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4-String Junction and Its Network

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ABSTRACT

We study a BPS configuration in which four strings (of different type) meet at a point in $N = 2, D = 8$ supergravity, i.e., the low energy effective theory of T^2-compactified type II string theory. We demonstrate that the charge conservation of the four strings implies the vanishing of the net force (due to the tensions of various strings) at the junction and vice versa, using the tension formula for $SL(3, Z)$ strings obtained recently by the present authors. We then show that a general 4-string junction preserves 1/8 of the spacetime supersymmetries. Using 4-string junctions as building blocks, we construct a string network which also preserves 1/8 of the spacetime supersymmetries.

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Type IIB string theory admits a stable BPS configuration in which three strings of different type meet at a point, i.e., a 3-string junction [1,2,3,4]. The existence of such a 3-string junction is due to the D-string picture [5] and the special properties of electrodynamics in two dimensions as well as the strong-weak U-duality $SL(2,Z)$ symmetry. D-string picture allows a F-string, i.e., (1,0)-string, ending on a D-string, i.e., (0,1)-string. The consistent coupling at the endpoint guarantees the charge conservation [6]. The charge at the endpoint of the F-string appears as a point source to the U(1) field on the D-string worldsheet. The Gauss law in one spatial dimension implies that the field strength F_{01} can be chosen as the constant charge flux on one side of the point charge along the D-string and zero on the other side [4]. This can also be understood from the picture that the other end of the F-string may be thought to end on the D-string at infinity, implying that the charge flux goes along the D-string only from the positive charge endpoint to the negative charge endpoint of the F-string at infinity. Due to this, one side of the original D-string can no longer be a D-string but a (1,1)-string (or (−1,−1)-string depending on the orientation) and the charge at the endpoint of F-string causes one half of the D-string to bend rigidly rather than to be spike-like as in the case for D p-brane for $p > 2$ [7], therefore giving rise to a stable 3-string junction. The U-duality symmetry $SL(2,Z)$ guarantees the existence of a general (p,q)-string with coprime integers p and q. Otherwise, the 3-string junction would not be possible. As an interesting application the 3-string junction has been used in [8] to construct the 1/4 BPS states in $D = 4$, $N = 4$ supersymmetric Yang-Mills theory. Also, the dynamics of 3-string junction has been studied in [9,10] and the M-theoretic origin of such junctions has been pointed out in [1,11,12].

The T^2-compactified Type II string theory admits a general stable BPS (q_1,q_2,q_3)-string configuration with any two of the three integers q_1, q_2 and q_3 being coprime [13]. All possible (q_1,q_2,q_3) strings fill in the various multiplets of the strong-weak symmetry $SL(3,Z)$ of the complete U-duality symmetry $SL(3,Z) \times SL(2,Z)$ in this theory (the strings are inert under the T-dual symmetry $SL(2,Z)$) and each of the strings breaks half of the spacetime supersymmetries. The three integral charges q_i are associated with the corresponding three 2-form potentials $B^{(i)}_{\mu\nu}$ $(i = 1,2,3)$ which form a triplet of the $SL(3,Z)$ in this theory. We therefore expect that we have three kinds of strings. In particular, the $(1,0,0)$-string is the T^2-compactified Type IIB F-string and the $(0,1,0)$-string is the T^2-compactified D-string while the $(0,0,1)$-string is the wrapped D 3-brane. If we draw the analogy between what we have here and what has been discussed in the previous paragraph for the 3-string junction, one natural conclusion is that we should have a 4-string junction in $D = 8$. The only thing which we have not explained well is how the $(1,0,0)$ and $(0,1,0)$ strings couple consistently with the $(0,0,1)$-string so that one can have a 4-string junction. This can be explained if we trace the origin of these strings back to $D = 10$. In other words, in order to have a stable BPS 4-string junction in $D = 8$, we should have a F-string and a D-string ending consistently on a D 3-brane in $D = 10$. This possibility certainly exists if one examines the effective D 3-brane worldvolume action [14,15] in which both the NSNS and RR 2-form potentials couple consistently with the U(1) gauge field on the worldvolume. Thanks to the recent work [16] where such a stable BPS configuration has been discovered explicitly and is shown to preserve 1/4 of the D 3-brane worldvolume supersymmetries. In general, a toroidal compactification on either supergravity theory or super p-brane action preserves the original supersymmetries. Loosely speaking, the 4-string junction can be viewed as the T^2-dimensionally reduced $D = 10$ BPS configuration of a F-string and a D-string ending on a D 3-brane as we will discuss in the following. This further convinces us that there should exist a 4-string junction in $D = 8$ which preserves 1/8 of the spacetime supersymmetries.

Let us explain further the 4-string junction from $D = 10$ picture. We start with a F-string and a D-string ending on a D 3-brane. To be specific, let us assume that the D 3-brane be along x^3, x^8 and x^9 directions, the F-string be along the x^1-direction with its endpoint on the D 3-brane at $x^3 = 0, x^8 = a, x^9 = 0$, and the D-string along the x^2-direction with its endpoint on the D 3-brane at $x^3 = 0, x^8 = 0, x^9 = b$ where a and b are two constants. If such a configuration is a stable BPS one, both the F-string and D-string are spike-like [16,7]. The charge at the F-string endpoint and the charge at the D-string endpoint appear to be electric and magnetic, respectively, to the U(1) field on the D 3-brane worldvolume.

Now let us compactify the coordinates x^8 and x^9 on a torus and insist that the 3-brane wrap on the torus. If we shrink the spacetime torus to zero, the 3-brane then becomes the (0,0,1)-string along x^3 in $D = 8$. At the same time, the F-string and D-string become (1,0,0)-string and (0,1,0)-string, respectively, in $D = 8$. Their endpoints must meet at $x^3 = 0$ ending on the (0,0,1)-string. The $D = 10$ type IIB NSNS and RR 2-form potentials are reduced to the $D = 8$ 2-form potentials $B_2^{(1)}$ and $B_2^{(2)}$, respectively. The U(1) gauge field on the D 3-brane worldvolume is reduced to the U(1) gauge field on the (0,0,1)-string worldsheet with the only field strength component F_{03}.

On reducing the D 3-brane worldvolume action to give the action describing the (0,0,1)-string in $D = 8$, we know that F_{03} couples to both $B_2^{(1)}$ and $B_2^{(2)}$. We also know that $B_2^{(1)}$ and $B_2^{(2)}$ determine the charges for (1,0,0)-string and (0,1,0)-string, respectively, in $D = 8$. So in $D = 8$, both the (1,0,0)-string and (0,1,0)-string endpoint charges provide point sources to the U(1) field strength F_{03} on the (0,0,1)-string worldsheet (the two charges are now located at the same point $x^3 = 0$)†.

With a similar discussion as we did above for the 3-string junction, we must conclude that the (0,0,1)-string cannot remain as it is when a (1,0,0)-string and a (0,1,0)-string end on the (0,0,1)-string at $x^3 = 0$. The two sides of the (0,0,1)-string around $x^3 = 0$ must bend rigidly according to the following two cases: 1) The $x^3 > 0$ (or $x^3 < 0$) side becomes (1,1,1)-string (or (-1,-1,-1)-string depending on the oritentation) while the other side remains as (0,0,1)-string if the two other ends of the (1,0,0) and (0,1,0) strings meet with the (0,0,1)-string at infinity on $x^3 > 0$ (or $x^3 < 0$) side. 2) The $x^3 > 0$ (or $x^3 < 0$) side becomes (1,0,1)-string and the other side becomes (0,1,1)-string if the other end of the (1,0,0)-string meets with the (0,0,1)-string at infinity on $x^3 > 0$ (or $x^3 < 0$) side while that of the (0,1,0)-string meets with the (0,0,1)-string at infinity on $x^3 < 0$ (or $x^3 > 0$) side. In each case, we have a 4-string junction.

The above discussion clearly indicates that there should exist a stable BPS 4-string junction in $D = 8$. However, at this point, the only thing that we are certain about is that the existence of various couplings indeed allows 4 strings in $D = 8$ to meet at one point. In other words, the charge conservation is not violated when 4 strings meet. Whether there actually exists a stable BPS 4-string junction requires us to show that the net-force due to string tensions at the junction should vanish and the junction should preserve some unbroken supersymmetries. We will show that the answers for both of these are positive.

If the four strings are of type $(q_1^{(i)}, q_2^{(i)}, q_3^{(i)})$ $(1 \leq i \leq 4)$, then the charge conservation [13,6,15,17] implies

$$\sum_{i=1}^{4} q_1^{(i)} = \sum_{i=1}^{4} q_2^{(i)} = \sum_{i=1}^{4} q_3^{(i)} = 0. \tag{1}$$

We recall from [13] that the tension for a (q_1, q_2, q_3)-string in string metric is

$$T_{(q_1,q_2,q_3)} = \sqrt{e^{-\phi_0+\sqrt{3}\varphi_0} q_3^2 + e^{-2\phi_0}(q_2 - \chi_{10}q_3)^2 + (q_1 - \chi_{30}q_2 - \chi_{20}q_3)^2}, \tag{2}$$

†In two dimensions, we do not have a distinction between electric and magnetic charges. But in the present case, we can still make a distinction between the two charges since their dependences on the string coupling are different when they act as sources to F_{03}.

where $\phi_0, \varphi_0, \chi_{10}, \chi_{20}$ and χ_{30} are the asymptotic values of the scalars parametrizing the coset $SL(3,R)/SO(3)$ in the $D = 8, N = 2$ supergravity. In particular, ϕ and χ_3 are the $D = 10$ type IIB dilaton and RR scalar, respectively. φ is another dilatonic scalar while χ_1 and χ_2 are axions, all of which are due to the T^2-compactification of type IIB supergravity. The above tension can be viewed as the magnitude of the following tension vector

$$\vec{T}_{(q_1,q_2,q_3)} = (q_1 - \chi_{30} q_2 - \chi_{20} q_3)\,\hat{e}_1 + e^{-\phi_0}(q_2 - \chi_{10} q_3)\,\hat{e}_2 + e^{-\phi_0/2+\sqrt{3}\varphi_0/2} q_3\,\hat{e}_3, \qquad (3)$$

where $\hat{e}_i$ $(i = 1,2,3)$ are the unit vectors along the x^i-directions, respectively. This corresponds to orienting the (q_1, q_2, q_3)-string along the direction of the tension vector. We use two simple examples to justify the above tension vector. First, the (1,0,0), (0,1,0) and (0,0,1) strings obtained from a F-string and a D-string ending on a D 3-brane by shrinking the spacetime torus to zero-size are perpendicular to each other (the axions are all set to zero). The above tension vector formula indeed shows this feature. Secondly, as discussed in [13], our tension formula reduces to the one for the type IIB (q_1, q_2)-string when φ, χ_1, χ_2 and q_3 are all set to zero. In this case, our above tension vector goes over to the one given by Sen [18] in discussing the 3-string junction network.

With the above tension vector, we have the zero net force automatically at the 4-string junction

$$\sum_{i=1}^{4} \vec{T}_{(q_1^{(i)},q_2^{(i)},q_3^{(i)})} = 0, \qquad (4)$$

provided the charge conservation (1) is satisfied. The converse is also true, i.e., the zero net force at the 4-string junction requires the charge conservation of eq.(1). This beautiful property further convinces us that the tension vector (3) is indeed correct.

We shall now show that a 4-string junction preserves 1/8 of the spacetime supersymmetries as expected. Before we do so, let us seek first the 1/2-unbroken supersymmetry condition for a stable BPS (1,0,0)-string configuration of the $D = 8, N = 2$ supergravity. This SUSY condition can be obtained from that of the F-string or (1,0)-string configuration in $D = 10$ type IIB supergravity. Let ϵ_L and ϵ_R be the two real Majorana-Weyl supersymmetry transformation parameters of type IIB string theory, associated with the left and the right moving sector of the worldsheet of the F-string. Assuming the F-string to be along x^1, we have the SUSY condition for the (1,0)-string configuration as[‡]

$$\epsilon_L + i\epsilon_R = \Gamma^2\Gamma^3\cdots\Gamma^9(\epsilon_L - i\epsilon_R), \tag{5}$$

where Γ^M $(M = 0, 1, \cdots 9)$ are the ten dimensional gamma matrices. The convenient representation for these matrices is the Majorana one in which Γ^0 is real and antisymmetric while the rest are real and symmetric. A concrete construction for the Γ^M is given in the appendix. In this representation, the above condition is equivalent to

$$\epsilon_L = \Gamma^2\Gamma^3\cdots\Gamma^9\epsilon_L, \qquad \epsilon_R = -\Gamma^2\Gamma^3\cdots\Gamma^9\epsilon_R. \tag{6}$$

If we are able to express the ϵ_L and ϵ_R in terms of the two pseudo Majorana supersymmetry transformation parameters $\epsilon^{(1)}$ and $\epsilon^{(2)}$ in $D = 8, N = 2$ supergravity and express Γ^M in terms of $D = 8$ gamma matrices γ^μ, then the above SUSY conditon is reduced to the one for the $(1, 0, 0)$-string configuration (this string is also along x^1). This has been done in detail in the appendix and the required SUSY condition is

$$\left(\begin{array}{c} \epsilon^{(1)} \\ \epsilon^{(2)} \end{array}\right) = \gamma^0\gamma^1 \otimes \sigma_1 \left(\begin{array}{c} \epsilon^{(1)} \\ \epsilon^{(2)} \end{array}\right), \tag{7}$$

where σ_1 is the 2×2 Pauli matrix.

As is well known in supergravity theories, the spinors are singlet for the non-compact Cremmer-Julia symmetry G but form certain representation of the local hidden symmetry H which is isomorphic to the maximal compact subgroup of G linearly realized on the physical states. In generating a general U-duality p-brane solution from a single charge solution as we did in [13], all we need to care about is the bosonic fields since the fermionic fields are always set to zero. Therefore, we do not need to use the "vielbein"-formalism for the scalar fields. In other words, we need to employ only the non-compact global Cremmer-Julia symmetry G to find the general U-duality p-brane solution. By this, we concluded in [13] that the general U-duality p-brane solution should preserve the same number of unbroken supersymmetries as that for the original single charge solution. Here the story is different. We no longer consider an isolated p-brane solution but a few strings meet at a point. Since the strings in a 4-string junction share the same $SL(3, R)/SO(3)$ moduli but different charges, they are physically inequivalent. Each of these strings can be obtained through a different global $SL(3, R)$ transformations. If we choose to work

[‡]Here and in the subsequent discussion, spinors in the corresponding SUSY condition all take their asymptotic values.

in the "vielbein"-formalism and in a unitary gauge, then an induced SU(2) ($\simeq SO(3)$) transformation will arise whenever a $SL(3,R)$ transformation is performed [19,20]. This induced SU(2) transformation will act on the pseudo Majorana spinors $\epsilon^{(1)}$ and $\epsilon^{(2)}$ which form a doublet of SU(2). So the SUSY condition for each of the 4 strings in a 4-string junction will be differnt. We will just make use of such differences to determine if any supersymmetry is preserved for a 4-string junction.

Since $\begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}$ is a doublet of SU(2), the SUSY condition (7) for the (1,0,0)-string configuration must be a special form of a general SU(2) covariant expression of a general (q_1, q_2, q_3)-string since they both preserve 1/2 of the spacetime supersymmetries and are related to each other by an induced SU(2) transformation. Examing the SUSY condition (7) for the (1,0,0)-string, we note that we already have an appearance of the SU(2) generator σ_1. Our hunch for the covariant SUSY condition for a general (q_1, q_2, q_3)-string along the x^1 axis is[§]

$$\begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} = \gamma^0 \gamma^1 \otimes \frac{V}{|V|} \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}, \tag{8}$$

where $V = V_a \sigma_a$ with V_a a SO(3) vector and σ_a $(a = 1,2,3)$ the three Pauli matrices. $|V|$ denotes the magnitude of V. Note that V_a depend only on the asymptotic values of $SL(3,R)/SO(3)$ moduli and the three charges q_1, q_2, q_3. Whether we succeed in reaching the above condition depends on if we can find the vector V_a.

We recall from [13] that the scalar matrix $\mathcal{M}_3$ parametrizing the coset $SL(3,R)/SO(3)$ is

$$\mathcal{M}_3 = e^{\frac{\varphi}{\sqrt{3}}} \begin{pmatrix} \begin{matrix} e^{-\phi} + \chi_3^2 e^{\phi} \\ +(\chi_2 + \chi_1\chi_3)^2 e^{-\sqrt{3}\varphi} \end{matrix} & \begin{matrix} \chi_3 e^{\phi} \\ +\chi_1(\chi_2 + \chi_1\chi_3) e^{-\sqrt{3}\varphi} \end{matrix} & (\chi_2 + \chi_1\chi_3) e^{-\sqrt{3}\varphi} \\ \begin{matrix} \chi_3 e^{\phi} \\ +\chi_1(\chi_2 + \chi_1\chi_3) e^{-\sqrt{3}\varphi} \end{matrix} & e^{\phi} + \chi_1^2 e^{-\sqrt{3}\varphi} & \chi_1 e^{-\sqrt{3}\varphi} \\ (\chi_2 + \chi_1\chi_3) e^{-\sqrt{3}\varphi} & \chi_1 e^{-\sqrt{3}\varphi} & e^{-\sqrt{3}\varphi} \end{pmatrix}. \tag{9}$$

The matrix $\mathcal{M}_3$ can be re-expressed in a 3-bein form as $\mathcal{M}_3 = \nu\nu^T$ with the 3-bein ν_{ia}. Here $i = 1,2,3$ are the $SL(3,R)$ indices while $a = 1,2,3$ are the $SO(3)$ indices. The

§ Similar SUSY condition was also given in a recent paper by Bhattacharya et al [21] in discussing the non-planar network of 3-string junctions. We, however, differ from theirs in the following ways. They used the product of gamma matrices in the transverse directions of the string in their SUSY condition while we have here the product of gamma matrices along the longitudinal directions of the string. The two are different since in D = 8, the pseudo Majorana spinors cannot be Weyl at the same time. The subsequent discussions are also differnt. They discussed the nonplanar network of 3-string junctions while we are here discussing a 4-string junction and its network.

3-bein ν transforms under the global $SL(3,R)$ and local $SO(3)$ as $\nu \to \Lambda\nu R$, with Λ a global $SL(3,R)$ matrix and R a $SO(3)$ rotation matrix. We also know that the charge triplet

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \tag{10}$$

transforms only under $SL(3,R)$ as $q \to \Lambda q$. So the $SO(3)$ vector V can be constructed as $V_a = (\nu_0^{-1})_{ai} q_i$ with

$$\nu^{-1} = e^{-\varphi/2\sqrt{3}} \begin{pmatrix} e^{\phi/2} & -\chi_3 e^{\phi/2} & -\chi_2 e^{\phi/2} \\ 0 & e^{-\phi/2} & -\chi_1 e^{-\phi/2} \\ 0 & 0 & e^{\sqrt{3}\varphi/2} \end{pmatrix}. \tag{11}$$

The subscirpt '0' always means that the scalars take their asymptotic values. The vector V_a transforms only under SO(3) as $V_a \to R_{ba} V_b$. The $SO(3)$ action on the vector V_a can also be realized on a $SU(2)$ basis through $V = V_a \sigma_a$ as $V \to U^+ V U$, with

$$U^+ \sigma_a U = R_{ab} \sigma_b, \tag{12}$$

where $U^+ = U^{-1}$ with U a SU(2) matrix.

Therefore, it can be shown that eq. (8) is indeed covariant under a SU(2) transformation U if the spinor doublet transforms as

$$\begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} \to U^{-1} \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}. \tag{13}$$

So far in discussing the SUSY condition (8), we always assume that the (q_1, q_2, q_3)-string be along the direction of x^1. We could not have found anything wrong with this assumption if we had never discussed various string junctions. We recall that the zero net force at a 4-string junction requires the orientation of the (q_1, q_2, q_3)-string be given by the tension vector (3). This implies that except for a $(q_1, 0, 0)$-string, a general (q_1, q_2, q_3)-string cannot be kept along the x^1-direction. Therefore, the superscript 1 in the γ^1-matrix in the SUSY condition (8) should be replaced by a label $1'$ characterizing the direction of the tension vector (3). We have

$$\gamma^{1'} = \sin\alpha \cos\beta\, \gamma^1 + \sin\alpha \sin\beta\, \gamma^2 + \cos\alpha\, \gamma^3, \tag{14}$$

where $\sin\alpha, \cos\alpha, \sin\beta, \cos\beta$ can be determined by the tension vector $\vec{T}_{(q_1,q_2,q_3)}$ as

$$\sin\alpha \cos\beta \quad = \quad \frac{q_1 - \chi_{30} q_2 - \chi_{20} q_3}{T_{(q_1,q_2,q_3)}},$$

$$\sin\alpha\sin\beta = \frac{e^{-\phi_0}\,(q_2 - \chi_{10} q_3)}{T_{(q_1,q_2,q_3)}},$$

$$\cos\alpha = \frac{e^{-\phi_0/2+\sqrt{3}\varphi_0/2}\,q_3}{T_{(q_1,q_2,q_3)}}, \tag{15}$$

where the tension $T_{(q_1,q_2,q_3)}$ is given by eq. (2). The tension vector can now be re-expressed as

$$\vec{T}_{(q_1,q_2,q_3)} = T_{(q_1,q_2,q_3)}(\sin\alpha\cos\beta\,\hat{e}_1 + \sin\alpha\sin\beta\,\hat{e}_2 + \cos\alpha\,\hat{e}_3). \tag{16}$$

If we introduce three Cartesian unit vectors $\hat{u}_a$ for the internal three dimensional tangent space of the coset $SL(3,R)/SO(3)$ space, then we can express V_a in a vector form as $\vec{V} = V_a\hat{u}_a$. Now if we employ eq. (11) to express the components $V_a = (\nu_0^{-1})_{ai}q_i$ explicitly, we have, using eq. (15),

$$\vec{V} = |V|(\sin\alpha\cos\beta\,\hat{u}_1 + \sin\alpha\sin\beta\,\hat{u}_2 + \cos\alpha\,\hat{u}_3), \tag{17}$$

where $|V|$, the magnitude of $\vec{V}$ or $V = V_a\sigma_a$, is actually the (q_1,q_2,q_3)-string tension written in Einstein metric [13]. Comparing eq. (16) with the above equation, we can easily see that the tension vector re-expressed in Einstein metric and the vector $\vec{V}$ behave in exactly the same way. The only difference is that they are expressed in two different spaces, namely, the tension vector is in three-dimensional space-time coordinate system whereas, the vector $\vec{V}$ is in the three-dimensional tangent space of the coset space $SL(3,R)/SO(3)$. Since both $\vec{T}_{(q_1,q_2,q_3)}$ and $\vec{V}$ are defined for arbitrary asymptotic values of the moduli and the three charges q_1, q_2, q_3, it would be strange if the two three-dimensional spaces are not identified given the above behavior even though we do not have to do so for the present purpose. The identification of these two spaces was also made in [21]. The similar behavior also occurs for a general type IIB (p,q)-string and the corresponding two 2-dimensional spaces were identified in the discussion of the planar string network in [18]. There must exist a deep reason for such an identification of two completely different spaces beyond what we have explained here. This may be connected to the well-known fact that there is a relationship between the unbroken SUSY condition and the so-called no-force condition.

Now we finally come to show that a general 4-string junction preserves 1/8 of the spacetime supersymmetries. Let us show this first for the simplest 4-string junction which consists of $(1,0,0), (0,1,0), (0,0,1)$ and $(-1,-1,-1)$ strings with $\chi_{10} = \chi_{20} = \chi_{30} = 0$. The SUSY condition for the $(1,0,0)$-string is, with $\gamma^{1'} = \gamma^1$ and $V/|V| = \sigma_1$,

$$\begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} = \gamma^0\gamma^1 \otimes \sigma_1 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}, \tag{18}$$

which is equivalent to $\epsilon^{(1)} = \gamma^0\gamma^1\epsilon^{(2)}$. Since $(\gamma^0\gamma^1)^2 = 1$, so one half of the spacetime supersymmeties is preseved by the $(1,0,0)$-string configuration. For the $(0,1,0)$-string, we have, $\gamma^{1'} = \gamma^2$ and $V/|V| = \sigma_2$. So the SUSY condition becomes,

$$\begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} = \gamma^0\gamma^2 \otimes \sigma_2 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}, \tag{19}$$

which is independent of the SUSY condition for the $(1,0,0)$-string. The above is equivalent to $\epsilon^{(1)} = -i\gamma^0\gamma^2\epsilon^{(2)}$. The above two independent SUSY conditions preserve 1/4 of the spacetime supersymmetries. For the $(0,0,1)$-string, we have, $\gamma^{1'} = \gamma^3$ and $V/|V| = \sigma_3$. The SUSY condition in this case has the form,

$$\begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} = \gamma^0\gamma^3 \otimes \sigma_3 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}, \tag{20}$$

which is equivalent to $\epsilon^{(1)} = \gamma^0\gamma^3\epsilon^{(1)}$ and $\epsilon^{(2)} = -\gamma^0\gamma^3\epsilon^{(2)}$. This breaks further 1/2 of what is left, i.e., preserving 1/8 of the spacetime supersymmetries. Does the $(-1,-1,-1)$-string break more supersymmetries? The SUSY condition for this string is automatically satisfied once we have all the above three conditions. In showing this, we need also the following deduced relations

$$\begin{aligned}
\gamma^1 \otimes \sigma_2 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} &= -\gamma^2 \otimes \sigma_1 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}, \\
\gamma^1 \otimes \sigma_3 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} &= -\gamma^3 \otimes \sigma_1 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}, \\
\gamma^2 \otimes \sigma_3 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} &= -\gamma^3 \otimes \sigma_2 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}.
\end{aligned} \tag{21}$$

In summary, the above 4-string junction indeed preserves 1/8 of the spacetime supersymmetries. The needed SUSY conditions are summarized here as

$$\begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} = \gamma^0\gamma^1 \otimes \sigma_1 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} = \gamma^0\gamma^2 \otimes \sigma_2 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix} = \gamma^0\gamma^3 \otimes \sigma_3 \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \end{pmatrix}. \tag{22}$$

One can show that the SUSY conditions for a general 4-string junction are exactly the ones given in eq. (22). One way to check this is to note that the three equations in (22) guarantee the SUSY condition to be satisfed for a general (q_1, q_2, q_3)-string with arbitrary asymptotic values of the moduli. Therefore, a general 4-string junction satisfying charge conservation (1) and zero net-force condition (4) always preserves 1/8 of the spacetime supersymmetries. So a 4-string junction can be a stable BPS configuration.

Our discussion for 4-string junctions also provides basis for the corresponding network in the same spirit of the planar string network discussed by Sen [18]. In the 4-string junction network, each 4-string junction satisfies the charge conservation (1) and zero net force condition (4), and preserves 1/8 of the spacetime supersymmetries. Since the SUSY conditions (22) are independent of the composition of the network, the 4-string junction network preserves also 1/8 of the spacetime supersymmetries. The orientations of links in the network are completely determined by the corresponding charge triplets associated with these links. Other topics such as string lattice and the compactification related to the string network can also be discussed in the similar spirit as in [18].

With this work, we may expect that there exist other kinds of stable BPS string junctions (with maximum number of strings allowable by the corresponding U-duality symmetry and charge conservation in each junction) and the corresponding network in $D \leq 7$. However, it appears that such a string junction and the corresponding network will exist only in $D = 7$ i.e. T^3-compactified type II string theory and not in $D < 7$. In the case of $D = 7$ we expect that there exists a 6-string junction. Both the 6-string junctions and the corresponding network preserves 1/32 of the spacetime supersymmetries. However, for $D < 7$, since the U-duality group becomes larger, the corresponding vector representation becomes bigger than the spatial dimensions of the theory. It also appears that no supersymmetries can be preseved for those junctions. Thus in those cases, it is not possible to form stable string junctions in a consistent fashion.

In conclusion, we have studied in this paper the BPS property of 4-string junction in $N = 2$, $D = 8$ supergravity which is the low energy effective theory of type II string theory compactified on T^2. The existence of 4-string junction in $D = 8$ has been argued to follow from the consistent coupling of F- and D-string to the D 3-brane of type IIB theory in $D = 10$. We have shown that the junction is stable since the net force due to the tensions of various strings at the junction vanishes as a consequence of charge conservation and vice-versa and the configuration preserves 1/8 of the space-time supersymmetries. We have also indicated how using the 4-string junction as the building block one can construct a string network and lattices. String junctions and the corresponding networks are stable but curious states that exist in string theory in various dimensions; however, their true utility either in the formulation of non-perturbative string field theory [3] or in the study of black holes [1,8] remains to be seen.

Appendix

In this appendix, we will demonstrate how to express the real Majorana-Weyl spinors ϵ_L and ϵ_R in $D = 10$ in terms of two pseudo Majorana spinors $\epsilon^{(1)}$ and $\epsilon^{(2)}$ in $D = 8$ and how to obtain eq. (7) from eq. (5). The spacetime signature is always chosen as $(-, ++\cdots+)$.

The eight 16×16 Dirac matrices Σ^i $(i = 1, 2, \cdots, 8)$ associated with $SO(8)$ can all be chosen as real and symmetric. An explicit representation for these Σ^i can be found, for example, on page 288 in [22]. These matrices satisfy the following Clifford algebra

$$\{\Sigma^i, \Sigma^j\} = 2\delta^{ij}. \tag{23}$$

The corresponding Σ^9 can be defined as

$$\Sigma^9 = \Sigma^1\Sigma^2 \cdots \Sigma^8, \tag{24}$$

which is real and symmetric and whose square is a unit matrix, i.e., $(\Sigma^9)^2 = 1$.

The SO(1,7) gamma matrices can therefore be defined as

$$\begin{aligned} \gamma^0 &= i\,\Sigma^1, \\ \gamma^{j-1} &= \Sigma^j, \qquad (j = 2, 3, \cdots, 8). \end{aligned} \tag{25}$$

In $D = 8$, one can show that there exist only either Weyl or pseudo Majorana spinors. If we choose to work in the above representation for the gamma matrices, a pseudo Majorana spinor η is defined as

$$\gamma^0\eta^* = \eta, \tag{26}$$

where $*$ denotes the complex conjugate. If we write $\eta = \lambda + i\zeta$ with λ and ζ as two real spinors, then the above equation says $\zeta = -i\gamma^0\lambda$, so a general pseudo Majorana spinor in $D = 8$ can be written as

$$\eta = (1 + \gamma^0)\lambda, \tag{27}$$

with λ an arbitrary real spinor in $D = 8$. Or we can express

$$\lambda = \frac{1}{2}(1 - \gamma^0)\eta. \tag{28}$$

As is well-known, a Majorana-Weyl spinor Ψ in $D = 10$ is real in a Majorana representation in which the gamma matrix Γ^0 is real and antisymmetric while the rest gamma matrices are real and symmetric. We can construct such a representation for Γ^M $(M = 0, 1, 2, \cdots, 9)$, which is useful for the purpose of this paper, as

$$
\begin{aligned}
\Gamma^0 &= -i\,\Sigma^1 \otimes \sigma_2, \\
\Gamma^{j-1} &= \Sigma^j \otimes I_2, \qquad (j = 2, \cdots, 9), \\
\Gamma^9 &= \Sigma^1 \otimes \sigma_3,
\end{aligned}
\tag{29}
$$

where Σ^9 is defined in eq. (24), I_2 is the 2×2 unit matrix and σ_2 and σ_3 are the usual Pauli matrices whose explicit forms are

$$
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{30}
$$

The corresponding Γ_{11} in this representation is

$$
\begin{aligned}
\Gamma_{11} &= \Gamma^0\Gamma^1 \cdots \Gamma^9, \\
&= \Sigma^1 \otimes \sigma_1, \\
&= \begin{pmatrix} 0 & \Sigma^1 \\ \Sigma^1 & 0 \end{pmatrix},
\end{aligned}
\tag{31}
$$

which is real and symmetric and $\Gamma_{11}^2 = 1$. A Majorana-Weyl spinor Ψ satisfies both $\Psi^* = \Psi$ and $\Gamma_{11}\Psi = \Psi$ where we choose the eigenvalue for Γ_{11} as 1. Such a spinor can be expressed in terms of a $D = 8$ real spinor, for example the λ, as

$$
\Psi = \begin{pmatrix} \lambda \\ \Sigma^1\lambda \end{pmatrix}. \tag{32}
$$

We are now ready to derive eq. (7) from eq. (5). If we write

$$
\epsilon_L = \begin{pmatrix} \lambda_L \\ \Sigma^1\lambda_L \end{pmatrix}, \qquad \epsilon_R = \begin{pmatrix} \lambda_R \\ \Sigma^1\lambda_R \end{pmatrix}, \tag{33}
$$

and notice that

$$
\begin{aligned}
\Gamma^2\Gamma^3 \cdots \Gamma^9 &= \Sigma^2 \otimes \sigma_3, \\
&= \begin{pmatrix} \Sigma^2 & 0 \\ 0 & -\Sigma^2 \end{pmatrix},
\end{aligned}
\tag{34}
$$

then eq. (5) is equivalent to the following equations

$$
\lambda_L = \Sigma^2\lambda_L, \qquad \lambda_R = -\Sigma^2\lambda_R. \tag{35}
$$

Note both λ_L and λ_R are arbitrary 16-component real spinors. If we express them in terms of the two $D = 8$ pseudo Majorana spinors $\epsilon^{(1)}$ and $\epsilon^{(2)}$, then the above SUSY condition should become the SUSY condition for the $(1,0,0)$-string configuration. Using eq. (28), we can identify

$$\begin{aligned}
\lambda_L &= \frac{1}{2}(1-\gamma^0)(\epsilon^{(1)}+\epsilon^{(2)}), \\
\lambda_R &= -\frac{1}{2}(1-\gamma^0)(\epsilon^{(1)}-\epsilon^{(2)}).
\end{aligned} \tag{36}$$

Substituting the above to eq. (35), we end up with eq. (7) where we have used $\gamma^1=\Sigma^2$.

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