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How Potential BLFs Can Help to Decide Under Incomplete Knowledge

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Abstract. In a Bipolar Leveled Framework (BLF) [7], the comparison of two candidates is done on the basis of the decision principles and inhibitions which are validated given the available knowledge-bases asso-ciated with each candidate. This article defines a refinement of the rules for comparing candidates by using the potential-BLFs which can be built according to what could additionally be learned about the candidates. We also propose a strategy for selecting the knowledge to acquire in order to better discriminate between candidates.

Keywords: Qualitative decision making · Bipolarity · Arguments Incomplete knowledge

1 Introduction

Making decision is both one of the most current task and one of the most difficult problem that human beings should face. Hence designing an intelligent system able to help people to make decisions is a very important challenge. Tchangani et al. [13] recall that decision analysis is a process requiring first to formulate the decision goals, then to identify the attributes that characterize the potential alternatives and then decide. In classical approaches about decision making (see e.g. the introductory book of [9]), the standard way is to use a utility function that evaluates the quality of each decision hence that helps to select the one that has the best utility. This utility function should be designed in order to take into account the uncertainty and the multi-criteria aspects of the problem.

Studies in Psychology (see e.g. [12]) have shown that decision making is often guided by affect. Even more, Slovic et al. [11] argue that "affect is essential to rational action" where affect is defined as "the specific quality of "goodness" and "badness", as felt consciously or not by the decision maker, and demarcating a positive or negative quality of a stimulus". Then it is natural to use a scale going from negative (bad) to positive (good) values, including a central neutral value, to encode the bipolarity of the affect. And even use a bipolar scale, indeed, it is often the case that human people evaluate the possible alternatives considering positive and negative aspects separately [5].

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In Artificial Intelligence literature some models have already been proposed based on a bipolar view of alternatives (see [8] for an aggregation function approach and [3] for a pairwise comparison approach). In this paper we further explore the Bipolar Leveled Framework (BLF), which is a new representation framework for decision making, first introduced in [2] and extended in [6,7]. The BLF is a bipolar structure that enables the human decision maker to *visualize the attributes and goals that are involved in the decision problem, together with their links and their importance levels*. The structure is bipolar in the sense that the goals are either positive (i.e. wished to be achieved, a decision that achieves that goal is good) or negative (i.e. dreaded to be achieved, a decision that achieves that goal, is bad). Information in a BLF is encoded under the form of "decision principles" (DP). A DP is a kind of argument linking a description of a factual situation (here the situation is a candidate, described by some attributes) to the achievement of a goal. Informally a BLF may be viewed as a kind of qualitative utility function with some extra features: (1) the links between attributes and goals are made explicit into the decision principles, (2) the fact that a decision principle can be inhibited in the presence of some attribute is represented by an arrow from the attribute to the DP, (3) the importance levels of decision principles are represented by the height of their position in the structure. For more details on the link between BLF and qualitative decision Theory see [6]. When an alternative is known, the attributes of this alternative define what is called a "Valid BLF" which is an instance of a generic BLF.

The problem addressed is the pairwise comparison of alternatives given a bipolar utility representation. More precisely, the aim of this paper is to study how the BLF can take into account the awareness of the user about the completeness of her knowledge. We propose a measure called "sensibility" that evaluates this awareness in terms of what is known about the alternative versus what could be known (given the generic BLF). Then we propose two different ways to deal with this *sensibility*. The first one aims at refining the comparison that could be done with the valid BLFs associated to the alternatives, by taking into account the potential BLFs that could be obtained if we had more available information on each alternative. The second one aims at helping the decision maker to choose which information is relevant in order to make the more reliable choice between two decisions, hence what information should be obtained before deciding.

In the next section we recall the BLF definitions introduced in [6,7]. The third section describes how to take into account the actual knowledge with regard to information that could be learned given the generic BLF, leading to define a potential BLF. The last section proposes two ways to take into account the potential BLF either for refining the ordering of candidates or to select which information has to be acquired in order to be more accurate in the comparison.

2 BLF: A Structure Encoding Decision Criteria

We consider a set $\mathscr{C}$ of candidates[1] about which some information is available. We propose two distinct languages in order to clearly differentiate beliefs (coming from observations) from desires (goals to be achieved when selecting a candidate): $\mathscr{L}_F$ (a propositional language based on a vocabulary $\mathcal{V}_F$) represents information about some features that are believed to hold for a candidate and $\mathscr{L}_G$ (another propositional language based on a distinct vocabulary $\mathcal{V}_G$) represents information about the achievement of some goals when a candidate is selected. In the propositional languages used here, the logical connectors "or", "and", "not" are denoted respectively by $\vee$, $\wedge$, and $\neg$. A *literal* is a propositional symbol x or its negation $\neg x$, the set of literals of $\mathscr{L}_G$ are denoted by LIT_G. Classical inference, logical equivalence and contradiction are denoted respectively by $\models$, $\equiv$, $\perp$.

In the following we denote by K a set of formulas representing the beliefs of an agent about the features that hold: hence $K \subseteq \mathscr{L}_F$ is the available information. Using the inference operator $\models$, the fact that a formula $\varphi \in \mathscr{L}_F$ holds[2] in K is written $K \models \varphi$.

2.1 BLF: Definitions [7]

The BLF is a structure that contains two kinds of information: decision principles and inhibitors. A decision principle can be viewed as a defeasible reason enabling to reach a conclusion about the achievement of a goal. More precisely, a decision principle is a pair (φ, g), it represents the default rule meaning that "if the formula φ is believed to hold for a candidate then the goal g is a priori believed to be achieved by selecting this candidate":

Definition 1 (decision principle (DP)). *A decision principle p is a pair $(\varphi, g) \in \mathscr{L}_F \times LIT_G$, where φ is the reason of p, denoted $reas(p)$ and g the conclusion of p, denoted $concl(p)$. $\mathcal{P}$ denotes the set of decision principles.*

Depending on whether the achievement of its goal is wished or dreaded, a decision principle may have either a positive or a negative polarity. Moreover some decision principles are more important than others because their goal is more important.

Definition 2 (polarity and importance). *A function $pol : \mathcal{V}_G \rightarrow \{\oplus, \ominus\}$ gives the polarity of a goal $g \in \mathcal{V}_G$, this function is extended to goal literals by $pol(\neg g) = -pol(g)$ with $-\oplus = \ominus$ and $-\ominus = \oplus$. A decision principle p is polarized accordingly to its goal: $pol(p) = pol(concl(p))$. The set of positive and negative goals are abbreviated $\overline{\oplus}$ and $\overline{\ominus}$ respectively: $\overline{\oplus} = \{g \in LIT_G : pol(g) = \oplus\}$ and $\overline{\ominus} = \{g \in LIT_G : pol(g) = \ominus\}$.*

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[1] Candidates are also called alternatives in the literature.

[2] The agent's knowledge K being considered to be certain, we write "φ holds" instead of "φ is believed to hold".

LIT_G is totally ordered by the relation $\preceq$ ("less or equally important than"). Decision principles are ordered accordingly: $(\varphi, g) \preceq (\psi, g')$ iff $g \preceq g'$.

The polarities and the relative importances of the goals in $\mathcal{V}_G$ are supposed to be given by the decision maker, *e.g.*, he may want to avoid to select an expensive hotel (hence "expensive hotel" can be a negative goal), while selecting a hotel where it is possible to swim can be a positive goal, moreover he may give more importance to swim than to pay less.

A decision principle (φ, g) is a defeasible piece of information because sometimes there may exist some reason φ' to believe that it does not apply in the situation, this reason is called an *inhibitor*.

The fact that ψ inhibits a decision principle (φ, g) is interpreted as follows: "when the decision maker only knows $\varphi \wedge \psi$ then he is no longer certain that g is achieved". In that case, the inhibition is represented with an arc towards the decision principle. The decision principles and their inhibitors are supposed to be given by the decision maker. An interpretation of decision principles in terms of possibility theory is described in [6].

We are now in position to define the BLF structure.

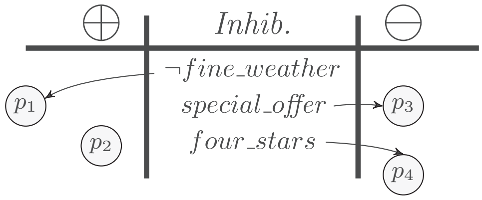
Definition 3 (BLF). *Given a set of goals $\mathcal{V}_G$, a BLF is a triplet $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ where $\mathcal{P}$ is a set of decision principles ordered[3] accordingly to their goals by $\preceq$ and with a polarity built on pol as defined in Definition 2, $\mathcal{R} \subseteq (\mathscr{L}_F \times \mathcal{P})$ is an inhibition relation.*

The four elements of the BLF are supposed to be available prior to the decision and to be settled for future decisions as if it was a kind of utility function. A graphical representation of a BLF is given below, it is a tripartite graph represented in three columns, the DPs with a positive level are situated on the left column, the inhibitors are in the middle, and the DPs with a negative polarity are situated on the right. The more important (positive and negative) DPs are in the higher part of the graph, equally important DPs are drawn at the same horizontal level. By convention the highest positive level is at the top left of the figure and the lowest negative level is at the bottom right. The height of the inhibitors is not significant only their existence is used.

Example 1. *Let us imagine an agent who wants to find an inexpensive hotel in which he can swim. This agent would also be happy to have free drinks but it is less important for him. $\mathcal{V}_G = \{swim, free\_drinks, expensive, crowded\}$, with $pol(swim) = pol(free\_drinks) = \oplus$ and $pol(expensive) = pol(crowded) = \ominus$ and $swim \simeq expensive \succ free\_drinks \succ crowded$. The possible pieces of information concern the following attributes: $\mathcal{V}_F = \{pool, open\_bar, four\_stars, fine\_weather, special\_offer\}$. The agent considers the following principles: $\mathcal{P} = \{p_1 = (pool, swim), p_2 = (open\_bar, free\_drinks), p_3 = (four\_stars, expensive), p_4 = (fine\_weather, crowded)\}$. When the weather is not fine then*

[3] The equivalence relation associated to $\preceq$ is denoted $\simeq$ ($x \simeq y \Leftrightarrow x \preceq y$ and $y \preceq x$) and the strict order is denoted $\prec$ ($x \prec y \Leftrightarrow x \preceq y$ and not $y \preceq x$).

the fact that there is a pool is not sufficient to ensure that the agent can swim, it means that there is an inhibition on p_1 *by* $\neg fine\_weather$*, and the DP* p_4 *that expresses that "if the weather is fine the hotel will be crowded" is inhibited when its a four stars hotel, and the DP* p_3 *is inhibited when the agent have a special offer, i.e.* $\mathcal{R} = \{(\neg fine\_weather,\ p_1),\ (four\_stars,\ p_4),\ (special\_offer,\ p_3)\}$.



In the following, the BLF $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ is set and we show how it can be used for comparing candidates. First, we present the available information and the notion of instantiated BLF, called valid-BLF.

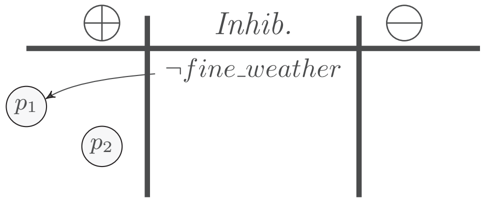
Given a candidate $c \in \mathscr{C}$, we consider that the knowledge of the decision maker about c has been gathered in a knowledge base K_c with $K_c \subseteq \mathscr{L}_F$. K_c is **supposed to be consistent**. Given a formula φ describing a configuration of features ($\varphi \in \mathscr{L}_F$), the decision maker can have three kinds of knowledge about c: φ holds for candidate c (i.e., $K_c \models \varphi$), or φ does not hold ($K_c \models \neg\varphi$) or the feature φ is unknown for c ($K_c \not\models \varphi$ and $K_c \not\models \neg\varphi$). When there is no ambiguity about the candidate c, K_c is denoted K.

Definition 4 (K-Valid-BLF). *Given a base* K*, a* K*-Valid-BLF is a quadruplet* $(\mathcal{P}_K, \mathcal{R}_K, pol, \preceq)$ *where*

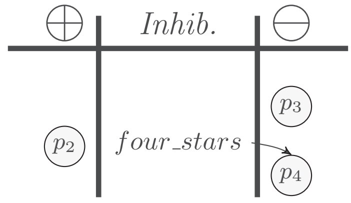
- $\mathcal{P}_K = \{(\varphi, g) \in \mathcal{P},\ s.t.\ K \models \varphi\}$ *is the set of DPs in* $\mathcal{P}$ *whose reason* φ *holds in* K*, called* valid-DP*s.*
- $\mathcal{R}_K = \{(\psi, p) \in \mathcal{R},\ s.t.\ K \models \psi\}$ *is the set of* valid inhibitions *wrt to* K*.*

When there is no ambiguity, we simply use "valid-BLF" instead of "K-Valid-BLF". The validity of a DP only depends on whether the features that constitute its reason φ hold or not, it does not depend on its goal g since the link between the reasons and the goal is given in the BLF (hence it is not questionable).

Example 2. *The agent has information about a hotel situated in a place where the weather will not be fine and that has a pool* ($reas(p_1)$) *and an open bar* ($reas(p_2)$)*:* $K_1 = \{\neg fine\_weather, pool, open\_bar\}$*. The* K_1*-Valid-BLF corresponding to what is known about this hotel is on the left. Now, we can consider another knowledge base* $K_2 = \{fine\_weather,\ four\_stars,\ open\_bar\}$ *describing a hotel that has an open-bar and that is located somewhere where the weather is nice but with no information about the existence of a pool, its associated* K_2*-Valid BLF is on the right.*



K_1-*Valid-BLF*



K_2-*Valid-BLF*

Now in the valid-BLF the principles that are not inhibited are the ones that are going to be trusted. A goal in $\mathcal{V}_G$ is said to be "realized" if there is a valid-DP that is not inhibited by any valid-inhibitor.

Definition 5 (realized goal). *Let g be a goal in LIT_G, g is* realized *w.r.t. a K-Valid-BLF $(\mathcal{P}_K, \mathcal{R}_K, pol, \preceq)$ iff $\exists(\varphi, g) \in \mathcal{P}_K$ and (φ, g) not inhibited in $\mathcal{R}_K$. The set of realized goals is denoted $\mathtt{R}_K$ (and simply $\mathtt{R}$ when there is no ambiguity about K) the positive and negative realized goals are denoted by $\mathtt{R}^{\oplus} = \mathtt{R} \cap \overline{\oplus}$ and $\mathtt{R}^{\ominus} = \mathtt{R} \cap \overline{\ominus}$ respectively.*

Example 3. *In the BLF with the knowledge K_1, concl(p_2) is the only realized goals. K_2 with the same initial BLF allows us to conclude that both concl(p_2) and concl(p_3) are realized. To summarize, the first valid-BLF has one positive realized goal: $\mathtt{R}_{K_1} = \{free\_drinks\}$, while the second valid-BLF has a positive and a negative realized goal, $\mathtt{R}_{K_2} = \{free\_drinks, expensive\}$. But the negative goal that is realized has greater importance for the agent than the positive one, hence he should prefer the first hotel.*

In the next section we show how to use a BLF in order to compare several candidates based on the goals that are realized in their corresponding valid-BLF.

2.2 Decision Rules for Comparing Candidates

In order to compare candidates we have to compare the levels of DPs that are valid, hence we are going to define an absolute scale of the levels of the goals in the BLF (this definition is straightforward from the BLF). We start by attributing levels to the goals starting from the least important ones that are assigned a level 1 and stepping by one each time the importance grows.

Definition 6 (levels of goals wrt a BLF). *Given a BLF $B = (\mathcal{P}, \mathcal{R}, pol, \prec)$ the levels of the goals of the BLF are defined by induction:*

- $L(B)_1 = \{g \in Goals(B) : \nexists g' \in Goals(B)$ *s.t.* $g' \prec g\}$
- $L(B)_{i+1} = \{g \in Goals(B) : \nexists g' \in Goals(B) \setminus (\bigcup_{k=1}^{i} L(B)_k)$ *s.t.* $g' \prec g\}$

where $Goals(B) = \bigcup_{p \in \mathcal{P}} concl(p)$

Given a set of goals $G \in Goals(B)$, we write $G_k = G \cap L(B)_k$, and the level of a goal $g \in Goals(B)$ is defined by $level(g) = k$ iff $g \in G_k$.

In [3], Bonnefon et al. introduce three decision rules called Pareto, Bipolar Possibility and Bipolar Leximin dominance relations. We recall only the Bipolar Leximin dominance relation below:

Definition 7 (BiLexi decision rule of [3]). *Given two candidates c and c' respectively described by K and K' with their associated realized goals $\mathtt{R} = \mathtt{R}_K$ and $\mathtt{R}' = \mathtt{R}_{K'}$, the Bipolar Leximin dominance relation denoted $\succeq_{BiLexi}$ is defined by:*

$$c \succeq_{BiLexi} c' \quad \textit{iff } |\mathtt{R}^{\oplus}_{\delta}| \geq |\mathtt{R}'^{\oplus}_{\delta}| \textit{ and } |\mathtt{R}^{\ominus}_{\delta}| \leq |\mathtt{R}'^{\ominus}_{\delta}|$$

where $\delta = argmax_{\lambda}(\{|\mathtt{R}^{\oplus}_{\lambda}| \neq |\mathtt{R}'^{\oplus}_{\lambda}| \text{ or } |\mathtt{R}^{\ominus}_{\lambda}| \neq |\mathtt{R}'^{\ominus}_{\lambda}|\})$.

Example 4. *As we expected, the hotel described by K_1 is preferred to the one described by K_2, wrt $\succeq_{Bilexi}$, since*
$\mathtt{R}^{\oplus}_{K_1} = \{free\_drinks\}$ $\mathtt{R}^{\ominus}_{K_1} = \varnothing$
$\mathtt{R}^{\oplus}_{K_2} = \{free\_drinks\}$ $\mathtt{R}^{\ominus}_{K_2} = \{expensive\}$
and $free\_drinks \in L(B)_2$, $expensive \in L(B)_3$. Hence, we have the same realized goals at level 1 and 2, hence $\delta = 3$.

3 Awareness and K-Potential-BLF

In [6], only the features that the agent knows are used to compare candidates, i.e., the decision is based on the K-Valid-BLF. It means that the knowledge about the potential existence of a DP or of an inhibition is not taken into account. Hence, the quality of the agent knowledge is not taken into account (see example below).

Example 5. *In Example 1 with $K_1 = \{\neg fine\_weather, pool, open\_bar\}$, the agent believes that the weather is not fine and that the hotel has a pool and has an open bar. If we compare this state of belief with the belief $K_3 = \{open\_bar, fine\_weather, special\_offer\}$, then the realized goal is the same ($concl(p2)$). However, knowing that there could be no possibility to swim in the first hotel while there could be a pool in the second one may incline the agent to prefer the second hotel.*

To refine the comparison of candidates, we propose to improve the evaluation of a candidate by evaluating the goals that *could be* realized under the actual knowledge K.

3.1 K-Potential-BLF

Potential DPs and potential inhibition relations are the ones that could belong to the valid BLF if we had more information. Hence they are the DPs and inhibition relations that can be consistently assumed to be valid. In other words, they are not proven to be not valid wrt to the agent knowledge about the candidate. A DP is proven not valid when the agent knows that its reason does not hold ($K \models \neg reas(p)$). An inhibition on a DP cannot be valid if the agent knows that the inhibitor does not hold. Hence a potential inhibition is an inhibition that is not proven impossible when the reason of the DP holds.

A K-Potential-BLF is made of potential DPs and potential inhibitions with respect to a knowledge base K.

Definition 8 (K-Potential-BLF). *Given a base K and a BLF $(\mathcal{P}, \mathcal{R}, pol, \preceq)$, the K-Potential-BLF associated to K is the quadruplet $(\widehat{\mathcal{P}}_K, \widehat{\mathcal{R}}_K, pol, \preceq)$ where*

- $\widehat{\mathcal{P}}_K$ *is the set of potential DPs defined by:*

$$\widehat{\mathcal{P}}_K = \{p \in \mathcal{P} \quad | \quad K \cup \{reas(p)\} \text{ is consistent}\}$$

- $\widehat{\mathcal{R}}_K$ *is the set of potential inhibition relations defined by:*

$$\widehat{\mathcal{R}}_K = \{(\psi, p) \in \mathcal{R} \mid K \cup \{reas(p) \wedge \psi\} \text{ is consistent}\}$$

Example 6. *The K_1-Potential-BLF associated to K_1 contains $\widehat{\mathcal{P}}_{K_1} = \{p_1, p_2, p_3\}$ and $\widehat{\mathcal{R}}_{K_1} = \{(\neg fine\_weather, p_1), (special\_offer, p_3)\}$.*

Now according to whether the BLF considered is the K-Valid-BLF or the K-Potential-BLF, some goals can be simply realized (we recall Definition 5) or necessarily/possibly/potentially realized.

Definition 9 (Potential realization). *A goal g in LIT_G can have eight statuses w.r.t. a knowledge base K and a BLF $(\mathcal{P}, \mathcal{R}, pol, \preceq)$:*

status	***Realized***	***Not realized***
notation	$g \in \mathtt{R}_K$	$g \in \overline{\mathtt{R}}_K$
definition	$\exists p \in \mathcal{P}_K$ *concl*$(p) = g$ *and* $\nexists(\psi, p) \in \mathcal{R}_K$	$\forall p \in \mathcal{P}_K$ *either concl*$(p) \neq g$ *or* $\exists(\psi, p) \in \mathcal{R}_K$
status	***Necessarily realized***	***Possibly not realized***
notation	$g \in N\mathtt{R}_K$	$g \in \mathit{\Pi}\overline{\mathtt{R}}_K$
definition	$\exists p \in \mathcal{P}_K$ *concl*$(p) = g$ *and* $\nexists(\psi, p) \in \widehat{\mathcal{R}}_K$	$\forall p \in \mathcal{P}_K$ *either concl*$(p) \neq g$ *or* $\exists(\psi, p) \in \widehat{\mathcal{R}}_K$
status	***Potentially realized***	***Potentially not realized***
notation	$g \in P\mathtt{R}_K$	$g \in P\overline{\mathtt{R}}_K$
definition	$\exists p \in \widehat{\mathcal{P}}_K$ *concl*$(p) = g$ *and* $\nexists(\psi, p) \in \widehat{\mathcal{R}}_K$	$\forall p \in \widehat{\mathcal{P}}_K$ *either concl*$(p) \neq g$ *or* $\exists(\psi, p) \in \widehat{\mathcal{R}}_K$
status	***Possibly realized***	***Necessarily not realized***
notation	$g \in \mathit{\Pi}\mathtt{R}_K$	$g \in N\overline{\mathtt{R}}_K$
definition	$\exists p \in \widehat{\mathcal{P}}_K$ *concl*$(p) = g$ *and* $\nexists(\psi, p) \in \mathcal{R}_K$	$\forall p \in \widehat{\mathcal{P}}_K$ *either concl*$(p) \neq g$ *or* $\exists(\psi, p) \in \mathcal{R}_K$

In other words, a *necessarily realized* goal is realized in the K-Valid BLF and has no potential inhibitor (i.e. no valid inhibitor and more information cannot bring anymore inhibitor). A *necessarily not realized* goal is either not achieved by any potential DP or it has a K-Valid inhibitor. A *possibly realized* goal is the conclusion of a DP whose reason could hold and for which no inhibition is known to hold.

Example 7. *We have already seen that* $\mathtt{R}_{K_1} = \{free\_drinks\}$*. We have also:* $N\mathtt{R}_{K_1} = \{free\_drinks\}$, $P\mathtt{R}_{K_1} = \{free\_drinks\}$, $\Pi\mathtt{R}_{K_1} = \{free\_drinks, expensive\}$.

3.2 Link Between K-Potential-BLF and K-Valid-BLF

In the following proposition we show that we have upper and lower bounds of the set of realized goals according to the potential knowledge.

Proposition 1. *For any BLF* $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ *and any knowledge base* K

$$\begin{array}{ll} N\mathtt{R}_K \subseteq \mathtt{R}_K \subseteq \Pi\mathtt{R}_K & \Pi\overline{\mathtt{R}}_K \subseteq \overline{\mathtt{R}}_K \subseteq N\overline{\mathtt{R}}_K \\ N\mathtt{R}_K \subseteq P\mathtt{R}_K \subseteq \Pi\mathtt{R}_K & \Pi\overline{\mathtt{R}}_K \subseteq P\overline{\mathtt{R}}_K \subseteq N\overline{\mathtt{R}}_K \end{array}$$

Since we are able to give an interval containing the realized goals, the confidence in the decision can be defined with respect to the size of this interval: the smallest the interval the surest the evaluation of the candidate (since learning the values of unknown features cannot change this evaluation). Hence we are going to define a measure that evaluates the size of this interval which is called sensibility of the BLF wrt knowledge.

Definition 10 (Sensitivity). *The sensitivity of a BLF* $B = (\mathcal{P}, \mathcal{R}, pol, \preceq)$ *wrt a knowledge base* K *is*

$$s(B, K) = |\Pi\mathtt{R}_K \setminus N\mathtt{R}_K|$$

The sensitivity is the number of goals that are possibly realized but not necessarily realized. The aim is to take this sensitivity into account while making a decision. In our example, $s(B, K_1) = 1$.

Definition 11. K *is a perfect knowledge wrt a BLF* $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ *iff* $\forall \varphi \in \bigcup_{p \in P}\{reas(p)\} \cup \bigcup_{(\psi,p)\in\mathcal{R}}\{\psi\}$*, either* $K \models \varphi$ *or* $K \models \neg\varphi$.

Note that it is not necessary to have perfect knowledge in order to have perfect information about the set of realized goals. In case of perfect knowledge the Valid BLF and the Potential BLF are equal, then all the eight statuses reduced to two, each goal is either necessarily realized or necessarily not realized.

Proposition 2. *For all BLF* $B = (\mathcal{P}, \mathcal{R}, pol, \preceq)$,
(K *is a perfect knowledge wrt* B*)* $\Rightarrow$ $(\widehat{\mathcal{P}}_K = \mathcal{P}_K$ *and* $\widehat{\mathcal{R}}_K = \mathcal{R}_K)$ $\Rightarrow s(B, K) = 0$.
But the converse does not necessarily hold.

4 K-Potential-BLF and Decision Making

In the BLF framework, a goal $g \in \mathtt{R}_K$ induces that "g is achieved" is the nominal conclusion. In other words, it is the conclusion drawn under the available knowledge K. Nevertheless, the ranking on goals obtained with $OM(\mathtt{R}_K)$ or $|\mathtt{R}_K|$ could be challenged when the quality of the knowledge is not the same for the candidates that we want to compare. In this section, we explore two different ways to exploit the K-Potential-BLF. The first way is to use the K-Potential-BLFs in case of equality or incomparability of two candidates wrt to their K-Valid-BLFs. Indeed, it allows us to use the three sets: the set of Necessarily (resp. Possibly and Potentially) realized goals $N\mathtt{R}_K$ (resp. $\Pi\mathtt{R}_K$ and $P\mathtt{R}_K$) additionally to the set of realized goals $\mathtt{R}_K$.

The second way aims at helping the decision maker to choose which information is relevant in order to make the more reliable choice between two candidates. When it is possible to acquire more information, it is fairer to obtain nearly the same level of sensitivity in the knowledge bases of the candidates to be compared.

4.1 Refining the Ordering of Candidates

In order to compare two candidates we should use one of the comparison operator recalled in Definition 7 on the Valid-BLFs of the candidates (hence on their respective realized goals). In case of equality or incomparability between two candidates the decision maker can use its awareness of the possible DPs given in the generic BLF. More precisely according to the decision maker's profile he may choose to use either $N\mathtt{R}_K$ (if "skeptic") or $\Pi\mathtt{R}_K$ (if "believer")[4]. The decision maker is called *skeptic* when he considers that a DP is valid and not inhibited only if this DPs remains valid and not inhibited whatever the missing information is, in accordance to the definition of Necessary realized goals (Definition 9). Similarly a *believer* considers that a DP is valid and not inhibited if there is a way to complete the missing information in order to make it possible.

4.2 Acquisition of Knowledge in Order to Discriminate Candidates

In this section, we consider that the decision maker is able to increase her knowledge K when she considers that this knowledge is not pertinent enough. After this acquisition K is increased into $K \cup \varphi$, the DM can compare the candidates by using the rules applied to the set $\mathtt{R}_{K\cup\varphi}$. In order to evaluate the quality of the knowledge available for each candidate, we can compare the sensitivity associated to their different knowledge bases. We may compare the candidates only if the knowledge about them has approximately the same sensitivity:

Definition 12. *Given a BLF $B = (\mathcal{P}, \mathcal{R}, pol, \preceq)$, a given constant ε and two knowledge bases K and K' describing two candidates c and c',*

- *c is* ε-sensitivity-comparable *to c' iff $|s(B, K) - s(B, K')| \leq \varepsilon$.*

[4] Note that the set $P\mathtt{R}$ is not meaningful in this context.

- c *is* $BiLexi$-preferred *to* c' with ε-sensitivity awareness *iff they are* ε-*sensitivity-comparable and* $c \succeq_{BiLexi} c'$

If the candidates are ε-sensitivity comparable but are equal wrt to BiLexi preference, or if we want to decrease the sensitivity then we have to choose the subject on which we have to increase our knowledge. The question to answer is "What is the most important goal that could be necessarily realized by adding only one formula φ to K which would not be possibly realized by adding $\neg\varphi$? and what is the simplest formula φ that could do that?"

Definition 13. *Given a BLF* $B = (\mathcal{P}, \mathcal{R}, pol, \preceq)$ *and a knowledge base* K*, a best-discriminating formula* (φ^*) *is a formula* $\varphi \in \mathscr{L}_F$ *such that:*

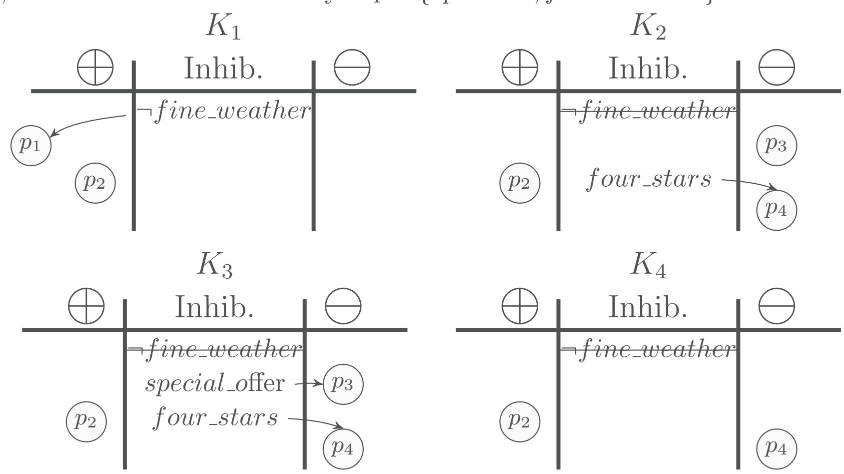
$$\varphi^* = \arg\max_{\varphi \in \mathscr{L}_F} \{k : level(g) = k \text{ and } g \in N\mathtt{R}_{K \cup \{\varphi\}} \text{ and } g \notin \Pi\mathtt{R}_{K \cup \{\neg\varphi\}}\}$$

The simplest-best-discriminating formula is a DNF[5] *best-discriminating formula that is not subsumed by any other DNF best-discriminating formula.*

We illustrate the two ways to use the K-Potential-BLF in the next section.

4.3 Example

We would like to compare 4 hotels: the three hotels described by K_1, K_2 and K_3, and a new one described by $K_4 = \{open\_bar; fine\_weather\}$:



If we apply the $BiLexi$ rule, we obtain $1 \sim_{BiLexi} 3 \succ_{BiLexi} 2 \succ_{BiLexi} 4$. The order relation between K_1 and K_2 can be refined by using the $BiLexi$ rule either on the set $N\mathtt{R}_K$ or on $\Pi\mathtt{R}_K$. The choice depends on the DM's profile: $N\mathtt{R}_K$ is taken if she is a skeptic and $\Pi\mathtt{R}_K$ if she is a believer. Using $N\mathtt{R}_K$, the ordering between candidate 1 and 3 remains the same but with $\Pi\mathtt{R}_K$, we get $1 \succ_{BiLexi} 3$.

[5] DNF: Disjunctive Normal Form.

Now, in case we can increase our knowledge. We can take into account the sensibility of the BLF associated to each candidate, which are $s(B, K_1) = 1$, $s(B, K_2) = 2$, $s(B, K_3) = 1$ and $s(B, K_4) = 3$. Note that Candidate 4 is very sensitive since its sensitivity is close to the maximal possible value of sensitivity (the number of goals). Hence, before concluding on the ordering the DM should increase her knowledge about candidate 4. She can investigate the feature *pool*: if the answer is Yes, then $swim \in N\mathtt{R}_{K_4 \cup \{pool\}}$ else $swim \notin \Pi\mathtt{R}_{K_4 \cup \{\neg pool\}}$. In the first case, 4 becomes the most preferred hotel otherwise it is the worst hotel.

5 Conclusion

The BLF is a visual tool made to help human decision makers in their tasks. Note that once the BLF is defined the decision is automatically computed, hence BLF can be used by artificial or human agents. In [6] we have already studied the comparison of two candidates a and b on the basis of the K_a-Valid-BLF and the K_b-Valid-BLF, that gathers the decision principles and inhibitions which are validated given the available knowledge-bases K_a and K_b associated with each candidate. In this paper, we have proposed a refinement of the rules for comparing candidates by using the *potential*-BLFs which can be built according to *what could additionally be learned* about the candidates.

We can consider that our approach is of the kind "compare then aggregate" in the sense that when we want to select one candidate among a set of candidates, we can do a pairwise comparison and then decide which candidate to elect. This last step is a kind of aggregation. Bonnefon et al. approach [3], and classical decision making methods like Electre [10], Promethee [4] and Condorcet [1] could be assigned to the same category of approaches where only [3] also uses bipolarity. Another approach of decision making is "aggregate then compare", it means that first candidates are given an absolute value and then the best one is selected. In this family of approaches we can find the weighted average method, Choquet integral-based methods (like [8]), the uninorm aggregation operators [14]. An interesting direction could be to override the pairwise comparisons done with the valid BLFs towards defining an absolute scale for ranking the candidates based on an aggregation function defined on BLFs.

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