Modèle markovien d'octroi de crédit en microfinance

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Rubrique

Markovian model for granting credit in microfinance

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RÉSUMÉ. En partant du modèle généralisé de Osman Khodr et Francine Diener [1], nous présentons un nouveau modèle qui répond aux attentes de l'institution de microfinance (IMF) et celle des emprunteurs et qui incorpore toutes les caractéristiques des populations pauvres, à savoir la tolérance en cas de défaut partiel et la possibilité d'avoir un prêt progressif de façon automatique. Ce modèle offrira aux institutions de microfinance un outil d'aide à la décision plus adapté à la réalité de la microfinance. Notre chaîne de Markov comprend plusieurs états associés à la situation économique de l'emprunteur dont trois types de bénéficiaires B_1 (état d'être bénéficiaire au temps $t = 0$), B_2 (état d'être bénéficiaire au temps $t = 1$) et I (état d'inclusion financière: bénéficiaire permanent), un état de demandeur A^1 et A^{T-1}($(T - 1)$ états d'exclus). Nous avons modélisé le comportement d'un emprunteur par un paramètre λ qui dépend de la probabilité α de réussite de l'emprunteur. A l'instant initial, $\lambda = \frac{1+\alpha}{1-\alpha}$, cette quantité change dès que l'emprunteur passe d'un état à un autre avec une probabilité de réussite différente de α. La décision de l'agence d'accorder un crédit dépend entièrement du paramètre λ qui est comparé aux valeurs-seuils subjectives fixées. La chance γ d'avoir un prêt (γ: probabilité de demande de crédit accordée) pour un emprunteur est fonction du paramètre λ, avec $\gamma = 1 - \frac{1}{\lambda}$.

ABSTRACT. Starting from the generalized model of Osman Khodr and Francine Diener [1], we present a new model that meets the expectations of the microfinance institution (MFI) and that of the borrowers and that incorporates all the characteristics of the poor, namely tolerance in case of partial default and the possibility of having a progressive loan automatically. This model will provide microfinance institutions with a decision support tool that is better adapted to the reality of microfinance. Our Markov chain consists of several statements associated with the economic status of the borrower including three types of recipients B_1 (state of being beneficiary at a time $t = 0$), B_2 (state to be beneficiary at a time $t = 1$) and I (state of financial inclusion: permanent beneficiary), an applicant state A^1 and A^{T-1} ($(T - 1)$ excluded states). We modeled a borrower's behavior by a λ parameter that depends on the borrower's α probability of success. At the initial time, $\lambda = \frac{1+\alpha}{1-\alpha}$, this quantity changes as soon as the borrower moves from one state to another with a probability of success different from α. The agency's decision to grant a credit depends entirely on the λ parameter which is compared to the set subjective threshold-values. The chance γ to have a loan (γ: probability of credit request granted) for a borrower depends on the parameter λ, with $\gamma = 1 - \frac{1}{\lambda}$.

MOTS-CLÉS : Microfinance, Décision d'octroi de crédit, chaîne de Markov, Prêt individuel, Incitation dynamique, Profit espéré actualisé

KEYWORDS : Microfinance, Credit Grant Decision, Markov Chain, Individual Loan, Dynamic Incentive, Updated Expected Profit

1. Introduction

The microfinance institution (MFI) is an important lever for the economic development of the poor. Despite this, some populations are not yet served by the system and others, already customers, are entering a vicious circle of debt. Seen from this angle, is it not necessary that it must question their model of granting credit to achieve not only this ultimate goal but also its sustainability ? Inspired by the learn method studied of study by the economist G.A. Tedeschi [7], Osman Khodr and Francine Diener [2] built an individual loan model using the spirit of successive loans as a dynamic incentive. Indeed, in the model of Osman Khodr and Francine Diener, the MFI is threatened with two types of non-repayment : strategic defects or defects resulting from an economic shock. According to this model, the loan agreement provides incentives to discourage borrowers from following strategic flaws while considering an inevitable economic shock. This model presents repayment incentives as the repeated interaction between borrower and lender : each repayment of the loan at maturity leads to a new loan contract while excluding the defaulting borrower, for some time, from the lending activity. Lansana Bangoura [5] stresses the importance of this to better manage credit and to overcome the difficulties faced by the microfinance institution (MFI). In this regard, he proposes as a possible solution the monetary bonus for the effort related to loan repayments.

The behavior of the borrower is unpredictable. This unpredictability makes the reality more complex. But a model is good if it makes this complexity simpler and understandable. The generalized model of Osman Khodr and Francine Diener does not take into account all the more advantageous techniques that can be applied in the lending activity. Their model incorporates no flexibility in the decision : a zero-tolerance model for the defaulting party (exclusion for a certain period of time), whether partial default or total default, and a loan model of the same size for all future stages of the decision for a successful borrower. It is therefore a question of extending the existing model for broad coverage and protection of participants. Thus, our objective is to build an individual loan model that meets the expectations of the MFI (dynamic incentive) and those of the borrowers (secure their daily life through a flexible offer adapted to their capacity to counter the permanent risk of falling into the state of indigence). This model will take into account all the characteristics of the poor and stigmatize them. Through this model, we hope to offer institutions a tool for decision-making more adapted to the reality of everyday life. We consider here the non-monetary effort bonus presented in Lansana Bangoura's work as an automatic increase in the amount of credit and a fall in interest rate. Like the Khodr and Diener model, we will use the Markov chain theory that we will not present here to achieve the desired result.

Hypotheses describing all the possible states at maturity and the different procedures applied according to the investment result, will be presented in the next section. Here we have taken up, among them, three hypotheses of Khodr and Diener. We will present in the third section our model of individual loan. We will finish our work by presenting the expected profit of the participants followed by a discussion.

2. Hypotheses

In addition to the assumptions made in the model of Khodr and Diener, as for our model, other hypotheses have been added :

– (**H1**) : The MFI finances only existing activities. Any credit request for a new project is not admissible.

– (**H2**) : The expected gross wealth $w_s = (w_1, w_2, ..., w_S)$ is a random variable dependent on the states of nature.

– (**H3**) : The interest rate r, $(0 < r \leq 1)$ may vary according to the state of nature (a low rate for a borrower who enters the circle of the permanent beneficiary (state I)). Let's note r_s the interest rate for a state's of nature.

– (**H4**)[2] : The borrower is successful if he pays back all of his loan, charged an interest rate r_s fixed at the time of the contract and automatically receives a new loan from an additional unit or from the same unit for the next period.

– (**H5**)[2] : The borrower is in default if he has not repaid, therefore, he will not receive a new loan for following $T(T \geq 1)$ periods.

– (**H6**)[2] : After the exclusion, he applies for a new loan and his chance depends on the number of eligible applicants and the limit on the number of borrowers in the loan portfolio. For the first period following the exlusion let's note their application acceptance probability as γ $(0 \leq \gamma \leq 1)$, while $1 - \gamma$ is the probability that his candidacy will be postponed for the next period, and so on.

– (**H7**) : It is only in the first contract that the IMF tolerates, at most, $d(1 \leq d \leq 3)$ partial defaults of repayement for a lump sum p as a penalty for each defect. If the borrower exceeds the number d authorized defects, they will be excluded for a period of time. Otherwise, a renewal of a new loan of the same size will be possible.

From these assumptions and from the fact that the borrower predicts his future from his present state but not from his past, we model the different states of a borrower by a Markov chain. The use of the Markovian process allows us to evaluate the expected inter-temporal profit of the borrower.

3. The model

3.1. Related works

Many economists have worked on dynamic lending incentives. Among them, we quote Hulme and Mosley [9], Armendariz de Aghion and Morduch [8], and Ghosh and Van Tassel [6] who, in their studies, built exclusively two-period models . Lansana Bangoura [5] builds an optimal loan contract model based on an effort oriented bonus to mitigate the problem of moral hazard and adverse selection. Tedeschi [7], for his part, adapted the two-step dynamic of Green and Porter [10] to build a model that takes into account all future stages of the borrower. For their part, Nahla Dhib, Francine Diener and Marc Diener [3] presented a four-state model that mathematically models, with the Markov chain, the evaluation of the impact of microcredit to reach a large number of individuals with low income. Convinced that some aspects of this will advance research in the field of mathematical modeling in microfinance, Osman Khodr and Francine Diener [2] have generalized the Tedeschi [7] model and the four state model of Nahla Dhib, Francine Diener and Marc Diener [3] while automatically renewing the loan and keeping the same size, when the borrower does not default and in the oppositi case nothing. Osman Khodr and Francine Diener put in their model the emphasis on determining the optimal contract between the borrower and the lender. They assume that, in all future stages, the contracting parties have an activity that lasts for several periods.

As this is an extension of the model of Osman Khodr and Francine Diener [2], our model stands out for its ability to better explain, on the one hand, the behavior the MFI should have in its moves to offer credit or loan and build a stable relation of trust with its customer, and, on the other hand, that of the business person in order to be able to benefit from the possibility a progressive and continuous financing if their project.

3.2. Presentation of the model

In entrepreneurship, it is universally accepted that the use of borrowed funds (combined funds : equity and borrowed funds) generates more profits than passed to equity. Thus, to undertake an income-generating activity, obtaining credit for financing is a prerequisite for success. On the other hand, to get a loan contract is to take risks related to the project to be financed. These risks are sometimes predictable and unpredictable hazards (states of nature) that interfere with the credit decision and shape the behavior of borrowers to meet their responsibility and commitment.

Let's note S the number if nature states. For each nature state s, two cases can arise, either the borrower repays his loan, or he fails. Our model provides a partial fault tolerance (Hypothesis **H7**), but the total failure costs it an exclusion for several periods and places it in a plaintiff state. Thus, the S states of nature are as follows :

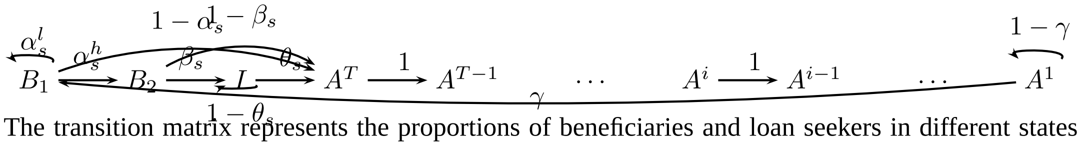
$$S = \{B_1, B_2, I, A^T, A^{(T-1)}, \dots, A^1\}$$

Where B_1 represents the profit state of the first loan, B_2 that of the second loan of the same unit as the previous one or more than one additional unit according to the needs of the contractor $(u(B_2) \geq u(B_1)$ where $u(B_1)$ denotes the unit loaned to the state B_1 (according to the Hypothesis **H5**). I is the state where the borrower automatically enters the circle of the permanent beneficiary. As for A^T, it is the state where the contractor will be excluded for T periods as a result of exceeding the number of defects d authorized repayment and A^1 is the state of a loan applicant. Since the decision to grant credit depends only on the beneficiary's recent past, the exclusive use of Markovian theory as a methodological tool has been of great help. In this respect, we have assigned to each state of nature a probability called the Markov chain probability describing all possible states of the beneficiary during the contract. Thus, the stochastic matrix P of the chain from one state to another is :

$$P = \begin{pmatrix} \alpha_s^l & \alpha_s^h & 0 & 1-\alpha_s & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \beta_s & 1-\beta_s & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1-\theta_s & \theta_s & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ \gamma & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1-\gamma \end{pmatrix}$$

And the following figure represents the Markov graph associated with the P pass matrix

Figure 1. *Markov graph with probabilities of transition from one state to another*



The transition matrix represents the proportions of beneficiaries and loan seekers in different states of nature. We are also seeking to know how these proportions will evolve over time. Moreover, institutions that provide business person with the means of financing their micro business are seeking a system of balance of credit. This system of balance allows the agency not to put too much emphasis on one parameter at the expense of another (for example, for a given MFI, increasing the probability α can be summed up as not granting only when it can secure a high chance of project success, but it will be at the expense of the γ probability since more credit applications would then have to be rejected). However, the rejection of a greater proportion of demands is displacing MFIs from their rationale : to promote access to finance for the entire poor population. The income that a borrower will earn on a unit invested is generally uncertain. The model therefore offers the borrower the default policy prevention constraint and the continvity constraint. This behavior is studied according to the states that occur in the sequence $(X_1, X_2, \cdots, X_n)$ of n transitions. The "Beneficiary-Beneficiary" event is a transition from having a loan and staying there. Let $\Omega = BB$ be this event and λ the parameter that models the behavior of a borrower, with $\lambda = \frac{N_\Omega}{N_B}$ where N_B is the number of times to have a loan in the sequence, $N_B = \sum_{n\geq 1} 1_{\{X_n = B\}}$ and N_Ω is the number of times to be successful in the sequence, $N_\Omega = \sum_{n\geq 1} 1_{\{X_n=B, X_{n+1}=B\}}$ and 1 is an indicator function.

Let's set $\tau = inf\{n \geq 1 : X_n = B\}$ and put the three states where the borrower has credit into a B state ($B = \{B_1, B_2, I\}$). Let S' denote the new states with $S' = \{B, A^T, A^{(T-1)}, \ldots, A^1\}$. At the initial time, the chain chooses a state according to the initial law μ, note B this state and stays there for a duration of geometric law on $\mathbb{N}^*$ of parameter $q(B, B)$. When this time has elapsed, the chain randomly chooses a new state following the probability $q(B, .)/(1 - q(B, B))$, with $q(B, B) \neq 1$.

Proposition 1. *: Let $(X_n, n \geq 1)$ a Markov chain with values in S.*

Let's put $S' = \{A^T, A^{(T-1)}, \ldots, A^1\}$, for $k \geq 2$, we have : $\{\tau = k\} = \{X_1 \in S'', \cdots, X_{k-1} \in S'', X_k = B\}$.

$$\begin{aligned}\mathbb{P}_B(\tau = k) &= \mathbb{P}_B(X_1 \in S'', \cdots, X_{k-1} \in S'', X_k = B)\\ &= (1-q)^{k-2}q\end{aligned}$$

So

$$\begin{aligned}\mathbb{E}(\tau_B) &= \textstyle\sum_{n \geq 2} k(1-q)^{k-2}q\\ &= \textstyle\sum_{n \geq 1}(k+1)(1-q)^{k-1}q\\ &= q(\textstyle\sum_{n \geq 1} k(1-q)^{k-1} + \sum_{n \geq 1}(1-q)^{k-1})\\ &= q(\tfrac{1}{q^2} + \tfrac{1}{q})\\ &= \tfrac{q+1}{q}\end{aligned}$$

From this $N_B \longmapsto \frac{q}{q+1}$ when $n \to \infty$.

Let E_k^B be the k^{th} entry time in B and let D_k^B be the duration of the k^{th} residence time in B. D_k^B is the integer N_Ω which checks : $\{X_{E_k^B} = B, \cdots, X_{E_k^B+i-1} = B, X_{E_k^B+i} \neq B\}$. $X_{E_k^B+D_k^B}$ is the place where the chain jumps out of B. With these ratings, we have :

$$\begin{aligned}\mathbb{P}_B(X_{E_k^B} = B, \cdots, X_{E_k^B+i} = B, X_{E_k^B+i+1} \neq B / E_k^B < \infty) &= \mathbb{P}_B(D_k^B = i+1 / E_k^B < \infty)\\ &= q^k(B,B)(1-q(B,B))\end{aligned}$$

So

$$\begin{aligned}\mathbb{E}(D_k^B) &= \textstyle\sum_{n \geq 1} kq^k(B,B)(1-q(B,B))\\ &= (1-q)q\textstyle\sum_{n \geq 1} kq^{k-1}\\ &= \tfrac{(1-q)q}{(1-q)^2}\\ &= \tfrac{q}{1-q}\end{aligned}$$

Hence $N_B \longmapsto \frac{q}{1-q}$ when $n \to \infty$. The parameter λ thus prends its value by the data of the parameter q. So, $\lambda = \frac{q+1}{1-q}$. By setting two thresholds λ_{min} and λ_{max}, she will make her decision as follows :

1) if $\lambda < \lambda_{min}$, the borrower will be excluded from the lending activity for T following periods (Assumptions **H5 et H6**).

2) if $\lambda > \lambda_{max}$, the borrower will receive a new loan of a higher amount according to his needs and/or benefit from a reduced rate (Hypothèses **H3 et H4**).

3) if $\lambda \in [\lambda_{min}, \lambda_{max}]$, the borrower will continue to benefit from the loan of the same amount and the same interest rate (Hypothesis **H7**).

3.3. Constraints imposed on model variables

According to the study by Osman Khodr and Francine Diener, three types of constraints are assigned to their model, namely : the participation constraint, the constraint of preventing the default strategy and the continuity constraint. As for our model, no change has been made to these constraints.

3.3.1. The constraint of participation

One of the necessary and sufficient conditions for granting credit is the financial viability of the project. A project is financially viable if the expected wealth will largely cover the loan repayment. Thus, assuming that the expected gross wealth in the s state of nature, of a unit invested, is $W_s (W_s \geq 0)$ positive on success and zero otherwise the participation constraint is as follows :

$$\begin{pmatrix} W_1 \\ \vdots \\ W_S \end{pmatrix} \geq \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} + \begin{pmatrix} r_1 \\ \vdots \\ r_S \end{pmatrix}$$

For the sake of simplification, we simply write : $W_s \geq 1 + r_s$.

3.3.2. The default policy prevention constraint

Given the high risk of non-repayment in microfinance, it seeks to protect itself while creating a dynamic incentive strategy : automatically benefit from a new loan in case of repayment and exclude for T periods in case of default . For a rational borrower who is risk averse, the restraint constraint that is interpreted as an opportunity cost is written :

$$W_s - (1 + r_s + C) + \delta V(s/s-1 = B) \geq W_s + \delta V(s/s-1 = A^T)$$

Where C is the incompressible consumption, δ the discount rate, $V(s/s-1 = B)$ the expected net future profit of a loan recipient, and $V(s)/s-1 = A^T)$ is that of an excluded for T periods.

The business person must choose after calculating the opportunity cost of continuing to be a credit recipient or falling into the applicant's state where he or she may remain for a lifetime. The establishment of the central credit risk allows to strengthen the default strategy prevention.

3.3.3. The continuity constraint

As a financial intermediary, each credit grant of one unit costs the agency, in a s state of nature, a certain sum z_s. In order to ensure the continuity of the loan activity, the expected reimbursement amount must cover the cost of the loan. Thus, the continuity constraint results :

$$\begin{pmatrix} \alpha_1 & \cdots & \alpha_S \end{pmatrix} \begin{pmatrix} 1 & & r_1 \\ \vdots & + & \vdots \\ 1 & & r_S \end{pmatrix} \geq \begin{pmatrix} 1 & & z_1 \\ \vdots & + & \vdots \\ 1 & & z_S \end{pmatrix}$$

Where α_s represents the probability of success of the borrower's project in the s state of nature. α_s is the probability that depends on the state s and values the outcome of the contractor's project. Thus, it is determined by the data of the $y(W_s, r_s)$ function of the net profit, from a unit loaned $u(s)$ to a possible state s invested in the project, before tax of the contractor : $\alpha_s = \frac{y(W_s, r_s)}{u(s)} = \frac{W_s - (1 + r_s + C)}{u(s)}$ [3] [1].

4. The participants expected profit

4.1. At the level of the borrower

4.1.1. Enjoy a period

If we consider a project for which we are considering S possible states of nature (eg favorable and neutral : possibility of repayment, unfavorable : default of repayment), with α_s the probability of success of this project and $1 - \alpha_s$ otherwise, the profit of a period, of a unit invested is :

$$\alpha_s[W_s - (1 + r_s) + C]$$

The approach consists of determining the different intertemporal benefits expected for all future periods and for each possible state of nature.

1. The authors evaluate the probability of success according to the investment result. We have adopted this formula by modifying certain aspects

4.1.2. Total profit return

Définition 1. *: Let $(X_t, t \geq 1)$ be a Markov chain. we assume that for any positive function, bounded f and for each trajectory $(X_m, X_{m+1}, \ldots)$ of the Markov chain, we define : Let $f : S * S \to \mathbb{R}$ describe the profit of an investment period, such as :*

$$f(s = B/s-1 = B_1) = \begin{cases} W_s - (1 + r_s + C + (p * d)) & \text{if } (s, s-1) = (B_1, B_1), d = 1 \text{ at } 3 \\ W_s - (1 + r_s + C) & \text{if } (s, s-1) = ((B_2, B_1) \text{and } (I, B_2)) \\ W_s - (1 + \bar{r_s} + C) & \text{if } (s, s-1) = (I, I) \\ 0 & \text{if not} \end{cases} \quad (1)$$

Where $\bar{r_s}$ is the reduced interest rate of a beneficiary who can build a stable relationship of trust with the MFI (with $\bar{r_s} \leq r_s$). This function represents the profit of passage between two states of the Markov chain.

Proposition 2. *: Let $(X_t, t \geq 1)$ be a Markov chain. We assume that for any positive function, bounded f and for each trajectory $(X_m, X_{m+1}, \ldots)$ of the Markov chain, we define :*

$$F(X_m, X_{m+1}, \ldots) = \sum_{t \geq 1} \delta^{t-m-1} f(X_{t-1}, X_t)$$

Demonstration : We will evaluate this function by using the Markov property and discounting the expected cash flow at the future of standard base period, with a discount rate $\delta (0 < \delta < 1)$, for this we note that :

$$\begin{aligned} F(X_m, X_{m+1}, \ldots) &= f(X_m, X_{m+1}) + \delta F(X_{m+1}, X_{m+2}, \ldots) \\ &= f(X_m, X_{m+1}) + \delta (f(X_{m+1}, X_{m+2}) + \delta F(X_{m+2}, X_{m+3}, \ldots)) \\ &= f(X_m, X_{m+1}) + \delta f(X_{m+1}, X_{m+2}) + \delta^2 (f(X_{m+2}, X_{m+3}) + \\ &\quad \delta F(X_{m+3}, X_{m+4}, \ldots)) \\ &= \sum_{t \geq 1} \delta^{t-m-1} f(X_{t-1}, X_t) \end{aligned}$$

Définition 2. *: Let $(X_t, t \geq 1)$ a Markov chain with values in a S state space compared to a filtration $(g_t; t \geq 1)$ transition matrix P, let m a time out compared to the same filtration, almost surely finished, the total net profit expected at time t depending on the state s of nature, $s \in S, (X_t = s)$ by $V_t : S \mapsto \mathbb{R}$, for any function f bounded is such that :*

$$V_t(s) = \mathbb{E}[F(X_t, X_{t+1}, \ldots, /X_t = s)] = V_0(s)$$

Indeed, we will proceed to the evaluation of the conditional expectation $\mathbb{E}[F(X_m, X_{m+1}, \ldots, X_{m+t}/g_m)]$. For any integer $t, \{m \leq t\}$ is a set of g_t or equivalently $\{m = t\}$ is a set of g_t and we suppose further that $\mathbb{P}(m < +\infty) = 1$. We note then that $\{m = k\}$ is in g_k and that $F(X_0, X_1, \ldots, X_t)$ is an independent random variable from g_k.

$$\begin{aligned} V_t(s/s-1) &= \mathbb{E}[F(X_m, X_{m+1}, \ldots, X_{m+t}/X_m = s-1)] \\ &= \sum_{k \geq 0} \mathbb{E}(\mathbb{P}(m = k) F(X_k, X_{k+1}, \ldots, X_{k+t}/X_k = s)) \\ &= f(X_m, X_{m+1}) + \delta f(X_{m+1}, X_{m+2}) + \delta^2 (f(X_{m+2}, X_{m+3}) + \\ &\quad (\sum_{k \geq 0} \mathbb{P}(m = k))(E(F(X_0, X_1, \ldots, X_t/X_0 = s))) \\ &= \sum_{t \geq 1} \delta^{t-m-1} f(X_{t-1}, X_t) \\ &= \mathbb{E}[F((X_0, X_1, \ldots, X_t/X_0 = s-1))] \\ &= V_0(s/s-1) \end{aligned}$$

This proves that $(X_m, X_{m+1}, \ldots, X_{m+t})$ has the same law as $(X_0, X_1, \ldots, X_t)$ and we can deduce that $V_t(s/s-1) = V_0(s/s-1)$. So just determine $V_0(s/s-1)$ for any state $s \in S$ of nature.

According to the proposal **(2)**,

$$V_0(s/s-1) = \mathbb{E}((f(X_0, X_1) + \delta F(X_1, X_2, \ldots))/X_0 = s-1)$$

Applying the conditional expectation and the Markov property, we obtain :

$$V_0(s/s-1) = \sum_{(s,s+1,\ldots\in S)}(f(s-1,s) + \delta V_0(s/s-1)) * \mathbb{P}_{s-1,s}$$

With $\mathbb{P}_{s-1,s} = \mathbb{P}((X_{t+1} = s/X_t = s-1))$ is the probability of transition, in a step, from state $s-1$ to state s. To determine the quantity $V_0(s/s-1)$, three cases can occur :

– For $\{s-1 = B_1\}$, two cases can occur :

- ***First case*** : if the borrower defaults either partially or completely, the only accessible states in the chain from the B_1 state are B_1 and A^T (i.e $s = B_1$ or A^T), with the following transition probabilities :

$$\mathbb{P}_{s-1,s} = \begin{cases} \alpha_s^l & \text{if } \lambda \in [\lambda_{min}, \lambda_{max}] \Rightarrow s = B_1 \\ 1-\alpha_s & \text{if } \lambda < \lambda_{min} \Rightarrow s = A^T \end{cases} \quad (2)$$

with α_s^h the probability associated with the total repayment of the debt and α_s^l that associated with the partial refund, and check the relations $\alpha_s = \alpha_s^l + \alpha_s^h$ and $\alpha_s^h > \alpha_s^l$.

So,

$$\begin{aligned} V_0(s/s-1 = B_1) &= (f(B_1,B_1) + \delta V_0(s = B_1/s-1 = B_1))\mathbb{P}_{B_1,B_1} + (f(B_1,A^T) \\ &+\delta V_0((s = A^T/s-1 = B_1))\mathbb{P}_{A^T,B_1} \\ &= \alpha_s^l(W_s - ((1+r_s) + C + (p*d)) + \delta V_0(s = B_1/s-1 = B_1)) \\ &+(1-\alpha_s)(\delta V_0(s = A^T/s-1 = B_1)) \end{aligned}$$

- ***Second case*** : the borrower repays in full his loan without default. The only accessible state from the state B_1 is B_2 with a transition probability $\mathbb{P}_{B_1B_2} = \alpha_s^h$ if of course $\lambda \in [\lambda_{min}, \lambda_{max}] \Rightarrow s = B_2$.

So,

$$\begin{aligned} V_0(s/s-1 = B_1) &= (f(B_1,B_2) + \delta V_0(s = B_2/s-1 = B_1))\mathbb{P}_{B_2,B_1} \\ &= \alpha_s^h(W_s - ((1+r_s) + C) + \delta V_0(s = B_2/s-1 = B_1)) \end{aligned}$$

– For $\{s-1 = B_2\}$, the only accessible states of the chain from the state B_2 are I and A^T (i.e $s = I$ or A^T), with following transition probabilities :

$$\mathbb{P}_{s-1,s} = \begin{cases} \beta_s & \text{if } \lambda \in [\lambda_{min}, \lambda_{max}] \Rightarrow s = I \\ 1-\beta_s & \text{if } \lambda < \lambda_{min} \Rightarrow s = A^T \end{cases} \quad (3)$$

Thus,

$$\begin{aligned} V_0(s/s-1 = B_2) &= (f(B_2,I) + \delta V_0(s = I/s-1 = B_2))\mathbb{P}_{I,B_2} + (f(B_2,A^T)) \\ &+\delta V_0((s = A^T/s-1 = B_2))P_{B_2,A^T} \\ &= \beta_s(W_s - (1+r_s) + \delta V_0(s = I/s-1 = B_2)) \\ &+(1-\beta_s)(\delta V_0(s = A^T/s-1 = B_2)) \end{aligned}$$

– For $\{s-1 = I\}$, the only accessible states of the chain from the state I are I and A^T (i.e $s = I$ or A^T), with the transition probabilities following :

$$\mathbb{P}_{s-1,s} = \begin{cases} 1-\theta_s & \text{if } \lambda > \lambda_{max} \Rightarrow s = I \\ \theta_s & \text{if } \lambda < \lambda_{min} \Rightarrow s = A^T \end{cases} \quad (4)$$

Thus,

$$\begin{aligned} V_0(s/s-1 = I) &= (f(I,I) + \delta V_0(s = I/s-1 = I))\mathbb{P}_{I,I} + (f(I,A^T) \\ &+\delta V_0((s = A^T/s-1 = I))\mathbb{P}_{I,A^T} \\ &= (1-\theta_s)(W_s - (1+r_s) + \delta V_0(s = I/s-1 = I)) \\ &+\theta_s(\delta V_0(s = A^T/s-1 = I)) \end{aligned}$$

– For $\left\{s-1 = A^i,\ i = T \text{ to } 2\right\}$, the chain stays for $T-1$ periods in its current state before the to leave. Thus, the only accessible state from the state A^i is A^{i-1}, with an almost sure probability worth 1 ($\mathbb{P}_{A^i,A^{i-1}} = 1$).

– For $\left\{s-1=A^1\right\}$, according to the hypothesis (**H6**), we have γ the transition probability from A^1 to $B_1(\mathbb{P}_{A^1,B_1}=\gamma)$ and $1-\gamma$ to stay in A^1 $(\mathbb{P}_{A^1,A^1}=1-\gamma)$. We will study for the Markov chain $(X_t, t\geq 1)$ on a state space S the time spent in the state A^1 before leaving it. We suppose to fix the ideas $X_0=A^1$ almost surely and we write $\tau=\{inf(t)\geq 1, X_t\neq A^1\}$ the first moment to be a loan recipient after being in the A^1 state, then τ is a time out.
We have of course, for $t=1$:

$$\mathbb{P}_{A^1}(\tau=1)=\mathbb{P}_{A^1}(X_1\neq A^1)=1-p(A^1,A^1)=1-\gamma$$

And more generally, for$t\geq 2$:

$$\begin{aligned}\mathbb{P}_{A^1}(\tau=t) \quad &=\mathbb{P}_{A^1}(X_1=A^1,X_2=A^1,\ldots,X_{t-1}=A^1,X_t\neq A^1)\\ &=\textstyle\sum_{k\geq 0}\mathbb{E}(\mathbb{P}(m=k)F(X_k,X_{k+1},\ldots,X_{k+t}/X_k=s))\\ &=\mathbb{P}_{A^1}(X_1=A^1,X_2=A^1,\ldots,X_{t-1}=A^1_t)\\ &-\mathbb{P}_{A^1}(X_1=A^1,X_2=A^1,\ldots,X_{t-1}=A^1,X_t=A^1)\\ &=((p(A^1,A^1))^{t-1})-((p(A^1,A^1))^t)\\ &=((p(A^1,A^1))^{t-1})(1-p(A^1,A^1))=((1-\gamma)^{t-1})\gamma\\ &=V_0(s/s-1)\end{aligned}$$

The law of τ is therefore a geometrical law. As $F(X_0,X_1,\ldots)=\delta^\tau F(X_\tau,X_{\tau+1},\ldots)$ and after this who is before :

$$\begin{aligned}\mathbb{P}_{A^1}(\tau=t) \quad &=\mathbb{E}(F(X_0,X_1,\ldots)/X_0=A^1)=A^1)\\ &=\mathbb{E}(\mathbb{E}(\delta^\tau F(X_\tau,X_{\tau+1},\ldots)/X_0=A^1,\ldots,X_{\tau-1}=A^1,X_\tau=s)/X_0=A^1)\\ &=\mathbb{E}(\mathbb{E}(\delta^\tau F(X_\tau,X_{\tau+1},\ldots)/X_\tau=s)/X_0=A^1)\\ &\mathbb{E}(\delta^\tau V_0(s/s-1=B)/X_0=A^1)\\ &=\mathbb{E}(\delta^\tau V_0(s/s-1=B))\\ &=((p(A^1,A^1))^{t-1})(1-p(A^1,A^1))=((1-\gamma)^{t-1})\gamma\\ &=V_0(s/s-1=B)E(\delta^\tau)\end{aligned}$$

Gold $\mathbb{E}(\delta^\tau)=\sum_{k\geq 1}\delta^k(1-\gamma)^{k-1}\gamma=\delta\gamma\sum_{k\geq 1}(\delta(1-\gamma))^{k-1}=\frac{\delta\gamma}{1-\delta(1-\gamma)}$, by crossing to the limit.

To achieve the desired result, the quantity must then be calculated $V_0(s/s-1=B_1)$.

$$\begin{aligned}V_0(s/s-1=B_1) \quad &=\alpha_s^l(W_s-((1+r_s)+C+(p*d))+\delta V_0(s=B_1/s-1=B_1))\\ &+(1-\alpha_s)(\delta V_0(s=A^T/s-1=B_1))\\ &=\alpha_s^l(W_s-((1+r_s)+C+(p*d)))+\alpha_s^l\delta V_0(s/s-1=B_1)\\ &+(1-\alpha_s)\delta^T V_0(s/s-1=B_1)E(\delta^\tau)\end{aligned}$$

$$\begin{aligned}V_0(s/s-1=B_1) \quad &=\alpha_s^l(W_s-((1+r_s)+C+(p*d)))\frac{1}{(1-\alpha_s^l\delta)-(1-\alpha_s)\delta^T\mathbb{E}(\delta^\tau)}\\ &=\frac{1}{1-\alpha_s^l\delta-(1-\alpha_s)\delta^T\frac{\delta\gamma}{1-\delta(1-\gamma)}}\alpha_s^l(W_s-((1+r_s)+C+(p*d)))\\ &=\frac{1}{\frac{(1-\alpha_s^l\delta)(1-\delta(1-\gamma))-\gamma(1-\alpha_s)\delta^{T+1}}{1-\delta(1-\gamma)}}\alpha_s^l(W_s-((1+r_s)+C+(p*d)))\\ &=\frac{1-\delta(1-\gamma)}{(1-\alpha_s^l\delta)(1-\delta(1-\gamma))-\gamma(1-\alpha_s)\delta^{T+1}}\alpha_s^l(W_s-((1+r_s)+C+(p*d)))\end{aligned}$$

And

$$\begin{aligned}V_0(s/s-1=B_1)= \quad &\alpha_s^h(W_s-((1+r_s)+C)+\delta V_0(s=B_2/s-1=B_1))\\ &=\frac{\alpha_s^h}{1-\delta\alpha_s^h}(W_s-((1+r_s)+C))\end{aligned}$$

By doing the same for $V_0(s/s-1=B_2)$ and $V_0(s/s-1=I)$, we get :

$$V_0(s/s-1=B_2)=\frac{1-\delta(1-\gamma)}{1-\delta(1-\gamma)-\beta_s\delta(1-\delta(1-\gamma))+\gamma(1-\beta_s)\delta^{T+1}}\beta_s(W_s-(1+r_s+C))$$

$$V_0(s/s-1=I)=\frac{1-\delta(1-\gamma)}{(1-\delta(1-\gamma))(1-\delta(1-\theta_s))-\gamma\theta_s\delta^{T+1}}(1-\theta_s)(W_s-((1+\bar{r_s})+C))$$

Where, finally :

Théorème 1. *:*

In the individual loan model defined by the Markov chain $(X_t, t \geq 1)$ with values in a state space S, the total expected profit of a borrower who is in the s state at the time t is given by :

$$V_t(s/s-1=B) = \begin{cases} \textit{if}\ (s,s-1)=(B_1,B_1),\ d=1\ \textit{at}\ 3\ : \\ \frac{1-\delta(1-\gamma)}{(1-\alpha_s^l\delta)(1-\delta(1-\gamma))-\gamma(1-\alpha_s)\delta^{T+1}}\alpha_s^l(W_s-((1+r_s)+C+(p*d))) \\ \textit{if}\ (s,s-1)=(B_2,B_1)\ : \\ \frac{\alpha_s^h}{1-\delta\alpha_s^h}(W_s-((1+r_s)+C)) \\ \textit{if}\ (s,s-1)=(I,B_2)\ : \\ \frac{1-\delta(1-\gamma)}{1-\delta(1-\gamma)-\beta_s\delta(1-\delta(1-\gamma))+\gamma(1-\beta_s)\delta^{T+1}}\beta_s(W_s-(1+r_s+C)) \\ \textit{if}\ (s,s-1)=(I,I)\ : \\ \frac{1-\delta(1-\gamma)}{(1-\delta(1-\gamma))(1-\delta(1-\theta_s))-\gamma\theta_s\delta^{T+1}}(1-\theta_s)(W_s-((1+\bar{r_s})+C)) \\ \textit{if}\ (s,s-1)=(A^i,I), i=\overline{T,1}\ : \\ \frac{\gamma\delta^i}{(1-\delta(1-\gamma))(1-\delta(1-\theta_s))-\gamma\theta_s\delta^{T+1}}(1-\theta_s)(W_s-((1+\bar{r_s})+C)) \end{cases} \quad (5)$$

Since the transition probabilities do not depend on time, the total expected profit is a random variable function dependent on the possible states of nature (Hypothesis **H2**) and the interest rate r_s.

4.2. At the level of the MFI

Through the dynamic incentive, for a borrower who repays at each time its loan, the credits are granted in a repetitive and progressive way, for amounts adapted to the capacities of its management. On the other hand, the MFI stops lending to the one who exceeds the number of allowed defaults. By nature, microfinance has to cover its costs with the proceeds of their activities. To achieve this, among the criteria for granting loans, the MFI focuses on the expected future profit of a loan applicant to guard against the risk of non-repayment. By granting credit only to the so-called good borrowers, the total expected profit of an MFI is determined as the result of their intertemporal investments.

Corollaire 1. *: In the individual loan model defined by the Markov chain $(X_t, t \geq 1)$ with values in a state space S, the expected total profit of the MFI that is in the state s at time t is given by :*

$$V_t(s/s-1=B) = \begin{cases} \textit{if}\ (s,s-1)=(B_1,B_1),\ d=1\ \textit{at}\ 3\ : \\ \frac{1-\delta(1-\gamma)}{(1-\alpha_s^l\delta)(1-\delta(1-\gamma))-\gamma(1-\alpha_s)\delta^{T+1}}(\alpha_s^l(1+r_s)-(1+z_s)) \\ \textit{if}\ (s,s-1)=(B_2,B_1)\ : \\ \frac{\alpha_s^h}{1-\delta\alpha_s^h}(\alpha_s^h(1+r_s)-(1+z_s)) \\ \textit{if}\ (s,s-1)=(I,B_2)\ : \\ \frac{1-\delta(1-\gamma)}{1-\delta(1-\gamma)-\beta_s\delta(1-\delta(1-\gamma))+\gamma(1-\beta_s)\delta^{T+1}}(\beta_s(1+r_s)-(1+z_s)) \\ \textit{if}\ (s,s-1)=(I,I)\ : \\ \frac{1-\delta(1-\gamma)}{(1-\delta(1-\gamma))(1-\delta(1-\theta_s))-\gamma\theta_s\delta^{T+1}}((1-\theta_s)(1+\bar{r_s})-(1+z_s)) \\ \textit{if}\ (s,s-1)=(A^i,I), i=\overline{T,1}\ : \\ \frac{\gamma\delta^i}{(1-\delta(1-\gamma))(1-\delta(1-\theta_s))-\gamma\theta_s\delta^{T+1}}((1-\theta_s)(1+\bar{r_s})-(1+z_s)) \end{cases} \quad (6)$$

The proof of this corollary is the same as the preceding theorem.

4.3. Algorithmic illustration

Let W_s^0, C, r_s and u_s input values.

1) Calculate α_s

2) Calculate λ and test the threshold value.

3) Funding Decision (Granted or Not Granted).

4) Refund test :

a) If $d > 4$ then the borrower is excluded ;

b) If $d \in [1, 3]$ then calculate $V(s, s-1)$ and go to step 5a ;

c) If $d = 0$ then calculate $V(s', s-1)$ and go to step 5b.

5) New funding request :

a) $W_s^t = W_s^{t-1} + V(s, s-1)$

b) $W_s^t = W_s^{t-1} + V(s', s-1)$

6) Go back to step 1 and so on.

5. Discussions

At the initial moment, the borrower submits his financing request to the institution of his free choice. The institution in turn undertakes to scrutinize the eligibility of the application. At this point, the parameter λ modeling the future behavior of the borrower allows you to choose between the state B_1 or A^1 where the borrower will land. The probability of applying for credit granted depends on γ, with $\gamma = 1 - \frac{1}{\lambda}$ (the existence condition of λ was given in the previous section) . The higher the λ parameter, the greater the chance of the borrower having a loan. The value of γ has hitherto been estimated from historical data by the institution. As for us, we assign a value to this parameter based on the future success of the borrower. Our model therefore leaves no guesswork as to the values taken from its parameters. By estimating the expected gross wealth W_s in $t + 1$, our model is similar to a decision tree that analyzes itself beginning with the end and going back in time.

We found that $V(A^1, I) <<< V(B_1, B_1) < V(B_2, B_1) < V(I, B_2) < V(I, I)$ in both camps. Results supported by those of Osman Khodr and Francine Diener (2015) [2]. On the other hand, our results are opposed to those found by Nahla Dhib, Francine Diener and Marc Diener (2013) [3] who in their four states model affirm that, whatever the factor of actualization, "for a micro business person with diminishing returns, his flow in the initial state is always profitable, while he realizes a V value in the weak and sometimes negative I state ".

Our model thus constructed satisfies all the hypotheses presented in this article. Our results show that the Microfinance institution is in symbiosis with its customers. Therefore, in order to maintain this relationship in perpetuity, microfinance is always looking for a stable credit granting system. The stability of the system means that the initial probability law towards which the system evolves remains unchanged over time. The Markov chain watched from the initial moment has the same law as the Markov chain watched from any instant t. We have therefore constructed a model tending towards a stationary distribution whatever the initial distribution of the beneficiaries, a result which is confirmed by those of Osman Khodr (2011) [4].

Every model is not free of limits. Our model does not claim in any way to remove the uncertainty associated with the valuation of future gross wealth that is part of the risk inherent in the granting of credit. In this respect, we have assumed that the expected gross wealth is a random variable without paying particular attention to its determination. In addition, the threshold values of the parameter λ are subjective values. The fate of the borrower costs a lot : a large range of threshold value penalizes the borrower (i.e λ_{max} high). It is clear that, whatever the threshold value, this parameter puts the decision maker neither in a pessimistic state nor in an optimistic state. The specificity of the probability distribution of the W_s random variable deserves to be underlined : $W_s^t = W_s^{t-1} + V_t(s/s-1) + \epsilon_t$, with $\epsilon_t \longmapsto \mathcal{N}(0, \sigma_W)$. The expected gross wealth of the year t is equal to the gross wealth of the year $t-1$ plus the expected net profit of the year t and a random variation with a distribution of zero mean and σ_W standard deviation.

6. Conclusion

This article is a proposal for a decision-making model that is getting closer and closer to the reality of microfinance. The profit patterns that can be achieved, whether from the borrower or lender point of view, depend on the set of possible states of nature. For this purpose, five profits are possible to determine as the borrower moves from one state to another.

The introduction of the parameter λ in this model avoids risk aversion (underestimation or overvaluation of the borrower). For the case of group loan, it will be the subject of another article later.

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