Analytical solution of shallow water equations for ideal dam-break flood along a wet bed slope

Bo Wang[a], Yunliang Chen[b], Yong Peng[c*], Jianmin Zhang[d], Yakun Guo[e]

[a]Professor, State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, Sichuan 610065, China. E-mail: wangbo@scu.edu.cn

[b]Professor, State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, Sichuan 610065, China. E-mail: 13541291332@163.com

[c]Associate Professor, State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, 610065, P. R. China. E-mail: pengyongscu@foxmail.com (Corresponding author)

[d]Professor, State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, Sichuan 610065, China. E-mail: zhangjianmin@scu.edu.cn

[e]Professor, Faculty of Engineering & Informatics, University of Bradford, BD7 1DP, UK. y.guo16@bradford.ac.uk

Abstract: The existing analytical solutions of dam-break flow do not consider simultaneously the effects of wet downstream bottom and bed slope on the dam-break wave propagation. In this study, a new analytical solution for the shallow-water equations (SWE) is developed to remove this limitation to simulate the wave caused by an instantaneous dam-break. The approach adopts the method of characteristics and has been applied to simulate the dam-break flows with different downstream water depths and slopes. The analytical solutions have been compared with predictions by the lattice Boltzmann method and the agreement is good. Although the proposed analytical solution treats an idealized case, it is nonetheless suitable for assessing the robustness and accuracy of numerical models based on the SWE without the frictional slope.

Keywords: Analytical solution; Dam-break; Rarefaction wave; Shock wave; Slope; Wet bed.

Introduction

Analytical studies of dam-break flows date back to over 120 years ago. Well-known analytical solutions for the dam-break flood waves in a horizontal channel include Ritter (1892) and Stoker (1957). Ritter (1892) derived an analytical solution for instantaneous dam-break flows on a frictionless and dry bed. For wet bed conditions downstream of the dam, a shock wave develops in the downstream region. To investigate the effect of the initial downstream water depth on the dam-break wave, Stoker (1957) proposed a theoretical solution which included three interrelated equations and three unknown variables. For the dam-break problem on a sloping bed, some exact and approximate analytical solutions exist (Dressler 1958; Hunt 1983; Fernandez-Feria 2006; Ancey et al. 2008; Chanson 2009). However, none of these considers simultaneously the effects of both the wet bed condition and the bottom slope on the dam-break flow.

In the present study, a new analytical solution of the shallow-water equations is proposed for an infinite volume of an ideal (frictionless) fluid released instantaneously from upstream of a dam with initial wet horizontal and sloping channel. The omission of the friction is based on the following considerations: (1) the frictional slope is a nonlinear term that hinders one from solving the Saint-Venant equations (SVE) analytically (Chanson 2009); (2) a frictionless fluid is often considered in the development of the analytical solutions for dam-break problems (Ritter 1892; Stoker 1957; Wu et al. 1999; Fernandez-Feria 2006; Ancey et al. 2008; Chen et al. 2011; Wang et al. 2017; Cozzolino et al. 2017); (3) dam-break flow can be considered as frictionless flow for relatively high flow velocity and little flow separation from solid boundaries (Batchelor 2000; Guo et al. 1998; Guo 2005), and (4) although a truly frictionless flow does not occur in nature, the frictionless case constitutes an unambiguous end-member as well as a clear target case for testing numerical models (Ancey et al. 2008). A typical example is that Zoppou and Roberts (2003) conducted an examination of the performance of twenty explicit numerical schemes used to solve the shallow water wave equations for simulating the dam-break problem by comparing the results from these schemes with analytical solutions and expected more analytical solutions to be developed for testing the numerical schemes. The proposed analytical solution is developed using the method of characteristics. Its performance will be examined by comparing with the numerical simulations based on the Lattice Boltzmann Method (LBM).

During the past two decades, the LBM has become a successful alternative numerical method for computational fluid dynamics. As a microscopic method, the LBM has been developed and used to successfully solve the shallow water equations (Zhou 2004; Peng et al. 2011a, b).

General models for dam-break wave

Figure 1 sketches the problem investigated in which x_M and x_N, respectively, denote the positions of the front and tail for the rarefaction wave; and x_R represents the location of the shock wave-front. The dam releases immediately the water at both sides initially stationary (i.e., $u_u = u_d = 0$) with two different water depths (i.e., $h_u > h_d > 0$) (Fig.1). After dam collapses, the flow can be divided into four zones: Zone 1 is the undisturbed far upstream; Zone 2 is a rarefaction wave; Zone 3 is a constant state where water is not at rest; and Zone 4 is the quiet downstream that is terminated on the upstream side by the shock wave. The water depths and flow velocities in Zones 2 and 3 are denoted as h and u as well as h_c and u_c respectively. ξ is the shock wave-front celerity.

Governing equation for rarefaction wave

The propagation of a dam-break wave is governed by the SVE (Chow 1959). For an infinitely long prismatic channel of mild constant slope, the SVE can be written if the effect of wall friction is neglected as follows:

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + \frac{A}{B}\frac{\partial u}{\partial x} = 0 \tag{1a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = gS_o \tag{1b}$$

Here h = the flow depth, x = the distance along flow direction, t = the time, g = gravity acceleration, u = the average flow velocity, B = the water surface width, A = the area of cross-section, and S_o = the bottom slope.

Applying the characteristic method to the system of partial differential equations (1a)–(1b), a characteristic system of equations results as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(u+\int_0^h\sqrt{g\frac{B}{A}}\mathrm{d}h\right)=gS_o \quad \text{along} \quad \frac{\mathrm{d}x}{\mathrm{d}t}=u+\sqrt{g\frac{A}{B}} \tag{2a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(u-\int_0^h\sqrt{g\frac{B}{A}}\mathrm{d}h\right)=gS_o \quad \text{along} \quad \frac{\mathrm{d}x}{\mathrm{d}t}=u-\sqrt{g\frac{A}{B}} \tag{2b}$$

Along the forward characteristics:

$$\left(u+\int_0^h\sqrt{g\frac{B}{A}}\mathrm{d}h\right)-\left(u_u+\int_0^{h_u}\sqrt{g\frac{B}{A}}\mathrm{d}h\right)=gS_o t \tag{3}$$

where subscript u = the undisturbed upstream reach, i.e., Zone 1. h_u and u_u = the initial water depth and flow velocity respectively in the undisturbed upstream reservoir.

As the upstream reach is undisturbed, u_u =0. Substituting it into Eq. (3) yields:

$$u=\int_0^{h_u}\sqrt{g\frac{B}{A}}\mathrm{d}h-\int_0^h\sqrt{g\frac{B}{A}}\mathrm{d}h+gS_o t \tag{4}$$

Substituting Eq. (4) into Eq. (2b) yields:

$$\int_0^h\sqrt{\frac{B}{A}}\mathrm{d}h+\sqrt{\frac{A}{B}}=\int_0^{h_u}\sqrt{\frac{B}{A}}\mathrm{d}h-\left(\frac{1}{\sqrt{g}}\frac{\mathrm{d}x}{\mathrm{d}t}-\sqrt{g}S_o t\right) \tag{5}$$

Eq. (5) is the transformed SVE and applies for the prismatic channel of arbitrary shape.

Governing equation for shock wave

The motion of the shock wave is described by the conservation equations for mass and momentum as:

$$A_c\left(u_c-\xi\right)=A_d\left(u_d-\xi\right) \tag{6}$$

$$A_d u_d\left(u_d-\xi\right)-A_c u_c\left(u_c-\xi\right)=g\left(A_c\bar{h}_c-A_d\bar{h}_d\right) \tag{7}$$

where subscripts c and d = the reaches upstream and downstream of the shock respectively; $\bar{h}$ = the centroid water depth for the cross section; and ξ is defined as:

$$\xi=\frac{\mathrm{d}x_R}{\mathrm{d}t} \tag{8}$$

In this study, the downstream flow is at rest, i.e., $u_d = 0$. Therefore Eqs. (6)–(7) simplify to:

$$\xi = \frac{A_c u_c}{A_c - A_d} \tag{9}$$

$$u_c = \left[g \frac{\left(A_c \bar{h}_c - A_d \bar{h}_d\right)\left(A_c - A_d\right)}{A_c A_d} \right]^{1/2} \tag{10}$$

The hydraulic characteristics, i.e., flow depth and velocity, are unique in the plane *N*–*N* connecting Zones 2 and 3. Applying Eq. (4) to this boundary condition yields the flow velocity in Zone 3:

$$u_c = \int_0^{h_u} \sqrt{g \frac{B}{A}} \mathrm{d}h - \int_0^{h_c} \sqrt{g \frac{B}{A}} \mathrm{d}h + g S_o t \tag{11}$$

Substituting Eq. (10) into Eq. (11) yields:

$$\int_0^{h_u} \sqrt{\frac{B}{A}} \mathrm{d}h - \int_0^{h_c} \sqrt{\frac{B}{A}} \mathrm{d}h + \sqrt{g} S_o t = \left[\frac{\left(A_c \bar{h}_c - A_d \bar{h}_d\right)\left(A_c - A_d\right)}{A_c A_d} \right]^{1/2} \tag{12}$$

Eq. (12) is the integral form of the momentum equation for the shock wave and applies for a channel of arbitrary shape.

Dam-break wave in wet rectangular channel

Analytical solution for rarefaction wave

For a rectangular channel, the cross-sectional area is the product of the flow depth and the water surface width. Applying this condition for the rarefaction wave, Eq. (5) simplifies to:

$$\int_0^{h} \frac{1}{\sqrt{h}} \mathrm{d}h + \sqrt{h} = \int_0^{h_u} \frac{1}{\sqrt{h}} \mathrm{d}h - \left(\frac{1}{\sqrt{g}} \frac{\mathrm{d}x}{\mathrm{d}t} - \sqrt{g} S_o t \right) \tag{13}$$

Integrating Eq. (13) yields the dimensionless flow depth:

$$h^* = \frac{h}{h_u} = \frac{1}{9} \left[2 - \frac{1}{\sqrt{g h_u}} \left(\frac{\mathrm{d}x}{\mathrm{d}t} - g S_o t \right) \right]^2 \tag{14}$$

Let

$$X = \frac{1}{\sqrt{gh_u}}\left(\frac{\mathrm{d}x}{\mathrm{d}t} - gS_o t\right) \tag{15}$$

$$X_1 = \frac{1}{\sqrt{gh_u}}\frac{\mathrm{d}x}{\mathrm{d}t} \tag{16}$$

$$X_2 = \sqrt{\frac{g}{h_u}} S_o t \tag{17}$$

where X_1 = a dimensionless distance and X_2 = a dimensionless variable to account for the effect of bed slope. Eq. (15) can then be rewritten as:

$$X = X_1 - X_2 \tag{18}$$

From Eqs. (14) and (15), one gets:

$$h^* = \frac{1}{9}(2 - X)^2 \tag{19}$$

Substituting Eq. (19) into Eq. (2b) yields the dimensionless velocity:

$$u^* = \frac{u}{\sqrt{gh_u}} = \frac{1}{3}\left[2(1 + X_1) + X_2\right] \tag{20}$$

For consistency, the flow discharge is normalized as:

$$\frac{Q}{A_u\sqrt{gh_u}} = \frac{Bhu}{B_u h_u \sqrt{gh_u}} \tag{21}$$

The top widths in Zones 1 and 2 are equal, i.e., $B = B_u$, thus the dimensionless discharge is:

$$Q^* = \frac{Q}{A_u\sqrt{gh_u}} = \frac{h}{h_u}\cdot\frac{u}{\sqrt{gh_u}} = h^*\cdot u^* = \frac{1}{27}(2 - X)^2\left[2(1 + X_1) + X_2\right] \tag{22}$$

Analytical solution for shock wave

For a rectangular channel, the centroid depth for the cross section is half of the flow depth. Applying this condition for the shock wave, Eq. (12) simplifies to:

$$\int_0^{h_u}\frac{1}{\sqrt{h}}\mathrm{d}h - \int_0^{h_c}\frac{1}{\sqrt{h}}\mathrm{d}h + \sqrt{g}S_o t = \left[\frac{(h_c^2 - h_d^2)(h_c - h_d)}{2h_c h_d}\right]^{1/2} \tag{23}$$

Integrating Eq. (23) yields:

$$\left(h_c^*\right)^3 - 9h_d^*\left(h_c^*\right)^2 + 8\left(2+\sqrt{\frac{g}{h_u}}S_o t\right)\left(h_d^*\right)\left(h_c^*\right)^{\frac{3}{2}} - \left(h_d^*\right)\left(h_d^* + 8 + 8\sqrt{\frac{g}{h_u}}S_o t + \frac{2}{h_u} g S_o^2 t^2\right)\left(h_c^*\right) + \left(h_d^*\right)^3 = 0 \tag{24}$$

where $h_c^* = h_c / h_u$ and $h_d^* = h_d / h_u$ = the dimensionless flow depths upstream and downstream of the shock wave respectively. Combining Eq. (17) with Eq. (24) yields the following equation:

$$\left(h_c^*\right)^3 - 9h_d^*\left(h_c^*\right)^2 + 8\left(2+X_2\right)\left(h_d^*\right)\left(h_c^*\right)^{\frac{3}{2}} - \left(h_d^*\right)\left(h_d^* + 8 + 8X_2 + 2X_2^2\right)\left(h_c^*\right) + \left(h_d^*\right)^3 = 0 \tag{25}$$

The hydraulic characteristics in Zone 2 are identical as those in Zone 3 at the junction, i.e., *N*–*N*. Therefore, applying Eq. (20) for the flow in junction yields the dimensionless velocity:

$$u_c^* = \frac{u_c}{\sqrt{gh_u}} = \frac{1}{3}\left[2\left(1+X_{1N}\right)+X_{2N}\right] \tag{26}$$

where X_{1N} and X_{2N} = dimensionless variables referring to the junction *N*–*N*, defined as:

$$X_{1N} = \frac{1}{\sqrt{gh_u}}\frac{\mathrm{d}x_N}{\mathrm{d}t} \tag{27}$$

$$X_{2N} = \sqrt{\frac{g}{h_u}}S_0 t \tag{28}$$

Appling Eq. (19) for the flow at the junction yields

$$h_c^* = \frac{1}{9}\left(2 - X_N\right)^2 \tag{29}$$

where X_N = a dimensionless variable referring to the junction *N*–*N* and can be expressed as:

$$X_N = X_{1N} - X_{2N} \tag{30}$$

Substituting Eqs. (29)-(30) into Eq. (26) yields:

$$u_c^* = X_{2N} + 2\left(1 - \sqrt{h_c^*}\right) \tag{31}$$

Equation (26) or (31) can be used to calculate the dimensionless velocity. Similarly, the dimensionless discharge is obtained by combining Eqs. (29) and (31):

$$Q_c^* = \frac{Q_c}{A_u\sqrt{gh_u}} = \frac{h_c}{h_u}\cdot\frac{u_c}{\sqrt{gh_u}} = h_c^*\left[X_{2N} + 2\left(1-\sqrt{h_c^*}\right)\right] \tag{32}$$

Substituting Eq. (31) into Eq. (9) yields the dimensionless wave-front celerity:

$$\xi^* = \frac{\xi}{\sqrt{gh_u}} = h_c^*\cdot\frac{X_{2N} + 2\left(1-\sqrt{h_c^*}\right)}{h_c^* - h_d^*} \tag{33}$$

Dam-break wave in slope and frictionless channel

To verify the accuracy of the proposed analytical solution, numerical simulations by the LBM are carried out correspondingly.

Lattice Boltzmann model for shallow water equations

In this study, the most common Lattice Boltzmann Model (D2Q9) is adopted and the lattice Boltzmann equation for 2D shallow water equations reads (Zhou 2004):

$$f_\alpha\left(\boldsymbol{x}+e_\alpha\Delta t, t+\Delta t\right) - f_\alpha\left(\boldsymbol{x}, t\right) = -\frac{1}{\tau}\left[f_\alpha\left(\boldsymbol{x}, t\right) - f_\alpha^{eq}(\boldsymbol{x}, t)\right] + \Delta t F_\alpha \tag{34}$$

where f_α and f_α^{eq} = the distribution functions, τ = the single relaxation time, e_α = the particle velocity, F_α = force term as defined by Peng et al. (2011a, b).

The fluid kinematic viscosity ν is defined as:

$$\nu = \frac{e^2\Delta t}{6}\left(2\tau - 1\right) \tag{35}$$

The local equilibrium distribution function f_α^{eq} in Eq. (34) is defined as:

$$f_\alpha^{eq} = \begin{cases} h - \dfrac{5gh^2}{6e^2} - \dfrac{2h}{3e^2}u_i u_i, & \alpha = 0 \\ \dfrac{gh^2}{6e^2} + \dfrac{h}{3e^2}e_{\alpha i}u_i + \dfrac{h}{2e^4}e_{\alpha i}e_{\alpha j}u_i u_j - \dfrac{h}{6e^2}u_i u_i, & \alpha = 1,3,5,7 \\ \dfrac{gh^2}{24e^2} + \dfrac{h}{12e^2}e_{\alpha i}u_i + \dfrac{h}{8e^4}e_{\alpha i}e_{\alpha j}u_i u_j - \dfrac{h}{24e^2}u_i u_i, & \alpha = 2,4,6,8 \end{cases} \tag{36}$$

The macroscopic water depth and velocity can be obtained from the following equations:

$$h = \sum_\alpha f_\alpha \tag{37}$$

$$u_i = \frac{1}{h}\sum_{\alpha} e_{\alpha i} f_{\alpha} \tag{38}$$

More details on LBM for shallow water equations can be found in Zhou (2004) and Peng et al. (2011a, b).

Comparison with simulations by LBM

Three smooth, rectangular flumes are 60 m long, 1m wide, and 1.2 m high with bottom slopes of 0.1, 0.3, and 0.5% respectively are used in experiments. A virtual dam located 30 m away from the downstream flume end is adopted to simulate an instantaneous dam failure. Three wet-bed conditions are tested: h_d = 0.12, 0.24 and 0.36 m, corresponding to h_d^* = 0.2, 0.4 and 0.6. Due to length limitation, only the simulated results for the case with h_d^* = 0.2 are shown in Figs. 2 as the results of other two cases are similar.

Figure 2 compares the water surface profiles predicted by the proposed analytical method and LBM for t =0.5, 1.5 and 3.0 s. Generally the water surface profile by the proposed analytical method agrees well with the prediction by LBM. In Zone 2, the water depth simulated by LBM is slightly smaller than the analytical solution. Both the front and the tail of the rarefaction wave captured from LBM propagate faster that those from the analytical model, resulting in an extension of rarefaction fans. In Zone 3, the water surface profiles predicted by LBM are under the analytical results. Compared with the numerical results, the evolution position of the shock wave in the analytical solution lags slightly and the difference between the analytical and LBM solution is smaller than 10% for all of three slopes during the first ten seconds. This difference may be due to the finite difference method adopted by the LBM model.

The discharges along the flume predicted by the proposed analytical method versus LBM are also shown in Fig. 2. It can be seen that the discharge increases as bottom slope of the flume increases. With the increase of downstream water depth, the flow discharge in Zone 3 generally decreases. The analytical solution captures the motion characteristics well. Based on the comparisons between analytical solutions and numerical results, it is clear that the analytical and simulated results agree well in both Zone 1 and Zone 2, while the numerical results are slightly higher than the analytical solutions in both Zone 3 and Zone 4, and the difference between them tends to increase with time. Especially in Zone 4, the analytical model assumes that the water body is not affected by the shock wave. Therefore, the water is stationary, and the corresponding discharge is zero. When the downstream water depth is small (e.g. $h_d^* = 0.2$), the analytical solution of discharge in Zone 3 agrees well with the numerical results. That demonstrates that the proposed analytical method can be applied to accurately predict the flood wave propagation generated by dam-break along a sloping and initially wet downstream bed.

Conclusions

In the present study, a new analytical solution, based on the method of characteristics, has been developed for the shallow-water equations (SWE) which can be used to validate numerical models based on the one-dimensional SWE as long as the term of the frictional slope is neglected. The analytical solution accurately predicts the effects of bottom slope and initial downstream water depth on the propagation of a flood wave generated by the dam-break, which is difficult to be achieved by previously analytical solutions. The propagation of the dam-break flood waves on sloping and wet beds predicted by the analytical model was compared with the numerical simulations based on the LBM. A satisfactory agreement between the analytical and numerical solutions is found in both Zones 1 and 2, while mild distinction exists in both Zones 3 and 4.

Acknowledgements

The authors would like to thank for the financial support of the National Key Research and Development Program of China (Grant No: 2018YFC1505000), National Natural Science Foundation of China (Grant Nos: 51879179; 51579166) and Sichuan Science and Technology Program (No. 2019JDTD0007). The research is also supported by the Open Fund from the State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University (SKHL1809).

Comments made reviewers have greatly improved the quality of the final paper.

Notation

The following symbols are used in this paper:

A = cross-sectional area;

B = water surface width;

e = particle velocity;

f = distribution function of particle;

F=force term;

g = gravity acceleration;

h = flow depth;

$\bar{h}$ = centroid water depth;

Q = flow discharge;

S_o = bottom slope;

t = time

Δt = time space

τ = single relaxation time

u = average flow velocity;

ν = fluid kinematic viscosity

$X = X_1 – X_2$;

X_1 = dimensionless distance;

X_2 = dimensionless variable accounting for bed slope effect;

x = distance along flow direction originated from dam site;

ξ = shock wave-front celerity.

Superscripts

eq = equilibrium ;

* = dimensionless quantity.

Subscripts

α = direction of lattice;

c = reach upstream of the shock;

d = reach downstream of the shock;

i= direction of velocity or force;

j= direction of velocity or force;

M = position of rarefaction wave front;

N = position of rarefaction wave tail;

R = location of shock wave-front; and

u = undisturbed upstream reach.

References

Ancey, C., R. M. Iverson, M. Rentschler, and R. P. Denlinger. 2008. "An exact solution for ideal dam-break floods on steep slopes." *Water Resources Research*, 44(1): 567–568.

Batchelor, G.K. 2000. An introduction to fluid dynamics, Cambridge University Press. Cambridge, New York.

Chanson, H. 2009. "Application of the method of characteristics to the dam break wave problem." *Journal of Hydraulic Research*, 47(1): 41–49.

Chen, Y. L., C. Wu, and B. Wang. 2011. "Similarity solution of dam-break flow on horizontal frictionless channel." *Journal of Hydraulic Research*, 49(3): 384–387.

Chow, V. T. 1959. *Open channel hydraulics*. New York: McGraw-Hill.

Cozzolino, L., V. Pepe, F. Morlando, L. Cimorelli, A. D'Aniello, R. Della Morte, and D. Pianese. 2017. "Exact solution of the dam-break problem for constrictions and obstructions in constant width rectangular channels". *Journal of Hydraulic Engineering*, 143(11): 04017047.

Dressler, R. H. 1958. "Unsteady non-linear waves in sloping channels." *Proceedings of the Royal*

Society of London. Series A, 247: 186–198.

Fernandez-Feria, R. 2006. “Dam-break flow for arbitrary slopes of the bottom.” *Journal of Engineering Mathematics*, 54: 319–331.

Guo, Y.K., Wen, X., Wu, C.G., and Fang, D. 1998. “Numerical modelling of spillway flow with free drop and initially unknown discharge.” *J. Hydraul. Res.*, 36(5), 785-801.

Guo, Y.K., 2005. “Numerical modelling of free overfall.” *J. Hydraul. Eng*., 131(2), 134-138.

Hunt, B. 1983. “Perturbation solution for dam-break floods.” *Journal of Hydraulic Engineering*, 110(8): 1053–1071.

Peng, Y., J. G. Zhou, and R. Burrows. 2011a. “Modelling the free surface flow in rectangular shallow basins by lattice Boltzmann method.” *Journal of Hydraulic Engineering*, 137(12): 1680–1685.

Peng, Y., J. G. Zhou, and R. Burrows. 2011b. “Modelling solute transport in shallow water with the lattice Boltzmann method.” *Computers and Fluids*, 50(1): 181–188.

Ritter, A. 1892. “Die Fortpflanzung der Wasserwellen.” *Zeitschrift Verein Deutscher Ingenieure* 36(33): 947–954 (in German).

Stoker, J. J. 1957. *Water waves: The mathematical theory with applications*. Interscience, New York.

Wang, B., Y. L. Chen, C. Wu, Y. Peng, X. Ma, and J. J. Song. 2017. “Analytical solution of dam-break flood wave propagation in a dry sloped channel with an irregular-shaped cross-section.” *Journal of Hydro-environment Research*, 14: 93–104.

Wu, C., G. Huang, and Y. Zheng. 1999. “Theoretical solution of dam-break shock wave.” *Journal of Hydraulic Engineering*, 125(11): 1210–1215.

Zhou, J. G. 2004. *Lattice Boltzmann Methods for Shallow Water Flows*. Springer.

Zhou, J.G., D. M. Causon, C. G. Mingham, and D. M. Ingram. 2001. “The surface gradient method for the treatment of source terms in the shallow water equations.” *Journal of Computational Physics*, 168: 1–25.

Zoppou, C., and S. Roberts. 2003. “Explicit schemes for dam-break simulations.” *Journal of*

Hydraulic Engineering, 129(1): 11–34.

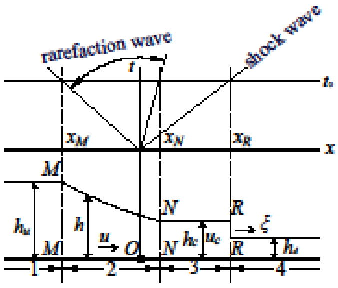

Figure 1. Definition sketch for dam-break flow after dam collapse

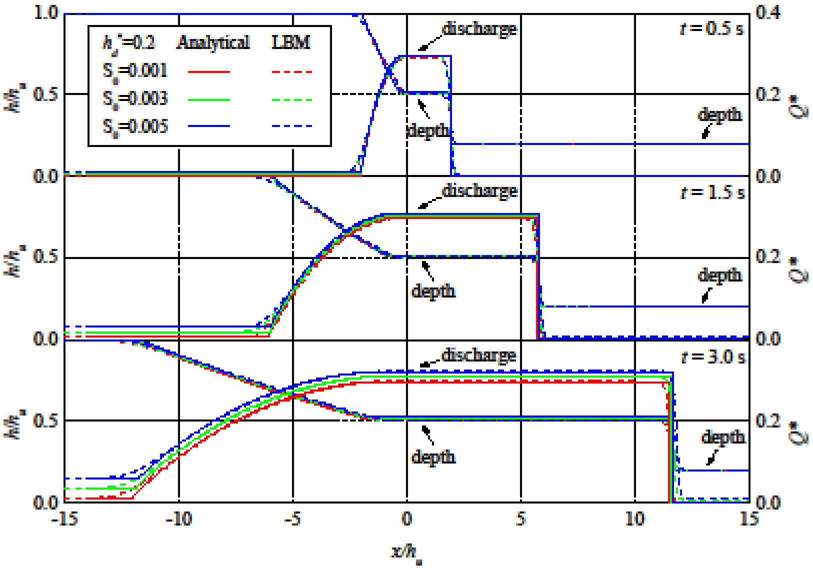

Figure 2. Profiles of dimensionless depth and discharge for $h_d^* = 0.2$.