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THE COUPLING OF YANG-MILLS TO EXTENDED OBJECTS

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ABSTRACT

The coupling of Yang-Mills fields to the heterotic string in bosonic formulation is generalized to extended objects of higher dimension (p-branes). For odd p, the Bianchi identities obeyed by the field strengths of the $(p+1)$-forms receive Chern-Simons corrections which, in the case of the 5–brane, are consistent with an earlier conjecture based on string/5-brane duality.

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I. Introduction

Although the effort to generalize the physics of superstrings to higher-dimensional objects, super-p-branes, has been an active area of research since 1986, it is only recently that attention has turned to incorporating internal symmetries. The heterotic string [1] provides the paradigm for such Yang-Mills couplings, and here the problem is well understood. We have the luxury of employing either a fermionic formulation in which chiral fermions on the 2-dimensional worldsheet carry the internal quantum numbers or a bosonic formulation where the basic variables can be either free bosons or else the coordinates on a simply-laced group manifold. To date, no analogous action has been found for p-branes even though the existence of a "heterotic fivebrane" was conjectured in 1987 [2]. However, now at least we have an existence proof: the heterotic fivebrane emerges as a soliton solution of the heterotic string [3]. A study of the zero-modes of this soliton suggests that the group manifold approach might be a good starting point for constructing the action. Here one must distinguish between the covariant Green-Schwarz action and the gauge-fixed action that decribes only physical degrees of freedom. In the former case, the problem is to generalize the D=10 spacetime supersymmetric and κ-invariant fivebrane action of [4] to include the internal degrees of freedom which correspond presumably to the group manifold of $SO(32)$ or $E_8 \times E_8$. In the latter case it is to find an action supersymmetric on the d=6 worldvolume, which would involve a non-linear σ model of a quaternionic Kahler manifold.

In this paper, we make a first step toward the construction of the heterotic fivebrane by adopting the group manifold approach to coupling Yang-Mills fields to bosonic extended objects. For generality, we consider a d-dimensional ($d = p + 1$) worldvolume and a D-dimensional spacetime. Let us begin by reviewing the bosonic sector of the heterotic string.

2. Coupling Yang-Mills Field to the String

The bosonic sector of the heterotic string may be described by the action $S_2 = S_2^K +$

S_2^W, where [5]

$$S_2^K = \int d^2\xi \Big\{ -\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\Big(\partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X) + \partial_i y^m \partial_j y^n g_{mn}(y)\Big)\Big\} \tag{2.1}$$

$$S_2^W = \int d^2\xi \Big\{ -\frac{1}{2}\epsilon^{ij}\Big(-\partial_i X^\mu \partial_j X^\nu B_{\mu\nu}(X) + \partial_i y^m \partial_j y^n b_{mn}(y)\Big)\Big\} \tag{2.2}$$

where ξ^i $(i = 0, 1)$ are the worldsheet coordinates, $x^\mu(\xi)$ $(\mu = 0, ..., 9)$ are the spacetime coordinates and $\gamma_{ij}(\xi)$ the worldsheet metric†. The first terms in S_2^K and S_2^W are just the usual Green-Schwarz couplings to the background spacetime metric $g_{\mu\nu}(X)$ and rank-2 antisymmetric tensor $B_{\mu\nu}(X)$. The second term in S_2^K describes a nonlinear σ-model on the compact semi-simple Lie group manifold G, where $y^m(\xi)$ $(m = 1, ..., \dim G)$ are the coordinates on G and $g_{mn}(y)$ is the bi-invariant metric. Introducing the left-invariant Killing vectors $K_m^a(y)$, we have ††

$$\begin{aligned} g_{mn} &= K_m^a K_n^a \\ \partial_m K_n^a - \partial_n K_m^a &= -f_{bc}^a K_m^b K_n^c, \end{aligned} \tag{2.3}$$

The second term in S_2^W is the WZW term, involving the rank-2 tensor $b_{mn}(y)$, for which

$$h_{mnp} \equiv 3\partial_{[m} b_{np]} - f_{abc} K_m^a K_n^b K_p^c = 0 \tag{2.4}$$

Strictly speaking, for the string to be heterotic, we require that the bosons $y^m(\xi)$ be chiral on the d=2 worldsheet. This is also required for κ-symmetry. Since in this paper we are primarily concerned with the bosonic sector of $(d-1)$-branes with $d \geq 2$, we shall omit this constraint. The action is invariant under rigid $G_L \times G_R$ transformations. For G_L they are

$$\delta y^m = K_m^a(y)\lambda^a \tag{2.5}$$

† Here, and in the rest of the paper, we set the dimensionful parameters as well as the possibly quantized coupling constants equal to one.

†† In our conventions, the generators of the group obey the algebra $[T_a, T_b] = f_{ab}^c T_c$. The raising and lowering of indices will be done with the invariant tensor d_{ab} defined by tr $T_a T_b = d_{ab}$.

In gauging G_L, however, by allowing $\lambda^a = \lambda^a(X)$, there is a subtlety. In the kinetic term S_2^K it is sufficient to introduce the covariantly transforming currents

$$J_i^a = \partial_i X^\mu A_\mu^a - \partial_i y^m K_m^a \tag{2.6}$$

where $A_\mu^a(X)$ are the Yang-Mills gauge-fields transforming as

$$\delta A_\mu^a = \partial_\mu \lambda^a + f_{bc}^a A_\mu^b \lambda^c \tag{2.7}$$

Thus the gauge invariant extension of (2.1) is

$$S_2^K = \int d^2\xi \Big\{ -\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\Big(\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} + J_i^a J_j^a\Big)\Big\} \tag{2.8}$$

In the WZW term, however, we have three terms [6]

$$S_2^W = \int d^2\xi \Big\{ -\frac{1}{2}\epsilon^{ij}\Big(-\partial_i X^\mu \partial_j X^\nu B_{\mu\nu} - 2\partial_i X^\mu A_\mu^a \partial_j y^m K_m^a + \partial_i y^m \partial_j y^n b_{mn}\Big)\Big\} \tag{2.9}$$

and the invariance can be achieved only by assigning a non-trivial transformation rule to the rank-2 tensor $B_{\mu\nu}$, namely

$$\delta B_{\mu\nu} = -A_\mu^a \partial_\nu \lambda^a + A_\nu^a \partial_\mu \lambda^a \tag{2.10}$$

In order to generalize this construction to d-dimensional extended objects in D space-time dimensions, it is useful to adopt a condensed notation to rewrite the string action in terms of building blocks which may readily admit higher dimensional generalizations. Introduce the Lie-algebra valued 1-forms

$$A = A_\mu^a(X) T^a \partial_i X^\mu d\xi^i \tag{2.11}$$

$$K = K_m^a(y) T^a \partial_i y^m d\xi^i \tag{2.12}$$

where T^a are the generators of G in the fundamental representation. As a consequence of (2.3), we have the Maurer-Cartan equation

$$dK + K^2 = 0, \tag{2.13}$$

where d is the exterior derivative $d = d\xi^i \partial_i = d\xi^i \frac{\partial}{\partial \xi^i}$. Furthermore, using (2.5) and (2.7), the gauge transformations of A and K can be expressed as follows

$$\delta A = d\lambda + [A, \lambda] \tag{2.14}$$

$$\delta K = d\lambda + [K, \lambda] \tag{2.15}$$

Note that the same parameter λ occurs in both of the transformation rules. As a consequence of this the combination $A - K$ transforms covariantly

$$\delta(A - K) = [A - K, \lambda] \tag{2.16}$$

In fact,

$$J_i^a T^a d\xi^i = A - K \equiv J \tag{2.17}$$

which makes manifest the covariant transformation character of J_i^a, and hence the gauge invariance of the kinetic action (2.8).

In order to write the WZW action in a compact form as well, let us also define the d-forms

$$\begin{aligned} B_d &= \frac{1}{d!} B_{\mu_1 \ldots \mu_d} \partial_{i_1} X^{\mu_1} \cdots \partial_{i_d} X^{\mu_d} d\xi^{i_1} \cdots d\xi^{i_d} \\ b_d &= \frac{1}{d!} b_{m_1 \ldots m_d} \partial_{i_1} y^{m_1} \cdots \partial_{i_d} y^{m_d} d\xi^{i_1} \cdots d\xi^{i_d} \end{aligned} \tag{2.18}$$

Then the WZW action (2.9) may be written

$$S_2^W = \int \left\{ B_2 + \mathrm{tr}(AK) - b_2 \right\} \tag{2.19}$$

It is useful to introduce the notation

$$C_2 \equiv \mathrm{tr}(AK) \tag{2.20}$$

Here, and in the rest of the paper, *tr* refers to trace in the *fundamental* representation. The gauge invariance of S_2^W can now be understood as follows. First, consider the gauge invariant polynomial $I_4(F)$, where $F = dA + A^2$. We then note the usual descent equations

$$I_4(F) = dI_3^0(F, A)$$

$$\delta I_3(F,A) = dI_2^1(F,A,\lambda) \tag{2.21}$$

The subscripts on I denote form degree and the superscripts count the number of gauge parameters λ. Next, in condensed notation (2.4) reads

$$h_3 \equiv db_2 + I_3^0(K) = 0 \tag{2.22}$$

and hence (up to a total derivative term)

$$\begin{aligned} \delta b_2 &= -\mathrm{tr}(K d\lambda) \\ &\equiv -I_2^1(K,\lambda) \end{aligned} \tag{2.23}$$

Then, from (2.10) and (2.11) we have

$$\begin{aligned} \delta B_2 &= -\mathrm{tr}(A d\lambda) \\ &\equiv -I_2^1(A) \end{aligned} \tag{2.24}$$

The total derivative term which we have dropped in (2.23) corresponds to a tensor gauge transformation of b_2. The action is, of course, invariant under these tensor gauge transformations as well as similar tensor gauge transformation of B_2. In the rest of this paper, we shall focus on the Yang-Mills gauge transformations. Finally the gauge transformation of C_2 is easily found to be

$$\delta C_2 = I_2^1(A,\lambda) - I_2^1(K,\lambda) \tag{2.25}$$

The manner in which the WZW action (2.19) is gauge invariant is now transparent, given the transformation rules (2.23), (2.24) and (2.25).

3. Coupling of Yang-Mills Field to the Higher Dimensional Extended Objects

The background fields in this case are the metric $g_{\mu\nu}$ ($\mu = 0, ...D-1$) and a rank-d antisymmetric tensor $B_{\mu_1 \dots \mu_d}(X)$. The generalization of the kinetic term (2.8) is obvious, namely

$$S_d^K = \int d^d\xi \Big\{ -\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\Big(\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} + J_i^a J_j^a\Big) + \frac{1}{2}(d-2)\sqrt{-\gamma}\Big\} \tag{3.1}$$

where ξ^i ($i = 0, ...d-1$) are the worldvolume coordinates. In order to generalize the results of the last section to higher dimensions, consider a gauge invariant polynomial $I_{2n+2}(F)$

in $(2n+2)$-dimensions. Since $I_{2n+2}(F)$ is closed and gauge invariant, we have the descent equations

$$
\begin{aligned}
I_{2n+2}(F) &= dI^0_{2n+1}(F,A) \\
\delta I^0_{2n+1}(F,A) &= dI^1_{2n}(F,A,\lambda)
\end{aligned}, \tag{3.2}
$$

Note that $I_{2n+2}(F)$ is an even form, and hence these descent equations are relevant for p-branes with p odd, since $d = p+1 = 2n$. We shall come back to the case of p-branes with p even. Note also that, since the curvature of K is vanishing, I^0_{2n+1} is an algebraic polynomial in K, and $dI^0_{2n+1}(K) = 0$. In analogy with (2.19) we propose the following WZW action

$$
S^W_{2n} = \int \left\{ B_{2n} + C_{2n}(A,K) - b_{2n} \right\} \equiv \int \mathcal{B}_{2n} \tag{3.3}
$$

where $C_{2n}(A,K)$, the analog of C_2 given in (2.20), is still to be determined. b_{2n} is again chosen so that

$$
h_{2n+1} \equiv db_{2n} + I^0_{2n+1}(K) = 0 \tag{3.4}
$$

The non-zero Chern-Simons forms $I^0_{2n+1}(K)$ are in one to one correspondence with the non-zero totally antisymmetric group invariant tensors of the group G. These are in turn generated by products of the primitive antisymmetric tensors of the group, which are (nearly) all of the form $\mathrm{tr}T^{[a_1}\cdots T^{a_{(2m+1)}]}$. Tables of the cohomology of the Lie algebras tell us which of these tensors are non-zero [7]. For example, we can construct an $SU(3)$ invariant 3-brane using $I^0_5 = a_5 \mathrm{tr}K^5$, and we can construct an $SO(2N)$ invariant 5-brane using $I^0_7 = a_7 \mathrm{tr}K^7$. Here a_{2n+1} are calculable constants. In some cases the WZW term b_{2n} does not exist, e.g. $G = SO(2N)$ (except for $N=3$) for the 3-brane and $G = E_8 \times E_8$ for the 5-brane. In the case of string, we know that global considerations play a role and yield a quantization condition on the coefficient of the WZW term [5]. We intend to return to such global questions for p-branes elsewhere.

The $(2n+1)$ forms db_{2n} and I_{2n+1} are of course defined on a $(2n+1)$ dimensional space whose boundary is the $2n$ dimensional worldvolume. A derivation similar to that of (2.23) yields the following transformation rule for b_{2n}

$$
\delta b_{2n} = -I^1_{2n}(K,\lambda) \tag{3.5}
$$

To achieve gauge invariance of the WZW action (3.2), in analogy with the string case, we propose the following Yang-Mills gauge transformation rules

$$\delta B_{2n} = -I^1_{2n}(A,\lambda) \tag{3.6}$$

$$\delta C_{2n} = I^1_{2n}(A,\lambda) - I^1_{2n}(K,\lambda) \tag{3.7}$$

Thus the problem of finding a gauge invariant coupling of the Yang-Mills field to a $(2n-1)$-brane has been essentially reduced to finding $C_{2n}(A,K)$ which transforms as in (3.7). It can be constructed as follows.

We first observe that since the Lagrangian $\mathcal{L} = \mathcal{B}_{2n}$ is gauge invariant up to a total derivative, its exterior derivative is gauge invariant, i.e. $\delta(d\mathcal{L}) = 0$. Hence $d\mathcal{L}$ can be written as a sum of separately gauge invariant pieces as follows

$$\mathcal{H}_{2n+1} \equiv d\mathcal{B}_{2n} = H_{2n+1} + R_{2n+1} \tag{3.8}$$

where we use (3.4) and

$$H_{2n+1} = dB_{2n} + I^0_{2n+1}(A) \tag{3.9}$$

$$R_{2n+1} = -I^0_{2n+1}(A) + I^0_{2n+1}(K) + dC_{2n}(A,K) \tag{3.10}$$

We can derive explicit formulae for expressions $R_{2n+1}(A,K)$ and $C_{2n}(A,K)$ which satisfy this equation in the following way. First introduce the following quantities.

$$\begin{aligned} A_t &= tA + (1-t)K \\ F_t &= dA_t + {A_t}^2 \\ &= tF + t(t-1)(A-K)^2 \end{aligned} \tag{3.11}$$

We then define the following operators

$$\begin{aligned} d_t &= dt\frac{d}{dt} \\ l_t &= dt(A-K)\frac{\partial}{\partial F_t} \end{aligned} \tag{3.12}$$

which, as shown in ref. [8], obey the following equation

$$d_t N = (l_t d - d l_t)N \tag{3.13}$$

for any local polynomial N in the forms A_t and F_t and the operators d, d_t and l_t. The operator l_t is a derivation which reduces the form degree of N in ξ by one and increases the form degree of N in t by one, by replacing a factor of F_t with $dt(A-K)$. We now choose

$$N = I^0_{2n+1}(F_t, A_t) \tag{3.14}$$

Substituting this into (3.13), and integrating from $t=0$ to $t=1$ we obtain

$$I^0_{2n+1}(A) - I^0_{2n+1}(K) = \int_0^1 l_t I_{2n+2}(F_t) - d\int_0^1 l_t I^0_{2n+1}(F_t, A_t) \tag{3.15}$$

The first term on the right hand side is manifestly gauge invariant. Thus, comparing with (3.10) we read off the expressions

$$R_{2n+1}(A,K) = \int_0^1 dtJ\frac{\partial}{\partial F_t} I_{2n+2}(F_t) \tag{3.16}$$

$$C_{2n}(A,K) = \int_0^1 dt\ J\frac{\partial}{\partial F_t} I^0_{2n+1}(F_t, A_t) \tag{3.17}$$

Since $R_{2n+1}(A,K)$ is gauge invariant, from (3.10) we now see that the variation of $C_{2n}(A,K)$ is indeed given by (3.7).

All invariants $I_{2k}(F)$ can be expressed as the products of the primitive invariants of lower rank P_{2n+2} given by

$$P_{2n+2}(F) = \mathrm{tr}F^{n+1} = d\omega^0_{2n+1} \tag{3.18}$$

A general formula for the Chern-Simons form ω^0_{2n+1} is well known,

$$\omega^0_{2n+1} = (n+1)\int_0^1 dt\ tr\Big(AF_t^n\Big), \tag{3.19}$$

where here $F_t = tF + t(t-1)A^2$. Some examples are

$$\omega^0_3 = \mathrm{tr}\Big(FA - \frac{1}{3}A^3\Big) \tag{3.20}$$

$$\omega^0_5 = \mathrm{tr}\Big(F^2A - \frac{1}{2}FA^3 + \frac{1}{10}A^5\Big) \tag{3.21}$$

$$\omega^0_7 = \mathrm{tr}\Big(F^3A - \frac{2}{5}F^2A^3 - \frac{1}{5}FAFA^2 + \frac{1}{5}FA^5 - \frac{1}{35}A^7\Big) \tag{3.22}$$

Using the formulae given above, we shall now work out explicitly the expressions for C_4 and C_6, occuring in the action for 3-branes, and 5-branes, respectively. In the case of 3-branes, as a starting point we consider

$$I_6 = c_1 \mathrm{tr} F^3 \tag{3.23}$$

From (3.17) we then obtain the result

$$C_4(A,K) = \frac{1}{2} c_1 \mathrm{tr} \Big\{ (FA + AF - A^3)K + \frac{1}{2} AKAK - AK^3 \Big\} \tag{3.24}$$

We can rewrite this result in many different ways by partially integrating and discarding total derivatives, which drop out in the action. In summary, the gauge invariant WZW action for the 3-brane is

$$S_4^W = \int \mathcal{B}_4, \tag{3.25}$$

where $\mathcal{B}_4$ is defined in (3.3). The case of 5-branes is somewhat more complicated. We can now consider the invariant

$$I_8(F) = c_1 trF^4 + c_2 (trF^2)^2, \tag{3.26}$$

where c_1 and c_2 are arbitrary constants. It is easily seen that

$$I_7^0 = c_1 \omega_7^0 + c_2 (trF^2) \omega_3^0 \tag{3.27}$$

Substituting this into (3.17), after a tedious but straightforward calculation we find the result

$$\begin{aligned} C_6(A,K) = & \Big(c_1 d^{efgh} + c_2 d^{(ef} d^{gh)} \Big) A^g K^h \Big\{ F^e F^f + \frac{1}{10} f^e_{ab} F^f \Big(3K^a K^b - 4K^a J^b + 4J^a J^b \Big) \\ & + \frac{1}{60} f^e_{ab} f^f_{cd} \Big(3K^a K^b K^c K^d + 6K^a K^b K^c J^d + 5K^a K^b J^c J^d + 4K^a J^b K^c J^d \\ & + 6K^a J^b J^c J^d + 3J^a J^b J^c J^d \Big) \Big\}, \end{aligned} \tag{3.28}$$

where $F^a = \mathrm{tr}(T^a F)$ and $d^{abcd} = \mathrm{tr}[T^{(a} T^b T^c T^{d)}]$. The gauge invariant WZW action for the 5-brane can then be written as

$$S_6^W = \int \mathcal{B}_6, \tag{3.29}$$

where $\mathcal{B}_6$ is defined in (3.2).

Let us now turn to the case of even p-branes with $p = 2n$. The kinetic action is given in (3.1). A rigidly G-invariant WZW term requires the existence of a rank $2n+2$ totally antisymmetric group invariant tensor, but for semi-simple groups these are absent until $p = 4$ and for simple groups G, they are absent until p = 6. For example for $p = 4$, and a group of the form $G = G_1 + G_2$, we could take $db_5 + \omega_3(K_1)\omega_3(K_2) = 0$; for p= 6 , with $G = SU(N), N \geq 3$, we could take $db_7 + \omega_3\omega_5 = 0$; and for p= 8 , with $G = SO(2N), N \geq 3$, we could take $db_9 + \omega_3\omega_7 = 0$.

However, most of the ingredients that went into the above construction of a *locally* G-invariant WZW action are only applicable for odd p-branes. For example, the nontrivially gauge invariant field strength H_{2n+1} which involves the Chern-Simons form $I^0_{2n+1}(F, A)$ has no analog for even p. This suggests that the field B_{2n+1} is inert under Yang-Mills gauge transformations. Therefore, the methods we used for odd p-branes have to be modified. To this end, we first observe that for even p-branes b_{2n+1} also satisfies $db_{2n+1} + I_{2n+2}(K) = 0$. In this case, $I_{2n+2}(K)$ can always be written as a product of an even number of primitive Chern-Simons forms $\omega_{2k+1}(K)$. Such factorizations follow from the cohomology of Lie algebras, and they can be deduced from ref. [7]. Consequently it is always true that b_{2n+1} factorizes as

$$b_{2n+1} = b_{2i_1} \prod_{k=2}^{2q} db_{2i_k}, \qquad \sum_1^{2q} i_k = n + 1 - q \tag{3.30}$$

This suggests that we introduce X-dependent lower rank antisymmetric tensor fields B_{2i_k} corresponding to each y-dependent one b_{2i_k}. This furthermore suggests that we use the forms $\mathcal{B}_{2n}$ as building blocks for a gauge invariant Lagrangian, since they have nice transformation properties and contain both B_{2i_k} and b_{2i_k}. We propose the following action for $p = 2n$

$$S^W_{2n+1} = \int \Big\{ B_{2n+1}(X) + \mathcal{B}_{2i_1} \prod_{k=2}^{2q} d\mathcal{B}_{2i_k} \Big\}, \qquad \sum_1^{2q} i_k = n + 1 - q \tag{3.31}$$

This action contains the rigid term (3.30), and it is indeed manifestly gauge invariant, since $\mathcal{B}_{2i_k}$ transforms into a total derivative.

We note that the factorization of the invariant tensor occuring on the right hand side of (3.31) as discussed above, can occur in some cases for odd p-branes as well, depending on the gauge group. In such cases, lower rank antisymmetric tensor fields $B_{2i_k}(X)$ can again be introduced, and gauge invariant actions of the type (3.31) can be written down.

Another generalization of the above construction is to introduce as a factor in the Lagrangian density the gauge invariant polynomials $I_{2i+2}(F)$ and lower rank tensors B_{2i+1} of odd degree that are taken to be inert under the Yang-Mills transformations. Putting all these together we arrive at a rather general form of the locally gauge invariant WZW term which can be written for both odd and even p-branes as follows

$$S^W_{2n+\epsilon} = \int \Big\{ \sum_i c_i \epsilon B_{2i+\epsilon}(X) I_{2n-2i}(F) + \sum_{\{i_k\}} c_{\{i_k\}} \mathcal{B}_{2i_1}\, I_{2i_2}(F)\, \mathcal{H}_{2i_3+1}\, \mathcal{H}_{2i_4+1} \cdots \mathcal{H}_{2i_k+1} \Big\}, \quad , \tag{3.32}$$

where $\epsilon = 0, 1$ corresponding to even and odd branes, respectively, c_i and $c_{\{i_k\}}$ are a set of arbitrary constants. Here $\{i_k\}$ is any partition and q is any integer such that $\sum_{k=1}^{2q+\epsilon} i_k = n+1-q$. Without loss of generality, we can define $\mathcal{B}_{2n},\ I_{2n}(A)$ and $\mathcal{H}_{2n+1}$ in terms of the primitive Chern-Simons form $\omega_{2k+1}(F,A)$ instead of $I_{2k+1}(F,A)$. This can be accomplished by field dependent redefinitions of higher rank forms $B_{2m}(X)$ in terms of the lower rank ones. For example, in the case of five-branes if we have the lower rank 2-form B_2 in addition to B_6, then the relevant redefinition is of the form $B_6 \to B_6 - c_2 I_4(F) B_2$.

4. Comments

In this paper we focused on generalizing the group manifold approach to Yang-Mills couplings, with semi-simple groups, and applying it to bosonic p-branes. There is clearly much scope for further work: including $U(1)$ groups, gauging both G_L and G_R, considering G/H coset spaces instead of group manifolds, including gravitational Chern-Simons corrections, and including supersymmetry. We do not anticipate any severe problems in these directions. Much more problematical, in our estimation, will be to preserve the κ-symmetry of the super p-branes when the Yang-Mills couplings are included. (For the case of string this has been done [9]). The solution to this latter problem is, of course, a prerequisite for constructing the action for the heterotic 5-brane, and testing the ideas that it might provide a dual description of the heterotic string [2,3,10]. We are encouraged, however, by the observation that the 5-brane Chern-Simons terms (3.27):

$$dH_7 = dI_7 = c_1 \mathrm{tr} F^4 + c_2 (\mathrm{tr} F^2)^2 \tag{4.1}$$

obtained in this paper are entirely consistent with an earlier conjecture based on string/fivebrane duality [10]. Recall that the string one-loop Green-Schwarz anomaly cancellation mechanism requires a correction term $B \wedge \mathrm{tr}(F \wedge F \wedge F \wedge F)$ in the $D = 10$ Lagrangian [11]. (For concreteness we focus on $SO(32)$). This corresponds to a string one-loop correction to the H_3 field equation, namely

$$d^*\left(e^{-\phi} H_3\right) = \frac{2\kappa^2}{3\alpha'(2\pi)^5}\mathrm{tr}F^4, \tag{4.2}$$

where ϕ is the dilaton and $\alpha' = \frac{1}{2\pi T_2}$ and T_2 is the string tension. But by string/fivebrane duality H_3 is related to H_7 of the fivebrane by $H_7 = e^{-\phi *} H_3$. Moreover, the string tension T_2 and the fivebrane tension T_6 are quantized according to $\kappa^2 T_2 T_6 = n\pi,\ n =$ integer. We may thus re-interpret (4.2) as a fivebrane tree-level correction to the H_7 Bianchi identity, namely [10]

$$dH_7 = n\frac{\beta'}{3}\mathrm{tr}F^4, \tag{4.3}$$

where $\beta' = \frac{1}{[(2\pi)^3 T_6]}$. This is consistent with (4.1). (In the case of the string the coefficient c_1 in $dH_3 = c_1 \mathrm{tr}F^2$ is quantized and fixed to be $c_1 = 2m\alpha', m =$ integer, by conformal invariance. This is also demanded by κ symmetry. We expect that κ invariance will lead to analogous restrictions on c_1 and c_2 in (4.1). In any case, it would appear that string/fivebrane duality requires $c_1 = n\beta'/3$ and $c_2 = 0$). That the classical fivebrane considerations of this paper should gel with quantum string effects represents, in our opinion, further circumstantial evidence in favour of string/fivebrane duality.

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