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A Brief Critical Introduction to the Ontological Argument and its Formalization: Anselm, Gaunilo, Descartes, Leibniz and Kant

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Abstract

The purpose of this paper is twofold. First, it aims at introducing the ontological argument through the analysis of five historical developments: Anselm's argument found in the second chapter of his Proslogion, Gaunilo's criticism of it, Descartes' version of the ontological argument found in his Meditations on First Philosophy, Leibniz's contribution to the debate on the ontological argument and his demonstration of the possibility of God, and Kant's famous criticisms against the (cartesian) ontological argument. Second, it intends to critically examine the enterprise of formally analyzing philosophical arguments and, as such, contribute in a small degree to the debate on the role of formalization in philosophy. My focus will be mainly on the drawbacks and limitations of such enterprise; as a guideline, I shall refer to a Carnapian, or Carnapian-like theory of argument analysis.

1 Introduction

The ontological argument is one of the most famous arguments (or family of arguments, to be more precise) in the history of philosophy. It was proposed in full-fledged form for the first time by Anselm of Canterbury, and either analyzed or reformulated by philosophers such as Descartes, Spinoza, Leibniz, Hume and Kant. Besides these classical approaches, so to speak, contemporary thinkers such as Norman Malcolm [22], Charles Hartshorne [11], David Lewis [21], Alvin Plantinga [29]) and Kurt Gödel [9] have either offered fresh views on the ontological argument or proposed new versions of it.

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The ontological argument is also perhaps the argument that has most attracted the attention of formal philosophers. Attempts to formally analyze the arguments attributed to Anselm, for instance, are abundant [11, pp. 49–57], [1, 26, 17, 24, 7] and [31, pp. 60–65]. Although there have been new formulations of the ontological argument directly embedded in formal frameworks,[1] the most common enterprise is still the formal analyses of traditional (and non-formal) versions of the ontological argument.

As far as formal analysis of existing philosophical arguments is concerned, some steps might be identified. First, there must be some sort of previous, informal analysis of the argument, meant to say, for example, what the premises and conclusion of the argument are, whether or not there are subsidiary arguments and hidden premises, etc. Second, there must be a formal language in which premises and conclusion are represented. Third, there might be an attempt to reconstruct the inferential steps of the original argument, possibly inside a specific theory of inference, be it proof theoretical or semantical or both. In a sense, the whole thing can be seen from the viewpoint of Carnap's project of conceptual explanation [5, pp. 1–18]. On one side, we have an argument, in general a prose text, whose relevant aspects — premises and conclusion, presuppositions, structure, etc. — are obscure and ambiguous. This would correspond to Carnap's notion of *explicandum*. On the other hand, we have the outcome of the analysis: a representation of the argument, possibly accompanied by a derivation, embedded in a formal framework, which is supposed to be a reconstruction, or to use Carnap's terminology, an explanation of the original argument. This is the *explicatum*.

Due to its exactness or formal feature, let us say, the *explicatum* is supposed not to have those obscure features of the *explicandum*. In particular, it must be evident in the *explicatum* the exact meaning of premises, conclusion and hidden presuppositions, the structure of the argument, and whether or not it is valid. The *explicatum* is also supposed to help in the evaluation of the reasonableness of the premises. This has to do with Carnap's second requirement: that the *explicatum* must be fruitful. Due to this, as well as to the very nature of formal reconstructions (Carnap would probably say their exactness) and the obscurity and incompleteness of informal arguments, the *explicatum* shall most probably have many features not shared by the original argument. However, this must not cause it to depart too much from the original argument, otherwise the former cannot be said to be an explanation of the latter. In Carnap's [5, p. 5] words, "the *explicatum* must be as close to or as similar with the *explicandum* as the latter's vagueness permits." To

[1] Gödel [9] and, to a lesser extent, Plantinga [29] are instances of this.

these three requirements — exactness, fruitfulness and similarity[2] — I will add a fourth one: that the *explicatum* should not be troublemaker, by which I mean that the *explicatum* or formal reconstruction should neither produce problems, confusing questions and unfruitful issues which are not already present in the *explicandum* nor obscure important and otherwise clear aspects of it.

The formal analysis of existing philosophical arguments can be categorized inside the umbrella of formalization in philosophy. As a methodology, the use of formal tools in philosophy has been the object of much debate in recent years [14, 10, 8]. Among other issues is the relation between formal philosophy and non-formal philosophy. Sven Hansson (2000) has rather dramatically put this as follows:

> Few issues in philosophical style and methodology are so controversial among philosophers as formalization. Some philosophers consider texts that make use of logical or mathematical notation as nonphilosophical and not worth reading, whereas others consider non-formal treatments as—at best—useful preparations for the real work to be done in a formal language. [...] This is unfortunate, since the value—or disvalue—of formalized methods is an important metaphilosophical issue that is worth systematic treatment. [...] It is urgently needed to revitalize formal philosophy and increase its interaction with non-formal philosophy. Technical developments should be focused on problems that have connections with philosophical issues.[3]

He correctly points out, although not that explicitly, that in order to revitalize formal philosophy and increase its interaction with non-formal philosophy, there must be a very clear understanding of the dangers and exaggerations of formalization [10, pp. 168–170].

The purpose of this paper is twofold. First, it aims at introducing the ontological argument through the analysis of five historical developments: Anselm's argument found in the second chapter of his *Proslogion*, Gaunilo's criticism of it, Descartes' version of the ontological argument found in his *Meditations on First Philosophy*, Leibniz's contribution to the debate on the ontological argument and his demonstration of the possibility of God, and Kant's famous criticisms against the (cartesian) ontological argument.

Second, it intends to critically examine the enterprise of formally analyzing philosophical arguments and, as such, contribute in a small degree to the debate on the role of formalization in philosophy. For this purpose, in my presentation of Anselm's

[2] There is a fourth requirement in Carnap's theory of conceptual explanation: simplicity [5, pp. 5–8].

[3] Hansson [10, pp. 162, 173].

argument and Gaunilo's criticism I shall refer to Robert Adam's (1971) pioneer work on the formalization of the ontological argument. Descartes' argument shall be introduced with the help of Howard Sobel's [31, pp. 31–40] analysis; as far as Leibniz's argument is concerned, I shall refer to Graham Oppy's [27, pp. 24–26] analysis, which, albeit not being a formal one, shall be useful as an instance of the first step in the task of formally analyzing an argument which I have mentioned above. My focus will be mainly on the drawbacks and limitations of these approaches as attempts to analyze existing philosophical arguments; as a guideline, I shall strongly refer to the Carnapian (or Carnapian-like) theory of argument analysis sketched above, specially its similarity and non-troublesome criteria.

The structure of the paper is as follows. In the next section I present Anselm's ontological argument, followed by Gaunilo's objection to it (Section 3)[4]. In Section 4, Descartes' version of the ontological argument is presented. In Section 5 I introduce Leibniz's contribution, which is followed by Kant's criticisms in Section 6. In Section 7 I lay down my concluding remarks about the enterprise of formally analyzing arguments.

2 Anselm

Although it is a consensus that a complete version of the ontological argument was first proposed by Anselm of Canterbury in his *Proslogion* (written between 1077 and 1078)[5], it is somehow controversial what the main argument is and where exactly in the text it is located.[6] Despite this, it is pretty safe to take the following extract from the second chapter of the Proslogion as describing the first, and surely the most famous, of Anselm's ontological arguments:

> (1) Well then, Lord, You who give understanding to faith, grant me that I may understand, as much as You see fit, that You exist as we believe You to exist, and that You are what we believe You to be. (2) Now we believe that You are something than which nothing greater can be thought. (3) Or can it be that a thing of such a nature does not exist, since "the Fool has said in his heart, there is no God?" (Psalms14, l.1, and 53, l. 1.) (4) But surely, when this same Fool hears what I

[4] The content of Sections 2 and 3 has been partially taken from [32].

[5] The basic ideas of the *Proslogion* were anticipated in one of Anselm's earlier writings, the *Monologion*.

[6] While some authors ([6, 2]) believe that the major argument is found in the second chapter of the Proslogion, others ([22, 2, 29]) claim that the main argument is a modal one occurring in the third chapter. Still others [18] claim that the second and third chapter, and perhaps the entire work, comprise a single argument.

> am talking about, namely, "something-than-which-nothing-greater-can-be-thought", he understands what he hears, and what he understands is in his mind (intellect, understanding), even if he does not understand that it actually exists. (5) For it is one thing for an object to exist in the mind, and another thing to understand that an object actually exists. (6) Thus, when a painter plans before hand what he is going to execute, he has (it) in his mind, but does not yet think that it actually exists because he has not yet executed it. (7) However, when he has actually painted it, then he both has it in his mind and understands that it exists because he has now made it. (8) Even the Fool, then, is forced to agree that something-than-which-nothing-greater-can-be-thought exists in the mind, since he understands this when he hears it, and whatever is understood is in the mind. (9) And surely that-than-which-a-greater-cannot-be-thought cannot exist in the mind alone. (10) For if it exists solely in the mind even, it can be thought to exist in reality also, which is greater. (11) If then that-than-which-a-greater-cannot-be-thought exists in the mind alone, this same that-than-which-a-greater-cannot-be-thought is that-than-which-a-greater-can-be-thought. (12) But this is obviously impossible. (13) Therefore there is absolutely no doubt that something-than-which-a-greater-cannot-be-thought exists both in the mind and in reality.[7]

Sentences (1) and (2) might be seen as an introduction to the argument. While (1) is a sort of opening statement, (2) is Anselm's famous definition of God: God is something than which nothing greater can be thought. (3) marks the proof style Anselm adopted: the reductio ad absurdum method; it states the reductio ad absurdum hypothesis, that is, the negation of what is supposed to be proved. Sentences (4) to (8) can be taken as a preliminary argument meant to prove a key premise of the argument: that something-than-which-nothing-greater-can-be-thought exists in the Fool's mind. (9) is an anticipation of the argument's conclusion: that God exists both in reality and in the understanding. (10) is the basic step of the argument: if this thing exists only in the mind, it can be thought to exist in reality also, and to exist in reality is greater. Sentence (11) states the consequence of what has been said so far: if this thing exists only in the mind, it will be at the same time that-than-which-a-greater-cannot-be-thought and that-than-which-a-greater-can-be-thought. But this, as sentence (12) says, is impossible. Therefore, the conclusion of the argument (13): that something-than-which-a-greater-cannot-be-thought exists both in the mind and in reality.

[7]Translation by M. J. Charlesworth [6].

In his pioneer work, Robert Adams [1, pp. 29–34] analyzes this argument as follows:

i There is, in the understanding at least, something than which nothing greater can be thought;

ii If it is even in the understanding alone, it can be thought to be in reality also;

iii which is greater;

iv There exists, therefore, ... both in the understanding and in reality, something than which a greater cannot be thought

As far as our numeration of Anselm's statements is concerned, (i) is (8), (ii) and (iii) are (10), and (iv) is (13). Adams still refers to a fourth premise which corresponds to the reductio ad absurdum hypothesis (3) and is used only at the time of reconstructing the derivation:

v There is no God.

A pertinent observation to be made about Adams's analysis concerns his choice of taking (8) as premise. As I have said, sentences from (4) to (8) can be taken very reasonably as a preliminary argument: while (4) is an anticipation of the conclusion and (8) is the conclusion, sentences (5) to (7) seem to be meant to support (8). That Adams skips this and takes (8) instead as premise is significant for a couple of reasons. First, although one could try to justify this move, the fact that Adams does not even mention it and simply ignores a good part of Anselm's original argument makes his analysis less faithful to it. Second, as an obvious consequence of that, Carnap's similarity criterion will probably not be satisfactorily met by Adams' reconstruction. Third, neglecting that Anselm himself tried to justify (8) has important consequences for evaluating the reasonableness of (Adam's reconstruction of) Anselm's argument. Some have argued that this premise is a very key one, and unless it is well justified, the argument as a whole might be accused of question-begging [30, pp. 37–52].

For the formalization proper, four predicates are used: $\boldsymbol{U}(x)$, meaning that x exists in the understanding, $\boldsymbol{R}(x)$, meaning that x exists in reality, $\boldsymbol{G}(x, y)$, meaning that x is greater than y, and $\boldsymbol{Q}(x, y)$, meaning that x is the magnitude of y. Besides them, Adams also uses a modal operator of possibility, represented here as $\Diamond$; $\Diamond\alpha$ means that it is possible that α,[8] which he takes as equivalent to *it can be thought that* α. Anselm's concept of a thing than which nothing greater can be though is represented as an abbreviation:

[8]Adams uses $\boldsymbol{M}$ instead of the symbol $\Diamond$.

$$\phi(x, m) =_{\text{def}} Q(m, x) \wedge \neg\Diamond\exists y\exists n(G(n, m) \wedge Q(n, y))$$

And here are the premises and conclusion of the argument:

I $\exists x\exists m(U(x) \wedge \phi(x, m))$

II $\forall x\forall m(U(x) \wedge \phi(x, m) \rightarrow \Diamond R(x))$

III $\forall x\forall m(\phi(x, m) \wedge \neg R(x) \rightarrow \neg\Diamond\neg(R(x) \rightarrow \exists n(G(n, m) \wedge Q(n, x))))$

IV $\exists x\exists m(U(x) \wedge R(x) \wedge \phi(x, m))$

$\phi(x, m)$ says that m is the magnitude of x and it is not possible that there is another thing, say y, whose magnitude n is greater than m. (I), (II) and (IV) are of easy understanding. (III) says that to every x and m, if x is God and m is his magnitude but he does not exist in reality, then it is not possible that the following proposition is false (that is to say, it is a necessary one): if x exists in reality, then its new magnitude, n, is greater than m. Adopting a counterfactual reading, (III) would mean the following: if God, whose magnitude is m, does not exist in reality, then would he exist in reality, his new magnitude, n, would be greater than m. The reductio ad absurdum hypothesis (v) is represented with the help of a constant, for it appears, one might argue, inside Anselm's talk-about-particulars discourse (see below):

V $\neg(a)$

Premise (III) — and the original sentence (10) in the argument — incorporates one of the most controversial issues in Anselm's argument, namely the doctrine that existence is something which 'produces' greatness:

(G) It is greater to exist in reality as well than to exist merely in the understanding.

In its turn, (G) might be understood in at least three different ways [23, pp. 90–91]:

(G1) Anything that exists both in reality and in the understanding is greater than anything that exists in the understanding alone.

(G2) Anything that exists both in reality and in the understanding is greater than the otherwise same kind of thing that exists in the understanding alone.

(G3) Anything that exists both in the understanding and in reality is greater than the otherwise exact same thing, if that thing exists merely in the understanding.

Adams picks (G3) as the correct or more suitable interpretation of (G). However, even considering its attempt to be as precise as possible, (G3) is still ambiguous with respect to one thing: are these two things we are comparing exactly the same object, or two objects which differ in one aspect only (existence)? Adams representation leaves no doubt: we are comparing the very and same object, the one referred to by variable x.

For the derivation, the following inference rules are used:[9]

M1. $\neg\Diamond\neg(\alpha \to \beta), \Diamond\alpha \vdash \Diamond\beta$

M2. $\exists x\Diamond\alpha(x) \vdash \Diamond\exists x\alpha(x)$

C1. $\exists x\alpha(x) \vdash \alpha(x/t)$

C2. $\forall x\alpha(x) \vdash \alpha(x/t)$

C3. $\alpha \wedge \beta, \beta \wedge \varphi \to \lambda \vdash \varphi \to \lambda$

C4. $\alpha(t) \vdash \exists x\alpha(t/x)$

C5. $\alpha \wedge \beta, \varphi \vdash \beta \wedge \varphi$

C6. If $\Gamma, \alpha \vdash \beta$ then $\Gamma \vdash \alpha \to \beta$

C7. $\neg\alpha \to \beta \wedge \neg\beta \vdash \alpha$

C8. $\alpha \wedge \beta, \varphi \vdash \alpha \wedge \varphi \wedge \beta$

MP. $\alpha, \alpha \to \beta \vdash \beta$

And here is the derivation:

1. $\exists x\exists m(U(x) \wedge \phi(x, m))$ Pr. (I)
2. $\forall x\forall m(U(x) \wedge \phi(x, m) \to \Diamond R(x))$ Pr.(II)
3. $\forall x\forall m(\phi(x, m) \wedge \neg R(x) \to \neg\Diamond\neg(R(x) \to \exists n(G(n, m) \wedge Q(n, x))))$ Pr. (III)

[9]Adams bases his formal treatment on Quine's *Methods of Logic*; I have here adopted a more standard notation. Some of these rules shall be used also in the coming sections. Due to their elementariness, I shall not bother neither to justify nor to prove them. Neither shall I take into consideration the provisos that some rules such as C1 and C2 are supposed to have, which depend on the particularities of the axiomatization at hand. If you wish, you could say that I am using a kind of a semi-formal approach here.

4.	$U(a) \wedge \phi(a,b)$	C1 (2x) 1[10]
5.	$U(a) \wedge \phi(a,b) \rightarrow \Diamond R(a)$	C2 (2x) 2
6.	$\Diamond R(a)$	MP 5,4
7.	$\phi(a,b) \wedge \neg R(a) \rightarrow \neg\Diamond\neg(R(a) \rightarrow \exists n(Gn,b) \wedge Q(n,a)))$	C2 (2x) 3
8.	$\neg R(a) \rightarrow \neg\Diamond\neg(R(a) \rightarrow \exists n(G(n,b) \wedge Q(n,a)))$	C3 4, 7
*9.	$\neg R(a)$	Pr. (V)
*10.	$\neg\Diamond\neg(R(a) \rightarrow \exists n(G(n,b) \wedge Q(n,a)))$	MP 8,9
*11.	$\Diamond\exists n(G(n,b) \wedge Q(n,a))$	M1 6,10
*12.	$\exists y\Diamond\exists n(G(n,b) \wedge Q(n,y))$	C4 11
*13.	$\Diamond\exists y\exists n(G(n,b) \wedge Q(n,y))$	M2 12
*14.	$U(a) \wedge Q(b,a) \wedge \neg\Diamond\exists y\exists n(G(n,b) \wedge Q(n,y))$	4[11]
*15.	$\Diamond\exists y\exists n(G(n,b) \wedge Q(n,y)) \wedge \neg\Diamond\exists y\exists x(G(n,b) \wedge Q(n,y))$	C5 13,14
16.	$\neg R(a) \rightarrow \Diamond\exists y\exists n(G(n,b) \wedge Q(n,y)) \wedge \neg\Diamond\exists y\exists x(G(n,b) \wedge Q(n,y))$	C6 9,15
17.	$R(a)$	C7 16
18.	$U(a) \wedge R(a) \wedge \phi(a,b)$	C8 4,17
19.	$\exists x\exists m(U(x) \wedge R(x) \wedge \phi(x,m))$	C4 (2x) 18

A couple of things have to be said about this reconstruction of Anselm's argument. First, it is exactly this: a reconstruction. At most, it might be taken as revealing the logic beyond Anselm's argument or unclosing all otherwise hidden logical steps needed to turn Anselm's argument into a valid one. Trivially Anselm's argument does not have this structure; at no point of the text do we find evidence for most of the steps and inference rules that Adams uses.

Despite of this, and this is the second point, Adams correctly represents two important structural features of Anselm's argument. First, starting from step 9, it uses the *reductio ad absurdum* method found in the original argument (it ends at

[10] Here "C1 (2x) 1" means that this step is justified by applying two times rule C1 to formula of step 1.

[11] Here is the unabbreviated form of 4.

15). Second, Anselm's original argument switches back and forth from a universal discourse to talk about particulars. From (4) to (8) he speaks about something than which nothing greater can be thought; however, from (9) to (12) he changes his discourse and starts speaking about that than which a greater cannot be thought; then, in (13), he goes back to talk about something than which nothing greater can be thought. Adams correctly represents this movement.[12]

Third, about Adams' use of the operator $\Diamond$, sure it is an interesting way to represent the expression "it can be thought that". However, it is significant that Anselm's original argument does not use any kind of modal construction. We might therefore once more bring into scene Carnap's similarity criterion. Moreover, from a logical point of view, taking "it can be thought that" to be equivalent to "it is possible that" has some worrisome consequences. Trivially, the correctness of his reconstruction depends on the validity of the modal inferences he uses. That they are valid when interpreting $\Diamond$ as "it is possible that" is not a big issue. But how about Adams' interpretation? Is the validity of these modal inferences automatically transferred when one interprets $\Diamond$ as "it can be thought that"? It is somehow *ad hoc* to arbitrarily assume that this question can be answered with a "yes".

3 Gaunilo

The very first objection to Anselm's argument[13] was given by one of his contemporaries, the Marmoutier monk Gaunilo, in a pamphlet entitled "On Behalf of the Fool". Here are Gaunilo's words:

> Consider this example: Certain people say that somewhere in the ocean there is a "Lost Island" [...] which is more abundantly filled with inestimable riches and delights than the Isles of the Blessed. [...] Suppose that one was to go on to say: You cannot doubt that this island, the most perfect of all lands, actually exists somewhere in reality, because it

[12]Using C1 and C2, he switches, in steps 4, 5 and 7, from a universal discourse to discourse about particulars (in the case, individuals a and b). Similarly to Anselm's original argument, all crucial *reductio ad absurdum* steps are done inside this particular discourse framework. Then, when he has proved that a exists in reality, he goes back in step 19, thought C4, to the universal type of discourse.

[13]Anselm's argument has been attacked on several different grounds. It might be objected, for instance, that the concept of greatness used by Anselm unjustifiably presupposes the existence of a maximum. How about if the order relation involved in such a concept is alike to the order of natural numbers, that is to say, how about if for every being we can think of, it is always possible to think of something greater than it? For the sake of space, I shall here mention only the objections related to the historical development I am following.

> undoubtedly stands in relation to your understanding. Since it is most excellent, not simply to stand in relation to the understanding, but to be in reality as well, therefore this island must necessarily be in reality. [...] If, I repeat, someone should wish by this argument to demonstrate to me that this island truly exists and is no longer to be doubted, I would think he were joking.[14]

Gaunilo's idea was to provide an argument which parallels Anselm's reasoning but which has an absurd conclusion. In order to reject the absurd conclusion that there exists such a lost perfect island, one has of course to reject the whole argument as invalid, even if she is unable to point out exactly what the defective steps in the argument are. But since the argument shares the same structure, so it is believed, than Anselm's argument, one is forced to also reject the latter argument along with its conclusion that there is something than which a greater cannot be thought.

In order to formalize Gaunilo's counter-argument, Adams [1, pp. 34–40] uses the following predicates: $I(x)$, meaning that x is an island, $L(x)$, meaning that x is a land or country, and $P(x)$, meaning that x has the profitable and delightful features attributed by legend to the lost island. The premises and conclusion of the argument, already formalized, are as follows:

(I) $\exists x(U(x) \wedge I(x) \wedge P(x) \wedge \neg\exists y(L(y) \wedge G(y, x)))$

(II) $\exists x(L(x) \wedge R(x))$

(III) $\forall x \forall y(L(x) \wedge R(x) \wedge I(y) \wedge \neg R(y) \to G(x, y))$

(IV) $\exists x(U(x) \wedge R(x) \wedge I(x) \wedge P(x) \wedge \neg\exists y(L(y) \wedge G(y, x)))$

(I) means that there is an individual x which exists in the understanding, is an island, has the profitable and delightful features attributed by legend to the lost island and, besides, there is no land greater than it. (II) says that there exists a real land. (III) says that any real land is greater than any island which does not exist in reality. The conclusion (IV) says that there exists such an island, both in the understanding and in reality, and that there is no greater land. Here is the *reductio ad absurdum* hypothesis:

(V) $\neg R(b)$

And here is the derivation:

1. $\exists x(U(x) \wedge I(x) \wedge P(x) \wedge \neg\exists y(Ly \wedge G(y, x)))$ Pr. (I)

[14] Hick and McGill [13, pp. 22-23].

2. $\exists x(L(x) \wedge R(x))$ Pr. (II)

3. $\forall x \forall y(L(x) \wedge R(x) \wedge I(y) \wedge \neg R(y) \rightarrow G(x, y))$ Pr (III)

4. $U(b) \wedge I(b) \wedge P(b) \wedge \neg \exists y(L(y) \wedge G(y, b)$ C1 1

5. $L(a) \wedge R(a)$ C1 2

6. $L(a) \wedge R(a) \wedge I(b) \wedge \neg R(b) \rightarrow G(a, b)$ C2 (2x) 3

7. $\neg R(b) \rightarrow G(a, b)$ C9 4,5,6

*8. $\neg R(b)$ Pr. (V)

*9. $G(a, b)$ MP 8, 7

*10. $L(a) \wedge G(a, b)$ C5 5,9

*11. $\exists y(L(y) \wedge G(y, b))$ C4 10

*12. $\exists y(L(y) \wedge G(y, b)) \wedge \neg \exists y(L(y) \wedge G(y, b)$ C5 4,11

13. $\neg R(b) \rightarrow \exists y(L(y) \wedge G(y, b)) \wedge \neg \exists y(L(y) \wedge G(y, b)$ C6 8, 12

14. $R(b)$ C7 13

15. $U(b) \wedge R(b) \wedge I(b) \wedge P(b) \wedge \neg \exists y(L(y) \wedge G(y, b))$ C8 4,14

16. $\exists x(U(x) \wedge R(x) \wedge I(x) \wedge P(x) \wedge \neg \exists y(L(y) \wedge G(y, x)))$ C4 15

where C9 is the following additional rule of inference:

C9. $\sigma_1 \wedge \beta \wedge \sigma_2, \alpha, \alpha \wedge \beta \wedge \lambda \rightarrow \varphi \vdash \lambda \rightarrow \varphi$

This is a valid argument. As far as Anselm's argument is concerned, despite the similarities (both proofs use the *reductio ad absurdum* method and the universal-to-particular-to-universal movement), it is pretty clear that both arguments have a quite different structure. In fact, the structure departure starts from the logical form of the premises: whereas Anselm spoke of a being whose greatness could not possibly be surpassed, Gaunilo speaks only of an island to which no country is superior.

Given this, it seems that Gaunilo's argument fails as a counter-argument to Anselm's. As I have said, a counter-argument in this sense is an argument that shares the same logical structure than the target argument, has true or reasonable premises and an absurd or patently false conclusion. But according to Adams' reconstructions both arguments have a quite different structure, which might allow us to conclude that, contrary to first appearances, Gaunilo did not succeed in refuting Anselm's argument.

This conclusion of course depends on the claim that Adams' formalization is a faithful and correct reconstruction of both Anselm's and Gaunilo's arguments. This of course is far from being trivial. As I have pointed out, Adams' reconstruction of Anselm's argument might be charged of departing too much from Anselm's original formulation, and therefore not satisfying Carnap's criterion of similarity to the *explanandum*. Besides, there are reconstructions according to which both arguments seem to share the same formal structure [27, pp. 17–18].

4 Descartes

Although writings such as the *Discourse on the Method* (1637) and *The Principles of Philosophy* (1644) discuss *a priori* arguments for the existence of God, Descartes' most referred version of the ontological argument appears in the fifth chapter of his *Meditations on First Philosophy*, first published in 1641. It is however controversial where exactly in the Fifth Meditation the argument (or arguments) is and how it shall be reconstructed. I shall take the following passage as the best representative of Descartes' formulation of his ontological argument:

> (1) [...] although it is not necessary that I should at any time entertain the notion of God, nevertheless whenever it happens that I think of a first and a sovereign Being, and, so to speak, derive the idea of Him from the storehouse of my mind, it is necessary that I should attribute to Him every sort of perfection, although I do not get so far as to enumerate them all, or to apply my mind to each one in particular. And this necessity suffices to make me conclude, (2) after having recognized that existence is a perfection, that (3) this first and sovereign Being really exists.[15]

Descartes' argument turns out to be a very simple one: the first premise (1) states that the idea or concept of God includes all perfections; from that, along with the second premise — that (2) existence is a perfection — we conclude that (3) God exists.[16]

In explaining the rationale behind this argument, it might be useful to refer to something which comes a little before this passage in the Fifth Meditation:

> But now, if just because I can draw the idea of something from my thought, it follows that all which I know clearly and distinctly as per-

[15]Translation by Elizabeth S. Haldane and G. R. T. Ross [19, p. 182].

[16]For alternative reconstructions of Descartes' ontological argument, see [27, pp. 20–24] and [25].

> taining to this object does really belong to it, may I not derive from this an argument demonstrating the existence of God? It is certain that I no less find the idea of God, that is to say, the idea of a supremely perfect Being, in me, than that of any figure or number whatever it is; and I do not know any less clearly and distinctly that an [actual and] eternal existence pertains to this nature than I know that all that which I am able to demonstrate of some figure or number truly pertains to the nature of this figure or number, and therefore, although all that I concluded in the preceding Meditations were found to be false, the existence of God would pass with me as at least as certain as I have ever held the truths of mathematics (which concern only numbers and figures) to be.[17]

Here Descartes invokes one of his key epistemological rules — that whatever I clearly and distinctly perceive to be contained in the idea of something is true of that thing — to introduce the possibility of an argument for the existence of God. In the same way that we arrive at basic truths about the nature of a figure or number, we might arrive at the conclusion that God exists simply by apprehending clearly and distinctively that existence pertains to the nature of such a supremely perfect being.

Although some have taken this passage as part of the argument itself [27, p. 21], it might be seen as providing a justification for the premises of the argument, first by offering the rule of clarity and distinctiveness as the epistemological support for the two premises, and second by giving a definition or explanation for the concept of God — a supremely perfect Being — which would render the first premise

(i) A supremely perfect being has every perfection.

quasi-tautological. Using this definition, the other premise and conclusion would be written as follows:

(ii) Existence is a perfection.

(iii) A supremely perfect being does exist.

This is the beginning of Sobel's reconstruction of Descartes argument [31, pp. 31–40].[18] Sobel points out that there is an ambiguity in (iii), which might be read either

[17]Translation by Elizabeth S. Haldane and G. R. T. Ross [19, pp. 180–181].

[18]Due to a passage in the Fifth Meditation where Descartes says that "it is no less repugnant to think of a God (that is, a supremely perfect being) lacking existence (that is, lacking some perfection), than it is to think of a mountain lacking a valley", Sobel takes him to be using a *reductio ad absurdum* proof style, adding then a fourth premise to the argument: A supremely perfect being does *not* exist. Even though one might argue against this analysis of Descartes' reasoning, it shall not interfere in my analysis of Sobel's contribution, for in order to get the contradiction one has to go through a direct proof, as Sobel does, and conclude that a supremely perfect being *does* exist.

as “Any supremely perfect being exists” or “At least one supremely perfect being exists”. After investigating the consequences of reconstructing the argument using the first reading, he (correctly) picks the second one as the most accurate analysis of Descartes’ argument.

For the formalization, Sobel uses three predicates — S, P and G — and a constant e. $S(x)$ means that x is a supremely perfect being, $P(x)$ that x is a perfection and $H(x, y)$ that x has property y; e means the property of existence. The argument is represented as follows:

(I) $\forall x(S(x) \rightarrow \forall y((P(y) \rightarrow H(x, y)))$

(II) $P(e)$

(III) $\exists x(S(x) \wedge H(x, e))$

The conclusion of Sobel’s analysis is that the argument is invalid: trivially, from the two premises we cannot arrive at (III); at most we reach at the conclusion that $\forall x(Sx \rightarrow H(x, e))$. But here Sobel’s analysis was extremely uncharitable to Descartes, to say the least. When a very key aspect of Descartes’ argument is considered, we see that Sobel’s analysis is faulty — it does not satisfactorily meet with the similarity criterion — and the argument straightforwardly valid.

Here is the key feature of Descartes’ argument that Sobel misses: If some object has the property of existence, it obviously must exist. In other words, there must be some kind of link between the property of existence and existence itself. From the perspective of the formalism Sobel uses, this means that there must be a connection between constant e and the existential quantifier $\exists$ (recall that they represent the very same notion, namely existence in reality). This might be expressed as follows:

(IV) $\forall x(H(x, e) \rightarrow \exists x H(x, e))$

This unfortunately does not solve the issue. First of all, even with this extra axiom we cannot derive (III): in order to conclude $\exists x H(x, e)$ we should have $H(d, e)$ for some object d, which we cannot, for if we had $H(d, e)$, we would have the hard part of the conclusion — $\exists x H(x, e)$ — and the argument would be patently circular. Second, and this is trivially related to the first point, $H(d, e) \rightarrow \exists x H(x, e)$ is an instance of the rule of existential generalization. (IV) is tautological and, as such, adds nothing to our set of premises. Since classical first order logic requires each singular term to denote an object in the domain of quantification, which is usually understood as the set of existing objects, it is vacuous to say of an object that if it has some property it exists: in order to have any property, it must already exist.

This inability to properly represent what seems to be a very key presupposition of Descartes' argument reveals that Sobel's approach is misguided and his reconstruction a troublemaker one. But what if we take seriously Descartes' claim, found in both extracts of the Meditations shown above, that the idea of God is in our minds? Is not Descartes presupposing here a kind of existence pretty much alike to Anselm's notion of existence in the understanding? It seems to me that textual evidence suggests a positive answer to these questions.

What follows is an attempt to consider these ruminations and fix the issue still inside the basic logical framework which Sobel uses, that is to say, first-order classical logic.[19] Taking the existential quantifier (and consequently the universal quantification) to refer to this Anselmian-like notion of existence — in cartesian terms, $\exists x \boldsymbol{A}(x)$ would mean that I can draw from my thought the idea of some x which has property $\boldsymbol{A}$ — and predicate $\boldsymbol{H}$ and constant $\boldsymbol{e}$ to the notion of existence in reality — $\boldsymbol{H}(x, e)$ means that x exists in reality — the argument could be rewritten as follows:

(I*) $\exists x \boldsymbol{S}(x)$

(I) $\forall x(\boldsymbol{S}(x) \to \forall y(\boldsymbol{P}(y) \to \boldsymbol{H}(x, y)))$

(II) $\boldsymbol{P}(e)$

(III) $\exists x(\boldsymbol{S}(x) \wedge \boldsymbol{H}(x, e))$

(I*) and (I) both represent premise (i). In the same way that there is an ambiguity in (iii), there is also an ambiguity in (i): it can be read as "Any supremely perfect being has every perfection" or "At least one supremely perfect being exists and it has every perfection." While the first reading is the one (and only one) that Sobel takes into account (to characterize the concept of a supreme being), the second one encapsulates the presupposition that a supreme being (or the idea of a supreme being, to be more precise) exists in our minds. Instead then of preferring one reading over the other, I take both of them into account; my reading of (i) in terms of (I*) and (I) is a compromise between these two interpretations.

The proof of the argument validity is straightforward:

1. $\exists x \boldsymbol{S}(x)$ — Pr. (I*)
2. $\forall x(\boldsymbol{S}(x) \to \forall y(\boldsymbol{P}(y) \to \boldsymbol{H}(x, y))$ — Pr. (I)
3. $\boldsymbol{P}(e)$ — Pr. (II)

[19]This is why I have left out free logic, which in this context could be a better representational tool than classical first order logic.

4. $S(d)$ C1 1

5. $S(d) \to \forall y(P(y) \to H(d, y))$ C2 2

6. $\forall y(P(y) \to H(d, y))$ MP 4,5

7. $P(e) \to H(d, e)$ C2 6

8. $H(d, e)$ MP 3,7

9. $S(d) \wedge H(d, e)$ C10 4,8

10. $\exists x(S(x) \wedge H(x, e))$ C4 9

In addition to the rules introduced in Section 2 in the context of Adam's reconstruction of Anselm's argument, an additional one has been used here:

C10. $\alpha, \beta \vdash \alpha \wedge \beta$

A more elegant second order version of this reconstruction of Descartes' argument would be as follows:

(*I) $\exists x S(x)$

(I) $\forall x(S(x) \to \forall Y(\mathsf{P}(Y) \to Y(x)))$

(II) $\mathsf{P}(E)$

(III) $\exists x(S(x) \wedge E(x))$

where $P(Y)$ is a second order predicate meaning that Y is a perfection and $E(x)$ is a first order predicate meaning that x exists in reality. The derivation then would be rewritten as follows:

1. $\exists x S(x)$ Pr. (I*)

2. $\forall x(S(x) \to \forall Y(\mathsf{P}(Y) \to Y(x)))$ Pr. (I)

3. $\mathsf{P}(E)$ Pr. (II)

4. $S(d)$ C1 1

5. $S(d) \to \forall Y(\mathsf{P}(Y) \to Y(d))$ C2 2

6. $\forall Y(\mathsf{P}(Y) \to Y(d))$ MP 4,5

7. $\mathsf{P}(E) \to E(d)$ C2^2 6

8. $E(d)$ MP 3,7

9. $S(d) \wedge E(d)$ C10 4,8

10. $\exists x(S(x) \wedge E(x))$ C4 9

, where C2^2 is second order version of C2.

5 Leibniz

Leibniz wrote about ontological arguments in many of his works, including *Monadology* (1714), *Theodicy* (1710), and *New Essays Concerning Human Understanding* (completed in 1704). Although he presented at least three different ontological proofs for the existence of God,[20] his most important contribution to the history of ontological arguments is his attempt to demonstrate the coherence of the concept of God. If this is not done, he argued, all ontological arguments are irremediably defective. In the "Meditations on Knowledge, Truth, and Ideas" of 1684, for instance, he analyzes what he calls the "old argument for the existence of God" as follows:

> The argument goes like this: Whatever follows from the idea or definition of a thing can be predicated of the thing. God is by definition the most perfect being, or the being nothing greater than which can be thought. Now, the idea of the most perfect being includes ideas of all perfections, and amongst these perfections is existence. So existence follows from the idea of God. Therefore [...] God exists. But this argument shows only that if God is possible then it follows that he exists. For we can't safely draw conclusions from definitions unless we know first that they are real definitions, that is, that they don't include any contradictions. If a definition does harbour a contradiction, we can infer contradictory conclusions from it, which is absurd.[21]

Leibniz is here charging the ontological argument of being incomplete. He is concerned that the concept of a being who possesses all perfections might not be consistent, that is to say, that two perfections A and B, say, are such that there cannot be an individual that possess A and B at the same time. In order for the concept of God as defined by Anselm and Descartes to be a "real definition", one has to show first

[20] Blumenfeld [4] tries to how that the three proofs are equivalent, something in which Leibniz himself believed, he claims.

[21] Translation by Jonathan Bennet [3].

that all perfections or 'greatness producers' are compossible, that is to say, that it is possible for all of them to be instantiated by one and the same individual. Unless this is shown, all the ontological arguments have reached is the following conclusion: if God is possible, then he exists. As far as Anselm's and Descartes's arguments are concerned, this implies that the reasonableness of

(I) $\exists x \exists m(U(x) \wedge \phi(x, m))$

and

(I*) $\exists x S(x)$

depend, respectively, on the truth of $\Diamond \exists x \exists m \phi(x, m)$ and $\Diamond \exists x S(x)$.

In order to fill in this gap, Leibniz presents the following argument, found in his *New Essays concerning Human Understanding*:

> I call every simple quality which is positive and absolute, or expresses whatever it expresses without any limits, a perfection. But a quality of this sort, because it is simple, is therefore irresolvable or indefinable, for otherwise, either it will not be a simple quality but an aggregate of many, or, if it is one, it will be circumscribed by limits and so be known through negations of further progress contrary to the hypothesis, for a purely positive quality was assumed. From these considerations it is not difficult to show that all perfections are compatible with each other or can exist in the same subject. For let the proposition be of this kind:
>
> A and B are incompatible
>
> (for understanding by A and B two simple forms of this kind or perfections, and it is the same if more are assumed like them), it is evident that it cannot be demonstrated without the resolution of the terms A and B, of each or both; for otherwise their nature would not enter into the ratiocination and the incompatibility could be demonstrated as well from any others as from themselves. But now (by hypothesis) they are irresolvable. Therefore this proposition cannot be demonstrated from these forms.[22]

This is the extract of Leibniz's work that Oppy analyzes. He informally reconstructs the argument as follows [27, p. 23]:

i. By definition, a perfection is a simple quality that is positive and absolute. (Definition)

[22] Translation by Alfred Langley [20, pp. 714–715].

ii. A simple quality that is positive and absolute is irresolvable or indefinable. (Premise — capable of further defense)

iii. *A* and *B* are perfections whose incompatibility can be demonstrated. (Hypothesis for *reductio*)

iv. In order to demonstrate the incompatibility of *A* and *B*, *A* and *B* must be resolved. (Premise)

v. Neither *A* nor *B* can be resolved. (From ii)

vi. (Hence) It cannot be demonstrated that *A* and *B* are incompatible. (From iii, iv, v by *reductio*)

Here is an attempt to turn this into a more formal and detailed account. Let $\blacksquare$ be a modal operator meant to represent the notion of demonstrability so that $\blacksquare\alpha$ means that α is demonstrable and $\mathsf{S}(Y), \mathsf{O}(Y), \mathsf{A}(Y)$ and $\mathsf{R}(Y)$ four second order predicates meaning, respectively, that Y is simple, Y is positive, Y is absolute and Y is resolvable. Besides, there are the following two definitions:

$$\mathsf{C}(X,Y) =_{\text{def}} \Diamond\exists z(X(z) \wedge Y(z))$$
$$\mathsf{P}(Y) =_{\text{def}} \mathsf{S}(Y) \wedge \mathsf{O}(Y) \wedge \mathsf{A}(Y)$$

$\mathsf{C}(X,Y)$ means that X and Y are two compossible (or compatible, in Oppy's terminology) properties and $\mathsf{P}(Y)$ that Y is a perfection. The premises of the argument would then be represented as follows:

(I) $\forall Y(\mathsf{P}(Y) \rightarrow \neg\mathsf{R}(Y))$

(II) $\mathsf{P}(A)$

(III) $\mathsf{P}(B)$

(IV) $\forall X \forall Y(\blacksquare\neg\mathsf{C}(X,Y) \rightarrow \mathsf{R}(X) \wedge \mathsf{R}(Y))$

(V) $\blacksquare\neg\mathsf{C}(A,B)$

(I) is premise (ii), (II) and (III) are (partially) (iii), (IV) is the universal form of (iv) and (V) is the hypothesis for *absurdum*, which is also contained in (iii). The conclusion is obviously

(VI) $\neg\blacksquare\neg\mathsf{C}(A,B)$

As for the derivation, the following additional rules shall be used:

C7*. $\alpha \to \beta \wedge \neg\beta \vdash \neg\alpha$

C11. $\neg\alpha \vdash \neg(\alpha \wedge \beta)$

C12. $\alpha, \beta \vdash \alpha \wedge \beta$

And here is the reconstruction of the derivation:

1.	$\forall Y(\mathsf{P}(Y) \to \neg\mathsf{R}(Y)$	Pr. (I)
2.	$\mathsf{P}(\boldsymbol{A})$	Pr. (II)
3.	$\mathsf{P}(\boldsymbol{A}) \to \neg\mathsf{R}(\boldsymbol{A})$	C2^2 1
4.	$\neg\mathsf{R}(\boldsymbol{A})$	MP 2,3
5.	$\mathsf{P}(\boldsymbol{B})$	Pr. (III)
6.	$\mathsf{P}(\boldsymbol{B}) \to \neg\mathsf{R}(\boldsymbol{B})$	C2^2 1
7.	$\neg\mathsf{R}(\boldsymbol{B})$	MP 5,6
8.	$\forall X \forall Y(\blacksquare\neg\mathsf{C}(X, Y) \to \mathsf{R}(X) \wedge \mathsf{R}(Y))$	Pr. (IV)
9.	$\forall Y(\blacksquare\neg(\mathsf{C}(\boldsymbol{A}, Y) \to \mathsf{R}(\boldsymbol{A}) \wedge \mathsf{R}(Y))$	C2^2 8
10.	$\blacksquare\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B}) \to \mathsf{R}(\boldsymbol{A}) \wedge \mathsf{R}(\boldsymbol{B})$	C2^2 9
*11.	$\blacksquare\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B})$	Pr. (V)
*12.	$\mathsf{R}(\boldsymbol{A}) \wedge \mathsf{R}(\boldsymbol{B})$	MP 11,10
*13.	$\neg(\mathsf{R}(\boldsymbol{A}) \wedge \mathsf{R}(\boldsymbol{B}))$	C11 4
*14.	$(\mathsf{R}(\boldsymbol{A}) \wedge \mathsf{R}(\boldsymbol{B})) \wedge \neg(\mathsf{R}(\boldsymbol{A}) \wedge \mathsf{R}(\boldsymbol{B}))$	C12 12,13
15.	$\blacksquare\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B}) \to (\mathsf{R}(\boldsymbol{A}) \wedge \mathsf{R}(\boldsymbol{B})) \wedge \neg(\mathsf{R}(\boldsymbol{A}) \wedge \mathsf{R}(\boldsymbol{B}))$	C6 11,14
16.	$\neg\blacksquare\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B})$	C7* 15

Oppy offers two criticisms against Leibniz's attempt to fix the ontological arguments. First, he correctly points out that showing that it cannot be demonstrated that A and B are incompatible is quite different from showing that $\boldsymbol{A}$ and $\boldsymbol{B}$ are compatible, what makes it obvious that the argument failed to reach the required conclusion [27, pp. 25–26].

Two observations are in order here. First of all, the conclusion that $\neg\blacksquare\neg \mathsf{C}(\boldsymbol{A}, \boldsymbol{B})$ might be quite relevant to the debate about the ontological argument. The way Leibniz puts the whole thing, saying that all the ontological arguments show is that if God is possible then it follows that he exists, implies that the ontological defender is the one who has to bear the burden of the proof. But it might also be argued, as Leibniz himself did, that "there is always a presumption on the side of possibility; that is to say, everything is held to be possible until its impossibility is proved".[23] In other words, possibility claims are blameless until the contrary is proven. The burden of the proof then, in this case the proof that the concept of God is indeed incoherent, is on the critic of the ontological argument. But if this is correct, then showing that the concept of God cannot be proved to be incoherent has the relevant consequence that the critic's movement to refute the argument is hopeless.[24]

Second, Oppy oddly overlooks what Leibniz writes right after the quotation he analyzes, where he clearly does offer an argument for the conclusion that $\boldsymbol{A}$ and $\boldsymbol{B}$ are compossible:

> But it might certainly be demonstrated by these if it were true, because it is not true *per se*, for all propositions necessarily true are either demonstrable or known per se. Therefore, this proposition is not necessarily true. Or if it is not necessary that $\boldsymbol{A}$ and $\boldsymbol{B}$ exist in the same subject, they cannot therefore exist in the same subject, and since the reasoning is the same as regards any other assumed qualities of this kind, therefore all perfections are compatible.[25]

Complementing Oppy's analysis then, we would have the two additional premises:

vii. A proposition is necessary only it is true (or known) *per se* or demonstrable.

viii. That $\boldsymbol{A}$ and $\boldsymbol{B}$ are incompatible is not true (or known) *per se*,

which might be formalized as follows:

[23]Quoted from Blumenfeld [4, p. 357]. Blumenfeld defends that there is an argument in Leibniz, which he calls Leibniz's fallback position, that, in the absence of proof, one ought to assume that God is possible.

[24]This can also be seen from another angle. From a dialectical point of view, there are two movements one can make to criticize a valid argument. The first is trying to show that one of its premises is false or is known to be false; if successful, this movement would be enough to claim that the argument has been refuted. The second, and more modest movement, is to question the truth of one of the premises on the grounds that the defender was unable to provide strong support or evidence for it or did not prove some non-obvious presupposition on which it depends; it puts the burden of the proof on the defender's shoulder, claiming the argument to be incomplete, so to speak, but does not serve as a final word on the correctness of the argument.

[25]Translation by Alfred Langley [20, p. 715].

VII. $\Box\alpha \rightarrow \blacksquare\alpha \lor \circ\alpha$

VIII. $\neg\circ\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B})$,

where $\circ\boldsymbol{\alpha}$ means that $\boldsymbol{\alpha}$ is true (or known) *per se* and $\circ\boldsymbol{\alpha}$ that $\boldsymbol{\alpha}$ is necessary; (VII) is a schema of premises, instead of a single premise.

Adding the following inference rules and axiom to our list of logical principles:

C13. $\alpha \rightarrow \beta \lor \varphi, \neg\beta, \neg\varphi \vdash \neg\alpha$

C14. $\neg\Box\neg\alpha \vdash \Diamond\alpha$

C15. $\Diamond\Diamond\alpha \vdash \Diamond\alpha$

we have a full Leibnizian derivation for the conclusion that $\boldsymbol{A}$ and $\boldsymbol{B}$ are compossible — $\mathsf{C}(\boldsymbol{A}, \boldsymbol{B})$ or $\Diamond\exists z(\boldsymbol{A}(z) \land \boldsymbol{B}(z))$ — as follows:

17.	$\Box\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B}) \rightarrow \blacksquare\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B}) \lor \circ\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B})$	Pr. (VII)
18.	$\neg\circ\neg\mathsf{C}(\boldsymbol{A}, \boldsymbol{B})$	Pr. (VIII)
19.	$\neg\Box\neg\Diamond\exists z(\boldsymbol{A}(z) \land \boldsymbol{B}(z))$	C13 16,17,18
20.	$\Diamond\Diamond\exists z(\boldsymbol{A}(z) \land \boldsymbol{B}(z))$	C14 19
21.	$\Diamond\exists z(\boldsymbol{A}(z) \land \boldsymbol{B}(z))$	C15 20

The second criticism Oppy offers is a threefold one [27, pp. 25–26]. First, even if one succeeds in showing that all simple, positive, absolute qualities are compatible, it seems there is still a hole in the ontological argument: one has to show that there are indeed simple, positive and absolute qualities. Second, given the nature of simple, positive, absolute qualities, there seems to be an epistemological problem about the possibility of reasonable belief in their existence.[26] Third, even if we grant that there are simple, positive, absolute qualities, the question can be raised whether existence is a simple, positive, absolute quality.

Although Oppy's criticism looks correct, it seems to be at odds with his analysis of the argument, and consequently with my own rendition of it. A good look at the first part of the derivation will suffice for one to see why: as far as premise

[26]He writes: "What grounds could one have for thinking that there are simple, positive, absolute qualities? There may only be the appearance of a problem here, since it seems reasonable to allow that reasonable belief need not require grounds. However, this problem does appear to threaten the dialectical value of the demonstration; it certainly seems that one could reasonably believe that there are no simple, positive, absolute qualities." [27, p. 25].

(I) — that every simple quality that is positive and absolute is irresolvable — is concerned, although we have applied it to both $\boldsymbol{A}$ and $\boldsymbol{B}$, only the result of applying it to $\boldsymbol{A}$ (which got us $\neg\boldsymbol{R}(A)$ at step 4) is effectively used in the derivation (step *13); steps 5 to 7 play no role whatsoever in the derivation and could, therefore, be harmlessly erased from it. Hence, as far as Leibniz's argument is concerned, only one of the properties needs effectively to be simple, positive and absolute. Based on this, someone might respond to Oppy's criticism as follows: contrary to what he says, one does not need to show that there *are* simple, positive and absolute qualities; all it is required is that one shows that one, and only one, of the divine properties is simple, positive and absolute, and it does need to be the property of existence.

Although surely sound from the point of view our logical analysis, there seems to be something wrong with this response. That only one of the divine properties needs to me simple, positive and absolute does not match with Leibniz account. The solution of this puzzle is in fact very simple: the analysis on which our formalization was based, Oppy's analysis, is faulty. Suppose that $\boldsymbol{A}$ is a simple, positive and absolute quality but $\boldsymbol{B}$ is not; while $\boldsymbol{B}$ is therefore resolvable, $\boldsymbol{A}$ is not. In addition, suppose that $\boldsymbol{B}$ is resolved into $\vec{\boldsymbol{A}}$ and $\boldsymbol{C}$, where $\vec{\boldsymbol{A}}$ is the complement of $\boldsymbol{A}$. It can therefore be demonstrated that $\boldsymbol{A}$ and $\boldsymbol{B}$ are incompatible, even though only $\boldsymbol{A}$ is unresolvable. What this shows is that in order to demonstrate that $\boldsymbol{A}$ and $\boldsymbol{B}$ are incompatible, $\boldsymbol{A}$ *or* $\boldsymbol{B}$ must be resolved, not $\boldsymbol{A}$ *and* $\boldsymbol{B}$. Therefore, the correct representation of premise (iv) is

iv. In order to demonstrate the incompatibility of $\boldsymbol{A}$ and $\boldsymbol{B}$, $\boldsymbol{A}$ *or* $\boldsymbol{B}$ must be resolved.

not "In order to demonstrate the incompatibility of $\boldsymbol{A}$ and $\boldsymbol{B}$, $\boldsymbol{A}$ *and* $\boldsymbol{B}$ must be resolved", as Oppy says.

Modifying (IV) accordingly —

(IV) $\forall X \forall Y(\blacksquare\neg \mathsf{C}(X,Y) \rightarrow \mathsf{R}(X) \vee \mathsf{R}(Y))$

we get a proper reconstruction of Leibniz' reasoning which is not susceptible to the mentioned response:[27]

1. $\forall Y(\mathsf{P}(Y) \rightarrow \neg\mathsf{R}(Y))$ Pr. (I)
2. $\mathsf{P}(A)$ Pr. (II)

[27]I am considering here (as Leibniz and Oppy did) only pairs of properties, which is obviously not general enough. For an account which considers not only pairs of properties but a potentially infinite numbers of divine properties, see [28].

3.	$\mathsf{P}(A) \to \neg\mathsf{R}(A)$	C2² 1
4.	$\neg\mathsf{R}(A)$	MP 2,3
5.	$\mathsf{P}(B)$	Pr. (III)
6.	$\mathsf{P}(B) \to \neg\mathsf{R}(B)$	C2² 1
7.	$\neg\mathsf{R}(B)$	MP 5,6
8.	$\forall X \forall Y(\blacksquare\neg\mathsf{C}(X, Y) \to \mathsf{R}(X) \vee \mathsf{R}(Y))$	Pr. (IV)
9.	$\forall Y(\blacksquare\neg\mathsf{C}(A, Y) \to \mathsf{R}(A) \vee \mathsf{R}(Y))$	C2² 8
10.	$\blacksquare\neg\mathsf{C}(A, B) \to \mathsf{R}(A) \vee \mathsf{R}(B)$	C2² 9
*11.	$\blacksquare\neg\mathsf{C}(A, B)$	Pr. (V)
*12.	$\mathsf{R}(A) \vee \mathsf{R}(B)$	MP 11,10
*13.	$\neg(\mathsf{R}(A) \vee \mathsf{R}(B))$	C16 4,7
*14.	$(\mathsf{R}(A) \vee \mathsf{R}(B)) \wedge \neg(\mathsf{R}(A) \vee \mathsf{R}(B))$	C12 12,13
15.	$\blacksquare\neg\mathsf{C}(A, B) \to (\mathsf{R}(A) \vee \mathsf{R}(B)) \wedge \neg(\mathsf{R}(A) \vee \mathsf{R}(B))$	C6 11,14
16.	$\neg\blacksquare\neg\mathsf{C}(A, B)$	C7* 16

where the additional rule of inference C16 is as follows:

C16. $\neg\alpha, \neg\beta \vdash \neg(\alpha \vee \beta)$

Oppy's mistake seems to be a good example of an informal analysis which gives rise to a formal reconstruction that does not meet the similarity criterion and, to the extent that the misrepresentation of premise (iv) gives rise to a fake response to one of Oppy's objections to Leibniz's solution, does not meet with the non-troublemaker criterion.

6 Kant

In his *Critique of Pure Reason* (1787, 2nd edition), Immanuel Kant presents three objections against what he calls "the ontological argument", which is, grossly speaking, the cartesian argument we discussed above. The first two critiques are general ones, addressed not to specific formulations of the ontological argument, but to any *a priori* argument for the existence of God. They follow the same general idea put forward by David Hume in his *Dialogues concerning Natural Religion* (1779):

> I shall begin with observing that there is an evident absurdity in pretending to demonstrate a matter of fact, or to prove it by any arguments a priori. Nothing is demonstrable unless the contrary implies a contradiction. Nothing that is distinctly conceivable implies a contradiction. Whatever we conceive as existent, we can also conceive as non-existent. There is no being, therefore, whose non-existence implies a contradiction. Consequently there is no being whose existence is demonstrable. I propose this argument as entirely decisive, and am willing to rest the whole controversy upon it.[28]

Otherwise said, since no existence claim is contradictory, for its negation is always possible, there cannot be any *a priori* proof for the existence of God or any other matter of fact.

Kant in some sense elaborates on this idea, now making use of the distinction between analytic and synthetic claims, the counterparts of Hume's relations of ideas and matters of fact, respectively. He follows Hume in maintaining that synthetic propositions can never be proved *a priori*; this is a prerogative of analytic propositions. Since existential claims are synthetic, he adds, it follows that no ontological proof of the existence of God is possible:

> If we admit, as every reasonable person must, that all existential propositions are synthetic, how can we profess to maintain that the predicate of existence cannot be rejected without contradiction? This is a feature which is found only in analytic propositions, and is precisely what constitutes their analytic character.[29]

But independently of the analytic-synthetic distinction, and this is Kant's second criticism, it is not difficult to see that negative existentials can never be contradictory. If we deny, say, that God is omnipotent, then we arrive at a contradiction, for we suppose that the property of omnipotence belongs to the very concept of an infinite being. But this is very different from denying that God exists; at this time, the instantiation in reality of the whole concept of God, with all its attributes, is denied, and this implies no contradiction whatsoever:

> To posit a triangle, and yet to reject its three angles, is self-contradictory, but there is no contradiction in rejecting the triangle together with its three angles. The same holds true of the concept of an absolutely necessary being. If its existence is rejected, we reject the thing itself with all

[28] Translation by H. Aiken [15, p. 58].

[29] Translation by N. Kemp-Smith [16, A598B626].

> its predicates; and no question of contradiction can then arise. There is nothing outside it that would then be contradicted, since the necessity of the thing is not supposed to be derived from anything external; nor is there anything internal that would be contradicted, since in rejecting the thing itself we have at the same time rejected all its internal properties. 'God is omnipotent' is a necessary judgement. The omnipotence cannot be rejected if we posit a Deity, that is, an infinite being; for the two concepts are identical. But if we say, 'There is no God', neither the omnipotence nor any other of its predicates is given; they are one and all rejected together with the subject; and there is therefore not the least contradiction in such a judgement.[30]

Not considering other issues such as the tenability of the analytic-synthetic distinction, these two objections have a very serious flaw: they completely overlook the ontological argument itself, or, in other words, what seems to be the very counter-example to the thesis (present in both objections) that no *a priori* proof of God is possible. Only in his third objection, which is by far the most famous, is that Kant addresses directly the (Cartesian) ontological argument:

> 'Being' is obviously not a real predicate; that is, it is not a concept of something which could be added to the concept of a thing. It is merely the positing of a thing, or of certain determinations, as existing in themselves. Logically, it is merely the copula of a judgement. The proposition 'God is omnipotent' contains two concepts, each of which has its object — God and omnipotence. The small word 'is' adds no new predicate, but only serves to posit the predicate in its relation to the subject. If, now, we take the subject (God) with all its predicates (among which is omnipotence), and say 'God is' or 'There is a God', we attach no new predicate to the concept of God, but only posit the subject itself with all its predicates, and indeed, posit it as an object that stands in relation to my concept. The content of both must be one and the same; nothing can have been added to the concept, which expresses merely what is possible, by my thinking its object (through the expression 'it is') as given absolutely. [. . .] By whatever and however many predicates we may think a thing — even if we completely determine it — we do not make the least addition to the thing when we further declare that this thing is. Otherwise, it would not be exactly the same thing that exists, but something more than we thought in the concept; and we could not,

[30] Translation by N. Kemp-Smith [16, A595B623].

> therefore, say that the exact object of my concept exists. If we think in a thing every feature of reality except one, the missing reality is not added by my saying that this defective thing exists.[31]

This is of course the famous existence-is-not-a-predicate criticism against the ontological argument. It is easy to see how it threatens Descartes' argument: if existence is not an authentic predicate, premise

(ii) Existence is a perfection.

is false, for in order for something to be a perfection it must be a predicate. By implication, it also threatens Sobel's formalization and my amendments of it.

The same however cannot be uncontroversially said about Anselm's formulation. Gareth Matthews, for example, has written as follows:

> He does [Anselm] not speak of adding the concept of existence, or even the concept of existence in reality, to the concept of God, or the concept of something than which nothing greater can be thought. What he does instead is to ask us to compare something existing merely in the understanding with something existing in reality as well. And the second, he says, is greater.[32]

Indeed, the key premise of Anselm's argument — premise (x) — and correlated doctrine (G) neither speak nor presuppose that existence is a property or perfection. Instead, they just make the comparative claim that it is greater to exist in reality than to exist merely in the understanding. So, it is not at all clear that Kant's criticism threatens Anselm's formulation.[33]

Oddly enough, it does threaten Adams' formulation of Anselm's argument. Since Adams represents the concepts of existence in reality and existence in the understanding with the help of logical predicates, *his formulation* naturally assumes that existence is a predicate, which implies the odd fact that while Anselm's formulation is at least defensible against Kant's critique, Adam's formalization of it is not. This is again a clear instance of a troublemaker *explicatum*.

It is important to keep in mind that the choice of representing the two existence concepts as logical predicates is exactly this: a technical choice. Many formalizations of Anselm's argument represent at least one of the concepts with the help of the existential quantifier. And in fact, it is not difficult to conceive an alternative version

[31] Translation by N. Kemp-Smith [16, A598B626-A600B628].

[32] [23, p. 90].

[33] For the same reason, Oppy's objection that one has to show that existence is a positive, simple and absolute property does not uncontroversially apply to Anselm's argument.

of Adam's formalization which represents none of the two existence concepts as properties. In order to illustrate this point, let me give a rough and somehow naïve sketch of what this version would look like.

First, we have to build an expanded first-order logic with two existential quantifiers, say, $\exists$ and $\reflectbox{$\exists$}$.[34] While $\exists$ is a broad quantifier ranging over a large domain $\boldsymbol{D}$, $\reflectbox{$\exists$}$ is a more restricted one ranging over domain $\boldsymbol{D'} \subseteq \boldsymbol{D}$. As far as Anselm's argument is concerned, $\boldsymbol{D}$ contains all objects, be them located in reality or in the understanding (it does not matter here who's understanding); $\boldsymbol{D'}$ contains only objects located in reality. Therefore, while $\exists x \boldsymbol{P}(x)$ means that x exists in reality or in the understanding and has property $\boldsymbol{P}, \reflectbox{$\exists$} x \boldsymbol{P}(x)$ means that x exists in reality and has property $\boldsymbol{P}$. Given this, we have two abbreviations:

$$\phi(x, m) =_{\mathrm{def}} Q(m, x) \land \neg\Diamond\exists y \exists n(G(n, m) \land Q(n, y))$$
$$\varepsilon(x) =_{\mathrm{def}} \reflectbox{$\exists$} y(y = x)$$

ϕ is the same as Adam's abbreviation. $\varepsilon(x)$ means that x exists in reality. The premises and conclusion are then represented as follows:

(I) $\exists x \exists m(\phi(x, m))$

(II) $\forall x \forall m(\phi(x, m) \to \Diamond\varepsilon(x)$

(III) $\forall x \forall m(\phi(x, m) \land \neg\varepsilon(x) \to \neg\Diamond\neg(\varepsilon(x) \to \exists n(G(n, m) \land Q(n, x)))$

(IV) $\reflectbox{$\exists$} x \exists m(\phi(x, m))$

7 Conclusion

The first aim of this paper was to provide a humble and somewhat historical introduction to the ontological argument. This was done by presenting the contributions of Anselm, Gaunilo, Descartes, Leibniz and Kant. Its second aim was to critically examine the enterprise of formally analyzing philosophical arguments and contribute in a small degree to the debate on the role of formalization in philosophy. For this purpose, I scrutinized Adam's formalization of Anselm's ontological argument and his formalization of Gaunilo's criticism against it, Sobel's formalization of Descartes'

[34] As far as I am concerned, I could not find any published formalization of such kind of logic. I however assume that building such a kind of logical system would not be a big issue. I base this assumption on two facts: the great diversity of existing multi-modal logics (that is to say, modal logics with more than one pair of modal operators) and the well-known equivalence between first-order logic with one variable and (mono) modal logic.

ontological argument and Oppy's analysis of Leibniz's proof for the possibility of God.

As I anticipated in the Introduction, my focus was mainly on the drawbacks and limitations of these approaches as attempts to analyze existing philosophical arguments. I have not however tried to put formal philosophy on trial. Au contraire, the modest critical analysis I have made in the previous sections and the synthesis I shall lay down here are meant to uncover some (quite trivial, one might say) dangers facing the enterprise of formally analyzing philosophical arguments. Besides, I have punctually called attention to the advantages of formalization (as in the disambiguation of (G) in Anselm's argument) and tried to amend (or show the possibility of amending) the approaches I have analyzed.

In one sense, all my criticisms had to do with Carnap's similarity criterion and my non-troublemaker criterion. For instance, although Adams' informal analysis of Anselm's argument incorporates many important elements of the original formulation, it unjustifiably ignores that Anselm did give an argument for premise (viii), not thus satisfying the similarity criterion. Sobel also slips into that in his informal analysis of Descartes' argument: he neglects Descartes' assumption that there must be a connection between the property of existence and existence itself. As we have seen, this had the consequence of turning Descartes' valid argument into an invalid one, which would classify his formal reconstruction as a troublemaker one. A similar problem occurs with Oppy's analysis of Leibniz's argument. As we have seen, besides being unfaithful to Leibniz's argument, Oppy's misrepresentation of premise (iv) gives rise to a fake response to one of Oppy's own objections to Leibniz's solution.

For the formal representation of arguments, here we also find violations of the similarity and non-troublemaker criteria. For instance, Adams equates "it can be thought that" with "it is possible that", representing premise (x) with the help of a possibility modal operator for which we find no hint in Anselm's original formulation. Furthermore, he does not provide any kind of justification for this theoretical decision of his. As I said, the *explicatum* is expected to depart in several aspects from the *explicadum*; however, for a big departure like this one is expected to provide strong philosophical arguments. Another troubling and in fact troublemaker aspect of Adam's formalization is his use of two predicates to represent the concepts of existence in reality and existence in the understanding. By introducing another element which was absent from Anselm's original formulation, he made the argument vulnerable to a criticism which the original formulation was at least defensible against.

As far as the similarity criterion is concerned, the reconstruction of the inferential steps of the argument seems to be the stage of the formal analysis of an existing argument which involves most foreign elements. The word "reconstruction" applies perhaps even more strongly here than in the other two stages. For instance, despite of correctly representing two structural features of Anselm's argument (the reductio ad absurdum strategy and the movement back and forth from a universal discourse to talk about particulars), Adams' derivation is mostly formed by elements trivially not found in Anselm's text. The same can be said about my formal analysis of the derivations of Descartes' and Leibniz's arguments.

The lessons here seem to be clear. From the informal analysis to the formal representation of the inferential steps, everything is but a reconstruction of the original argument. As such, in some degree or other they will depart from the original formulation. But if the departure is too much — this is the first lesson — it is hard to see how the formal argument at hand might be taken as a formalization of the original argument. As a consequence of this, there will be hardly any hope of effectively contributing to the philosophical debate involving the argument: unless it is uncontroversial that the formalization is a reconstruction of the argument at hand, it is very hard to see how it will shed noteworthy light on the issues involving the argument. The similarity criterion should therefore be seen as an indispensable desideratum of any formal reconstruction.

The second lesson is related to the non-troublemaker criterion. What is the point of formally reconstructing an argument if the reconstruction produces confusing questions and unfruitful issues which are not found in the original formulation of the argument or obscure important aspects of it? Besides being a technical contribution, a formal analysis of an argument must also be a philosophical contribution to the debate over the argument in question; this is my version of Carnap's fruitfulness criterion. Its purposes should be to clarify, to shed some light on the philosophical issues involving the argument, and not to introduce new, and we might say, artificial problems.

The only way to avoid these two complications — this is the third and last lesson — is that the reconstruction be itself a philosophical endeavor. As soon as one starts a formal reconstruction of an existing argument, many theoretical choices will be made. But they should be rationally, philosophically justified. Formal philosophy must still be philosophy. *Ad hoc* and unjustified theoretical decisions should be reduced to the minimum, and when they are unavoidable, attention should be called upon them; a formal reconstruction should be aware of its own limitations. Only if this is done can the formal analysis be a real philosophical contribution. Then perhaps we might have hopes to revitalize formal philosophy and increase its interaction with non-formal philosophy.

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