add-in, called *TPX* (Thermodynamic Properties for Excel), that determines the thermodynamic properties of selected gases ($H_2O$, $H_2$, $O_2$, $N_2$, and $CH_4$) and refrigerant R-134a. A number of property add-ins have also been developed for research applications [17-19]. For industrial applications, the American National Institute of Standards and Technology (NIST)  developed the *REFPROP* add-in that provides thermophysical properties of various refrigerants and their mixtures [20]. An open-source alternative to *REFPROP*, called *CoolProp*, was developed by Bell [21] at the University of Liege, Belgium. A commercial alternative to *REFPROP* is provided by Optimized Thermal Systems called *XProps* [22].

The Excel-based modelling platform used in this book requires, in addition to Excel and Solver, an educational Excel add-in called Thermax. Thermax provides property functions for 12 ideal gases (air, $N_2$, $O_2$, $H_2$, CO, $CO_2$, $H_2O$, NO, $NO_2$, $S_2$, $SO_2$, and $SO_3$), saturated water and superheated steam, 6 refrigerants (R134a, R22, R410A, R717, R718, and R744), humid air for psychrometric analyses, and air at standard atmospheric pressure. Thermax also provides two interpolation functions and a Newton-Raphson solver for nonlinear equations that further enhance the usefulness of the modelling platform for thermofluid analyses. The nonlinear equations and any additional functions required by the analyses are developed by using VBA. Table 1.1 summarises the roles of the four components of the Excel-based modelling platform as used in this book.

## 1.3.  Closure

The following nine chapters of the book are grouped into three main parts. The first part of the book consists of Chapters 2, 3, and 4 that describe the four components of the Excel-based modelling platform in more details. Chapter 2 describes the features and built-in functions of Excel that are mostly needed for thermofluid analyses. This chapter also illustrates the use of Excel's iterative tools; Goal Seek and circular calculations. Chapter 3 introduces the Solver add-in and shows how its three solution methods can be used for solving different types of computer-based problems. The chapter also shows how VBA can be used for developing user-defined functions. Chapter 4 describes the

Thermax add-in and shows how its functions can be used in Excel's formulae. Relevant exercises are given at the end of each chapter in this part to train the students for using the different components of the modelling platform.

Table 1.1. Roles of the four components of the Excel-based modelling platform

| Component | Role |
|---|---|
| Excel user-interface | • Provides the basic functions needed for thermofluid analyses including general mathematical functions, matrix-operation functions, and logical-operation functions.<br>• Provides graphical tools for data charting and trendlining<br>• Provides the Goal Seek command for performing unconstrained iterative solutions involving a single parameter |
| Solver | • Allows constrained iterative solutions involving multiple parameters<br>• Allows constrained optimisation analyses<br>• Offers three search options that suit different types of problems |
| Thermax add-in | • Provides the physical properties of ideal gases, water and superheated steam, six refrigerants (R134a, R22, R410A, R717, R718, and R744), humid air for psychrometric analyses, and air at atmospheric pressure<br>• Provides a Newton-Raphson solver for non-linear equations such as the Colebrook equation and the SRK equation<br>• Provides two interpolation functions for tabulated data |
| VBA | • Can be used to develop additional fluid property functions if needed<br>• Can be used to develop user-defined functions for dealing with non-linear equations and large systems of linear equations involved in iterative solutions or optimisation analyses<br>• Can be used to develop macros for repetitive calculations |

The second part of the book consists of Chapters 5 and 6 that show how the Excel-based modelling platform can be used to perform basic types of computer-aided thermofluid analyses. Chapter 5 shows how Excel's Goal Seek command and Solver can be used to solve problems that require iterative solutions in the fields of fluid dynamics, heat-transfer, and thermo-dynamics. Chapter 6 focuses on optimisation analyses of thermofluid systems and shows how Solver can be used to deal with those involving a single design parameter, such as the optimum thickness of insulation for a pipe, and those involving multiple design variables. This chapter also shows how the default settings of the GRG Nonlinear method can be adjusted when it fails to reach a solution and demonstrates the use of the Evolutionary method instead of the GRG Nonlinear method in optimisation analyses. Exercises are given at the end of each chapter in this part to train the students to use the modelling platform in relevant analyses.

Chapters 7, 8, and 9 form the third part of the book. Each one of these three chapters focuses on one type of thermofluid analyses. Chapter 7 deals with thermodynamic first-law and second-law analyses of power generation and refrigeration cycles using property add-ins. Chapter 8 that focuses on fluid-dynamic analyses illustrates the use of Goal Seek and Solver to deal with the analyses of multi-pipe and pump-pipe systems. Different pipe and pump arrangements are analysed in this chapter to determine the system's friction losses, power requirement, and operating point. Chapter 9 deals with conduction heat-transfer analyses by using the finite-difference method. The three chapters of this last part end with more challenging exercises that can be used as mini projects for students' evaluation.

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# 2

# Excel

Excel is the backbone of the modelling platform used in this book for thermofluid analyses. As a general-purpose application, Excel is equipped with numerous features and functions that can be utilised by various users for the presentation and analysis of their data. This chapter focuses on its features that are mostly needed in this book for building a modelling platform for thermofluid analyses. These include Excel's user-interface, its formulae and built-in functions, and its graphical tools and trendline feature. The chapter highlights the use of cell-labelling instead of the commonly-used referencing by location and illustrates the use of Excel's matrix functions for the solution of linear systems of equations and the use of Goal Seek and circular calculations for the solution of nonlinear equations. Finally, the section on Excel's graphical tools demonstrates the use of the trendline feature for data curve-fitting.

## 2.1. Elements of Excel's user-interface

Excel's user-interface allows us to store and manipulate our data by providing a large set of analytical functions and several tools. It also provides numerous commands for adjusting the appearance of the workspace and presenting the primary data and the analysis results in various forms. Figure 2.1 shows a screenshot of an Excel sheet that stores the scores obtained by a group of students in one semester.
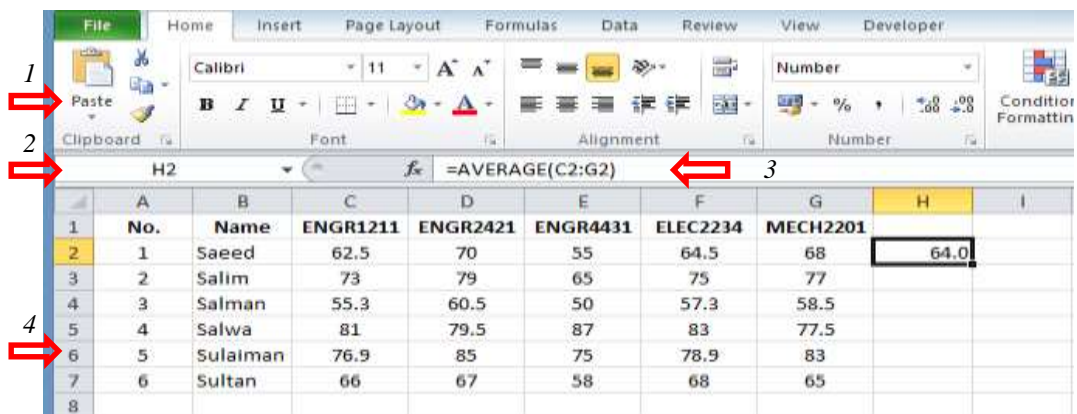


Figure 2.1. The main elements of Excel's user-interface

To allow easy access to the large number of functions tools and commands provided by Excel, its interface is divided into a number of elements with different purposes. Figure 2.1 shows four of these elements which are:

1. The ribbon
2. The name box
3. The formula bar
4. The workspace

The **Ribbon**, which occupies the top part of the sheet, organises the numerous commands provided by Excel into nine "tabs" including the **File**, **Home**, and **Insert** tabs. Each tab consists of a number of command-groups that have a common purpose. For example, the **File** tab consists of four groups of commands called **Clipboard**, **Font**, **Alignment**, and **Number**. The **Workspace**, which is the main part of the sheet, is divided into a grid of columns and rows so that the intersections of the rows with columns form separate "cells". A cell is referred to by a letter that represents its column followed by a number that represents its row, e.g. A1, B3, H2, etc. As the figure shows, a cell can contain a character data, such as "Saeed" and "Salim", or a numerical data, such as 62.5 and 70. A cell can also contain a formula for data manipulation using the numerous built-in functions provided by Excel. The formula in cell H2 calculates the average mark for the first student in the list; "Saeed". While the **Name box** shows the location of the current cell, the **Formula bar** shows the formula typed in the cell. The role of the **Formula bar** will be explained in more details in the following section.

## 2.2. Excel's formulae

The formula bar in Figure 2.1 reveals the formula typed in cell H2 that uses the built-in function "AVERAGE" to determine the average score of the first student in the list (Saeed) in the five subjects (64.0). Note that a formula is preceded by the equal sign "=". In general, Excel's formulae consist of cell references, built-in functions, and mathematical or logical operators. For illustration, let us write a formula that calculates the area of a circle from its radius and use this formula to determine the area of a circle with a radius of 5 m. To do this, open a new Excel sheet and type the number 5, which is the radius of the circle, in cell A1 as shown in Figure 2.2.
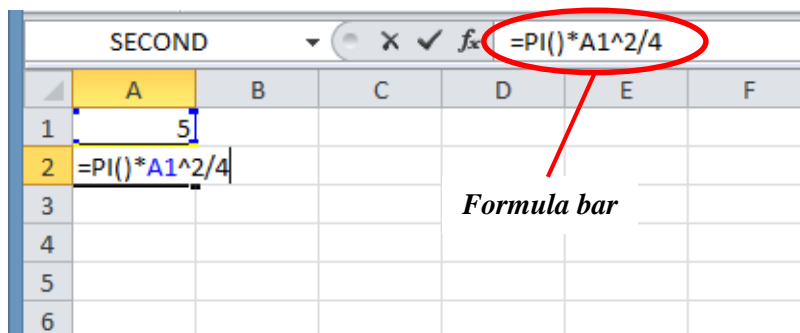


Figure 2.2. Writing an Excel formula to determine the area of a circle

Now, go to cell A2 and type the formula "**=PI()\*A1^2/4**". The function "PI()" is a built-in function that returns the value of Archimedes' constant π. The formula also contains a reference to cell A1 that stores the value of the circle's radius, the multiplication operator *, the division operator /, the power operator ^, and the constants 2 and 4. Note that the formula is shown in the formula bar which can be used to edit the formula. Pressing the **Enter** key after typing the formula, the result shown in Figure 2.3 is obtained in cell A2; which is 19.63495 square meters.

Figure 2.3. The completed Excel sheet with formula that determines the area of a circle

The following example shows how Excel's formulae and built-in functions can be used in a typical thermofluid analysis.

**Example 2.1. Determining the error of the ideal-gas law for refrigerant R134a**
Develop an Excel sheet that calculates the specific volume ($v$) of refrigerant R134a from the ideal-gas law at a pressure of 200 kPa ($T_{sat}$ = -10.09°C) and temperatures in the range 0°C to 100°C (273 to 373 K). Compare your results with the tabulated data.

**Solution**
Figure 2.4 shows the Excel sheet prepared for this example. The pressure (P), the gas constant (R), and the temperature (T) are stored in columns A, B, and C, respectively. Column D stores the values of $v$ obtained from property tables and column E stores the corresponding values obtained from the ideal-gas law:

$$v = RT / P \tag{2.1}$$

where, $P$ and $T$ are the absolute pressure and temperature, respectively, and $R$ is the gas constant (for R134a $R$ = 0.08149 kJ/kg.K). The percentage error of the ideal-gas law in estimating the specific volume is given by:

$$Error = \frac{v_{Ideal} - v_{Table}}{v_{Table}} \times 100 \tag{2.2}$$

To determine the percentage error at 273K, go to cell F2 and type the following formula which is equivalent to Equation (2.2):

$$=(E2 - D2)/D2*100$$

When you press the **Enter** key, the number **6.566** will appear in cell F2 as shown in Figure 2.4.

| | F2 | | | $f_x$ | =(E2-D2)/D2*100 | | |
|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G |
| 1 | P | R | T | v_table | v_ideal | error_v_ideal | |
| 2 | 200 | 0.08149 | 273 | 0.10438 | 0.1112339 | 6.5662483 | |
| 3 | 200 | 0.08149 | 283 | 0.10922 | 0.1153084 | 5.5743911 | |
| 4 | 200 | 0.08149 | 293 | 0.11394 | 0.1193829 | 4.776944 | |
| 5 | 200 | 0.08149 | 303 | 0.11856 | 0.1234574 | 4.1306933 | |
| 6 | 200 | 0.08149 | 313 | 0.12311 | 0.1275319 | 3.5917878 | |
| 7 | 200 | 0.08149 | 323 | 0.12758 | 0.1316064 | 3.1559414 | |
| 8 | 200 | 0.08149 | 333 | 0.13201 | 0.1356809 | 2.7807363 | |
| 9 | 200 | 0.08149 | 343 | 0.13639 | 0.1397554 | 2.4674463 | |
| 10 | 200 | 0.08149 | 353 | 0.14073 | 0.1438299 | 2.2026931 | |
| 11 | 200 | 0.08149 | 363 | 0.14504 | 0.1479044 | 1.974869 | |
| 12 | 200 | 0.08149 | 373 | 0.14932 | 0.1519789 | 1.7806389 | |
| 13 | | | | | | | |

Figure 2.4.The sheet developed for determining error in the ideal-gas law for R134a

Note that the formula shown in the formula bar represents Equation (2.2) when the temperature is 273K. To find the percentage errors at other temperatures, you can simply copy the formula in cell F2 and paste it on cells F3 to F12. Values of the calculated errors show that the maximum error occurs at the lowest temperature, which is 273K. The error decreases gradually as the temperature increases.

### 2.3. Use of cell labels

Reference to the cell by its relative location in the sheet, e.g., A5 and C3, suits perfectly statistical analyses in which the same formula is applied to a large body of data that is stored column-wise or row-wise. For example, we want to determine the average value, maximum value, or minimum value of the data. Example 2.1 illustrated this situation. However, thermofluid analyses usually involve a large number of formulae but a small set of data, e.g. the diameter of a pipe, the density or viscosity of a fluid, the effectiveness of a heat exchanger, etc. For such analyses, it is more convenient to give the cell a meaningful name or "label" that matches its content. The label can then be used as reference to the cell instead of its relative location. This method makes it easier to interpret Excel formulae and recognise the quantities involved in them.

For the purpose of illustration, suppose that we want to compare the density of air before and after an isentropic compression process from an initial condition of $P_1 = 100$ kPa, $T_1 = 300K$ to a final pressure of $P_2 = 800$ kPA. The two densities can be calculated from the ideal-gas law as follows:

$$\rho_1 = P_1 / RT_1 \hspace{6cm} (2.3)$$

$$\rho_2 = P_2 / RT_2 \hspace{6cm} (2.4)$$

Where $R$ is the gas constant for air (0.287 kJ/kg.K).

For an isentropic process, $T_2$ is related to $T_1$ according to the following approximate relationship:

$$T_2 = T_1 \times (P_2 / P_1)^{\frac{k-1}{k}} \tag{2.5}$$

Where $k$ is the ratio of specific heat at constant pressure ($c_p$) and at constant volume ($c_v$). For air, $k$ can be taken as 1.4.

Figure 2.5 shows the sheet prepared for this case. Note that respective labels are typed in the column to the left of the different pressures and temperatures, while the corresponding units are written in the column to the right of each quantity. This is also done to the other quantities in the calculations.

| | F4 | | | $f_x$ | =B4*B7^((B8-1)/B8) | | |
|---|---|---|---|---|---|---|---|
| ⊿ | A | B | C | D | E | F | G |
| 1 | Air density before and after an isentriopic compression | | | | | | |
| 2 | | | | | | | |
| 3 | P_1 | 100 | kPa | | P_2 | 800 | kPa |
| 4 | T_1 | 300 | K | | T_2 | 543.434 | K |
| 5 | | | | | | | |
| 6 | R_ | 0.287 | kJ/kg.K | | Density_1 | 1.16144 | m3/kg |
| 7 | P_r | 8 | | | Density_2 | 5.12934 | m3/kg |
| 8 | k_ | 1.4 | | | | | |
| 9 | | | | | | | |
| 10 | | | | | | | |

Figure 2.5. Excel sheet for calculating the air densities before and after compression

Placing the cursor on cell F4 makes the formula bar reveal the formula used in the calculation of the temperature $T_2$, which is:

=B4*B7^((B8-1)/B8)

The above formula can be made more understandable by using meaningful labels to refer to the different cells involved. To do that, select the cells in columns A and B as shown in Figure 2.6, then go to **Formulas** and, at the **Name Manager**, select **Create from Selection**. When the form shown in Figure 2.6 appears to you, tick the "Left column" option. Pressing the "OK" button will make Excel create names for the different values in the selection box according to the labels written on the left column. The cell F3 that stores the value of $P\_2$ can also be associated with its corresponding label in cell E3. Now, type the formula in cell F4 that determines $T\_2$ as:

=T_1*P_r^((k_-1)/k_)

The formula bar in the sheet shown in Figure 2.7 reveals the formula with the corresponding labels instead of location references. Labelled formulae are easier to edit than those using location referencing particularly when intricate formulas are involved.
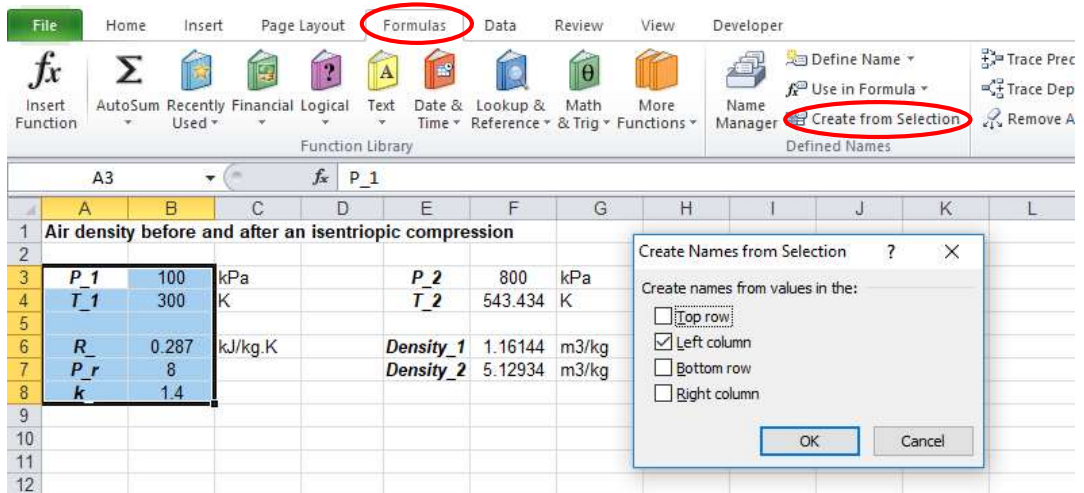


Figure 2.6. Creating names for a selected group of cells
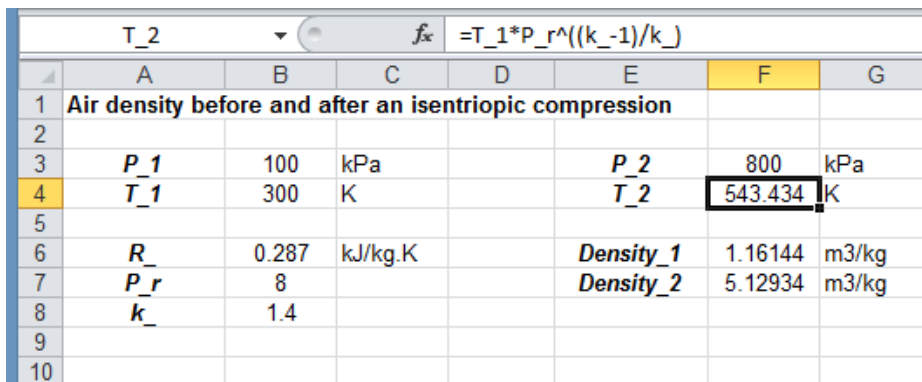


Figure 2.7. Formulae using cells labels instead of locations

There are a couple of rules that have to be observed when using cell labels. When naming your cells, choose suitable representative names for the variables involved, e.g. P_1 and T_1 for $P_1$ and $T_1$. Note that Excel does not accept "P1" or "T1" as labels since these can be confused with usual cell references by locations. In this case, Excel automatically changes the labels to "P1_" and "T1_". Also note that if you copy a formula that uses the usual referencing by location in another cell, you will get a different answer, but if you copy a labelled formula and paste it in any other cell, you will get the same answer. To reveal or hide all the formulae in the sheet, press the control key (ctrl) with the tilde key (~). A more detailed discussion of Excel formulae can be found in Walkenbach [1] or the numerous online help sources.

## 2.4. Excel's built-in functions

Excel provides a large library of built-in functions for data manipulation like the "AVERAGE" function and other functions needed in engineering analyses like the "PI" "SIN", and "COS" functions. To see the full range of Excel built-in functions, type "=" in any Excel cell as shown in Figure 2.8 and then press the "**Insert Function**" *fx* button in the formula bar. The dialog box shown in Figure 2.9 will appear to you. This dialog box allows you to select from various categories of built-in functions.
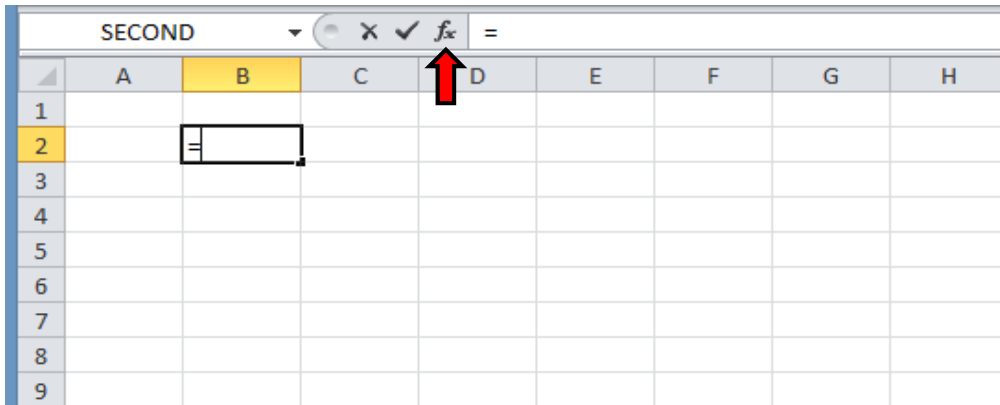
Figure 2.8. Using Excel's built-in functions
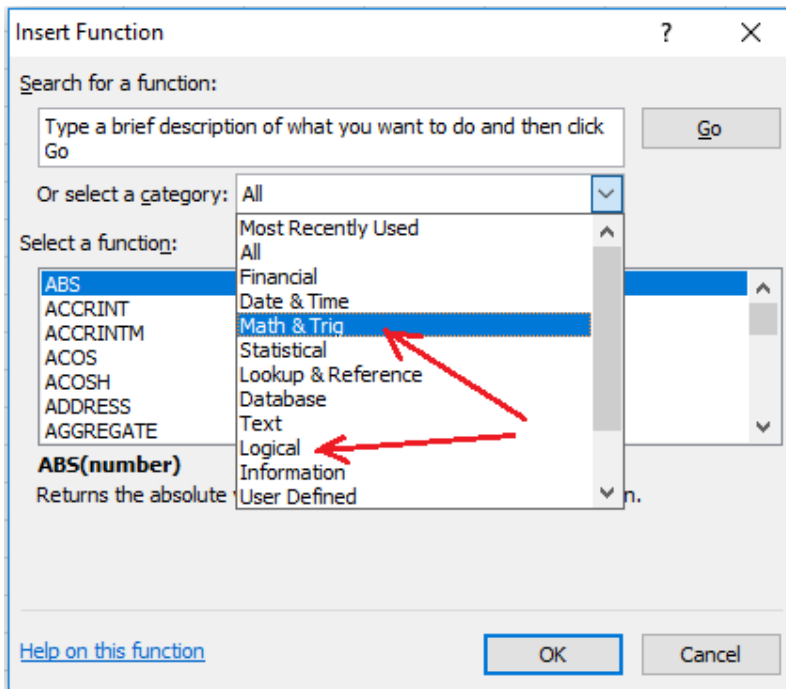
Figure 2.9. Categories of Excel's built-in functions

The "**Math & Trig**" group includes the mathematical and trigonometric functions used in different types of engineering analyses, including thermofluid analyses. Figure 2.10 shows some of the numerous functions in this group. Note that the dialog box gives a brief explanation of each function. For example, the explanation given to the ABS function is that it returns the absolute value of a number. The functions ACOS, ASIN, and ATAN apply the familiar inverse trigonometric functions: $\cos^{-1}$, $\sin^{-1}$, and $\tan^{-1}$, respectively. By scrolling down the list, you can find many other functions frequently used in engineering calculations. The following sections focus on two types of functions that deserve a special attention, which are (a) the logical functions and (b) the functions for matrix operations.
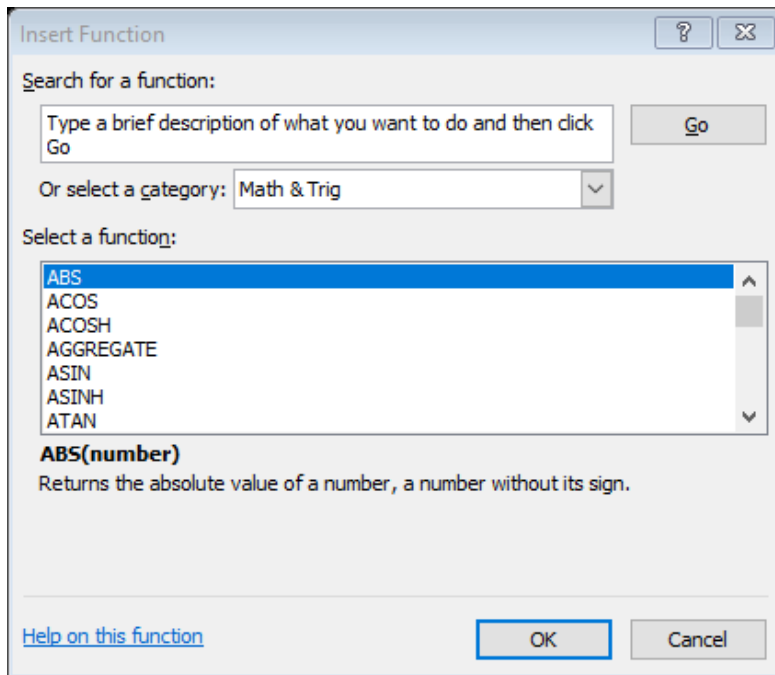


Figure 2.10. Common mathematical functions supported by Excel

### 2.4.1. Logical functions

Logical functions are needed frequently in thermofluid analyses. For example, before we can use the Darcy-Weisbach equation to determine the major friction loss in a pipe we have to establish whether the flow is laminar or turbulent. The flow remains laminar before the Reynolds number (Re) reaches a certain value, which is usually taken as 2,000. There is a transitional region between laminar and turbulent flows when $2000 < Re < 3,000$. Beyond $Re = 3000$, the flow is considered fully turbulent. Suppose that we want to write an Excel formula to tell us the type of flow from the given value of the Reynolds number. A simple IF function that gives only two choices is as follows:

=IF(**logical_test**,[value_if_true],value_if_false])

Using the simple IF function, we can write the following formula:

  =IF(**Re<=2000**, "Laminar", "Turbulent or transitional")

Note the quotation marks around "Laminar" and "Turbulent or transitional". However, the above formula does not tell us whether the flow is turbulent or transitional. The required general formula can be written by using the following nested IF function:

  =IF(**Re<=2000**, "Laminar", IF(**Re>=3000**, "Turbulent", "Transitional"))

Figure 2.11 shows an Excel sheet containing the above formula (shown in the formula bar) and the response of the formula when Re = 500, which is "Laminar". Excel supports six other logical functions; AND, FALSE, IFERROR, NOT, OR and TRUE that can be combined in the same formula so as to handle more intricate choices.
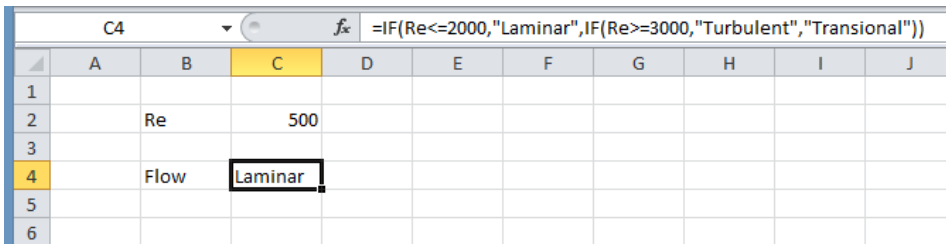


Figure 2.11. A formula using the nested IF function to determine the type of flow

### 2.4.2. Functions for matrix operations

A group of adjacent cells can be treated as a matrix or a vector and Excel formulae allow for the addition, subtraction, and multiplication of these matrices and vectors according to the established rules of matrix operations. For example, matrix [A] and vector {b} shown in Figure 2.12 can be multiplied and the result stored in a third vector {c} by using the matrix function MMULT. The procedure is as follows:

1. After keying in the data of matrix [A] and vector {b} as shown in Figure 2.12, position the cursor at cell H3 and type the formula: =MMULT(B3:D5;F3:F5).
2. Now press ENTER key and cell H3 will take the value 14, which the result of multiplying the first row of the matrix with the vector {b} (Figure 2.13).

The other two elements of the result vector will not appear automatically. To view the complete results vector, do as follows:

3. Select the cells H3:H5 (Figure 2.14),
4. Press the function key F2 once and then simultaneously hold the (SHIFT + CONTROL) keys together and press ENTER. The complete result vector {c} will now appear as shown in Figure 2.15.
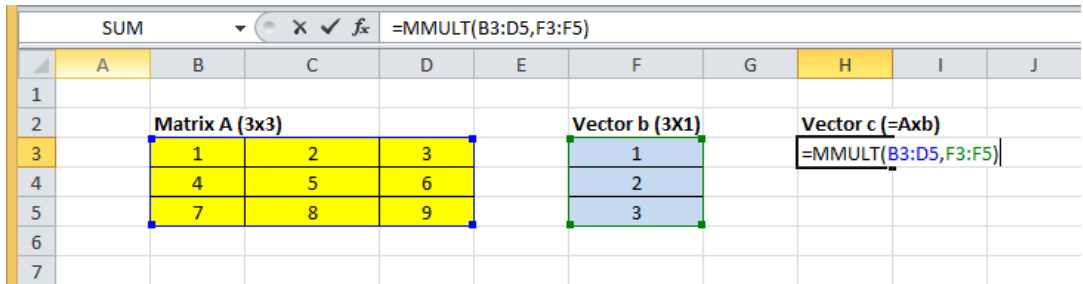
Figure 2.12. Step 1 of using the matrix multiplication function
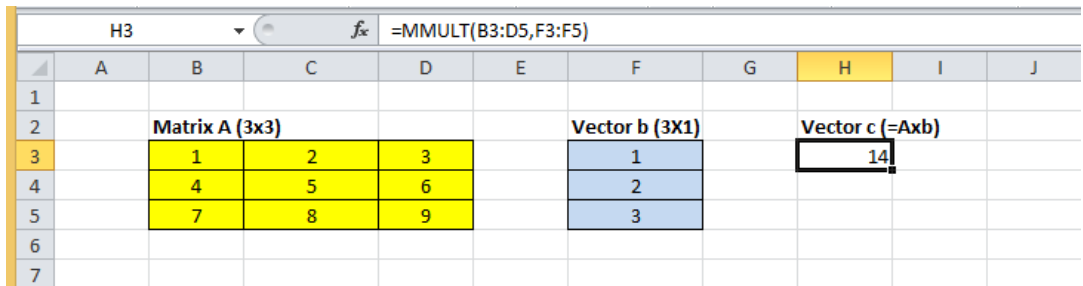


Figure 2.13. Step 2 of using the matrix multiplication function
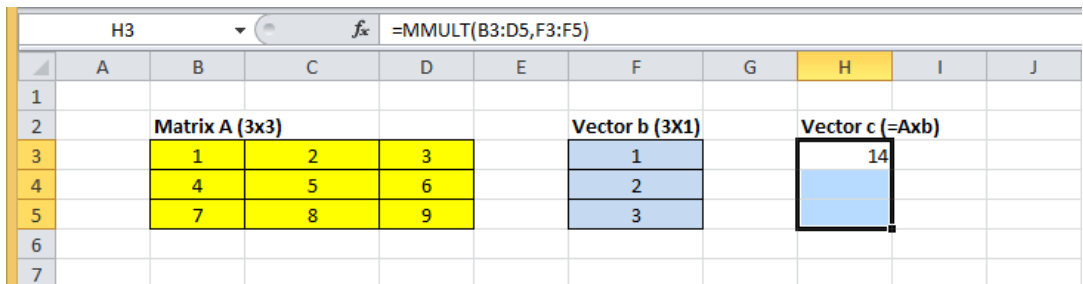


Figure 2.14. Step 3 of using the matrix multiplication function
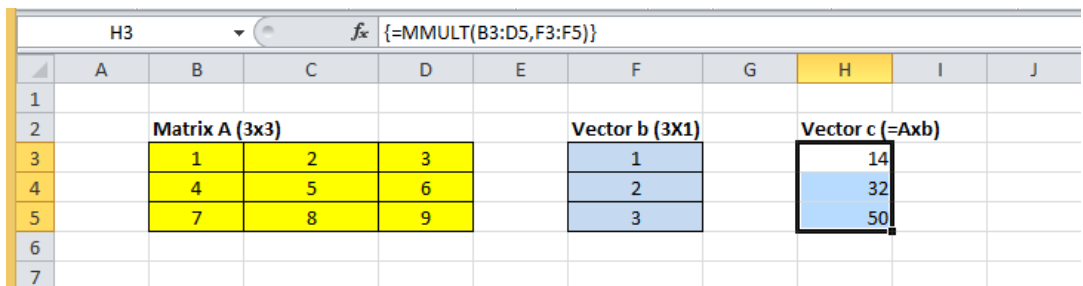


Figure 2.15. Step 4 of using the matrix multiplication function

Another important matrix-operation function provided by Excel is the matrix-inversion function "MINVERSE". The following example illustrates the use of this function which is needed for the solution of linear systems of equations.

**Example 2.2. Using the matrix inversion function**

Develop and Excel sheet to find the inverse of matrix [A] given by:

$$[A] = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 5 & 6 \\ 7 & 0 & 5 \end{bmatrix}$$

**Solution**

The first step is to enter the elements of the matrix as shown in Figure 2.16. After entering the data, go to cell F2 and type the formula "=MINVERSE(B2:D4)". When you press ENTER, this cell will have the value -0.3125, which is the first element of $[A]^{-1}$ shown in Figure 2.17. Starting with the formula in cell F2, select the range F2 to H4 as shown in the figure. Press and release the function key F2 and then simultaneously hold the CTRL+SHIFT keys and press ENTER. Other elements of the inverse matrix $[A]^{-1}$ will then appear as shown in Figure 2.18. You can check the answer by finding the product of matrix [A] with its inverse by using the MMULT functions. The procedure is illustrated by Figures 2.19 to 2.21. As should be expected, Figure 2.21 shows that the resultant matrix is the identity matrix.
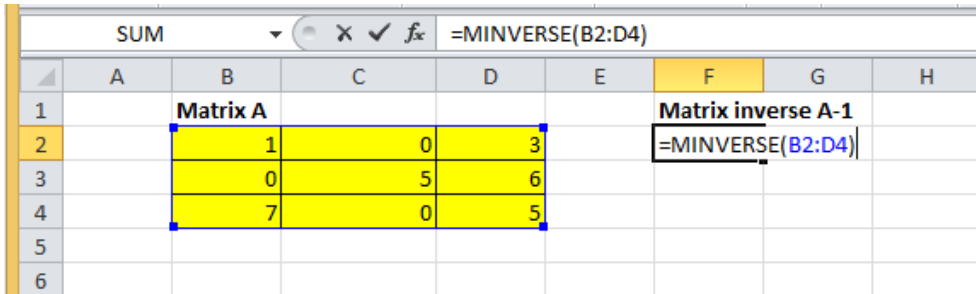


Figure 2.16. Step 1 of using the MINVERSE function
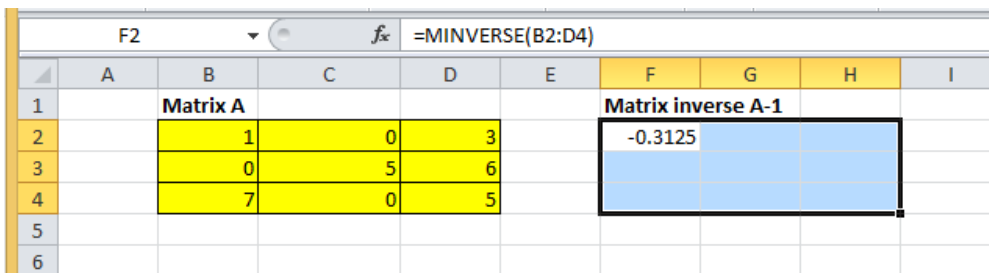


Figure 2.17. Step 3 of using the MINVERSE function

Figure 2.18. The complete inversed matrix $[A]^{-1}$



Figure 2.19. Multiplying matrix $[A]$ by its inverse $[A]^{-1}$



Figure 2.20. The first element of the identity matrix

## 2.5. Solution of linear system of equations

Systems of linear equations arise, for example, in the numerical solution of the heat conduction equation using the finite-difference method. Small and medium-sized linear systems of equations can be solved with Excel by applying the matrix-inversion method. Consider the following linear system written in matrix notation:

$$[A]\{x\} = \{y\}$$

(2.6)

Where $[A]$ is the coefficient matrix, $\{x\}$ the vector of unknowns, and $\{y\}$ the right-side or "load" vector.

Figure 2.21. The complete identity matrix

The solution vector $\{x\}$ can be obtained as follows:

$$\{x\} = [A]^{-1}\{y\} \tag{2.7}$$

Where $[A]^{-1}$ is the inverse of matrix $[A]$. The following example illustrates the procedure.

### Example 2.3. Solution of a system of linear equations
Find the values of $x_i$ in the following system of linear equations:

$$
\begin{bmatrix}
14 & 14 & -9 & 3 & -5 \\
14 & 52 & -15 & 2 & -32 \\
-9 & -15 & 36 & -5 & 16 \\
3 & 2 & -5 & 47 & 49 \\
-5 & -32 & 16 & 49 & 79
\end{bmatrix}
\begin{Bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{Bmatrix}
=
\begin{Bmatrix}
-15 \\
-100 \\
106 \\
329 \\
463
\end{Bmatrix} \tag{2.8}
$$

### Solution
Note that the system is symmetric; which is typically the case with linear systems that arise in the solution of heat-conduction problems by the finite-difference method. For large systems of equations, symmetry of the system can be utilised for reducing the required computer memory by storing only one half of the coefficient matrix. However, for small systems, such as the one considered here, it is more convenient to use the matrix inversion method using Excel functions. Figure 2.22 shows the Excel sheet that stores both the coefficient matrix $[A]$ and the load vector $\{y\}$. The inverse of the coefficient matrix $[A]^{-1}$ was obtained by following the procedure described above and stored below the coefficient matrix as shown in the figure. The inverse matrix $[A]^{-1}$ is then multiplied by the load vector $\{y\}$ and the result stored below the load vector as shown in Figure 2.23. The complete solution is shown in Figure 2.24. The first element is practically zero and, therefore, the answer is $\{x\} = (0, 1, 2, 3, 4)$.

Figure 2.22. The coefficient matrix [A], the load vector {y}, and the inverse matrix $[A]^{-1}$



Figure 2.23. Multiplying the inverse matrix $[A]^{-1}$ with the load vector {y}



Figure 2.24. The complete solution vector {x}

## 2.6. Itearative solutions with Excel

Iterative solutions are required in thermofluid analyses when the analytical model involves a nonlinear equation, such as the SRK equation, or when there is interdependency between fluid and flow parameters, as in type 2 and type 3 flow problems. Excel's user-interface offers two methods for obtaining iterative solutions: by using the **Goal Seek** command or by using **circular calculations**. In what follows these two methods will be explained with the help of simple examples.

### 2.6.1.   Itearative solutions with Goal Seek

The Goal Seek command is a simple, yet very useful tool for "What-if" analyses. It is used for finding the value of an independent variable ($x$) that yields a specified value of a dependent variable ($y$). The following example illustrates how the Goal Seek command can be used to solve a nonlinear equation.

**Example 2.4. Solution of a nonlinear equation by Goal Seek**

Two variable $x$ and $y$ are related according to the following quadratic equation:

$$y = x^2 - 2x + 1 \qquad\qquad -1.0 \le x \le 3 \tag{2.9}$$

It is required to find the two values of $x$ that yields $y = 1.5$.

**Solution**

Using the conventional analytical method, the two values can be found by substituting $y$ in Equation (2.9) by 1.5. This leads to the following quadratic equation:

$$x^2 - 2x - 0.5 = 0$$

The two roots of this equation using the standard solution method are:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 + 4 \times 1 \times 0.5}}{2 \times 1} = \frac{2 \pm \sqrt{6}}{2}$$

Or:

$$x_1 = -0.22474$$

$$x_2 = 2.2247.$$

We will now solve the problem by using the Goal Seek command. Figure 2.25 shows the Excel sheet prepared for this purpose in which $y$ is plotted at various values of $x$. The figure shows that the two values of $x$ that yield $y = 1.5$ are approximately -0.2 and 2.2.
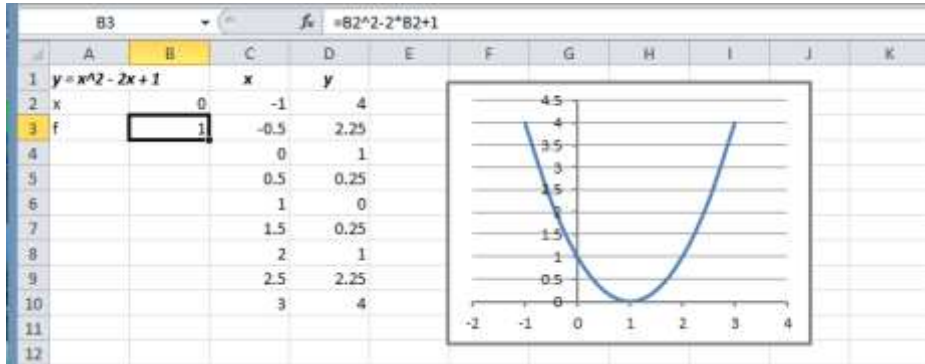
Figure 2.25. Excel sheet for finding the roots of a nonlinear equation

To solve the problem by using Goal Seek, enter an initial guess for $x$ in cell B2, say 0, and then enter the following formula that calculates $y$ in cell B3:

**= B2^2 – 2\*B2 +1**

Note that the formula bar reveals the above formula when the cursor is placed at cell B3. To activate the Goal Seek command, go to the **Data** tab, select the **What-If-Analysis** option in the **Data Tools** group and then select **Goal Seek,** as shown in Figure 2.26. The Goal Seek dialog form shown in Figure 2.27 will ask you to select the "**Set cell**", i.e. the cell that contains the dependent variable, which is B3 in this case.



Figure 2.26. Activation of the Goal Seek command
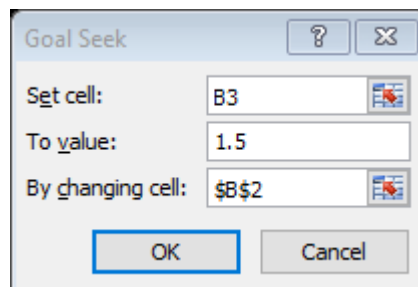


Figure 2.27. Goal Seek Set-up for finding the root of a nonlinear equation

You also have to specify the value sought for this cell and then select the adjustable cell that stores the value of the parameter to be changed. In this case, we seek the value in the Set Cell B3 to be 1.5 by changing the value of cell B2. Providing this information to Goal Seek and then pressing the "**OK**" button, will trigger the tool to iterate by changing the value in the adjustable cell (B2) until the Set cell (B3) acquires the required value. As shown in Figure 2.28, the answer obtained by Goal Seek is $x = -0.22474$ which agrees with the first root obtained analytically.
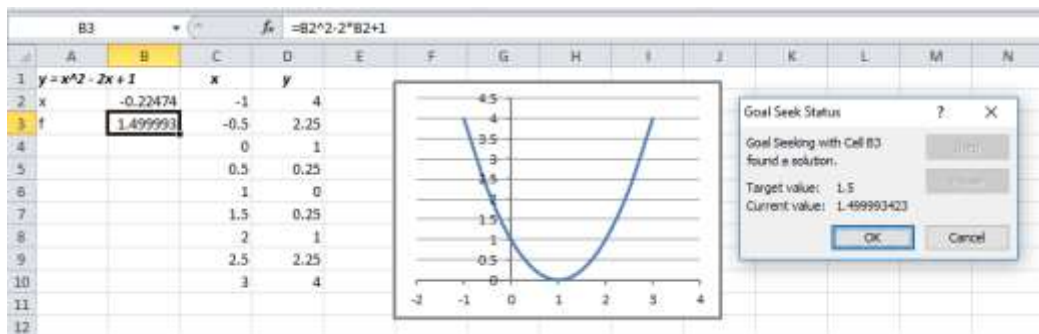


Figure 2.28. Goal Seek solution with an initial value of $x = 0$

Goal Seek determined the first solution and not the second one because it starts the iterative process with the initially specified value in the changeable cell, which is $x = 0.0$, and gives the solution that is closer to it. To determine the second solution, we have to start with a suitable initial guess. Figure 2.29 shows the same Excel sheet with the initial value of $x$ changed to 1.0 and the set-up for Goal Seek before the solution. Figure 2.30 shows the new solution found by Goal Seek, which is $x = 2.2249$.
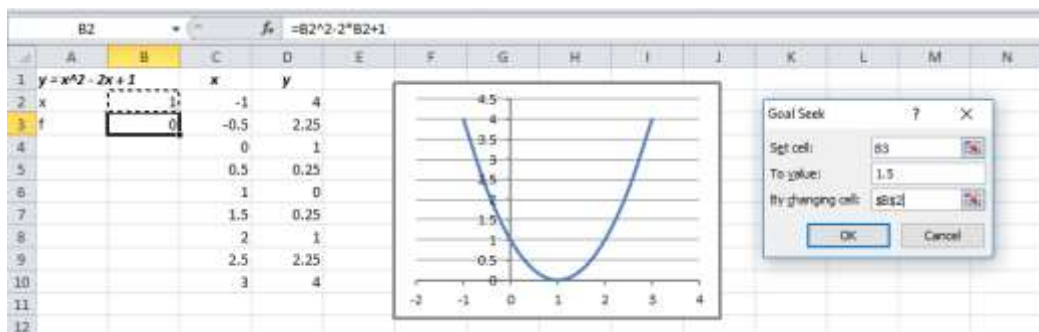


Figure 2.29. Goal Seek set-up with an initial value of $x = 1$

Note that the second Goal Seek solution deviates slightly from the analytical solution of $x=2.2247$. The precision of Goal Seek solution can be improved by adjusting Excel's default parameters for iterative solutions. This can be done by selecting **Options** from the **File** tab and then selecting **Formulas**. You can now increase the allowable maximum number of iterations from the default value of 100 and reduce the maximum change from the default value of 0.001.
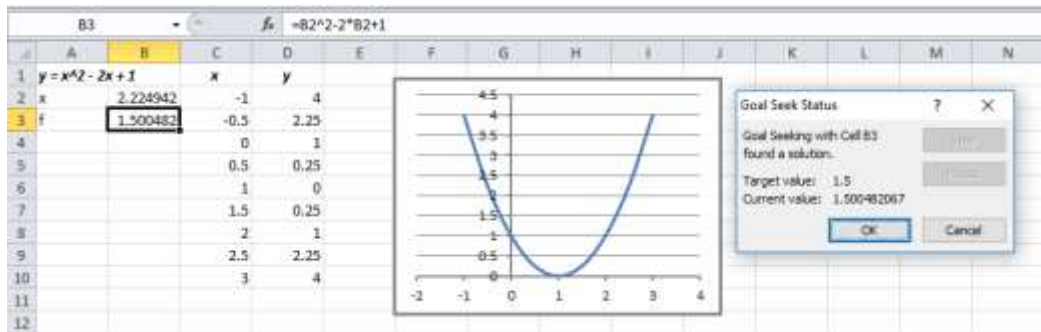
Figure 2.30. Goal Seek solution with an initial value of $x = 1$

## 2.6.2. Iterative solution with circular calculations

A circular calculation occurs when an Excel formula refers to its own cell in a direct or indirect way. When this happens, Excel prompts the user that there is a circular refrence and then indicates the cells involved. By allowing Excel to perform circular calculations, it will itearate until all the parameters involved are satisfied. The following example illustrates this feature.

## Example 2.5. Determining the final temperature of heated air

A closed system contains one kg of air initially at 300K. 100 kJ of heat is added to the air at constant pressure. Determine the final temperature of air if its molar specific-heat ($\tilde{c}_p$) varies with temperature according to the following formula:

$$\tilde{c}_p = a + bT + cT^2 + dT^3 \quad \text{[kJ/kmol]} \tag{2.10}$$

Where $a = 28.11$, $b = 1.97 \times 10^{-03}$, $c = 4.80 \times 10^{-06}$, and $d = -1.97 \times 10^{-09}$.

## Solution

From the defenition of specific heat, the final temperature ($T_2$) is given by:

$$T_2 = T_1 + Q / \left( \tilde{c}_p / M \right) \tag{2.11}$$

Where $T_1$ is the initial temperature, $Q$ is the amount of heat added, and $M$ is the molar mass for air, taken as 29. If we ignored the variation of $\tilde{c}_p$ with temperature and determined the final temperature based on its value at $T_1$ only, the answer would be $T_2 = 399.73$K. However, we can be more accurate by determining $\tilde{c}_p$ from Equation (2.10) by using the average temperature, $T_{avr} = (T_1 + T_2)/2$. Figure 2.31 shows the Excel sheet developed for this method which reveals the formulae inserted in cells F2, F4, and F6. As soon as we type Equation (2.11) in cell F6, Excel makes the warning message that there is a circulare refernce as shown in Figure 2.32. The circular reference occurs because $T_2$ depends on $\tilde{c}_p$ according to Equation (2.11) while $\tilde{c}_p$ itself depends on $T_2$

according to Equation (2.10). If we press the "OK" button shown in Figure 2.32, the cells involved in the circular reference whill be identified as shown in Figure 2.33. In this case, three cells are involved in the circular reference, which are F2, F4, and F6.



Figure 2.31. Excel sheet developed for Example 2.5
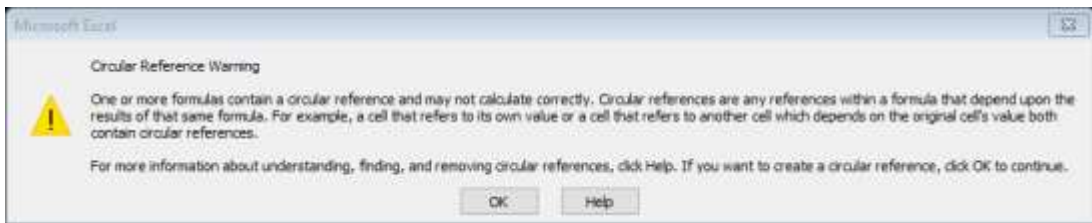


Figure 2.32. The circular-reference prompt



Figure 2.33. The cells involved in the circular reference

Excel can solve this problem and determine the values of both $T_2$ and $\tilde{c}_p$ that satisfy the relevant equations if allowed to iterate. To allow this option, go to **File** and select **Options**. The **Backstage View** form shown in Figure 2.34 will appear to you. Select **Formulas**, then the form will appear as shown in Figure 2.35. Enable iterative calculations by ticking the box indicated in the figure $\boxed{\surd}$ and press the "OK" button. Excel can now find the correct values of $T_2$ and $c_p$ by iteration. Figure 2.36 shows the solution found by Excel, which is $T_2 = 398.976K$.
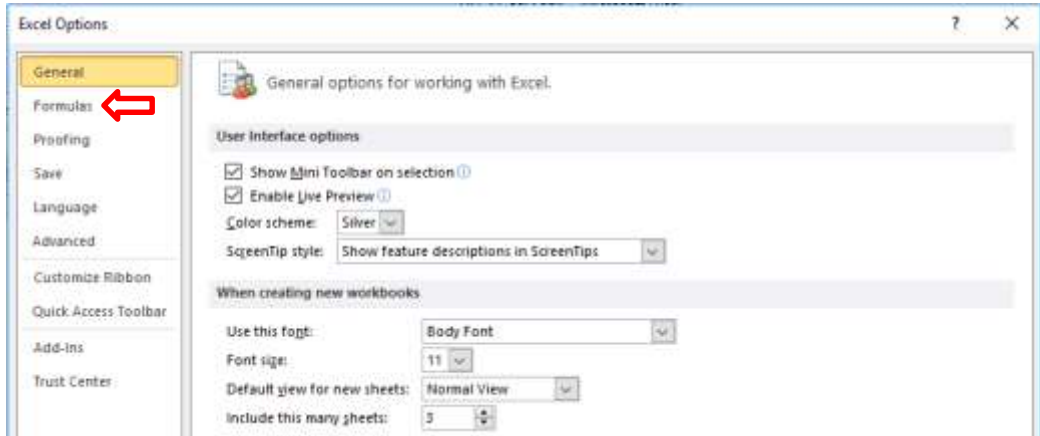
Figure 2.34. Selecting Excel's option (Formulas)
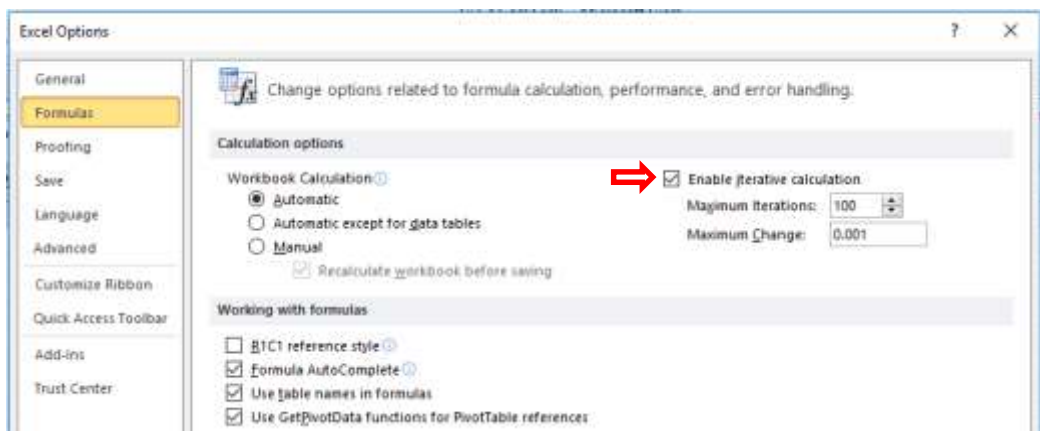


Figure 2.35. Enabling iterative calculations from Excel's Formulas option



Figure 2.36. Solution of Example 2.5 by circular calculations

Although the subtle nature of circular calculations can be useful in certain situations, various type of thermofluid analyses that require iterative solutions can be solved more easily with Goal Seek or the Solver add-in as shown in Chapters 5 and 8. Try to solve this example by using Goal Seek and compare the results (Problem 2.10).

**2.7. Excel's graphical tools for data presentation and analysis**

Excel has numerous graphical tools for the visualisation and analysis of numerical data. Using these tools, the data can be presented in a variety of charts. Figure 2.37 shows one type of Excel charts that displays the annual variation of temperature and relative humidity at one location in a certain day. The figure shows a line chart in which the temperature is scaled on the primary *y*-axis (on the left) while the humidity is scaled on the secondary *y*-axis (on the right). This arrangement is useful for displaying two or more types of data that differ significantly in their magnitudes.
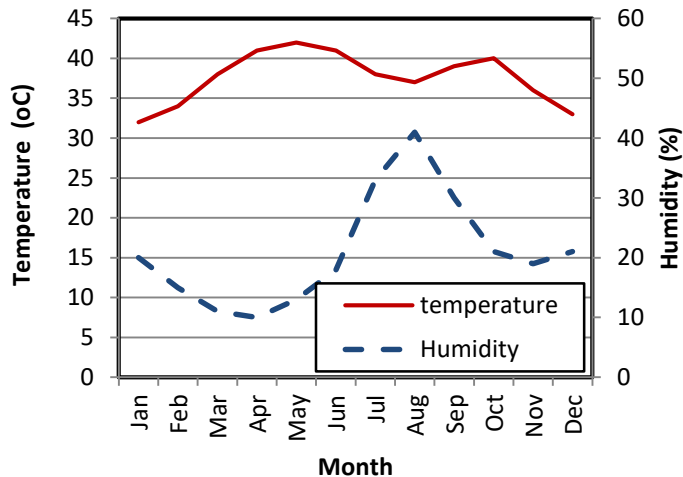


Figure 2.37. An example of line charts

Excel supports other types of charts that allow the user to select the most appropriate way to display his/her data in the form of bar, area, or scatter charts. For more information about the different types of Excel's charts, the reader can refer to specialised references such as Walkenbach [2]. A number of tutorials and videos that show how to create different types of charts can also be found in the internet.

Excel's graphical tools provide a curve-fitting capability of numerical data by using the Trendline feature. This particular capability is useful for computer-aided thermofluid analyses because it can be used to convert tabulated data into analytical equations that suit computer-based methods better. The trendline feature provides a number of options, which include exponential, linear, logarithmic, polynomial, and power equations as shown in Figure 2.38. For example, Table 2.1 shows properties of saturated water in the range 0.001°C – 60°C. Values of the saturation pressure ($P_{sat}$) and saturated liquid enthalpy ($h_f$) are used in psychrometric analyses of air-conditioning applications, but for computer-based analyses it is useful to convert these data into equations. First, we have to create line charts for the two properties as shown in Figure 2.39.
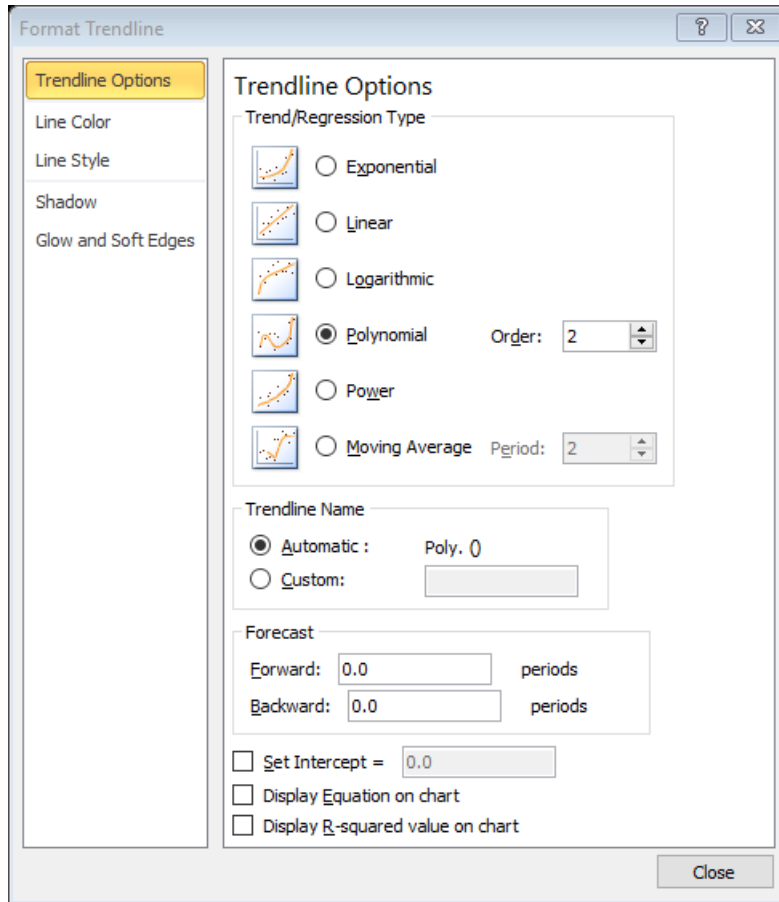
Figure 2.38. The Format Trendline window

Table 2.1. Properties of saturated water at temperatures in the range 0ºC- 60ºC taken from Cengel and Boles [3]

| $T$ºC | $P_{sat}$ [kPa] | $v_f$ [m³/kg] | $v_g$ [m³/kg] | $u_f$ [kJ/kg] | $u_g$ [kJ/kg] | $h_f$ [kJ/kg] | $h_g$ [kJ/kg] | $s_f$ [kJ/kg.K] | $s_g$ [kJ/kg.K] |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.6117 | 0.001000 | 206.00 | 0.000 | 2374.9 | 0.001 | 2500.9 | 0.0000 | 9.1556 |
| 5 | 0.8725 | 0.001000 | 147.03 | 21.019 | 2381.8 | 21.020 | 2510.1 | 0.0763 | 9.0249 |
| 10 | 1.2281 | 0.001000 | 106.32 | 42.020 | 2388.7 | 42.022 | 2519.2 | 0.1511 | 8.8999 |
| 15 | 1.7057 | 0.001001 | 77.885 | 62.980 | 2395.5 | 62.982 | 2528.3 | 0.2245 | 8.7803 |
| 20 | 2.3392 | 0.001002 | 57.762 | 83.913 | 2402.3 | 83.915 | 2537.4 | 0.2965 | 8.6661 |
| 25 | 3.1698 | 0.001003 | 43.340 | 104.83 | 2409.1 | 104.83 | 2546.5 | 0.3672 | 8.5567 |
| 30 | 4.2469 | 0.001004 | 32.879 | 125.73 | 2415.9 | 125.74 | 2555.6 | 0.4368 | 8.4520 |
| 35 | 5.6291 | 0.001006 | 25.205 | 146.63 | 2422.7 | 146.64 | 2564.6 | 0.5051 | 8.3517 |
| 40 | 7.3851 | 0.001008 | 19.515 | 167.53 | 2429.4 | 167.53 | 2573.5 | 0.5724 | 8.2556 |
| 45 | 9.5953 | 0.001010 | 15.251 | 188.43 | 2436.1 | 188.44 | 2582.4 | 0.6386 | 8.1633 |
| 50 | 12.352 | 0.001012 | 12.026 | 209.33 | 2442.7 | 209.34 | 2591.3 | 0.7038 | 8.0748 |
| 55 | 15.763 | 0.001015 | 9.5639 | 230.24 | 2449.3 | 230.26 | 2600.1 | 0.7680 | 7.9898 |
| 60 | 19.947 | 0.001017 | 7.6670 | 251.16 | 2455.9 | 251.18 | 2608.8 | 0.8313 | 7.9082 |
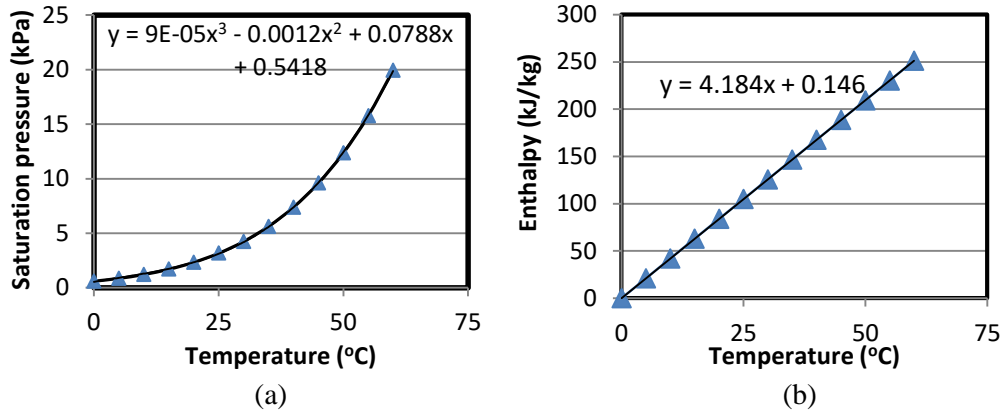
Figure 2.39. Fitting trendlines on tabulated data for water of
(a) saturation pressure and (b) saturated liquid enthalpy

Trendlines can then be added on the line charts. Figures 2.39.a and 2.39.b also show the corresponding trendline equations of the tabulated data as determined by using polynomial equations. A linear equation is adequate for the $h_f$ data since its variation over the given temperature range is mild (Figure 2.39.b), but a third-order polynomial is required to represent the variation of $P_{sat}$ with temperature (Figure 2.39.a).

## 2.8. Closure

This chapter described the main features of Excel needed for thermofluid analyses. The chapter highlighted the importance of using cell labelling with Excel's formulae and illustrated the use of Excel's general mathematical functions and logical functions. The chapter also showed how Excel's functions for matrix operations can be used for solving linear systems of equations and demonstrated the use of its iterative tools, Goal Seek and circular calculations. In spite of its simplicity, the Goal Seek command is very useful for computer-aided thermofluid analyses. As shown in later chapters of this book, it can be used for solving problems that require iterative solutions, such as type-2 and type-3 pipe flow problems, and flow analyses of multi-pipe arrangements. Finally, the chapter illustrated the usefulness of Excel's charting tools for computer-based thermofluid analyses particularly the trendline feature.

It should be mentioned that the **Developer tab** in Excel's user-interface provides a number of useful features that can be used to enhance the performance and user-friendliness of Excel as modelling platform for thermofluid analyses. For example, **macros** are useful when conducting repetitive calculations and parametric analyses. For more information about these features, the reader can refer to specialised references or the internet.

## References
1.  J. Walkenbach, *Excel 2010 Formulas*, Wiley Publishing Inc., 2010
2.  J. Walkenbach, *Excel 2007 Charts*, Wiley Publishing Inc., 2007

3. Y. A. Cengel and M. A. Boles. *Thermodynamics an Engineering Approach*, McGraw-Hill, 7th Edition, 2007

4. S. C. Chapra and R. P. Canale, Numerical Methods for Engineers, 6th Edition, McGraw Hill, 2010

**Exercises**

1. The following table shows the measured values of the temperature by two different methods compared to the correct values. Find the average error for each method.

| *Correct T* (ºC) | *Method* 1 | *Method* 2 |
|---|---|---|
| 0 | 0.1044 | 0.1112 |
| 10 | 10.1092 | 10.1153 |
| 20 | 20.1139 | 20.1194 |
| 30 | 30.1186 | 30.1235 |
| 40 | 40.1231 | 40.1275 |
| 50 | 50.1276 | 50.1316 |
| 60 | 60.1320 | 60.1357 |
| 70 | 70.1364 | 70.1397 |
| 80 | 80.1407 | 80.1438 |
| 90 | 90.1450 | 90.1479 |
| 100 | 100.1493 | 100.1520 |

2. Suppose that we have the data shown on the following table for the saturation pressure of a certain fluid. Use a nested IF statement to develop an interpolation formula that determines the saturation pressure for any temperature in the range 5ºC $\leq T \leq$ 30ºC.

| $T$(ºC) | $P_{sat}$ (kPa) |
|---|---|
| 5 | 0.872 |
| 10 | 1.228 |
| 15 | 1.705 |
| 20 | 2.339 |
| 25 | 3.169 |
| 30 | 4.246 |

3. A system of algebraic equations can be expressed in matrix form as follows:

$$\begin{bmatrix} 70 & 1 & 0 \\ 60 & -1 & 1 \\ 40 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 636 \\ 518 \\ 307 \end{Bmatrix}$$

Solve the system of equations to determine the values of the three unknowns $a$, $b$, and $c$. This exercise is based on Example 9.11 in Chapra and Canale [4]. The answer is: $a = 8.5941$, $b=34.4118$, and $c = 36.7647$.

4.  The following system of linear equations resulted from a finite-difference solution of the heat transfer in the triangular fin shown in the following figure.



|  0      1      2      3      4      5 |
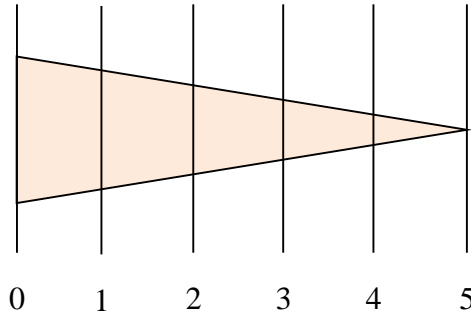
Solution of the system provides the temperatures in °C at different distances from the fin base as shown in the accompanying figure. Rewrite the systems in the matrix form $[A]\{x\} = \{b\}$ and use Excel functions to solve it.

$-8.008\ T_1 + 3.5\ T_2 = -900.209$

$3.5\ T_1 -6.008\ T_2 + 2.5\ T_3 = -0.209$

$2.5\ T_2 -4.008\ T_3 + 1.5\ T_4 = -0.209$

$1.5\ T_3 -2.008\ T_4 + 0.5\ T_5 = -0.209$

$T_4 - 1.008\ T_5 = -0.209$

5.  Adopting suitable names in your formulae, prepare an Excel sheet for calculating the frictional loss ($h_f$) in a circular pipe of diameter $D$, length $L$, and roughness $k_s$. Use your sheet to determine $h_f$ in the following cases:

(a) $D = 25$ cm, $L = 150$ m, $V = 2$ m/s, $k_s = 0.045$ mm, carrying water at 20°C.
(b) $D = 25$ cm, $L = 150$ m, $V = 0.2$ m/s, $k_s = 0.045$ mm, carrying oil at 20°C.
(c) $D = 25$ cm, $L = 150$ m, $V = 7$ m/s, $k_s = 0.045$ mm, carrying air at 20°C.

Determine the values of the kinematic viscosity from relevant property tables and take the transition of flow from laminar to turbulent to occur at Re = 4,000.

6.  Using a line chart, plot the variation of sine $\theta$ for $-180 \le \theta \le 180$ in steps of 10° then add cosine $\theta$ on the same chart.

7.  Using the data shown in Table 2.1, make a line chart for vf and vg. Add polynomial trendlines for both and comment on the trendlines equations.

8.  The table below shows some of the thermo-physical properties of air at atmospheric pressure and different temperatures. Use Excel charts to show the variation of the properties $\rho$, $\beta$, cp, k, $\alpha$, $\mu$, $\nu$, and Pr with temperature and use trendline to obtain suitable equations for these properties.

| $T$ (K) | $\rho$ (kg/m³) | $\beta \times 10^3$ (1/K) | $c_p$ (J/kg.K) | $k$ (W/m.K) | $\alpha$ (m²/s) | $\mu \times 10^6$ (N S/m²) | $\nu \times 10^6$ (m²/s) | Pr |
|---|---|---|---|---|---|---|---|---|
| 273 | 1.252 | 3.66 | 1011 | 0.0237 | 19.2 | 17.456 | 13.9 | 0.71 |
| 293 | 1.164 | 3.41 | 1012 | 0.0251 | 22.0 | 18.240 | 15.7 | 0.71 |
| 313 | 1.092 | 3.19 | 1014 | 0.0265 | 24.8 | 19.123 | 17.6 | 0.71 |
| 333 | 1.025 | 3.00 | 1017 | 0.0279 | 27.6 | 19.907 | 19.4 | 0.71 |
| 353 | 0.968 | 2.83 | 1019 | 0.0293 | 30.6 | 20.790 | 21.5 | 0.71 |
| 373 | 0.916 | 2.68 | 1022 | 0.0307 | 33.6 | 21.673 | 23.6 | 0.71 |
| 473 | 0.723 | 2.11 | 1035 | 0.0370 | 49.7 | 25.693 | 35.5 | 0.71 |
| 573 | 0.596 | 1.75 | 1047 | 0.0429 | 68.9 | 29.322 | 49.2 | 0.71 |
| 673 | 0.508 | 1.49 | 1059 | 0.0485 | 89.4 | 32.754 | 64.6 | 0.72 |
| 773 | 0.442 | 1.29 | 1076 | 0.0540 | 113.2 | 35.794 | 81.0 | 0.72 |

9.  Using the Excel sheet developed for Example 2.4, check the accuracy of the solution obtained by Goal Seek when the spreadsheet iteration parameters are adjusted such that the maximum number of iterations is increased to 10,000 and the maximum change is reduced to 0.00001.

10. Solve Example 2.5 by using the Goal Seek command instead of circular calculations.

11. The volume $V$ of liquid in a spherical tank of radius $r$ is related to the depth $h$ of the liquid by:

$$V = \pi h^2(3r - h)/3$$

Using Excel and the Goal Seek command, determine $h$ given $r=1$ m and $V = 0.5$ m³. This exercise is based on Problem 8.9 in Chapra and Canale [4]. Answer: $h = 0.431$ m.

# 3

# Solver and VBA

This chapter focuses on two auxiliary components of the Excel-based modelling platform for thermofluid analyses, which are Solver and VBA. Developed by Frontline Systems [1], Solver enables Excel to perform iterative solutions and optimisation analyses with multiple adjustable cells. It offers three solution methods that suit different types of problems including a deterministic gradient method, a linear-programming method, and a stochastic evolutionary method. This chapter shows how Solver can be activated and used for solving single nonlinear equations and systems of linear equations and performing optimisation analyses. The Chapter describes the settings of Solver's solution options and illustrates the use of its GRG Nonlinear method and the Evolutionary method. Finally, the chapter shows how VBA can be used for developing custom functions not provided by Excel.

### 3.1. Activation of Solver

Like the Goal Seek command, Solver is found in the **Data tab** as shown in Figure 3.1. If it doesn't appear in your Data tab, then you need to activate it as follows:

1. Go to **File** and then click **Options**.
2. Select **Add-Ins**. From the **Manage** option at the bottom of the menu select **Excel Add-ins** and then press "**Go**". The **Add-Ins** dialog box shown in Figure 3.2 will appear to you.
3. To add **Solver** to the add-ins menu, tick (√) on the "**Solver"** option. Return to the data tab.
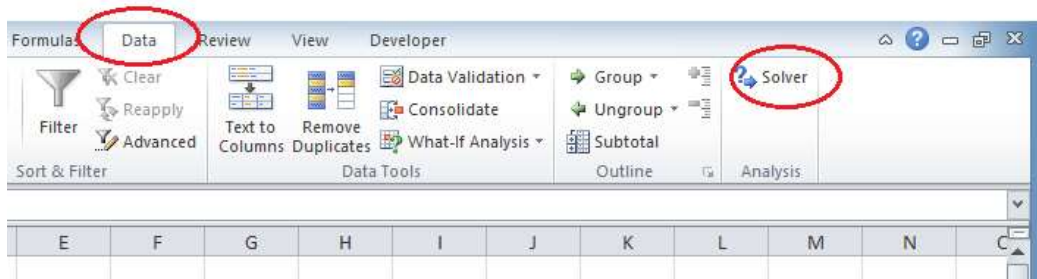


Figure 3.1. The Solver add-in in the Data tab

When you click the **Solve**r button from the Data tab, **Solver Parameters** dialog box shown in Figure 3.3 will appear to you. This dialog box helps you to specify the required outcome of a formula in one cell called the **objective cell** by adjusting the values of a group of cells, called **decision variables** or variable cells, which are directly or indirectly related to the formula in the objective cell. As shown on the parameters dialog box, you can specify **constraints** on the values of the decision variables. Compared to the Goal Seek command, Solver offers the following advantages:

1. While Goal-Seek can only be used for simple problems that involve one decision variable, Solver can deal with more difficult problems in which the objective cell is affected by many decision variables.
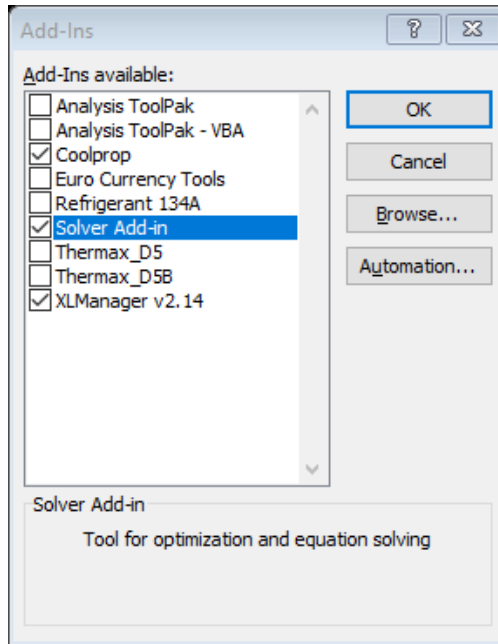
Figure 3.2. Activating Solver from the menu of Excel add-ins
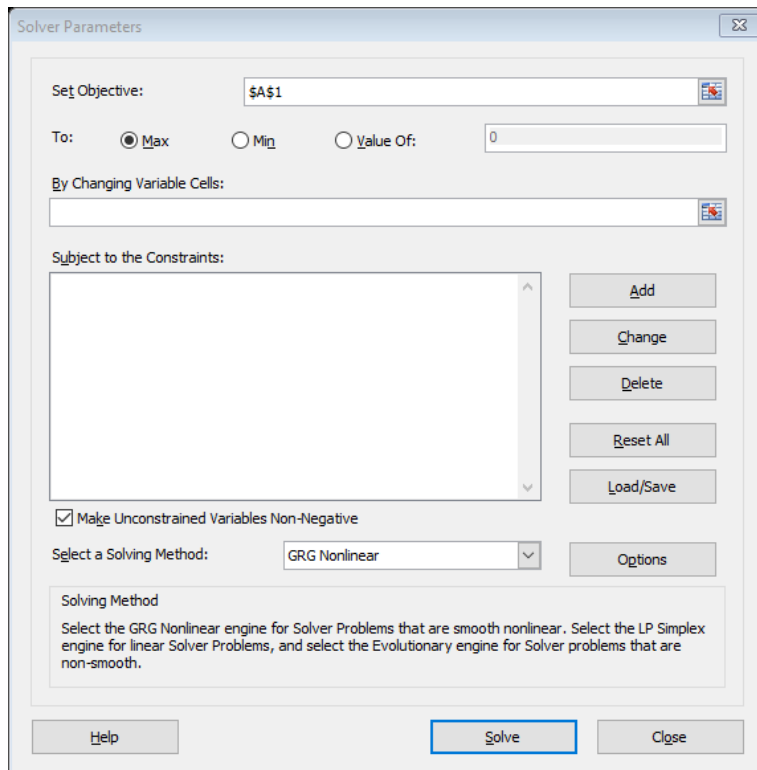


Figure 3.3. Solver parameters dialog box

2. Goal Seek allows only a required value of the objective cell to be achieved, but Solver also enables us to perform an optimisation analysis by finding the maximum or minimum value for the formula in the objective cell.
3. With Solver, we can apply constraints on the solution, which is not possible with Goal Seek. Constraints are needed for some optimisation analyses and iterative solutions.
4. Solver is that it allows the user to select the appropriate solving method for his or her problem from three options.

The version of Solver that comes with Excel 2010 and later editions offers the following three search methods:

1. **GRG Nonlinear**. A Generalized Reduced Gradient method (which is the method selected in Figure 3.3).
2. **Evolutionary.** An Evolutionary method.
3. **Simplex LP.** A linear programming method.

Both the **GRG Nonlinear** method and the **Evolutionary** method are used for non-linear problems, while the **Simplex LP** method is suitable for linear problems. Solver uses the **GRG Nonlinear** method by default. This section shows how the three solution methods can be used to solve different types of problems.

## 3.2.    The GRG Nonlinear method

The GRG Nonlinear method is a mathematical method that involves the determination of the function's gradient like the Steepest Descent method [2]. Therefore, it is suitable for problems involving functions that have continuous and smooth variations. The following sections show how the method can be used for the solution of nonlinear equations and optimisation analyses.

### 3.2.1.    Solution of nonlinear equations

To illustrate this option, let us reconsider Equation (2.9) in Example 2.4 which has two solutions. We can use the Excel sheet developed previously for the solution with Goal Seek. Solver constraints allow us to search for the first or the second solution at a time. Let us initially find the second solution, i.e. $x \geq 0$. To do so, select Solver from the Data tab and fill its parameters dialog-box by entering the following information:

| | |
|---|---|
| Set Objective: | Type **B3**, select **Value of**, and type **1.5** for this option since want the value of the function in cell B3 to be 1.5. |
| By Changing Variable Cells: | Type **B2**, which is the cell that stores the value of the independent variable $x$ |
| Subject to the Constraints: | Add two constraints that specify the minimum and maximum values of $x$, e.g. $x \geq 0$ and $x \leq 3$ |
| Select a Solving Method: | Select the **GRG Nonlinear** option |

The upper part of the completed parameters dialog box will be as shown in Figure 3.4. Note that the two constraints imposed on the solution correspond to $x \geq 0$ and $x \leq 3$. When you press the **Solve** button in the dialog box, Solver will iterate to determine the value of the only variable cell that yield the targeted objective within the limits specified by the constraints. As Figure 3.5 shows, the solution determined by Solver is $x = 2.22474$, $y = 1.5$, which is the second analytical solution found in Example 2.4. To keep this solution, press the "OK" button in the **Solver Results** dialog box. Now try to find the first solution by modifying the two constraints to: $x \geq -1$ and $x \leq 0$.
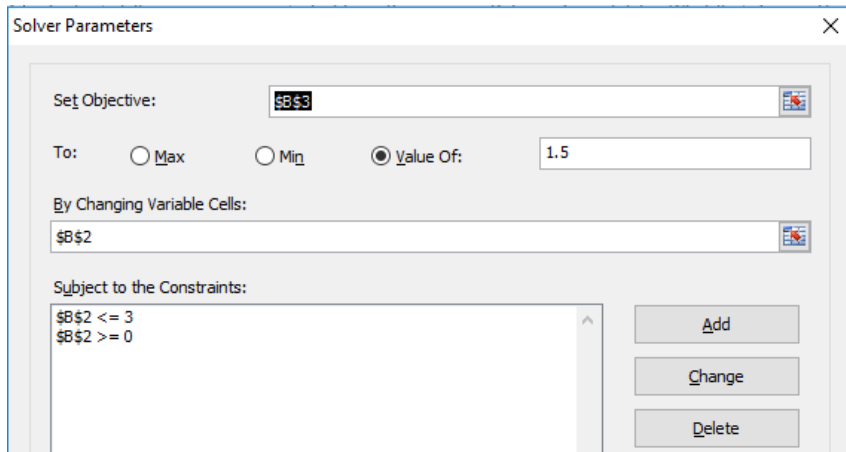


Figure 3.4. The completed Solver dialog box for Example 2.4 with $x \geq 0$
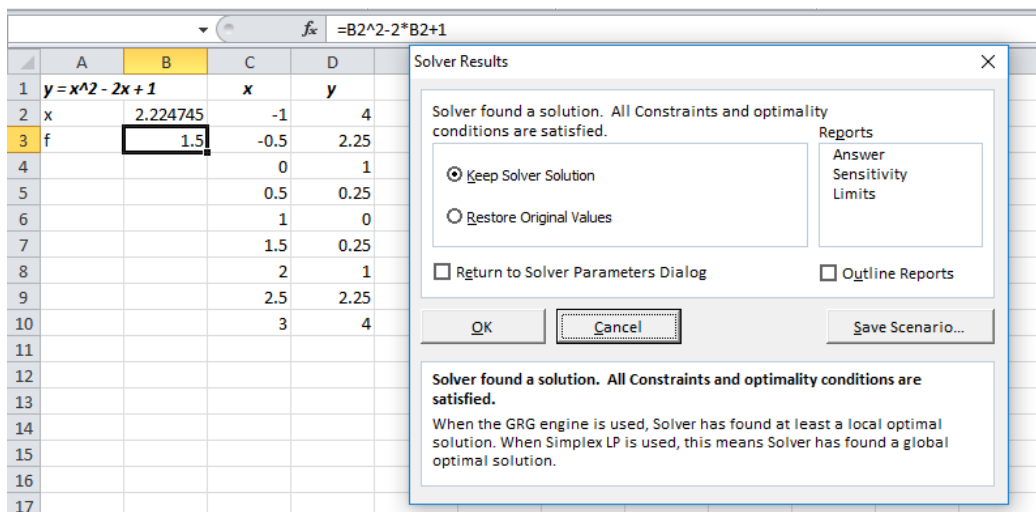


Figure 3.5. Solver solution for Example 2.4 with $x \geq 0$

### 3.2.2. Optimisation analyses

An optimisation problem requires the function in the objective cell to be maximised or minimised. For example, the thermodynamic optimisation of a power-generation plant requires its thermal efficiency to be maximised, while the economic optimisation of

pipe insulation requires its total cost to be minimised. The following example illustrates the use of the **GRG Nonlinear** method in optimisation analyses.

**Example 3.1. Finding the minimum value of a smooth function**
Find the minimum value of the following quadratic function $f$ in the specified range.

$$f(x) = x^2 - 2x - 1; \qquad -2 \le x \le 3 \tag{3.1}$$

**Solution**
Figure 3.6 shows the Excel sheet developed for this example that calculates the values of $f$ at different values of $x$. The line chart in Figure 3.6 shows the variation of $f$ with $x$ from which we can see that the minimum value of $f$ is -2 and occurs at $x = 1$. Note the curser is placed on cell **B6** to reveal the formula $fx = $ **B3^2-2\*B3−1**. We will now use Solver to determine the minimum value of the function. Figure 3.7 shows the completed Solver's parameters dialog box. Press the "Solve" button and Solver will iterate to find the solution shown in Figure 3.8. As the figure shows, the answer found by Solver, which is $x = 1, f = -2$, agrees with the graphical solution shown in Figure 3.6.
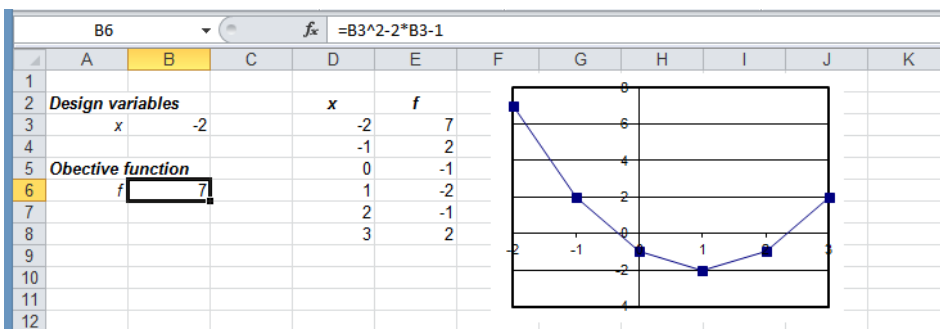


Figure 3.6. Excel sheet for determining the local minimum of the quadratic function
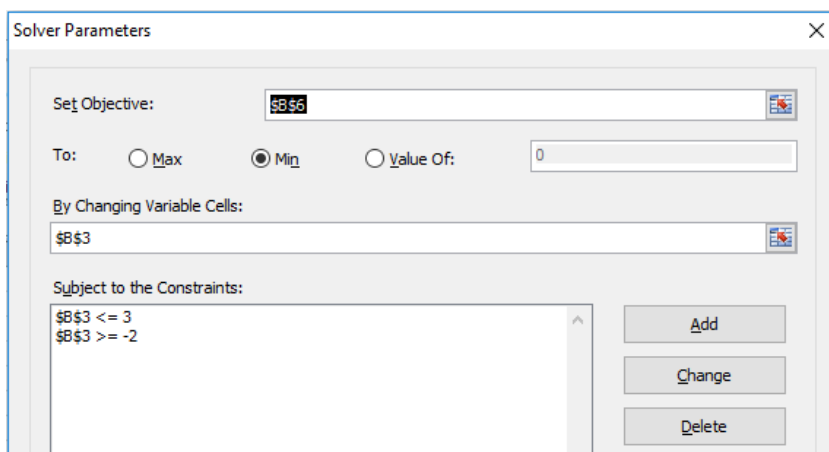


Figure 3.7. The completed Solver dialog box for Example 3.1

Figure 3.8. Solver solution for Example 3.1

## 3.3.  The Simplex LP method

This Solver option provides an alternative method for solving small systems of linear equations to that described in the previous chapter by using Excel's matrix functions. To illustrate this method, reconsider the problem of Example 2.3. Figure 3.9 shows a new Excel sheet that has been prepared for the present method.



Figure 3.9. Excel sheet for solving Example 2.3 with Solver

The top part of the sheet stores the coefficient matrix [A] and the right-hand vector {y} of the system of linear equations to be solved. The procedure starts with a guessed solution which is stored in a vector {x0} in cell F9:F13. All the elements of this vector are given a value of 1 as shown in Figure 3.9. The coefficient matric [A] is then multiplied with the guessed vector {x0} by using Excel's "MMULT" function and the result stored in cells H9:H13. If this initial guess were correct, the multiplication [A]{x0} would be the same as the true right-hand side vector, i.e,:

$$[A]\{x0\} = \{y\} \tag{3.2}$$

However, Figure 3.9 shows that the vector [A]{x0} is different form the true right-hand side vector {y} stored in cells H2:H6. Solver can now be used to adjust the variable cells D9:D13 so that all elements of the vector [A]{x0} become equal to their counterparts in vector {y}, i.e:

H9 = H2
H10 = H3
H11 = H4
H12 = H5
H13 = H6

Solver set-up for this task is shown in Figure 3.10. Note that the objective cell is left blank. In this case, Solver will iterate to find the values of the decision variables that satisfy all the imposed constraints. Also note that the Simplex LP method is selected as the solution option.
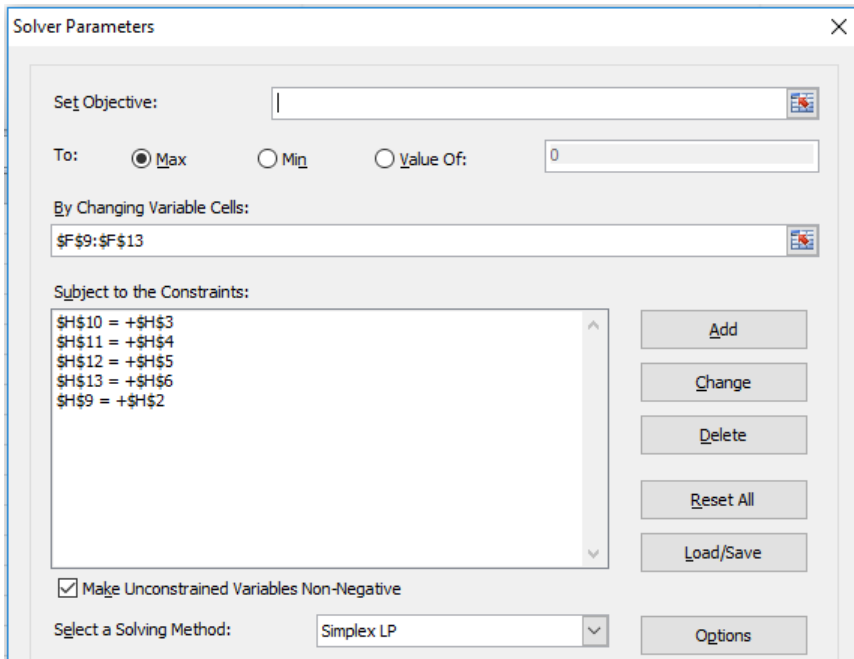


Figure 3.10. Solver set-up for Example 2.3 with the Simplex LP method

The solution found by Solver using the above set-up is shown in Figure 3.11. All the elements of the [A]{x0} are now equal to their corresponding elements in the vector {y}. The first element of the solution vector, which is $-6.6 \times 10^{-16}$, is practically zero. Therefore, the solution is [0,1,2,3,4], which is the same as that obtained in Example 2.3 by using the matrix inversion method. You can now try to use the other two solution methods of Solver to solve this problem. Solver can also be used for solving systems of simultaneous nonlinear equations (refer to Problem 3.7).
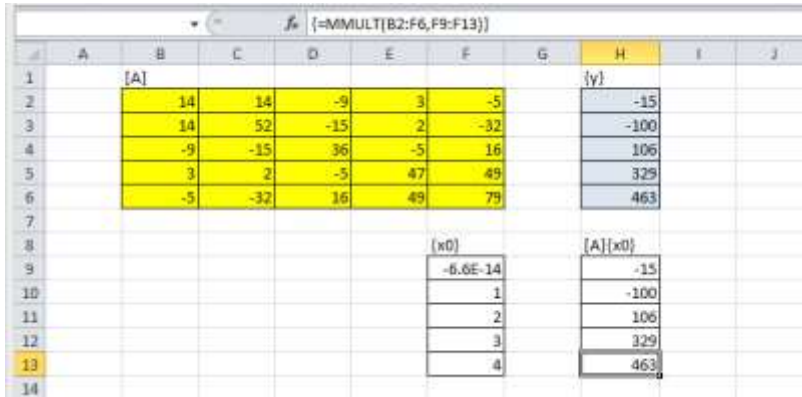
Figure 3.11. Solution of Example 2.3 with the Simplex LP method

### 3.4. The Evolutionary method

When the function to be optimised has more than one optimum point, the GRG Nonlinear method can only find the one that is closest to the initial guess; which may not be the global minimum. For such functions, Solver provides the **Evolutionary** method that adopts a variety of genetic algorithms and local search methods [3]. The following example illustrates the capability of the Evolutionary method to find the global minimum of a simple function.

### Example 3.2. Finding the global minimum of a function

Determine the global minimum value for the following function:

$$f(x) = x \cos(x) \qquad 3 \leq x \leq 14 \tag{3.3}$$

### Solution

Figure 3.12 shows the Excel sheet developed for solving this example. The insert shows that the function has two minima in the specified range of $x$; one at $x \approx 5$ and another at $x \approx 11$. At the initially-specified value ox $x = 3$, the function has a value of 0.42336. Let us first try to solve the problem with the GRG Nonlinear method. Figure 3.13 shows the completed Solver parameters dialog-box with two constraints that specify the upper and lower limits for $x$. From Figure 3.14 that shows the solution found by Solver by using the GRG Nonlinear method it is clear that Solver found the local minimum which is nearer to the initial guess and not the global minimum. In order to locate the global minimum by the GRG Nonlinear method, the solution has to be started with an initial guess that is nearer to the global minimum, e.g., $x = 9$. The advantage of the Evolutionary method is that such an arrangement is not required. With this method we may choose to specify bounds on variables (see Section 3.6). Since the set-up shown in Figure 3.13 already specifies upper and lower bounds on $x$, we only need to change the solution method to "Evolutionary". Figure 3.15, which shows the solution obtained by this method, shows that the method produced the global minmum.
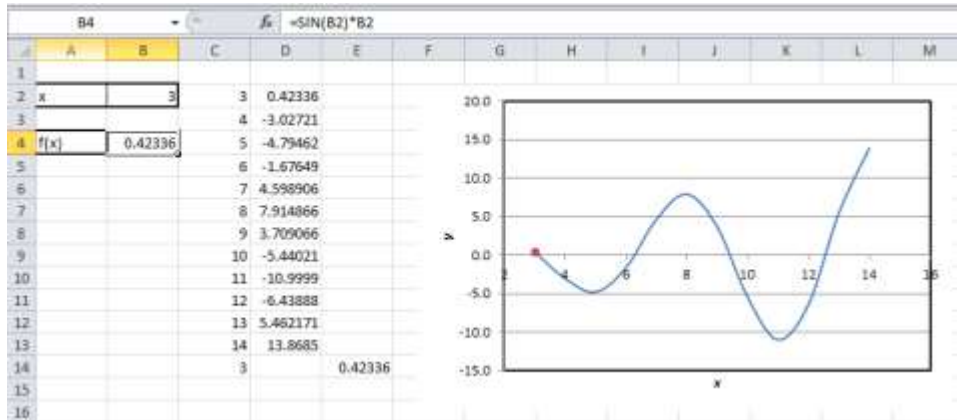
Figure 3.12. The Excel sheet for Example 3.2
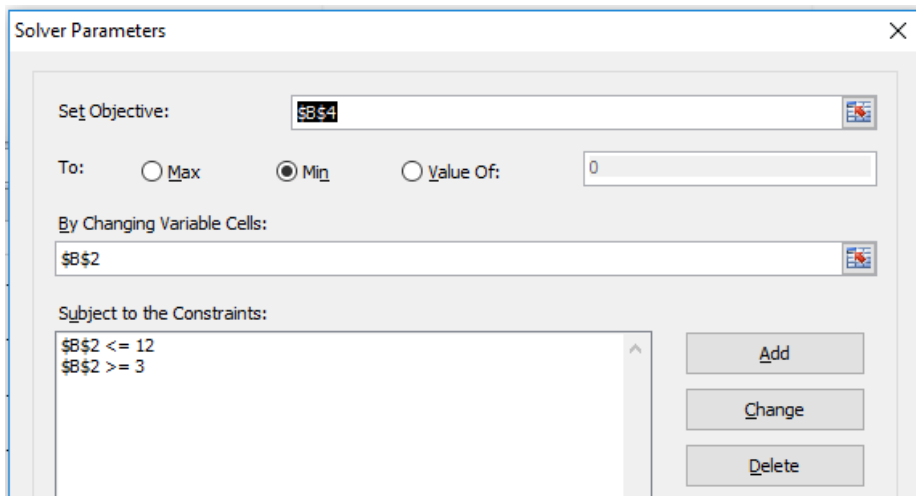


Figure 3.13. Solver set-up for Example 3.2 with GRG Nonlinear method
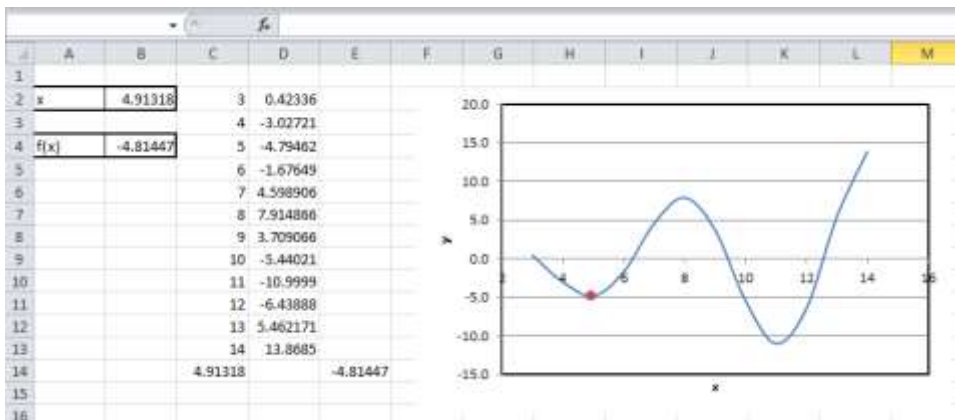


Figure 3.14. Solver solution for Example 3.2 with the GRG Nonlinear method
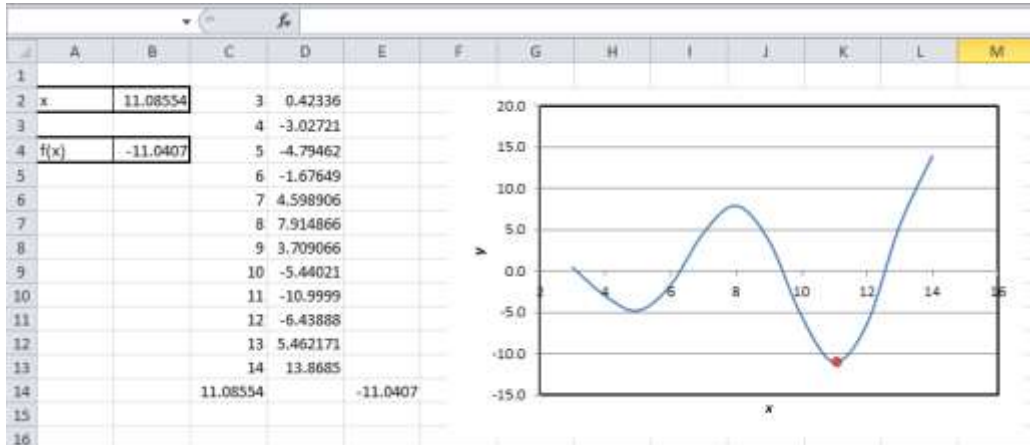
Figure 3.15. Solver solution for Example 3.2 with the Evolutionary method

While the solution with the GRG Nonlinear method took less than a second that of the Evolutionary method took one minute and 35 seconds on the same computer. As shown in the following sections, the computer time of the Evolutionary method can be reduced by adjusting the default options of the method. The method is particularly useful for optimisation analyses that involve non-smooth and discontinuous functions, which are difficult to solve with the GRG Nonlinear method. Section 3.6 compares the GRG Nonlinear method and the Evolutionary method by considering an optimsation analyses that is more relevant to thermofluid systems.

## 3.5. The default settings of Solver options

Solver gives it user some control over how its three solution methods work by allowing certain options in these methods to be adjusted. The default settings of these options are usually satisfactory, but they may have to be changed in order to reduce the computation time or increase the precision of the solution for some problems. Sometimes, Solver fails altogether to find the solution if the default options are not changed. While some adjustable options are common to all three solution methods, others are particular to the GRG Nonlinear method or the Evolutionary method. By clicking the "**Options**" button in Solver's parameters dialog box as shown in Figure 3.16, the dialog box shown in Figure 3.17 will appear to you. This figure shows the default settings of the options that are common to all three solution methods. The general options shown in Figure 3.17 are used without change in most of the analyses presented in this book. For example, the default value for the maximum number iterations, which is 100, is adequate for all the analyses presented in later chapters of the book. However, certain analyses required the automatic-scaling option to be used. The automatic-scaling option enables Solver to handle a poorly-scaled model, i.e. a model in which the values of the objective and constraint functions differ by several orders of magnitude, by allowing the values of the objective and constraint functions to be scaled internally in order to minimise the differences between them.
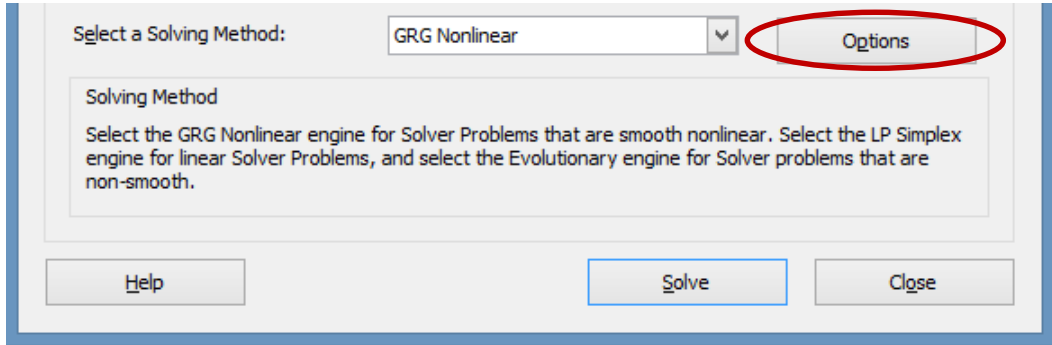
Figure 3.16. Solver options in the Properties dialog box
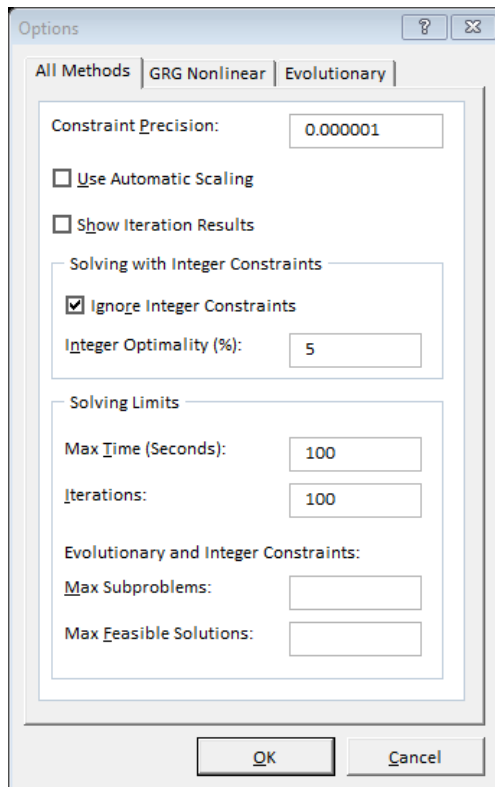


Figure 3.17. Default Solver options adopted in the analyses for all solution methods

Figures 3.18.a and 3.18.b show the default settings which are particular to the GRG Nonlinear method and to the Evolutionary method, respectively. Figure 3.18.a shows that the GRG Nonlinear method uses the forward difference (FD) approximation of derivatives by default. This option is kept unchanged in all the analyses presented in this book. A case is considered in the following section in which the GRG Nonlinear method needed the automatic-scaling option in order to reach the solution.

<div align="center">(a)                                    (b)</div>
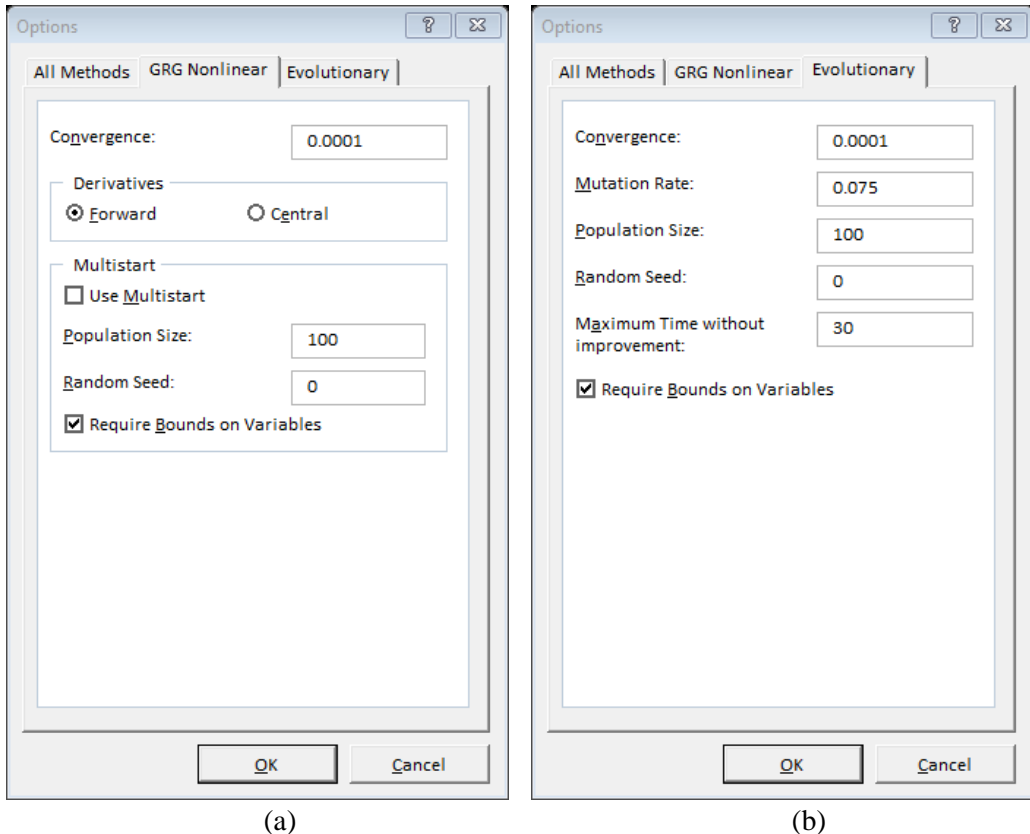
Figure 3.18. The default Solver options specific to: (a) the GRG Nonlinear method and (b) the Evolutionary method

Figure 3.18.b shows the default settings used by the Evolutionary method. According to this set-up, the population size is 100, the maximum allowable time without improvement is 50 seconds, and upper and lower bounds on variables are required. As shown in the following section, the time required by the Evolutionary method can be reduced by adjusting the population size or the maximum allowable time. Because of its long computer time, only few cases in this book used the Evolutionary method.

## 3.6. Optimisation with the GRG Nonlinear and Evolutionary methods

The GRG Nonlinear method and the Evolutionary method apply two fundamentally different methods of optimisation. While the GRG Nonlinear method applies a deterministic numerical method, which is the generalised reduced gradient method [4], the Evolutionary method applies a group of stochastic search algorithms [5]. This section compares the two optimisation methods by considering the case of optimising the thickness of insulation around an air-conditioning duct. The following example is based on Example 4.1 in Janna [6].

**Example 3.3. Determining the optimum thickness of insulation around a duct**

Figure 3.19 shows a circular duct of external diameter $D_1$ that transports hot air-conditioning air. The difference in temperature between the air inside the duct and the surroundings causes heat losses to the surroundings. This heat loss can be minimised by using insulation of outside diameter $D_2$.
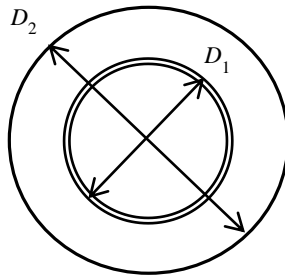


Figure 3.19. Schematic of the insulated duct

It is required to determine the most economical thickness of insulation based on the following information:

(a)    The cost of pumping the air through the duct ($C_p$) in \$/year is given by:

$$C_p = 3 \times 10^{-6} D_1^{-5}$$
(3.4)

(b)    The cost of heating the air ($C_h$) in \$/year is given by:

$$C_h = \frac{9}{\delta}$$
(3.5)

In which $\delta$ is the insulation thickness ($\delta = D_2 - D_1$) in meters.

(c)    Due to space limitations, the outside diameter of the insulation $D_2$ cannot exceed 12 cm.

**The analytical model**

The total cost ($C_T$) is given by the summation of the pumping and heating costs:

$$C_T = \frac{3 \times 10^{-6}}{D_1^5} + \frac{9}{\delta} = \frac{3 \times 10^{-6}}{D_1^5} + \frac{9}{(D_2 - D_1)}$$
(3.6)

By imposing the requirement that the maximum diameter $D_2$ should not exceed 0.12 m, the total cost becomes;

$$C_T = \frac{3 \times 10^{-6}}{D_1^5} + \frac{9}{(0.12 - D_1)} \tag{3.7}$$

Differentiating Equation (3.7) with respect to $D_1$ and equating the result to zero, Janna [6] obtained the following solution for $D_1$:

$$D_1 = \left[ 1.67 \times 10^{-6} (0.12 - D_1)^2 \right]^{1/6} \tag{3.8}$$

The solution Equation (3.8) would give the required value of $D_1$, but it is nonlinear. Therefore, it requires an iterative solution. By using an iterative method, Janna [6] determined the optimum diameter as $D_1 = 0.045$ m. Excel can easily perform the iterative solution of Equation (3.8) with Goal Seek. However, Solver provides an alternative method for solving the problem by minimising the basic function in Equation (3.6) without any differentiation.

**Solution with the GRG Nonlinear method**
Figure 3.20 shows the Excel sheet developed for this example. Note that the figure reveals the formulae used in the calculations part in which cell labelling has been used. The data part includes only the value of $D_2$ (D_2) = 0.12 m. The sheet gives the total cost for a guessed inner diameter $D_1$ (D_1) of 0.1 m. At this guessed diameter, the insulation thickness $\delta$ ($t$) is 2 cm and the total cost is 450.3$.

| | C_Total | | | $f_x$ | =C_p+c_h | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J |
| 1 | | | | | | | | | | |
| 2 | D_2 | 0.12 | | D_1 | 0.1 | | C_Total | 450.3 | =C_p+c_h | |
| 3 | | | | | | | | | | |
| 4 | | | | t | 0.02 | =D_2-D_1 | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | C_p | 0.3 | =0.000003/D_1^5 | | | | |
| 7 | | | | c_h | 450 | =9/t | | | | |
| 8 | | | | | | | | | | |

Figure 3.20. Excel sheet for optimisation of the insulated duct

The diameter that minimises the total cost can be found by using Solver. Figure 3.21 shows Solver Parameters dialog box for finding the values of $D_1$ (in the adjustable cell) that minimises the total cost (in the target cell). Initial trials with the GRG Nonlinear method showed that no solution can be obtained by using the default options shown in Figures 3.17 and 3.18.a (i.e. without automatic scaling) without imposing a lower limit on $D_1$. Therefore, a lower limit of 1 cm was imposed on $D_1$ as shown in Figure 3.21. The solution obtained by Solver with this method and setup is shown in Figure 3.22. The optimum value thus obtained for the inner diameter, which is $D_1 = 0.045763$ m, agrees well with the value obtained by Janna [6]. The GRG Nonlinear method can solve

the problem without imposing a lower limit on $D_1$, but by selecting the "Use Automatic Scaling" option (see Problem 3.6).
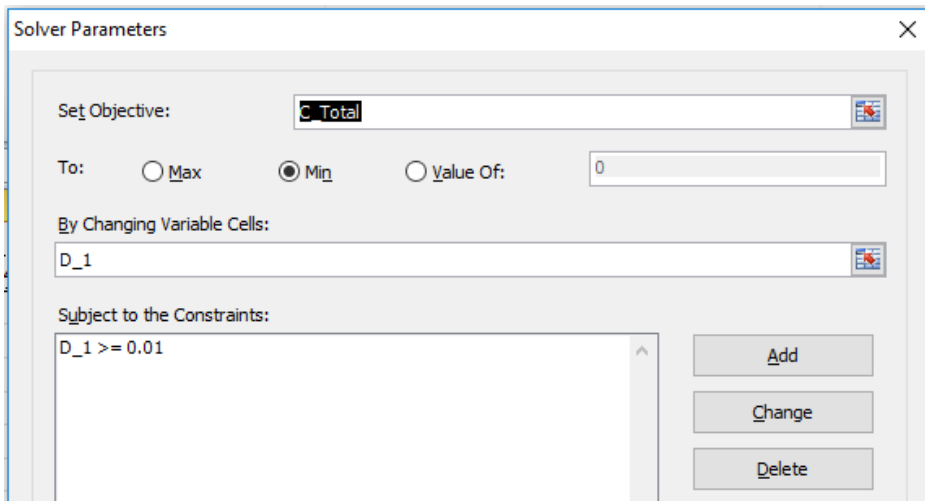


Figure 3.21. Solver set-up for the insulated duct optimisation with the GRG Nonlinear method



Figure 3.22. Optimised solution for Example 3.3 with the GRG Nonlinear method

**Solution with the Evolutionary method**
The same sheet that used the GRG Nonlinear method can be used with the Evolutionary method. As shown in Figure 3.18.b, imposing upper and lower bounds on the problem variables is also optional with the Evolutionary method. However, initial trials with this method showed that no solution can be obtained without specifying limits on $D_1$, whether automatic-scaling is used or not. Therefore, both upper and lower limits were used with this method as shown in Figure 3.23. The upper limit is specified such that $D_1$ cannot exceed $D_2$. As Figure 3.24 shows, the solution obtained with the Evolutionary method is the same as that reached by the GRG Nonlinear method. However, with the default number of the population size of 100 the Evolutionary method required more than two minutes of computer time. By reducing the population size to 10, the method

reached the same solution in less than a second. Chapter 6 gives more examples of using the GRG Nonlinear method and the Evolutionary method for optimisation analyses of thermofluid systems.
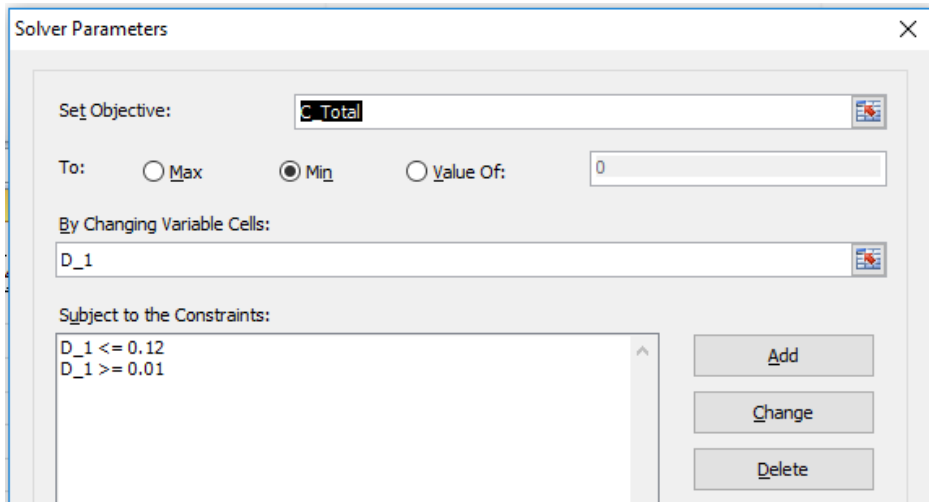


Figure 3.23. Solver set-up for the insulated duct optimisation with the Evolutionary method



Figure 3.24. Optimised solution for Example 3.3 with the Evolutionary method

## 3.7.   VBA and the development of user-defined functions

VBA is a programming language that can be used for the development of customised functions, called user-defined functions (UDFs), or macros. The process of activating and using VBA for developing UDFs can be illustrated by means of a simple example. Suppose that we want to write a function for determining the area $A$ of a circle given its diameter $D$ by using the following equation:

$$A = \pi D^2 / 4 \tag{3.9}$$

As shown in Figure 3.25, VBA is found on the left side of the **Developer tab**. This tab gives many other development tools. If the Developer tab is not shown in the ribbon of your Excel sheet, you can make it available by going to **File**, selecting **Options**, and then the **Customise Ribbon** from the **Backstage View** shown in Figure 3.26.
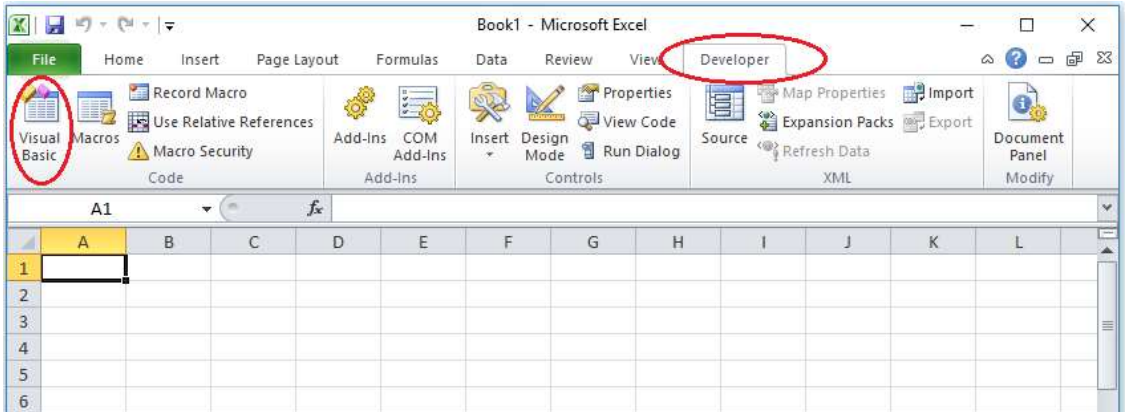


Figure 3.25. Selection of VBA from the Developer tab
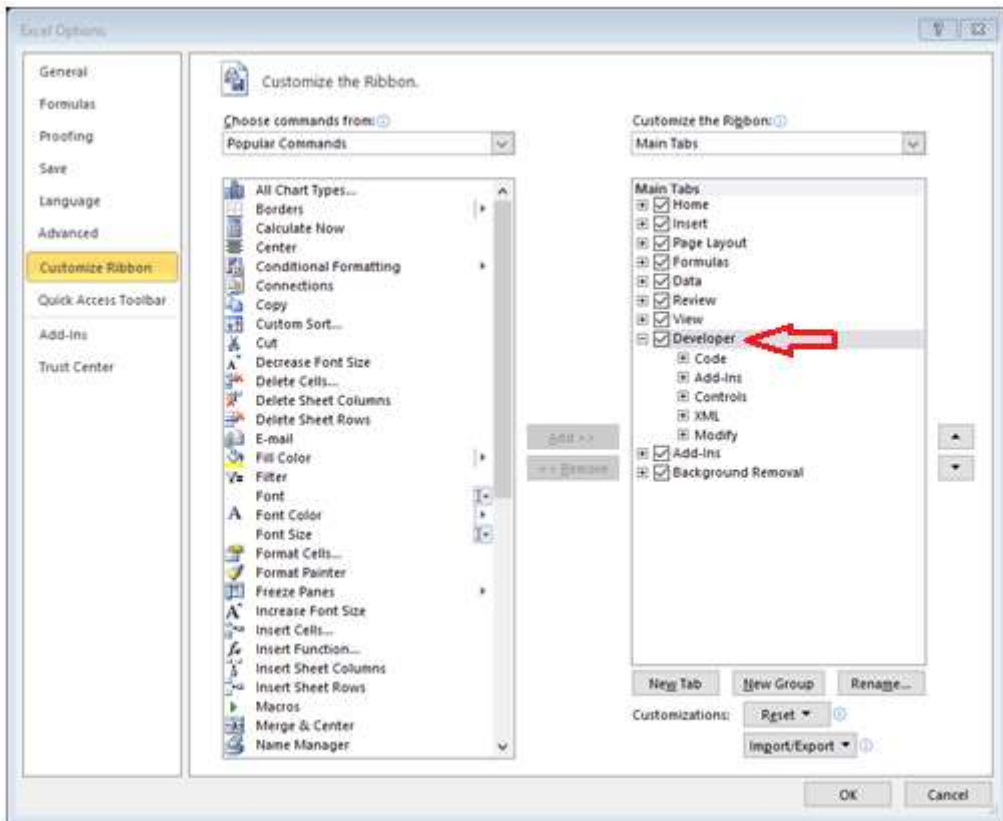


Figure 3.26. Adding VBA to the Developer tab

In the **Main Tabs** list, select the **Developer** check box as shown in the figure, and then click "**OK**". The Developer tab will now be shown in the ribbon of your Excel sheet. To start writing the UDF, go to **Developer** tab menu and select **Visual Basic**. The Visual Basic editor will appear to you as shown in Figure 3.27. Select **Insert →  Module** and the blank page shown in Figure 3.28 will be open for you to type the VBA code.
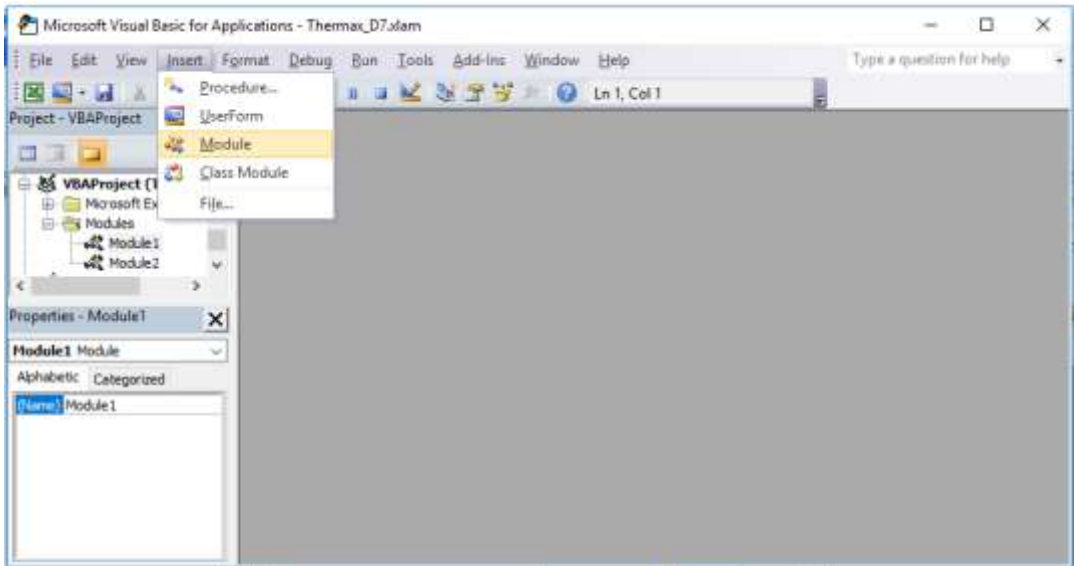


Figure 3.27. Inserting a new module



Figure 3.28. A new VBA module

The following VBA code is the required the UDF that determines the circle's area from its diameter according to Equation (3.9):

```
Function Circ_area(Dia)
Pi = 3.141593
Circ_area = Pi * Dia ^2 / 4
End Function
```

Note that the first line in the code starts with the word "**Function**" and then adds a name for the function and specifies the required input parameters. The name you give to your new function will be used to call it from Excel UI. Therefore, this name should clearly indicate the purpose of its use. In the present case, the name given to the function is "Circ_area" to indicate that it calculates the area of a circle. The function has only one input parameter, which is the circle's diameter (Dia). As soon as you type the first line of the code and press the "**Enter**" key, the editor will automatically add the **End** line of the function. Now, type the rest of the code as shown in Figure 3.29.
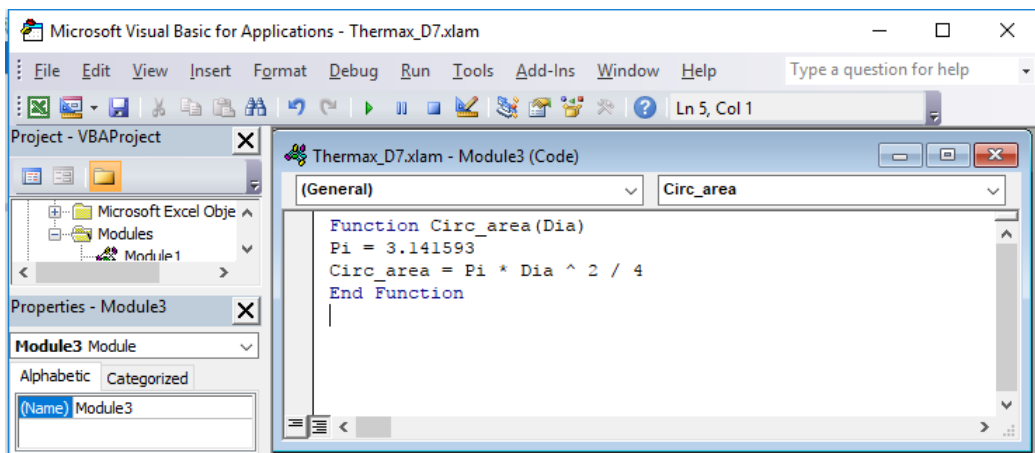


Figure 3.29. A UDF for caclcuating the area of a circle with a given diameter

After typing the code correctly, the function can be used via Excel UI just like any built-in function as shown in Figure 3.30. Note that the formula bar in Figure 3.30 reveals the formula in cell B2 as:

**= Circ_area(10**)

Where the number 10 refers to the diameter of the circle. You can now check the output of your user-defined function, which is 314.1593 square units, by calculating the circle's area with a normal Excel formula. Finally, note that VBA does not provide a built-in function for the constant Pi ($\pi$) and, therefore, you have to assign a value for it as shown in Figure 3.29. Alternatively, you can use the one provided by Excel.

Figure 3.30. Using the "Circ_area" function in Excel

In thermofluid analyses, VBA is useful for developing UDFs for fluid properties. For example, the molar specific-heat at constant pressure ($\tilde{c}_p$) for an ideal gas is given by the following formula [7]:

$$\tilde{c}_p = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \qquad \text{[kJ/kmol.K]} \qquad (3.10)$$

Where $T$ is the absolute temperature and $a_0$, $a_1$, $a_2$, and $a_3$ are constants that have different values for different gases. For air, the constants are 28.11, $0.1967 \times 10^{-2}$, $0.4802 \times 10^{-5}$, and $-1.966 \times 10^{-9}$ in this order. Figure 3.31 shows the VBA code for the UDF "cp_air" that determines $\tilde{c}_p$ for air based on Equation (3.10) and the formula bar in Figure 3.32 shows how the function can be used in an Excel formula to determine $\tilde{c}_p$ for air at 300K. The value returned by the function is 29.0771 kJ/kmol.K.
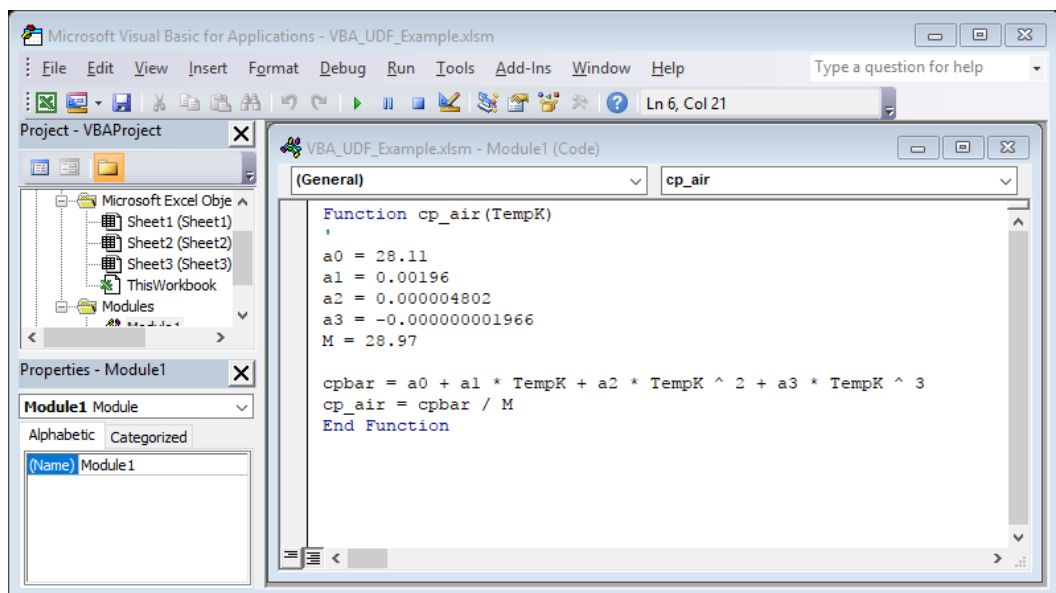


Figure 3.31. A UDF for caclcuating the molar specific-heat for air

Figure 3.32. Using the cp_air function in Excel

## 3.8.   Closure

This chapter introduced the Solver add-in that enables Excel to perform constrained iterative solutions and optimisation analyses involving multiple parameters. The chapter briefly described the three solution methods provided by solver, which are the GRG Nonlinear method, the Evolutionary method, and the Simplex LP method, and showed how these methods can be used for solving nonlinear equations and systems of linear equation and performing optimisation analyses. The use of Solver for optimisation analyses of thermofluid systems is demonstrated by considering the case of insulating a duct that carries hot air for air-conditioning. This analysis showsed that the default set-up of Solver may have to be adjusted to allow automatic-scalling in order to perform the analysis by the GRG Nonlinear method or to reduce the population size in the Evolutionary method in order to reduce the computer time. Solver can also deal with multi-variable optimisation analyses as shown in later chapters of the book.

The chapter also showed how VBA can be used for developing user-defined functions not provided by Excel. In thermofluid analyses, this is needed for the development of custom functions for fluid properties. As an example, a custom function was developed for determining the specific heat of air from the ideal-gas law. Another situation that requires the development of user-defined functions with VBA is faced when the analytical model involves complex equations, implicit nonlinear equations, or a complicated logical branching. In such cases, Excel formulae which are confined to separate cells become too restrictive and inconvenient to use. VBA can then be used to develop a suitable user-defined function that performs the complicated calculations and passes the outcome to Excel. More information about the VBA language can be found in specialised references [8-10].

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**Exercises**

1. Draw a line chart with Excel to show the variation of the following function in the range $-5 \leq x \leq 5$:

$$f(x) = x^3 - 13x - 12$$

Use Solver with proper bounds to determine the roots of this equation. This exercise is based on Example 6.2 in Chapra and Canale [11].

2. A system of algebraic equations can be expressed in matrix form as follows:

$$\begin{bmatrix} 70 & 1 & 0 \\ 60 & -1 & 1 \\ 40 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 636 \\ 518 \\ 307 \end{Bmatrix}$$

Solve the system of equations by using Solver to determine the values of the three unknowns $a$, $b$, and $c$. Based on Example 9.11 in Chapra and Canale [11]. The answer is: $a = 8.5941$, $b = 34.4118$, and $c = 36.7647$.

3. Draw a line chart with Excel to show the variation of the following function in the range $0 \leq x \leq 4$:

$$f(x) = 2 \sin x - x^2/10$$

Use Solver to find the maximum of the function in the same range. Based on Example 13.1 in Chapra and Canale [11]. The answer is: $f(x) = 1.7757$ at $x = 1.4276$.

4. The following figure is a plot of the function:

$$f = e^{\theta} \sin(2\theta)$$

Use Solver to find:

    a) The minimum value of the function and the corresponding angle
    b) The maximum value of the function and the corresponding angle
    c) The angle at which value of the function equals 4
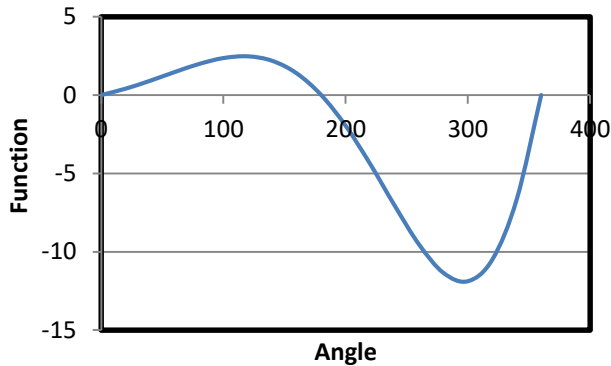


Figure P1.4. A composite function

5. Using the Excel sheet developed to solve Example 2.4 by the GRG Nonlinear method, study the effect of using central-difference approximation of derivatives instead of the default forward-difference approximation on the solution.
6. Using the Excel sheet developed for the solution of Example 3.3 with the GRG Nonlinear method, show that the same solution shown in Figure 3.22 can be obtained without imposing a lower limit on $D_1$ by using automatic scaling.
7. Consider the following set of simultaneous nonlinear equations:

$$x^2 + xy = 10 \tag{A}$$

$$y + 3xy^2 = 57 \tag{B}$$

To solve the system with Solver, rearrange the equations as follows:

$$u(x, y) = x^2 + xy - 10 = 0 \tag{C}$$

$$v(x, y) = y + 3xy^2 - 57 = 0 \tag{D}$$

Create two cells (B1 and B2) to hold initial guesses for x and y. Enter the function values themselves, $u(x, y)$ and $v(x, y)$ into two other cells (B3 and B4). The initial

guesses may result in function values of $u$ and $v$ that are far from zero. Determine the sum of the function squares, i.e. $u^2 + v^2$, and store it in cell B5. Use Solver to find the values of $x$ and $y$ in cells B1 and B2 (the Changing cells) that make the value in cell B5 (the objective cell) equal to zero. Using this procedure, find the roots of the above system starting with initial guesses of $x = 1$ and $y = 3.5$. Based on Example 6.5 in Chapra and Canale [11]. The correct pair of roots is $x=2$ and $y=3$.

8. The pipe shown in Figure 3.19 has an external diameter $D_1 = 4.6$ cm. The pipe is surrounded by an insulation material with outside diameter $D_2$. The cost of pumping the fluid through the pipe ($Cp$) and the cost of heating the fluid ($Ch$) are given by Equations (3.5) and (3.6) in Examples 3.3, respectively. The total cost ($C_T$) includes the cost of the insulation itself, which is given by:

$$C_s = 500\,\delta$$

Use Solver with the GRG Nonlinear method to determine the optimum thickness of insulation.

9. Solve Problem 3.8 with the Evolutionary method using lower and upper bounds for $D_2$ of 5 cm and 30 cm respectively.

10. The volume $V$ of liquid in a spherical tank of radius $r$ is related to the depth $h$ of the liquid by:

$$V = \pi h^2 (3r - h)/3$$

Using VBA, develop a user-defined function that determines $h$ at any given values of $r$ [m] and $V$ [m$^3$]. Check your function at $r=1$ m and $V = 0.5$ m$^3$. Answer: $h = 0.431$ m. Develop user-defined functions with VBA for determining the specific enthalpy and entropy of superheated steam from its pressure and temperature.

11. Using tabulated data for refrigerant R134a, develop user-defined functions with VBA for determining properties, e.g., enthalpy and entropy, of saturated liquid/vapour R134a from its temperature or pressure.

12. Using suitable formulae for superheated refrigerant R134a, develop user-defined functions with VBA for determining properties, e.g. enthalpy and entropy, of superheated R134a from its temperature and pressure.

# 4

# The Thermax add-in

Equipped with numerous built-in functions, the Goal Seek command and the Solver add-in, Excel is a powerful modelling platform for fluid-flow and heat-transfer analyses. However, for thermodynamic analyses Excel's capacity is limited by the lack of built-in functions for fluid properties. Another limitation of Excel for thermofluid analyses even for fluid-flow and heat-transfer analyses is that Excel's cell-confined formula becomes too restrictive for model development when an iterative solution involves a nonlinear equation like the Colebrook equation. This chapter presents the Thermax add-in that provides custom functions for the properties of ideal gases, saturated water and superheated steam, six commonly used refrigerants, humid air for psychrometric analyses, and air at standard atmospheric pressure. In addition to its property functions, Thermax provides two interpolation functions for general use and an internal Newton-Raphson solver for nonlinear equations that can be used to deal with the linear equation in an iterative solution. The chapter describes the procedure for installing the add-in and using its functions and numerical tools in Excel's formulae.

## 4.1. Thermax property functions

Thermax provides five groups of property functions for: (i) ideal gases, (ii) water and superheated steam, (iii) vapour-compression refrigerants, (iv) humid air for psychrometric analyses and (v) air at atmospheric pressure. To easily select the required function from the large number of functions provided by the add-in, Thermax adopts a style for naming the functions that indicates the group and the input and output parameters of each function. This section describes the adopted name-style and shows how the functions in the five groups are formulated.

### 4.1.1.  Name style for Thermax property functions

The name of any property function consists of three distinct parts as follows:

1. The first part indicates the function's group: "Gas" for ideal gases, "Wat" for water, "Ref" for refrigerants, and "Air" for air at atmospheric pressure.
2. The second part indicates the function's output property, e.g. "h" for enthalpy and "s" for entropy.
3. The third part indicates the function's input parameters, .e.g. "P" for pressure and "T" for temperature.

For example, consider the two functions shown in Figure 4.1. The top function, **Wath_Px**, determines the enthalpy of saturated water at a pressure of 500 kPa and quality of 0.8. The first three letters in the function's name refer to its group (Wat) immediately followed by the function's output (h). An underscore precedes the function's two input arguments, which are the pressure (P) and quality (x). Similarly, the name of the bottom function, **Refs_PT**, tells that it belongs to the refrigerants group and that it determines the entropy (s) of refrigerant R134a from its pressure (P) and temperature (T). This function requires three input parameters which are (i) the refrigerant name, (ii) the pressure, and (iii) the temperature. Table 4.1 shows more examples for the functions with their intended usage.
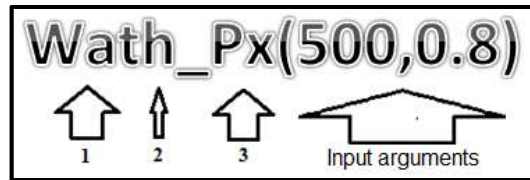
(a)
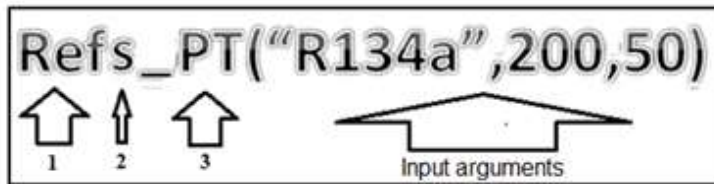


(b)

Figure 4.1. Examples of Thermax functions

Table 4.1. Examples of Thermax property functions with their output arguments

| # | Thermax function | Output |
|---|---|---|
| 1 | Gash_TK("Air",350) | Determines the enthalpy ($h$) for air at 350K |
| 2 | Wats_Px(300,0.5) | Determines the entropy ($s$) of saturated water at a pressure of 300 kPa and quality of 0.5 |
| 3 | Wath_PT(90,150) | Determines the enthalpy of superheated steam at 90 kPa and 150°C |
| 4 | RefPsat_T("R134a",-5) | Determines the saturation pressure ($P_{sat}$) for refrigerant R134a at -5°C. |
| 5 | PsyRh_PTSh(101,30, 0.001) | Determines relative humidity ($\phi$) of humid air at 101 kPa, 30°C, and specific humidity ($\omega$) of 0.001 kg/kg |
| 6 | Airdv_T(25) | Determines the dynamic viscosity of air at standard atmospheric pressure and 25°C. |

The following points should be noted regarding the name style adopted by Thermax:

1. The output properties in all the functions are represented by one or two letters, e.g. Gash_TK and GasTK_h, except those for the saturation pressure and saturation temperature in the WAT and Ref groups and the air density in the Air-group, which are named WatPsat_T, WatTsat_P, RefPsat_T, RefTsat_P, and Airrho_T.
2. The Gas-group and the Ref-group require the name of the gas or the refrigerant fluid as the first input parameter as shown in Figure 4.1.b.
3. The unit for pressure in both the Wat-group and the Ref-group is kPa.
4. To indicate that absolute temperature is used in the Gas-group, the temperature is represented by "TK". However, "T" represents temperature in °C in the other three other groups.

### 4.1.2.  Functions for ideal gases

Properties of ideal gases at a given temperature are obtained by suitable integration of Equation (3.10) for the molar specific heat at constant pressure ($\tilde{c}_p$ ). Accordingly, the molar enthalpy ($\tilde{h}$ ), molar internal energy ($\tilde{u}$ ), and temperature-dependent molar entropy change ($\tilde{s}^0$) of an ideal gas are calculated from [1]:

$$\tilde{h} = \tilde{h}_0 + \int_{T_0}^{T} \tilde{c}_p(T)dT \tag{4.1}$$

$$\tilde{u} = \tilde{u}_0 + \int_{T_0}^{T} \tilde{c}_v(T)dT = \tilde{u}_0 + \int_{1}^{2} (\tilde{c}_p(T) - R_u)dT \tag{4.2}$$

$$\tilde{s}^0 = \tilde{s}_0^0 + \int_{T_0}^{T} \frac{\tilde{c}_p(T)}{T}dT \tag{4.3}$$

Where, $\tilde{h}_0, \tilde{u}_0$, and $\tilde{s}_0^0$ are pre-specified values at a reference temperature ($T_0$) and $R_u$ is the universal gas constant. For $\tilde{h}$ and $\tilde{u}$, the reference temperature is taken as 300K and the corresponding values of $\tilde{h}_0$ and $\tilde{u}_0$ are those given by Cengel and Boles [1]. However, for $\tilde{s}^0$ the reference temperature is taken as 298K and the corresponding value of $\tilde{s}_0^0$ is the absolute entropy. Rather than the molar properties given above, Thermax functions return the ideal-gase properties per kg, i.e. $h$, $u$ and $s^0$, i.e.:

$$h = \tilde{h} / M \tag{4.4}$$

$$u = \tilde{u} / M \tag{4.5}$$

$$s^0 = \tilde{s}^0 / M \tag{4.6}$$

Where $M$ is the molar mass of the gas. The relative pressure ($P_r$) and relative specific volume ($v_r$) are then obtained from [1]:

$$P_r = \exp(s°/R) \tag{4.7}$$

$$v_r = T / P_r \tag{4.8}$$

Note that $P_r$ is a dimensionless quantity but $v_r$ is not. Based on the above equations, this group provides 12 functions that give properties of the 12 ideal gases listed in Table 4.2. Table 4.3 lists the 12 functions and shows the input and output of each function and their relevant units.

Table 4.2. The ideal gases supported by Thermax

| #  | Gas              | Thermax name |
|----|------------------|--------------|
| 1  | Nitrogen         | N2           |
| 2  | Oxygen           | O2           |
| 3  | Air              | Air          |
| 4  | Hydrogen         | H2           |
| 5  | Carbon monoxide  | CO           |
| 6  | Carbon dioxide   | CO2          |
| 7  | Water vapour     | H2O          |
| 8  | Nitric oxide     | NO           |
| 9  | Nitrogen dioxide | NO2          |
| 10 | Sulphur          | S2           |
| 11 | Sulphur dioxide  | SO2          |
| 12 | Sulphur trioxide | SO3          |

Table 4.3. Functions for thermodynamic properties of gases

| #  | Function  | Input/Unit                | Output/Unit         |
|----|-----------|---------------------------|---------------------|
| 1  | GasM      | Gas name                  | $M$ [-]             |
| 2  | Gascp_TK  | Gas name, $T$[K]          | $c_p$ [kJ/kg.K]     |
| 3  | Gash_TK   | Gas name, $T$[K]          | $h$ [kJ/kg]         |
| 4  | Gasu_TK   | Gas name, $T$[K]          | $u$ [kJ/kg]         |
| 5  | Gass0_TK  | Gas name, $T$[K]          | $s^0$ [kJ/kg.K]     |
| 6  | GasPr_TK  | Gas name, $T$[K]          | $P_r$ [-]           |
| 7  | Gasvr_TK  | Gas name, $T$[K]          | $v_r$ [K]           |
| 8  | GasTK_h   | Gas name, $h$[kJ/kg]      | $T$ [K]             |
| 9  | GasTK_u   | Gas name, $u$[kJ/kg]      | $T$ [K]             |
| 10 | GasTK_s0  | Gas name, $s^0$[kJ/kg.K]  | $T$ [K]             |
| 11 | GasTK_Pr  | Gas name, $P_r$ [-]       | $T$ [K]             |
| 12 | GasTK_vr  | Gas name, $v_r$ [K]       | $T$ [K]             |

The first five functions shown in Table 4.3 determine the molar mass ($M$), specific heat ($c_p$), enthalpy ($h$), internal energy ($u$), and the part of entropy change due to temperature change ($s^0$). The following two functions determine the relative pressure ($P_r$) and relative volume ($v_r$). The last five functions in Table 4.3 are inversion functions that determine the temperature of the ideal gas by iteration from its enthalpy ($h$), internal energy ($u$), temperature-dependent entropy change ($s^0$), relative pressure ($P_r$) or relative specific volume ($v_r$). Note that the temperature is represented by the letters "TK" in the names of all the functions in this group. With the exception of the first function, GasM, all the functions require the absolute temperature as inputs. An auxiliary custom function named "Gas_data" stores the values of the four coefficients $a_0$, $a_1$, $a_2$, and $a_3$ in Equation (3.10) for the twelve ideal gases. Values of the coefficients for the different gases were obtained from Cengel and Boles [1].

### 4.1.3.   Functions for saturated water and superheated steam

This group includes 23 functions that return the properties of saturated water-steam mixtures and superheated steam. These functions are divided into two subgroups: (a) functions that determine the properties of saturated water and (b) functions that determine the properties of superheated steam given the pressure and another property. Properties of compressed liquid water are approximated by their corresponding values of saturated liquid water at the given temperature.

### a)  Properties of saturated water/steam mixtures

Table 4.4 shows the input and output parameters of 8 property functions that determine the properties of saturated water/steam mixtures at a given temperature, in °C, with their relevant units. The first function "WatPsat_T" returns the saturation pressure at a given temperature. The letters "f" and "g" before the underscore in the following 7 functions, e.g., "Wathf_T" and "Wathg_T", refer to the saturated liquid state and saturated vapour states, respectively. The corresponding custom functions that provide properties of saturated water-steam mixture at a given pressure, in kPa, as listed in Table 4.5 with their relevant input and output parameters.

Table 4.4. Property functions for saturated water/steam at a given temperature in °C

| #  | Function  | Output/Unit         |
|----|-----------|---------------------|
| 1  | WatPsat_T | $p_s$ [kPa]         |
| 2  | Wathf_T   | $h_f$ [kJ/kg]       |
| 3  | Wathg_T   | $h_g$ [kJ/kg]       |
| 4  | Wathfg_T  | $h_{fg}$ [kJ/kg]    |
| 5  | Watvf_T   | $v_f$ [m³/kg]       |
| 6  | Watvg_T   | $v_g$ [m³/kg]       |
| 7  | Watsf_T   | $s_f$ [kJ/kg.K]     |
| 8  | Watsg_T   | $s_g$ [kJ/kg.K]     |

Table 4.5. Property functions for saturated water/steam at a given pressure in kPa

| #  | Function  | Output/Unit         |
|----|-----------|---------------------|
| 1  | WatTsat_P | $T_s$ [°C]          |
| 2  | Wathf_P   | $h_f$ [kJ/kg]       |
| 3  | Wathg_P   | $h_g$ [kJ/kg]       |
| 4  | Wathfg_P  | $h_{fg}$ [kJ/kg]    |
| 5  | Watvf_P   | $v_f$ [m³/kg]       |
| 6  | Watvg_P   | $v_g$ [m³/kg]       |
| 7  | Watsf_P   | $s_f$ [kJ/kg.K]     |
| 8  | Watsg_P   | $s_g$ [kJ/kg.K]     |

### b)  Properties of superheated steam and compressed liquid water

For superheated steam, the water group provides 7 functions that return the properties at a given pressure and another property as shown in Table 4.6. The formulae used in these

functions are the same as those used for superheated refrigerants which will be described in the following section. These functions also determine properties of compressed liquid water by calling those of saturated liquid water at the given temperature shown in Table 4.4.

Table 4.6. Properties of superheated steam given the pressure in kPa and another property

| # | Function | Input/Unit | Output/Unit |
|---|----------|-----------|-------------|
| 1 | Watv_PT | $P$, $T$[ºC] | $v$ [m³/kg] |
| 2 | Wath_PT | $P$, $T$[ºC] | $h$ [kJ/kg] |
| 3 | Wats_PT | $P$, $T$[ºC] | $s$ [kJ/kg.K] |
| 4 | WatT_Ph | $P$, $h$[kJ/kg] | $T$ [⁰C] |
| 5 | WatT_Ps | $P$, $s$[kJ/kg.K] | $T$ [⁰C] |
| 6 | Wath_Ps | $P$, $s$[kJ/kg.K] | $h$ [kJ/kg] |
| 7 | Wats_Ph | $P$, $h$[kJ/kg] | $s$ [kJ/kg.K] |

### 4.1.4.   Functions for refrigerants

This group of Thermax functions deal with the properties of three synthetic refrigerants, R22, R134a, and R410A (R32/R125 - 50/50), and three natural refrigerants, R717 (ammonia), R718 (water), and R744 (carbon-dioxide). The group includes 25 functions that can be divided into two subgroups: (a) functions that determine the properties of saturated refrigerants and (b) functions that determine the properties of superheated refrigerants from the given pressure and another property.

### a)   Properties of saturated liquid/vapour refrigerants

Table 4.7 lists nine functions that determine the thermo-physical properties of saturated refrigerants at a given pressure while Table 4.8 lists nine functions that determine the same properties at a given temperature. These functions do not use mathematical formulae, but interpolate the tabulated data provided by ASHRAE [2] for the six refrigerants by a linear interpolation function. The functions return warning messages if the given temperature, pressure, or quality is beyond the expected range.

Table 4.7. Properties of saturated refrigerants at a given pressure in kPa and quality

| # | Function | Input | Output | Unit |
|---|----------|-------|--------|------|
| 1 | RefTsat_P | *Refrigerant name, P, x* | *Saturation temperature, $T_{sat}$* | [⁰C] |
| 2 | Refv_Px | *Refrigerant name, P, x* | *Specific volume, v* | [m³/kg] |
| 3 | Refh_Px | *Refrigerant name, P, x* | *Specific enthalpy, h* | [kJ/kg] |
| 4 | Refs_Px | *Refrigerant name, P, x* | *Specific entropy, s* | [kJ/kg.K] |
| 5 | Refcp_Px | *Refrigerant name, P, x* | *Specific heat, $c_p$* | [kJ/kg.K] |
| 6 | Refvs_Px | *Refrigerant name, P, x* | *Velocity of sound* | [m/s] |
| 7 | Refdv_Px | *Refrigerant name, P, x* | *Dynamic viscosity* | [µPa·s] |
| 8 | Refk_Px | *Refrigerant name, P, x* | *Thermal conductivity* | [mW/(m·K)] |
| 9 | Refst_P | *Refrigerant nam, P* | *Surface tension* | [mN/m] |

Table 4.8. Properties of saturated refrigerants at a given temperature in ºC and quality

| # | Function | Input | Output | Unit |
|---|----------|-------|--------|------|
| 1 | RefPsat_T | *Refrigerant name, T, x* | *Saturation temperature, $T_{sat}$* | [$^0$C] |
| 2 | Refv_Tx | *Refrigerant name, T, x* | *Specific volume, v* | [m$^3$/kg] |
| 3 | Refh_Tx | *Refrigerant name, T, x* | *Specific enthalpy, h* | [kJ/kg] |
| 4 | Refs_Tx | *Refrigerant name, T, x* | *Specific entropy, s* | [kJ/kg.K] |
| 5 | Refcp_Tx | *Refrigerant name, T, x* | *Specific heat, $c_p$* | [kJ/kg.K] |
| 6 | Refvs_Tx | *Refrigerant name, T, x* | *Velocity of sound* | [m/s] |
| 7 | Refdv_Tx | *Refrigerant name, T, x* | *Dynamic viscosity* | [μPa·s] |
| 8 | Refk_Tx | *Refrigerant name, T, x* | *Thermal conductivity* | [mW/(m·K)] |
| 9 | Refst_T | *Refrigerant name, T* | *Surface tension* | [mN/m] |

## b) Properties of superheated refrigerants

Thermax functions that determine the thermodynamic properties of superheated refrigerant use the following mathematical formulae.

*Specific volume*

The molar specific volume ($\tilde{v}$) of a superheated refrigerant is obtained from the Soave-Redlich-Kwong equation of state [1]:

$$P = \frac{R_u T}{\tilde{v} - b} - \frac{a\alpha}{\tilde{v}(\tilde{v} + b)} \tag{4.9}$$

where, $R_u$ is the universal gas constant, $P$ is the absolute pressure, and $T$ is the absolute temperature. The constants $a$, $b$ and $\alpha$, which depend on the refrigerant's pressure and temperature at the critical point, are given by:

$$a = 0.4278 \, R_u^2 \, T_c^2 / P_c \tag{4.10}$$

$$b = 0.0867 \, R_u \, T_c / P_c \tag{4.11}$$

$$\alpha = \left[1 + S\left(1 - \sqrt{T_r}\right)\right]^2 \tag{4.12}$$

where, $T_c$ and $P_c$ are the temperature and pressure at the critical point, $T_r = T/T_c$ is the reduced temperature, and $S$ is a function of the acentric factor ($\omega$) for the given refrigerant:

$$S = 0.48508 + 1.55171 \, \omega - 0.15613 \, \omega^2 \tag{4.13}$$

The acentric factor itself can be calculated from [3]:

$$\omega = -\log_{10}\left(p_r^{sat}\right)-1, \ \text{at} \ T_r = 0.7 \tag{4.14}$$

Where $P_r^{sat} = P_{sat}/P_c$ is the reduced saturation pressure. Values of $T_c$, $P_c$, and $\omega$ for the six refrigerants supported by Thermax are shown in Table 4.9.

Table 4.9. Values of $T_c$, $P_c$, $\omega$ and $M$ for six refrigerants

| # | Refrigerant | Formula | $M$ | $T_c$ [K] | $P_c$ [MPa] | $\omega$ |
|---|---|---|---|---|---|---|
| 1 | R134a | $CF_3CH_2F$ | 102.03 | 374.2 | 4.059 | 0.3268 |
| 2 | R22 | $CHClF_2$ | 86.48 | 369.0 | 4.98 | 0.221 |
| 3 | R410A | $CH_2F_2,/CHF_2CF_3$ | 72.585 | 344.51 | 4.903 | 0.296 |
| 4 | R717 | $NH_3$ | 17.03 | 405.5 | 11.33 | 0.256 |
| 5 | R718 | $H_2O$ | 18.0 | 647.1 | 22.06 | 0.3443 |
| 6 | R744 | $CO_2$ | 44.0 | 304.1 | 7.38 | 0.239 |

Equation (4.9) is a non-linear equation in $\tilde{v}$ that requires a numerical solution. In the present add-in the equation is solved by using the Newton-Raphson method. The specific volume ($v$), in m³/kg, can be obtained from:

$$v = \tilde{v}/M \tag{4.15}$$

Where $M$ is the molar mass of the refrigerant and its values for the six refrigerants are shown in Table 4.9. The accuracy of the above relation in determining the specific volume was demonstrated by El-Awad [4].

*Enthalpy at a given pressure and temperature or entropy*
Enthalpy ($h$) of a superheated refrigerant at a given pressure and temperature is determined from the following relationship [4, 5]:

$$h = h_g + Cp_g^*\left(T - T_s\right) \tag{4.16}$$

Where $h_g$ and $T_s$ are the enthalpy and temperature of the saturated vapour refrigerant at the given pressure while $Cp_g^*$ is the value of its specific heat evaluated at a skewed pressure ($P^*$) given by:

$$P^* = z \times P \tag{4.17}$$

Where $z$ is an adjusting factor the value of which can be taken as 0.5 [4,5]. Values of $h_g$, $T_s$, and $Cp_g^*$ are obtained from ASHRAE data for saturated refrigerants.

*Entropy*

Entropy (*s*) of a superheated refrigerant at a given pressure and temperature is determined from the following relationship:

$$s = s_g + \frac{Cp_g^*(T - T_s)}{(T_{av} + 273)}$$  (4.18)

Where $s_g$ is the entropy of saturated vapour refrigerant at the given pressure, $Cp_g^*$ ,is the value of the specific heat evaluated at the reduced pressure ($P^*$), and $T_{av}$ is an average temperature calculated as follows:

$$T_{av} = (T + T_s)/2$$  (4.19)

*Temperature at a given pressure and enthalpy or entropy*
Thermax group of functions for refrigerants properties also determine the temperature of a superheated refrigerant given its pressure and enthalpy or pressure and entropy. In the first case, Equation (4.16) is rearranged as follows:

$$T = T_s + (h - h_g)/Cp_g^*$$  (4.20)

Where $T_s$ and $h_g$ are values of the saturation temperature and enthalpy of saturated refrigerant vapour at the given pressure, but $Cp_g^*$ is the value of the specific heat of saturated refrigerant vapour determined at the reduced pressure $P^*$. Similarly, when the pressure and entropy of the superheated refrigerant are known and its temperature is to be determined, the following equation is used:

$$T = (T_g + 273)e^{\frac{s - s_g}{Cp_g^*}} - 273$$  (4.21)

Where $T_g$ and $s_g$ are the temperature and entropy of saturated vapour refrigerant at the given pressure, while $Cp_g^*$ is the value of the specific heat of saturated refrigerant vapour determined at the reduced pressure $P^*$. Table 4.10 lists seven functions that apply the above formulae to deal with superheated vapours of refrigerants.

Table 4.10. Properties of superheated refrigerants given the pressure in kPa

| # | Function | Input/unit | Output/Unit |
|---|---|---|---|
| 1 | Refv_PT | *Refrigerant name, P, T* [ºC] | *v* [m³/kg] |
| 2 | Refh_PT | *Refrigerant name, P, T* [ºC] | *h* [kJ/kg] |
| 3 | Refs_PT | *Refrigerant name, P, T* [ºC] | *s* [kJ/kg.K] |
| 4 | RefT_Ph | *Refrigerant name, P, h* [kJ/kg] | *T* [ºC] |
| 5 | RefT_Ps | *Refrigerant name, P, s* [kJ/kg.K] | *T* [ºC] |
| 6 | Refh_Ps | *Refrigerant name, P, s* [kJ/kg.K] | *h* [kJ/kg] |
| 7 | Refs_Ph | *Refrigerant name, P, h* [kJ/kg] | *s* [kJ/kg.K] |

The first six functions in Table 4.10 use Equations (4.15) to (4.21) to determine property values. The last function is an inversion function that uses an iterative tool. All the functions apply for pressures and temperatures not higher than the critical values of the given refrigerant.

### 4.1.5.   Functions for psychrometic analyses (Psy)

Functions of this group are based on the property relations commonly used in psychrometric analyses [1]. Accordingly, the enthalpy ($h$) of an atmospheric mixture of dry air and water-vapour is calculated from:

$$h = c_{pa}T + \omega h_v \qquad (4.22)$$

Where, $c_{pa}$ is the specific heat at constant pressure for dry air ($c_{pa}$ =1.005 kJ/kg.K), $T$ is the temperature in °C, $\omega$ is the absolute humidity, and $h_v$ is the enthalpy of water vapour at the air temperature and partial-pressure of the water-vapour ($P_v$). The absolute humidity ($\omega$) in Equation (4.22) is determined from:

$$\omega = \frac{0.622 \, P_v}{P - P_v} \qquad (4.23)$$

Where, $P$ is the total pressure of the air-water-vapour mixture and $P_v$ is the partial pressure of water-vapour in the air. The partial pressure of water vapour itself is determined from:

$$P_v = \frac{\phi}{100} P_{sat} \qquad (4.24)$$

Where $\phi$ is the relative humidity and $P_{sat}$ is the saturation pressure of water at the given dry-bulb temperature. The enthalpy of saturated liquid water ($h_v$) in Equation (4.22) is approximated by the enthalpy of saturated water-vapour at the given temperature ($h_g$) and calculated as follows:

$$h_v \approx h_g = 2500.09 + 1.82T \qquad (4.25)$$

Property functions of this group also use the following relationship between the relative and specific humidities:

$$\phi = \frac{\omega P}{(0.622 + \omega)P_g} \qquad (4.26)$$

Table 4.11 lists 14 functions that are included in psychrometry group together with their input and output arguments. The letters in the function names have the following

meanings: h (specific enthalpy), Db (dry bulb), Dp (dew point), P (pressure), Rh (releative humidity), Sh (specific humidity), v (specfic volume), Wb (wet bulp).

Table 4.11. Function for psychrometry analyses given the pressure in kPa and two other properties

| # | Function | Input/unit | Output/unit |
|---|----------|-----------|-------------|
| 1 | Psyv_PDbRh | $P$, $T_{db}$ [ºC], $\phi$ [%] | $v$ [m³/kg] |
| 2 | PsyRh_PDbSh | $P$, $T_{db}$ [ºC], $\omega$[kg/kg] | $\phi$ [%] |
| 3 | PsyRh_PDbWb | $P$, $T_{db}$ [ºC], $T_{wb}$ [ºC] | $\phi$ [%] |
| 4 | PsySh_PDbRh | $P$, $T$ [ºC], $\phi$ [%] | $\omega$ [kg/kg] |
| 5 | PsySh_PDbWb | $P$, $T_{db}$ [ºC], $T_{wb}$ [ºC] | $\omega$ [kg/kg] |
| 6 | Psyh_PDbSh | $P$, $T$ [ºC], $\omega$[kg/kg] | $h$ [kJ/kg] |
| 7 | Psyh_PDbRh | $P$, $T$ [ºC], $\phi$ [%] | $h$ [kJ/kg] |
| 8 | PsyDp_PDbRh | $P$, $T_{db}$ [ºC], $\phi$ [%] | $T_{dp}$ [ºC] |
| 9 | PsyDp_PDbWb | $P$, $T_{db}$ [ºC], $T_{wb}$ [ºC] | $T_{dp}$ [ºC] |
| 10 | PsyDb_PRhSh | $P$, $\phi$ [%], $\omega$[kg/kg] | $T_{db}$ [ºC] |
| 11 | PsyDb_PhSh | $P$, $h$ [kJ/kg], $\omega$[kg/kg] | $T_{db}$ [ºC] |
| 12 | PsyWb_PDbRh | $P$, $T_{db}$ [ºC], $\phi$ [%] | $T_{wb}$ [ºC] |
| 13 | PsyWb_PDbSh | $P$, $T_{db}$ [ºC], $\omega$[kg/kg] | $T_{wb}$ [ºC] |
| 14 | PsyWb_PRhSh | $P$, $\phi$ [%], $\omega$[kg/kg] | $T_{wb}$ [ºC] |

This group also needs functions that determine certain properties of saturated liquid water, viz. $P_{sat}$, $T_{sat}$ and $h_l$. The group has its own functions that determine these properties for temperatures in the range 0 – 100ºC as met in common air-conditioning practice. The saturation pressure ($P_{sat}$) and saturation temperature ($T_{sat}$) are obtained from the following Antoine equations [6]:

$$P_{sat} = 0.1333 \times 10^{A - \frac{B}{C+T}}$$
(4.27)

$$T_{sat} = B / \left[ A - \log\left( \frac{P}{0.1333} \right) \right] - C$$
(4.28)

Where, $T$ is in ºC, $P$ in kPa and the three constants $A$, $B$, and $C$, respectively, have values of 8.07131, 1730.63, 233.426 for 1< $T$ < 100ºC and 8.14019, 1810.94, 244.485 for 99 < $T$ < 374ºC. Enthalpy of saturated liquid water at a given temperature is obtained from the following equation which was obtained by curve-fitting the data using Excel's trendline feature:

$$h_L = 0.146 + 4.184T$$
(4.29)

The functions in this group that use Equations (4.27), (2.18) and (4.29) make the group independent from the water group. Case 5 in Table 4.1 shows how the function

**PsyRh_PTSh** can be used to determine the relative humidity for air given its pressure, temperature, and specific humidity.

### 4.1.6.    Functions for air at standard atmospheric pressure (Air)

This group of functions provides the thermo-pysical properties of air at standard atmospheric pressure as required by fluid-flow and heat-transfer analyses. Based on the tabulated data given by Cengel and Ghajar [7], the functions use the linear interpolation function to determine the air desnity ($\rho$), specific heat ($c_p$), thermal conductivity ($k$), thermal diffusivity ($\alpha$), dynamic viscosity ($\mu$), kinematic viscosity ($\nu$) and Prandtl number ($Pr$) at temperatures in the range -150ºC to 2000ºC. Table 2.17 shows the names of the seven functions in this group with their corresponding output properties. Unlike the functions in the Gas-group, the temperature in this group is given in ºC. Also note that this group provides its own function for determining the specific-heat at constant pressure ($c_p$) for air.

Table 2.17. Properties of air at 1 atm pressure given the temperature in ºC

| # | Function | Output | Output unit |
|---|----------|--------|-------------|
| 1 | Airrho_T | Density ($\rho$) | kg/m$^3$ |
| 2 | Aircp_T | Specific heat ($c_p$) | J/Kg.ºC |
| 3 | Airk_T | Thermal conductivity ($k$) | W/m.ºC |
| 4 | Airdf_T | Thermal diffusivity ($\alpha$) | m$^2$/s |
| 5 | Airdv_T | Dynamic viscosity ($\mu$) | kg/m·s |
| 6 | Airkv_T | Kinematic viscosity ($\nu$) | m$^2$/s |
| 7 | AirPr_T | Prandtl number (Pr) | - |

### 4.2. Installation and use of Thermax property functions

Excel comes with a number of add-ins that have been developed for general use, including the Solver add-in. Active add-ins are automatically loaded when Excel starts up. Before Thermax can be recognised by Excel you have to save it in your computer and then install it. To do that, open the **Thermax.xla** file and then save it in your computer as an "**Excel Add-in**". Recent Excel versions locate all add-ins in a certain folder in the computer that depends on the version of Excel you are using and automatically directs you to the appropriate location. Save the add-in and **restart** Excel in order to activate it. Open a new Excel sheet and then do the following:

1.    Go to **File** and then click **Options**.
2.    Select **Add-Ins**. From the **Manage** ribbon at the bottom of the menu select **Excel Add-ins** and then press **Go**.
3.    The pull-down menu shown in Figure 4.2 will appear to you. To add **Thermax** to the add-ins menu, tick ($\sqrt{}$) the corresponding box.

If for any reason you saved the add-in in a location that is different from the default folder, then click on **Browse** and search for it in the destination folder and select it.

Having installed the add-in, its functions can be used in Excel's formulae just like its built-in functions. Thermax functions are named in a way that helps you to easily identify the required function via Excel's user-interface without having to memorise all the names of the functions and required input. The following sections illustrate two methods for using the add-in functions in Excel formulae.
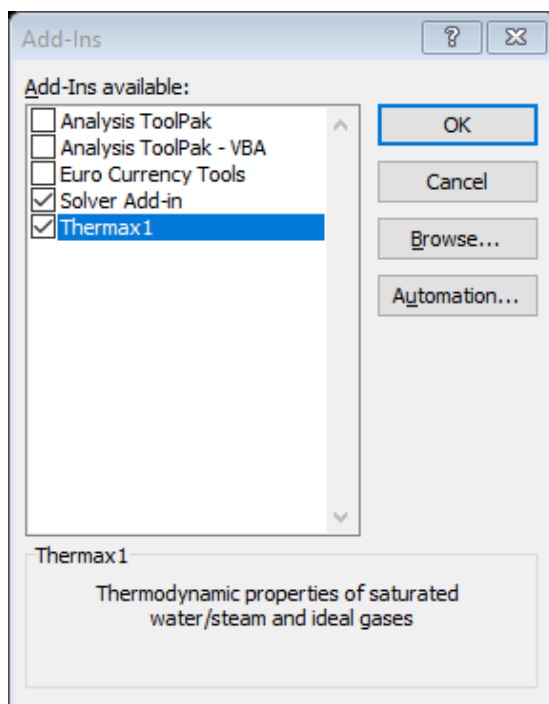


Figure 4.2. Adding Thermax to the menu of Excel add-ins

### 4.2.1.   Accessing Thermax functions via the Function Wizard

To illustrate this method, let us start a formula by entering the equal sign (=) in any cell (say cell B2). If you now press the *fx* button in the formula ribbon, the **Function Wizard** shown in Figure 4.3 will come up. The Function Wizard firstly lists the various categories of built-in functions provided by Excel, e.g. financial, mathematical, statistical, etc. Scroll down the list of function categories and select the **User-defined** functions. Then, all the functions provided by Thermax will be listed alphabetically as shown in Figure 4.4. The first function in the list, **Air_Data**, is the auxiliary function that stores the data for the seven thermophysical properties of air at standard atmospheric pressure. This function is called by other functions in the Air-group to obtain the values of these properties at the required temperature. To start using the add-in functions, select the function number 4 in the list, which is the function **Airk_T** that determines the thermal conductivity ($k$) of air at a given temperature. Upon pressing the OK button, the **Function Arguments** box shown in Figure 4.5 will appear to you.
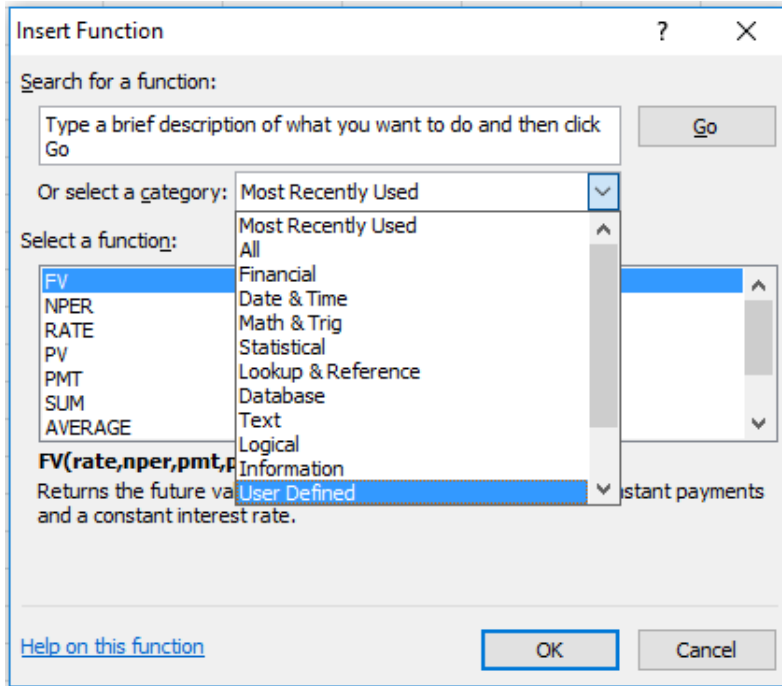
Figure 4.3. Finding the add-in user-defined functions in the Function Wizard
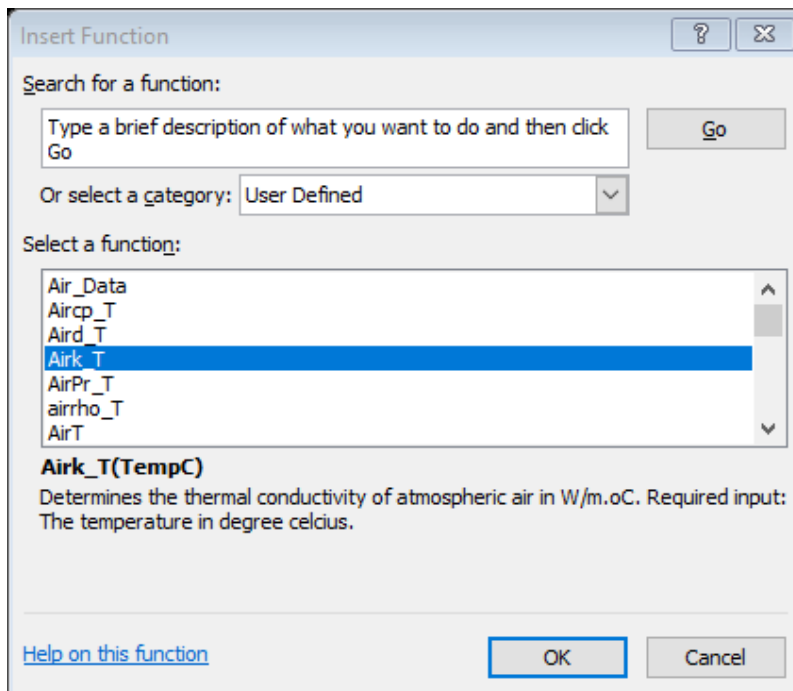


Figure 4.4. Thermax functions listed alphabetically in the User Defined category
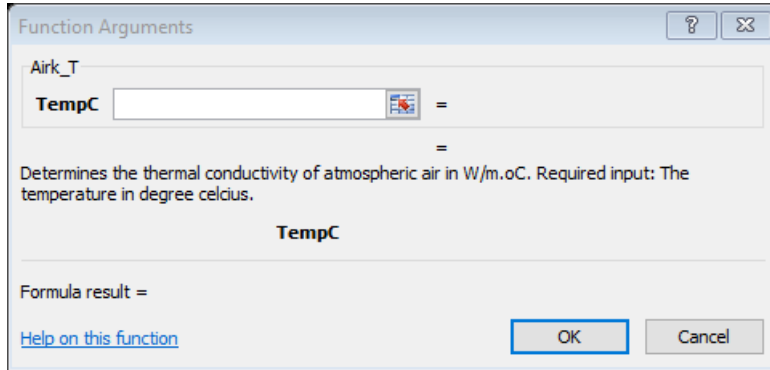
Figure 4.5. The Function Arguments box for the "Airk_T" function

Figure 4.5 shows that this function has one input parameter, which is the temperature in °C "TempC", and gives a brief description of its intended use. Let us use the function to determine the thermal conductivity for air at 25°C. Fill the form by entering thr value of the temperature, 25, as shown in Figure 4.6. Note that the formula ribbon now shows the formula in cell B2, which is "=Airk_T(25)". The form also shows the calculated value of $k$, which is 0.02551 W/m.°C. When you press the "OK" button, this value will appear in the cell B2. Check this value with the tabulated data and try other functions.
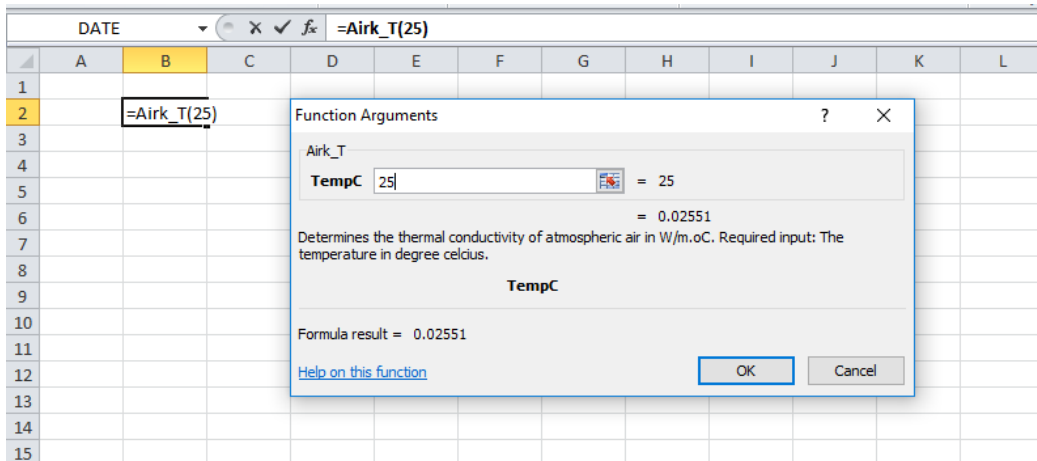


Figure 4.6. Using the function "Airk_T" to determine the thermal conductivity of atmospheric air at 25°C

### 4.2.2. Direct use of Thermax functions in Excel formulae

It is not necessary to follow the lengthy procedure described above by using the Function Wizard in order to use the add-in property functions. Instead, one can simply type in the formula that contains the property function in the required Excel cell. For illustration, suppose that we want to determine the ideal-gas temperature of carbon-dioxide ($CO_2$) at which the value of its enthalpy ($h$) equals 750 kJ/kg. Obviously, in this case, we need to use the function in the "Gas" group that determines the temperature for

a given enthalpy value. Therefore, we start an Excel formula by typing the equal sign "=" in any cell. As shown in Figure 4.7, as soon as we type the prefix "Gas" after the equal sign, the user-interface will display all Thermax functions in the ideal-gases group for us to select from.
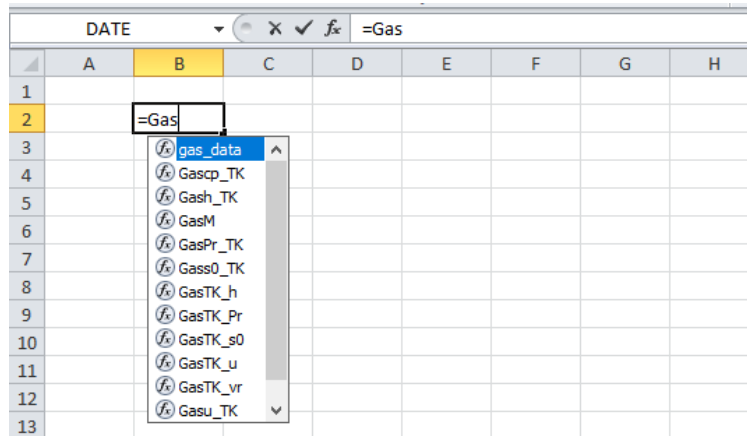


Figure 4.7. Excel UI showing all the functions in the Gas-group

Since the property we want to find is the temperature, which the Gas-group functions require in absolute degree, we continue the name of the function by adding the letters "TK" immediately after the three-letter prefix "Gas" followed by an underscore. As shown in Figure 4.8, the user-interface then lists only the five functions in Table 4.3 that determine the gas temperature given $h$, $P_r$, $s^0$, $u$, or $v_r$.
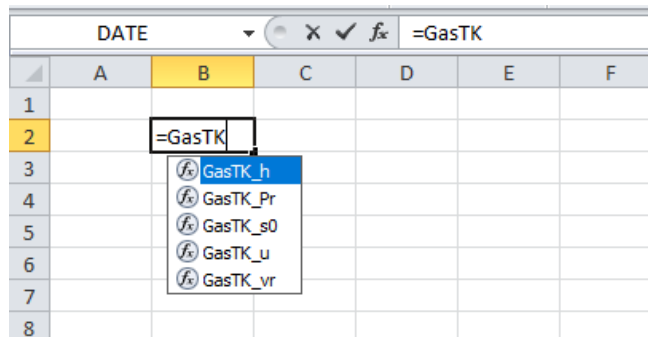


Figure 4.8. UI showing only the five functions that determine the temperature of an ideal gas from known values of $h$, $p_r$, $s^0$, $u$, or $v_r$

Since we want to find the temperature from a known value of enthalpy, we have to select the "**GasTK_h**" function. This function requires as input the name of the gas, which is "CO2", and the value of enthalpy, which is 750 kJ/kg, as shown in Figure 4.9. Pressing the "Enter" key after entering the required data, the function will calculate the corresponding temperature. As shown in Figure 4.10, the answer is 817.5544K.
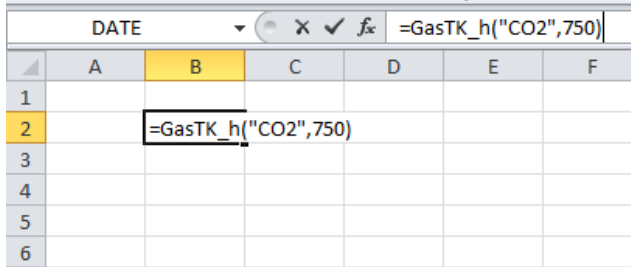
Figure 4.9. The required input for the GasTK_h function
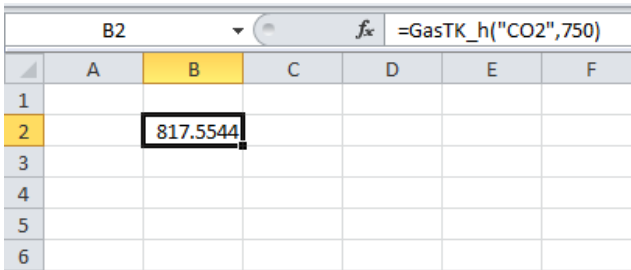


Figure 4.10. The temperature determined by the GasTK_h function

The following example shows how property functions in the Gas-group can be used for applying the exact, variable specific-heat method in thermodynamic analyses.

**Example 4.1. Thermodynamic analysis with the exact method**
Figure 4.11 shows a well-insulated piston–cylinder device that initially contains 0.1 m³ of air at 100 kPa, 330K. Fifty kJ of heat is transferred to the air causing the air to expand at constant pressure. Treating air as an ideal gas, determine the final temperature inside the cylinder using: (a) the approximate constant-specific heat method, with $c_p= 1.005$ kJ/kg.K, and (b) the exact variable specific-heat method with Thermax functions. Also, determine the error of the approximate method in determining the final temperature.
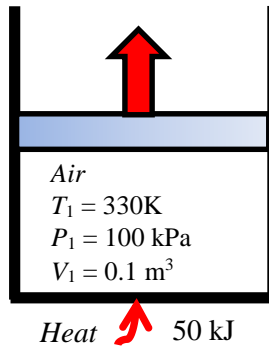


Figure 4.11. Schematic for Example 4.1

**Solution**

Using the approximate constant specific heat method, the final temperature is determined from:

$$T_2 = T_1 + \frac{Q}{m \times c_p}$$                                        (4.30)

Where $m$ is the mass of air in the piston-cylinder device, $c_p$ is the the specific heat at constant pressure, and $Q$ is the amount of heat added. The mass of air is calculated from the ideal-gas law:

$$m = \frac{P_1 V_1}{R \times T_1}$$                                        (4.31)

Where $R$ is the gas constant for air, $R = 0.287$ kJ/kg.K. Substituting for $P_1$, $V_1$, $R$ and $T_1$ in Equation (4.31), leads to $m = 0.105585$ kg and substituting for $m$ in Equation (4.30) gives $T_2 = 801.194$K.

To apply the exact method by using the add-in functions in the Gas group, we first determine the final enthalpy, $h_2$, by applying the first law of thermodynamics to the closed system:

$$h_2 = h_1 + Q / m$$                                        (4.32)

Once $h_2$ is found, the final temperature, $T_2$, can be determined by using the function **GasTK_h** in the Gas group.

Figure 4.12 shows the Excel sheet developed for this example. The given data are inserted at the left-hand side of the sheet together with the gas constant (R_) and specific heat (cp) for air. The calculations part is divided into two parts that determine the final temperature according to the approximate method and the exact method using the corresponding equations given above.



| | H8 | | | $f_x$ | =(T_2approx-T_2exact)/T_2exact*100 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K |
| 1 | | | | Approxiamte method | | | Exact method | | | | |
| 2 | T_1 | 330 | K | m | | 0.106 | =P_1*V_1/(R_*T_1) | h_1 | 330.027 | =Gash_TK("Air",T_1) | |
| 3 | P_1 | 100 | kPa | | | | | | | | |
| 4 | V_1 | 0.1 | m3 | T_2approx | 801.194 | =T_1+Q/(m*cp) | h_2 | 803.577 | =h_1+Q/m | |
| 5 | | | | | | | | | | | |
| 6 | Q | 50 | kJ | | | | T_2exact | 781.644 | =GasTK_h("Air",h_2) | |
| 7 | | | | | | | | | | | |
| 8 | R_ | 0.287 | kJ/kg.K | | | | Error | | 2.501 | =(T_2approx-T_2exact)/T_2exact*100 | |
| 9 | cp | 1.005 | kJ/kg.K | | | | | | | | |
| 10 | | | | | | | | | | | |

Figure 4.12. Excel sheet developed for Example 4.1

Figure 4.12 also reveals the formulae typed in these calculations. As the figure shows, the answer found by the approximate method for $T_2$ is 801.2K, while the exact method determines the final temperature as 781.6K. Thus, the approximate constant specific-heat method results in an error of 2.5%.

## 4.3. Numerical functions provided by Thermax

In addition to its five groups of property functions, Thermax provides two interpolation functions for tabulated data and a Newton-Raphson solver for nonlinear equations such as the Soave-Redlich-Kwong equation of state (Equation 1.39). This section shows how to use these additional functions by means of simple relevant examples.

### 4.3.1. The interpolation functions

The two interpolation functions, called **Interpol1** and **Interpol2**, perform linear and quadratic interpolation of tabulated data. These functions, which are listed in Appendix E, enable tabulated fluid properties to be used in parametric studies, iterative solutions, or optimisation analyses. For example, Figure 4.13 shows an Excel sheet in which three properties of engine oil; density ($\rho$), specific heat ($c_p$), and kinematic viscosity ($v$) are given at various temperatures. The interpolation functions can be used to calculate these properties at any temperature within the range of tabulated data. Suppose that we want to determine the viscosity at 90°C by using these functions.



| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| | H7 | | | $f_x$ | =Interpl1(T_,B5:B13,C5:C13,9) | | | | | |
| 1 | | Engine oil (unused) | | From Holman p 657 | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | ToC | ρ [kg/m3] | cp [kJ/kg.oC] | v [m2/s] | | | | | |
| 4 | | | | | | | T_ | 90 | oC | |
| 5 | | 0 | 899.12 | 1.796 | 0.00428 | | | | | |
| 6 | | 20 | 888.23 | 1.88 | 0.0009 | | | Interpl1 | Interpl2 | |
| 7 | | 40 | 876.05 | 1.964 | 0.00024 | | Density | 846.015 | 846.465 | |
| 8 | | 60 | 864.04 | 2.047 | 8.39E-05 | | Sp. Heat | 2.175 | 2.172 | |
| 9 | | 80 | 852.02 | 2.131 | 3.75E-05 | | Viscosity | 2.89E-05 | 2.92E-05 | |
| 10 | | 100 | 840.01 | 2.219 | 2.03E-05 | | | | | |
| 11 | | 120 | 828.96 | 2.307 | 1.24E-05 | | | | | |
| 12 | | 140 | 816.94 | 2.395 | 8.00E-06 | | | | | |
| 13 | | 160 | 805.89 | 2.483 | 5.60E-06 | | | | | |
| 14 | | | | | | | | | | |

Figure 4.13. Using the interpolation functions for tabulated data

Both **Interpol1** and **Interpol2** require four input arguments referred to as: X, XX, YY, and Ndata. Their meanings are as follows:

- X passes the value of the independent property (i.e. temperature) at which the value of the dependent property (i.e. viscosity) is to be determined.
- XX and YY are vectors that store the tabulated data (in this case, the temperature, B5:B13, and viscosity values, E5:E13,).
- Ndata passes the number of entries in the tabulated data. For the tabulated data shown in Figure 4.13, Ndata = 9.

The formula bar in Figure 4.13 reveals the formula in which the linear interpolation function **Interpol1** is used to determine the viscosity at 90ºC. Both the linear and quadratic interpolation functions return the warning message "**Given data is out of range**" if the temperature lies outside the tabulated range. The following example shows how the interpolation functions can be used in a thermofluid analysis.

### Example 4.2. Effect of oil temperature on the drag force over a flat plate

Engine oil at 20°C flows over the upper surface of a 5-m-long flat plate as shown in Figure 4.14. If the oil velocity is 2 m/s, determine the total drag force ($F_D$) per unit width of the entire plate. Also, plot the variation of $F_D$ with oil temperatures in the range 40 - 150ºC.
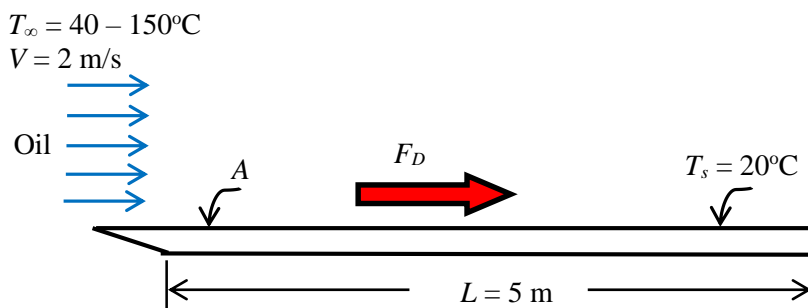


Figure 4.14. Schematic for Example 4.2 (adapted from Cengel and Ghajar [7])

### The analytical model

The drag force ($F_D$) over a flat plate is due to friction, which is given by [7]:

$$F_D = C_f A \rho V^2 / 2 \tag{4.34}$$

Where $C_f$ is the friction coefficient (not to be confused with the friction factor $f$), $A$ is the surface area of the plate, and $\rho$ and $V$ are the density and velocity of oil, respectively. The value of the friction coefficient depends on whether is flow is laminar or turbulent as indicated by the Reynolds number ($Re_L$). For the flow over a flat plate, the critical Reynolds number above which the flow becomes turbulent is about $5 \times 10^5$. For laminar flows, the friction coefficient is given by:

$$C_f = 1.328 \text{Re}_L^{-0.5} \qquad \text{Re}_L < 5 \times 10^5 \tag{4.35}$$

For turbulent flows, it is given by:

$$C_f = 0.074 \text{Re}_L^{-0.2} \qquad 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \tag{4.36}$$

Note that fluid properties are evaluated at the average temperature $(T_s+T_\infty)/2$. For example, if the oil temperature is 60°C, the oil viscosity and density are evaluated at 40°C.

**The Excel sheet**

Figure 4.15 shows the Excel sheet developed for this example. The data part shows the given information concerning the oil temperature (T_oil), plate temperature (T_plate), oil velocity (Velocity), and plate length (Length). Based on the specified oil temperature, the sheet calculates the average temperature (T_average) and the corresponding oil density (ρ_oil) and kinematic viscosity (v_oil). The sheet then calculates the Reynolds number (Re) and friction coefficient (Cf). The single final result, which is the drag force (F_D), is shown on the right-side of the sheet. The sheet reveals the formulae entered in each cell in the calculations part and the formula bar reveals that used for determining the drag force in cell I2.

| | | | | $f_x$ | =Cf*(Length*1)*ρ_oil*(Velocity^2)/2 | | |
|---|---|---|---|---|---|---|---|
| | B | C | D | E | F | G | H | I |
| 1 | | | | | | | | |
| 2 | T_oil | 40 | oC | T_average | 30 | =(T_oil+T_plate)/2 | F_D | 88.445 N |
| 3 | T_plate | 20 | oC | ρ_oil | 882.14 | =Interpl1(T_average,Data!I5:I13,Data!J5:J13,9) | | |
| 4 | Velocity | 2 | m/s | v_oil | 0.00057 | =Interpl1(T_average,Data!I5:I13,Data!L5:L13,9) | | |
| 5 | Length | 5 | m | | | | | |
| 6 | | | | Re | 17543.86 | =Velocity*Length/v_oil | | |
| 7 | | | | Cf | 0.010026 | =IF(Re<500000,1.328*Re^-0.5,0.074*Re^-0.2) | | |
| 8 | | | | | | | | |

Figure 4.15. The Excel sheet developed for Example 4.2

The friction coefficient is calculated depending on the value of the Reynolds number by using the following nested-If formula in cell F7:

Cf=IF(Re<500000,1.328*Re^-0.5,0.074*Re^-0.2)

The above formula determines whether to use Equation (4.35) or Equation (4.36) to calculate the friction coefficient depending on whether the flow is laminar or turbulent. Figure 4.15 shows the calculations at an oil temperature of 40°C, which is the initial oil temperature in the required range. At the average temperature, which is 30°C, the sheet uses the linear interpolation function **Iinterpol1** to determine the values of the oil

density and viscosity. At this temperature the calculated drag force, F_D, is 88.445 N. By inserting a different value for the oil temperature, say 50ºC, the sheet automatically updates its calculations. Figure 4.16 shows a plot of the drag force determined at different oil temperatures in the range 40ºC to 150ºC. The figure shows that the drag force drops exponentially with temperature, approaching a value of about 20 N.
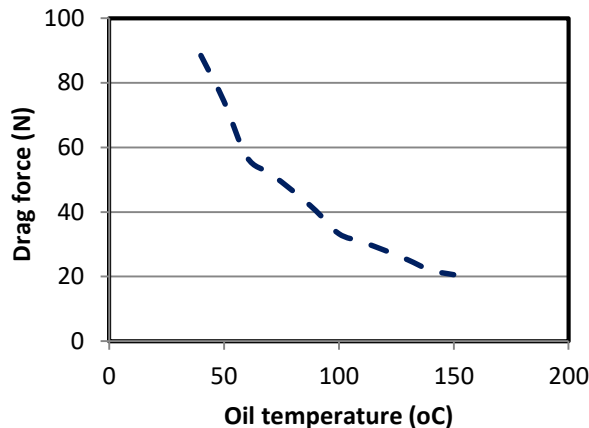


Figure 4.16. Variation of the drag force with oil temperature

The usefulness of the linear and quadratic interpolation functions in thermofluid analyses goes beyond that of determining fluid properties since these functions are also useful for interpolation of other types of tabulated data that is required by such analyses. Table 4.12 shows the cost per meter of galvanised-steel air-conditioning ducts for different diameters of the duct. In this case, the interpolation functions permit automatic determination of the duct diameter from the tabulated data, which is useful for optimisation analyses that involve the duct diameter.

Table 4.12. Cost of galvanised-steel air-conditioning ducts

| Duct diameter (m) | Cost per linear meter ($) |
|---|---|
| 0.10 | 9.0 |
| 0.15 | 11.5 |
| 0.20 | 14.5 |
| 0.25 | 17.0 |
| 0.30 | 22.5 |
| 0.35 | 29.0 |
| 0.40 | 34.0 |
| 0.45 | 40.0 |
| 0.50 | 50.0 |

### 4.3.2. The Newton-Raphson solver

Excel's cell formulae become too restrictive and inconvenient to use when the analytical model involves an implicit equation such as the following Colebrook equation that determines the Darcy friction factor ($f$) in a turbulent pipe flow:

$$\sqrt{\frac{1}{f}} = -2.0 \log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \tag{1.35}$$

Where $D$ is the pipe's diameter, $\varepsilon$ is the roughness of the pipe material, and Re is the Reynolds number. Since the equation involves $f$ on both sides, it needs to be solved iteratively. Determining $f$ from Equation (1.35) will be inconvenient particularly when dealing with type-2 or type-3 pipe-flow problems, which themselves require iterative solutions, since we have to deal with two nested iterations; an inner iteration to determine $f$ and an outer iteration to determine the pipe's diameter or the flow rate.

Another nonlinear equation that is even more difficult to fit in an Excel formula is the following Benedict-Webb-Rubin (BWR) equation of state [1]:

$$P = \frac{R_u T}{\tilde{v}} + \left(B_0 R_u T - A_0 - \frac{C_0}{T^2}\right)\frac{1}{\tilde{v}^2} + \frac{bR_u T - a}{\tilde{v}^3} + \frac{a\alpha}{\tilde{v}^6} + \frac{c}{\tilde{v}^3 T^2}\left(1 + \frac{\gamma}{\tilde{v}^2}\right)e^{-\gamma/\tilde{v}^2} \tag{4.37}$$

Where $R_u$ is the universal gas constant, $P$ is the absolute pressure, $T$ is the absolute temperature, and $\tilde{v}$ is the molar specific volume. The BWR equation, which is one of the most accurate equations of state, is implicit in $\tilde{v}$ and, therefore, cannot be used directly in an Excel formula to determine the molar specific volume.

Thermax provides a third VBA function for solving nonlinear equations such as the Colebrook equation and the BWR equation. The function, called NRM, requires four input arguments: fun$, x0, Var1, and Var2. The first argument, fun$, passes the name of the user-defined function developed separately for the particular nonlinear equation to be solved; e.g. the SRK equation or the BWR equation. The second argument, x0, is an initial guess for the dependent variable in the equation; i.e. $f$ in Equation (1.35) and $\tilde{v}$ in Equation (4.37). The third and fourth arguments, Var1 and Var2, are values of two independent variables in the nonlinear equation; i.e. $\varepsilon/D$ and Re in Equation (1.35) and $P$ and $T$ in Equation (4.37). To illustrate the use of the NRM function, let us use it to compare the specific volumes of carbon dioxide ($CO_2$) as determined by the ideal-gas law and the BWR equation of state at different temperatures. The following VBA function is that for the BWR equation of state taking the different constants in the equation as those of carbon dioxide ($CO_2$):

```
Function BWR(x, P, T)
Dim Ru, a, A0, b, B0, c, C0, alfa, gama
```

'Carbon dioxide, CO2
Ru = 8314
a = 13.86
A0 = 277.3
b = 0.00721
B0 = 0.04991
c = 1511000.0
C0 = 14040000.0
alfa = 0.0000847
gama = 0.00539
BWR = P - Ru * T / x + (B0 * Ru * T - A0 - C0 / T ^ 2) / (x ^ 2) + (b * Ru * T
- a) / x ^ 3 + a * alfa / x ^ 6 + c / (x ^ 3 * T ^ 2) * (1 + gama / x ^ 2) * Exp(-
gama / x ^ 2)
End Function

Figure 4.17 shows an Excel sheet that determines the specific volume of carbon dioxide at 0.2 MPa and temperatures in the range 273 – 373K by using the ideal gas law and by using the BWR equation of state and Table 4.13 reveals the formulae used in the sheet.

| | F3 | | | $f_x$ | =NRM("BWR",E3,P,D3) | | |
|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H |
| 1 | | | | | | | | |
| 2 | | | | T | v_ideal | v_BWR | error_v_ideal | |
| 3 | P_ | 200000 | Pa | 273 | 11.34861 | 11.29804143 | 0.44759 | |
| 4 | | | | 283 | 11.76431 | 11.71375945 | 0.43155 | |
| 5 | Ru | 8314 | J/kmol·K | 293 | 12.18001 | 12.12947677 | 0.41662 | |
| 6 | | | | 303 | 12.59571 | 12.54519338 | 0.40268 | |
| 7 | | | | 313 | 13.01141 | 12.96090929 | 0.38964 | |
| 8 | | | | 323 | 13.42711 | 13.37662453 | 0.37742 | |
| 9 | | | | 333 | 13.84281 | 13.79233913 | 0.36593 | |
| 10 | | | | 343 | 14.25851 | 14.2080531 | 0.35513 | |
| 11 | | | | 353 | 14.67421 | 14.62376648 | 0.34494 | |
| 12 | | | | 363 | 15.08991 | 15.0394793 | 0.33532 | |
| 13 | | | | 373 | 15.50561 | 15.45519157 | 0.32622 | |
| 14 | | | | | | | | |

Figure 4.17. Excel sheet for determining the specific volume by the ideal-gas law compared to the BWR equation of state

The formula bar in Figure 4.17 reveals the following Excel formula in cell F3 that uses the NRM solver to calculate $\tilde{v}$ at 273K:

D4=NRM("BWR",E3,P,D3)

Note that the first argument passed to the NRM function is the name of the UDF written for the nonlinear equation, which in this case is the BWR function. Also, note that the value determined by the ideal-gas law in cell E3 is used as an initial guess for the BWR equation. Figure 4.18 shows how the error of the ideal-gas law in estimating the specific volume increases at the low temperatures.

Table 4.13. The formulae used in the Excel sheet using the NRM solver

| T | v_ideal | v_BWR | error_v_ideal |
|---|---------|-------|---------------|
| 273 | =Ru*D3/P | =NRM("BWR",E3,P,D3) | =ABS(F3-E3)/F3*100 |
| 283 | =Ru*D4/P | =NRM("BWR",E4,P,D4) | =ABS(F4-E4)/F4*100 |
| 293 | =Ru*D5/P | =NRM("BWR",E5,P,D5) | =ABS(F5-E5)/F5*100 |
| 303 | =Ru*D6/P | =NRM("BWR",E6,P,D6) | =ABS(F6-E6)/F6*100 |
| 313 | =Ru*D7/P | =NRM("BWR",E7,P,D7) | =ABS(F7-E7)/F7*100 |
| 323 | =Ru*D8/P | =NRM("BWR",E8,P,D8) | =ABS(F8-E8)/F8*100 |
| 333 | =Ru*D9/P | =NRM("BWR",E9,P,D9) | =ABS(F9-E9)/F9*100 |
| 343 | =Ru*D10/P | =NRM("BWR",E10,P,D10) | =ABS(F10-E10)/F10*100 |
| 353 | =Ru*D11/P | =NRM("BWR",E11,P,D11) | =ABS(F11-E11)/F11*100 |
| 363 | =Ru*D12/P | =NRM("BWR",E12,P,D12) | =ABS(F12-E12)/F12*100 |
| 373 | =Ru*D13/P | =NRM("BWR",E13,P,D13) | =ABS(F13-E13)/F13*100 |
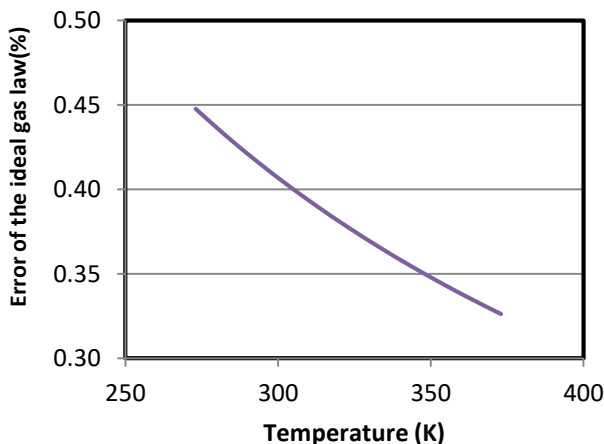


Figure 4.18. Errors in the specific volume estimations by the ideal-gas law compared to the BWR equation of state

## 4.4. Closure

The chapter introduced the Thermax add-in that provides property functions for 12 ideal gases, saturated water and superheated steam, the six refrigerants R134a, R22, R410A, R717, R718, and R744, humid air for psychrometric analyses, and air at

atmospheric pressure. Thermax property functions are named in a way that makes it easy to find the appropriate function via Excel's user-interface without having to memorise the names of all the functions. Thermax also provides two custom functions for data interpolation and a solver for nonlinear equations based on the Newton-Raphson method. While the interpolation functions are useful for including additional fluid properties or other tabulated data in a thermofluid analysis, the Newton-Raphson solver removes the restriction of Excel's formula when an optimisation analysis or an iterative solution involves a nonlinear equation.

## References

[1] Y. A. Cengel and M. A. Boles. *Thermodynamics an Engineering Approach*, McGraw-Hill, 7th Edition, 2007.

[2] American Society of Heating, Refrigeration and Air-Conditioning Engineers, (ASHRAE), Handbook of fundamentals, Atlanta, 2017.

[3] Wikipedia contributors, "Accentric factor", Wikipedia, The Free Incyclopedia, internet: https://en.wikipedia.org/wiki/Acentric_factor, (last acessed November 30, 2015.

[4] M.M. El-Awad, An Excel Add-In for Refrigerants Properties and its Use for optimising Multi-Stage Compression Refrigeration Cycles, the Electronic Journal of Spreadsheets in Education (eJSiE), Vol. 12, Iss. 1, Article 3, 2019.

[5] M.M. El-Awad, M.S. Al Nabhani, K.S. Al Hinai, A. Younis, Development and Validation of an Excel Add-In for determining the Properties of Various Refrigerants, Proceedings of the 1st National Conference on Applied Science, Engineering & Technology 2019, CASET – 2K19, 11th June 2019, Ibri, Oman.

[6] Wikipedia contributors, "Antoine equation", Wikipedia, The Free Incyclopedia, internet, https://en.wikipedia.org/wiki/Antoine_equation  (Last accessed November 23, 2015).

[7] Y. A. Cengel and A. J. Ghajar. *Heat and Mass Transfer: Fundamentals and Applications*. McGraw Hill, 2011.

## Exercises

1. Complete the following table by specifying the usage of the given Thermax function according to its name:

| # | Function | Output property | Input property/properties |
|---|----------|-----------------|---------------------------|
| 1 | Watvf_T | | |
| 2 | Gascp_TK | | |
| 3 | WatTs_P | | |
| 4 | GasTK_s0 | | |
| 5 | WatT_Ph | | |
| 6 | PsyDb_PRhSh | | |

2.  Name the appropriate Thermax function to determine the following tasks:

| # | Task | Thermax function |
|---|------|------------------|
| 1 | The enthalpy of saturated liquid water at 25°C | |
| 2 | The internal energy of oxygen at 300K | |
| 3 | The entropy of saturated steam at 10 kPa | |
| 4 | The absolute temperature at which the relative pressure of air is equal to 15 | |
| 5 | The saturation pressure for water at 150°C | |
| 6 | To determine the dry-bulb temperature of humid air at $P = 101$ kPa, $\phi = 50\%$, $\omega = 0.01$ | |

3.  Using Thermax functions, determine the following fluid properties: Also, determine the same properties by using the appropriate property tables and compare their values with those determined previously by using Thermax functions.

| # | Fluid | Property | Given properties |
|---|-------|----------|------------------|
| 1 | Water | Enthalpy ($h$) | 100 kPa, $x = 0.5$ |
| 2 | Air | Relative volume ($v_r$) | 700K |
| 3 | Steam | Entropy ($s$) | 100°C, $x = 1.0$ |
| 4 | Nitrogen | Temperature ($T$) | 350K |
| 5 | Liquid water | Enthalpy ($h$) | 10 kPa |
| 6 | Carbon dioxide | Specific heat ($c_p$) | 450K |
| 7 | Water | Specific volume ($v$) | $P=200$ kPa, $x = 0.7$ |
| 8 | Hydrogen | Temperature ($T$) | *Enthalpy h = 2000 kJ/kg* |

4.  Appendix A shows properties of air at atmospheric pressure and different temperatures. Enter the data into Excel and use the interpolation functions **Interpol1** or **Interpol2** to determine these properties at 360K and 500K.

5.  An equation of state that is simpler than the BWR equation, yet more accurate than the ideal-gas law, is the following Soave-Redlich-Kwong (SRK) equation of state:

$$P = \frac{R_u T}{\tilde{v} - b} - \frac{a\alpha}{\tilde{v}(\tilde{v} + b)} \qquad (1.39)$$

Where the constants $a$, $b$ and $\alpha$ are fluid-dependent. Following the method described in Section 4.5.2, develop a user-defined function that can be used with the Newton-Raphson Solver to calculate the specific volume for refrigerant R-12 from the SRK equation of state at a pressure of 200 kPa and temperatures in the range 0 - 50°C. On a suitable chart, compare values of the specific volume thus obtained by those calculated by the ideal-gas equation of state.

6.  Develop a user-defined function with VBA to determine the friction factor from the Colebrook equation, Equation (1.35), and use it with the NRM solver to determine the frictional losses ($h_f$) in a circular pipe in the following case:

$D = 25$ cm, $L = 150$ m, $V = 7$ m/s, $k_s = 0.045$ mm, carrying air at 20°C.

7.  Using the data for properties of air at atmospheric pressure, develop a user-defined function that can be used for determining the kinematic viscosity of air at any given temperature in the range 200 – 1000K.

   (a)  By using the trendline feature of Excel
   (b)  By using the linear-interpolation function (Interpl) listed in Appendix E.

# 5

# Iterative solutions

Thermofluid analyses frequently require iterative solutions that can be time-consuming and inaccurate if only hand calculation and property tables are used. This chapter shows how such analyses can be solved by using the iterative tools provided by Excel; the Goal Seek command and the Solver add-in. Chapters 2 and 3 illustrated the use of these tools for the solution of simple nonlinear equations. The advantage of Solver over Goal Seek is that it enables iterative solutions involving multiple variables and allows constraints to be applied to the solution. When the problem involves a nonlinear equation, such as the Colebrook equation, the problem becomes difficult to solve with only Excel's iterative tools. For such problems, the chapter shows how the Newton-Raphson solver provided by Thermax can be used to deal with the nonlinear equation while Goal Seek or Solver performs the main iteration.

## 5.1. Simple iterative solutions by using Goal Seek

Despite of its simplicity, the Goal Seek command can be used to solve the majority of thermofluid problem that require iterative solutions. This section presents three examples that demonstrate the use of Goal Seek for iterative solutions of fluid-dynamics, thermodynamics, and heat-transfer problems.

### 5.1.1.  Type-2 and type-3 pipe-flow analyses

Frictional head loss ($h_f$) in a pipe depends on a number of factors that characterise the pipe itself as well as the velocity and viscosity of the fluid being transported. For a straight pipe with no fittings carrying a viscous Newtonian and incompressible fluid, the Darcy-Weisbach equation states:

$$h_f = f \, \frac{L}{D} \frac{V^2}{2g} \tag{1.21}$$

where $f$ is the Darcy friction factor, $L$ the length of the pipe, $D$ its diameter, $V$ the fluid velocity, and $g$ the gravitational acceleration constant. The friction factor can be obtained from Equation (1.22) if the flow is laminar and from Equation (1.24) or (1.25) if it is turbulent.

Practical pipe-flow problems can be divided into three types [1]:

1. Type-1 problem - requires the determination of $h_f$ when both the pipe's diameter and fluid velocity (or flow rate) are known.
2. Type-2 problem - requires the flow rate for a specified $h_f$ and pipe diameter to be determined.
3. Type 3 problem - requires the pipe diameter to be determined for a given $h_f$ and flow rate.

Type-1 problems can be solved in a straight-forward manner by using Equation (1.21) to determine the friction head loss. However, both type-2 and type-3 problems require

iterative solutions because the Reynolds number and, therefore, the friction factor cannot be determined in advance. In the case of type-2 problems (i.e. unknown velocity), the iterative procedure can be avoided by using extended Moody diagrams that require the determination of the following dimensionless parameter [2]:

$$\text{Re} \, f^{\,0.5} = \frac{D^{1.5}}{\nu} \left( \frac{2gh_f}{L} \right)^{0.5}. \tag{5.1}$$

Apart from the inaccuracy of visual chart interpolation, the procedure is difficult to adopt in optimisation or parametric analyses. By using the Goal Seek command, both type-2 and type-3 problems can be solved more easily and accurately. The following example, which is based on Example 8.4 in Cengel and Cimbala [1], demonstrates this method by solving a type-3 problem.

### Example 5.1. Solution of type-3 pipe flow problems

Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct ($\varepsilon$=0.000045 m) as shown in Figure 5.1 at a rate, $\dot{V}$, of 0.35 m³/s. If the head loss in the duct is not to exceed 20 m, determine the smallest required diameter for the duct.
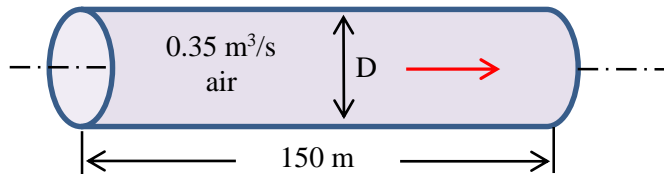


Figure 5.1. Schematic for Example 5.1 (adapted from Cengel and Cimbala [1])

### Solution:

The problem can be solved by calculating the friction head loss at different diameters of the duct and then selecting the diameter that gives the required head loss of 20 m. The iterative solution proceeds as follows:

1.   Select a diameter for the inner pipe ($D$).
2.   Calculate the flow areas of the pipe and velocity of the hot air, $V = \dot{V}/A$.
3.   Calculate the Reynolds number in the pipe, $\text{Re} = VD/\nu$.
4.   Calculate the friction factor ($f$) from Equation (1.22), (1.24), or (1.25).
5.   Calculate the friction head loss ($h_f$) from Equation (1.21).
6.   If $h_f \neq 20$ m, repeat steps 1 to 5.

Figure 5.2 shows the Excel sheet developed for this example which is divided into three parts: (i) problem data (ii) calculations, and (iii) results. The data part shows the

information given in the question. The value of the kinematic viscosity of air at 35°C ($v$ = 1.655x10$^{-5}$ m²/s) was obtained from Cengel and Cimbala [1] and fixed throughout the calculations. Cell-labelling is applied in the formulae and Figure 5.2 reveals the formulae used in each cell of the calculations part.
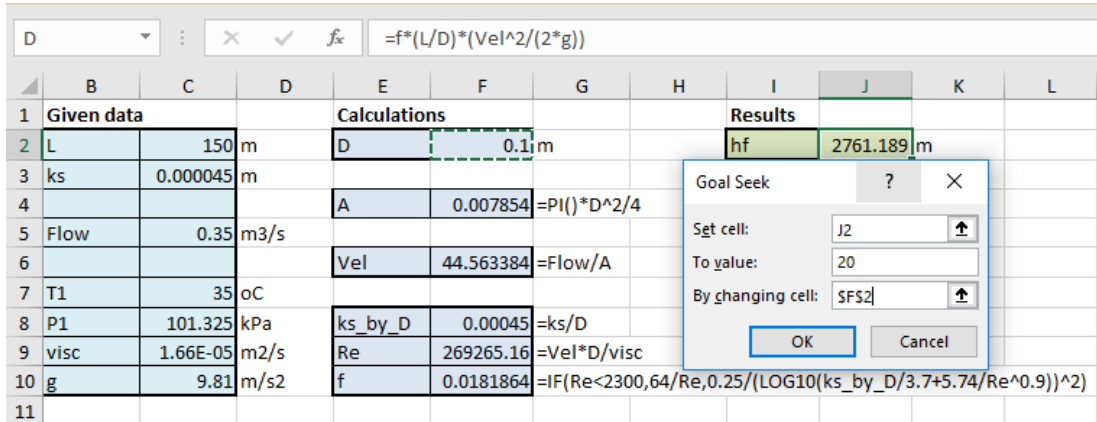


Figure 5.2. Excel sheet and Goal Seek set-up for Example 5.1

As Figure 5.2 shows, for an assumed duct diameter of 0.1 m the friction head loss exceeds 2761 m. Figure 5.2 also shows the completed Goal Seek dialog box that requires Goal Seek to change the diameter in cell F2 and iterate until the friction head loss in cell J2 attains the required value of 20 m. Figure 5.3 shows the answer found by Goal Seek, which is $D \geq 0.27$ m. This answer agrees with that given by Cengel and Cimbala [1]. A similar procedure can be used to solve type-2 flow problems by iterating over the flow rate instead of the diameter.
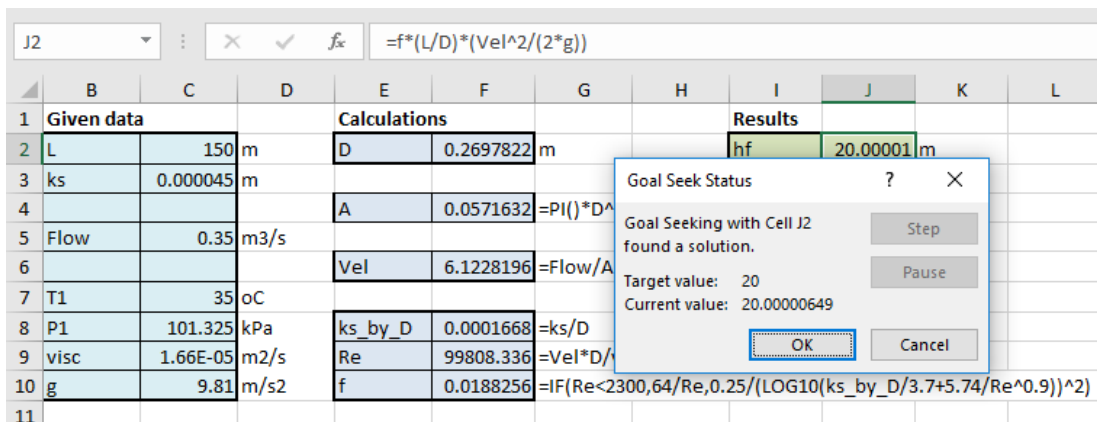


Figure 5.3. Goal Seek solution for Example 5.1

### 5.1.2.   Thermodynamic analyses involving ideal-gas mixtures

Thermodynamic analyses apply many simplifications and idealisations without which most of these analyses, if not all, would require iterative solutions. For example, a

common simplificationan in these analyses is the use of constant specific heats for gases. Example 4.1 showed how the property function provided by Thermax enable the exact method of analysis with variable specific heats to be used instead of the approximate method. Another commonly used approximation in thermodynamic analyses is to treat air as a pure gas even though it is mainly a mixture of nitrogen and oxygen gases. Computer-aided analyses with property functions enable a more realistic approximation to be used by treating air as a mixture of gases instead of a single gas. However, this method involves an iterative solution when the temperature of the gas mixture is not known but has to be determined. The following example shows how the method can be applied by using the Goal Seek command.

**Example 5.2. Constant-pressure expansion of a heated oxygen-nitrogen mixture**
Figure 5.4 shows a piston–cylinder device that initially contains a mixture of oxygen and nitrogen with 21% oxygen and 79% nitrogen by volume. Initially at 100 kPa, 330K, the mixture occupies 0.1 m³. Fifty kJ of heat is then transferred to the mixture causing it to expand at constant pressure. Treating oxygen and nitrogen as ideal gases, determine the final temperature of the mixture inside the cylinder.
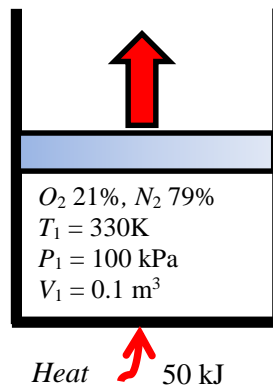


$O_2$ 21%, $N_2$ 79%
$T_1 = 330K$
$P_1 = 100$ kPa
$V_1 = 0.1$ m³

*Heat* 50 kJ

Figure 5.4. Schematic diagram for Example 5.2

**Solution**
This problem is basically the same as that given in Example 4.1, but the air is now treated as a mixture of $O_2$ and $N_2$ and not as a single pure gas. The solution procedure also applies the first-law of thermodynamics, but the law is now applied as follows:

$$Q = m_{O2}\left(h_{2\_O2} - h_{1\_O2}\right) + m_{N2}\left(h_{2\_N2} - h_{1\_N2}\right) \tag{5.2}$$

Where $Q$ is the amount of heat added, $m_{O2}$ and $m_{N2}$ are the masses of oxygen and nitrogen in the device, $h_{1\_O2}$ and $h_{2\_O2}$ are enthalpies of oxygen at the initial and final temperatures, respectively, and $h_{1\_N2}$ and $h_{2\_N2}$ are the corresponding enthalpies for nitrogen. The enthalpies of $O_2$ and $N_2$ in Equation (5.2) can be determined by using the

relevant Thermax function, **Gash_TK**, and the masses $m_{O2}$ and $m_{N2}$ can be obtained from the ideal-gas law using the corresponding partial pressures as follows:

$$m_{O2} = \frac{0.21 P_1 V_1}{R_{O2} T_1}$$                                   (5.3)

$$m_{N2} = \frac{0.79 P_1 V_1}{R_{N2} T_1}$$                                   (5.4)

Where $R_{O2}$ and $R_{N2}$ are the gas constants for oxygen and nitrogen, which are 0.2598 kJ/kg.K and 0.2968 kJ/kg.K, respectively. The correct value of the final temperature is that at which the amount of heat added ($Q$) is the same as the given value, which is 50 kJ. This can be determined by using Goal Seek.

Figure 5.5 shows the Excel sheet developed for this example. The data part includes the initial pressure, temperature, and volume of the gas mixture together with the mole fractions and gas constants of oxygen and nitrogen. The initial partial pressures of oxygen and nitrogen, P1_O2 and P1_N2, are calculated from the total initial pressure (P_1) and the respective volume fractions, y_O2 and y_N2, as shown in cells E2 and E3, respectively. The sheet then determines the masses of the two gases in the mixture (m_O2 and m_N2) in cells E5 and E6, respectively, and the total mass (m_total) in cell E8. Note that the value of the total mass, 0.1052 kg, is slightly different from that obtained in Example 4.1, which is 0.106 kg.
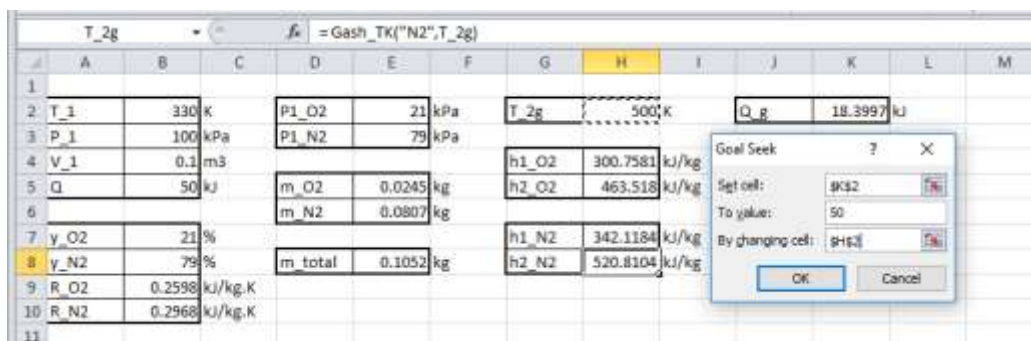


Figure 5.5. The Excel sheet developed for Example 5.2 by using Thermax functions

The calculations start with a guessed value for the final temperature, T_2g, which is 500K. Equation (5.2) is then used to determine the total amount heat added in the process (Q_g). The initial and final enthalpies of oxygen and nitrogen are determined by using the function **Gash_TK** at the corresponding temperatures. Note that the sheet determines the total amout of heat as 18.4 kJ, which is less than the actual values of 50 kJ. To find the appropriate final temperature, the guessed temperature T_2g has to be adjusted by Goal Seek so that the value of Q_2g equals 50 kJ. Figure 5.5 shows the required Goal Seek set-up and Figure 5.6 shows the solution obtained by Goal Seek, which  is 780.444 K. The value determined in Example 4.1 for $T_2$ by using the exact

method was 781.6K, while the value determined by using the approximate method was 801.2K. Although these results confirm the the accuracy of treating air as a single pure gas and using the exact method of analysis, the deviation from the present model and that of Exampe 4.1 is expected to increase as the amout of heating increases.
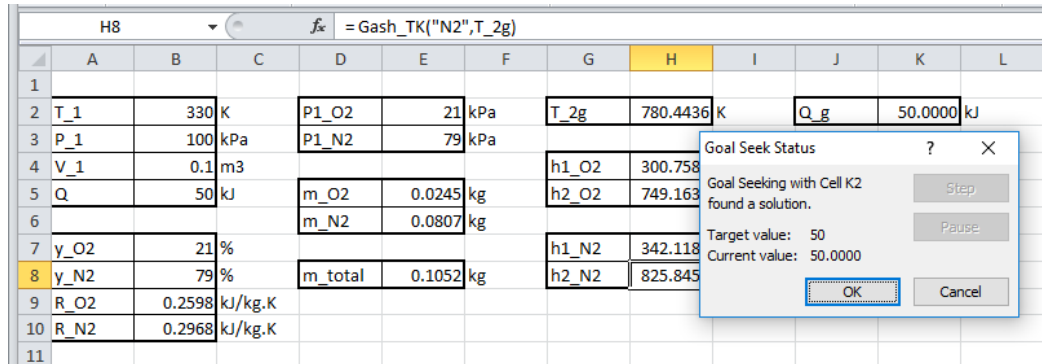


Figure 5.6. Goal Seek solution for Exampe 6.2 by using Thermax functions

### 5.1.3.  Convection heat-transfer analyses

Many types of heat-transfer problems require iterative solutions, particularly those of convection heat-transfer. Although dimensional-analysis techniques are used in some cases to prepare specific charts that enable the iterative solutions to be avoided, visual interpolations of the charts can be inaccurate. The following example shows how Excel's Goal Seek command provides an alternative solution method for such analyses that is easier and more accurate. The example is based on Example 10.1 in Holman [3].

### Example 5.3. Overall heat-transfer coefficient for pipe in air

Hot water at 98°C flows through a 2-in schedule 40 horizontal steel pipe ($k$ =54 W/m·°C) and is exposed to atmospheric air at 20°C as shown in Figure 5.7. The water velocity is 25 cm/s. Calculate:

> (a) the rate of heat-transfer through the pipe,
> (b) the temperatures at the inside and outside surfaces of the pipe, and
> (c) the overall heat-transfer coefficient based on the outer area of the pipe.

Properties of water at 98°C are: $\rho = 960$ kg/m³, $\mu = 2.82$ x $10^{-4}$ kg/m.s, $k = 0.68$ W/m.°C, Pr = 1.76. For a 2-in schedule 40 pipe, $D_i = 5.25$ cm and $D_o = 6.033$ cm.

### The analytical model

The rate of heat-transfer through the pipe is given by:

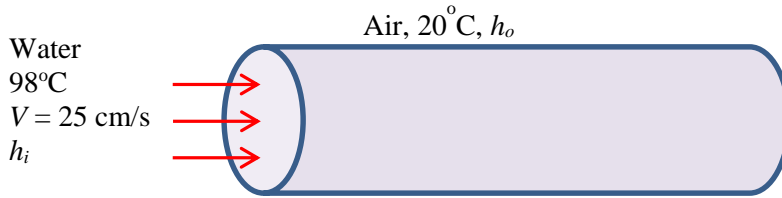$$Q = \left(T_w - T_\infty\right)/ R_{th} \tag{5.5}$$

Figure 5.7. Schematic for Example 5.3

Where $T_w$ and $T_\infty$ are the water temperature and air-temperature, respectively, and $R_{th}$ is the total thermal resistance to heat-transfer that consists of the thermal resistances due to heat-transfer by convection inside the pipe ($R_i$), by conduction through the steel pipe ($R_p$), and by convection from outside the pipe ($R_o$). The three resistances are given by:

$$R_i = \frac{1}{A_i h_i} \tag{5.6}$$

$$R_p = \frac{\ln(D_i / D_o)}{2\pi k} \tag{5.7}$$

$$R_o = \frac{1}{A_o h_o} \tag{5.8}$$

Where $A_i$ and $A_o$ are the inside and outside areas of the pipe and $h_i$ and $h_o$ are the corresponding heat-transfer coefficiens. The internal heat-transfer coefficien $h_i$ is determined from the corresponding Nusselt number ($Nu$):

$$h_i = Nu \frac{k_w}{D_i} \tag{5.9}$$

Where, $k_w$ is the thermal conductivity of water. The Nusselt number is determined from emprirical equations depending on the type of the flow, i.e. natural or forced, laminar or turbulent. For the turbulent forced internal flow (to be confirmed later), $Nu$ is obtained from the Dittus-Boelter equation, Equation (1.31), with $n = 0.4$ following Holman [3]:

$$Nu = 0.023 \, \mathrm{Re}^{0.8} \, \mathrm{Pr}^{0.4} \tag{5.10}$$

Where, Re and Pr are the Reynolds number and Prandtl number, respectively.

For the external flow, Holman [3] used the following simplified equation for free laminar convection from a horizontal pipe to air at atmospheric pressure:

$$h_o = 1.32\left(\frac{\Delta T}{D}\right)^{1/4} = 1.32\left(\frac{T_o - T_\infty}{D_o}\right)^{1/4} \tag{5.11}$$

Both $R_i$ and $R_p$ can be determined directly from the given data, but $R_o$ depends on $h_o$ which cannot be determined directly because $T_o$ is not known. Therefore, the problem has to be solved by adopting an iterative approach. A value for $T_o$ is assumed based on which $h_o$ is determined and, consequently, $Q$. The value of $Q$ thus obtained can be used to calculate corresponding values for $T_i$ and $T_o$ from:

$$T_i = T_w - Q.R_i \tag{5.12}$$

$$T_o = T_i + Q.R_p \tag{5.13}$$

If the guessed value for $T_o$ is correct, it will be the same as that obtained from Equation (5.13). Otherwise, a new guess for $T_o$ has to be made repeatedly until this condition is met. Once this is achieved, the overall heat-transfer coefficient ($U_o$) based on the outside area ($A_o$) can be obtained from:

$$U_o = \frac{1}{A_o\left(R_i + R_o + R_p\right)} \tag{5.14}$$

**Solution with Excel**
The Excel sheet developed for this example is shown in Figure 5.8. The given information about the pipe, water, and air properties are entered in the data part on the left side of the sheet. The cells are labelled and the figure shows the formulae used in the calculations. The calculations part at the central part of the sheet starts with a guessesd value for the pipe's outside temperature (T_og) of 50ºC. Based on this value, the sheet determines the outside heat-transfer coefficient (h_o) from Equation (5.11) and the thermal resistance associated with it (R_o) from Equation (5.8). Following the analytical model described above, the sheet determines the three thermal resistances (R_i, R_p, and R_o), and then calculates the rate of heat-transfer (Q), inside temperature (T_i), outside temperature (T_o), and overall-heat transfer coefficient (U).

As Figure 5.8 shows, the value of T_o calculated from Equation (5.13) is 97.876ºC, which is different from the initially guessed value (T_og = 50ºC). The formula bar reveals the formula entered in cell H12 that calculates the difference between the calculated exit temperature (T_o) and the guessd value (T_og) as a fraction of T_og. The exit temperature that makes the difference vanishes can be found by using the Goal Seek command and Figure 5.8 shows the required set-up. The solution found by Goal Seek is shown in Figure 5.9. Table 5.1 that comapres the presents results with those given by Holman [3] confirms the accuracy of the iterative solution with Goal Seek.
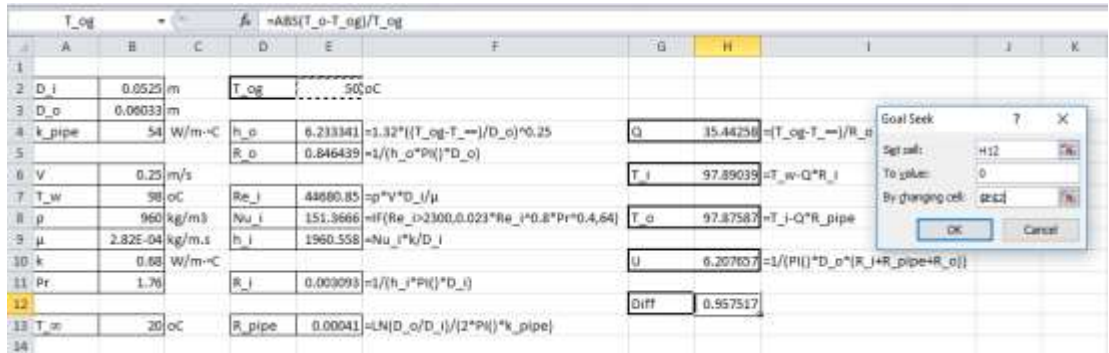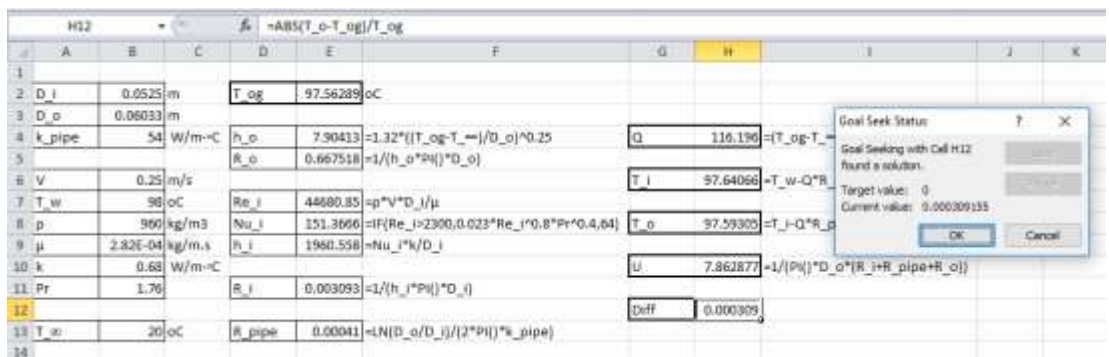
Figure 5.8. Excel sheet developed for Example 5.3



Figure 5.9. Solution obtained by Goal Seek for Example 5.3

Table 5.1. Comparison of the presnt solution with that given by Holman [3]

|  | Present solution | Holman [3] |
|---|---|---|
| $T_i$ | 97.64 | 97.65 |
| $T_o$ | 97.59 | 97.6 |
| $h_i$ | 1960.56 | 1961.0 |
| $h_o$ | 7.90 | 7.91 |
| $U_o$ | 7.86 | 7.87 |

## 5.2. Constrained iterative solutions with Solver

The previous sections demonstrated the adequacy of Excel's Goal Seek command for obtaining iterative solutions of different types of thermofluid problems. However, Solver offers greater flexibility than Goal Seek by allowing multiple changeable cells and by allowing constraints to be specified in the iterative process. This section illustrates the need for these additional features in thermofluid analyses by means of two examples from the areas of fluid dynamics and thermodynamics. The first example, which is basically a type-2 pipe flow problem, is based on a similar example given by

Schumack [4]. The second example demonstrates the usefulness of Solver for a constrained iterative solution of Example 5.4 given by Cengel and Boles [5].

**Example 5.4.  Determining the maximum water flow rate to avoid cavitation**
Water at 20ºC ($\gamma$ = 9810 N/m³ and $v$ =1.006x10⁻⁶ m²/s) is to be pumped from a large reservoir via a pump-pipe system as shown in Figure 5.10. The pump is positioned vertically at a level which is 9 m above the surface of the reservoir and horizontally at 1 m from the vertical section of the pipe. The pipe is made of commercial steel pipe ($\varepsilon$ = 0.046 mm) and has a 2″ nominal diameter.
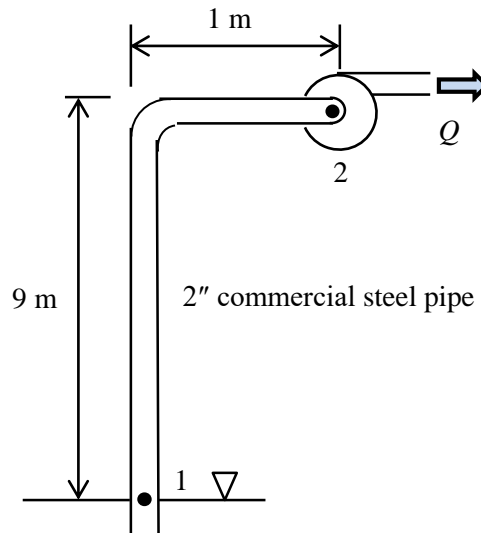


Figure 5.10.  Schematic for the pump-pipe system in Example 5.4

Determine the maximum allowable water flow rate ($Q$) that satisfies the following restrictions:

1. To avoid cavitation, the pressure at the pump inlet must be greater than the saturation pressure of water at 20ºC, which is 2.338 kPa.
2. For economic considerations, the water velocity ($V$) is to be in the range 1.4-2.8 m.

**The analytical model**
The energy equation between the pipe inlet (point 1) and the pump inlet (point 2) is:

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_f \qquad (5.15)$$

Where $\gamma$ stands for the specific weight of water, $z$ for the elevation, $V$ for the water velocity, $g$ for the gravitational acceleration, and $h_f$ for the friction loss in the pipe. For

suction from a large reservoir $V_1 = 0$. Taking point 1 as a reference, i.e. $z_1 = 0$, and noting that the water velocity in the pipe is uniform, i.e. $V_2 = V$, the energy equation becomes:

$$p_2 = \gamma \left( \frac{p_1}{\gamma} - z_2 - \frac{V^2}{2g} - h_f \right) \qquad (5.16)$$

The velocity $V$ is related to the pipe diameter ($D$) and water flow rate ($Q$) as follows:

$$V = 4Q / \pi D^2 \qquad (5.17)$$

Neglecting minor losses, the friction loss can be calculated from the Darcy-Weisbach equation which needs an auxiliary formula to determine the friction factor ($f$) as described in Chapter 1 for both laminar and turbulent flows.

**Solution with Excel**
Figure 5.11 shows the Excel sheet developed for this example. The data part on the left side stores the problem data such as the diameter, roughness, and length of the pipe, etc. The central part stores a guessed value for the water velocity (V=1.0 m/s) in cell E2. Based on the guessed water velocity, this part also performs the necessary calculations according to the analytical model given above. Figure 5.11 reveals the formulae used in these calculations. Note that an IF statement is used to calculate the friction factor (f) depending on the value of the Reynolds number (Re). Cell E6 calculates the friction loss (hf). Based on the calculated value of friction loss, the pressure at point 2 (P_2) is calculated from Equation (5.16) and stored in cell E7.

| | Q | | | $f_x$ | =0.25*PI()*D^2*V | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K |
| 1 | | | | | | | | | | | |
| 2 | D | 0.05252 m | | V | 1.00 m/s | | | Q | 0.0021664 m3/s | | |
| 3 | ε | 0.000046 m | | | | | | | | | |
| 4 | L | 10 m | | Re | 52520 =V*D/v | | | | | | |
| 5 | Z_2 | 9 m | | f | 0.023651 =IF(Re<2000,64/Re,0.25/(LOG10(ε/3.7/D+5.74/Re^0.9))^2) | | | | | | |
| 6 | P_1 | 100 kPa | | hf | 0.229523 =f*L/D*V^2/(2*gc) | | | | | | |
| 7 | v | 1.00E-06 Pa.s | | P_2 | 8.958382 =(P_1*1000/γ-Z_2-V^2/(2*gc)-hf)*γ/1000 | | | | | | |
| 8 | γ | 9810 N/m3 | | | | | | | | | |
| 9 | gc | 9.81 m/s2 | | | | | | | | | |
| 10 | | | | | | | | | | | |

Figure 5.11.  Excel sheet developed for Example 5.4

The right side of the sheet contains the single cell I2 that determines the flow rate (Q). The formula in this cell is shown in the formula bar. Based on the assumed water velocity of 1.0 m/s, the calculated values of hf and P_2 are 0.2295 m and 8.958 kPa, respectively. Since the pressure at point 2 is higher than the minimum design level of 2.338 kPa, while the water velocity (V) is less than the minimum economic value of 1.4

m, there is room to increase the flow rate. The task can be left to Solver and Figure 5.12 shows the required set-up.
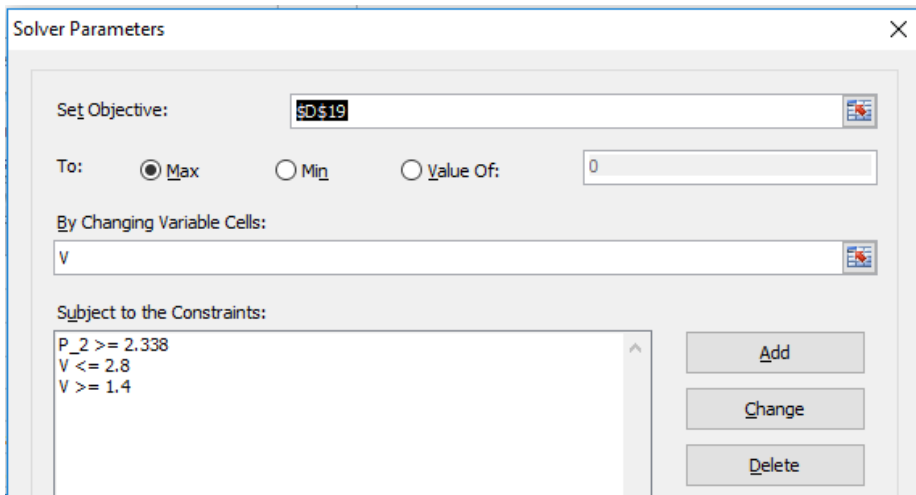


Figure 5.12. Solver parameters dialog box for Example 5.4

The above set-up requires Solver to maximise the value of the flow rate Q while satisfying the three constraints shown in the figure. The first constraint on the iterative solution requires the value of P_2 in cell E7 to be higher than or equal to the specified value of 2.338 kPa, which is the minimum pressure level required to prevent cavitation. The two other constraints are to satisfy the limits on the water velocity imposed by economic limits, i.e. $1.4 \text{ m} \leq V \leq 2.8 \text{ m}$. Pressing the "Solve" button will trigger Solver to search for the solution. The solution found by Solver using the GRG Nonlinear method is shown in Figure 5.13.



Figure 5.13. Solver solution for Example 5.4

The value determined by Solver for the water velocity is 1.90 m/s. Note that this velocity lies within the limits imposed by the economic constraint. The corresponding flow rate, which is 0.00413 m³/s, is the maximum flow rate to be recommended.

**Example 5.5. Restrained expansion of air inside a piston-cylinder device**

Figure 5.14 shows a piston–cylinder device that initially contains 0.05 m³ of air at 200 kPa, 317K. At this state, a linear spring is touching the piston but exerting no force on it. Now, 72.7 kJ of heat is transferred to the air, causing the piston to rise and compress the spring that has a constant of 150 kN/m.
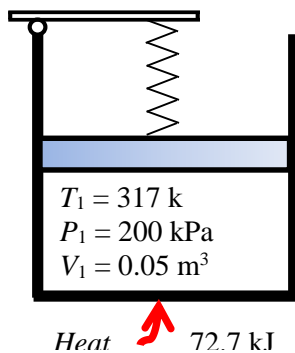


Figure 5.14. Schematic and pressure-volume diagrams for Example 5.5 (adopted from Cengel and Boles [5])

If the cross-sectional area of the piston is 0.25 m², determine the final volume, pressure, and temperature of the air inside the cylinder. Air can be treated as an ideal gas with a specific heat at constant volume ($c_v$) that varies linearly with the temperature according to the formula:

$$c_v = 0.645 + 0.0002T \tag{5.18}$$

Where $T$ is the temperature in K and $c_v$ is in kJ/kg.K.

**Solution**

Unlike the present example, Cengel and Boles [5] specified the final volume to be 0.1 m³ instead of the amount of heat added. When the final volume (or final pressure) is given, the problem can be solved in a straightforward manner without iteration by using the ideal-gas law. However, in the present example $T_2$, $V_2$, and $P_2$ at the final state all depend on the amount of heat added. The specific value of 72.7 kJ given in this example has been chosen so that the final volume will be 0.1 m³ as specified by Cengel and Boles [5]. Therefore, the final pressure on the piston and the total work should be the same as those obtained by Cengel and Boles [5] even though the formulations of the two examples are different.

**The analytical model**

Like Example 5.2, this problem can be solved by using the first-law of thermodynamics together with the ideal-gas law. However, the addition of the linear spring in this example introduces a new factor, which is the variation of pressure with air expansion.

As in Example 5.2, the variation of the specific heat with temperature makes it necessary to adopt an iterative solution approach. Since the iterative process here involves not only the temperature but also the volume (or pressure), the Goal Seek command is inadequate. Therefore, this example requires Solver since it allows the iterative procedure to start by assumed values for both the final temperature ($T_2^*$) and the final volume ($V_2^*$). The final pressure $P_2$ is given by:

$$P_2 = P_1 - \frac{k\Delta x}{A} \tag{5.19}$$

Where $A$ is the base area of the piston and $\Delta x$ is the reduction in the spring's length given by:

$$\Delta x = \frac{V_2^* - V_1}{A} \tag{5.20}$$

The total work ($W$), i.e. the summation of the air expansion work and the work done against the spring, can now be obtained from:

$$W = \frac{(P_1 + P_2)}{2}\left(V_2^* - V_1\right) \tag{5.21}$$

The final temperature can be determined by applying the first-law of thermodynamics to the piston-cylinder device as a closed system:

$$Q - W = m(u_2 - u_1) = mc_v(T_2 - T_1) \tag{5.22}$$

Where $Q$ is the amount of heat added, $u$ is the internal energy, $m$ is the mass of air inside the cylinder, and $\bar{c}_v$ is the average specific heat of air at constant volume. The mass and specific heat of air can be obtained from:

$$m = P_1 V_1 / RT_1 \tag{5.23}$$

$$\bar{c}_v = 0.6 + 0.0002\left(T_1 + T_2^*\right)/2 \tag{5.24}$$

Rearranging Equation (5.22), the final temperature $T_2$ is given by:

$$T_2 = T_1 + \frac{Q - W}{m\bar{c}_v} \tag{5.25}$$

Using the values obtained for $T_2$ and $P_2$, the final volume $V_2$ can be determined from the ideal-gas law:

$$V_2 = mRT_2 / P_2 \tag{5.26}$$

If the initially guessed volumes of $T_2^*$ and $V_2^*$ are correct, then they will be the same as $T_2$ and $V_2$ obtained from Equation (5.23) and Equation (5.25), respectively. Otherwise, new values for $T_2^*$ and $V_2^*$ have to be used until the difference between the calculated and guessed values becomes negligibly small.

**Solution with Excel**

Figure 5.15 shows the Excel sheet developed for this example. The left side of the sheet accommodates the problem data. The calculations part start by an assumed values for the final temperature (T_2g = 500K) and final volume V_2g = 0.15 m³. Based on the assumed final volume, the sheet determines the compression of the spring (Δx), spring force (Fspring), final pressure (P_2), and total work involved (Work). The final temperature (T_2) is then calculated from the first-law according to Equation (5.24), and the final volume (V_2) from the ideal-gas law, Equation (5.26). As Figure, 5.15 shows, the calculated values T_2 and V_2 are different from the initial values T_2g and V_2g. Solver can now be used to adjust the guessed value of T_2g and V_2g until they become the same as the calculated values.

| | T_2 | | $f_x$ | =T_1+(Q-Work)/m/Cv | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J |
| 1 | | | | | | | | | | |
| 2 | | P_1 | 200 kPa | | T_2g | 500 | | T_2 | 826.5425 K | |
| 3 | | T_1 | 317 kPa | | V_2g | 0.15 | | V_2 | 0.059259 m3 | |
| 4 | | V_1 | 0.05 m3 | | | | | | | |
| 5 | | | | | m | 0.109915 | =P_1*V_1/Rgas/T_1 | | | |
| 6 | | Q | 72.7 kJ | | | | | | | |
| 7 | | Area | 0.25 m | | Δx | 0.4 | =(V_2g-V_1)/Area | | | |
| 8 | | | | | Fspring | 60 | =kspring*Δx | | | |
| 9 | | kspring | 150 kN/m | | P_2 | 440 | =P_1+Fspring/Area | | | |
| 10 | | | | | Work | 32 | =0.5*(P_1+P_2)*(V_2g-V_1) | | | |
| 11 | | Rgas | 0.287 kJ/kg.K | | | | | | | |
| 12 | | | | | Cv | 0.7267 | =0.645+0.0002*(T_1+T_2g)/2 | | | |
| 13 | | | | | | | | | | |

Figure 5.15. Excel sheet developed for Example 5.5

Figure 5.16 shows the set-up that requires Solver to change the values of T_2g and V_2g in cells F2 and F3, respectively, until two constraints are satisfied: (i) T_2 = T_2g and (ii) V_2=V_2g. Note that the "Set Objective" option has been left blank. The "Changing Variable Cells" are F2 and F3. Figure 5.17 shows the solution obtained by the GRG Nonlinear method of Solver, which is T_2g = 1014.864K and V_2g = 0.1 m³. At this state, the final pressure on the piston is 320.0 kPa and the total work is 13.0 kJ.

These values agree with their corresponding values given by Cengel and Boles [5] whose analysis also gave $P_2 = 320$ kPa and $W = 13$ kJ.
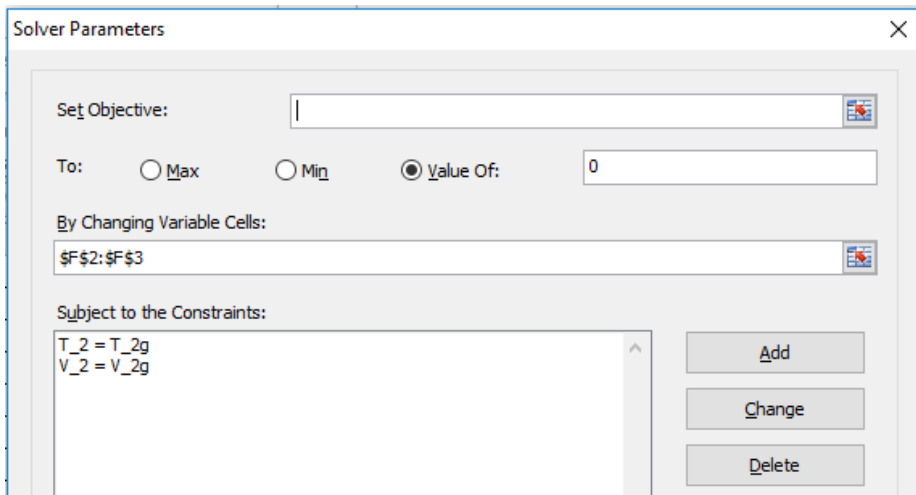


Figure 5.16. Solver set-up for Example 5.5



Figure 5.17. Solver solution for Example 5.5

## 5.3.  Iterative solutions involving nonlinear equations

Several nonlinear equations are met in thermofluid analyses because of the characteristics of a process or properties of the fluid involved. For example, to determine the head loss due to friction ($h_f$) in Example 5.1 the friction factor ($f$) for the turbulent pipe flow was obtained from Equation (1.25) - which is an explicit equation. As mentioned in Chapter 1,  $f$ can be determined more accurately for a turbulent pipe flow by using the Colebrook equation [1]:

$$\sqrt{\frac{1}{f}} = -2.0 \log \left( \frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right) \tag{1.35}$$

Since the Colebrook equation involves the friction factor $f$ in both sides, it needs to be solved iteratively in order to determine $f$. Therefore, for type-2 and type-3 flow problems using the Colebrook equation to determine the friction factor would involve two nested iterations; an inside iteration to determine $f$ and an outside iteration to determine the pipe's diameter or flow rate. This situation also occurs in optimisation analyses that use the Colebrook equation.

Chapter 4 illustrated the use of the NRM solver provided by Thermax by considering the Benedict-Webb-Rubin equation. The NRM solver can also be used to solve the Colebrook equation with type-2 and type-3 problems. For illustration, let us reconsider Example 5.1 and solve it by the using the Colebrook equation to determine $f$ instead of the Swamee-Jain equation. As mentioned in Chapter 4, the NRM solver requires the intended nonlinear equation to be written as a separate user-defined VBA function. The needed VBA function for the Colebrook equation is listed below:

```
Function colebrook(x, e, Re)
' Colebrook equation for the friction factor
        colebrook = 1/Sqr(x) + (2/log(10))*Log(e /3.7 + 2.51/Re/Sqr(x))
End Function
```

Note that, unlike in an Excel formula, in VBA syntax the term "log" is used for the natural logarithm "ln". Figure 5.18 shows the Excel sheet developed for solving Example 5.1 with the Colebrook equation.

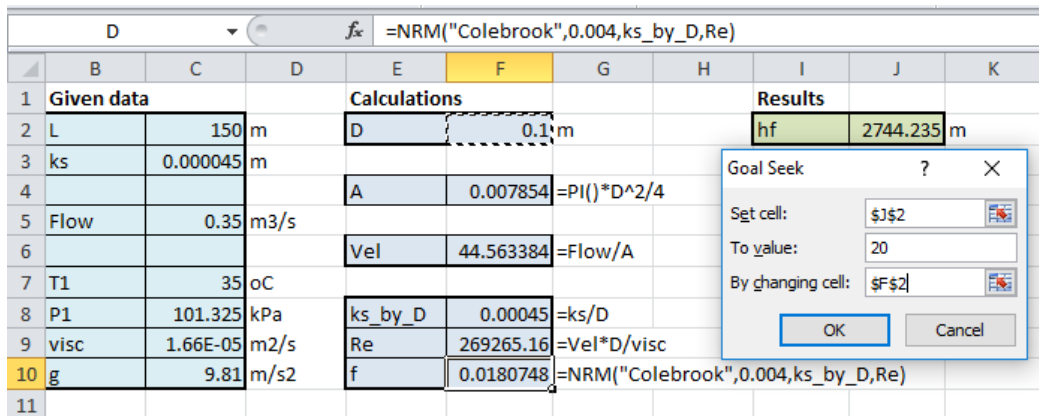| | D | | | | $f_x$ | =NRM("Colebrook",0.004,ks_by_D,Re) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | B | C | D | E | F | G | H | I | J | K |
| 1 | Given data | | | Calculations | | | | Results | | |
| 2 | L | 150 | m | D | 0.1 | m | | hf | 2744.235 | m |
| 3 | ks | 0.000045 | m | | | | | Goal Seek | ? | × |
| 4 | | | | A | 0.007854 | =PI()*D^2/4 | | | | |
| 5 | Flow | 0.35 | m3/s | | | | | Set cell: | $J$2 | |
| 6 | | | | Vel | 44.563384 | =Flow/A | | To value: | 20 | |
| 7 | T1 | 35 | oC | | | | | By changing cell: | $F$2 | |
| 8 | P1 | 101.325 | kPa | ks_by_D | 0.00045 | =ks/D | | OK | Cancel | |
| 9 | visc | 1.66E-05 | m2/s | Re | 269265.16 | =Vel*D/visc | | | | |
| 10 | g | 9.81 | m/s2 | f | 0.0180748 | =NRM("Colebrook",0.004,ks_by_D,Re) | | | | |
| 11 | | | | | | | | | | |

Figure 5.18. Excel sheet for Example 5.1 using the Colebrook equation

The only difference between this sheet shown in Figure 5.2 is the content of the cell F9 that calculates friction factor. As Figure 5.18 shows, the formula typed in this cell now reads:

"=NRM ("colebrook",0.004, $\varepsilon$_by_D, Re)".

The first input to the NRM function, "colebrook", refers to the function that contains the Colebrook equation while the second input, 0.004, is an initial guess for $f$. The last two arguments, **ε_by_D** and **Re**, respectively, are labels for the cells that hold values of the roughness-diameter ratio ($ε/D$) and the Reynolds number ($Re$) at which $f$ is to be determined. Figure 5.18, which shows the calculations for a selected diameter of 0.1 m, shows that the value of the friction factor obtained by the Colebrook equation is 0.018075. The diameter that keeps the losses below 20 m can be determined by using Goal Seek as in Example 5.1. Figure 5.18 also shows Goal Seek set-up for finding the value of $D$ that makes the friction head loss equal to 20 m. As Figure 5.19 shows, the answer found by Goal Seek is $D \geq 0.27$ m, which is the same answer obtained earlier in Example 5.1.
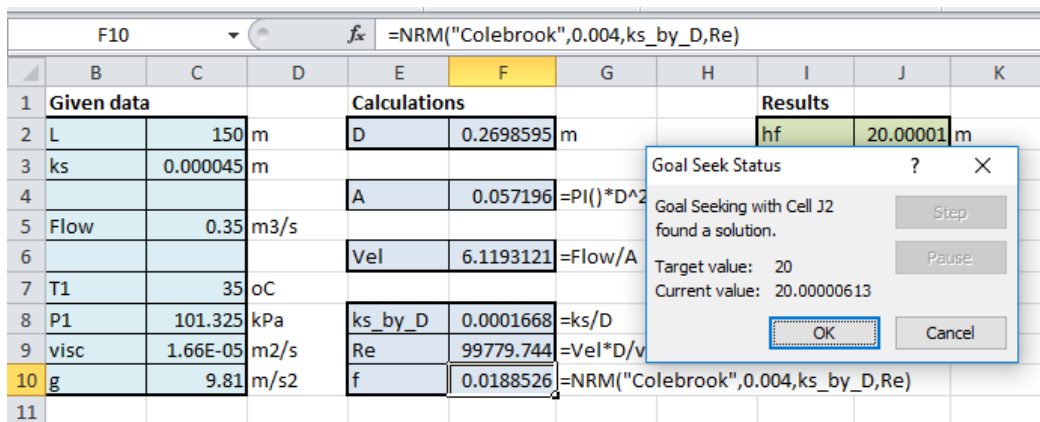


Figure 5.19. Goal Seek solution for Example 5.1 using the Colebrook equation

## 5.4. Closure

This chapter dealt with thermofluid analyses that require iterative solutions and showed how Excel's Goal Seek command and Solver can be used for solving typical problems from the areas of fluid dynamics, thermodynamics, and heat-transfer. The cases considered include a type-3 flow problem, expansion of an oxygen-nitrogen ideal-gas mixture in a closed system, and a convective heat-transfer analysis of a pipe heat-exchanger. Because of its simplicity, Goal Seek should be the first option to consider for an iterative solution that involves a single parameter. Solver can be used to solve more difficult problems because it can handle multiple changeable cells and allows constraints to be applied to the iterative process. The chapter also showed how the Newton-Raphson solver provided by Thermax can be used to deal with the analyses that involve nonlinear equations such as the Colebrook equation.

## References

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[2]  C. T. Crowe, D. F. Elger, B. C. Williams, and J. A. Roberson, *Engineering Fluid Mechanics*, 9th edition, John Wiley, 2009.

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[4]   M. Schumack, Solution of complex pipe flow problems using spreadsheets in an introductory fluid mechanics course, Proceedings of the 2004 American Society for Engineering Education Annual Conference & Exposition, Session 3666, Pages 9.1108.1 - 9.1108.13. Available at https://peer.asae.org/solution-of-complex-pipe-flow-problems-using-spreadsheets-in-an-introductory-fluid-mechanics-course

[5]   Y. A. Cengel, and M. A. Boles, *Thermodynamics. An Engineering Approach*. 5th edition, McGraw-Hill, New York, 2006.

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[7]   M.M. El-Awad, Use of Excel's 'Goal Seek' feature for Thermal-Fluid Calculations, the Electronic Journal of Spreadsheets in Education (eJSiE), Vol. 9, Iss. 1, Article 3, 2016. Available at: http://sie.scholasticahq.com/article/4657, last accessed Aug, 4, 2019.

**Exercises**

1.   Consider the problem in Example 5.1. Suppose that the only available pipe diameter is 20 m and we want to maintain the same maximum limit on the friction head loss of 20 m by reducing the water flow rate. Using the Excel sheet developed for this example, determine the water flow rate that gives the required result. Answer: 0.157 $m^3$.

2.   Using the two Excel sheets developed for Example 5.2, determine the final temperature for air by the two methods when the amount of heat added is 50, 100, 150, and 200 kJ. Also calculate the final temperature from Equation (5.4) by using a constant specific heat ($c_p$) of 1.043 kJ/kg.K. Plot the values obtained for the final temperature ($T_2$) with the amount of heat added by the three methods and comment on the result.

3.   A gas mixture consisting of $O_2$ and $CO_2$ with mole fractions 0.2 and 0.8, respectively, expands isentropically and at steady state through a nozzle from 700 K, 500 kPa to an exit pressure of 100 kPa as shown in the figure. Determine the temperature at the nozzle exit, in K.
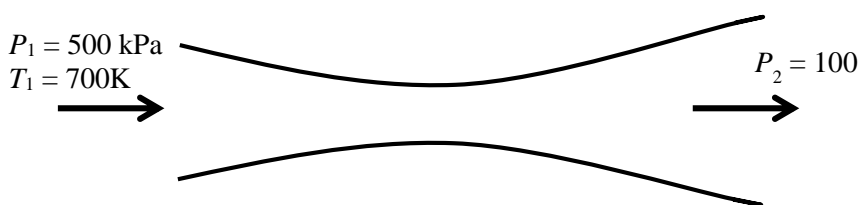


Figure P5.3. Isentropic expansion in a nozzle

This exercise is based on Example 12.4 in Moran and Shapiro [6]. Using the approximate constant-specific heat method, the exit temperature ($T_2$) can be determined from:

$$T_2 = T_1 \times \left(P_2 / P_1\right)^{\frac{k-1}{k}} \tag{A}$$

Where $k$ is the ratio of the specific heats for the mixture. Using $k$ = 1.304, the resulting exit temperature is 480.9K. Using the exact variable specific heat method, $T_2$ is determined by requiring that the total entropy change is zero, i.e:

$$y_{O2}\left[s^0_{O2}(T_2) - s^0_{O2}(T_1) - R_{O2} \ln \frac{P_2}{P_1}\right] + y_{CO2}\left[s^0_{CO2}(T_2) - s^0_{CO2}(T_1) - R_{CO2} \ln \frac{P_2}{P_1}\right] = 0 \tag{B}$$

Where $y_{O2}$ and $y_{CO2}$ are the volume fractions of $O_2$ and $C_{O2}$, respectively, and $R_{O2}$ and $R_{CO2}$ are the molar masses for $O_2$ and $C_{O2}$, respectively. The values of $s^0_{O2}$ and $s^0_{CO2}$ can be determined by using the relevant function provided by Thermax. Equation (B), that requires an iterative solution, can be solved by using the Goal Seek command. Answer: $T_2$ = 514.05K.

4. Steam is be condensed at 30°C on the shell side of the multi-pass shell-and-tube heat exchanger shown in Figure P5.4. The condenser has 8-tube-passes with 50 tubes in each pass. Its overall heat transfer coefficient is 1000 W/m²·°C. Cooling water ($Cp$ = 4180 J/kg·°C) enters the tubes at 15°C at a rate of 55,000 kg/h. The tubes are thin-walled, and have a diameter of 1.5 cm and length of 2 m per pass. Develp and Excel shhet to determine the outlet temperature of the cooling water by using Goal Seek and the LMTD method instead of the ε-NTU method [7].
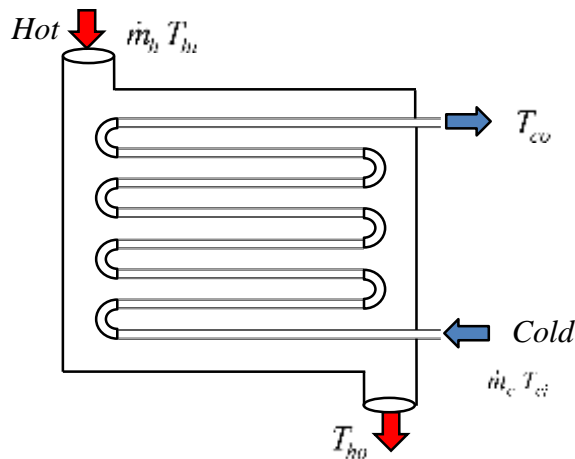


Figure P5.4. A multi-pass shell-and-tube heat exchanger

5.  Reconsider the problem in Example 5.5. Show that an alternative solution of this problem that also takes into consideration the variation of specific heat for air with temperature can be obtained by using the ideal-gas property functions provided by Thermax instead of Equation (5.18). Show that this solution can be obtained by using the Goal Seek command instead or Solver and compare your solution with that given in Example 5.5.

6.  Consider the semi-infinite slab shown in the figure that is suddenly exposed to convection environment at $T_\infty$. The temperature ($T$) at a depth $x$ from the surface at any time is given by [3]:

$$\frac{T - T_i}{T_\infty - T_i} = 1 - erf(X) - \left[ \exp\left( \frac{hx}{k} + \frac{h^2 \alpha \tau}{k^2} \right) \right] \times \left[ 1 - erf\left( X + \frac{h\sqrt{\alpha\tau}}{k} \right) \right] \qquad (A)$$

Where $\alpha$ and $k$ are the diffusivity and thermal conductivity of the slap material, respectively, and:

$$X = \left( 2\sqrt{\alpha\tau} \right)$$

$T_i$ = initial temperature of solid
$T_\infty$ = environmental temperature
$\tau$ = elapsed time in seconds
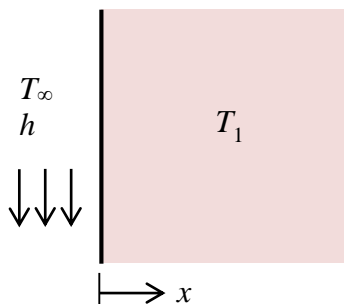


Figure P5.6. Semi-infinite slab with convection heat-transfer

 Equation (A) requires an iterative procedure because the time ($\tau$) appears in both terms on the right-hand side of the equation.

A large slab of aluminium ($k = 215$ W/m.ºC, $\alpha = 8.4 \times 10^{-5}$ m²/s) at a uniform temperature of 200ºC is suddenly exposed to a convection-surface environment of 70ºC with a heat-transfer coefficient of 525 W/m²·ºC. Calculate the time required for the temperature to reach 120ºC at the depth of 4.0 cm for this circumstance.

This problem is based on Example 4-5 in Holman [3]. His answer is approximately 3000 seconds.

7. Water at 60°C enters a tube of 3-cm diameter at a mean flow velocity of 1.2 cm/s. If the tube is 3.0 m long and the wall temperature is constant at 80°C, what will be the exit water temperature?

Use Goal Seek to perform the iterative solution of this problem. To determine the viscosity of water at any temperature, develop a user-defined function based on the data shown in Appendix B. This exercise is based on Example 5.2 in Holman [3]. Answer: 73.0°C.

# 6

# Optimisation analyses of thermofluid systems

The cost of energy constitutes a major part of the total lifetime costs of thermofluid systems. Therefore, a compromise must be made between the lifetime savings in energy cost that result from improving the efficiency of these systems and the additional costs of such improvements. Optimisation analyses of thermofluid systems aim to determine the most desirable design or operating conditions for these systems. This chapter shows how the Excel-based platform described in this book can be used for optimisation analyses of thermofluid systems. Initially, the chapter highlights the advantages of the Excel-aided optimisation method over the traditional analytical method by considering a two-stage air-compression system. The optimisation method with Excel is then applied to selected thermofluid systems that involve single and multiple design variables. Finally, the chapter demonstrates the use of the Evolutionary method in thermofluid optimisation analyses by solving two examples previously solved by the GRG Nonlinear method and comparing the results and computer time.

## 6.1. Analytical vs. Excel-Aided optimisation of thermofluid systems

The analytical optimisation process involves two main steps: (a) development of an objective function that can be used to analyse the system's performance in terms of its design and operating parameters and (b) using calculus methods to find the values of these parameters that maximise or minimise the objective function. An important advantage of the Excel-based method that uses Solver in the second step is that the optimisation process can easily take into consideration any relevant constraints such as physical constraints (spatial or temporal), economical constraints, or safety constraints. To highlight the advantages of the Excel-aided optimisation method, let us use the two methods to determine the intermediate pressure between the two stages in the air-compression system shown in Figure 1.1 that minimises the total compression work.

## 6.1.1. The analytical optimisation method

The development of the objective function in the analytical method that uses calculus techniques to determine the maximum or minimum value of the function usually requires simplifying assumptions to be made regarding properties of the materials and fluids involved in the system so that an analytical solution can be obtained. Chapter 1 showed that the total compression work ($w_{total}$) of the two-stage system is given by:

$$w_{total} = c_p T_1 \left[ 2 - \left( \frac{P_i}{P_1} \right)^{\frac{k-1}{k}} - \left( \frac{P_4}{P_i} \right)^{\frac{k-1}{k}} \right] \tag{6.1}$$

Where $k$ is the ratio of the specific heat for air at constant pressure ($c_p$) to that at constant volume ($c_v$). Equation (6.1) is the objective function for optimisation and we are required to determine the intermediate pressure ($P_i$) that minimises the total compression work. Designating the pressure ratio in the first and second compressor stages by $r_{p1}$ and $r_{p2}$, respectively, and the ratio $(k-1)/k$ by α, Equation (6.1) can be put in the following simpler form:

$$w_{total} = c_p T_1 \left[ 2 - r_{p1}^{\alpha} - r_{p2}^{\alpha} \right] \tag{6.2}$$

The pressure ratio in the second stage $r_{p2}$ can be replaced by the total pressure ratio ($r_p$) and the pressure ratio in the first stage ($r_{p1}$) as follows:

$$r_{p2} = \frac{P_4}{P_i} = \left( \frac{P_4}{P_1} \right) \times \left( \frac{P_1}{P_i} \right) = \frac{r_p}{r_{p1}} \tag{6.3}$$

Accordingly, Equations (6.3) can be written as:

$$w_{total} = c_p T_1 \left[ 2 - r_{p1}^{\alpha} - \left( \frac{r_p}{r_{p1}} \right)^{\alpha} \right] \tag{6.4}$$

The analytical solution to the optimisation problem is obtained by determining the intermediate pressure such that:

$$\frac{dw_{total}}{dr_{p1}} = 0 \tag{6.5}$$

For convenience, let us replace $r_{p1}$ in Equation (6.4) by $x$. Then, Equation (6.5) leads to:

$$\frac{d}{dx} c_p T_1 \left[ 2 - x^{\alpha} - \left( \frac{r_p}{x} \right)^{\alpha} \right] = 0 \tag{6.6}$$

After differentiation, we get:

$$\left[ 0 - \alpha x^{\alpha-1} - r_p^{\alpha} (-\alpha) x^{-\alpha-1} \right] = 0 \tag{6.7}$$

Dividing by $\alpha$ and rearranging:

$$x^{\alpha-1} = r_p^{\alpha} x^{-\alpha-1} \tag{6.8}$$

Multiplying both sides of the equation by $x^{(\alpha+1)}$, we finally get:

$$x^{2\alpha} = r_p^{\alpha}, \text{ or } x = \sqrt{r_p} \tag{6.9}$$

Thus, the optimum intermediate pressure ratio is such that:

$$r_{p1} = r_{p2} = \sqrt{r_p} \tag{6.10}$$

This solution indicates that, in order to minimise the total compression work, the overall pressure ratio should be divided equally between the two compression stages. For example, consider a two-stage air compression system designed to receive air at 100 kPa, 300K and discharge it at 900 kPa. Using Equation (6.10), the optimum pressure ratios $r_{p1}$ and $r_{p2}$ are such that:

$$r_{p1} = r_{p2} = \sqrt{9} = 3$$

The total compression work ($w$) can be determined from rom Equation (6.1). Taking $c_p$ for air as 1.005 kJ/kg.K and $k$ as 1.4, gives:

$$w_{total} = 1.005 \times 300 \left[ 2 - 2(3)^{\frac{1.4-1}{1.4}} \right] \qquad = \text{-222.35 kJ}$$

### 6.1.2. The Excel-aided optimisation method

The analytical optimisation method described above required the following simplifications and idealisations in order to develop the objective function:

1. Ideal isentropic compression processes in both stages
2. Constant specific heat for air
3. Perfect intercooling, i.e. $T_3 = T_1$
4. Zero pressure losses in the intercooler, i.e. $P_3 = P_2$

These simplifications and idealisations can undermine the accuracy of the results obtained. The following example demonstrates how the Excel-based platform enables a more realistic method for solving the optimisation problem by avoiding all the four simplifying assumptions mentioned above.

### Example 6.1. Excel-aided optimisation of a two-stage air compressor

An air compressor with two stages of compression takes atmospheric air at 300K and 100 kPa and delivers it at a final pressure of 900 kPa. The adiabatic efficiency of both stages of the compressor ($\eta_c$) is 85%. The intercooler that cools the air after the first stage causes a pressure drop of 5 kPa and has effectiveness ($\varepsilon$) of 85%. Determine the exit pressure of the first stage ($P_x$) that minimises the total compression work.

Unlike the analytical model, we don't take the compression process to be ideal, but allow for the actual friction losses in the process by using $\eta_c = 85\%$. We also allow for the imperfections of the intercooler by accounting for its pressure losses and heat-transfer effectiveness. By using Thermax functions for ideal gases, we also don't have to use the approximate method that assumes constant specific heats for air.

**The analytical model**

Figure 6.1 shows the *T-s* diagram for the two-stage compression process in this example. Given the values of $T_1$, $P_1$, $T_3$, and $P_4$, the first step in the solution is to assume a value for the intermediate pressure ($P_x$). Using the exact variable-specific-heat method, the relative pressure for air at state 1 ($P_{r1}$) is determined by using the relevant function provided by Thermax, which is **GasPr_TK**.
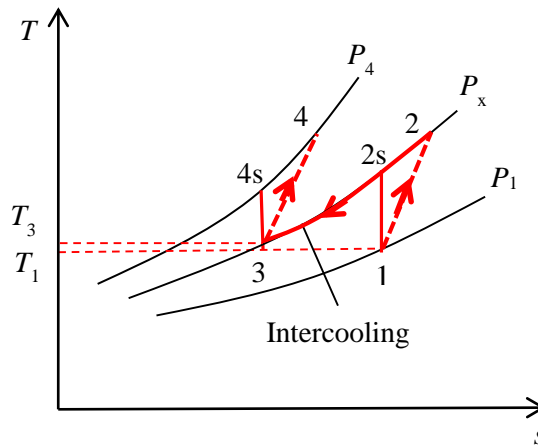


Figure 6.1. *T-s* diagram for the two-stage air compressor with
inter-stage intercooling

The relative pressure at state 2s ($P_{r2s}$) can then be calculated from the following relationship [1]:

$$P_{r2s} = P_{r1} \times \frac{P_x}{P_1} \tag{6.11}$$

Having determined the value of $P_{r2s}$, the ideal isentropic temperature after the first compression stage $T_{2s}$ can be determined by using the add-in function **GasTK_Pr**. Enthalpy values at states 1 and 2s can be calculated by using the **Gash_TK** function. Enthalpy at the actual temperature ($T_2$) can now be determined as follows:

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c \tag{6.12}$$

Where $\eta_c$ is the adiabatic efficiency of the compressor. Therefore, the compression work in the first stage ($w_{c1}$) is given by:

$$w_{c1} = h_2 - h_1 \tag{6.13}$$

The temperature and pressure before the second compressor stage are given by:

$$T_3 = T_2 - \varepsilon(T_2 - T_{wi})$$ (6.14)

$$P_3 = P_2 - \Delta P_{IC}$$ (6.15)

The compression work in the second stage can also be calculated by using the exact method to determine the ideal exit temperature $T_{4s}$ and actual exit enthalpy $h_4$:

$$P_{r4s} = P_{r3} \times \frac{P_{4s}}{P_3}$$ (6.16)

$$h_4 = h_3 + (h_{4s} - h_3)/\eta_c$$ (6.17)

$$w_{c2} = h_4 - h_3$$ (6.18)

The total compression work ($w_{total}$) is the summation of the work in the two stages:

$$w_{total} = w_{c1} + w_{c2}.$$ (6.19)

**The Excel model and solution**

Figure 6.2 shows the sheet prepared for solving this example. The sheet reveals the formulae entered in each cell in the calculations part, while the formula bar shows the formula that calculates the total compression work (**W_c** =**w_c1+w_c2**) in cell L2. Cell labelling has been used to clarify the formulae. Three Thermax functions are needed as shown in Table 6.1. Figure 6.2 shows the results of calculations at a "guessed" intermediate pressure of 200 kPa that gives a total compression work (W_c) of 277.85 kJ. Figure 6.3 shows the total compression work obtained at various assumed values of the intermediate pressure (P_i).

| | L2 | | | $f_x$ | =w_c1+w_c2 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | B | C | D | E | F | G | H | I | J | K | L |
| 1 | | | | | | | | | | | |
| 2 | T_1 | 300 | K | P_i | 200 | kPa | T_3 | 309.8113 | K | W_c | 277.8516 |
| 3 | P_1 | 100 | kPa | | | | P_3 | 195 | kPa | | |
| 4 | P_4 | 900 | kPa | h_1 | 299.8455 | =Gash_TK("Air",T_1) | h_3 | 309.7009 | =Gash_TK("Air",T_3) | | |
| 5 | | | | Pr_1 | 1.383536 | =GasPr_TK("Air",T_1) | Pr_3 | 1.548483 | =GasPr_TK("Air",T_3) | | |
| 6 | η_c | 0.85 | | P_r2 | 2.767073 | =Pr_1*(P_i/P_1) | P_r4 | 7.146844 | =Pr_3*(P_4/P_3) | | |
| 7 | T_wi | 300 | oC | T_2 | 365.4084 | =GasTK_Pr("Air",P_r2) | T_4 | 476.8719 | =GasTK_Pr("Air",P_r4) | | |
| 8 | ε | 0.85 | | h_2 | 365.8286 | =Gash_TK("Air",T_2) | h_4 | 479.8916 | =Gash_TK("Air",T_4) | | |
| 9 | ΔP | 5 | kPa | h_2a | 377.4727 | =h_1+(h_2-h_1)/η_c | h_4a | 509.9253 | =h_3+(h_4-h_3)/η_c | | |
| 10 | | | | w_c1 | 77.62727 | =h_2a-h_1 | w_c2 | 200.2244 | =h_4a-h_3 | | |
| 11 | | | | | | | | | | | |

Figure 6.2. Excel sheet developed for Example 6.1

Table 6.1. Usage of Thermax functions in the Excel model

| # | Function | Usage |
|---|----------|-------|
| 1 | GasPr_TK | $P_{r1}$, $P_{r3}$ |
| 2 | GasTK_Pr | $T_2$, $T_4$ |
| 3 | Gash_TK | $h_1$, $h_2$, $h_3$, $h_4$ |
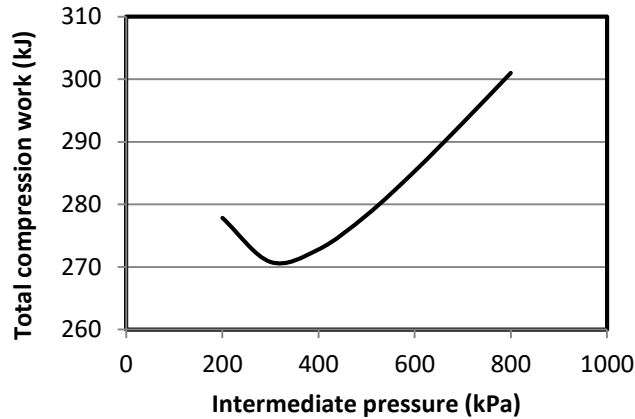


Figure 6.3. Variation of the total compression work with the intermediate pressure

Figure 6.3 indicates that the total work has a minimum value of about 310 kPa. The exact value of the intermediate pressure that minimises the total work can be determined by using Solver. Figure 6.4 shows Solver Parameters set-up that requires Solver to minimise the total compression work calculated in cell L2 by adjusting the value of the intermediate pressure (P_i). Figure 6.5 shows the solution found by Solver with this set-up using the GRG Nonlinear method.
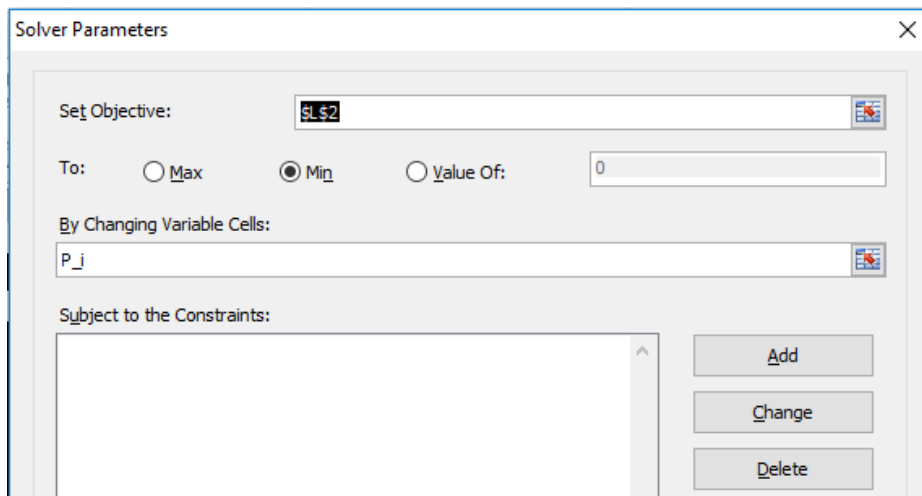


Figure 6.4. Solver Parameters set-up for Example 6.1

Figure 6.5. Solver solution for Example 6.1

As shown in Figure 6.5, the solution found by solver is 316.88 kPa at which the total compression work is 270.75 kJ. These values of the optimum intermediate pressure and the corresponding compression work are higher than the corresponding values of 300 kPa and 222.35 kJ given by the analytical method.

This example demonstrates three advantages for the computer-aided optimisation over the analytical method. Firstly, the computer-aided method is easier to use because it is applied to the basic objective function without differentiation. Secondly, it leads to more realistic results because it can take into consideration the pressure and heat-transfer losses in the compressors and the intercooler. Thirdly, accounting for the variation of the specific heat with temperature makes the computer-aided method more accurate than the analytical method that uses a constant specific heat. Optimisation analyses of thermofluid systems may also involve one or more of the following complications that make the analytical method practically impossible to apply:

1.  The model may involve discrete-valued variables, such as the cost of material or equipment, which are difficult to handle using analytical methods.
2.  The model may involve too many details that require the lengthy manipulations. This makes analytical optimisation both tedious and inaccurate.
3.  The analytical method cannot deal with complex systems that involve multiple parameters in the objective function.

As the following sections illustrate, with these complications the computer-aided method becomes the only viable method for optimisation.

## 6.2.    Optimisation with a single design variable

Although many optimisation analyses of thermofluid systems involve a single design parameter, such as the thickness of insulation or diameter of a pipeline, the nonlinearity of the equations involved or the variation of fluid properties with temperature and pressure make it practically impossible to solve the optimisation problem analytically. Four cases are considered in this section to illustrate how the Excel-based platform can be used to perform the optimisation analyses. All four optimisation analyses are

performed by using the GRG Nonlinear method of Solver and the default settings shown in Figures 3.17 and 3.18.a.

### 6.2.1. Optimum thickness of insulation

A thick layer of insulation maximises the resistance to heat-transfer and minimises the heat loss, but the cost of insulation also increases as its thickness is increased. Therefore, there is an optimum thickness beyond which adding more insulation becomes economically unprofitable. The following example, which is based on Example 6.1 in Eastop and Croft [2], illustrates the methodology of determining the optimum thickness insulation by using the Excel-aided optimisation method.

### Example 6.2. Optimum thickness of insulation

A steel pipe carries wet steam from a gas-fired boiler through a small workshop to a process plant. It is proposed to insulate the pipe using glass fibre insulation with an aluminium alloy casing. The cost of insulation is given below.

| Thickness of insulation ($\delta$) /mm | 19 | 25 | 32 | 38 | 50 | 60 |
|---|---|---|---|---|---|---|
| Cost per metre length ($C_i$)/p | 476 | 531 | 632 | 763 | 1007 | 1280 |

Determine the most economic thickness of insulation based on the following data:

> Pipe outside diameter ($D_1$) = 60.3 mm
> Heat transfer coefficient for outside surface of insulation ($h_o$) = 10 W/m$^2$.K
> Thermal conductivity of insulation ($k$) = 0.07 W/m.K
> Steam temperature ($T_s$) = 200°C
> Temperature of air in workshop ($T_\infty$) = 15°C
> Boiler efficiency ($\eta$) = 80%
> Price of gas ($c_g$) = 0.3 p/MJ
> Operation time of plant ($t$) = 3000 h per annum
> Lifetime of insulation ($N$) = 5 years

### The analytical model

Figure 6.6 shows a schematic of the system that consists of a pipe of external diameter $D_1$ surrounded by an insulation material with a thickness $\delta = D_2 - D_1$. The heat loss through the insulation per 1 meter length of the pipe is given by:

$$Q = \frac{T_s - T_\infty}{R_{th}} \quad \text{W/m} \tag{6.20}$$

Where $R_{th}$ is the total thermal resistance to heat transfer by conduction through the pipe and the insulation and by convection and radiation to the surrounding air.
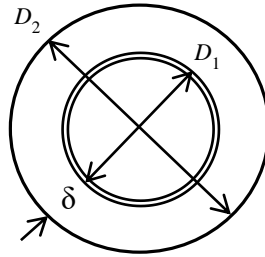
Figure 6.6. Dimensions of the pipe and its insulation

Neglecting the thermal resistance to heat transfer by conduction through the pipe and by radiation from the outer insulation surface, the total thermal resistance becomes that due to heat transfer by conduction through the insulation and by convection from the surface of insulation casing to the surrounding air, i.e.:

$$R_{th} = \frac{\ln(D_2 / D_1)}{2\pi k} + \frac{1}{h_o \pi D_2} \qquad (\text{m.K/W}) \qquad\qquad (6.21)$$

Where $D_2$ is the outside diameter of the insulation given by:

$$D_2 = D_1 + 2\delta \quad (\text{m}) \qquad\qquad (6.22)$$

The total annualised cost ($C_T$) is the summation of the annual heating cost and the annualised cost of insulation given by:

$$C_T = \alpha \times C_i + C_h \qquad\qquad (6.23)$$

Where $C_i$ is the cost of insulation per meter length of the pipe and $\alpha$ is the amortisation rate, which can be approximated by the following relationship:

$$\alpha = 1 / N \qquad\qquad (6.24)$$

Where $N$ is the lifetime of insulation or period after which capital cost is written off. The annual cost of heat lost to the surrounding ($C_h$) is given by:

$$C_h = (Q / 10^6) \times c_g \times (t \times 3600) / \eta \qquad (\text{p/m}) \qquad\qquad (6.25)$$

Although the objective function for optimisation in this case, which is Equation (6.23), is a simple one, it is easier to handle the discrete data for the cost of insulation by using the computer-aided method of optimisation than by using the analytical method.

**The Excel sheet**

Figure 6.7 shows the Excel sheet developed for this example in which the problem data are shown on the left side of the sheet (column B). The calculations start with a guessed insulation thickness ($\delta$) equal to 25 mm. Based on this assumed thickness, the sheet determines the thermal resistances due to conduction (R_ins) and due to convection (R_con). From the total resistance (R_total = R_ins + R_con), the sheet calculates the rate of heat transfer (Q) according to Equation (6.20). From Q and $\delta$, the sheet determines the annual cost of heat-loss (C_heat) and annualised cost of insulation (C_ins), respectively. Finally, the sheet determines the total annualised cost (C_total) for the given insulation thickness according to Equation (6.23).

| | C_total | | ▾ | | $f_x$ =C_heat+C_ins | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K | L |
| 1 | | | | | | | | | | | | |
| 2 | D_1 | 60.3 | mm | δ | 25 | mm | C_heat | 450.92787 | =Q*time*3600*(c_gas/1000000)/η_boiler | | | |
| 3 | h_o | 10 | | | | | ci_ins | 539.265 | =359.42+1.3713*δ+0.2329*δ^2 | | | |
| 4 | k_ins | 0.07 | | D_2 | 110.3 | =D_1+2*δ | C_ins | 107.853 | =α*ci_ins | | | |
| 5 | T_steam | 200 | | R_ins | 1.372988 | =LN(D_2/D_1)/(2*PI()*k_ins) | | | | | | |
| 6 | T_air | 15 | | R_conv | 0.288586 | =1/(h_o*PI()*D_2/1000) | C_total | 558.78087 | =C_heat+C_ins | | | |
| 7 | η_boiler | 0.8 | | R_total | 1.661574 | =R_ins+R_conv | | | | | | |
| 8 | c_gas | 0.3 | | | | | | | | | | |
| 9 | time | 3000 | | Q | 111.3402 | =(T_steam-T_air)/R_total | | | | | | |
| 10 | α | 0.20 | | | | | | | | | | |
| 11 | | | | | | | | | | | | |

Figure 6.7. Excel sheet for the optimisation of insulated pipe

The rate of heat loss (Q) is calculated as 111.3 W, which results in an annual cost (C_total) of 450.9 p per metre length of the pipe. Adding the annualised cost of insulation of 107.9 p, the total cost becomes 558.8 p. To allow the sheet to determine the cost of insulation automatically for other values of $\delta$, the following polynomial was obtained by using Excel trendline feature to curve-fit the price data given above:

$$C_i = 359.42 + 1.3713\delta + 0.2329\delta^2 \qquad (6.26)$$

As shown in Figure 6.7, the above formula is entered in cell H3 that calculates the initial cost (ci_ins). Figure 6.8 shows the variation of the cost of the annualised insulation cost (C_ins), annual heat-loss (C_heat), and the total annualised cost (C_total) with the thickness of insulation ($\delta$) as obtained by changing the insulation thickness from 0 to 60 mm. It can be seen from the figure that the total cost has a minimum value at an insulation thickness which is slightly greater than 40 mm. From a graph similar to that shown in Figure 6.8, Eastop and Croft selected 50-mm as the optimum thickness of insulation. The optimum insulation thickness can be determined more accurately by using Solver. Figure 6.9 shows the set-up for Solver Parameters dialog box to find the optimum value of $\delta$ (in the adjustable cell) by minimising the total annualised cost C_total (in the target cell). Based on the given data, two constraints have been added to limit the thickness of insulation within the range $19 \leq \delta \leq 60$ mm. The solution determined by Solver is shown in Figure 6.10.
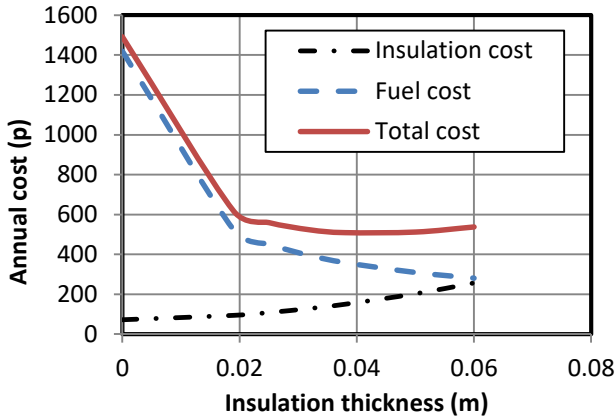
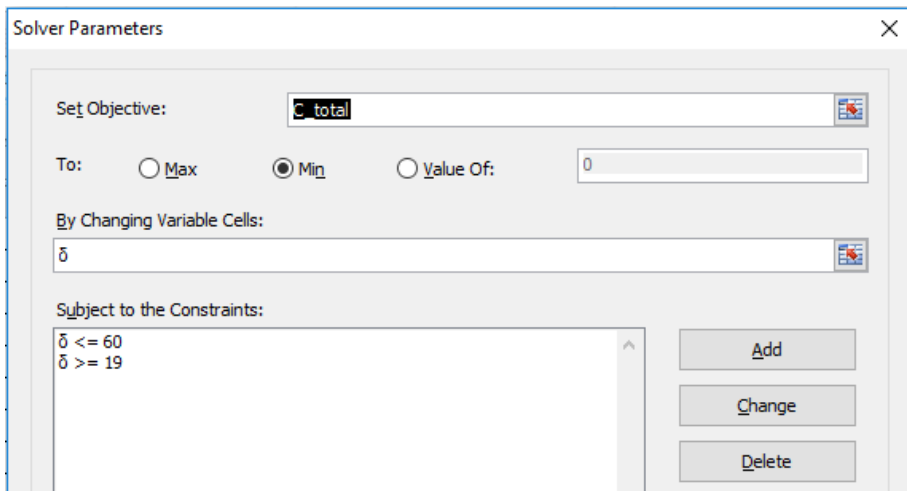Figure 6.8. Insulation cost against the thickness δ



Figure 6.9. Solver set-up for insulation optimisation



Figure 6.10. Optimised solution for the insulation thickness

Figure 6.10 shows that Solver determined the optimum thickness of insulation as 42.8 mm. Since this thickness is not available in the given data, the nearest larger thickness

should be selected; which is 50-mm. The thicker insulation is selected so as to allow for any possible increase in fuel cost in the future.

### 6.2.2. Optimisation of a single heat-exchanger

Heat exchangers are used in many industrial and power generation plants and optimisation of both the design and operation of heat-exchangers is needed for satisfactory performance of these plants. This section shows how the total life-time cost of a single heat- exchanger can be optimised by using Excel and Solver. The example considered in this case is based on Example 9.6 in Janna [3].

### Example 6.3. Optimum outlet temperature of the cooling water

A shell and tube heat exchanger uses water as a cooling medium to dispose of 1000 kW of heat from a stream of hot water. The hot water enters the exchanger at 93.3ºC and leaves at 37.8ºC. The cooling water enters the exchanger at 26.7ºC. Use the data given below to calculate the optimum cooling water outlet temperature:

$U = 613$ W/m$^2$.K
$F = 0.817$
$c_w = \$1.32/\text{m}^3$
$c_o = \$215/(\text{m}^2 .\text{yr})$
$\tau = 7800$ hr/yr,

Where $U$ and $F$ are the overall heat exchanger coefficient and correction factor in Equation (1.36), respectively, $c_w$ is the unit cost of producing the cooling water, and $c_o$ is unit cost of operating the heat-exchanger. Take the density of cold water ($\rho_c$) and its specific heat ($C_{pc}$) as 994 kg/m$^3$ and 4179 J/kg.K, respectively.

### The analytical model

In this case, both the inlet and exit temperatures of the hot water are given as well as the inlet temperature of the cold water. The total rate of heat transfer in the heat exchanger ($\dot{Q}$) is also given. What is then required is to determine the optimum exit temperature and mass flow rate of the cold water that minimises the total cost. The total annual cost of the heat-exchangers ($C_T$) consists of two parts: (i) the annual cost of cooling water ($C_{AW}$) and (ii) the annual cost of operating the exchanger ($C_{AO}$):

$$C_T = C_{AW} + C_{AO} \tag{6.27}$$

The annual cost of the cooling water $C_{AW}$ is given by:

$$C_{AW} = 3600 \frac{\dot{m}}{\rho} c_w \tau \tag{6.28}$$

Where $c_w$ is the cost of water per m$^3$, $\tau$ is the total number of operation per year, and the mass flow rate of cooling water ($\dot{m}$) is in kg/s. Taking the time-value of money in consideration, the annual cost of operating the exchanger is given by:

$$C_{AO} = Ac_o \tag{6.29}$$

Where $c_o$ is the annual cost of operating the exchanger per m$^2$ of its area.

Both $C_{AW}$ and $C_{AO}$ depend on the cooling water exit temperature ($T_o$). Starting the iterative optimisation process by an assumed value of this temperature, the mass flow rate of the cold water ($\dot{m}_c$) can be obtained from the following equation:

$$\dot{m} = \dot{Q} / C_{pc}\left(T_{co} - T_{ci}\right) \tag{6.30}$$

Where $c_p$, $T_{ci}$, and $T_{co}$ are the specific heat and inlet and exit temperatures of the cold water, respectively. From Equation (D.4), the surface area ($A$) of the heat-exchanger is:

$$A = \dot{Q} / UF\Delta T_{lm} \tag{6.31}$$

Where $\Delta T_{lm}$ is the log-mean temperature difference (LMTD) defined in Appendix D by Figure D.2 and Equation (D.3).

The above analytical model indicates that, for a given rate of heat rejection to the cooling water, the water outlet temperature depends on the water flow rate and the area of the heat-exchanger. Increasing the water flow rate will increase the cost of cooling water, but reduce the heat-exchanger area and the resulting annual operation cost. The optimum water outlet temperature is that at which the total cost is minimal.

### Development of the Excel sheet

Figure 6.11 shows the Excel sheet developed for this example. The left side of the sheet stores the given value for the rate of heat-transfer (Q), the heat exchanger data, the properties of the hot and cold water, and the water and operation costs involved in the analysis. The calculations part starts with an assumed exit temperature for the cold water (T_c2) which is given a value of 30°C. Based on this assumption, the sheet calculates the log-mean temperature difference (LMTD), the area of the heat-exchanger (A_HX), the mass flow rate of cold water (m_c), the volume flow rate of cold water (V_c), the annual water and operation costs (C_AW and C_HX), and the total annual cost (C_T). Figure 6.11 also reveals the formulae involved in the calculations by the side of their respective cells. Based on the assumed outlet temperature of the cold water, Figure 6.11 shows that the annual cost of water is $61,453.36, the annual cost of operating the exchanger is $14,317.57, and the resulting total annual cost is $75,770.93.

Figure 6.11. The Excel sheet developed for Example 6.3

The sheet was used to calculate the heat-exchanger costs at different values of the cooling-water outlet temperature and the results are shown in Figure 6.12. The figure shows that, as the water temperature increases, the annual cost of water decreases but the annual cost of operating the exchanger increases. The total annual cost initially decreases as the water outlet temperature is increased but then increases. From the figure, it can be seen that the optimum water outlet temperature is in the range 45 to 60°C. A more accurate estimation of the optimum cooling-water outlet temperature can be determined by using Solver. Figure 6.13 shows the required set-up in which two constraints have been imposed on the optimisation process to ensure a physically meaningful result. Figure 6.14 shows sheet with the solution found by Solver. The figure shows that the optimum cooling-water outlet temperature is 53.27°C at which the total annual cost is $26,666.4.
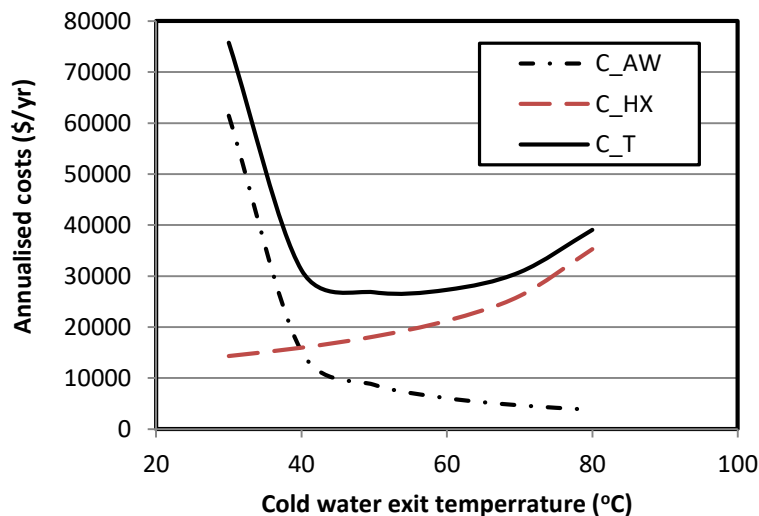


Figure 6.12. The variation of the total cost with the cooling water outlet temperature
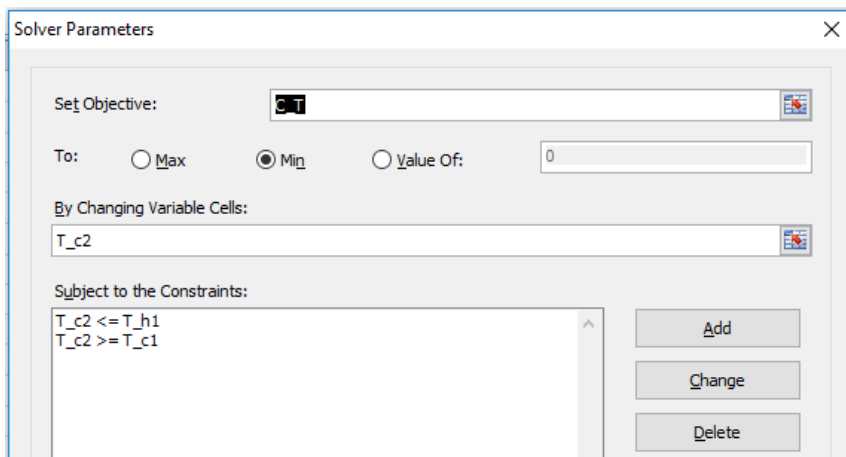
Figure 6.13. Solver set-up for Example 6.3



Figure 6.14. Solver solution for Example 6.3

## 6.2.3. Optimisation of two heat-exchangers in series

Li and Priddy [4] presented a hot water generation system that consists of two heaters connected in series as shown in Figure 6.15. The incoming water enters at $T_{wi}$ and its temperature is raised to an intermediate temperature $T_x$ as it flows through the first heater. The water is then heated in the second heater to the desired level $T_{wo}$. Two streams of saturated steam flow through the two heaters and heat the incoming water. The pressures of two streams are $P_{slp}$ and $P_{shp}$ for the first and second heaters, respectively. The corresponding saturation temperatures are $T_{slp}$ and $T_{shp}$ for the low-pressure and high-pressure, respectively. The high-pressure steam costs more than the low-pressure steam. The high-pressure heater is also expected to cost more to purchase than the low-pressure heater. Therefore, the total cost, which is the summation of the initial cost and steam cost, depends on how the heat load is distributed between the two heaters. It is required to determine the optimal sizes of the two heaters that minimise the total annual relative cost of the system.

High-pressure steam    Low-pressure steam

$T_{s,hp}$    $T_{s,1p}$

$T_{wo}$    $T_x$    $T_{wi}$    Water

HP heater    LP heater

$T_{s,h}$    $T_{s,1}$

Saturated liquid water
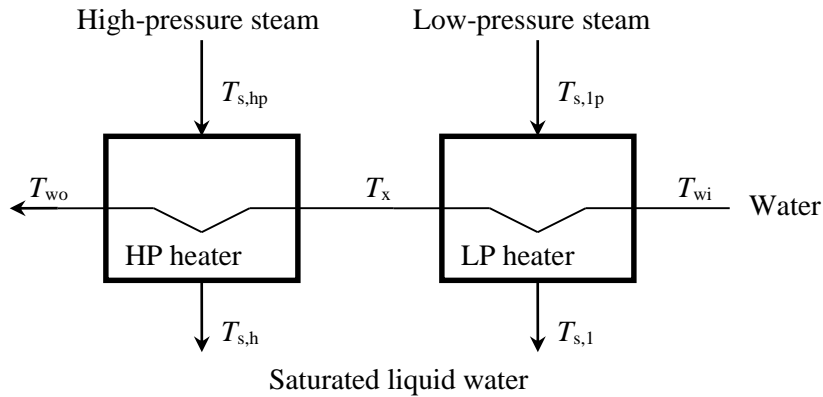
Figure 6.15. Schematic of the hot-water generation system
(adapted from Li and Priddy [4])

**The analytical model**

The cost of each heater is the sum of its annualised initial cost ($C_i$) and annual steam cost ($C_s$). For the system that consists of two heaters, the total cost ($C$) is given by:

$$C = (C_i + C_s)_{hp} + (C_i + C_s)_{lp} \tag{6.32}$$

Where $C_i$ is the annual initial cost (\$/year) and $C_s$ is the annual steam cost (\$/year) and the subscripts $hp$ and $lp$ refer to the high-pressure heater and the low-pressure heater, respectively. For a given heat exchanger, the steam cost is determined from the heat transfer rate in the heater, which is also the amount of heat transferred to the water. Using the notation shown in Figure 6.15, the rates of heat transfer in the two heaters can be expressed as:

$$\dot{Q}_{lp} = \dot{m} C_p \left( T_x - T_{wi} \right) \tag{6.33}$$

$$\dot{Q}_{hp} = \dot{m} C_p \left( T_{wo} - T_x \right) \tag{6.34}$$

Where $\dot{m}$ is the water mass flow rate. The annual steam costs for both heaters are:

$$C_{s,lp} = a_{lp} \dot{Q}_{lp} . \tau \tag{6.35}$$

$$C_{s,hp} = a_{hp} \dot{Q}_{hp} . \tau \tag{6.36}$$

Where $a$ is the steam cost in \$/kJ and $\tau$ is the operating hours per year. To determine the annual initial cost of the two heaters, we must calculate their heat-transfer surface areas.

Using Equation (D.4) with $F = 1$, the surface of the low-pressure heater ($A_{lp}$) is given by:

$$A_{lp} = \dot{Q}_{lp} / \left[ U_{lp} \Delta T_{lm,lp} \right]$$ (6.37)

Where $\Delta T_{lm}$ is the log-mean temperature difference which for the low-pressure heater is defined as:

$$\Delta T_{lm,lp} = \frac{\Delta T_{lp,1} - \Delta T_{lp,2}}{\ln\left(\Delta T_{lp,1} / \Delta T_{lp,2}\right)},$$ (6.38)

Where,

$$\Delta T_{lp,1} = \left(T_{slp} - T_x\right)$$ (6.39)

$$\Delta T_{lp,2} = \left(T_{slp} - T_{wi}\right)$$ (6.40)

The relative cost of the low-pressure heater is simply the product of the heat transfer surface area ($A_{ip}$) and the unit cost per square meter ($b_{lp}$). If we apply the annual fixed charge rate, the annual relative cost becomes:

$$C_{i,lp} = b_{lp} A_{lp}.\beta$$ (6.41)

Where $\beta$ is the annual fixed charge rate. Combining Equations (6.37) and (6.41) gives:

$$C_{i,lp} = \frac{b_{lp}.\beta.\dot{Q}_{lp}}{U_{lp}} \frac{\ln\left[\Delta T_{lp,1} / \Delta T_{lp,2}\right]}{\Delta T_{lp,1} - \Delta T_{lp,2}}$$ (6.42)

Similarly, the annual relative cost for the high-pressure heater is given by:

$$C_{i,hp} = \frac{b_{hp}.\beta.\dot{Q}_{hp}}{U_{hp}} \frac{\ln\left[\Delta T_{hp,1} / \Delta T_{hp,2}\right]}{\Delta T_{hp,1} - \Delta T_{hp,2}}$$ (6.43)

Where,

$$\Delta T_{hp,1} = \left(T_{shp} - T_{wo}\right)$$ (6.44)

$$\Delta T_{hp,2} = \left(T_{shp} - T_x\right)$$ (6.45)

Equation (6.32), which is the objective function for optimisation, requires lengthy calculations in order to determine the initial costs and the steam costs for both heaters. Since the total cost depends on the intermediate temperature $T_x$, trying to obtain the optimum value of $T_x$ by the analytical method is bound to be a tedious and most likely inaccurate procedure. The following numerical example illustrates the application of the computer-aided method of optimisation with Excel and Solver.

## Example 6.4. Design analysis of a hot water generation system

It is required to determine the optimum intermediate temperature and pressure for the two-stage hot water generation system shown in Figure 6.15 based on the following data:

| | Low-pressure heater | High-pressure heater |
|---|---|---|
| Steam pressure [kPa] | 350 | 550 |
| Steam cost [$/10^6 kJ] | 5.7 | 5.72 |
| Heater surface cost [$/m²] | 108.7 | 109.8 |

| | |
|---|---|
| Water flow rate ($\dot{m}$) | 180,000 kg/hr |
| Water inlet temperature ($T_{wi}$) | 65°C |
| Water outlet temperature ($T_{wo}$) | 150°C |
| Overall heat transfer coefficient ($U$) | 10220.667 kJ/hr-m²-°C |
| Annual fixed charge rate ($\beta$) | 20% |
| Operation hours ($\tau$) | 2000 h/yr |

## Excel implementation

Figure 6.16 shows the Excel sheet developed for this case. The top-left part of the sheet shows the data provided above. The bottom-right part of the sheet shows calculations of the steam and relative initial cost for the two heaters and their total for each heater. The figure shows the calculations and total relative cost of the system at the top right-hand side of the sheet for an intermediate temperature $T_x = 125$°C. We can determine the optimal sizes of the two heaters by varying the intermediate temperature $T_x$ and finding the value at which the annual relative cost is minimal. Table 6.2 shows the heat load distributions and surface areas of the two heaters for $T_x$ in the range 65-135°C. It is seen from the table that the heat load supported by the low-pressure heater increases as the intermediate temperature $T_x$ increases. Because of its large temperature differences, however, the high-pressure heater always has the advantage over the low pressure heater in terms of heat transfer surface area required. For instance, at $T_x = 65$°C, i.e. when the heat load is solely supported by the high-pressure heater, the heater surface area is approximately 207.41 m². At $T_x = 135$°C, i.e. when supported mostly by the low-pressure, the total surface area becomes 217.95+97.51 = 315.46 m².

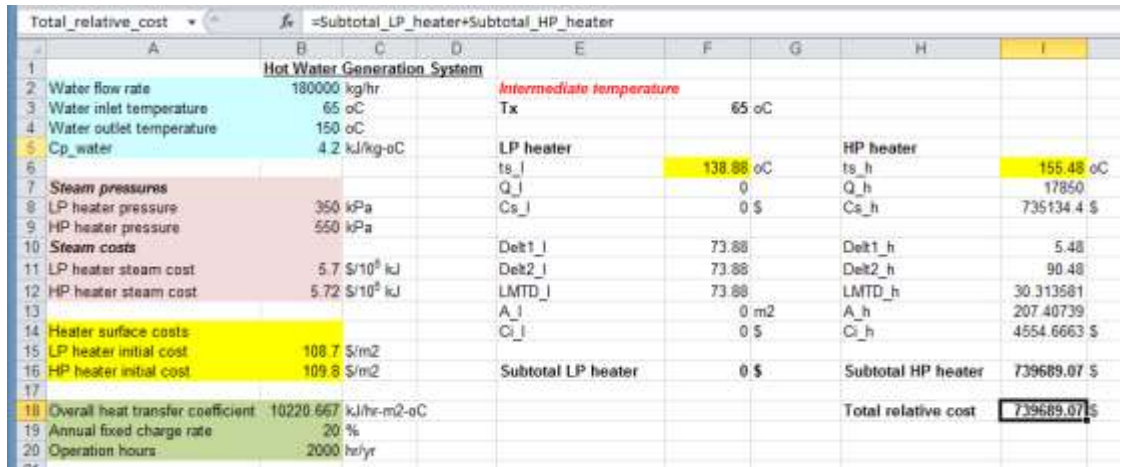Figure 6.16. Excel sheet for Example 6.4

Table 6.2. Heat load distributions and heater sizes

| $T_x$ [°C] | Heat load $Q$ [kW] | | Heater surface area [m²] | |
|---|---|---|---|---|
| | LP | HP | LP | HP |
| 65 | 0 | 17850 | 0 | 207.41 |
| 75 | 2100 | 15750 | 10.76 | 198.74 |
| 85 | 4200 | 13650 | 23.35 | 188.93 |
| 95 | 6300 | 11550 | 38.54 | 177.61 |
| 105 | 8400 | 9450 | 57.67 | 164.24 |
| 115 | 10500 | 7350 | 83.54 | 147.91 |
| 125 | 12600 | 5250 | 123.67 | 126.93 |
| 135 | 14700 | 3150 | 217.95 | 97.51 |

Figure 6.17 shows the variation with $T_x$ of the combined initial cost (C_initial), the combined steam cost (C_steam), and the total combined cost (C_total). The figure shows that the minimum combined initial cost is obtained at $T_x = 65$°C, while the minimum combined steam cost is obtained at $T_x =135$°C. If only the initial cost is considered, then a single high-pressure heater would be preferred for the hot water production. On the other hand, if only steam cost is considered, then only a single low-pressure heater is to be selected. However, the variation of the total relative cost with $T_x$ indicates that the minimum total cost occurs at an intermediate temperature which is around 115°C. At this temperature, the figures in Table 6.2 indicate that the high-pressure heater will take one-third of the total heat load while the low-pressure heater will supply the remaining two-thirds. To precisely determine the optimum value of $T_x$, we can use Solver. Figure 6.18 shows the prepared Solver Parameters dialog box for the determination of the temperature $T_x$ that minimises the total cost. Two constraints have been inserted which are:
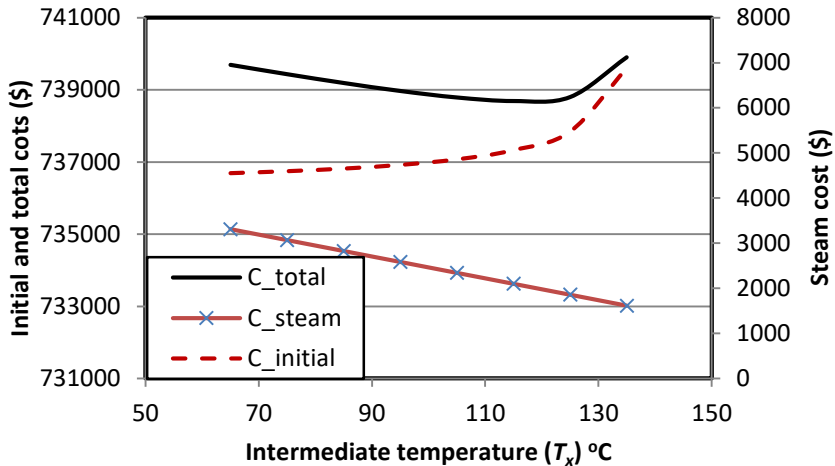
Figure 6.17. Variation of the systems costs with the intermediate temperature $T_x$
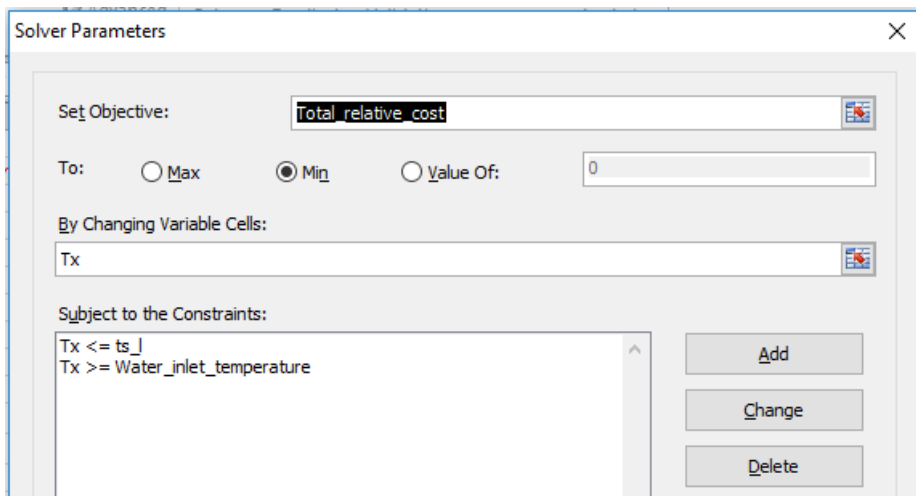


Figure 6.18. Solver parameters dialog box for determining the optimum intermediate temperature for two heat-exchangers in series

$$T_x \leq T_{s,l}$$
$$T_x \geq T_{w,in}$$

Pressing the **Solve** button at the bottom of the dialog box will trigger Solver to iterate and search for the value of $T_x$ that makes the total cost assume its minimum value. Figure 6.19 shows the solution found by Solver, which is $T_x$=116.5°C. At the optimum temperature, the surface area of the LP heater is 88.428 m$^2$ while that of the HP heater is 145.07 m$^2$. The relative cost of the LP heater is 446,004.8\$ and that of the HP heater is 292,679.7\$, giving a total relative cost of 738,684.4\$. This result shows that steam cost is an important variable in the optimisation of the hot-water production system. When the steam cost for low-pressure heater is substantially lower than that for high-

pressure heater, it will definitely favour the use of a low-pressure heater. This is particularly true when the hot water system is operated on a continuous basis.



Figure 6.19. Solver solution for Example 6.4

### 6.2.4. Optimisation of a rectangular fin

Fins are added to heat-transfer surfaces so as to increase the rate of heat transfer from these surfaces by forced convection. A good example of the application of fins is in car radiators which cool the engine's cooling water. Since the addition of fins increases the weight of the radiator, the improvement in heat-transfer achieved by adding the fins must be weighed against the additional cost and weight that they cause. In the following example, adapted from Bejan et al [2], it is required to optimise the geometry of a straight rectangular fin for the maximum possible heat-transfer rate. The example illustrates how Solver can be obtaind to determine the optimum geometry for the fin.

### Example 6.5. Optimisation of a rectangular fin

An electronic package includes several parallel straight rectangular (plate) fins such as the one shown in Figure 6.20. The width of each fin $W = 2.2$ cm and the fins are swept by forced air with $U_\infty = 1.75$ m/s and $T_\infty = 20^{\circ}$C. The fin material is aluminium. Weight limitations on the overall package permit the use of only 1 g aluminium for each fin. Determine the plate fin thickness ($t$) and length ($L$) that maximise the heat transfer rate extracted per fin and the corresponding heat transfer rate from the fin per degree of the temperature difference at the base ($T_b - T_\infty$).

### Analytical model

The rate of heat transfer from a fin ($\dot{Q}_{fin}$) is given by Equation (6.9). Using the notation of Figure 6.20, the area and efficiency of the straight rectangular fin can be obtained from [1]:
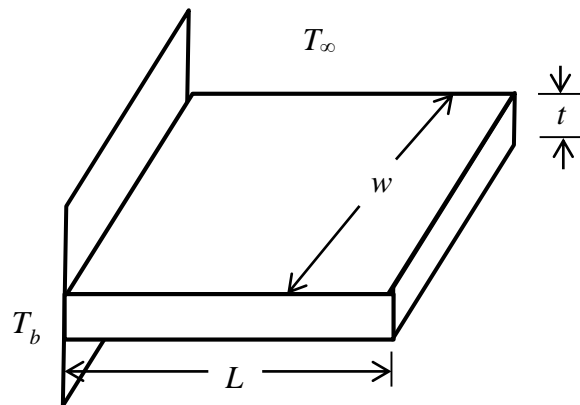
Figure 6.20. A straight rectangular fin

$$A_{fin} = 2wL_c \tag{6.46}$$

$$L_c = L + t/2 \tag{6.46}$$

For a rectangular fin the fin efficiency is given by:

$$\eta_{fin} = \frac{\tanh(mL_c)}{mL_c} \tag{6.48}$$

The constamt $m$ in Equation (6.13) is defined as:

$$m = \sqrt{2h/kt} \tag{6.49}$$

Where, $k$ is the thermal conductivity of the fin's material. The coefficient of heat transfer $h$ is depends on the type of air-flow over the fin, which is established by calculating the Reynolds number ($Re = \rho U_\infty L / \mu$). For a laminar flow, $h$ is calculated from [1]:

$$h = 0.664 \frac{k_{air}}{w} Re^{1/2} Pr^{1/3} \tag{6.50}$$

Where, $k_{air}$ and Pr are the thermal conductivity and Prandtl number of air, respectively. For a turbulent flow…

**Excel implementation**
Figure 6.21 shows the Excel sheet prepared for the solution of this case problem. The given data are entered on the left-side column of the sheet. The sheet then calculates the

Reynolds number (Re), heat-transfer coefficient (h) and volume of the fin (Vol). In the present case, the mass of the fin is not to exceed 1 g of aluminium. The thickness of the fin is initially assumed and given the value of $t=0.0002$ m. The length of the fin is then calculated from:

$$L = \frac{V}{wt} = \frac{0.001}{\rho_{al} wt} \tag{6.51}$$

Where, $\rho_{al}$ is the mass density of aluminium. Accordingly, the fin efficiency and heat-transfer from the fin are determined from Equations (6.12.c) and (6.10), respectively. As shown in Figure 6.21, the resulting rate of heat transfer is 0.037 W.Solver can now be used to find the combination of $t$ and $L$ that gives the maximum possible rate of heat transfer from the fin. Figure 6.22 shows the setting up of the dialog box for the determination of the thickness that maximises the rate of heat transfer from the fin.

| | Qfin | | $f_x$ | =ηfin*Qmax | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K |
| 1 | | | | | | | | | | | |
| 2 | | mass | 1 | g | | | Re | 2026.315789 | | ηfin | 0.290395 | |
| 3 | | rho | 2.707 | g/m3 | | | h | 34.09582157 | | Qmax | 0.126104 | W |
| 4 | | k_air | 0.028 | | | | Vol | 3.69413E-07 | m3 | Qfin | 0.03662 | W |
| 5 | | k_Al | 204 | | | | | | | | | |
| 6 | | U∞ | 1.75 | m/s | | | thickness | 0.0002 | m | | | |
| 7 | | Width | 0.022 | m | | | Length | 0.083957417 | m | | | |
| 8 | | kvisc | 1.90E-05 | | | | | | | | | |
| 9 | | Pr | 0.72 | | | | m | 40.8823165 | | | | |
| 10 | | Tbase | 21 | oC | | | Lc | 0.084057417 | m | | | |
| 11 | | T∞ | 20 | oC | | | Afin | 0.003698526 | m2 | | | |
| 12 | | | | | | | | | | | | |

Figure 6.21. Excel sheet for example 6.5

Solver Parameters ✕

Set Objective:     Qfin

To:   ⦿ Max   ◯ Min   ◯ Value Of:   0

By Changing Variable Cells:
thickness

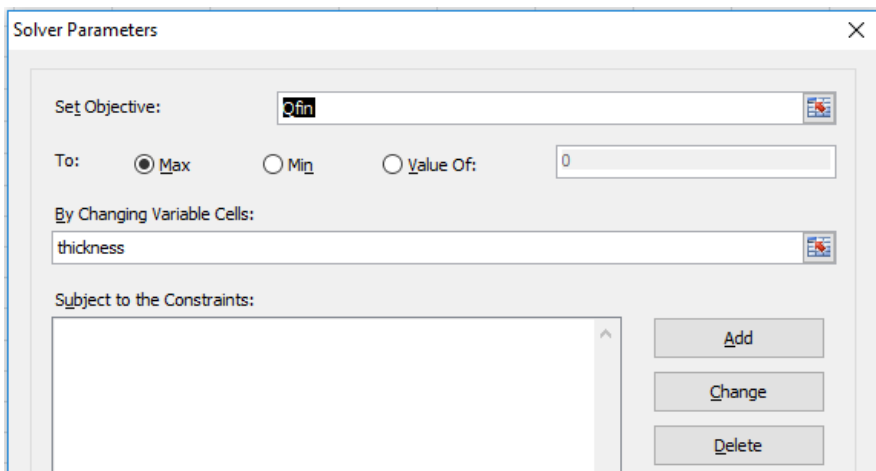Subject to the Constraints:

Add

Change

Delete

Figure 6.22. Solver parameters dialog box for Example 6.5

No constraints have been inserted for this case. Pressing the **Solve** button at the bottom of the dialog box will trigger Solver to iterate and search for the value of $t$ that maximise the fin's heat transfer rate, which in this case is $t$=0.00036 m or 0.36 mm (Figure 6.23). The corresponding length is 0.046 m or 4.6 cm and the resulting heat transfer rate is 0.044 W.

| | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $f_x$ | =ηfin*Qmax | | | | | |
| 1 | | | | | | | | | | | |
| 2 | | mass | 1 | g | | Re | 2026.315789 | | ηfin | 0.627877 | |
| 3 | | rho | 2.707 | g/m3 | | h | 34.09582157 | | Qmax | 0.069867 | W |
| 4 | | k_air | 0.028 | | | Vol | 3.69413E-07 | m3 | Qfin | 0.043868 | W |
| 5 | | k_Al | 204 | | | | | | | | |
| 6 | | U∞ | 1.75 | m/s | | thickness | 0.000361963 | m | | | |
| 7 | | Width | 0.022 | m | | Length | 0.046390023 | m | | | |
| 8 | | kvisc | 1.90E-05 | | | | | | | | |
| 9 | | Pr | 0.72 | | | m | 30.38912908 | | | | |
| 10 | | Tbase | 21 | oC | | Lc | 0.046571004 | m | | | |
| 11 | | T∞ | 20 | oC | | Afin | 0.002049124 | m2 | | | |
| 12 | | | | | | | | | | | |

Figure 6.23. Solver solution to Example 6.5

## 6.3. Multi-variable optimisation

All the cases considered so far involved a single variable for optimisation, e.g., the intermediate pressure ($P_x$) in Example 6.1, the thickness of insulation in Example 6.2, and the optimum cold-water outlet temperature ($T_{co}$) in Example 6.3. This section demonstrates the capability of the Excel-Solver combination to perform multi-variable optimisation of thermofluid systems which is the ultimate application of computer-based methods in thermofluid analyses. The objective of optimisation for the two cases to be considered is to minimise the total lifetime cost of the system, which achieved by using the GRG Nonlinear method of Solver.

### 6.3.1. Optimisation of the cooling tower for a power plant

Steam-turbine power generation plants that consume large amounts of fuel energy have to reject significant fractions of that energy to the atmosphere because of the second law of thermodynamics. The waste heat is usually released to a near-by river or lake or rejected directly into the atmospheric air by means of a cooling tower. Depending on the space and fund available for the installation, the cooling tower can be of the forced or natural types. Since the cost of the cooling system constitutes a major fraction of the total installation cost, the optimisation of its components is important for the economic feasibility of the whole plant. Example 6.3 considered earlier dealt with the optimisation of the heat-exchanger that transfers the rejected heat from the power plant condenser to the cooling water. The following example focuses on the cooling tower and illustrates the use of Excel for its optimisation. The example is based on Problem 11.7 given by Stoecker [5].

**Example 6.6. Optimisation of the cooling tower for a power plant**

A cooling-tower is to be used for removing 14 MW of heat rejected by the condenser of a steam-turbine power plant as shown in Figure 6.24. The total lifetime cost of the system consists of three parts: (a) the first cost of cooling tower ($C_1$), (b) the lifetime cost of pumping power ($C_2$) and (c) the lifetime penalty in power production due to elevation of temperature of cooling water ($C_3$).
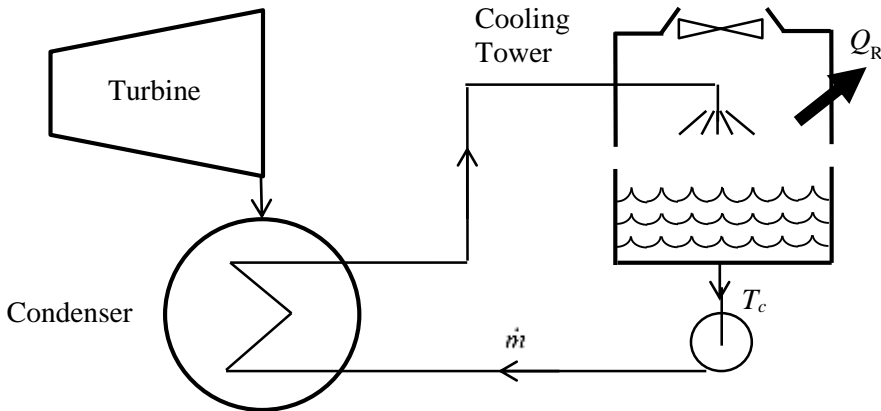


Figure 6.24. Schematic for the cooling system in Example 6.6

The three components of the total cost are estimated by the following formulae:

$$C_1 = 800\,A^{0.6} \tag{6.52}$$

$$C_2 = 0.0005\,\dot{m}^3 \tag{6.53}$$

$$C_3 = 270 T_c \tag{6.54}$$

Where $A$ is the cooling-tower area in m$^2$, $\dot{m}$ is the flow rate of cooling water in kg/s, and $T_c$ is the temperature of water entering the condenser in °C. The rate of heat rejection ($Q_R$) is also known to be related to the three factors by the following empirical formula:

$$Q_R = 3.7\dot{m}^{1.2}AT_c = 14\text{x}10^6\text{W} \tag{6.55}$$

It is required to determine the values of the three design variables $A$, $\dot{m}$, and $T_c$, that minimise the total cost of the cooling system ($C_T$) given by:

$$C_T = C_1 + C_2 + C_3 \tag{6.56}$$

**Solution**

The objective function for optimisation in this case is Equation (6.56). Although three variables are involved in the optimisation process, which are $A$, $\dot{m}$, and $T_c$, Equation (6.55) can be used to reduce the number to two variables only, e.g. $A$ and $\dot{m}$. Figure 6.25 shows the Excel sheet developed for this example. The only data in this problem is the rate of heat rejection (Qcooling) which is assigned the given value of 14 MW. The intermediate calculations start with initially assumed values for the water flow rate (m_w=500 kg/s) and the area of the cooling tower (A=150 m²). Based on these assumed values, Equation (6.55) is used to calculate the cooling-water temperature (T_cw) and then the three costs given by Equations (6.52) - (6.54). Figure 6.25 reveals the formulae used in these calculations. As Figure 6.25 shows, the cooling-water turned out to be about 14.56ºC and the resulting total lifetime cost (C_T) is $82,601.68. The values of m_w and A that minimise the total cost C_T can be found by using Solver and Figure 6.26 shows the required set-up for Solver Parameters dialog box.



Figure 6.25. The Excel sheet for Example 6.6
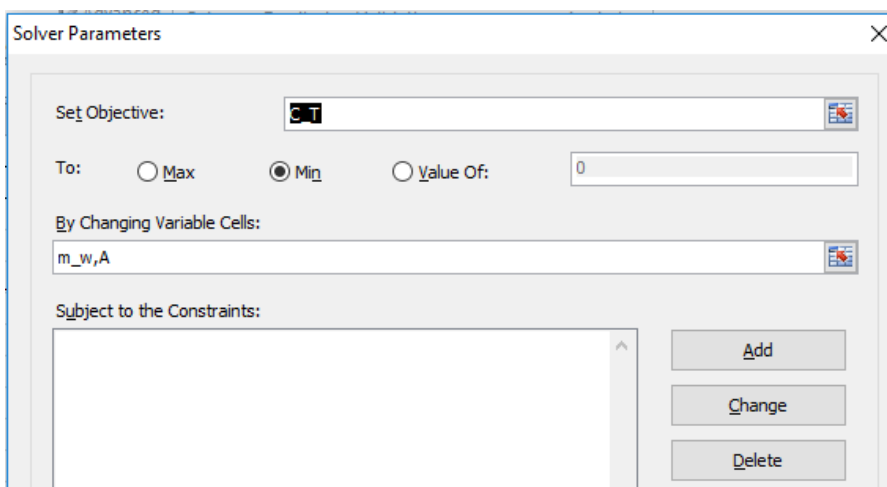


Figure 6.26. Set-up of Solver Parameters dialog box for Example 6.6

The set-up in Figure 6.26 requires Solver to minimise the total cost (C_T) by adjusting the values of the two parameters, m_w and A. Although no constraints have been specified, a better practice would be to apply reasonable upper and lower limits on the two optimisation variables. The solution found by Solver is shown in Figure 6.27. As the figure shows, the optimum mass flow rate for the cooling water is 202.52 kg/s and the optimum area of the cooling tower is 167.95 m². The value obtained for the water mass flow rate agrees well with that given by Stoecker [5], which is 202.6 kg/s. Stoecker [5] did not give the optimum value for the heat-exchanger area.

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $f_x$ | =C_ct+C_pump+C_penalty | | | | |
| 1 | | | | | | | | | |
| 2 | Qcooling | | 14 MW | m_w | 202.5247 | kg/s | C_T | 31842.78 | |
| 3 | | | | A | 167.9469 | m2 | | | |
| 4 | | | | | | | | | |
| 5 | | | | T_cw | 38.45746 | =Qcooling*1000000/(3.7*m_w^1.2*A) | | | |
| 6 | | | | | | | | | |
| 7 | | | | C_ct | 17305.86 | =800*A^0.6 | | | |
| 8 | | | | C_pump | 4153.405 | =0.0005*m_w^3 | | | |
| 9 | | | | C_penalty | 10383.51 | =270*T_cw | | | |
| 10 | | | | | | | | | |

Figure 6.27. Solution of Example 6.6 obtained with Solver

### 6.3.2. Optimisation of an air-conditioning duct

The second example for multi-variable optimisation with Excel-Solver is that of an insulated air-conditioning duct for which the objective function for optimisation involves both the diameter ($D$) and thickness of insulation ($\delta$). In this case, we wish to install the largest possible duct diameter so as to reduce the friction loss, but this would adversely increase the heat loss from the duct because of the larger surface area. Since the duct is insulated and the economical thickness of insulation also depends on the duct's diameter, the optimisation process for determining the minimum total cost involves two parameters, $D$ and $\delta$. Moreover, the duct has two sections with different diameters and, therefore, the process involves four parameters; $D_1$, $\delta_1$, $D_2$ and $\delta_2$. In this case, the computer-aided optimisation method is the only viable method to be used.

### Example 6.7. Optimisation of an air-conditioning duct

Conditioned air enters the 30-m air-handling duct shown in Figure 6.28 at $P_1 = 100$ kPa and $T_1 = 80$°C. The flow rate at the entrance is $Q_1 = 0.7$ m³/s, part of which ($Q_3 = 0.3$ m³/s) is discharged at a point 16 m downstream of the duct entrance. The remaining part ($Q_2 = 0.4$ m³/s) is discharged at the end. Ambient temperature ($T_{\infty 2}$) is 15°C and the outside heat-transfer coefficient $h_2$ is 30 W/m².°C. The air duct is to be assembled from 1-m-long prefabricated units made of 3-mm galvanized sheet metal and, to minimise heat losses to the surroundings, it is decided to insulate the duct with fiberglass.
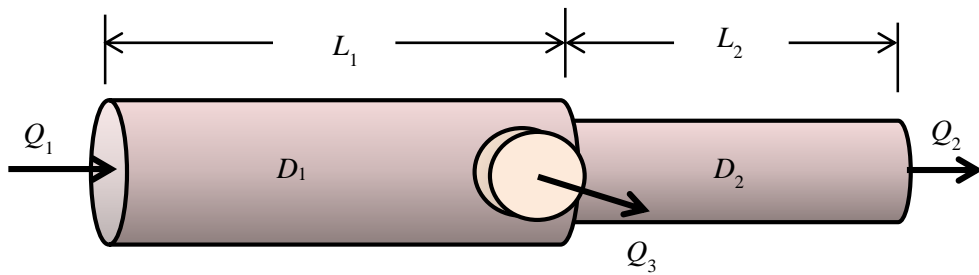
Figure 6.28. The uninsulated air-conditioning duct

It is required to determine the diameters of the two duct sections ($D_1$ and $D_2$) and the thicknesses of insulation ($\delta_1$ and $\delta_2$) that minimise the total owning cost based on the data provided below.

*Duct data*:
Thermal conductivity ($k_d$) = 18 W/m.ºC
Duct roughness ($\varepsilon$) = 0.045mm
Cost of 1-m unit ($c_d$) is as shown in Table 6.3.

Table 6.3. Unit cost of the duct body [6]

| Diameter ($D_i$) in m | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 |
|---|---|---|---|---|---|---|---|---|---|
| Cost per 1 m length ($) | 9 | 11.5 | 14.5 | 17 | 22.5 | 29 | 34 | 40 | 50 |

*Insulation (Fiberglas) data*:
Thermal conductivity ($k_s$) = 0.04 W/m.ºC
Insulation cost ($c_{s1}$): 30 \$/m$^2$ per cm of insulation
Labour cost ($c_{s2}$): 10 \$/m$^2$ (irrespective of thickness)

*Operation data*:
365 days per year 24 hours per day

*Energy costs*:
Cost of electricity ($c_E$): 0.12 \$/kWh
Cost of fuel ($c_F$): 0.5 \$/therm (1 therm = 105500 kJ)
*Capital recovery factor* ($i$) = 0.15

The idea of this example stemmed from a case given by Janna [3] which could be solved analytically. The addition of insulation to the duct in the present example makes the optimisation process involves four parameters instead of two as in the case considered by Janna [3] and, therefore, the example is difficult to solve analytically.

**The analytical model**

The total annual cost of the insulated air-duct ($C_{Total}$) consists of three components as expressed by:

$$C_{Total} = C_I \times i + C_E + C_F \qquad (\$) \qquad\qquad (6.57)$$

Where:

> $C_I$ = initial cost of the duct itself plus the cost of insulation
> $i$ = capital recovery factor
> $C_E$ = cost of electricity consumed by the fan in order to overcome friction in the duct
> $C_F$ = cost of fuel needed to make-up for the heat loss to the surrounding air

How the three components of the total cost are evaluated is explained below.

a)  Initial cost

The initial cost ($C_I$) has two parts: (1) the cost of the duct itself ($C_{duct}$) and (2) the cost of insulation ($C_{ins}$). The two parts are given by:

$$C_{duct} = \left(D_1 \times c_{d,1}\right)\times L_1 + \left(D_2 \times c_{d,2}\right)\times L_2 \qquad\qquad (6.58)$$

$$C_{ins} = \left[\left(\pi D_1 L_1\right)\times \delta_1 + \left(\pi D_2 L_2\right)\times \delta_2\right]\times c_{s1} + \left[\left(\pi D_1 L_1\right)+\left(\pi D_2 L_2\right)\right]\times c_{s2} \qquad (6.59)$$

Where:

> $c_d$ = cost of 1-m duct unit which depends on the diameter ($\$/m$)
>
> $\delta_1$ = thickness of insulation in the first section of the duct (m)
>
> $\delta_2$ = thickness of insulation in the second section of the duct (m)
>
> $c_{s,1}$ = cost of insulation per $m^2$ that varies with insulation thickness ($\$/m^2$.cm)
>
> $c_{s,2}$ = cost of labour per $m^2$ that depends on insulated surface area only ($\$/m^2$)

b)  Annual cost of electricity:

$$C_E = \dot{W}_{fan}(kW) \times time(hr) \times c_E (\$/kW.hr) \qquad (\$) \qquad\qquad (6.60)$$

Where $\dot{W}_{fan}$ is the power consumed by the air-circulation fan and $c_E$ is the electricity tariff in $\$/kW.hr$. The power of the circulation fan depends on the friction head losses ($h_f$) in both sections of the duct which are given by the Darcy-Weisbach equation, Equation (1.21). The friction factor ($f$) in each section of the duct, which depends on

the Reynolds number in the section, is obtained from the Swamee-Jain equation (Equation 1.25). The total power consumed by the air-circulation fan ($\dot{W}_{fan}$) is then determined as follows:

$$\dot{W}_{fan} = \frac{\rho_{air} g \left( h_{f,1} Q_1 + h_{f,2} Q_2 \right)}{1000} \quad \text{(kW)} \tag{6.61}$$

c)   Annual cost of fuel:

$$C_F = \frac{\dot{Q}_{Total}[W] \times t[hr]}{1000 \times 105500} \times c_F \quad (\$) \tag{6.62}$$

Where $c_F$ is the cost of fuel in \$/therm and t is the total number of operation hours in a year. The total heat loss ($\dot{Q}_{Total}$) is the sum of the heat loss in both sections, i.e. $\dot{Q}_{Total} = \dot{Q}_1 + \dot{Q}_2$, where the heat loss ($\dot{Q}$) in each section is calculated according to the formula:

$$\dot{Q} = \left( T_{\infty 1} - T_{\infty 2} \right) / R_{th} \quad \text{(W)} \tag{6.63}$$

Where $T_{\infty 1}$ and $T_{\infty 2}$ are the inside and outside air temperatures and $R_{th}$ is the total thermal resistance of the series resistances shown in Figure 6.29.
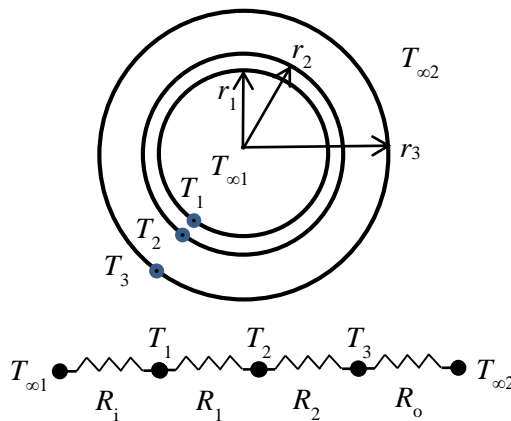


Figure 6.29. Total thermal resistance of the insulated duct

The total thermal resistance ($R_{th}$) is given by:

$$R_{th} = \frac{1}{2\pi r_1 h_1 L} + \frac{\ln\left( r_2 / r_1 \right)}{2\pi k_d L} + \frac{\ln\left( r_3 / r_2 \right)}{2\pi k_s L} + \frac{1}{2\pi r_3 h_2 L} \quad \text{(m.K/W)} \tag{6.64}$$

Where $r_1$, $r_2$ and $r_3$ are the radii shown in Figure 6.29. Note that the value of the outside heat-transfer coefficient ($h_2$) is constant, but the inside heat-transfer coefficient ($h_1$), which depends on the air velocity, changes with the inside diameter of the duct and therefore, has to be determined from the Nusselt number ($Nu$). For fully developed turbulent flow in tubes, the Nusselt number is calculated from the Dittus–Boelter equation, Equation (1.31), with $n = 0.3$ [7]:

$$Nu = 0.023 \, \text{Re}_i^{0.8} \, \text{Pr}^{0.3} \tag{6.65}$$

Where Re and Pr are the Reynolds and Prandtl number, respectively.

$$h_1 = \frac{k_{air}}{D_1} Nu \qquad (\text{W/m}^2 \cdot {}^\circ\text{C}) \tag{6.66}$$

**Excel sheet**

Figure 6.30 shows the Excel sheet developed for this example. The problem data are shown in the two columns at the left side of the sheet. The calculations part has two columns for the two sections of the duct. The calculations stat by guessed values for the two diameters (D_1 and D_2) and two insulation thicknesses (δ_1 and δ_2) in both sections of the duct. The results part determines the total friction loss (hf_total), the fan power (Power), and the different costs involved. The formula bar shows how the annual cost of electricity is determined. The sheet determines the total annual cost (C_total) for the guessed values of D_1=0.3m, D_2=0.2m, δ_1=0.1m and δ_2=0.1m.



Figure 6.30. Excel sheet developed for Example 6.7

To allow Excel to automatically calculate the cost of duct unit ($c_d$) when the two duct diameters are changed, the following equation for $c_d$ was obtained by using Excel's trendline feature from the data shown in Table 6.3:

$$c_d = 7.6881 - 1.7814D + 169.91D^2 \tag{6.67}$$

Solver can now be used to find the optimum values of D_1, D_2, δ_1 and δ_2. As shown in Figure 6.31, the set-up box for Solver requires it to minimise the total cost (C_total), which is the target cell, by changing the values of the two diameters (D_1 and D_2) and the two insulation thicknesses (δ_1 and δ_2), which are the adjustable cells. Figure 6.32 shows the Excel sheet with the solution found by Solver. The optimised dimensions found by Solver are shown in Table 6.4 which also shows the different cost involved. The nearest dimeters are D_1=0.4 m and D_2=0.3 m. Both insulation thicknesses are ≈ 0.3 m. The total annual cost sums up to 479.7 $.
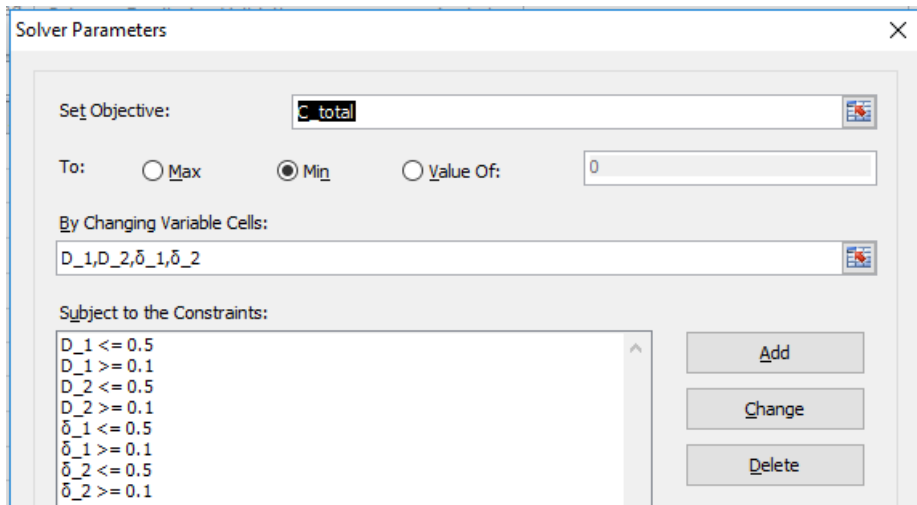


Figure 6.31. Solver set-up for Example 6.7

As a rule-of-thump, air-conditioning engineers frequently determine the duct areas from the ratio of flow rates. Accordingly, the duct diameters $D_1$ and $D_2$ are related as follows:

$$D_2 = \sqrt{Q_3 / Q_1} \times D_1 \tag{6.68}$$

Equation (6.68) allows us to solve the optimisation problem with only three variables, which are $D_1$, $\delta_1$ and $\delta_2$. The solution determined by Solver with $D_1$, $\delta_1$ and $\delta_2$ as adjustable cells is also shown in Table 6.4. Comparison with the solution obtained without Equation (6.68) indicates that the rule-of-thump leads to a larger $D_1$ and a smaller $D_2$. Although the insulation thicknesses on the two section are only marginally affected, the figures in the table show that the total cost (486.2$) has increased due to increases in the initial cost of the duct (132.1$) as well as the annual cost of electricity

(186.4$). By suitably adjusting the given data, the Excel sheet can be used to study the effects of electricity cost, fuel cost, or capital recovery factor on the opimised solution.
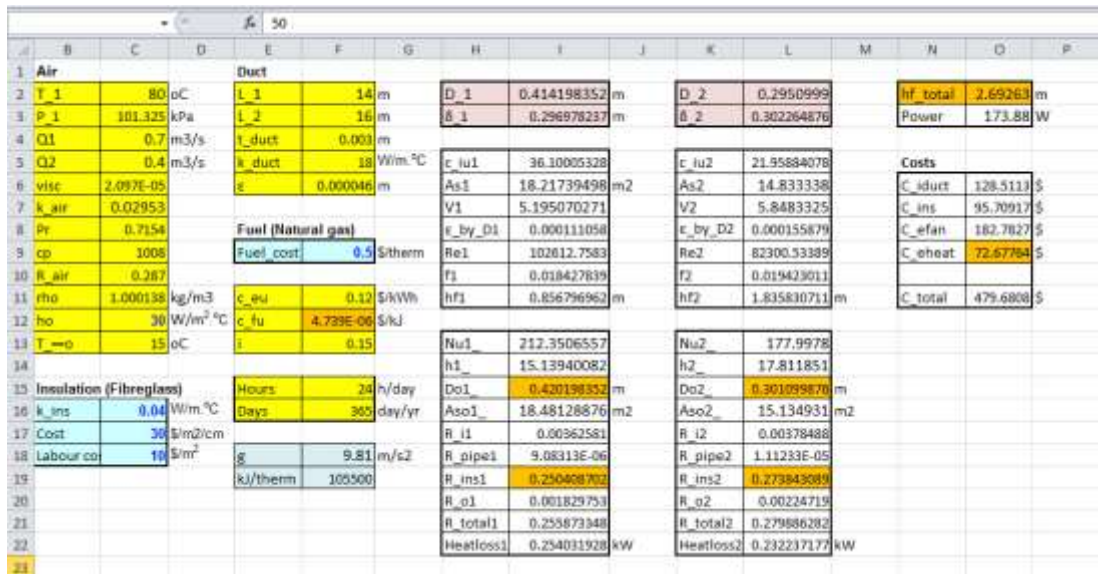


Figure 6.32. Solver solution for Example 6.7

Table 6.4. Solver solution for the insulated duct

|  | Optimised values determined by Solver without Equation (6.68) | Optimised values determined by Solver with Equation (6.68) |
|---|---|---|
| $D_1$(m) | 0.4142 | 0.4525 |
| $D_2$(m) | 0.2951 | 0.2586 |
| $\delta_1$ (m) | 0.2970 | 0.2953 |
| $\delta_2$ (m) | 0.3023 | 0.3056 |
| $C_{duct}$($) | 128.5113 | 132.1191 |
| $C_{ins}$($) | 95.7092 | 95.2828 |
| $C_E$ ($) | 182.7827 | 186.4329 |
| $C_F$ ($) | 72.6776 | 72.3330 |
| $C_{Total}$($) | 479.6808 | 486.1678 |

## 6.4. Optimisation with the Evolutionary method

As demonstrated in Chapter 3, the Evolutionary method has an important advantage over the GRG Nonlinear method which is the ability to find the global optimal point. Although this advantage is particularly important in economic optimisation analyses, the method has not been used in the previous sections because of its longer computation time compared the GRG Nonlinear method. For the purpose of illustration, two examples previously solved by the GRG Nonlinear method were solved by the

Evolutionary method. The two examples are Example 6.3 that involved a single optimisation variable and Example 6.7 that involved four optimisation variables. The same sheets developed for these two examples were used to perform the optimisation analyses by changing the solution method from "GRG Nonlinear" to "Evolutionary". Figure 6.33 shows the solution obtained with the Evolutionary method for Example 6.3, while Figure 6.34 shows the method's solution of Example 6.7.



Figure 6.33. Solver solution for Example 6.3 with the Evolutionary method



Figure 6.34. Solver solution for Example 6.7 with the Evolutionary method

Both solutions were obtained with the default set-up shown in Figure 3.17 and Figure 3.18.b. As Figures 7.29 and 7.30 show, the two solutions are identical to those obtained earlier with the GRG Nonlinear method shown in Figures 7.14 and 7.35, respectively. With the default options of the Evolutionary method shown in Figure 3.18.b, Solver took more than 100 seconds to solve Example 6.3 which required less than one second

with the GRG Nonlinear method. Example 6.7 required more than 110 seconds with the Evolutionary method. As mentioned in Chapter 3, the computational time of the Evolutionary method can be reduced by adjusting the default set-up for the method shown in Figure 3.18.b to reduce the population size from 100 to 10. With this adjustment, the Evolutionary method took about one second to solve Example 6.3, but took about 24 seconds to solve Example 6.7 that involved four parameters.

## 6.5.   Closure

This chapter showed how the Excel-based platform can be utilised for optimisation analyses of thermofluid systems. The chapter initially highlighted the limitations of traditional optimisation methods that apply calculus techniques and the advantages of the computer-aided optimisation method. Four examples were then considered that dealt with optimisation analyses involving a single design parameter, but the computer-aided optimisation method was indispensable because the mathematical model either involved lengthy calculations, nonlinear equations, or discrete-valued variables. The advantage of the computer-aided optimisation method becomes more appreciable when the optimisation process involves multiple parameters. In this respect, the chapter showed how Solver can be used to handle optimisation analyses of insulating an air-conditioning duct consisting of two sections with different diameters. Finally, the chapter considered using the Evolutionary method instead of the GRG Nonlinear method. It was shown that the computer time of the Evolutionary method can be reduced by suitable adjustment to the default values of its options.

## References

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## Exercises

1.   Using the analytical optimisation procedure, show that the compression ratios that minimise the total work input for in a multi-stage compression systems with $n$ compression stages and a maximum compression ratio $r_{p,max}$ are given by:

$$r_{p1} = r_{p2} = .... = r_{p,n-1} = r_{p,n} = \sqrt[n]{r_{p,max}}$$

2.  An air compressor with three stages of compression takes atmospheric air at 300K and 100 kPa and delivers it at a final pressure of 900 kPa. The adiabatic efficiency of the three compressors ($\eta_c$) is 85%. Each one of the intercoolers that cool the air after the first and second stages causes a pressure drop of 5 kPa and has effectiveness ($\varepsilon$) of 85%. By suitably extending the Excel sheet developed for Example 6.1, determine the exit pressures of the first and second stages, $P_x$ and $P_y$, respectively, that minimise the total compression work.

3.  Air, initially at a pressure of 175 kPa and a temperature of 50°C, is to be compressed to a final pressure of 17,500 kPa by means of a two-stage compression system that consists of a centrifugal compressor in series with a reciprocating compressor as shown in Figure P6.3.
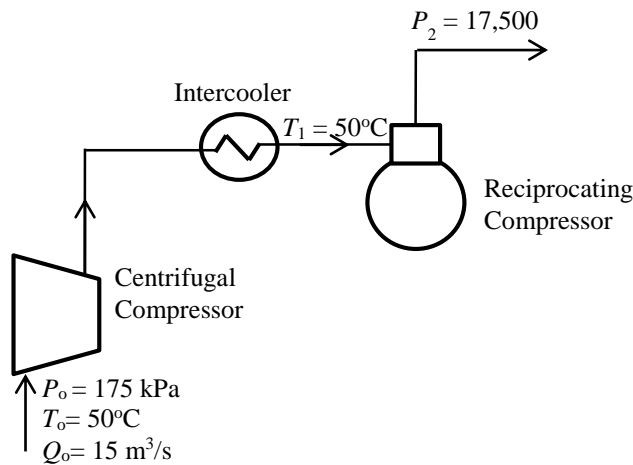


Figure P6.3. The air compression system with centrifugal and reciprocating compressors in series

The advantage of this arrangement is that centrifugal compressors can handle high-volume flow rates but develop only low pressure ratios while reciprocating compressors are suited to low-volume flow rates but can develop high pressure ratios. The initial flow rate of air is 15 m³/s and its temperature after the intercooler can be taken as the same as the inlet temperature of 50°C.

The first costs of the centrifugal and reciprocating compressors can be represented by the following formulae:

$$C_c = 70Q_o + 1600 \frac{P_1}{P_0} \quad (\$) \tag{A}$$

$$C_r = 200Q_1 + 800 \frac{P_2}{P_1} \quad (\$) \tag{B}$$

Where $C_c$ and $C_r$ are the first costs of the centrifugal and reciprocating compressors, respectively, and $Q_0$ and $Q_1$ are the corresponding volume flow rates.

(a) Treating air as a perfect gas, develop the relevant analytical model and objective function for optimisation in terms of the pressures and flow rates appearing in Equations (A) and (B).
(b) Use Solver to determine the optimum intermediate pressures and the corresponding total initial cost of the system.

This exercise is based on Problem 8.5 in Stoecker [5], p 178, Answer: minimum cost = $24,100

4.  A gas turbine operates on a regenerative Brayton cycle with air entering the compressor at 100 kPa, 300 K. As shown in Figure P6.4, the hot exhaust gas is used to preheat the compressed air before the combustion chamber. The combustion gases leave the combustion chamber (CC) at 1400 K. Take the regenerator effectiveness as 80% and the isentropic efficiency of both compressor and turbine as 75%. Pressure losses can be neglected.
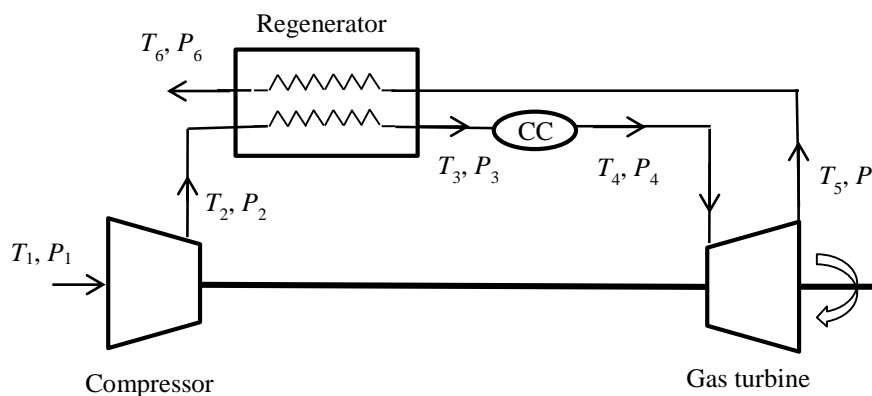


Figure P6.4. The regenerative gas-turbine system

(a) Treating the working fluid (air) as an ideal gas and using the exact method of analysis and the functions provided Thermax for ideal gases, develop a model that determines the thermal efficiency of the regenerative gas-turbine cycle.
(b) By varying the pressure ratio from 2 to 16, study the effect of pressure ratio on the thermal efficiency of the plant.
(c) Use Solver to determine the pressure ratio that maximizes thermal efficiency.

5.  Consider the steam power plant that operates with one closed feedwater heater as shown in Figure P6.5. Steam enters the high-pressure turbine at 15 MPa and 600°C where it expands to a pressure of 700 kPa after which it is returned to the steam generator house for reheating to a temperature of 550°C. The steam then expands

in the low-pressure turbine and then condensed in the condenser at a pressure of 10 kPa. A fraction $y$ of the steam leaves the low-pressure turbine at an intermediate pressure $P_i$ which is passed to the closed feedwater heater.
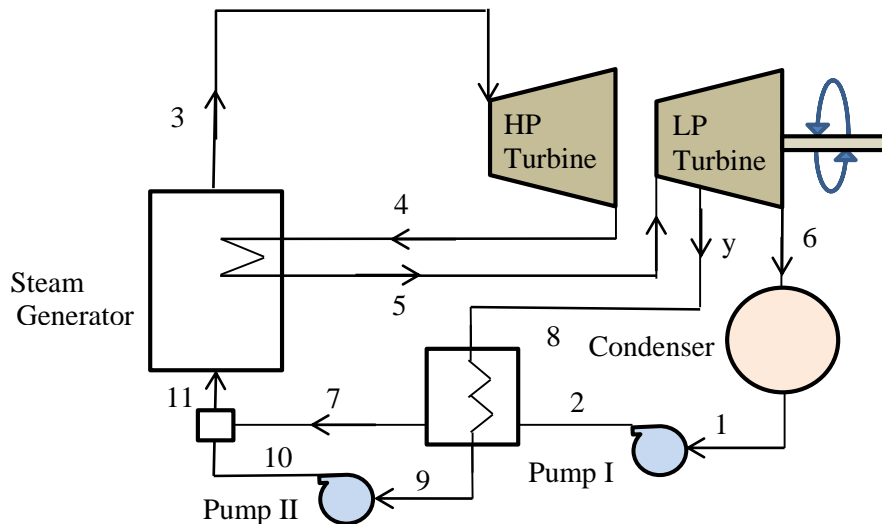


Figure P6.5. Schematic diagram of a steam-turbine power plant with superheating and reheating

Using reasonable upper and lower limits on the value of $P_i$, develop an Excel-Thermax sheet to determine the value of $P_i$ that maximises the thermal efficiency of the cycle and determine the corresponding fraction $y$ of steam extracted from the turbine.

6. Two identical pumps operate in parallel as shown in Figure P6.6 to deliver a total of 0.01 m³/s of water at 15°C. The head losses $h_{L1}$, $h_{L2}$ in the lines depend upon volumetric flow rates $Q_1$, $Q_2$ according to the following formulae:

$$h_{L1} = 2.1 \times 10^7 Q_1^2 \text{ kPa} \qquad\qquad\qquad\qquad (A)$$

$$h_{L2} = 3.6 \times 10^7 Q_2^2 \text{ kPa} \qquad\qquad\qquad\qquad (B)$$

Where $Q_1$ and $Q_2$ are the respective flow rates in cubic meters per second.

(a) Develop the mathematical model for determining the flow rates delivered by the two pumps that minimize the total pumping power.
(b) Develop the Excel sheet with appropriate water properties and use Solver to determine the optimum values for the pumps flows.
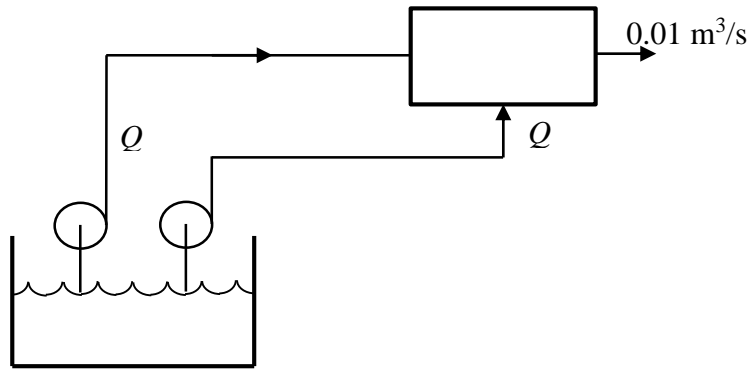
Figure P6.6. Water delivery system with two pumps in parallel

This exercise is based on Problem 8.3 in Stoecker [5], p. 177. Answer: $Q_1$=0.00567 m$^3$/s, $Q_2$=0.00433 m$^3$/s.

Hint:

$$\dot{W}_P = \rho g(Q_1 h_{L1} + Q_2 h_{L2})/1000 \text{ kW} \tag{C}$$

7.  Figure P6.7 shows a circular air duct system is to be design based on the following data:

$Q_1 = 2.4$ m$^3$/s, $Q_2 = 0.6$ m$^3$/s, $Q_3 = 1.0$ m$^3$/s, $Q_4 = 0.8$ m$^3$/s,
Quantity of sheet metal available for the system, 60 m$^2$.
Use a constant friction factor $f = 0.02$
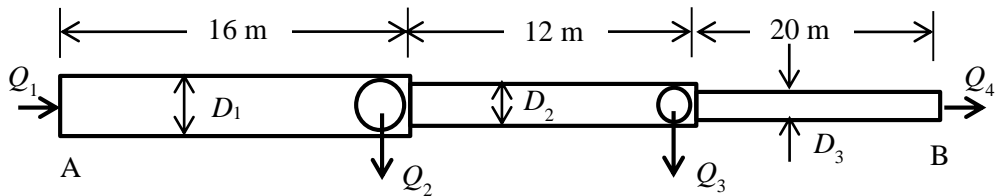Air density $\rho$, 1.2 kg/m$^3$



Figure P6.7. The air duct system

It is required to determine the values of the three diameters $D_1$, $D_2$ and $D_3$ that minimize the drop in static pressure between points A and B.

(a)  Set up the objective function and constraints in terms of $D_1$, $D_2$ and $D_3$.
(b)  Using Excel and Solver, determine the optimal diameters and the minimum total head loss.

Neglect the influence of changes in velocity pressure
Neglect the pressure drop in the straight section past an outlet

This exercise is based on Problem 8.9 in Stoecker [5], p 182. Answer: $D_1 = 0.468$ m $D_2 = 0.426$m and $D_3 = 0.325$ m. $h_f = 17.203$ m.