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PCA-based Approach for Inhomogeneous PSF Estimation and Partial Volume Correction in PET

Zacharie Irace, Anthonin Reilhac, Bruno Mendez de Vigo, Hadj Batatia and Nicolas Costes

***Abstract*—The quality of the measurements obtained in Positron Emission Tomography (PET) is severely limited by partial volume effects (PVE). This study proposes a Partial Volume Correction (PVC) technique that considers the spatial variability of the system's Point Spread Function (PSF) across the Field Of View (FOV). The proposed PSF model uses Principal Component Analysis to express their variability according to a small number of components called eigen-PSF, forming an orthonormal basis. The interpolation of the coordinates of these PSFs in this created basis allows their precise estimation across the FOV of the system. The resulting image degradation model can be expressed as a weighted sum of convolutions that can be integrated efficiently into classical PVC algorithms. Initial results shows accurate PSF estimation as well as significant image restoration.**

***Index Terms*—Partial Volume Effect, Point Spread Function, Deconvolution, PET**

I. Introduction

MEASUREMENTS obtained in Positron Emission Tomography (PET) are severely affected by partial volume effects (PVE) [1]. When the impulse response or Point Spread Function (PSF) of the system is assumed to be spatially homogeneous inside the Field of View (FOV), Partial Volume Correction (PVC) techniques mainly rely on deconvolution algorithms. However, because of the geometry of the scanner's detectors, the PVE behaves significantly inhomogeneously within the FOV and the assumption of the spatially invariance of the PSF should be relaxed for improved results. Some studies have presented parametric models of the PSF, most often with a 2D or 3D Gaussian [2] that has the advantage to speed-up the PVC process. Non-parametric methods are preferred to model the PSF with more precision, but are often limited by the computational time of the PVC.

This paper proposes a fast PVC method that rely on an accurate non-parametric model of spatially variant PSF. The PSF at any position of the FOV is estimated by interpolation from a set of PSFs previously measured on a regular grid of source points. The representation of the spatial variability of the PSFs is assessed by mean of Principal Component Analysis (PCA), which has already been studied in the field of astronomy [3], but not, to our knowledge, in medical imaging.

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Z. Irace, A. Reilhac, B. Mendez de Vigo and N. Costes are with CERMEP-Imagerie du vivant, Lyon, France and Neurodis Foundation, Lyon, France (e-mail: {irace, anthonin, costes}@cermep.fr).

H. Batatia is with ENSEEIHT, University of Toulouse, France, (e-mail: hadj.batatia@enseeiht.fr).

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II. Method

When the PSF is inhomogeneous inside the Field of View (FOV), the acquired image I can be modeled at each location x of the FOV by:

$$I(x) = \sum_{x' \in \Omega} h_{x'}(x) \cdot \hat{I}(x') \qquad (1)$$

where $\hat{I}$ is the original image, Ω is the image domain and $h_{x'}(\cdot)$ is the PSF at point x'. We assume that the PSF has been measured at P known positions $\{x_p\}_{p=1\cdots P}$ within the FOV. For simplicity, we denote $h_p(\cdot) = h_{x_p}(\cdot)$ the PSF at point x_p and $H = \{h_p\}_{p=1\cdots P}$ the set of known PSFs. The problems are 1) to estimate $h_x(\cdot)$ at any point x knowing the discrete set of measurements H and 2) to estimate $\hat{I}$.

A. Principal Component Analysis

Principal Component Analysis (PCA), also known as the Karhunen-Loève transform, is a widely used method to estimate the variations of data around their mean. The set of observations is decomposed into linearly uncorrelated variables called principal components, or in this case *eigen-PSFs*, that define an orthonormal basis, and that are by nature non-parametric, and suits the data at best. Accordingly, a PSF h_x at any location x can be expressed as a linear combination of *eigen-PSFs* $\{\phi_k\}_{k=1\cdots K}$. In addition, the variance representing the spatial variations of the PSF are generally fairly well represented by the first components, so the dimension of the data can be significantly reduced by projecting the PSF into the K first dimensions of the new basis.

$$h_x = h_{\text{mean}} + \sum_{p=1}^{P} w_p(x) \cdot \phi_p \simeq h_{\text{mean}} + \sum_{k=1}^{K} w_k(x) \cdot \phi_k \qquad (2)$$

where ϕ_k is the k^{th} eigen-PSF, $\{w_k(x)\}_{k=1\cdots K}$ are the component weights of the PSF h_x in the restricted basis and $K \ll P$.

In practice, the eigenvalues and the associated eigenvectors can be obtained by computing a Single Value Decomposition (SVD) on the data matrix H. The decomposition returns the new basis as well as the component weights of the measured set of PSF $w_k(x_p)_{k=1\cdots K}^{p=1\cdots P}$. To estimate the PSF h_x at any location x, one can estimate its coefficients $w_k(x)$ by performing traditional linear interpolation.

B. Partial Volume Correction

From 1 and 2 we derive:

$$I = \left(h_{\text{mean}} \otimes \hat{I}\right) + \sum_{k=1}^{K} \left(\phi_k \otimes \hat{I}_k\right) \qquad (3)$$

where $\otimes$ is the usual convolution operator and $\hat{I}_k(x) = w_k(x) \cdot \hat{I}(x)$ is the original image where pixels have been weighted over its full domain by the coefficient field $w_k(x)$. In other terms, the measured image I can be seen as a sum of weighted original images $\hat{I}_k$, each of which having been convolved by the invariant kernel ϕ_k. Provided that most deconvolution algorithms rely on an iterative scheme including convolution and that ϕ_k and w_k are known, problem 1 of restoring $\hat{I}$ from a spatially-variant PSF can be solved by performing $K+1$ parallel convolutions (with spatially invariant kernels) associated to the K principal components.

III. Validation

A. PSF estimation

Imaging data have been generated by the *PET-SORTEO* simulation software, modeling the Siemens mMR scanner [4]. A learning set of 35 source points have been simulated on a regular grid at radii($\rho = [0; 30; 60; 90; 140; 210; 280]$mm) and depth ($z = [0; 30; 60; 90; 120]$mm) from the center of the FOV. This initial set has been extended to a grid covering the whole FOV by exploiting spatial symmetries. The images have been reconstructed by Filtered Back-Projection (FBP) with dimensions $256 \times 256 \times 127$ pixels, then cropped to a $17 \times 17 \times 17$ window.

PCA have been performed on the obtained images, 9 components were sufficient to assess 95% of the variability. The first 4 eigen-PSFs and the associated coefficient fields are displayed on figure 1. One sees that the coefficients are

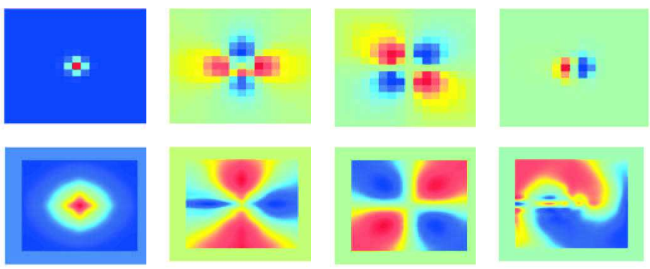

Fig. 1. First 4 eigen-PSFs (top) and associated coefficient fields (bottom).

regular, which legitimates the use of interpolation.

To validate the PSF estimation method, a set of 50 point sources have been generated at random positions of the FOV. The measured PSF has been compared to different PSFs:

- pcaPSF: the proposed spatially-variant estimation
- meanPSF: the mean PSF of the initial set
- centerPSF: the PSF measured at the center of the FOV
- GaussPSF: a parametric Gaussian PSF

Figure 2 shows an example of a 2D-slice of the measured PSF, and the associated estimations. One notice that in this case, the shape of the measured PSF is hardly approximated by a Gaussian. Figure 3 plots the estimation error in function of the radial distance from the FOV. As expected, the center PSF reproduces fairly well the PSFs that are near the center but the estimation error grows rapidly as their distance to the center increases. The spatially-invariant PSF meanPSF and

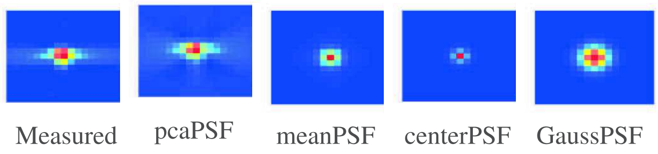

Fig. 2. Example of measured and estimated PSF from different methods.

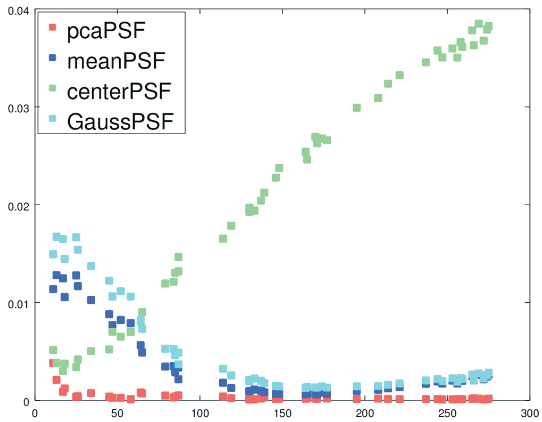

Fig. 3. Estimation error in function of radial distance to the center of the FOV.

gaussPSF both give globally good approximations across the FOV but struggle to model the PSF that are to near or to far from the center. In contrast, the proposed PSF model presents good approximation on the whole FOV and outperforms the other models on each of the 50 instances. Table I sums up the estimation errors between the measured and estimated PSFs from the different approaches. The proposed method provides

Method	Sum of estimation errors
pcaPSF	0.0204
meanPSF	0.1861
centerPSF	1.0747
gaussPSF	0.2551

TABLE I

Sum over the 50 estimation errors of the PSF estimations.

estimates that are more accurate than spatially-invariant PSFs across the field of view.

B. Partial Volume Correction

A numerical Derenzo phantom has been simulated. The reconstructed image (see Fig. 4) illustrates how the spatially-variant PSF alters the homogeneity of the measured activity.

The restoration results obtained after 5 Landweber iterations integrating the proposed PSF model have been compared by those obtained by classical Landweber deconvolution with several invariant PSFs. Restored images are displayed in Fig. 5 and the associated profiles are shown in Fig. 6.

As expected, when using the spatially-invariant centerPSF, the activity is correctly restored near the system center but the activity is underestimated away from the center. This

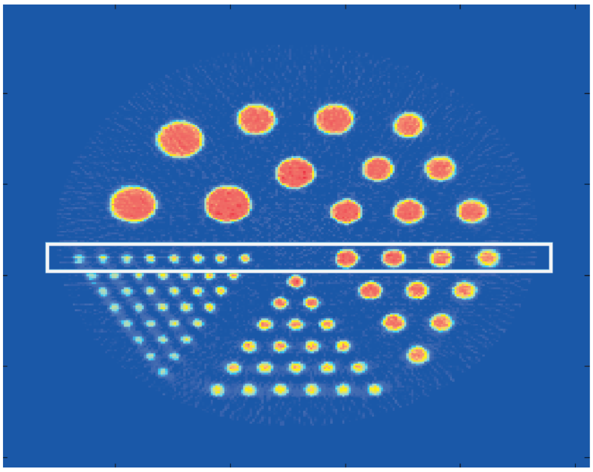

Fig. 4. Reconstructed Derenzo phantom. The theoretical activity is constant in each sphere but the spatially-variant PSF produces inhomogeneities in the measured activity.

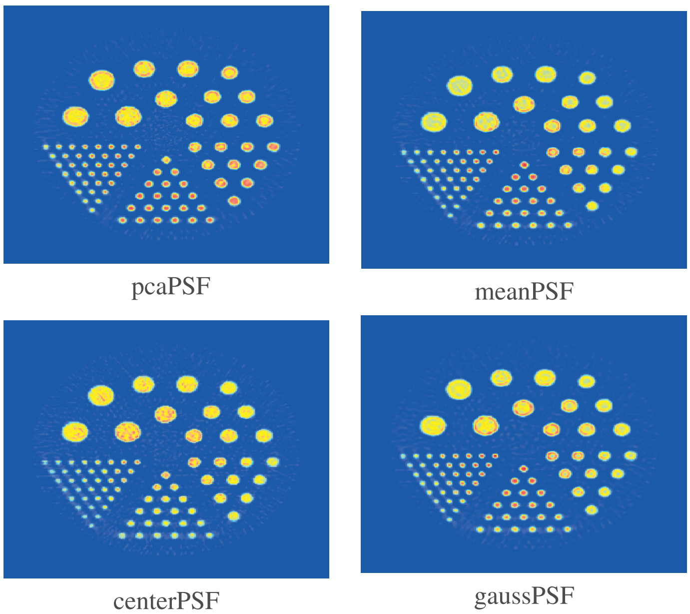

Fig. 5. Restored images obtained with the different models of PSF.

phenomenon is also seen when using the parametric Gaussian model. Besides, when using the meanPSF, the restored activities are overestimated near the center and underestimated away from the center of the FOV. The proposed method provides a restored image whose activities are more faithful to the theoretical values. Furthermore, contrary to the PVC methods based on a spatially invariant PSF, restored activities are homogeneous.

IV. Conclusion

In this work, we proposed a novel way to account for the spatially-variant nature of the system's PSF that allows fast and accurate partial volume correction. The presented results have been obtained without any regularization. Note however that the generalization of the classical deconvolution algorithms

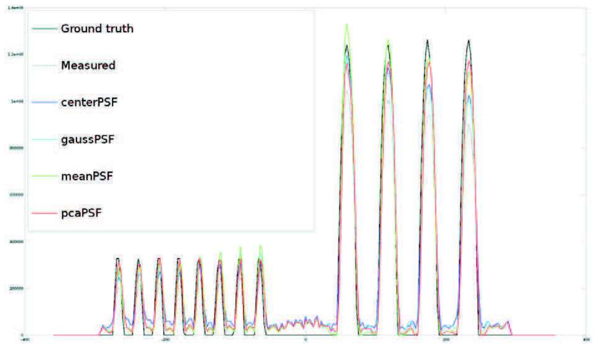

Fig. 6. Profiles corresponding to the selected area in Fig. 4, of the theoretical activity the measured activity, and of the restored images obtained with the different PSF models.

integrating the spatially-variant model of the PSF also allows the usage of regularization to reduce noise and improve the quantification results.

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