The Minimal Solution to the μ/B_μ Problem in Gauge Mediation

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Abstract

We provide a minimal solution to the μ/B_μ problem in the gauge mediated supersymmetry breaking by introducing a Standard Model singlet filed S with a mass around the messenger scale which couples to the Higgs and messenger fields. This singlet is nearly supersymmetric and acquires a relatively small Vacuum Expectation Value (VEV) from its radiatively generated tadpole term. Consequently, both μ and B_μ parameters receive the tree-level and one-loop contributions, which are comparable due to the small S VEV. Because there exists a proper cancellation in such two kinds of contributions to B_μ, we can have a viable Higgs sector for electroweak symmetry breaking.

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I. INTRODUCTION

Gauge mediated supersymmetry breaking (GMSB) [1, 2] provides an extremely appealing mechanism to solve the flavor problem elegantly in the Minimal Supersymetric Standard Model (MSSM). However, the well-known μ problem, namely the origin of the unique electroweak (EW)-scale mass parameter μ in the supersymmetric Higgs mass superpotential term $\mu H_u H_d$, cannot be addressed naturally in the Minimal Gauge Mediation (MGM) framework. To solve such a μ problem, the authors in Ref. [3] propose to couple the two Higgs doublets with the hidden sector messengers and μ can be generated at one-loop level as follows

$$\mu \sim \frac{\lambda_u \lambda_d}{16\pi^2} \frac{F}{M}, \tag{1}$$

where $\lambda_{u,d}$ are some Yukawa couplings in the hidden sector, M denotes the messenger mass scale and $\sqrt{|F|}$ is the effective supersymmetry (SUSY) breaking scale in the hidden sector. However, in this way the soft Higgs mixing term $B_\mu H_u H_d$ is simultaneously generated, leading to the relation $B_\mu \sim \mu F/M$. Then, for a phenomenologically preferred $\mu \sim 100 - 1000\,\text{GeV}$, the B_μ parameter will be several orders larger than μ^2, rendering the electroweak symmetry breaking (EWSB) problematic. This is the so-called μ/B_μ problem in the GMSB.

Various attempts have been made to solve this μ/B_μ problem [4–9]. For example, one may use a complicate dynamical structure to generate μ at one loop whereas B_μ does not appear until two loops [3–5] or even vanishes at the messenger boundary [9]. One can also work in the next-to-minimal supersymmetric standard model (NMSSM) [10] to generate the μ parameter from the Vacuum Expectation Value (VEV) of a dynamical field, which, however, is proved to be difficult to generate a large enough μ in the pure GMSB [11, 12] (This problem could be solved through coupling the singlet field to hidden sectors [13]). In addition, the renormalization of the strong dynamics in the hidden sector may affect differently on the μ and B_μ parameters [7, 8], but such models with strong dynamics suffer from the incalculable problem. An interesting observation was made in Ref. [8], which found that a desired Higgs sector with successful EWSB does not always need a small B_μ while a large B_μ together with a correspondingly large $m_{H_d}^2$ can also make EWSB work. Such a scenario does not require new fine-tuning and has distinct phenomenology [14].

Based on the above progresses, we in this paper attempt to explore a minimal solution to the μ/B_μ problem. For this purpose, we try to introduce the minimal degrees of freedom with the minimal new scales. But we require the calculability and perturbativity up to the Grand Unified Theory (GUT) scale. We find that we can make it by extending the minimal gauge mediation with a Standard Model (SM) singlet field S around the messenger scale which couples to the Higgs and messenger fields. The heavy singlet S is nearly supersymmetric,

i.e., the mass of its scalar component is almost equal to its fermionic component. And it obtains a relatively small VEV due to the radiatively generated tadpole term. Therefore, both μ and B_μ parameters can receive the tree-level and one-loop contributions that are comparable due to the small S VEV. Our crucial observation is that such two kinds of contributions to B_μ can allow for a proper cancellation. Thus, a viable Higgs sector for EWSB can be realized.

The paper is organized as follows. In Section II we present the minimal solution. In Section III we investigate the EWSB from the generated Higgs sector and comment on the supersymmetric CP problem. The Conclusion is made in Section IV.

II. THE MINIMAL SOLUTION TO THE μ/B_μ PROBLEM

A. Minimal Gauge Mediation

We start from the superpotential in the minimal gauge mediation

$$W_h = X\phi\bar{\phi} \tag{2}$$

with the spurion field $X = M + \theta^2 F$ and N pairs of messengers $(\phi, \bar{\phi})$ which fill the $(5, \bar{5})$ representations of $SU(5)$ gauge group. The supersymmetry breaking soft mass spectrum at the messenger scale is given by

$$M_a \simeq N\frac{\alpha_a}{4\pi}\Lambda, \tag{3}$$

$$m_{\tilde{f}}^2 \simeq 2N \sum_a C_a^f \left(\frac{\alpha_a}{4\pi}\right)^2 \Lambda^2, \tag{4}$$

where $\Lambda \equiv F/M$, $a = 1$, 2, 3, α_1, α_2 and α_3 are respectively the gauge couplings for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, and C_a^f denote the quadratic Casimir invariant for the particle $\tilde{f}$ with respect to three SM gauge groups.

B. Higgs-Messenger Mixings and Tree-Level μ/B_μ terms

The Higgs doublets H_u and H_d just assemble a pair of messengers: they have a supersymmetric mass term $\mu H_u H_d$ as well as a SUSY-breaking term $B_\mu H_u H_d$. Naively, one expects that coupling them directly to the Goldstino field X via $\lambda X H_u H_d$ may solve the μ/B_μ−problem, provided that $\mu = \lambda M \sim \mathcal{O}(m_Z)$. However, this does not work because the robust relation $B_\mu/\mu^2 = \Lambda/\mu$ still holds even if λ can be as small as we want.

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In order to see how the Higgs fields can feel the SUSY-breaking effect through small Higgs-messenger mixings, we consider an example

$$W_{HV} = X\phi\bar{\phi} + \left(v_1 H_u \phi_L + v_2 H_d \bar{\phi}_L\right), \tag{5}$$

where the $SU(2)_L$ doublet components of the messengers are denoted as $(\phi_L, \bar{\phi}_L)$, and v_1 and v_2 are the introduced new scales. Integrating out the messengers $(\phi_L, \bar{\phi}_L)$ at tree level, we obtain μ and B_μ as follows

$$\mu = -\frac{v_1 v_2}{M},$$
$$B_\mu = -\frac{v_1 v_2}{M^2} \times F = \mu\Lambda, \tag{6}$$

which are valid at small mixing limits $v_{1,2} \ll M$ ($v_{1,2}$ are naturally small because in the UV completed models they are induced by radiative corrections). We see that the μ/B_μ problem remains unsolved due to the relation $B_\mu = \Lambda\mu$.

Some comments on the above results are in order. First, the SUSY breaking parameter B_μ is proportional to the extent of SUSY-breaking F/M^2 while the supersymmetric parameter μ is not related to SUSY-breaking. Second, the above result is easy to understand. Rotating the Higgs doublets and messenger doublets to the mass eigenstates (with one eigenvalue μ), we obtain an effective coupling

$$W_{HV} \supset \left(F_{\phi u} F_{\bar{\phi}d}\right) X H_u H_d, \tag{7}$$

where we use the same notation for the fields before and after rotation. Here $F_{\phi_{u,d}}(v_1, v_2, M)$ are the light doublet fractions contained in the messengers. We just recover the model at the beginning of this section by setting $\lambda = F_{\phi u} F_{\bar{\phi}d}$.

By the way, in the above proposal some ad hoc new scales $v_{1,2}$ have to be introduced by hand. So it seems that we just trade one problem with another. In our new attempt we will try to dynamically generate such a new scale from the VEV of a new SM singlet field S. Consequently, additional contributions arise from one loop, which can naturally (when we consider the complete dynamics of S) reduce the large tree-level B_μ.

C. Dynamical Mechanism and One-Loop Contributions to μ/B_μ

In order to overcome the tree-level relation between μ and B_μ, we dynamically generate the scales $v_{1,2}$ by choosing the superpotential as follows

$$W = \lambda_u S H_u \phi_L + \lambda_d S H_d \bar{\phi}_L + \lambda X \phi\bar{\phi} + f(S). \tag{8}$$

where λ_u, λ_d, and λ are Yukawa couplings. Also, $f(S)$ denotes the complete dynamics of S which generates a VEV for S at $\langle S \rangle \equiv v$ (the complete form for $f(S)$ will be specified in the next Section). Then we have $v_{1,2} = \lambda_{1,2}v$. The key dynamical information of S can be parameterized as

$$W \supset \lambda_u \mathcal{S} H_u \phi_L + \lambda_d \mathcal{S} H_d \bar{\phi}_L + \lambda X \phi \bar{\phi} + \frac{1}{2} M_S \mathcal{S}^2 + \lambda_S \mathcal{S}^3, \tag{9}$$

where we have made the shift $S \to v + \mathcal{S}$ and simply assumed the effective theory for S is a polynomial. Note that in our approach S is supersymmetric (we will turn back to this point later), which is contrary to the cases discussed in Ref. [14, 15]. This will lead to much different results between our work and previous studies. A crucial observation from Eq. (9)
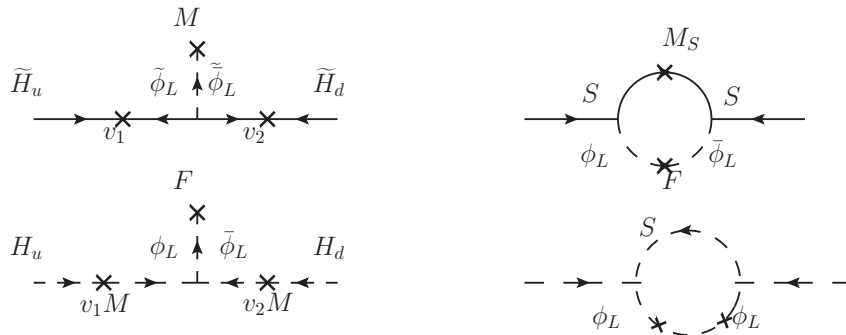

FIG. 1: The generation of μ/B_μ at tree and loop level.

is that with a dynamical S propagating in the loop, there will be one-loop contributions to the μ/B_μ parameters, which are proportional to F. Now we calculate the contributions from the one-loop diagrams shown in Fig. 1. We start from Eq. (9) and neglect the small Higgs-messenger mixings. Note that the dimensionless parameter λ_S is irrelevant because it does not enter into the one-loop diagrams. The analytical results are given by

$$\Delta\mu = -\frac{\lambda_u \lambda_d}{16\pi^2} f_1(x)\Lambda, \tag{10}$$

$$\Delta B_\mu = -\frac{\lambda_u \lambda_d}{16\pi^2} f_2(x)\Lambda^2, \tag{11}$$

where $x \equiv M_S/M$, and the functions $f_{1,2}(x)$ are defined by [27]

$$f_1(x) = \frac{x}{(x^2-1)^2}(x^2 \log x^2 - x^2 + 1),$$

$$f_2(x) = \frac{x}{(x^2-1)^3}(-2x^2 \log x^2 + x^4 - 1). \tag{12}$$

Notice that these one-loop contributions are proportional to $M_{\mathcal{S}}$. This can be easily understood because turning off the mass term of $\mathcal{S}$ will render the model a global $U(1)$ symmetry

under which $\mathcal{S}$ is charged, aside from the irrelevant term $\lambda_S \mathcal{S}^3$. Now the total values of μ and B_μ are given by

$$\mu = -\lambda_u \lambda_d \left[f_v + f_1(x) \right] \frac{\Lambda}{16\pi^2}, \tag{13}$$

$$B_\mu = \mu \frac{f_v + f_2(x)}{f_v + f_1(x)} \Lambda, \tag{14}$$

where $f_v = 16\pi^2 v^2/F$. Since B_μ receives contributions at both tree level and one loop, these contributions may cancel each other to some extent for a negative F. In particular, if the S VEV is small, f_v and $f_2(x)$ can be comparable.

Through the Yukawa interactions the soft masses of H_u and H_d can also get the one-loop (with $\mathcal{S}$ running in the loops) non-holomorphic contributions, given at the leading order by

$$\Delta m_{H_u}^2 = \frac{\lambda_u^2}{16\pi^2} \Lambda^2 g(x), \quad \Delta m_{H_d}^2 = \frac{\lambda_d^2}{16\pi^2} \Lambda^2 g(x), \tag{15}$$

where $g(x)$ is defined as

$$g(x) = \frac{x^2}{(x^2-1)^3} \left[(1+x^2) \ln x^2 + 2(1-x^2) \right]. \tag{16}$$

Note that there are also tree-level soft masses for $H_{u,d}$ which are proportion to $v^4 F^2/M^6$ and much smaller than the above one-loop contributions, so they can be omitted safely. It is well known that in the Yukawa-deflected models [16–18] such one-loop soft masses vanish at the leading order of SUSY-breaking and they are generated only at two-loop level through the wave function renormalization [19, 20]. Here in our model the one-loop contributions survive by virtue of the fact that S does not couple to the (single) spurion field X and thus avoid the accidental cancellation (we will turn to this point later). In addition, the trilinear soft terms $A_u(h_u)_i \widetilde{Q}_i H_u \widetilde{U}_i^c$ are also generated

$$A_{u,d} = \frac{\lambda_{u,d}^2}{16\pi^2} h(x) \Lambda, \tag{17}$$

with the function $h(x)$ defined as

$$h(x) = \frac{1}{(x^2-1)^2} \left[(x^2-1) - x^2 \log x^2 \right]. \tag{18}$$

As shown later, these trilinear soft terms do not play a significant role in our discussions.

In the above results we defined four functions. While the function $h(x)$ is negative, the other three functions $f_{1,2}(x)$ and $g(x)$, relevant to the Higgs parameters, are semi-positive. Especially, $h(x)$ is not proportional to x, and thus it is the most important function in the small x region. Such properties may be useful, for example, may help to lift up the lightest CP-even Higgs boson mass. Note that for $f_{1,2}(x)$ and $g(x)$ there is a relation

$$g(x) = \frac{f_1(x) - f_2(x)}{x} \tag{19}$$

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and further they satisfy

$$0 < g(x) < f_2(x) < f_1(x) < 1. \tag{20}$$

For $x \ll 1$, we have $g(x) \ll f_1(x) \simeq f_2(x) \ll 1$, it is easy to see that $B_\mu \sim \Lambda\mu$ from Eq. (13), so in this paper we will turn to the regions $x > 0.2$; while for a large x we have the hierarchy $g(x) \ll f_2(x) \ll f_1(x) \ll 1$, which is helpful to solve the μ/B_μ−problem. These properties are shown Fig. 2.
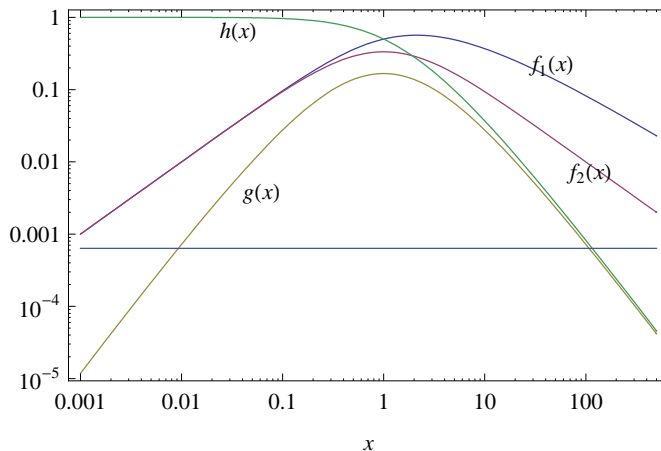

FIG. 2: Plot of the functions $f_1(x)$, $f_2(x)$, $g(x)$ and $|h(x)|$.

In the Higgs sector there are four effective free parameters: λ_u, λ_d, f_v and x. Especially f_v plays an important role in the cancellation between the tree-level and one-loop contributions of B_μ because it only appears at tree level. It is convenient to define

$$f_v + f_2(x) \equiv a_B g(x), \tag{21}$$

and hereafter we use a_B to replace the parameter f_v. If $f_v > 0$, we obtain $a_B > f_2(x)/g(x)$, which slowly varies with x, reaching a minimal value around 2 when x is about 1 (see Fig. 2). By contrast, if $a_B < 2$, we should have the cancellation between f_v and $f_2(x)$. Naively, a very small a_B implies a severe tuning and w will turn to this problem later. Now the parameters μ, B_μ, $\Delta m^2_{H_u}$ and $\Delta m^2_{H_d}$ can be written as

$$\mu = -g(x)\lambda_u\lambda_d(x + a_B)\frac{\Lambda}{16\pi^2}, \quad B_\mu = \frac{a_B}{x + a_B}\mu\Lambda,$$

$$\Delta m^2_{H_u} \simeq -\frac{1}{|r|(x + a_B)}\mu\Lambda, \quad \Delta m^2_{H_d} \simeq -\frac{|r|}{x + a_B}\mu\Lambda, \tag{22}$$

where $r \equiv \lambda_d/\lambda_u$, and in our convention $B_\mu > 0$. Here we see that $g(x)$ can be absorbed into the Yukawa couplings via the rescaling $\lambda_{u,d} \to g^{1/2}(x)\lambda_{u,d}$. Although the value of $g(x)$ is irrelevant to our solution, it affects the perturbativity of the couplings. As shown in Fig. 2,

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$g^{1/2}(x) \lesssim 0.3$. From the above expressions we see that for a large x, only the parameter μ is enhanced while the other three parameters including B_μ are suppressed. So it is possible to suppress B_μ relative to μ, and then the μ/B_μ problem is solved.

Note that with only the one-loop contributions the parameters B_μ, $m_{H_u}^2$ and $m_{H_d}^2$ are usually correlated undesirably, leading to an uncontrollably large B_μ. As discussed in [14], in order to break such a correlation, some new fields and scales have to be introduced. However, in our proposed minimal solution, since B_μ receives both the tree-level and one-loop contributions and these two kinds of contributions can cancel each other to some extent if they are comparable, such a correlation can be relaxed, albeit with reasonable fine-tuning.

D. The Complete Model

In the above we have presented the viable minimal solution to the μ/B_μ problem through a proper cancellation between the tree-level and one-loop contributions of B_μ. In this solution, we have a key assumption that the hidden singlet S develops a small VEV $v \ll M$. Now we discuss the generation of this small scale.

Since S is a singlet, at the renormalizable level we can in general expect the supersymmetric effective theory of S takes a form of

$$W = aM^2 S - \frac{M_S S^2}{2} + \frac{\lambda_S S^3}{3}, \tag{23}$$

where a and λ_S are dimensionless parameters, and M_S is a mass parameter. Of course, some term(s) can be forbidden by imposing some proper symmetry. In the following we discuss a simple case with $\lambda_S = 0$. Then the $F-$flatness of S determines a VEV: $v = aM^2/M_S$. On the other hand, for the cancellation between tree and loop contributions we have

$$f_v \sim -f_2(x) \Rightarrow a \sim \frac{x\sqrt{f_2(x)}}{4\pi} \left(\frac{\Lambda}{M}\right)^{1/2}. \tag{24}$$

In our interested region $f_2(x) \sim \mathcal{O}(10^{-2}) - \mathcal{O}(10^{-1})$, the value of a should be highly suppressed especially when we have a large messenger scale.

Motivated by an very small a, here we give a simple realization. It is natural to conjecture that a small VEV of S is driven by a small tadpole term, generated by radiative mechanism. So we consider a simple model with the following superpotential in the hidden sector

$$W_{hidden} = \lambda_u S H_u \phi_L + \lambda_d S H_d \bar{\phi}_L + \kappa S \phi \bar{\phi} + \frac{1}{2} M_S S^2 + X \phi \bar{\phi}. \tag{25}$$

It conserves a discrete Z_2 symmetry

$$S \to -S, \quad X \to -X,$$
$$\phi \to -\phi, \quad H_d \to -H_d, \tag{26}$$

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with other fields invariant under this symmetry. With such a Z_2 symmetry the bare term $\mu H_u H_d$ is forbidden. In this model the singlet S (as the spurion field) couples to the messenger pair directly and consequently a term $\int d^4\theta \epsilon (SX^\dagger + XS^\dagger)$ is generated. Replacing the spurion field $F-$term VEV, we get the small tadpole term $(\epsilon F)S + h.c.$, with ϵ given by

$$\epsilon \simeq N_f \frac{\kappa}{16\pi^2} \log \frac{\Lambda_{\text{UV}}}{M}, \tag{27}$$

where Λ_{UV} is the scale at the limit of zero mixing, N_f is the number of the fields running in the loop ($N_f = 5$ in the minimal case). Naturally it is expected that $\epsilon \sim \mathcal{O}(0.1)$. Now the effective parameter $a = \epsilon F/M^2 = \epsilon \Lambda/M$. From the constraint in Eq. (24), we require $\epsilon \sim x \left(f_2(x)\frac{M}{\Lambda} \right)^{1/2}/4\pi$. Apparently, in this mechanism M should be relatively light, say $\lesssim 10^9$ GeV, so as to be consistent with Eq. (27). Otherwise, the generated tadpole may not enable f_v to cancel $f_2(x)$.

Noet that although only the tadpole mechanism for the generation of the S VEV is discussed above (we believe it is the simplest way without involving other unknown dynamics), one can consider other possibilities, e.g., $a = 0$, $\lambda_S \neq 0$, and M_S originates from the $U(1)_R$ symmetry breaking. There exists a vacuum with $F_S = 0$ where S obtains a small VEV.

III. DISCUSSIONS ON LOW ENERGY PHENOMENOLOGY

In this Section we discuss some low energy phenomenology of our model. We will first discuss whether or not our model can give successful EWSB. Then we will make a brief comment on the supersymmetric CP-problem, which cannot be solved automatically in our model.

A. Electroweak Symmetry Breaking

1. Requirement of electroweak symmetry breaking

In the MSSM a viable Higgs sector triggering successful EWSB implies that at the EW-scale they satisfy the following two minimizing equations

$$\sin 2\beta = \frac{2B_\mu}{m_{H_u}^2 + m_{H_d}^2 + 2\mu^2}, \tag{28}$$

$$\frac{m_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - \tan^2\beta \, m_{H_u}^2}{\tan^2\beta - 1}, \tag{29}$$

where $\tan\beta$ is the ratio of Higgs VEVs, and m_Z is the Z boson mass. All the parameters take values at the EW scale. Among them, $m_{H_u}^2(m_Z)$ is driven negative by the stop RGE

effect, and leads to the well-known radiative EWSB

$$m^2_{H_u}(m_Z) = \Delta m^2_{H_u} + m^2_{H_u}(M) - \frac{3\alpha_t}{\pi} m^2_{\tilde{t}} \log \frac{M}{m_{\tilde{t}}} , \tag{30}$$

where α_t is the top quark Yukawa coupling. Also, the second term in the r.h.s is the pure GMSB contribution given in Eq. (3), and the stop mass $m_{\tilde{t}}$ is defined as the geometric mean of the two stop masses. In addition, the EW-scale μ and B_μ can be approximated to be the value at the boundary since they are not quite sensitive to the RGE evolutions. At the leading order their RGEs are given by [25]

$$\frac{d\mu}{dt} \simeq \frac{3(\alpha_t - \alpha_2)}{4\pi} \mu, \tag{31}$$

$$\frac{dB_\mu}{dt} \simeq \frac{3(\alpha_t - \alpha_2)}{4\pi} B_\mu + \left(\frac{6\alpha_t}{4\pi} A_u + \frac{6\alpha_b}{4\pi} A_d + \frac{6\alpha_2}{4\pi} M_2 \right) \mu, \tag{32}$$

where $t \equiv \log \frac{Q}{Q_0}$ with Q the running scale and Q_0 the referred scale, $A_{u,d}$ are the trilinear soft terms of the stops and sbottoms, and M_2 is the $SU(2)_L$ gaugino mass. As shown later, in the viable parameter space μ is small while B_μ is large, the RGE correction can be ignored in our analysis.

Note that in the MSSM we need a heavy stop $m_{\tilde{t}} \sim \mathcal{O}(1)$ TeV to lift up the lightest CP-even Higgs boson h mass to satisfy the LEP bound $m_h > 114.4$ GeV. So in the pure GMSB $-m^2_{H_u}(m_Z)$ will be of the stop mass scale (see Eq.30) which is far above m_Z and thus implies a fine-tuning between μ^2 and $m^2_{H_u}$ (the tuning extent is roughly measured by $\eta \equiv m^2_Z/m^2_{\tilde{t}}$) to satisfy Eq. (29). This is the so-called little hierarchy problem of the MSSM.

To satisfy the two equations in Eqs.(28) and (29), the Higgs parameters can be classified into the following three scenarios with different phenomenological consequences (a similar but more complicate classification can be found in Ref. [22]):

I: Large μ and large $|m^2_{H_u}(m_Z)|$. In the ordinary GMSB like MGM, μ is an input parameter which is tuned to cancel the large value of $m^2_{H_u}(m_Z)$

$$-m^2_{H_u}(m_Z) - \mu^2 \simeq m^2_Z/2. \tag{33}$$

In this scenario the lightest neutralino is bino-like, while the Higgsino-like neutralino/chargino are as heavy as $\sim \mathcal{O}(1)$ TeV. As for the Higgs spectrum, apart from a light Higgs h, other Higgs states are nearly degenerate and quite heavy

$$m_A \simeq m_{H^0} \simeq m_{H^\pm} \simeq \max\{|\mu|, m_{H_d}(m_Z)\} \gtrsim \mathcal{O}(1\,\text{TeV}). \tag{34}$$

So in this scenario we may only have a light Higgs boson at the LHC.

II: Small μ and small $m^2_{H_d}(m_Z)$. In a large class of models with dynamical μ/B_μ origin, due to the Higgs-messenger direct coupling, there are extra (probably large by virtue of being generated at one-loop level) contributions to Higgs soft masses at the boundary. Consequently, it is likely that such extra positive contribution $\Delta m^2_{H_u}$ may cancel a large portion of the stop RGE contribution, leading to $m^2_{H_u}(m_Z) \sim -\mathcal{O}(m^2_Z)$. Note that this does not mean that the fine-tuning problem is solved because we just shift the tuning to a new place, *i.e.*, a unnatural cancellation between $\Delta m^2_{H_u}$ and the stop RGE contribution. Evidently, μ should be around m_Z as well. If further $m^2_{H_d}(m_Z)$ is also small (*e.g.*, determined by pure GMSB or even smaller for some other reasons), then B_μ must be small in light of Eq. (28). In realistic models such a scenario may incur new fine-tuning. This scenario is characterized by no very heavy states in the EW-sector, indicating a more interesting discovery potential at the early LHC run.

III: Small μ and large $m^2_{H_d}(m_Z)$. For a large $m^2_{H_d}(m_Z)$ (maybe due to the large new soft term $\Delta m^2_{H_d}$), Eq. (28) can be satisfied even with a large B_μ [8]. This can be realized in the lopsided GMSB [14], which dose not incur new fine-tuning but in general requires a hierarchy $\lambda_u \ll \lambda_d$. Moreover, this kind of models also predict a heavy Higgs spectrum.

2. Realization of electroweak symmetry breaking in our model

Now we show that our minimal model can realize electroweak symmetry breaking via scenario-III listed above without too much difficulty. First we check the constraints on the Higgs parameters:

- The collider gives a lower bound on the chargino mass, which requires $|\mu| > 100$ GeV. And $\Delta m^2_{H_u}$ should roughly be below $(1\text{ TeV})^2$ so as to keep Eq. (30) negative to trigger EWSB (actually, a negative $m^2_{H_u}(m_Z)$ helps to trigger EWSB but is not a necessary condition). Then we have

$$16\pi^2 \frac{10^2\,\text{GeV}}{\Lambda}\frac{1}{|(x+a_B)r|} \lesssim g(x)\lambda^2_u \lesssim 16\pi^2\frac{3\alpha_t}{\pi}\frac{m^2_{\tilde{t}}}{\Lambda^2}\log\frac{M}{m_{\tilde{t}}} \equiv I_u, \tag{35}$$

As an illustration we take $m_{\tilde{t}} = 1.5$ TeV and $\Lambda = 100$ TeV, which gives an upper bound $I_u \simeq 0.08$. For the scale M we take $M = M_{\text{GUT}}$ for a general analysis in this subsection and a smaller M such as 10^9 GeV discussed previously will not change our conclusion. And roughly we can get $|(x+a_B)r| \gtrsim \mathcal{O}(1)$.

Note that the above constraint also implies that the RGE effect in $\mu/B\mu$ (see Eq. 31) is small. Since $|\Delta m^2_{H_u}| = |A_u\Lambda g(x)/h(x)| \lesssim 10^6\,\text{GeV}^2$, we have $A_u \sim \mathcal{O}(10)$ GeV in

the interesting region of $x > 1$. Then the RGE of B_μ is dominated by gaugino effect and thus is small. So in our model B_μ is dominated by its boundary value.

- In the MGM, the requirement of $U(1)_Y$ anomaly free ensures the traceless of the sparticles masses at the scale M, namely $\mathcal{S}_Y = \mathrm{Tr}(Y_{\tilde{f}} m_{\tilde{f}}^2) = 0$, with tracing over all $U(1)_Y$-charged sparticles. However, this sum rule is violated due to the generation of new soft masses from Yukawa interactions, e.g., $\mathcal{S}_Y \simeq -\Delta m_{H_d}^2$. Then it induces an effective Fayet-Iliopoulos (FI) term for the $U(1)_Y$-charged fields via RGE running [25]:

$$\delta m_{\tilde{f}}^{2(\mathrm{FI})} \simeq -\frac{3Y_{\tilde{f}}\alpha_1\mathcal{S}_Y}{10\pi} \log\frac{M}{\mu_0}. \tag{36}$$

For the fields carrying negative hypercharge (for example the left-handed sleptons), their soft masses will be decreased considerably (recall that $\Delta m_{H_d}^2$ is generated at one loop). So to avoid tachyonic sleptons, λ_d is upper bounded by

$$g(x)\lambda_d^2 \lesssim \frac{5\alpha_2^2}{8\pi\alpha_1}\frac{16\pi^2}{L_M} \equiv I_d, \tag{37}$$

where $L_M = \log\frac{M}{m_{\tilde{t}}}$. Roughly, $I_d \simeq 0.1$ and thus Δm_{H_d} is below several TeVs.

- Since both λ_u and λ_d are bounded above, then for a fixed x we get an upper bound on μ

$$|\mu| \lesssim \min\{I_u|r|, \frac{I_d}{|r|}\}|x + a_B|\frac{\Lambda}{16\pi^2}. \tag{38}$$

¿From this naive estimation, $|\mu|$ gets its maximal value for $r \simeq \sqrt{I_d/I_u}$

$$|\mu| \lesssim (x + a_B)\sqrt{I_d I_u}\frac{\Lambda}{16\pi^2}. \tag{39}$$

The bound is saturated if and only if $g(x)\lambda_{u,d}^2 = I_{u,d}$. Taking $\lambda_{u,d} < 4\pi$ (purterbativity) into consideration, then $g(x) > \max\{I_u, I_d\}/(4\pi)^2$ and thus $x \sim 10^{-2} - 10^2$. And it allows for a maximal $|\mu| \simeq 10$ TeV when $x \simeq 10^2$. Note that if the messenger scale is orders lower, then $\lambda_{u,d} \lesssim 1$ for keeping perturbative until GUT scale and also I_u will be reduced, so the maximal value of μ will be smaller.

With the constraints in Eqs. (35), (37) and (39), we now investigate the EWSB in our model. First let us consider a TeV scale $|\mu|$ in Scenario-I. If $x \sim \mathcal{O}(1)$ ($x + a_B \sim \mathcal{O}(1)$), as shown from Eq. (22), we can have a TeV scale $|\mu|$. But then, independent of the value of $|r|$, we run into either an excessively large $\Delta m_{H_d}^2$ or $\Delta m_{H_u}^2$. Therefore the case with $x \sim \mathcal{O}(1)$ is in contradiction with Eqs. (35) and (37). If x is large, e.g., $x \sim 50$, then from Fig. 2 we

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see a suppressed $g(x) \sim 10^{-3}$. In this case, in order to have a TeV-scale $|\mu|$, we require large $|\lambda_u \lambda_d|$ from Eq. (33)

$$|\lambda_u \lambda_d| \simeq \frac{1}{g(x)(x + a_B)} \left(16\pi^2 \frac{m_{\tilde{t}}}{\Lambda}\right) \left(\frac{3\alpha_t}{\pi} L_M\right)^{1/2} = \frac{4\pi \sqrt{I_u}}{g(x)x}, \tag{40}$$

where $\sqrt{I_u} \simeq 0.3$ and thus $|\lambda_u \lambda_d| \sim \mathcal{O}(10)$. This can be only allowed by GUT scale messengers. However, in our model the messenger scale is below 10^9 GeV, as discussed below Eq. (27).

In addition, from Eqs. (28) and (29) we obtain

$$1/\tan\beta \simeq \frac{a_B}{|r| + (x + a_B)\sqrt{I_u}/4\pi}. \tag{41}$$

It is well known that in the MSSM a large $\tan\beta$ is favored to push up the lightest Higgs boson mass. So we require $\tan\beta \gtrsim 10$ and $a_B \sim \mathcal{O}(0.1)$.

Next consider the small μ scenarios: Scenario-II and Scenario-III. Again we start from the Higgs soft masses in Eq. (22). First of all, recall that a TeV-scale $\Delta m_{H_u}^2$ is required to cancel the stop RGE contribution, namely $1/(|r|(x + a_B)) \sim \mathcal{O}(0.1)$. If we further want to make $\Delta m_{H_d}^2$ far below TeV scale, then typically we need $x \sim 100$ and $|r| \sim 0.1$. For a moderate B_μ, an small a_B of order $\mathcal{O}(0.01)$ is needed. At the same time due to the fact that $\lambda_u \lambda_d \sim 1$ to obtain a proper μ, a large λ_u is unavoidable. So it is beyond our minimal dynamical model neither.

Next we turn to the most attractive case in which μ is small but Δm_{H_d} is at TeV scale, namely the lopsided scenario. In this case the EW-breaking equations in Eqs. (28) and (29) can be approximated as follows

$$1/\tan\beta \simeq \frac{B_\mu}{m_{H_d}^2(m_Z)} \left[1 + \mathcal{O}(m_Z^2/m_{H_d}^2(m_Z), 1/\tan^2\beta)\right], \tag{42}$$

$$B_\mu^2 \simeq \left(\mu^2 + m_{H_u}^2(m_Z)\right) m_{H_d}^2(m_Z), \tag{43}$$

which are valid for large $m_{H_d}^2(m_Z)$ and large $\tan\beta$. Then we can obtain

$$a_B = \frac{|r|}{\tan\beta} \ll 1, \quad \lambda_u^2 \simeq \frac{I_u}{g(x)\left(1 - a_B^2\right)} \approx \frac{I_u}{g(x)}, \tag{44}$$

where we have ignored the RGE corrections to μ and B_μ, and also neglected pure gauge terms at the boundary such as $m_{H_d}^2$ and $m_{H_d}^2$. Now $\mu \simeq r(x + a_B)I_u\Lambda/16\pi^2 \sim 100$ GeV means $rx \gtrsim \mathcal{O}(1)$. Clearly, the desired parameter values are

$$|r| \sim 1, \, x \sim 1 \Rightarrow \lambda_u^2 \sim 1, \, a_B \simeq \tan^{-1}\beta \sim 0.1. \tag{45}$$

Next let us turn attention to the fine tuning during the tree loop cancelation in the complete model. The final B_μ is

$$B_\mu = \Delta_{tree} + \Delta_{loop} = \frac{-\lambda_u \lambda_d}{16\pi^2} [\frac{N_F^2 \kappa^2 F^3}{16\pi^2 M^2 M_S^2}(\log \frac{\Lambda_{UV}}{M})^2 + f_2(\frac{M_S}{M})\frac{F^2}{M^2}] \qquad (46)$$

The tuning is $\frac{B_\mu}{\Delta_{loop}}$ approximately, which is about 10% in our scenario. So it is acceptable. But the tuning increases as $\tan\beta$ gets large, and thus a moderate large $\tan\beta$ is preferred. Note that although such a tuning is needed in order to reduce B_μ considerably, we do not require a hierarchy between λ_u and λ_d.

Finally, we summarize the characteristics of our minimal model as well as the most natural parameter space

- We have a small $\mu \simeq 100$ GeV but a large m_{H_d} at TeV scale, namely favors the lopsided GSMB scenario.

- In turn, the mass scale of the singlet S is around the messenger scale $i.e.$, $x \simeq 1$. A high messenger scale is favored in light of perturbativity of λ_u and λ_d. But the messenger scale should low enough if the minimal model with loop induced tadpole is viable.

- The model works at the price of introducing a new source of tuning around $\sim 10\%$, but not worse than the original one. Such kind of fine-tuning is acceptable.

B. The Supersymmetric CP Problem

Finally we discuss the supersymmetric CP-problem in our minimal proposal. The GMSB elegantly solves the flavor problem by virtue of the flavor blindness of gauge interactions. However, in the MSSM the CP violation still arises via the flavor-conserving interactions, $e.g.$, the generation of the electric dipole moment (EDM) of the fermions via exchanging sparticles. In a class of GMSB models with vanishing boundary B_μ and trilinear $A-$terms, μ can be rotated to be real via $U(1)_{PQ}-$rotation while the induced phases of B_μ and A via gaugino RGE effects are the same as the phase of the gauginos and can be rotated away through $U(1)_R$ rotation. So this kind of model is CP safe [23, 24].

However, in general dynamical models explaining μ/B_μ origin, new CP phases are expected and thus the phases of B_μ and A are usually irrelevant to the gaugino phases. So these models could have the supersymmetric CP problem. In our model described in Eq. (25), one cannot take M_S and a simultaneously real by rephasing S, and consequently, the μ/B_μ may have different phases. To overcome this problem, one may need to consider some more complicated hidden sector (e.g., see Ref. [26]) that is beyond our scope.

IV. CONCLUSION

We explored the minimal solution to the μ/B_μ problem in gauge mediation. We introduced a SM singlet field S at about the messenger scale which couples to the Higgs and messenger fields. This singlet is nearly supersymmetric and obtains a relatively small VEV from its radiatively generated tadpole term. Consequently, both μ and B_μ parameters receive the tree-level and one-loop contributions, which are comparable if the S VEV is small. These two kinds of contributions allow for a proper cancellation for B_μ and thus provide a viable Higgs sector for the EWSB.

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