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Introduction



❑ Multichannel active noise control (ANC) systems

- Better performance when we need to create large-size quiet zone.
- Applications:



Interior of Vehicles



Range Hood



Infant Incubator



Air Conditioner

Introduction



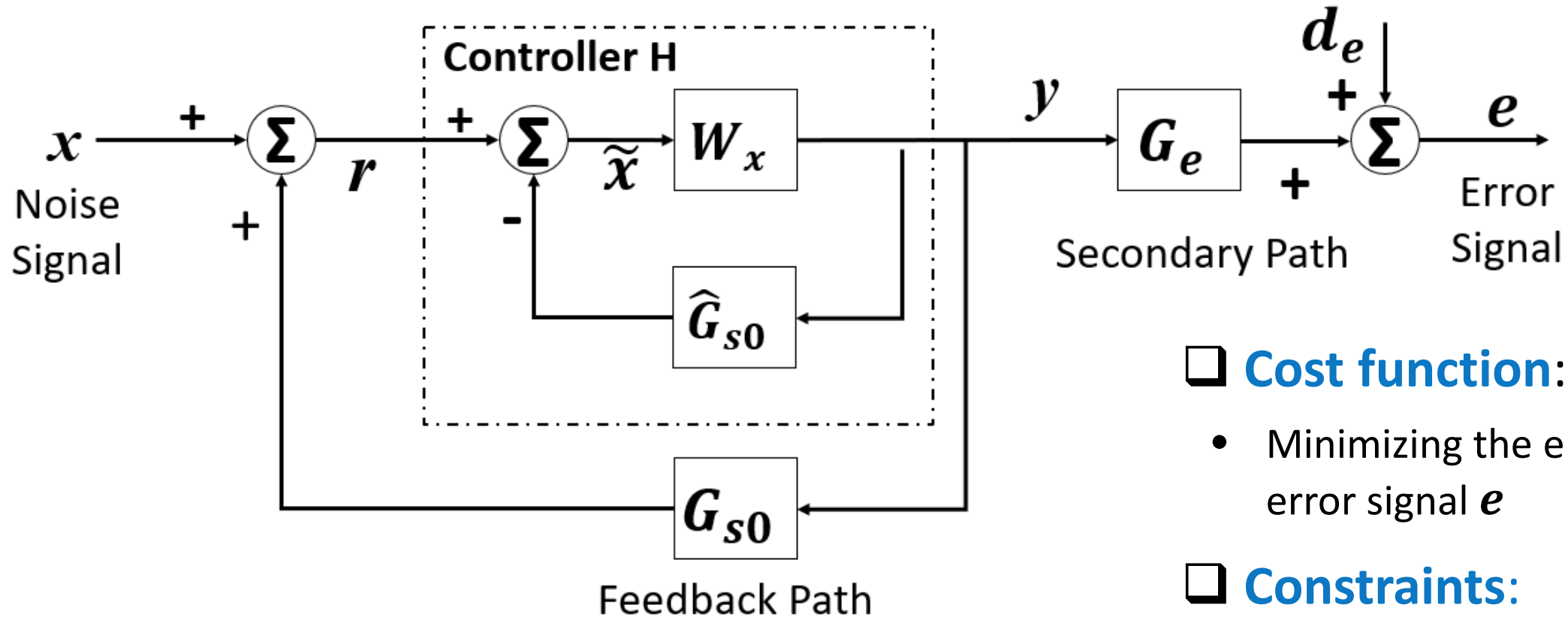
□ Motivation of using frequency domain design

- Easier to specify frequency dependent constraints.
- Constraints in one frequency band will not affect performance of other bands.
- Usually, better ANC performance.
- Convenient to design sub-band filter structure

□ Motivation of using cone programming

- The computational complexity is usually significant for frequency domain design method.
- In recent study of convex optimization, very efficient algorithms were developed for cone programming.
- Many optimization problems can be converted to cone programming.
- Potential to perform adaptive control in frequency domain with multiple constraints.

Active Noise Control System



❑ Cost function:

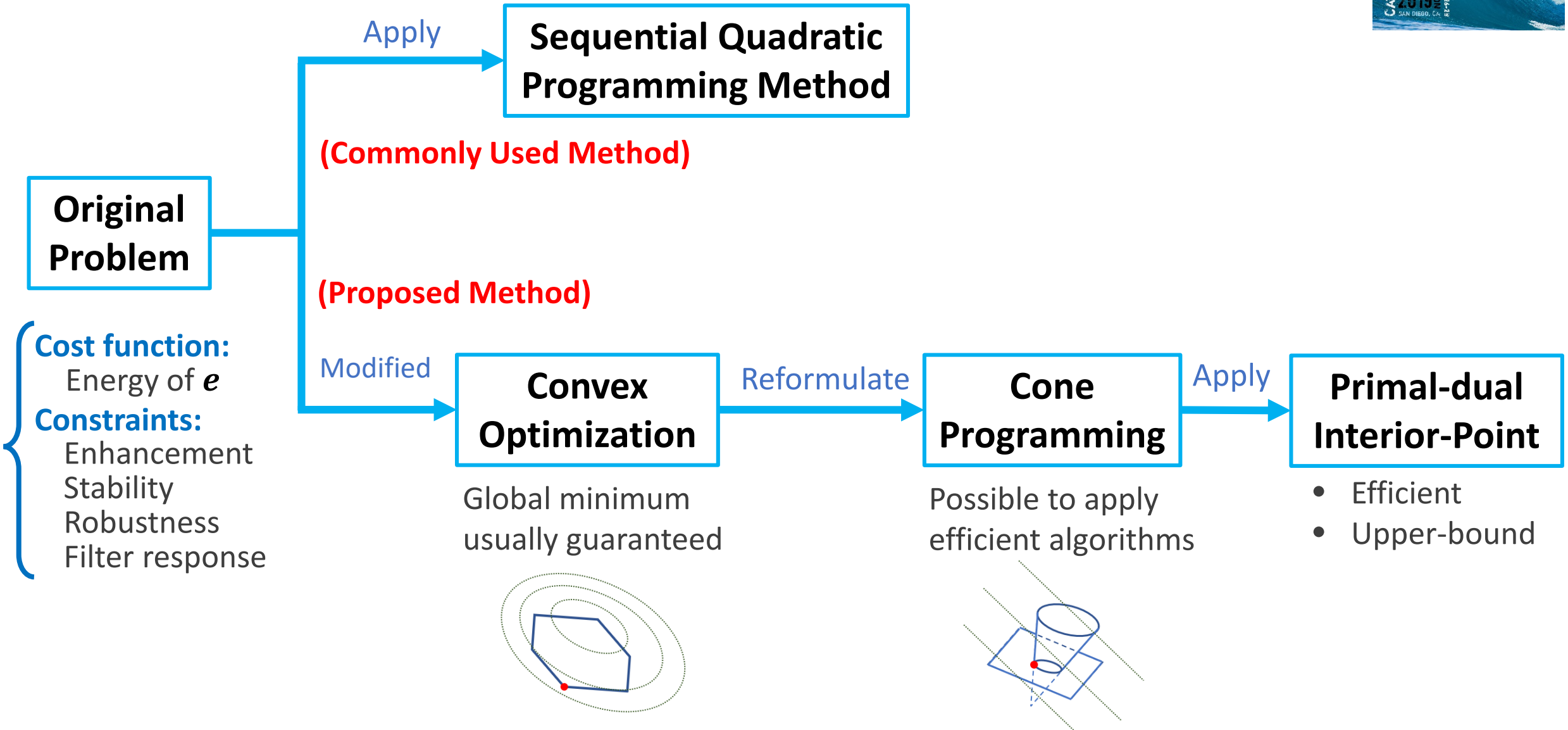
- Minimizing the energy of error signal e

❑ Constraints:

- Disturbance enhancement
- Stability
- Robustness
- Filter response

(Non-adaptive control is considered in the current work)

Overview of Reformulation



Original Problem



Cost function:

$$\sum_{k=k_1}^{k_2} \text{tr}[E(f_k)E(f_k)^H]$$



Total energy of e cross all frequencies

Constraints:

Enhancement: Normalized energy of e:

$$\text{tr}[E(f_k)E(f_k)^H]D_e(f_k) \leq A_e$$

Stability: Use Nyquist criterion:

$$\min \left(\text{Re} \left(\lambda \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) \right) > -1$$

Robustness: M - Δ structure and small gain theory:

$$\max \left(\sigma \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) B(f_k) \leq 1$$

Filter response: The magnitude of frequency response:

$$|W_{x_{i,j}}(f_k)| \leq C(f_k)$$

Original Problem



Cost function: Total energy of e :

$$\sum_{k=k_1}^{k_2} \text{tr}[E(f_k)E(f_k)^H],$$

Constraints:

Enhancement:

$$\text{tr}[E(f_k)E(f_k)^H]D_e(f_k) \leq A_e \quad \rightarrow \quad \text{Normalized energy of } e \text{ at each frequency}$$

Stability: Use Nyquist criterion:

$$\min \left(\text{Re} \left(\lambda \left(W_x(f_k)\hat{G}_{s0}(f_k) \right) \right) \right) > -1$$

Robustness: M - Δ structure and small gain theory:

$$\max \left(\sigma \left(W_x(f_k)\hat{G}_{s0}(f_k) \right) \right) B(f_k) \leq 1$$

Filter response: The magnitude of frequency response:

$$|W_{x_{i,j}}(f_k)| \leq C(f_k)$$

Original Problem



Cost function: Total energy of e :

$$\sum_{k=k_1}^{k_2} \text{tr}[E(f_k)E(f_k)^H],$$

Constraints:

Enhancement: Normalized energy of e :

$$\text{tr}[E(f_k)E(f_k)^H]D_e(f_k) \leq A_e$$

Stability:

$$\min \left(\text{Re} \left(\lambda \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) \right) > -1 \quad \rightarrow \quad \text{Nyquist criterion, on the right of -1 point}$$

Robustness: M - Δ structure and small gain theory:

$$\max \left(\sigma \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) B(f_k) \leq 1$$

Filter response: The magnitude of frequency response:

$$|W_{x_{i,j}}(f_k)| \leq C(f_k)$$

Original Problem



Cost function: Total energy of e :

$$\sum_{k=k_1}^{k_2} \text{tr}[E(f_k)E(f_k)^H],$$

Constraints:

Enhancement: Normalized energy of e :

$$\text{tr}[E(f_k)E(f_k)^H]D_e(f_k) \leq A_e$$

Stability: Use Nyquist criterion:

$$\min \left(\text{Re} \left(\lambda \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) \right) > -1$$

Robustness:

$$\max \left(\sigma \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) B(f_k) \leq 1 \quad \Rightarrow \quad \mathbf{M-\Delta \text{ structure and small gain theory}}$$

Filter response: The magnitude of frequency response:

$$|W_{x_{i,j}}(f_k)| \leq C(f_k)$$

Original Problem

Cost function: Total energy of e :

$$\sum_{k=k_1}^{k_2} \text{tr}[E(f_k)E(f_k)^H],$$

Constraints:

Enhancement: Normalized energy of e :

$$\text{tr}[E(f_k)E(f_k)^H]D_e(f_k) \leq A_e$$

Stability: Use Nyquist criterion:

$$\min \left(\text{Re} \left(\lambda \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) \right) > -1$$

Robustness: M - Δ structure and small gain theory:

$$\max \left(\sigma \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) B(f_k) \leq 1$$

Filter response:

$$\left| W_{x_{i,j}}(f_k) \right| \leq C(f_k) \quad \rightarrow$$

The magnitude of frequency response



Modification



Original Problem

Cost function: Total energy of e:

$$\sum_{k=k_1}^{k_2} \text{tr}[E(f_k)E(f_k)^H],$$

Constraints:

Enhancement: Normalized energy of e:

$$\text{tr}[E(f_k)E(f_k)^H]D_e(f_k) \leq A_e$$

Stability: Use Nyquist criterion:

$$\min \left(\text{Re} \left(\lambda \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) \right) > -1$$

Robustness: M - Δ structure and small gain theory:

$$\max \left(\sigma \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) B(f_k) \leq 1$$

Filter response: The magnitude of frequency response:

$$|W_{x_{i,j}}(f_k)| \leq C(f_k)$$



Standard General Convex Problem

Cost function: $f_0(x)$

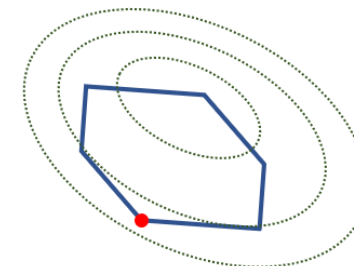
Constraints: $f_i(x) \leq 0, \quad i = 1, 2, 3 \dots$

$$Ax = b$$

$f_0(x)$ to be a convex function

$f_i(x)$ to be a convex function

A, b to be a constant matrix and vector



Modification



Original Problem ➔ General Convex Problem

Cost function: $w^T \left(\sum_{k=k_1}^{k_2} A_J(f_k) \right) w + 2\text{Re} \left(\sum_{k=k_1}^{k_2} b_J^T(f_k) \right) w + \sum_{k=k_1}^{k_2} c_J(f_k)$ ➔ • Quadratic ✓
 • Hessian $A_J(f_k)$ positive semidefinite Convex

Constraints:

Enhancement: $w^T A_J(f_k) w + 2\text{Re}(b_J^T(f_k) w) + c_J(f_k) - \frac{A_e}{D_e(f_k)} \leq 0$ ➔ • Quadratic ✓
 • Hessian $A_J(f_k)$ positive semidefinite Convex

Robustness: $\max \left(\sigma \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) B(f_k) - 1 \leq 0$ ➔ Matrix norm Convex ✓

Filter response: $\|F_z(f_k) w_{F_{i,j}}\|_2 - C(f_k) \leq 0$ ➔ Vector norm Convex ✓

Stability: $\min \left(\text{Re} \left(\lambda \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) \right) > -1$ ➔ NOT Convex ✗

↳ Replace by its upper-bound:

$\max \left(\lambda \left(\frac{-W_x(f_k) \hat{G}_{s0}(f_k) + \left(-W_x(f_k) \hat{G}_{s0}(f_k) \right)^H}{2} \right) \right) - (1 - \epsilon_s) \leq 0$ ➔ Easy to prove convexity by $\max(\lambda(A)) = \sup_{\|x\|_2=1} x^T A x$ Convex ✓

Reformulation

Convex Problem

Cost function:

$$w^T \left(\sum_{k=k_1}^{k_2} A_J(f_k) \right) w + 2\text{Re} \left(\sum_{k=k_1}^{k_2} b_J^T(f_k) \right) w + \sum_{k=k_1}^{k_2} c_J(f_k) \quad \text{Quadratic}$$

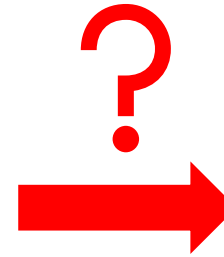
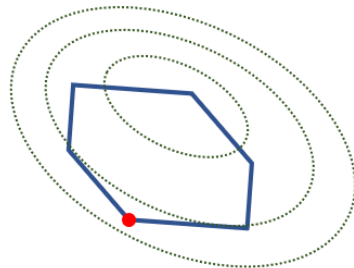
Constraints:

$$w^T A_J(f_k) w + 2\text{Re}(b_J^T(f_k) w) + c_J(f_k) - \frac{A_e}{D_e(f_k)} \leq 0 \quad \text{Quadratic}$$

$$\max \left(\lambda \left(\frac{-W_x(f_k) \hat{G}_{s0}(f_k) + (-W_x(f_k) \hat{G}_{s0}(f_k))^H}{2} \right) \right) - (1 - \epsilon_s) \leq 0 \quad \text{Max Eigenvalue}$$

$$\max \left(\sigma \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) B(f_k) - 1 \leq 0 \quad \text{Max Singular Value}$$

$$\|F_Z(f_k) w_{F_{i,j}}\|_2 - C(f_k) \leq 0 \quad \text{Vector Norm}$$



Standard Cone Programming

Cost function: $c^T x$

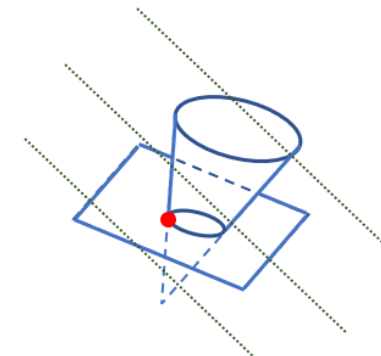
Constraints: $x \in K_i, \quad i = 1, 2, 3 \dots$

$$Ax = b$$

c to be a constant vector

K_i to be a convex cone

A, b to be a constant matrix and vector



Reformulation

Convex Problem ➔ Cone Programming



- Reformulate quadratic cost function

Cost function: $x^T A x + b^T x + c$



Cost function: $t_0 + b^T x$

➔ Linear cost function

Constraints: $\|\sqrt{A} x\|_2 \leq \sqrt{t_0} \tilde{t}_0$
 $\tilde{t}_0 = 1$

➔ Rotated second-order cone

➔ Linear constraint

- Reformulate quadratic constraints

Constraints: $x^T A x + b^T x + c \leq 0$



Constraints: $t_1 + b^T x + c = 0$

➔ Linear constraint

$\|\sqrt{A} x\|_2 \leq \sqrt{t_1} \tilde{t}_1$

➔ Rotated second-order cone

$\tilde{t}_1 = 1$

➔ Linear constraint

- The vector norm itself meets second-order cone requirement

Reformulation



Convex Problem \rightarrow Cone Programming

- Reformulate eigenvalue constraints

Constraints: $\max\left(\lambda\left(\frac{A(x) + A(x)^H}{2}\right)\right) - \epsilon \leq 0$ \rightarrow Each $A_{i,j}$ is the linearly related to x

\updownarrow

Constraints: $-A(x) - A(x)^H + 2\epsilon I \succcurlyeq 0$ \rightarrow Positive semidefinite cone
Each $A_{i,j}$ is the linearly related to x

- Reformulate singular value constraints

Constraints: $\max(\sigma(A(x))) - \epsilon \leq 0$ \rightarrow Each $A_{i,j}$ is the linearly related to x

\updownarrow

Constraints: $\begin{bmatrix} \epsilon I & A(x) \\ A(x)^H & \epsilon I \end{bmatrix} \succcurlyeq 0$ \rightarrow Positive semidefinite cone
Each $A_{i,j}$ is the linearly related to x

Reformulation

Convex Problem

Cost function:

$$w^T \left(\sum_{k=k_1}^{k_2} A_J(f_k) \right) w + 2\text{Re} \left(\sum_{k=k_1}^{k_2} b_J^T(f_k) \right) w + \sum_{k=k_1}^{k_2} c_J(f_k)$$

Constraints:

$$w^T A_J(f_k) w + 2\text{Re}(b_J^T(f_k) w) + c_J(f_k) - \frac{A_e}{D_e(f_k)} \leq 0$$

$$\max \left(\lambda \left(\frac{-W_x(f_k) \hat{G}_{s0}(f_k) + (-W_x(f_k) \hat{G}_{s0}(f_k))^H}{2} \right) \right) - (1 - \epsilon_s) \leq 0$$

$$\max \left(\sigma \left(W_x(f_k) \hat{G}_{s0}(f_k) \right) \right) B(f_k) - 1 \leq 0$$

$$\|F_Z(f_k) w_{F_{i,j}}\|_2 - C(f_k) \leq 0$$



Cone Programming

Cost function:

$$t_0 + 2\text{Re} \left(\sum_{k=k_1}^{k_2} b_J^T(f_k) \right) w$$

Constraints:

$$\|M_0 w\|_2 \leq \sqrt{t_0 \tilde{t}_0}, \quad \tilde{t}_0 = 1$$

$$t_{1,k} + 2\text{Re}(b_J^T(f_k) w) + c_J(f_k) - \frac{A_e}{D_e(k)} = 0$$

$$\|M_{1,k} w\|_2 \leq \sqrt{t_{1,k} \tilde{t}_{1,k}}, \quad \tilde{t}_{1,k} = 1$$

$$W_x(f_k) \hat{G}_{s0}(f_k) + (W_x(f_k) \hat{G}_{s0}(f_k))^H + 2(1 - \epsilon_s) \geq 0$$

$$\begin{bmatrix} \frac{1}{B(k)} I_{N_s} & W_x(k) \hat{G}_{s0}(k) \\ (W_x(k) \hat{G}_{s0}(k))^H & \frac{1}{B(k)} I_{N_s} \end{bmatrix} \geq 0$$

$$\|F_Z(f_k) w_{F_{i,j}}\|_2 \leq t_{3,k}, \quad t_{3,k} = C(f_k)$$



Results

Off-line Simulation based on experimental data

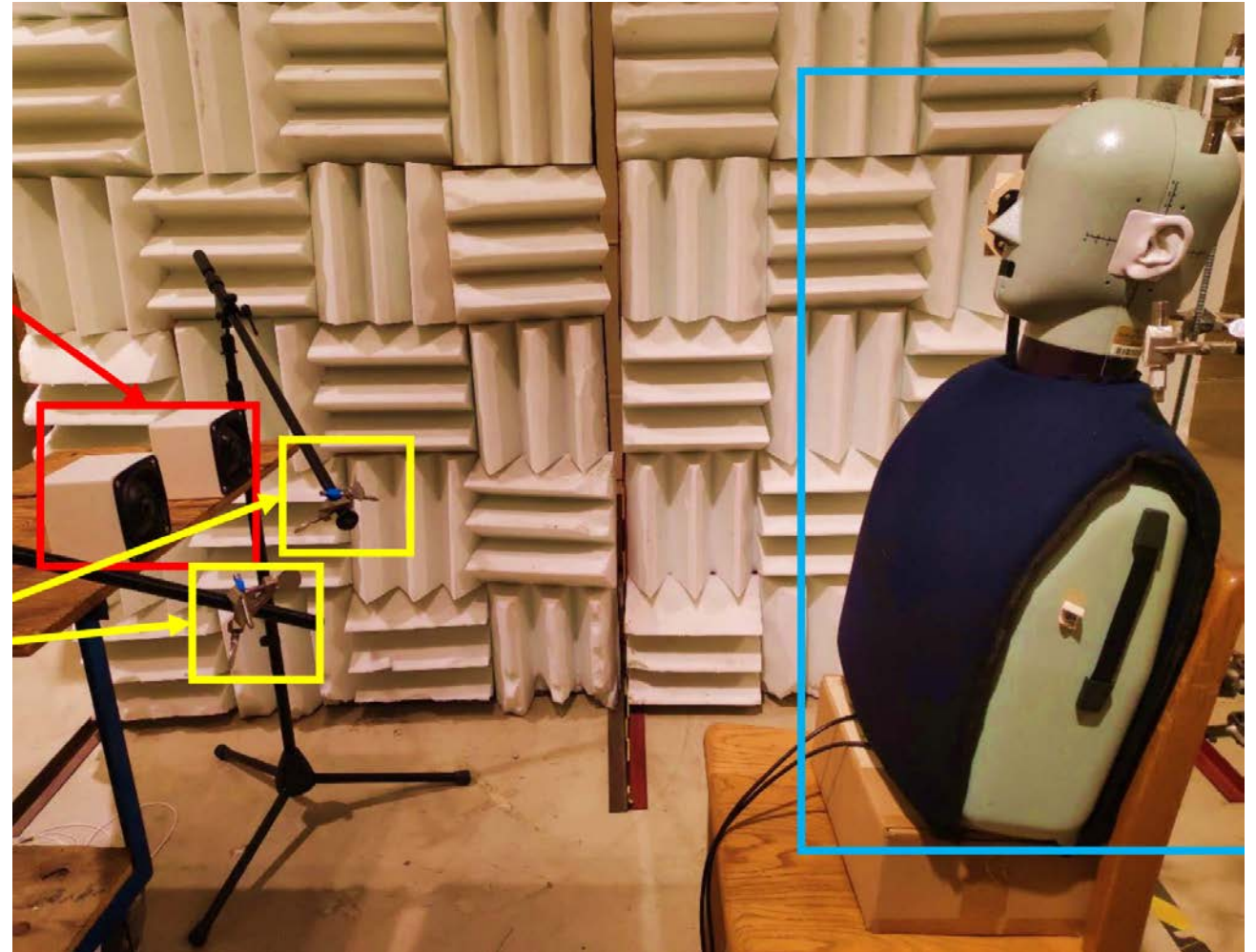
Experiment description:

- 2 reference microphones
- 2 control loudspeakers
- 2 error microphones
- sampling frequency is 8000 Hz

Red: Noise source

Yellow: Reference Microphones

Blue: Dummy, place for error microphone



Results



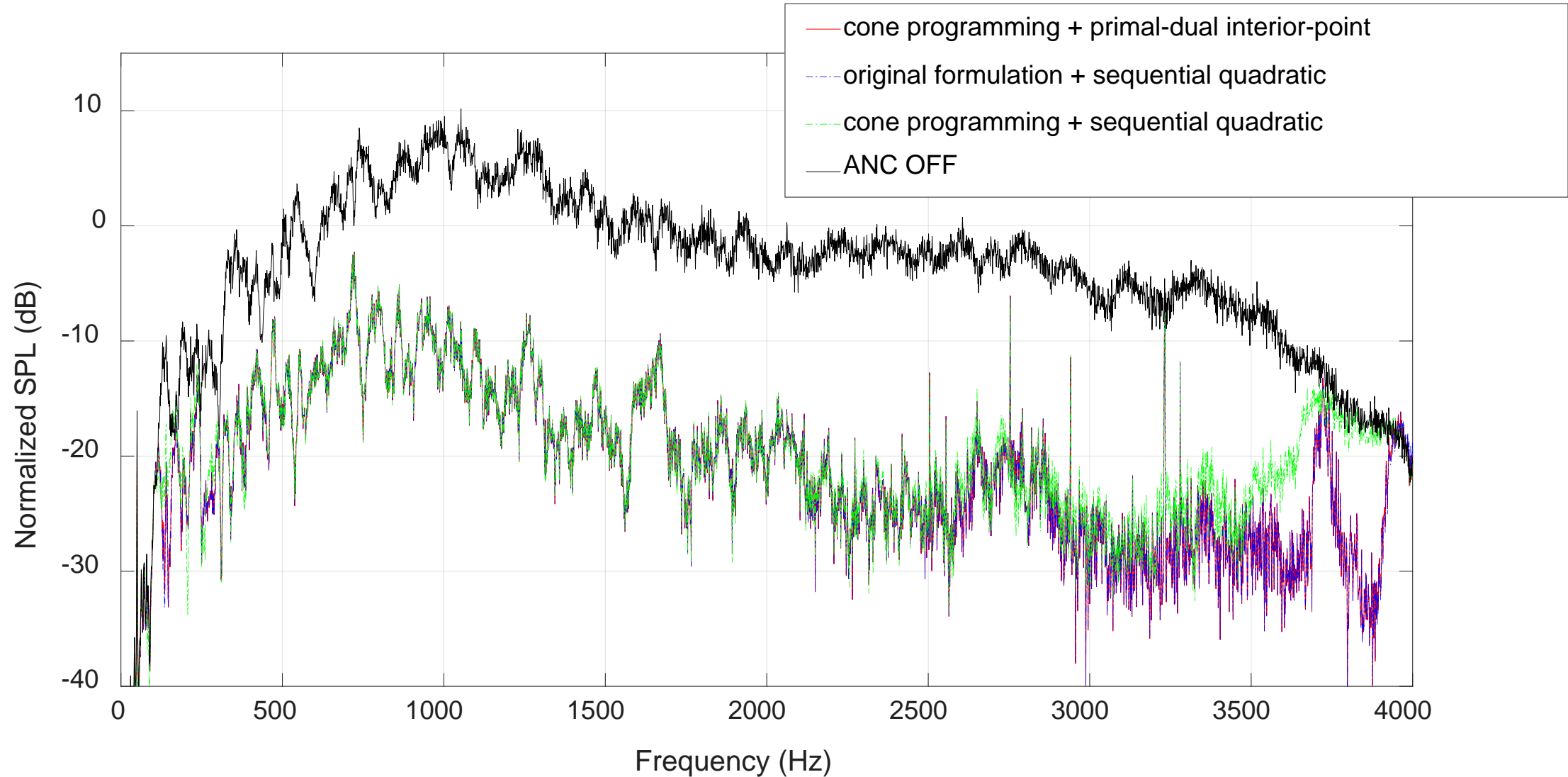
Table: Computation time for two problem sizes using different formulation-algorithm combinations

FIR length	cone programming + primal-dual interior-point	original formulation + sequential quadratic	cone programming + sequential quadratic
64	8.0 s	1790.4 s	1943.2 s
128	28.4 s	7504.9 s	5980.7 s

It is more **efficient**, if the filter design problem is reformulated to **cone programming** and solved by the **primal-dual interior-point method** (although the scale of the problem is much larger).

Results

Simulation of the attenuation performance for FIR length 128 using experimental data



Conclusions



- The ANC filter design problem can be modified and reformulated to a cone programming problem.
- The calculation using the primal-dual interior-point method for cone programming can be faster, compared with that using the commonly used sequential quadratic programming method.
- In the future, if the efficiency of this method can be further improved, it is possible to consider making this filter design problem adaptive.

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Thank you !

