RAPID PROTOTYPING USING FRACTAL GEOMETRY

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Abstract

The paper proposes a method for rapid prototyping (RP) fractal geometry represented objects. RP technology has made possible the physical fabrication of solid freeform objects. However, contemporary CAD/CAM/RP systems are developed for Euclidean geometry and are incapable of handling self-similar fractal objects (e.g. jewelry products). To address the problem, a Radial-Annular Tree (RAT) data structure is proposed to represent Iterated Function Systems (IFS) fractal curves. RP toolpaths can then be generated from the RAT data structure. Geometry modeled in the RAT data structure can also be combined with Euclidean geometry from traditional CAD systems to make aesthetic patterns for the jewelry industry.

Introduction

Rapid prototyping (RP) technology has made possible the physical fabrication of solid freeform objects. The input to RP is either a solid model or closed surface model prepared from computer-aided design (CAD) systems. However, the contemporary CAD or RP systems are developed for Euclidean geometry. It is thus beneficial to widen the geometric scope of CAD and RP to non-Euclidean geometry. For instance, many products in jewelry industry will adopt aesthetic appealing patterns existed in nature. In mathematics, natural objects, likes clouds, terrain, trees, can be described by fractal geometry. In order to fabricate jewelry prototype, a CAD model for fractal geometry is needed.

Kerekes [1] is probably the first to generate a fractal tree structure in three dimensions with Stereolithography (SL) RP machine. The example is generated without any help from CAD as the structure is extremely complex. However, Kerekes has not reported much details. Lee [2] has reviewed various commercial CAD systems being used for fractal design. All software have functions for 2-D fractal image generation and processing, but none of them has addressed the 3-D fractal object rapid prototyping. In jewelry making, Lee *et al* [3] has proposed an integrated CAD and RP approach. The discussion is only based on Euclidean geometry domain. In fact, CAD and RP systems available in the market for jewelry making cannot handle fractal geometry yet.

In this research, RP of fractal geometry is achieved indirectly. Fractal geometry is first modeled into some computer understandable format. The information is then imported to Euclidean CAD. Conventional CAD modeling and editing operations are applied before RP tool path is generated for making the physical fractal geometry prototype.

The Fractal Geometry

From the foundation work of Mandelbrot [4], fractal geometry provides a way to describe and to model aesthetic object, especially for those not easily created by Euclidean geometry. Fractal is by definition a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension. It is used to describe the fragmentation of an object. A common property of fractal object is self-similar, i.e. a fractal is a shape made of parts similar to the whole in some way. One deterministic approach to generate or to approximate a mathematical fractal object is to use **Iteration function system** (**IFS**) [5]. An IFS is a family of specified contraction mappings that map a whole object onto the parts, unionize all the parts and iteration of these mappings will result in convergence to an invariant set [6], i.e. a fractal. From the viewpoint of geometric modeling, unionization in IFS is regularized in order to maintain the validity of the operands in further iteration. Thus the equation of **Regularized Iterated Function System** generation is

$$F^k(S) = F(F^{k-1}(S)) = \bigcup_{i=1}^{n}{}^{*} f_i(F^{k-1}(S)) \qquad k = 2, 3, \ldots \quad (1)$$

where S = self-similar segment, F= iterated function, and n=iteration number.

In this research, focus will be put on 2D fractal curves. In particular, the quadric Koch curve will be used to illustrate the methodology. Fractal curves are self-similar and continuous, but they are nowhere differentiable. The quadric Koch curve consists of a *generator* and an *initiator*. The generator is a multi-segment polyline (S) which is also used to derive the IFS. The initiator is a square which is used to derive transformations for replicating other copies. Figure 1 shows an 18-segment quadric Koch curve. The IFS has totally eighteen functions $F=\{f_1, \ldots, f_{18}\}$ that will contribute to a combined transformation from the whole S to each parts $f_i(S)$. According to Equation (1), the number of transformations in the IFS equals to eighteen (i.e. n = 18), such that

$$F^k(S) = F(F^{k-1}(S)) = \bigcup_{i=1}^{18}{}^{*} f_i(F^{k-1}(S)) \qquad (2)$$

The Radial-Annular Tree data structure

In order to represent an IFS generated fractal curve in a computational form, a RAT data structure is developed [7]. The RAT data structure is based on a vertex-link scheme. The data node of RAT consist of three elements, a set of coordinate values, a *sibling* pointer and a *child* pointer. The geometry information of a vertex is its coordinate values. The sibling pointer points to the next node in same level. The child pointer points to the next higher-level node in order to inherit lower level information. Figure 1 also illustrates the RAT levels corresponding to the 18-segment quadric Koch curve. To generate the tool path for RP, the circumferential nodes of the RAT will be traversed.

Capability of RAT data structure

In addition to direct making of fractal curve, the RAT data structure can also combine with Euclidean geometry in traditional CAD systems to model aesthetic patterns for jewelry industry.

Fractal curve on free-form surface

A fractal pattern for engraving or embossing can be prepared by projecting a fractal curve onto a free-form surface. Figure 2 shows an example which parallel projects an 18-segment quadric Koch curve onto a NURBS surface. $\vec{P}_{Hole}$ denotes a punctured NURBS surface while $\check{P}_{Trim}$ denotes a trimmed NURBS surface with the projected fractal curve $\check{P}_{fractal}$. $\check{P}_{Hole}$ is formed by subtracting the original NURBS surface with a projected volume of fractal curve while $\check{P}_{Trim}$ is formed by subtracting the original NURBS surface with $\check{P}_{Hole}$. Mathematically,

$$\check{P}_{Hole} = \check{P}_{NURBS} - Vol_{proj}\left(\check{P}_{fractal}\right)$$

$$\check{P}_{Trim} = \check{P}_{NURBS} - \check{P}_{Hole} = \check{P}_{NURBS} - \left[\check{P}_{NURBS} - Vol_{proj}\left(\check{P}_{fractal}\right)\right]$$

The interior of the projected fractal curve $\check{P}_{fractal}$ forms $Vol_{proj}\left(\check{P}_{fractal}\right)$ by intersecting with the modeling work space, W, i.e.

$$Vol_{proj}\left(\check{P}_{fractal}\right) = \left\{(x, y, z) \mid \exists (x, y, z') \in \check{P}_{fractal}\right\} \cap W \qquad (3)$$

where z' is arbitrary selected with no duplication. In practice, the coordinate values of the RAT represented $\check{P}_{fractal}$ are translated, scaled and projected onto the NURBS surface. Thus, a variant of the 18-segment quadric Koch curve with piecewise free-form curve segment is generated.

Fractal solid modeling

In order to construct a solid model with fractal boundary, the RAT represented fractal curve can be transformed in Euclidean CAD system. The fractal curve is treated as the operand point set for extrusion, revolution, lofting, sweep, etc. [8]. Figure 3 shows the workflow of embedding the RAT represented fractal curve into contemporary CAD work flow. Figure 4 shows a twist-extruded fractal solid with twice iterated 18-segment quadric Koch curve. Figure 5 shows a twist-extruded fractal solid with offset twisting vector. Figure 4 shows a revolved fractal solid in three quarters turn. Figures 7 and 8 show a lofted fractal solid and a spirally lofted fractal solid respectively. In the above illustration examples, the RAT information of a particular fractal curve level is communicated with Euclidean CAD system via IGES.

Discussion and Conclusion

A workflow for rapid prototyping fractal geometry has been outlined. This is demonstrated with a 2D quadric Koch curve. It can be seen that the RAT data structure plays a central role in bridging the gap between fractal geometry and Euclidean geometry. Figure 8 shows other fractal curves [9] with different generators and/or different initiators that can be constructed using the generation scheme. However, the RAT data structure is applicable for fractal curves that can be formulated in IFS. Other data structure may be needed for non-IFS fractal curve.

The approach also provides a user-friendly way to design jewelry products, such as pendant, button, bangle/bracelet, brooch, medal and cufflinks. To further extend the geometric coverage, true fractal solids like Meger sponge [10] can also be integrated with Euclidean CAD.

Acknowledgement

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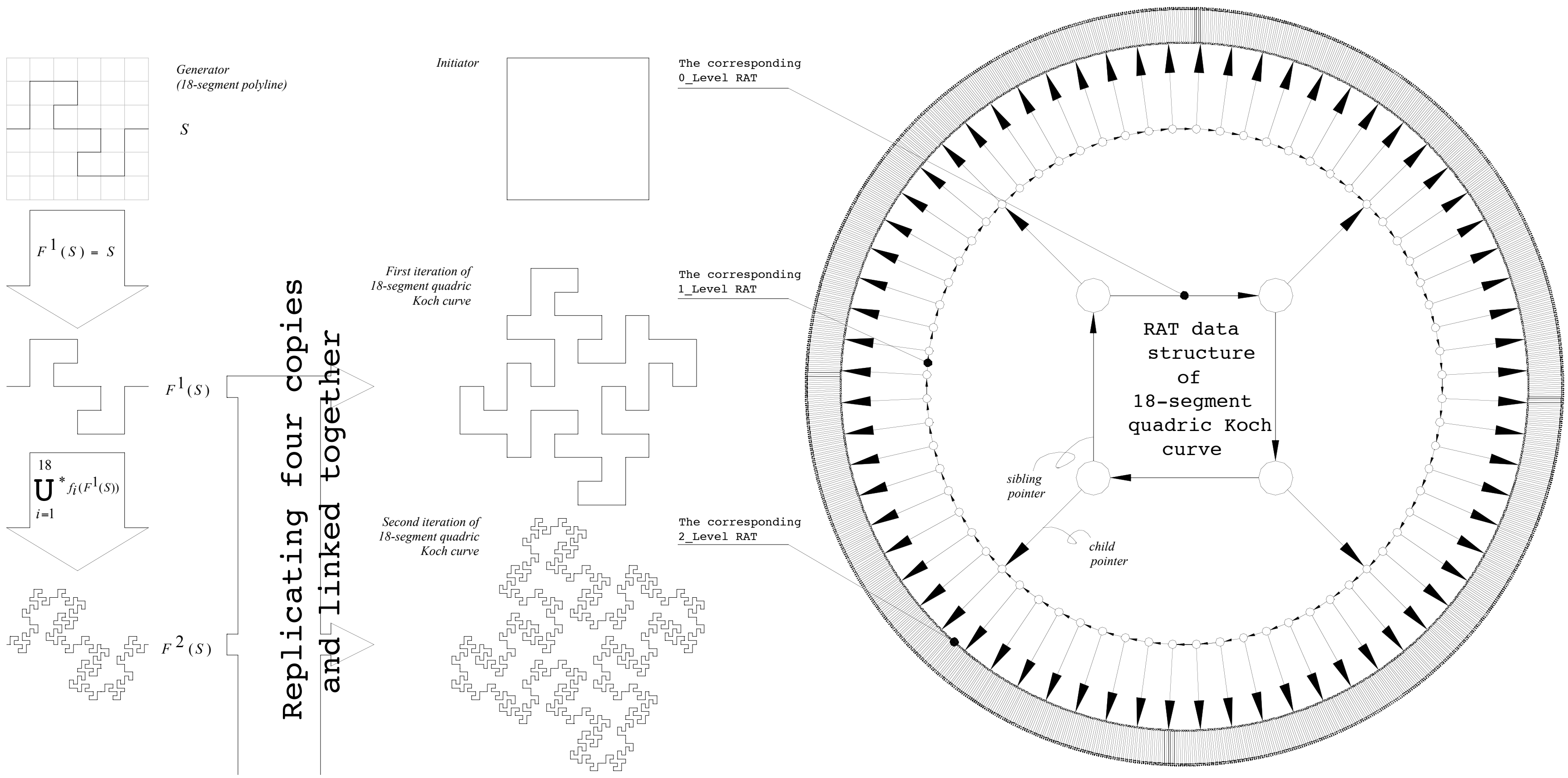


Figure 1. An 18-segment quadric Koch curve and the corresponding Radial-Annular Tree data structure.

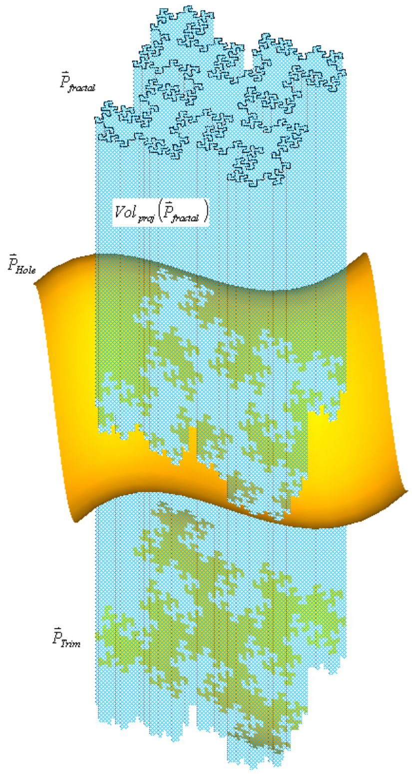


Figure 2. Free-form surface trimmed with a projected Koch snowflake curve.

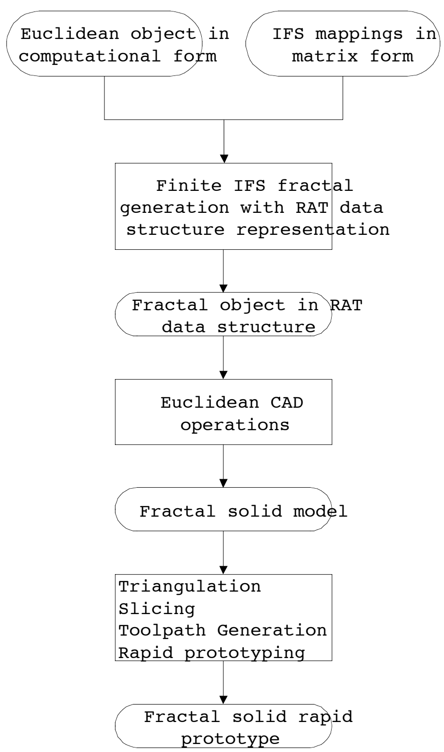


Figure 3. Fractal solid rapid prototyping workflow.

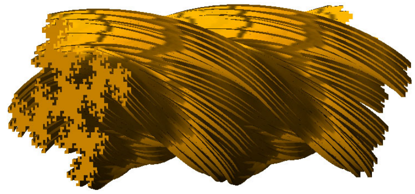


Figure 4. Twisted extrusion with coaxial twisting vector.

Figure 5. Twisted extrusion with offset twisting vector.

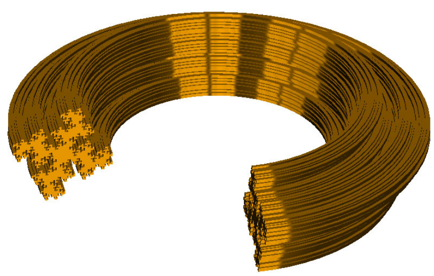


Figure 6. Revolution generated fractal solid.

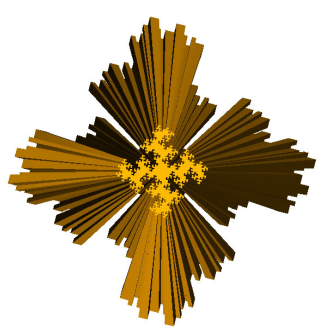
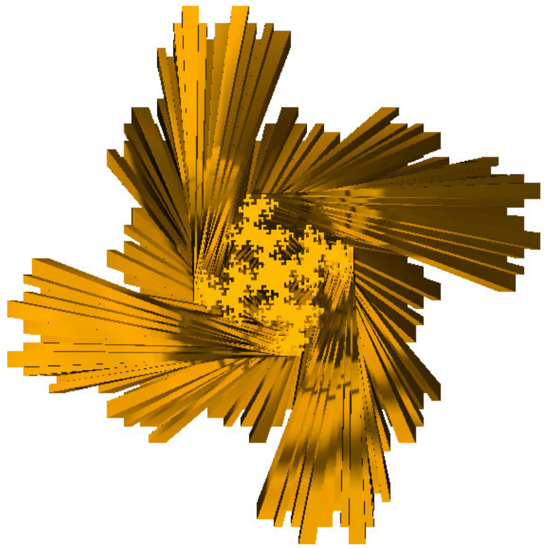


Figure 7. Lofting with cross-sections in different scales.



(a) CAD model

(b) FDM prototype

Figure 8. Lofting with cross-sections in different scales and orientations.

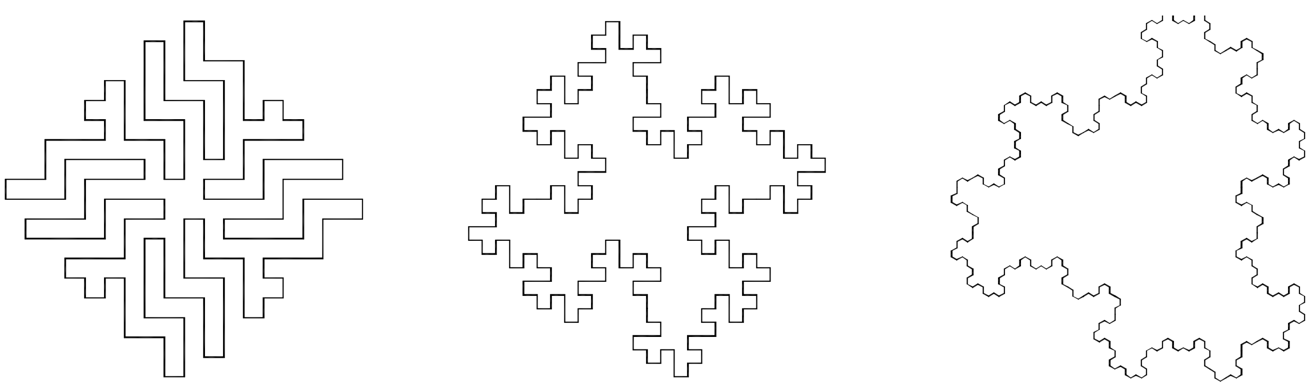


Figure 9. Other quadric Koch curves (a, b) and Fudgeflake boundary curve (c).