Andrew Aberdein & Matthew Inglis

Advances in Experimental Philosophy of Logic and Mathematics

Introduction

There has been very little overt discussion of the experimental philosophy of logic or mathematics. So it may be tempting to assume that application of the methods of experimental philosophy to logic or the philosophy of mathematics is impractical or unavailing. That this would be a mistake is exhibited by at least three trends in recent research: a renewed interest in historical antecedents of experimental philosophy in philosophical logic; a 'practice turn' in the philosophies of mathematics and logic; and philosophical interest in a substantial body of work in adjacent disciplines, such as the psychology of reasoning and mathematics education. Before turning to the specific contribution that we hope this book will make, we will offer a snapshot of each trend and address how they intersect with some of the standard criticisms of experimental philosophy. Firstly, although experimental philosophy is often thought of as a twenty-first-century phenomenon primarily focussed on questions in ethics and epistemology, it has some important anticipations in earlier projects in the philosophy of logic. The most significant is the work of Arne Naess and the Oslo Group (Naess, 1938, 1959, 1982; Tönnessen, 1951). For instance, Ingemund Gullvåg argued that to understand the meaning of a word such as 'truth', it was 'hardly sufficient that a single person registers his own reactions to this or that sentence, or makes pronouncements based on intuitions' (Gullvåg, 1955, 343). Instead, the Oslo group argued, systematic empirical investigations were required. The connections between the ‘empirical semantics’ developed by the Oslo group and experimental philosophy have now begun to be made explicit by historians of philosophy and further developed by a new generation of researchers (Murphy, 2014; Barnard and Ulatowski, 2016; Chapman, 2018). This productive connection between the empirical methods of two different generations is continued in Barnard and Ulatowski’s chapter in the present volume, discussed in greater detail below.

Secondly, in recent decades there has been a ‘practice turn’ in the philosophy of mathematics, focussing on how mathematical research is actually conducted, rather than on the search for foundations for mathematics (Van Kerkhove and Van Bendegem, 2007; Mancosu, 2008). This has naturally led to an interest in empirical data about mathematical practice, a programme dubbed ‘Empirical Philosophy of Mathematics’ by some of its practitioners (Buldt et al., 2008; Löwe et al., 2010; Pantsar, 2015). There are several distinct axes along which the connections between the philosophy of mathematical practice and empirical work have been drawn. A significant body of work applies cognitive science research on mathematical reasoning to philosophical questions (Pease et al., 2013). This includes work on the status of mathematical knowledge (Cappelletti and Giardino, 2007; Pantsar, 2014); on the symbol systems of mathematics (De Cruz and De Smedt, 2013; Dutilh Novaes, 2013; Marghetis and Núñez, 2013); and on the role of diagrams and visualization in mathematics (Giaquinto, 2007; Hamami and Mumma, 2013). Moreover, modern mathematicians increasingly employ online tools for collaboration. This produces a considerable amount of potential data for researchers interested in mathematical practice, giving rise to another strategy for the investigation of that practice (Martin and Pease, 2013; Martin, 2015; Pease et al., 2017). Although logical practice may have received less attention than its mathematical counterpart, some researchers in the philosophy of logic have pursued a practice turn of their own, modelled on that in the philosophy of mathematics (Dutilh Novaes, 2012). As with its sister programme in philosophy of mathematics, advocates of the philosophy of logical practice stress that too much attention has been paid to foundational issues at the expense of philosophical questions that arise elsewhere, such as in the application of logic to artificial intelligence, game theory, linguistics, and other disciplines. For example, the burgeoning research programme of 'argumentation mining', which applies corpus-based techniques to extract and analyse arguments across large bodies of text, may be seen as a logical counterpart to the use of big data techniques in analysis of mathematical practice (Moens, 2018).

Thirdly, there is a growing awareness of how much research in adjacent disciplines has anticipated the research questions of experimental philosophy of logic and mathematics. Philosophers of logic have an extensive body of research on the psychology of reasoning to draw upon (Johnson-Laird, 2006). Lately, some work in the intersection of philosophy of logic and psychology of reasoning has made the relationship to experimental philosophy explicit (Pfeifer, 2012; Pfeifer and Douven, 2014; Ripley, 2016). There is also a substantial research tradition in mathematics education that addresses questions of immediate relevance to the philosophy of mathematical practice (Heinze, 2010; Weber et al., 2014; Weber and Mejía-

Ramos, 2015; Alcock et al., 2016). Understanding mathematical practice is important for education researchers for at least two reasons. First, understanding the behaviour of expert mathematicians helps to decide what the purpose of a mathematics curriculum should be. If a particular activity is highly valued in expert mathematical practice then this perhaps provides a reason for mathematics students to be exposed to some appropriate version of it (see, for example, Ball and Bass, 2000; Harel and Sowder, 2007; Lampert, 1990; Weber et al., 2014). Second, studying the in-the-moment strategies adopted by expert practitioners (in any domain) might provide suggestions for how to develop interventions that assist learners to develop expertise. An example of this approach can be found in the work of Alcock, Hodds, Roy and Inglis (2015). They studied the reading behaviour of research mathematicians, and used these insights to develop training materials that encouraged undergraduates to adopt similar strategies. These training materials significantly increased the amount students learned from reading a mathematical text.

In addition, there is now an emerging tradition of interdisciplinary work, applying quantitative techniques to address traditionally philosophical questions, such as mathematical aesthetics (Inglis and Aberdein, 2015, 2016). Some of this work has been presented as an enquiry into ‘mathematical cultures’ (Löwe, 2016; Larvor, 2016). Likewise, Reuben Hersh, one of the forerunners of the practice turn, has lately called for ‘a unified, distinct scholarly activity of mathematics studies: the study of mathematical activity and behavior’ (Hersh, 2017, 335). We regard the present volume as, in part, a contribution to the integrative work required for this project.

The advent of experimental philosophy has not been without controversy and has provoked a salutary debate on the proper methods of philosophical enquiry. One of the most prominent critiques is the ‘expertise defence’ of traditional philosophical practice (Nado, 2014; Mizrahi, 2015). This maintains that surveys of non-philosophers have limited bearing on the arguments of philosophers since, as experts, philosophers can be expected to be immune from the errors and biases exhibited by non-experts. This debate has given rise to a substantial literature. However, the experimental philosophies of mathematics and logic seem to have ready responses to the expertise defence. Many studies of mathematical practice focus on professional mathematicians, placing the expertise of the participants essentially beyond dispute. Nonetheless, this is not universally true; for instance, some philosophers (for example, De Cruz, 2016) have used results from the numerical cognition literature to draw conclusions about the ontology of natural numbers. Participants in numerical cognition studies include non-mathematical adults, children and even non-human animals.

An important difference between mainstream experimental philosophy and work focused on mathematics is that studies in the latter tradition typically ask their participants—be they mathematicians, children or animals—about mathematics, not about philosophy. (This is just as well, for in Hersh’s famous formulation, ‘the typical working mathematician is a Platonist on weekdays and a formalist on Sundays’ (Hersh, 1979, 32). Such insouciance would not bode well for the resolution of philosophical dilemmas.) In this respect experimental philosophy of mathematics is similar to psychological work on reasoning relevant to debates in the philosophy of logic. Here too participants are typically drawn from a more general population. (Although there clearly is such a thing as logical expertise; for a start, people can be trained to be better at logical reasoning (Attridge et al., 2016).) Just as mathematicians/children/fish are asked about mathematics not philosophy, participants in reasoning studies are asked object-level questions about everyday reasoning, not specialised questions about logical hypotheses that might predict or explain such reasoning. On this basis David Ripley has argued that these studies are better placed to answer the expertise objection than studies relevant to debates in ethics or epistemology (Ripley, 2016).

Nonetheless, there is a substantial body of psychological research that reveals a divergence between best practice in reasoning (at least, as defined by logicians) and how lay people actually reason. There is also a substantial body of work critiquing these results. Broadly speaking, they lend themselves to four possible responses:

1. Lay people are to blame: they routinely make damaging errors in their inferential practices;
2. Psychologists are to blame: they fail to understand the relationship between formal and informal reasoning, and thereby design experiments which show only that good reasoners can be hoodwinked by artificial examples;
3. Logicians are to blame: they persist in defending systems of formal inference which do not describe the legitimate inferential practices of ordinary folk;

4. No one is to blame: logicians might well be right that formal logic is the best way to reason, but in many (perhaps most) real world circumstances it takes too much cognitive effort to do so.

Many such studies, especially in early psychology of reasoning work, are presented as supporting the first response. However, they can often be reinterpreted in support of one of the others. In particular, much research of this sort is implicitly deductivist (and often classicist): it presumes that the best account of human inference will always be deductive logic (and often that classical logic is the best or only viable system of deductive logic). Hence such work is undermined by the successful modelling of informal, non-deductive patterns of inference in argumentation theory (Zenker, 2018) or non-classical logics (Aberdein and Read, 2009). The moral may be that, as with many sciences, theoretical and empirical approaches should be mutually reinforcing: logicians need the empirical research conducted by psychologists of reasoning to corroborate their claim of faithfulness to actual reasoning; psychology of reasoning needs to be informed by current research in logic if it is to stay relevant.

Recent research suggests that even preverbal children can exhibit behaviour consistent with logical reasoning (Cesana-Arlotti et al., 2018). Children as young as twelve months were presented with stimuli either complying with or violating simple inferential rules, such as disjunctive syllogism, $p \lor q, \neg p \vdash q$. That they looked longer at violating cases than they did at stimuli consistent with those rules, just as adults do, suggests that they found those cases incongruous. This provides an echo of a far older debate.

The ancient logician Chrysippus argued that dogs employ disjunctive syllogism, since a scent hound, tracking a quarry to a crossroads and eliminating all but one of the exits, will (or so Chrysippus claims) immediately take the last exit without further checks of the trail. The story has been retold many times, with at least four different morals:

1. dogs use logic, so they are as clever as humans;
2. dogs use logic, so using logic is nothing special;
3. dogs reason well enough without logic;
4. dogs reason better for not having logic (for details, see Aberdein, 2008).

The third option may be closest to Chrysippus's own; it may also be the best take on the empirical research. That is, such studies do not attribute conscious, reflective awareness of any system of logic to dogs (or infants). Rather, they demonstrate that logic succeeds in tracking the pre-theoretical reasoning not just of the logically educated, but of pretty much anyone capable of rational thought.

It is sometimes argued that logic, as an a priori discipline, is immune from revisionary pressures that apply to natural science. If this is so, then there may be little room for empirical research in logic. On the other hand, there is a tradition, associated with W. V. O. Quine in particular, of treating logic as continuous with the natural sciences (Bryant, 2017). In recent years, this debate has been characterized in terms of 'anti-exceptionalism' about logic (Hjortland, 2017; Read, 2018). However, we do not need to resolve the debate in order to observe that it is less damaging to our concerns than it may first appear. Even if one concedes that the truths of logic are analytic and necessary—that is, true in virtue of their meaning and such that they could not have been different—our knowledge of these truths is still fallible. So we may expect the methods whereby we come to know these truths to have much in common with the methods whereby we learn truths in the natural sciences, even though the truths of those disciplines are neither analytic nor necessary.

This collection is intended to consolidate and develop the three trends identified above: the reappraisal of the Oslo Group; the practice turn in the philosophies of logic and mathematics; and the reintegration into these philosophies of empirical work from adjacent disciplines. The ten chapters are divided equally between the philosophies of mathematics and logic. Their authors include some of the leading figures in each of the areas of research discussed above. Several chapters are methodological analyses of the applicability of empirical techniques to these areas of philosophy, but many (also) include actual empirical results. They demonstrate a wide variety of different empirical methods, including experiments, surveys, and data-mining.

Benedikt Löwe and Bart Van Kerkhove's chapter, 'Methodological triangulation in empirical philosophy of mathematics', is written by two of the leading figures in the philosophy of mathematical practice. They survey the uses that have been found for a variety of different empirical methods in philosophy, emphasising that the experimental method in the strict sense is only one of them. They argue for methodological triangulation in empirical philosophy, that is, the employment of a battery of different empirical methods to compensate for the biases and limitations implicit in any one of them. Their paper provides a helpful introduction to the potential that empirical methods offer for the philosopher of mathematics (or logic). In particular, they rehearse a sartorial analogy that Löwe has proposed elsewhere for the different levels of integration between philosophy and empirical methods (Löwe, 2016, 36). He distinguishes 'ready-to-wear', the philosophical exploitation of existing, independently conducted empirical research, from 'bespoke', which involves more direct collaboration, such as philosophers designing projects to be conducted by empirical researchers, and 'do-it-yourself' (homespun?), in which the philosopher conducts all aspects of the research. (We might add that such cross-disciplinary work can cut both ways: empirical researchers can develop the interest and expertise necessary to address philosophical questions. Indeed, some philosophical questions, including many posed by the philosophies of mathematical and logical practice, are already within the remit of nearby empirical disciplines.) It is important to stress, as Löwe and Van Kerkhove do, that this is not a hierarchy of quality. If you are lucky enough to find off-the-peg clothes that are a good fit, they may be much better value than bespoke. And making your own clothes is unlikely to have a good outcome unless you acquire significant expertise. Likewise, when existing empirical studies address the right questions, ready-to-wear studies can be highly effective. The remaining chapters in this collection report on studies of all three varieties.

Helen De Cruz's work in the philosophy of mathematics has long made use of empirical results (De Cruz, 2006, 2016). Her chapter, 'Animal cognition, species invariantism and mathematical realism', is a notable piece of ready-to-wear empirical philosophy. She uses a variety of results from numerical cognition (especially neurological and animal work) to tackle a recently influential argument in the philosophy of mathematics. The 'evolutionary debunking' argument against moral realism suggests that our moral beliefs cannot be objectively true if they are the result of a highly contingent evolutionary process. If we had evolved from animals with very different social behaviour (and there are many such species) then we would have a quite different set of moral intuitions, so why imagine that those intuitions track the truth? Mathematical realism, the view that our mathematical beliefs are objectively true, has obvious similarities to moral realism. So might there not be an analogous evolutionary debunking argument against mathematical realism too? However, De Cruz demonstrates that there is significant evidence that the mathematical behaviour of animals is substantially convergent, which suggests that the analogy fails; if anything the empirical data provide support for mathematical realism.

The next chapter, 'The beauty (?) of mathematical proofs', also makes extensive use of existing empirical research. We have already noted Catarina Dutilh Novaes's research on logical practice; besides the history and philosophy of logic she also works on social epistemology and the philosophy of mathematics, as in this chapter. She coordinates a number of disparate literatures to propose a novel approach to the aesthetics of mathematical proof grounded in empirical work on affective responses to unexpectedness. The key idea is that in many situations mathematical judgments bring together epistemic and aesthetic components, and that we should not be surprised by this.

The next two chapters are drawn from the bespoke tradition: they report on original studies that were conducted by the authors to address (at least) philosophical questions. Both chapters look at aspects of visual reasoning in mathematics. This has been a controversial subject: an influential view maintains that visuals should play no role in mathematical proof, but a growing body of work suggests that this is an unrealistic, indeed harmful, idealization (Larvor, 2013, 2018). Josephine Relaford-Doyle and Rafael Núñez are both cognitive scientists—the latter is a co-author of a landmark in the application of cognitive science to mathematics (Lakoff and Núñez, 2000). Their chapter, 'Can a picture prove a theorem? Using empirical methods to investigate visual proofs by induction', reports an empirical study that investigates how undergraduate students with and without formal mathematical training use images to justify mathematical claims. They focus on visual induction proofs, and find results that challenge James Brown's (non-empirical) claim that such proofs are immediately understandable for people without mathematical training (Brown, 1997).

Keith Weber and Juan Pablo Mejía-Ramos are mathematics educators. In their chapter, 'An empirical study on the admissibility of graphical inferences in mathematical proofs', they investigate the admissibility of graphical inferences in proofs in real analysis. They conclude that the type of graphical inference is important to consider when addressing their question. In particular, Weber and Mejía-Ramos find support for the importance of distinguishing between metrical and non-metrical graphical inferences

(Larvor, 2018). A metrical graphical inference is one that depends for its success on the measurements of angles, lengths, and so on, being precisely correct, whereas a non-metrical graphical inference does not; that is, the latter sort of inference is unaffected by local deformations in the diagrams at issue.

Where the first five chapters focus primarily on the philosophy of mathematics, the remaining five concentrate on the philosophy of logic. In their chapter, 'Does anyone really think that ⟨⌜*p*⌝ is true if and only if *p*⟩?', the philosophers Robert Barnard and Joseph Ulatowski link together Arne Naess's early empirical work, their own recent replications of some of these results, and the contemporary debate on deflationary accounts of truth. As well as noting this chapter's contribution to philosophical theory, given the ongoing replication crisis in psychology (Chambers, 2017), it is worth explicitly remarking upon and celebrating Barnard and Ulatowski's successful replication of Naess's early findings.

Igor Douven's research lies at the intersection of several fields, including formal epistemology and cognitive science. His chapter, 'New foundations for fuzzy set theory', seeks to rehabilitate fuzzy set theory as an account of vagueness by grounding it in recent empirical work on conceptual spaces. Conceptual spaces were developed by the cognitive scientist Peter Gärdenfors as a geometrical framework for the qualitative comparison of concepts along multiple dimensions (Gärdenfors, 2000). Douven argues that seeing fuzzy membership as the distance of a point from a prototypical point in such a space is a productive approach to fuzzy set theory and, moreover, that there is empirical support for adopting this view from work conducted by cognitive psychologists.

The philosopher Moti Mizrahi's chapter, 'What isn't obvious about "obvious": A big data approach to philosophy of logic', uses a corpus linguistics approach to investigate the obviousness or otherwise of logic. Mizrahi reasons that if logic really was obvious, then the frequency with which logicians use the word 'obvious' should correlate with deductive indicator words such as 'necessary' and 'certainly' but not with inductive indicator words such as 'probably' or 'likely'. By analysing a large corpus of text drawn from research papers published in logic, philosophy, mathematics and biology journals, Mizrahi empirically tests these predictions. While he finds some support for the predictions, he also finds some results that require further explanation.

David Over and Nicole Cruz are both psychologists of reasoning. Their chapter, 'Philosophy and the psychology of conditional reasoning', is a wide-ranging discussion of recent empirical work on conditional statements and its relationship to philosophy. The authors cite an extensive array of empirical studies to argue in favour of a Bayesian account of conditionals and against a mental model account. They conclude that much psychological research on conditionals has paid too little attention to philosophical and logical work. Remedying that oversight has already led to improved empirical studies, and promises to go further.

In their chapter, 'Folk judgments about conditional excluded middle', the philosophers Michael J. Shaffer and James Beebe employ empirical studies to motivate a novel analysis of so-called Bizet/Verdi conditionals:

- If Bizet and Verdi had been compatriots, Bizet would have been Italian.
- If Bizet and Verdi had been compatriots, Verdi would have been French.

Across three experiments Shaffer and Beebe find evidence for Alchourrón et al.'s (1985) 'belief revision' theory of counterfactuals, in line with the tradition of the Ramsey Test. Interestingly, they reject the alternative accounts from Lewis (1973) and Stalnaker (1981) by coordinating analyses from both quantitative and qualitative data.

The experimental philosophy of logic and mathematics has been quietly thriving for some time. We hope that this collection will form an indispensable resource for future research in the field.

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