

ePub^{WU} Institutional Repository

Lena Silbermayr

A review of non-cooperative newsvendor games with horizontal inventory interactions

Article (Draft)
(Refereed)

Original Citation:

Silbermayr, Lena
(2019)

A review of non-cooperative newsvendor games with horizontal inventory interactions.

Omega - International Journal of Management Science.

pp. 1-44. ISSN 0305-0483

This version is available at: <https://epub.wu.ac.at/7215/>

Available in ePub^{WU}: October 2019

License: [Creative Commons: Attribution 4.0 International \(CC BY 4.0\)](#)

ePub^{WU}, the institutional repository of the WU Vienna University of Economics and Business, is provided by the University Library and the IT-Services. The aim is to enable open access to the scholarly output of the WU.

This document is an early version circulated as work in progress.

Journal Pre-proof

A review of non-cooperative newsvendor games with horizontal inventory interactions

Lena Silbermayr

PII: S0305-0483(18)31340-9
DOI: <https://doi.org/10.1016/j.omega.2019.102148>
Reference: OME 102148



To appear in: *Omega*

Received date: 23 November 2018

Accepted date: 15 October 2019

Please cite this article as: Lena Silbermayr, A review of non-cooperative newsvendor games with horizontal inventory interactions, *Omega* (2019), doi: <https://doi.org/10.1016/j.omega.2019.102148>

This is a PDF file of an article that has undergone enhancements after acceptance, such as the addition of a cover page and metadata, and formatting for readability, but it is not yet the definitive version of record. This version will undergo additional copyediting, typesetting and review before it is published in its final form, but we are providing this version to give early visibility of the article. Please note that, during the production process, errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

© 2019 Published by Elsevier Ltd.

Highlights

- The use of non-cooperative game theory with the focus on horizontal inventory interactions among multiple independent newsvendors is reviewed.
- A framework for identifying the different types of horizontal inventory interactions is developed. Different modelling approaches and conditions required for existence and uniqueness of Nash equilibrium are discussed.
- An up-to-date literature review is provided and important future research directions are discussed.

A review of non-cooperative newsvendor games with horizontal inventory interactions

Lena Silbermayr*

*Department of Information Systems and Operations,

WU Vienna University of Economics and Business, Welthandelsplatz 1, 1020 Wien, Austria

Abstract

There are numerous applications of game theory in the analysis of supply chains where multiple actors interact with each other in order to reach their own objectives. In this paper we review the use of non-cooperative game theory in inventory management within the newsvendor framework describing a single period inventory control model with the focus on horizontal interactions among multiple independent newsvendors. We develop a framework for identifying these types of horizontal interactions including, for example, the models with the possibility of inventory sharing via transshipments, and situations with substitutable products sold by multiple newsvendors. Based on this framework, we discuss and relate the results of prior research and identify future research opportunities.

Keywords: newsvendor; non-cooperative simultaneous game; transshipments; substitution; horizontal coordination

1. Introduction

The newsvendor model is a single-period inventory problem, where the decision maker places an inventory order in advance of the selling season before demand is realized. As demand is not known in advance the amount of stock to be kept becomes a challenging decision. Excess inventory results in high inventory related costs while shortages results in low service levels and lost revenues.

Whenever there are multiple newsvendors (retailers) selling same or related products to customers then any *horizontal interaction* between the retailers has to be taken into con-

sideration when making inventory decisions. In practice, several forms and varying degrees of horizontal interactions can occur. For example, inventory aggregation or risk pooling through inventory transshipments, enables retailers to pool their demand risks while increasing profits and service levels. That is, a retailer might use the option to transship excess inventory to a retailer facing a shortage to benefit from reduced leftover inventory. The transshipment receiving retailer, on the other hand, can benefit from increased sales. Another possible form of horizontal interaction is product substitution where customers, in case of a stockout, find a substitute for the product at another retailer.

In many real situations retailers in the same supply chain echelon that interact in such ways are independent (*decentralized*) players. They act independently by optimizing their own objectives. This constitutes a *non-cooperative static game* where retailers in advance of the selling season make their decisions simultaneously taking any inventory interaction into consideration.

Previous works surveying the application of non-cooperative newsvendor games are, for example, Cachon (2003) who focuses on vertical coordination between supplier and buyer (newsvendor) or Cachon and Netessine (2006) and Chinchuluun et al. (2008) who provide tutorials on non-cooperative newsvendor games with horizontal interactions where two newsvendors compete on product availability. There are also reviews on inventory transshipments. Paterson et al. (2011) review inventory models with lateral transshipments in general, in centralized and decentralized supply chains. Huang (2013) considers transshipments and substitution between retailers focusing on supply chain relationships between the manufacturer and the retailers.

This paper fills a gap in the literature by providing a review on quantitative models for multiple independent newsvendors with any type of horizontal inventory interactions. In the last years, especially due to new business models and advanced information technologies several papers dealing with the application of non-cooperative newsvendor games focusing on several aspects leading to horizontal inventory interactions have appeared. What is still missing is a structured classification of the different aspects and applications that enable

horizontal inventory interactions in a supply chain with multiple independent retailers in order to provide an overview on the main findings in this field and to discuss future research opportunities.

To summarize, the contributions of this paper are as follows:

- We describe and classify the specific settings where game theory applications to horizontal inventory interactions arise.
- We discuss the modelling approaches and conditions required for the existence and uniqueness of Nash equilibrium under each setting.
- We address horizontal coordination and discuss possible coordination mechanisms that can be applied for horizontal supply chain relationships.
- We provide an up-to-date literature review summarizing the important findings in that field.
- We identify gaps and discuss important future research directions.

The reminder of this paper is organized as follows. In Section 2 we introduce the classical newsvendor model and the basic concepts of non-cooperative game theory and discuss possible interactions between decentralized newsvendors that may result in inventory interactions leading to the review structure and methodology used in this paper. In Sections 3 and 4 the different models are analyzed using non-cooperative game theory. In each section we introduce some basic concepts of the non-cooperative games and then proceed to review and discuss the related literature in the respective field. Section 5 discusses miscellaneous aspects with combinations or comparisons of different models discussed in Sections 3 and 4. Finally, Section 6 concludes the paper and discusses directions for future research.

2. Preliminaries

We first introduce the classical newsvendor model, discuss possible interactions among multiple decentralized newsvendors in a non-cooperative fashion and provide a brief overview

on basic game theoretic concepts in this context. Then, the structure of the literature review and the review methodology is presented.

2.1. The classical newsvendor model

Consider a single period inventory model with n retailers $i = 1, 2, \dots, n$. The retailers act to maximize their own profits. They face random demands D_i with marginal distribution function F_{D_i} and demands can be correlated. The product is sold at a selling price r_i per unit and purchased at c_i per unit.

In the classical newsvendor setting, if demand of retailer i , D_i , turns out to be larger than the stocking quantity Q_i , unsatisfied demand is assumed to be immediately lost and a penalty cost of p_i is incurred per unsatisfied demand. On the other hand, if demand turns out to be smaller than the stocking quantity, leftovers are salvaged at a value of s_i per unit where $r_i > c_i > s_i$. All parameters as well as marginal and joint distributions of demands are common knowledge.¹

In this case, the expected profit of a retailer i is

$$\Pi_i^{NV}(Q_i) = E(r_i \min(D_i, Q_i) + s_i(Q_i - D_i)^+ - p_i(D_i - Q_i)^+ - c_i Q_i), \quad (1)$$

where we define $(X)^+ = \max(X, 0)$. The expected profit function is concave and the optimal order quantity satisfies

$$F_{D_i}(Q_i) = \frac{r_i - c_i + p_i}{r_i - s_i + p_i}, \quad (2)$$

where the right-hand side of (2) is the newsvendor's critical ratio.

However, there are several settings which cause interaction among the retailers when there are shortages and leftovers at different retailers. For example, the retailers with a shortage can procure goods from the retailers with leftovers to satisfy some of their customer demand, which describes a system with lateral transshipments; or customers

¹For games with incomplete informations we refer to e.g. Wu and Parlar (2011).

whose demand could not be satisfied might search for a retailer with leftover inventory and make a purchase in this substitute location. Because of these possible interactions, a newsvendor has to consider the decisions of other newsvendors which gives rise to a game theoretic approach. The common characteristic of the different settings is that the unsatisfied demand of one retailer can be satisfied by another retailer. Therefore the risks related to shortages and leftovers differ from the *classical* newsvendor setting.

2.2. Basic concepts in non-cooperative game theory

We give a short overview of the basic concepts in non-cooperative *static* games with *complete information*, i.e. all players are in possession of all information in the game. Since our focus is on single-period newsvendor games, only one-shot games (i.e. games with only one play-through) are discussed and we will directly link the basic concepts to this setting. For further details we refer to e.g. Friedman (1990), Cachon and Netessine (2006) and Chinchuluun et al. (2008).

A game has a set of rational decision makers, called *players*, denoted by retailer $i = 1, \dots, n$, a set of *strategies* available to each retailer denoted by stocking quantities Q_i , and *payoffs* received by each player given by the expected profits Π_i . In a non-cooperative static game the strategies (stocking quantities) are chosen simultaneously and the players are unable to make binding commitments before choosing their strategies.

The rational outcome of such a game is the *Nash equilibrium* which is characterized by solving the system of n *best response functions* of all players. The best response function of player i given vector of fixed strategies of the other players, \mathbf{Q}_{-i} gives player i 's response that maximizes its payoff (i.e. $Q_i^*(\mathbf{Q}_{-i}) = \max_{Q_i} \Pi_i(Q_i, \mathbf{Q}_{-i})$), hence, it is typically defined by the players' first order conditions. A Nash equilibrium $Q_i^*(\mathbf{Q}_{-i}^*)$ is considered a stable outcome as no player has incentive to deviate from his strategy choice.

The *existence* of a Nash equilibrium is guaranteed when the players' payoffs Π_i are concave with respect to Q_i . The *uniqueness* of an equilibrium can be proven e.g. through contraction mapping argument showing that the best response mapping is a contraction, which then implies that the mapping has a unique fixed point. In other words, one has to

verify that no column sum or no row sum of the matrix of derivatives of the best response functions exceeds one (Cachon and Netessine, 2006).

In general *decentralized* decision making results in supply chain inefficiency, since the Nash equilibrium usually is not equal to the system optimal solution where a *centralized* decision maker maximizes the total payoff of all players.

2.3. Review structure and methodology

We group different settings which give rise to interaction among decentralized newsvendors in two categories: 1) *designed interaction* and 2) *customer driven interaction*. The first category includes settings where the newsvendors design operations in such a way that the supply chain design enables risk pooling via inventory sharing. Lateral transshipments among retailers (i.e. virtual pooling), and physical centralization of stocks at a central warehouse (i.e. physical pooling) belong to this first category. Customer driven interaction, on the other hand, stems from the perceived substitutability and complementarity of different products and/or sellers. If the customers are willing to substitute products and/or sellers, a shortage at one retailer causes higher demand at other retailers. On the other hand, if products are complementary, shortage at one retailer hinders sales of another complementary retailer.

One of the main differences among the two categories is that for the designed interaction there is a necessity to develop horizontal contracts so that the horizontal interaction through inventory pooling can take place. If the benefits of the designed system are not appropriately distributed, the players might decide not to participate in the game at all. Then, their decisions can be properly modelled using the classical newsvendor setting. On the other hand, the customer driven interaction is an external factor which should not be ignored. Accordingly, the literature on the first category (designed interaction) puts some focus on horizontal coordination and contracting while the literature on the second category (customer driven interaction) does not deal with horizontal contracting.

The structure of our literature review is shown in Table 1. In the case of a designed interaction through transshipments we explicitly discuss a two-retailer and $n > 2$ retailer

problem, since the general model with $n > 2$ retailers includes an additional complexity of how to allocate residual stocks to stocked-out retailers. For the physical centralization example the conclusions are similar, hence, they will not be repeatedly discussed in such a detail. In the final Section 5 papers dealing with a combination or comparison of designed and customer driven interactions will be discussed.

Designed interaction (Section 3)	Transshipments (Section 3.1)	Two retailers (Section 3.1.1) $n > 2$ retailers (Section 3.1.2)
	Physical centralization (Section 3.2)	
Customer driven interaction (Section 4)	Substitution (Section 4.1) Complementarity (Section 4.2)	
Combination/Comparison of designed and customer driven interaction (Section 5)		

Table 1: Organization of literature review including section numbers.

The inclusion criteria for this review are articles studying horizontal inventory interactions among multiple decentralized newsvendors that make non-cooperative stocking decisions. Exclusion criteria applied for this review are articles focusing on purely cooperative operational decisions and information asymmetries or incomplete information. Since we put our focus on inventory-related implications of horizontal interactions we also exclude articles with price competition.

3. Designed interaction

In this section we consider non-cooperative risk pooling games resulting from the sellers designed operations that enable inventory sharing. For independent retailers selling identical products the most commonly studied setting in this category is the lateral transshipments system which is the focus of Section 3.1. Another possible setting to exploit the

risk pooling benefit is the physical consolidation of inventories at a central location (see Section 3.2). The detailed classification of the literature discussed in this section is shown in Table 3.

3.1. Transshipments

First, we discuss virtual pooling through transshipments, i.e. residual inventory at one retailer is shipped to another retailer that faces a stockout.

3.1.1. Two retailers

Transshipments from i to j (throughout the paper, when using this indexing, we assume $i \neq j$) incurs a transshipment cost c_{ij} per unit. Since i and j are independent a transshipment price r_{ij} per unit is charged by retailer i and paid by j . If there is still unsatisfied demand at i after the transshipments are realized, then a shortage penalty cost of p_i is incurred.

We assume that $c_i < c_j + c_{ji}$, $s_j > s_i + c_{ji}$, and $r_i + p_i < r_j + p_j + c_{ji}$. These conditions guarantee that it is not beneficial to always purchase and/or salvage through the other retailer, and to sell to the other retailer instead of own customers. To ensure mutually profitable transshipments we need to assume that the unit transshipment price $r_{ij} \in [s_i + c_{ij}, r_j + p_j]$, where $s_i + c_{ij} < r_j + p_j$ (see Rudi et al. (2001); Hu et al. (2007)), then neither retailer is worse off by performing transshipments. We will call the mutually beneficial transshipment system as bidirectional setting.

For given order quantities Q_i and Q_j , transshipments from i to j are $T_{ij} = \min((D_j - Q_j)^+, (Q_i - D_i)^+)$. Sales at retailer i are $S_i = \min(D_i, Q_i) + T_{ji}$, leftovers at retailer i are $L_i = (Q_i - D_i - T_{ij})^+$, and unsatisfied demand at retailer i is $P_i = (D_i - Q_i - T_{ji})^+$.

When the two retailers make their ordering decisions locally, in a decentralized manner, then the expected profit of retailer i is

$$\Pi_i(Q_i, Q_j) = E(r_i S_i + (r_{ij} - c_{ij})T_{ij} - c_i Q_i - r_{ji} T_{ji} + s_i L_i - p_i P_i). \quad (3)$$

A special case of the problem where transshipments are only possible in one direction

has been studied in some recent papers (Seifert et al., 2006; He et al., 2014; Arikan and Silbermayr, 2018). Under the so-called unidirectional transshipments setting, we assume that transshipments are only allowed from retailer i to retailer j , i.e. $T_{ji} = 0$. Unidirectional transshipments can be applicable, for example, in omni-channel systems with online and offline stores where in-store customers in contrast to online customers are not willing to wait a regular shipment time for product delivery (Seifert et al., 2006). Other reasons might be that certain transshipment routes are impossible, due to e.g. high cost, unavailable transport or different proximities, limiting inventory pooling (Axsäter, 2003; Smirnov and Gerchak, 2014, 2016).

Equilibrium analysis

For the bidirectional setting, following Huang et al. (2011), we define $D_i^e = D_i + (D_j - Q_j)^+$ as the effective demand for retailer i . It includes the initial demand at retailer i and all the secondary demand, i.e. the unsatisfied demand at other retailers which can be potentially satisfied by retailer i through transshipment (or substitution discussed in Section 4.1). Similarly, $D_i^n = D_i - (Q_j - D_j)^+$ is the net demand at retailer i , which is the initial demand minus the part which can be potentially satisfied by retailer j (Arikan and Silbermayr, 2018). Note that $D_i^e \geq D_i \geq D_i^n$ and, hence, $F_{D_i^e}(Q_i) \leq F_{D_i}(Q_i) \leq F_{D_i^n}(Q_i)$.

Hu et al. (2007) show that the expected profit under decentralized decision making of retailer i , Equation (3), is concave in Q_i . The first order conditions characterizing the optimal order quantities $Q_i^*(Q_j)$ for $i = 1, 2$ are

$$\begin{aligned} (r_i - c_i + p_i) - (r_{ji} - r_{ij} + c_{ij})F_{D_i}(Q_i) - (r_i - r_{ji} + p_i)F_{D_i^n}(Q_i) \\ - (r_{ij} - c_{ij} - s_i)F_{D_i^e}(Q_i) = 0. \end{aligned} \quad (4)$$

The existence of the Nash equilibrium is guaranteed by the concavity of the expected profit functions, and the uniqueness of the equilibrium is shown by a contraction mapping argument (see e.g. Cachon and Netessine (2006) for further details). The proof follows along the lines of the proof of Proposition 1 in Rudi et al. (2001) (see also Arikan and

Silbermayr (2018)). Implicit differentiation of Equation (4) results in

$$\frac{\partial Q_i}{\partial Q_j} = - \frac{(r_{ij} - s_i - c_{ij})g_{D_i^e}(Q_i) + (r_i + p_i - r_{ji})g_{D_i^n}(Q_i)}{(r_i + p_i - s_i)f_{D_i}(Q_i) + (r_{ij} - s_i - c_{ij})f_{D_i^e}(Q_i) + (r_i + p_i - r_{ji})f_{D_i^n}(Q_i)}. \quad (5)$$

For D_i^e , define $f_{D_i^e}(Q_i) \doteq \partial F_{D_i^e}(Q_i)/\partial Q_i$ and $g_{D_i^e}(Q_i) \doteq \partial F_{D_i^e}(Q_i)/\partial Q_j$, and similarly for D_i^n . From the definitions of the effective demand and the net demand, one can derive $F_{D_i^e}(Q_i) \leq F_{D_i}(Q_i) \leq F_{D_i^n}(Q_i)$, which leads to $f_{D_i^e}(Q_i) \geq g_{D_i^e}(Q_i)$ and $f_{D_i^n}(Q_i) \geq g_{D_i^n}(Q_i)$, respectively. Combining these properties, one can conclude that $-1 \leq \frac{\partial Q_i}{\partial Q_j} \leq 0$. That is, the slopes of the best response functions are less than one in absolute terms, which is sufficient condition for the uniqueness of the Nash equilibrium (Cachon and Netessine, 2006).

Note that the analysis regarding existence and uniqueness of the Nash equilibrium for the unidirectional transshipment setting is similar.

Horizontal coordination

In a system with decentralized retailers that maximize their own profits, there are incentive problems that prevent coordination. However, there is literature that deals with horizontal coordination discussing mechanisms or contracts that can be designed in order to reach system optimal solutions in which a single decision maker acts so as to maximize the total profit of the system. If the ordering decisions are centrally made then there is no need to charge a transshipment price and the total expected profit is

$$\Pi^C(Q_i, Q_j) = E\left(\sum_{i=1}^2 (r_i S_i - c_{ij} T_{ij} - c_i Q_i + s_i L_i - p_i P_i)\right). \quad (6)$$

The total expected profit given in Equation (6) is concave (Rudi et al., 2001) and the first order conditions characterizing the optimal order quantities Q_i^{*C} for $i = 1, 2$ are

$$\begin{aligned} (r_i - c_i + p_i) - (c_{ij} - r_j + c_{ji} + s_j - p_j)F_{D_i}(Q_i) - (r_i + p_i - c_{ji} - s_j)F_{D_i^n}(Q_i) \\ - (r_j + p_j - c_{ij} - s_i)F_{D_i^e}(Q_i) = 0. \end{aligned} \quad (7)$$

As discussed before, players, without coordination, only focus on their own objectives which leads to an inefficiency in the supply chain (see difference between Equation (4) and Equation (7)), i.e. they cannot achieve the system optimal solution of a centralized decision maker. Literature on supply chain coordination seeks to find simple mechanisms (contracts) that provide incentives for the players to coordinate the decentralized system and achieve the system optimum.

Rudi et al. (2001) study the simple transshipment price contract discussed above that from the practical perspective is easy to execute, i.e. a retailer facing a stockout has to pay a predetermined unit transshipment price in order to receive leftovers from the other retailer. Rudi et al. (2001) show that the decentralized system can be coordinated by appropriately set transshipment prices equating Equation (4) and Equation (7) and solving for the transshipment price r_{ij} .

Hu et al. (2007) provide examples which show that such coordinating prices may not exist in several cases considering a more general model than Rudi et al. (2001) including uncertain capacity. Especially with increasing asymmetries in the economic parameters for the two retailers, coordination of bidirectional transshipments may not be possible by varying the transshipment prices. For example, Arikan and Silbermayr (2018) numerically show that coordinating transshipment prices might only exist for situations with low demand correlation between the retailers when they differ in their shortage cost. That is, coordination is only possible in cases where transshipments are very beneficial. A sufficient condition that coordinating transshipment prices exist is that the retailers are symmetric (Hu et al., 2007).

Unlike Hu et al. (2007) that assume that retailers subject to uncertain capacity are supplied by two independent suppliers, Lee and Park (2016) consider a two-retailer transshipment game supplied by a single supplier with uncertain capacity. Hence their model includes competition for supplier capacity and inventory sharing through transshipments. They show that retailers increase their orders when capacity is allocated proportional to the orders (i.e. rationing game) and that coordinating transshipment prices exist in a more

limited range due to the capacity uncertainty.

Shao et al. (2011) examine transshipment incentives under the transshipment price contract for a fully decentralized two-echelon supply chain including also the manufacturer's decision that distributes the product through two identical retailers. They show that if the manufacturer has control, it will set a high transshipment price in order to increase its profit. If, however, the retailers have control they prefer low transshipment prices and as a result the manufacturer might prefer dealing with centralized retailers.

Feng et al. (2018) consider the same transshipment price contract, but under partial backordering. That is, the transshipment quantity from i to j is $T_{ij} = \min(\delta(D_j - Q_j)^+, (Q_i - D_i)^+)$, with δ being the fraction of backordered demands. However, they do not discuss the coordinating transshipment prices under this setting.

Hezarkhani and Kubiak (2010) find a coordinating contract with an implicit pricing mechanism where transshipment prices depend on the retailers' inventory decisions. Thereby, they split up the contract into two phases. In the first phase inventory dependent transshipment prices are set and then in a second phase - after deciding on inventories but before demand realization - the negotiated transshipment prices are fixed. The contract has the desirable property of flexibility as it allows to arbitrarily divide the total expected profit between the two retailers depending on the bargaining power of the retailers. This flexibility is not provided by the coordinating transshipment price contract since it leads to a single split-up of the total expected profit.

The unidirectional transshipment system ($T_{ji} = 0$) cannot be coordinated at all with an unit transshipment price contract (see Arıkan and Silbermayr (2018)). This is similar to the setting under a wholesale price contract in a two echelon system with a supplier and a buyer (e.g. Cachon (2003)) or to the bidirectional transshipment setting of Hu et al. (2007) with asymmetric parameters between two retailers. Limiting transshipments in only one direction causes extreme asymmetry in the system. Arıkan and Silbermayr (2018) study a number of simple contracts that coordinate the unidirectional transshipment setting. These contracts include a combination of transshipment price r_{ij} , leftover subsidy τ^L per unit

of leftover at the transshipment giver i paid by the transshipment receiving retailer j and shortage subsidy τ^S per unit of shortage in j paid by i . Contract types including (r_{ij}, τ^L) or (r_{ij}, τ^S) can not guarantee that the contract is beneficial for both parties. This problem is shown to be overcome by designing a contract with three terms (r_{ij}, τ^L, τ^S) which can achieve coordination such that both parties are better off compared to a no-transshipment setting. Such a contract again has the desirable property of flexibility allowing an arbitrary division of the total expected profit.

3.1.2. $n > 2$ retailers

So far we have focused on the case of two retailers. Analyzing a transshipment system with more than two decentralized players is known to be a nontrivial task (Rudi et al. (2001); Huang and Sošić (2010a); Shao et al. (2011); Arikan and Silberman (2018)). In the case of two retailers it is clear that any leftovers of one retailer are transshipped to the other retailer facing a stockout. In the $n > 2$ retailer case, however, if there is more than one retailer facing a stockout any leftovers have to be allocated according to a specific allocation rule. In a non-cooperative environment, finding a proper allocation rule is challenging.

For simplicity and tractability Huang and Sošić (2010a), Shao et al. (2011) and Arikan and Silberman (2018) extend their analytical tractable two-retailer models by numerically investigating a proportional allocation rule and assume $n > 2$ symmetric retailers. That is, a retailer facing a stockout (leftover) receives transshipments (transships excess inventory) proportional to its excess demand (inventory), if total excess demand (inventory) of all retailers is higher than their total excess inventory (demand). Arikan and Silberman (2018) show that, for both the bidirectional and unidirectional setting, most of the results in the two retailer case also hold for the case with $n > 2$ newsvendors (this is also concluded by Huang and Sošić (2010a) and Shao et al. (2011) for the bidirectional transshipment setting). For example, the transshipment price contract can coordinate the symmetric bidirectional multi-retailer transshipment system. However, similar to the two-retailer setting, the system with n retailers might not be coordinated via simple unit transshipment

prices as soon as certain asymmetries are included in the system. In practice, however, there may be certain asymmetries in the supply chain. Reasons can be differences in retailers' size and/or proximity to transportation hubs or differences in the retailers' shortage cost (Axsäter, 2003; Kranenburg and Van Houtum, 2009). Ben-Zvi and Gerchak (2012) provide an example where a hospital and adjacent pharmacy share inventory of medical equipment or materials, where a shortage of an item means something different at the two locations.

Hanany et al. (2010) develop a transshipment pricing mechanism with a transshipment fund, that unlike the unit transshipment price contract discussed in Rudi et al. (2001) and others for the case of 2 retailers, always coordinates the general n -retailer transshipment problem in a fully non-cooperative setting. Retailers make initial payments to the fund (i.e. a third party financial entity that contracts with the retailers on transshipment payments) and after demand realization, transshipped residuals and payments from the fund to the retailers dependent on the retailers' announcements about excess supply or demand and are specified according to a predetermined rule. This mechanism creates a large set of feasible transshipment payments that coordinate the n -retailer transshipment problem.

Another stream of literature discusses two-stage non-cooperative/cooperative games, so-called 'biform games' (Brandenburger and Stuart (2007)), with non-cooperative inventory decisions in the first stage and cooperative shipping decisions in the second stage termed 'coopetition' (for a review on cooperative game theory and inventory management see e.g. Nagarajan and Sošić (2008), Fiestras-Janeiro et al. (2011) and Montrucchio et al. (2012)). The main difference to the literature discussed before is that although the allocation mechanism is as-well defined before demand realization the transshipment prices are set after demand realization (ex post) and are agreed on cooperatively. Anupindi et al. (2001) analyze a very general framework with n decentralized newsvendors that share all residual inventories through such a two-stage non-cooperative/cooperative game. They show that the core of the game is non-empty and that there exists an allocation mechanism in the form of a fractional allocation rule with a side payment scheme that achieves a coordinated solution for inventory deployment and allocation. The authors, however,

point out the main difficulty in such allocation problems. That is, while the inventory decision can be done without any agreement with the other players in the game, for the allocation decision, in contrast, the players must find a mutual consent. Huang and Sošić (2010a) compare the performance of the ex ante transshipment price method of Rudi et al. (2001) and others against the ex post allocation rule of Anupindi et al. (2001) showing that neither allocation method dominates the other.

Granot and Sošić (2003) extend the model of Anupindi et al. (2001) including the retailers' decision of how much residual inventory to share with the other players. They present a three-stage model, where in the first stage retailers make inventory decision before demand realization, in the second stage (after demand realization) they decide on how much residual stock to share and in the third stage, inventories are transshipped and profits allocated according to an allocation rule. They discuss the impact of different allocation rules on the retailers willingness to share residual inventory with others in the second stage and if coordination can be achieved. Yan and Zhao (2015) develop a mechanism to coordinate n retailers that will completely share their residuals by involving a third party (e.g. a manufacturer) who subsidizes the transshipment profit allocation.

Huang and Sošić (2010b) compare the single-shot transshipment game of n newsvendors with a game that is repeated infinitely many times. They show that it could be a subgame-perfect Nash equilibrium to share all the residuals at the second stage in the repeated game if the discount factor is large enough.

3.2. *Physical centralization of inventories*

Physical aggregation is another common practice of inventory pooling. Instead of having local inventories for each retailer, inventories are stored at a central storage facility (warehouse). Usually the storage cost at the centralized facility is lower than the total storage cost at the retailers. However, physical centralization also increases response time and transportation cost to customers (Chopra and Meindl, 2016).

The majority of literature dealing with the physical centralization of inventories assume either a central control (e.g. Eppen, 1979; Chen and Lin, 1989; Gerchak and He, 2003;

Corbett and Rajaram, 2006) or a cooperative newsvendor game (e.g. Hartman et al., 2000; Hartman and Dror, 2005). This may be caused by the fact that the physical sharing of a storage location and consequently a joint stock and ownership can be critical for independent retailers. Further, one again needs a proper allocation rule for assigning the joint stock to the retailers.

In practice, however, there do exist situations where the warehouse is the only source for replenishment in case of a stockout. For independent retailers this is especially appropriate when organizational structures, like franchising arrangements, inhibit transshipments among retailers (Wee and Dada, 2005).

Although in respect to the design of the network the non-cooperative game with physical centralization of inventories is different from the game discussed in Section 3.1, mathematically they are related to each other regarding existence and uniqueness of the Nash equilibrium (see Silbermayr and Gerchak (2019)). Hence, in this section we briefly discuss main differences and findings in literature.

In terms of modelling the main differences compared to the setting discussed in Section 3.1 are that i) there is a single per unit purchase cost c at the central storage location for all retailers, ii) the final allocation of stocks to the retailers or customers may include additional transportation cost due to the centralization and iii) there might be a compensation cost for using the other firms contribution to the common stock (Silbermayr and Gerchak, 2019). Now, in a non-cooperative environment each retailer i decides on his contribution Q_i to the joint stock based on uncertain demand D_i (Ben-Zvi and Gerchak, 2012). The retailers make their decisions simultaneously. After demand is realized each retailer i first receives the minimum of the order Q_i and the demand D_i . Then, residual stock is allocated to retailers facing a shortage according to a given allocation rule.

Consider the model of Ben-Zvi and Gerchak (2012) with two independent firms differing in their shortage cost. If a retailer faces a stockout situation, then he will receive an allocation proportional to its contribution, i.e. $Q_i/(Q_i + Q_j)$. Ben-Zvi and Gerchak (2012) provide first order conditions of such a model and prove existence and uniqueness of a Nash

equilibrium. Gerchak (2016) analyze the consequences of such a non-cooperative game with a modified scheme that is beneficial to all parties relative to a no-pooling situation. Ben-Zvi and Gerchak (2012) and Gerchak (2016) assume that it is costless for retailer i to use residual stock from j 's contribution to the joint stock Q_j .

Netessine and Rudi (2006) study the practice of drop-shipping for multiple decentralized newsvendors and a wholesaler as a non-cooperative game. They analyze a combined strategy where retailers use own inventories and drop-shipping stocked at a central location owned by the wholesaler. The difference to the literature discussed before is that centralized inventory decision is done by another independent player at an upstream echelon and not by the independent retailers. They discuss structural properties of the equilibrium solution and show that such a combined strategy often benefits retailers as well as wholesaler.

Silbermayr and Gerchak (2019) address horizontal coordination in a model with two independent retailers that might partially pool their inventories. Each retailer decides on the quantity stocked at its local storage facility and on its contribution to the joint warehouse. They assume a non-negative compensation cost for using the other firms contribution to the pool. Transshipments between the retailers are prohibitively expensive. They show that, similar to the transshipment setting with a transshipment price contract, a system with physical centralization of inventories can be coordinated if retailers appropriately set a compensation cost for using the other retailers' residual stock at the warehouse.

The general framework of Anupindi et al. (2001) with n newsvendors constituting a biform game with an ex post inventory allocation rule also considers physical centralization at one or several jointly owned warehouses in addition to transshipments between retailers. They introduce the notion of 'claims' that establish ownership for each unit of inventory in the system, regardless of its locations. After demand is realized the claim holder owns the right to determine how its purchased units are to be used. This reduces the inventory decision (at the first stage of the biform game) to a non-cooperative game in spite of the presence of common inventory in central warehouses with joint ownership. That is, with

'claims' the ex post allocation rule allows for horizontal coordination also under physical centralization of inventories.

A summary of literature on horizontal contracts that coordinate a system of independent retailers that physically or virtually pool their inventories is given in Table 2. We report whether the contract has been studied analytically for the 2 or n retailer case and whether it has the flexibility to arbitrarily divide the total profit between the retailers.

Horizontal contract type	Inventory interaction	No. retailers	Flexible profit div.	Paper
Transshipment price (ex ante)	Transsh.	2	no	Rudi et al. (2001), Hu et al. (2007), Huang and Sošić (2010a), Shao et al. (2011), Lee and Park (2016), Arıkan and Silbermayr (2018), Silbermayr and Gerchak (2019)
Compensation cost for using centralized stock	Physical cent.	2	no	
Implicit transsh. price depend. on inventory dec.	Transsh.	2	yes	Hezarkhani and Kubiak (2010)
Combining transsh. price, leftover & shortage subsidy	Unid.-transsh.	2	yes	Arıkan and Silbermayr (2018)
Transshipment fund arranged by third party	Transsh.	n	yes	Hanany et al. (2010)
Ex post allocation rule (biform game)	Transsh. (Physical cent.)	n	yes	Anupindi et al. (2001), Granot and Sošić (2003), Huang and Sošić (2010a), Huang and Sošić (2010b), Yan and Zhao (2015)

Table 2: Overview for literature on horizontal contracts between retailers.

4. Customer driven interaction

In this section we consider non-cooperative newsvendor games that arise from distinct customer driven interactions, whereby the sales of one product affects the sales of another product. First, there is product substitution where an unsatisfied customer is willing to buy a substitute product at another retailer, i.e. a system with competition between the newsvendors (Section 4.1). Then, there is product complementarity where an unsatisfied customers of one product hinders the sales of a complementary product (Section 4.2). The detailed classification of the literature discussed in this section is shown in Table 4.

4.1. Substitution

The general setting is based on n competing retailers selling substitutable products. As in the classical newsvendor model, at the beginning of the selling period each newsvendor decides about own stocking quantity in advance of demand realization. If the demand

for i 'th newsvendor turns out to be larger than its stocking quantity, $D_i > Q_i$, then a customer might be willing to search and buy the product at the other retailers. If the customer cannot find the product at retailer j he can continue searching in other retail locations (e.g. Mahajan and Van Ryzin (2001)) or he stops and his demand remains unsatisfied. In the following we discuss the latter setting where the search is over after one attempt. The search is generally modelled such that a proportion a_{ij} of customers demanding a product at newsvendor i will take product at newsvendor j as substitute.

Modelling the proportion of the unsatisfied demand of i switching to j by $0 < a_{ij} \leq 1$ is general enough to model partial substitution, full substitution and one-way substitution of demand. For an overview we refer to Turken et al. (2012). Under partial substitution only a fraction of demands will switch to a competitor, i.e. $0 < a_{ij} < 1$. Full substitution means that all customers of i are willing to accept the product j when retailer i is out of stock, i.e. $a_{ij} = 1$. Finally, one-way substitution expresses a system where, for example, customer i will switch to j with probability a_{ij} in case of a stockout but not vice versa, i.e. $0 < a_{ij} \leq 1$ and $a_{ji} = 0$. This can be the case if the product of j has a higher quality than the product of i . One might additionally need an adjustment cost whenever the lower quality product will be substituted by the higher quality product (Deflem and Van Nieuwenhuyse, 2013). Note that one-way substitution can also be related to unidirectional transshipments discussed in Section 3.1.

One of the earliest works under this setting is Parlar (1988) which considers only two competing newsvendors, but highlights that in practice substitution often takes place between different products sold by independent retailers. Wang and Parlar (1994) extend the work of Parlar (1988) to three competing newsvendors. Li and Ha (2008) extend the two retailer case selling substitutable products including reactive capacity to fill uncertain demand in addition to initial inventory. They show that additional reactive capacity has a positive competitive effect on a retailer but a negative effect on the competitor. Wu et al. (2019) explore the two competing retailer case with asymmetric bargaining power of the retailers where the weak retailer is capital-constrained and takes a trade credit with

a wholesale price dictated by the manufacturer. The trade credit can be used by the manufacturer as a strategic response to the bargaining power of the other retailer.

Mahajan and Van Ryzin (2001), Netessine and Rudi (2003) and Huang et al. (2011), for example, study the problem for any number of newsvendors. Lippman and McCardle (1997) study n competing newsvendors with a probabilistic aggregate industry demand that is allocated across firms by a predefined rule (e.g. deterministic or random splitting). Cachon (2003) studies supplier retailer coordination with n competing retailers where stochastic industry demand is divided proportional to the retailers' inventory levels.

For the general model with n retailer facing random demands $D_i, i = 1, \dots, n$ and substitution, the effective demand is $D_i^e = D_i + \sum_{j \neq i} a_{ji}(D_j - Q_j)^+$ while the net demand does not play a role in this customer search setting.

There are slight differences in the literature with respect to modelling the total shortage cost. A limiting case would be a newsvendor with emergency orders. Some (e.g. Netessine and Rudi (2003)) do not consider any shortage costs. Parlar (1988), Wang and Parlar (1994) and Qi et al. (2015), for example, assume that the shortage cost is only incurred for the unsatisfied demand from initial customers, i.e. retailer i incurs a total shortage cost of $p_i(D_i - Q_i)^+$. On the other hand, Lippman and McCardle (1997) and Huang et al. (2011) assume a shortage cost for all the unsatisfied demand both from initial and secondary customers, i.e. retailer i incurs a total shortage cost of $p_i(D_i^e - Q_i)^+$. In the latter case, the expected profit of retailer i is

$$\Pi_i = E(r_i \min(D_i^e, Q_i) - c_i Q_i + s_i(Q_i - D_i^e)^+ - p_i(D_i^e - Q_i)^+). \quad (8)$$

If the shortage cost is only incurred for the unsatisfied demand from initial customers then the last term in Equation (8) has to be replaced by $p_i(D_i - Q_i)^+$.

Equilibrium analysis

From the modelling perspective the advantage of considering shortage cost $p_i(D_i^e - Q_i)^+$ is the fact that expected profit Π_i only includes D_i^e as the random component while the

other one has both D_i^e and D_i . As a result, the first model gives a more compact formulation of the optimality condition. The optimal order quantity Q_i^* satisfies

$$Pr(D_i^e < Q_i^*) = F_{D_i^e}(Q_i^*) = \frac{r_i - c_i + p_i}{r_i - s_i + p_i}. \quad (9)$$

Hence, the critical ratio of retailer i under substitution is equal to the critical ratio of the classical newsvendor in Equation (2), but the demand distribution is different.

Π_i is concave in Q_i and the slope of the best response function is

$$\frac{\partial Q_i}{\partial Q_j} = -a_{ji} \frac{g_{D_i^e}(Q_i)}{f_{D_i^e}(Q_i)}. \quad (10)$$

Since $f_{D_i^e}(Q_i) \geq g_{D_i^e}(Q_i)$, the slope is non-positive and smaller than one in absolute terms.

On the other hand, under the model with shortage cost $p_i(D_i - Q_i)^+$ the optimality condition is

$$r_i(1 - F_{D_i^e}(Q_i^*)) - p_i(1 - F_{D_i}(Q_i^*)) + s_i F_{D_i^e}(Q_i^*) - c_i = 0 \quad (11)$$

and the slope of the best response function is

$$\frac{\partial Q_i}{\partial Q_j} = -a_{ji} \frac{(r_i - s_i)g_{D_i^e}(Q_i)}{(r_i - s_i)f_{D_i^e}(Q_i) + p_i f_{D_i}(Q_i)}. \quad (12)$$

Similarly, the slope is non-positive and smaller than one in absolute terms.

Concavity of Π_i guarantees the existence of a Nash equilibrium under both models. The uniqueness of the equilibrium is again proven using the contraction mapping argument. In order to show that the mapping is a contraction it is sufficient to show that no column or row sum of matrix J exceeds one, where J is the Jacobian of the best response mapping. Under both models, $|\frac{\partial Q_i}{\partial Q_j}| < a_{ji}$, and the following holds for all columns i :

$$\sum_{j=1, \dots, n} |\frac{\partial Q_i}{\partial Q_j}| < \sum_{j=1, \dots, n} a_{ji}. \quad (13)$$

Note that the same holds for the row sums.

Therefore, if either $\sum_{j=1,\dots,n} a_{ji} < 1$ for all i or $\sum_{i=1,\dots,n} a_{ji} < 1$ for all j , then there exists a unique Nash equilibrium.

Horizontal coordination

In the centralized case, when all products/locations are managed by a single decision maker, the total expected profit with shortage cost $p_i(D_i^e - Q_i)^+$ is

$$\Pi^C = E\left(\sum_{i=1}^n r_i \min(D_i^e, Q_i) - c_i Q_i + s_i(Q_i - D_i^e)^+ - p_i(D_i^e - Q_i)^+\right). \quad (14)$$

Equation (14) is concave for $n = 2$ (see Parlar and Goyal (1984)) and $n = 3$ and partial substitution (see Ernst and Kouvelis (1999)). Netessine and Rudi (2003), however, show that the profit function might not be concave and not even quasiconcave in a setting with $n > 2$ retailers and full substitution considering the analytically tractable deterministic analog of Equation (14) and numerical experiments. Hence, the first order condition does not guarantee the global optimum in such settings. Their numerical experiments, however, indicate that for reasonable demands where the coefficient of variation is more than 0.1 the objective function is concave in Q_i for $i = 1, \dots, n$.

The first order necessary optimality conditions for $i = 1, \dots, n$ are given by

$$Pr(D_i^e < Q_i) - Pr(D_i < Q_i < D_i^e) + \sum_{i \neq j} \frac{r_j - s_j + p_j}{r_i - s_i + p_i} a_{ij} Pr(D_j^e < Q_j, D_i > Q_i) - \frac{r_i - c_i + p_i}{r_i - s_i + p_i} = 0. \quad (15)$$

If the other approach with shortage cost $p_i(D_i - Q_i)^+$ is taken the first order conditions are given by

$$Pr(D_i^e < Q_i) - Pr(D_i < Q_i < D_i^e) + \sum_{i \neq j} \frac{r_j - s_j}{r_i - s_i} a_{ij} Pr(D_j^e < Q_j, D_i > Q_i) + \frac{p_i}{r_i - s_i} Pr(D_i < Q_i) - \frac{r_i - c_i + p_i}{r_i - s_i} = 0. \quad (16)$$

Comparing the first order conditions under centralized control (Equations (15) and (16)) with those under decentralized control (Equations (9) and (11)), respectively, it can be concluded that the system is suboptimal in the competitive newsvendor case with product/demand substitution (e.g. Netessine and Rudi (2003) and Huang et al. (2011)). Compared to the centralized case, in competition each retailer receives less expected profit. That is, competition among retailers introduces challenges for supply chain coordination (Cachon, 2003).

Lippman and McCardle (1997) find that competition makes the retailers order more in anticipation of overflow demand from the competitors who may stockout. This effect is also noticed by e.g. Mahajan and Van Ryzin (2001) and Cachon (2003). Netessine and Rudi (2003) show that although competition compared to centralized control usually leads to overstocking, there are also some cases with the counterintuitive situation that competition leads to understocking. Qi et al. (2015) who consider a two-echelon framework with one manufacturer and two retailers show that the total stock of two competing newsvendors is decreasing in the (endogenous) wholesale price and that competition may result in lower total supply chain efficiency compared to a centralized system.

As already discussed in Section 2 the customer driven interaction is an external factor and no literature on horizontal contract design exist in this setting. Coordination under substitution is only addressed through vertical contracts with manufacturer-retailer competition, as for example in Boyaci (2005) who studies a multiple-channel distribution system in which a manufacturer sells its product through an independent retailer and its wholly-owned channel with customer spill-over. While simple vertical contracts fail to coordinate such a system, they show that a penalty contract and a two-part compensation-commission contract under which the retailer earns revenues only as commissions on his channel sales that exceed a flexible target can coordinate the supply chain.

4.2. Complementarity

Unlike product substitution where an unsatisfied customer chooses to buy a substitute product which benefits the seller of the substitute, the existence of cross-selling or com-

plementarity may produce opposite effects (Zhang et al., 2014). Complementarity implies that customers are willing to purchase some related products together with the original and if one of the products is out of stock the related product is no longer demanded. Especially for electronic products sold by independent companies the complementarity may be an issue. Netessine and Zhang (2005) give a few examples such as, e.g., music players whose sales are impacted by the availability of compatible rerecorded discs.

The topic has been mainly studied in areas such as e.g. customer behavior or marketing. Within the newsvendor framework, however, it has been rarely studied. In particular, we only came across Netessine and Zhang (2005) and Zhang et al. (2014) that deal with cross-selling/complementarity in non-cooperative newsvendor problems.

Let a_{ij} be the decreased demand for item i caused by a stock out situation of item j , then the expected profit of a newsvendor with cross-selling is obtained by replacing the effective demand D_i^e in the expected profit functions defined in Section 4.1 by the net demand $D_i^n = D_i - \sum_{i \neq j} a_{ji}(D_j - Q_j)^+$. Hence, cross-selling is complementary to substitution where a newsvendor has to consider the effective demand when making the inventory decision. Note that Netessine and Zhang (2005) describe the net demand for i as $D_i^n = D_i - \sum_{i \neq j} a_{ji}(D_j - Q_j)^+ + \sum_{i \neq j} a_{ji}D_j$, where a_{ij} is the fraction of customers willing to purchase from retailer i will also purchase from retailer j but only if retailer i has the product on stock.

Similar to the substitution setting, the optimal order quantity $Q_i^*(Q_j)$ under complementarity and shortage cost $p_i(D_i^n - Q_i)^+$ satisfies

$$Pr(D_i^n < Q_i^*) = F_{D_i^n}(Q_i^*) = \frac{r_i - c_i + p_i}{r_i - s_i + p_i}. \quad (17)$$

The critical ratio of retailer i under complementarity again is equal to the critical ratio of the classical newsvendor in Equation (2), but the demand D_i is replaced by the net demand D_i^n .

Again a unique Nash equilibrium for the game exists if $\sum_{j=1, \dots, n} a_{ji} < 1$ for all i or

$\sum_{i=1,\dots,n} a_{ji} < 1$ for all j (Zhang et al., 2014).

Assuming a centralized decision maker under complementarity a unique solution is guaranteed (Netessine and Zhang, 2005; Zhang et al., 2014), while under substitution this is not the case (see discussion in Section 4.1). Comparing the first order conditions of the centralized and decentralized system it can again be concluded that the system with independent players is not coordinated. Netessine and Zhang (2005) and Zhang et al. (2014) show that decentralization in such a setting leads to understocking. Netessine and Zhang (2005) also discuss the intuitive result that the expected profits in the decentralized case are lower than in the centralized case, while the numerical results of Zhang et al. (2014) show that in some cases the opposite is true, i.e. more sales do not mean higher profits under complementarity.

Netessine and Zhang (2005) also discuss the implication of complementarity on a two-echelon supply chain including the manufacturer's decision. They conclude that competition on complements induces both retailers and the wholesaler to coordinate the supply chain.

5. Combination or comparison of designed and customer driven interactions

There exist some work that deal with the i) combination of or ii) comparison between the different settings of non-cooperative games with horizontal inventory interactions. Anupindi and Bassok (1999) study a decentralized two-echelon supply chain with one manufacturer and two retailers and partial substitution. They discuss the impact of the retailers designed operations, in particular centralization of stocks at a warehouse versus decentralized stocks at the retail level, on the supply chain when a fraction α of customers who do not find the good at their retailer attempt to buy the good at the other retailer. In the system with centralization of stocks they assume a single decision maker (central control). This assumption implies that centralization of stocks is always more beneficial for the retailers. However, looking at the total supply chain profit including the manufacturers profit they show that it may not always increase upon centralization of stocks by retailers

as for certain α the expected sales in the decentralized system with substitution is larger. Çömez et al. (2012) study decentralized retailers with customer over-spills and transshipments. They assume that transshipments do not occur after all demands are realized, but a transshipment request is sent after each customer arrival at a stock-out retailer. The other retailer has the flexibility to reject or accept each request individually. They show that the optimal transshipment policy is characterized by the inventory holdback levels.

Zou et al. (2010) compare two alternative scenarios of a system with two retailers and a common manufacturer: one with transshipments and another without transshipments but substitution. They discuss the impact of the customer switching rate and the transshipment price on the benefit of transshipments compared to a scenario with substitution. Chen et al. (2016) also compare the two scenarios but they additionally include a customer's willingness to wait in case the product is not available and the possibility to arrange a fast-shipment directly to the customer from the supplier arranged by the stocked out retailer. They find that when fast-ship participation rate in the substitution scenario is high, the supplier tends to prefer the retailers to transship, while the retailers prefer substitution. Li and Li (2018) study a decentralized supply chain with one manufacturer and two symmetric retailers comparing transshipments with substitution, i.e. without transshipments a fraction of unsatisfied customers buy the product at the competing retailer. They show that as long as the transshipment price is properly chosen the retailers will always prefer transshipments independent of the proportion of customers switching to the competitor. They also study how the power structure of the supply chain members affects the transshipment decision. If the manufacturer has power to control transshipment prices he will choose transshipments over substitution and set a high transshipment price which increases the retailers inventories. If the manufacturer can only decide whether to transship or not, then he will avoid transshipment if the customer search probability is high.

6. Summary and future research directions

6.1. Summary

In practice retailers are often independent players in the supply chain. Due to risk pooling or customer interactions between independent retailers the inventory decision of one retailer may affect the decision of another. Hence, non-cooperative game theory is a tool that can be applied in this context. This paper provides an overview of the specific settings where non-cooperative static game theory with horizontal inventory interactions among newsvendors can be applied and discusses findings in the literature. Thereby, we distinguish between inventory interactions that are caused i) through the design of the retail-networks, i.e. local storage with inventory sharing through transshipments or inventory sharing through physical centralization of inventories and ii) through the customer driven interactions whereby retailers sell substitutable or complementary products. For each specific setting we discuss the conditions required for the existence and uniqueness of a Nash equilibrium in such a non-cooperative game. Further, we analyze the differences and relations between the different settings and discuss the main findings in the literature. We also compare the decentralized systems to a centrally controlled systems in order to emphasize the impact of horizontal coordination and horizontal contracting in such games.

To summarize, in this survey we review the contributions to date for the practical settings of horizontal inventory interactions between independent newsvendors and also give a comparison with a system of a single decision maker. From the managerial perspective, the reviewed mathematical models and their solutions provide important insights for inventory managers into the horizontal interactions in decentralized retailer networks. The findings provide guidance for both practice and future research how retailers considering the impact of these interactions can improve supply chain performance. The presented models can serve as building blocks for topics that have not been addressed so far and deserve further investigation.

6.2. Future research directions

The findings of this review of the literature on non-cooperative newsvendor games with horizontal inventory interactions open various promising future research directions.

1. *Combining vertical and horizontal relationships in the supply chain:* Although some work already discusses the vertical relationships, e.g. with a supplier or a manufacturer, in addition to the horizontal relationships we have focused on, there is still a lack of fully understanding how the vertical relations impact the supply chain performance under inventory pooling and competition, respectively. There is also potential in comparing all the individual settings discussed here in order to get a better understanding under which particular scenario a focus should either be put on the designed interaction or the customer driven interaction in a system with decentralized retailers.
2. *Horizontal contracting:* There is also a lot of potential in searching for more horizontal contracts that coordinate the supply chain, since most of the contracts that have been addressed so far are either not always coordinating or they are very complex and difficult to implement in practice. One has to find simple contracts that are flexible in allocating the total profits arbitrarily between the independent newsvendors. Consider the simple ex ante transshipment price contract that might not coordinate the decentralized transshipment system as soon as certain asymmetries among retailers arise (Hu et al., 2007). This is due to the implied unbalanced risk sharing between the retailers whenever they are asymmetric. For example, the retailers could add another payment, e.g. on leftovers or stockouts, in addition to the transshipment price in order to balance the risk between them (see also Arikan and Silbermayr (2018)). This would also lead to more flexibility as the introduction of another price also allows to arbitrarily divide the total expected profit between the retailers.
3. *Omni-channel supply chains and off-price retailers:* The settings reviewed here are also relevant in today's omni-channel supply chains that include online-shops and/or different costumer segments where the non-cooperative game-theoretic framework can be applied (see e.g. Seifert et al. (2006) and Zhao et al. (2016)). Online and

offline retailers need to compete in new and innovative ways (Brynjolfsson et al., 2013). In addition to e-commerce, off-price retailers such as T.J. Maxx and Ross have seen their market share increase greatly in the last three decades (Khouja et al., 2018). Regular retailers are transshipping leftover inventory to off-price retailers offering the retailers a useful sale channel. Off-price retailers offer these products at considerable price discounts, which puts them into a unique competitive position (Hess and Ring, 2014). It would be interesting to model such settings as a non-cooperative newsvendor game. To summarize, using the existing concepts reviewed here and apply them to the nowadays challenging supply chain structures of omni-channel retailing and/or the presence of off-price retailers in order to provide decision support is a promising research field.

4. *New technologies and data driven processes:* Recent technology advances are creating new business models presenting new challenges to retailers. For example, smart phones enable tracking of customers and also remove barriers from retailers such as geography and customer ignorance (Brynjolfsson et al., 2013). Large amounts of valuable data from e.g. social, mobile or local channels provide potential for better understanding customer interactions and effectively control inventory levels. However, they are also creating a new competitive environment for retailers. Hence, retailers that are able to analyze these new data can gain competitive advantage in the new business environment (Zhan and Tan, 2018). It would be interesting to integrate big data from internal and external sources into the existing models to generate new business models for retailers in competitive environments. Choi et al. (2018) discuss how different types of big data techniques can be applied in modern operations management. From the inventory related side new technologies such as e.g. 3D printing, also known as additive manufacturing, with its built to order fashion also lead to new challenges (Chen et al., 2017). Retailers will have to rethink their traditional approaches with this new technology; however, it could also lead to new ways of horizontal collaboration between independent retailers if they are willing to

share these new technological resources or raw materials.

5. *Sustainability of horizontal inventory interaction:* When looking at the designed operations, inventory pooling is known to be a best practice from the economic perspective, however its consequence on the environment and the impact on the product carbon footprint of the product should also be taken into consideration (Silbermayr et al., 2017). It is well-known that inventory pooling leads to higher expected profits than no pooling. However, the consequences on the environmental sustainability of such horizontal interactions for independent players has not been analyzed yet.
6. *Rules for inventory allocations:* Especially in networks with multiple (more than two) retailers, understanding how to design and negotiate proper allocation rules for assigning residual inventory or demand to the individual retailers has to be advanced. While there is some work on allocation mechanisms, the relevant factors that influence the performance of the mechanism, and the problem of putting the right incentives to the retailers in order to engage in horizontal interactions deserve future exploration. This is especially relevant in the case of physical pooling, where there are additional issues in sharing joint warehouses in a decentralized manner. Empirical investigations could help in order to better understand the main challenges and extend the theory in such settings.
7. *Newsvendors with alternative optimization objectives:* Previous work on the newsvendor problem has mainly focused on analytical approaches assuming risk neutral expected profit maximizing decision makers (Wang et al., 2012). The findings of this review of the literature are all based on maximizing the expected profits. There exist some work in the setting with independent risk-averse or loss-averse retailers selling substitutable products (e.g. Wu et al., 2014, Wang, 2010, Liu et al., 2013, Shen et al., 2016). Future research could focus on using alternative risk preferences rather than risk neutrality to describe risk behavior under both designed and customer driven inventory interactions. This would change the (equilibrium) inventory quantities under

decentralized and centralized supply chains depending on the retailers' risk attitudes. However, still very little is known about actual risk attitudes of the newsvendors in a decentralized setting.

8. *Behavioral newsvendors:* The success of the existing operations management tools reviewed in this paper and the accuracy of its theory rely heavily on understanding human behaviour (Bendoly et al., 2006). There is a great potential in studying the human factor in an environment with independent players that optimize their own objectives in order to understand decision makers' attitudes and extend the standard theory on non-cooperative newsvendor games. Ovchinnikov et al. (2015) is the first study that analyzes the human factor through an laboratory experiment for the setting with two newsvendors selling substitutable products. Their experimental result shows that on the aggregate level the participants did not respond to the other players actions. This was also shown in Zhao and Zhao (2016), Feng and Zhang (2017), Kirshner and Ovchinnikov (2018), Villa and Castañeda (2018) and Quiroga et al. (2019). The use of human experiments can help to relate the empirical findings to the analytical tools discussed here and extend them by including behavioral factors. Since studying the behavioral aspects in decentralized supply chains is just at its beginning it would be very interesting to further explore the human factor whenever horizontal interaction in a decentralized environment appear in order to integrate the experimental observations into the existing tools. Thereby, one has to analyze the aggregate and also the individual level outcomes of the experiments. Note that, for example, a newsvendor with per unit penalty cost discussed here has been defined in the seminal work of Schweitzer and Cachon (2000) as a newsvendor with stockout-averse preferences. Ho et al. (2010) and Kirshner and Ovchinnikov (2018), for example, show by means of experiments that the psychological aversion to left-overs is greater than the disutility for stockouts. Therefore, it would be interesting to study this further by extending the centralized multi-retailer inventory model of Ho et al. (2010) to a setting with decentralized retailers. This is an important re-

search direction and might lead to some additional models and tools that can assist in designing and negotiating such interactions in practice.

Acknowledgments

The author would like to thank Emel Arikan, Yigal Gerchak, Werner Jammerneegg, the editor and the referees for their valuable comments and suggestions.

References

- Anupindi, R., Bassok, Y., 1999. Centralization of stocks: Retailers vs. manufacturer. *Management Science* 45 (2), 178–191.
- Anupindi, R., Bassok, Y., Zemel, E., 2001. A general framework for the study of decentralized distribution systems. *Manufacturing & Service Operations Management* 3 (4), 349–368.
- Arikan, E., Silbermayr, L., 2018. Risk pooling via unidirectional inventory transshipments in a decentralized supply chain. *International Journal of Production Research* 56 (17), 5593–5610.
- Axsäter, S., 2003. Evaluation of unidirectional lateral transshipments and substitutions in inventory systems. *European Journal of Operational Research* 149 (2), 438–447.
- Ben-Zvi, N., Gerchak, Y., 2012. Inventory centralization in a newsvendor setting when shortage costs differ: Priorities and costs allocation. In: Choi, T.-M. (Ed.), *Handbook of Newsvendor Problems*. Springer, pp. 263–276.
- Bendoly, E., Donohue, K., Schultz, K. L., 2006. Behavior in operations management: Assessing recent findings and revisiting old assumptions. *Journal of Operations Management* 24 (6), 737–752.
- Boyaci, T., 2005. Competitive stocking and coordination in a multiple-channel distribution system. *IIE Transactions* 37 (5), 407–427.

- Brandenburger, A., Stuart, H., 2007. Biform games. *Management Science* 53 (4), 537–549.
- Brynjolfsson, E., Hu, Y. J., Rahman, M. S., 2013. Competing in the age of omnichannel retailing. *MIT Sloan Management Review*.
- Cachon, G. P., 2003. Supply chain coordination with contracts. *Handbooks in Operations Research and Management Science* 11, 227–339.
- Cachon, G. P., Netessine, S., 2006. Game theory in supply chain analysis. *Tutorials in Operations Research: Models, Methods, and Applications for Innovative Decision Making*, 200–233.
- Chen, H.-W., Gupta, D., Gurnani, H., 2016. Balancing inventory and stockout risk in retail supply chains using fast-ship. *Production and Operations Management* 25 (12), 2103–2115.
- Chen, L., Cui, Y., Lee, H. L., 2017. Retailing with 3d printing. working paper, 1–56.
- Chen, M. S., Lin, C. T., 1989. Effects of centralization on expected costs in a multi-location newsboy problem. *Journal of Operational Research Society* 40, 597–602.
- Chinchuluun, A., Karakitsiou, A., Mavrommati, A., 2008. Game theory models and their applications in inventory management and supply chain. In: A. Chinchuluun, P. Pardalos, A. Migdalas, and L. Pitsoulis (Ed.), *Pareto Optimality, Game Theory And Equilibria*. Springer, pp. 833–865.
- Choi, T.-M., Wallace, S. W., Wang, Y., 2018. Big data analytics in operations management. *Production and Operations Management* 27 (10), 1868–1883.
- Chopra, S., Meindl, P., 2016. Supply chain management. *Strategy, planning & operation*. Vol. 6. Springer.
- Çömez, N., Stecké, K. E., Çakanyıldırım, M., 2012. In-season transshipments among competitive retailers. *Manufacturing & Service Operations Management* 14 (2), 290–300.

- Corbett, C., Rajaram, K., 2006. A generalization of the inventory pooling effect to non-normal dependent demand. *Manufacturing & Service Operations Management* 8 (4), 351–358.
- Deflem, Y., Van Nieuwenhuyse, I., 2013. Managing inventories with one-way substitution: A newsvendor analysis. *European Journal of Operational Research* 228 (3), 484–493.
- Eppen, G., 1979. Effects of centralization on expected costs in a multi-location newsboy problem. *Management Science* 25 (5), 498–501.
- Ernst, R., Kouvelis, P., 1999. The effects of selling packaged goods on inventory decisions. *Management Science* 45 (8), 1142–1155.
- Feng, P., Wu, F., Fung, R. Y., Jia, T., 2018. Evaluation of two transshipment policies in a two-location decentralized inventory system under partial backordering. *Transportation Research Part E: Logistics and Transportation Review* 118, 207–224.
- Feng, T., Zhang, Y., 2017. Modeling strategic behavior in the competitive newsvendor problem: an experimental investigation. *Production and Operations Management* 26 (7), 1383–1398.
- Fiestras-Janeiro, M. G., García-Jurado, I., Meca, A., Mosquera, M. A., 2011. Cooperative game theory and inventory management. *European Journal of Operational Research* 210 (3), 459–466.
- Friedman, J. W., 1990. *Game theory with applications to economics*. Oxford University Press, USA.
- Gerchak, Y., 2016. Allocating pooled inventory according to contributions and entitlements. *Decision Making in Manufacturing and Services* 9 (1), 51–54.
- Gerchak, Y., He, Q.-M., 2003. On the relation between the benefits of risk pooling and the variability of demand. *IIE Transactions* 35 (11), 1027–1031.

- Granot, D., Sošić, G., 2003. A three-stage model for a decentralized distribution system of retailers. *Operations Research* 51 (5), 771–784.
- Hanany, E., Tzur, M., Levran, A., 2010. The transshipment fund mechanism: Coordinating the decentralized multilocation transshipment problem. *Naval Research Logistics (NRL)* 57 (4), 342–353.
- Hartman, B. C., Dror, M., 2005. Allocation of gains from inventory centralization in newsvendor environments. *IIE Transactions* 37 (2), 93–107.
- Hartman, B. C., Dror, M., Shaked, M., 2000. Cores of inventory centralization games. *Games and Economic Behavior* 31 (1), 26–49.
- He, Y., Zhang, P., Yao, Y., 2014. Unidirectional transshipment policies in a dual-channel supply chain. *Economic Modelling* 40, 259–268.
- Hess, R. L., Ring, L., 2014. Off-price versus price-off: Use of discriminant analysis to identify competitive differences across retail formats. *International Journal of Retail & Distribution Management* 42 (10), 902–928.
- Hezarkhani, B., Kubiak, W., 2010. A coordinating contract for transshipment in a two-company supply chain. *European Journal of Operational Research* 207 (1), 232–237.
- Ho, T.-H., Lim, N., Cui, T. H., 2010. Reference dependence in multilocation newsvendor models: A structural analysis. *Management Science* 56 (11), 1891–1910.
- Hu, X., Duenyas, I., Kapuscinski, R., 2007. Existence of coordinating transshipment prices in a two-location inventory model. *Management Science* 53 (8), 1289–1302.
- Huang, D., Zhou, H., Zhao, Q.-H., 2011. A competitive multiple-product newsboy problem with partial product substitution. *Omega* 39 (3), 302–312.
- Huang, X., 2013. A review on policies and supply chain relationships under inventory transshipment. *Bulletin of Statistics and Operations Research* 29 (1), 21–42.

- Huang, X., Sošić, G., 2010b. Repeated newsvendor game with transshipments under dual allocations. *European Journal of Operational Research* 204 (2), 274–284.
- Huang, X., Sošić, G., 2010a. Cooperation between multiple newsvendors with warehouses. *Manufacturing & Service Operations Management* 12 (2), 299 – 318.
- Khouja, M., Liu, X., Zhou, J., 2018. To sell or not to sell to an off-price retailer in the presence of strategic consumers. *Omega*, 1–17.
- Kirshner, S. N., Ovchinnikov, A., 2018. Heterogeneity of reference effects in the competitive newsvendor problem. *Manufacturing & Service Operations Management*.
- Kranenburg, A., Van Houtum, G.-J., 2009. A new partial pooling structure for spare parts networks. *European Journal of Operational Research* 199 (3), 908–921.
- Lee, C., Park, K. S., 2016. Inventory and transshipment decisions in the rationing game under capacity uncertainty. *Omega* 65, 82–97.
- Li, M., Li, T., 2018. Consumer search, transshipment, and bargaining power in a supply chain. *International Journal of Production Research* 56 (10), 3423–3438.
- Li, Q., Ha, A. Y., 2008. Reactive capacity and inventory competition under demand substitution. *IIE Transactions* 40 (8), 707–717.
- Lippman, S. A., McCardle, K. F., 1997. The competitive newsboy. *Operations Research* 45 (1), 54–65.
- Liu, W., Song, S., Wu, C., 2013. Impact of loss aversion on the newsvendor game with product substitution. *International Journal of Production Economics* 141 (1), 352 – 359.
- Mahajan, S., Van Ryzin, G., 2001. Inventory competition under dynamic consumer choice. *Operations Research* 49 (5), 646–657.
- Montrucchio, L., Norde, H., Özen, U., Scarsini, M., Slikker, M., 2012. Cooperative newsvendor games: A review. In: Choi, T.-M. (Ed.), *Handbook of Newsvendor Problems*. Springer, pp. 137–162.

- Nagarajan, M., Sošić, G., 2008. Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *European Journal of Operational Research* 187 (3), 719–745.
- Netessine, S., Rudi, N., 2003. Centralized and competitive inventory models with demand substitution. *Operations Research* 51 (2), 329–335.
- Netessine, S., Rudi, N., 2006. Supply chain choice on the internet. *Management Science* 52 (6), 844–864.
- Netessine, S., Zhang, F., 2005. Positive vs. negative externalities in inventory management: Implications for supply chain design. *Manufacturing & Service Operations Management* 7 (1), 58–73.
- Ovchinnikov, A., Moritz, B., Quiroga, B. F., 2015. How to compete against a behavioral newsvendor. *Production and Operations Management* 24 (11), 1783–1793.
- Parlar, M., 1988. Game theoretic analysis of the substitutable product inventory problem with random demands. *Naval Research Logistics (NRL)* 35 (3), 397–409.
- Parlar, M., Goyal, S., 1984. Optimal ordering decisions for two substitutable products with stochastic demand. *OPSEARCH* 21 (1), 1–15.
- Paterson, C., Kiesmüller, G., Teunter, R., Glazebrook, K., 2011. Inventory models with lateral transshipments: A review. *European Journal of Operational Research* 210 (2), 125–136.
- Qi, Y., Ni, W., Shi, K., 2015. Game theoretic analysis of one manufacturer two retailer supply chain with customer market search. *International Journal of Production Economics* 164, 57–64.
- Quiroga, B. F., Moritz, B., Ovchinnikov, A., 2019. Behavioral ordering, competition and profits: An experimental investigation. *Production and Operations Management*.

- Rudi, N., Kapur, S., Pyke, D. F., 2001. A two-location inventory model with transshipment and local decision making. *Management Science* 47 (12), 1668–1680.
- Schweitzer, M. E., Cachon, G. P., 2000. Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence. *Management Science* 46 (3), 404–420.
- Seifert, R. W., Thonemann, U. W., Sieke, M. A., 2006. Integrating direct and indirect sales channels under decentralized decision-making. *International Journal of Production Economics* 103 (1), 209–229.
- Shao, J., Krishnan, H., McCormick, S. T., 2011. Incentives for transshipment in a supply chain with decentralized retailers. *Manufacturing & Service Operations Management* 13 (3), 361–372.
- Shen, Y., Xie, J., Li, T., 2016. The risk-averse newsvendor game with competition on demand. *Journal of Industrial & Management Optimization* 12 (3), 931–947.
- Silbermayr, L., Gerchak, Y., 2019. Partial pooling by independent firms with allocation according to contribution to pool. *International Journal of Production Economics* 218, 375–385.
- Silbermayr, L., Jammernegg, W., Kischka, P., 2017. Inventory pooling with environmental constraints using copulas. *European Journal of Operational Research* 263 (2), 479–492.
- Smirnov, D., Gerchak, Y., 2014. Inventory sharing via circular unidirectional chaining. *European Journal of Operational Research* 237 (2), 474–486.
- Smirnov, D., Gerchak, Y., 2016. Inventory sharing via circular bidirectional chaining. *International Journal of Production Economics* 179, 141–152.
- Turken, N., Tan, Y., Vakharia, A. J., Wang, L., Wang, R., Yenipazarli, A., 2012. The multi-product newsvendor problem: Review, extensions, and directions for future research. In: Choi, T.-M. (Ed.), *Handbook of Newsvendor Problems*. Springer, pp. 3–39.

- Villa, S., Castañeda, J. A., 2018. Transshipments in supply chains: A behavioral investigation. *European Journal of Operational Research* 269 (2), 715–729.
- Wang, C., Webster, S., Zhang, S., 2012. Newsvendor models with alternative risk preferences within expected utility theory and prospect theory frameworks. In: Choi, T.-M. (Ed.), *Handbook of Newsvendor Problems*. Springer, pp. 177–196.
- Wang, C. X., 2010. The loss-averse newsvendor game. *International Journal of Production Economics* 124 (2), 448 – 452.
- Wang, Q., Parlar, M., 1994. A three-person game theory model arising in stochastic inventory control theory. *European Journal of Operational Research* 76 (1), 83–97.
- Wee, K. E., Dada, M., 2005. Optimal policies for transshipping inventory in a retail network. *Management Science* 51 (10), 1519–1533.
- Wu, D., Zhang, B., Baron, O., 2019. A trade credit model with asymmetric competing retailers. *Production and Operations Management* 28 (1), 206–231.
- Wu, H., Parlar, M., 2011. Games with incomplete information: A simplified exposition with inventory management applications. *International Journal of Production Economics* 133 (2), 562–577.
- Wu, M., Zhu, S. X., Teunter, R. H., 2014. A risk-averse competitive newsvendor problem under the cvar criterion. *International Journal of Production Economics* 156, 13 – 23.
- Yan, X., Zhao, H., 2015. Inventory sharing and coordination among n independent retailers. *European Journal of Operational Research* 243 (2), 576–587.
- Zhan, Y., Tan, K. H., 2018. An analytic infrastructure for harvesting big data to enhance supply chain performance. *European Journal of Operational Research*.
- Zhang, R.-Q., Zhang, L.-K., Zhou, W.-H., Saigal, R., Wang, H.-W., 2014. The multi-item newsvendor model with cross-selling and the solution when demand is jointly normally distributed. *European Journal of Operational Research* 236 (1), 147–159.

Zhao, F., Wu, D., Liang, L., Dolgui, A., 2016. Lateral inventory transshipment problem in online-to-offline supply chain. *International Journal of Production Research* 54 (7), 1951–1963.

Zhao, Y., Zhao, X., 2016. How a competing environment influences newsvendor ordering decisions. *International Journal of Production Research* 54 (1), 204–214.

Zou, L., Dresner, M., Windle, R., 2010. A two-location inventory model with transshipments in a competitive environment. *International Journal of Production Economics* 125 (2), 235–250.

Journal Pre-proof

	No. dec. newsv.	Non-ident. newsv.	Transsh.	Physical Pooling	Comp. to central con.	Coord. mechanism	Incl. upstream dec.	Specifics
Anupindi et al. (2001)	n	✓	✓	✓	✓	B		
Rudi et al. (2001)	2	✓	✓		✓	T		
Granot and Sošić (2003)	n	✓	✓		✓	B		
Netessine and Rudi (2006)	n			✓	✓		✓	drop-shipping
Hu et al. (2007)	2	✓	✓		✓	T		uncertain supplier capacity
Hanany et al. (2010)	n	✓	✓		✓	F		
Hezarkhani and Kubiak (2010)	2	✓	✓		✓	I		
Huang and Sošić (2010a)	n	✓	✓		✓	T, B		
Huang and Sošić (2010b)	n	✓	✓		✓	B		repeated game
Shao et al. (2011)	2		✓		✓	T	✓	
Ben-Zvi and Gerchak (2012)	2	✓		✓	✓			
Yan and Zhao (2015)	n	✓	✓		✓	B		
Gerchak (2016)	2			✓				
Lee and Park (2016)	2	✓	✓		✓	T		uncertain supplier capacity (rationing game)
Arikan and Silberman (2018)	2	✓	✓		✓	T+S+L		unidirectional transshipments
Feng et al. (2018)	2	✓	✓		✓	T		partial backordering
Silberman and Gerchak (2019)	2	✓		✓	✓	C		partial pooling

Coordination mechanism: T: transshipment price (ex ante), B: biform (non-cooperative/cooperative) game,
I: implicit pricing, F: transshipment fund, S: shortage subsidy, L: leftover subsidy, C: compensation cost

Table 3: Overview for the literature on designed horizontal interactions.

	No. dec. newsv.	Non-ident. newsv.	Substitutes	Complements	Comp. to central con.	Incl. upstream dec.	Specifics
Parlar (1988)	2	✓	✓		✓		
Wang and Parlar (1994)	3	✓	✓		✓		
Lippman and McCardle (1997)	n		✓				splitting rule for industry demand allocation
Mahajan and Van Ryzin (2001)	n	✓	✓		✓		dynamic consumer choice
Cachon (2003)	n		✓		✓	✓	industry demand allocated propor. to inventory
Netessine and Rudi (2003)	n	✓	✓		✓		
Netessine and Zhang (2005)	n	✓	✓	✓	✓	✓	complements vs. substitutes
Li and Ha (2008)	2	✓	✓				reactive capacity
Huang et al. (2011)	n	✓	✓		✓		
Zhang et al. (2014)	2	✓	✓	✓	✓		
Qi et al. (2015)	n	✓	✓		✓	✓	
Wu et al. (2019)	2	✓	✓		✓	✓	dominant and weak (capital-contained) retailer
Anupindi and Bassok (1999)	2	✓	✓		✓	✓	physical pooling vs. decentralized stocks
Zou et al. (2010)	2	✓	✓				substitution vs. transshipments
Çömez et al. (2012)	2	✓	✓				transshipment request after each customer arrival
Chen et al. (2016)	2	✓	✓		✓	✓	transshipments vs. substitution (incl. fast-ship option)
Li and Li (2018)	2		✓			✓	transshipments vs. substitution

Table 4: Overview for the literature on customer driven interactions and on combination/comparison of designed and customer driven interaction.