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Partial wave treatment of Supersymmetric Dark Matter in the presence of CP -violation

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Abstract

We present an improved partial wave analysis of the dominant LSP annihilation channel to a fermion-antifermion pair which avoids the non-relativistic expansion being therefore applicable near thresholds and poles. The method we develop allows of contributions of any partial wave in the total angular momentum J in contrast to partial wave analyses in terms of the orbital angular momentum L of the initial state, which is usually truncated to p-waves, and yields very accurate results. The method is formulated in such a way as to allow easy handling of CP -violating phases residing in supersymmetric parameters. We apply this refined partial wave technique in order to calculate the neutralino relic density in the constrained MSSM (CMSSM) in the presence of CP -violating terms occurring in the Higgs - mixing parameter μ and trilinear A coupling for large $\tan\beta$. The inclusion of CP -violating phases in μ and A does not upset significantly the picture and the annihilation of the LSP's to a $b\bar{b}$, through Higgs exchange, is still the dominant mechanism in obtaining cosmologically acceptable neutralino relic densities in regions far from the stau-coannihilation and the 'focus point'. Significant changes can occur if we allow for phases in the gaugino masses and in particular the gluino mass.

1 Introduction

In the framework of Supersymmetric Theories, in which R - parity is conserved, the lightest of the neutralinos, $\tilde{\chi}$, which is also the lightest supersymmetric particle (LSP) is a promising candidate for cold Dark Matter (DM) [1].

The knowledge of its relic density today combined with recent experimental observations, especially those coming from WMAP [2,3], put severe constraints on the parameters of the theory which if combined with other data enhance the possibilities of discovering Supersymmetry in future accelerator experiments.

In the framework of the Constrained Minimal Supersymmetric Standard Model small values of the neutralino relic density, $\Omega_{\tilde{\chi}} h_0^2$, in the vicinity of 10% range as recent astrophysical data on Cold Dark Matter suggest [4–6], [3], are obtained in regions of the parameter space which belong to either the neutralino - stau coannihilation region [7,8], or to the focus point [9,10], or to a region in the neighborhood of the line along which the pseudoscalar Higgs has a mass approximately twice that of the LSP mass [5,11–16], which is characterized by large $\tan\beta$. In this region the LSP annihilation fusion to a pseudoscalar Higgs, which subsequently decays into a $b\bar{b}$ pair, dominates the annihilation process. Interestingly enough in the same large $\tan\beta$ region the elastic LSP - nucleon cross section receives one of its highest possible values close to the sensitivity limits put by future experiments which will directly search for DM [15,17]. In this region as well as in the neutralino - stau coannihilation region the LSP is mainly bino. In the focus point region the neutralino LSP bears a relatively sizeable higgsino component and its pair annihilation cross section through the Z and/or the light Higgs boson exchange becomes significant, resulting to cosmologically acceptable relic density and high values for the LSP-nucleon cross section.

In order to calculate the LSP relic density, one has to solve the Boltzmann equation having as input the thermal average $\langle\sigma v_{rel}\rangle$ of the LSP's annihilation cross section [1, 18–30] $\sigma(s)$ times their relative velocity v_{rel} , if we are in a region of the parameter space where coannihilation effects are negligible. Its computation demands integration over the center of mass scattering angle $\cos\theta_{CM}$ of the transition matrix element squared summed over the final spins followed by an additional integration over s through which the aforementioned thermal average is defined.

If one follows the standard trace technique for calculating the amplitudes many interference terms are encountered since the squares of the annihilation amplitudes are summed in a coherent way. However this can be successfully accomplished as has been

shown elsewhere [28–30].

In this paper we shall pursue a partial wave analysis in the total angular momentum J , which can be considered as complementary to the trace technique and it improves analogous treatments existing in literature. In a partial wave expansion the integration over the scattering angle $\cos\theta_{\text{CM}}$ is carried out trivially, since the amplitudes of different J are summed incoherently in the total cross section avoiding numerous interference terms. This allows a better theoretical and numerical control of the annihilation process.

In addition the analysis, which we employ in this paper, has the following advantages:

- i) It avoids the non-relativistic expansion resulting to more accurate numerical treatment of the thermally averaged cross section especially near poles some of which, as those of Higgs bosons, are significant in obtaining small LSP relic densities in accord with recent cosmological data. The partial wave schemes encountered in literature usually expand the total cross section in powers of the relative velocity v_{rel} keeping terms up to $\mathcal{O}(v_{\text{rel}}^2)$ ¹.
- ii) It allows contributions of partial waves of any J , where J is the total angular momentum, offering a better approximation scheme of the partial wave series. The existing partial wave treatments are usually truncated to include terms up to p-waves in the orbital angular momentum L of the initial state which are included in $J \leq 2$.
- iii) The scheme is formulated in such a way as to allow easy handling of CP -violations residing in the superpotential Higgs mixing parameter μ and the trilinear A soft couplings and/or in the presence of additional supersymmetric CP -violating phases occurring in nonminimal models.

The scheme we present can be directly compared to the exact results one obtains using the trace technique and from this the importance of terms larger than $J = 2$, omitted in other approaches, can be sought. As a preview, our analysis reveals that the thermal average of the cross sections times the relative velocity of the annihilating LSP's obtained using $J \leq 2$ in the partial wave series approximates the exact result to an astonishingly good accuracy in the entire region of the constrained MSSM parameter space for temperatures that are relevant for the calculation of the relic density. We shall comment on this later on.

¹In the first of reference [11] such an expansion holds and the problems associated with the poles and thresholds are avoided by not expanding the kinematical factors β_f and the s-channel propagators.

In this paper paying special emphasis to the mechanism for the LSP annihilation through the Higgs boson exchange, we apply this refined technique in order to calculate the neutralino relic density in the CMSSM in the presence of CP -violating terms residing in the μ and trilinear A parameters of the CMSSM [31] taking also into account the effect of the mixing occurring in the Higgs sector. The effect of the coannihilation processes, in regions of the parameter space where this applies, in the context of this partial wave approach, will be the subject of a forthcoming publication.

This paper is organized as follows: In section 2 we give a brief outline of the helicity technique. In subsection 2.1 we discuss the s- channel contributions of Higgs and Z - boson exchanges while in 2.2 we present an analysis of the t and u - channel sfermion exchanges. In section 3 we briefly discuss the handling of the thermally averaged cross section and in section 4 we apply this technique when CP -violating phases are present. We end up with the conclusions presented in section 5.

2 The Helicity Formalism

In this section we will briefly outline the helicity formalism relevant to our calculation of the amplitude for the annihilation of two LSP's to a fermion - antifermion pair. We should evaluate the annihilation amplitudes for the above processes distinguishing initial and final helicity states. The Dirac spinor of a free fermion with mass m , 3-momentum \vec{p} and energy E is:

$$u(p) = N \begin{pmatrix} \xi \\ \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \xi \end{pmatrix} \quad (1)$$

where ξ is a two-component spinor. The overall normalization factor $N = (\frac{E + m}{2m})^{1/2}$ is consistent with $u\bar{u} = 1$. In order to find the spin - $\frac{1}{2}$ helicity states, we choose the two - component spinor ξ to be eigenstates of the helicity operator $\Lambda = \frac{1}{2} \vec{\sigma} \cdot \hat{p}$:

$$\Lambda \xi_\lambda = \lambda \xi_\lambda \quad , \lambda = \pm \frac{1}{2} \quad . \quad (2)$$

\hat{p} is the unit momentum vector, $\hat{p} = (\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta)$, and the helicity eigenstates are found to be,

$$\xi_+ = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix} \quad \text{and} \quad \xi_- = \begin{pmatrix} -e^{-i\varphi} \sin \frac{\theta}{2} \\ \cos \frac{\theta}{2} \end{pmatrix} \quad (3)$$

corresponding to spinors of helicities $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively. Therefore the one - particle helicity states for a particle, $u(p, \pm)$, or antiparticle, $v(p, \pm)$, with helicities $\pm\frac{1}{2}$ are given by

$$u(p, \pm) = N \begin{pmatrix} \xi_{\pm} \\ \pm \frac{|\vec{\mathbf{p}}|}{E+m} \xi_{\pm} \end{pmatrix}, \quad v(p, \pm) = N \begin{pmatrix} \frac{|\vec{\mathbf{p}}|}{E+m} \xi_{\mp} \\ \mp \xi_{\mp} \end{pmatrix}. \quad (4)$$

Note that $v(p, \pm) = C\bar{u}^T(p, \pm)$.

For the construction of a two - particle helicity state, one has to define appropriately the helicity spinors for the second fermion, i.e for the fermion carrying momentum $-\vec{p}$ in the center of mass frame. If the momentum of the first particle \vec{p} is pointing in the direction specified by the angles (θ, φ) , then the momentum of the second particle is in the direction specified by $(\pi - \theta, \varphi + \pi)$. The corresponding helicity spinors for the second fermion are,

$$\xi'_+ = \begin{pmatrix} -e^{-i\varphi} \sin \frac{\theta}{2} \\ \theta \\ \cos \frac{\theta}{2} \end{pmatrix} \quad \text{and} \quad \xi'_- = \begin{pmatrix} \cos \frac{\theta}{2} \\ \theta \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix}. \quad (5)$$

To comply with the Jacob-Wick phase convention these have been multiplied by the appropriate phase factors $-2\lambda e^{-i2\lambda\varphi}$.

Any helicity amplitude can then be expressed as a sum over the total angular momentum J ,

$$\mathcal{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2}(s, \theta, \varphi) = \sum_J (2J+1) d_{\mu\mu'}^{(J)}(\theta) \mathcal{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J)}(s) e^{i(\mu-\mu')\phi}, \quad (6)$$

where $\mu = \lambda_1 - \lambda_2$ and $\mu' = \lambda_3 - \lambda_4$. In this equation $\mathcal{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J)}$ is the reduced matrix element, and the rotational functions $d_{\mu\mu'}^{(J)}(\theta)$ satisfy the orthogonality relations

$$\int_{-1}^{+1} d(\cos\theta) d_{\mu\mu'}^{(J)*}(\theta) d_{\mu\mu'}^{(J')}(\theta) = \frac{2}{2J+1} \delta_{JJ'}, \quad (7)$$

so the interference terms are avoided when integrating the amplitudes squared.

We are primarily interested in studying the annihilation of two neutralinos into a fermion-antifermion pair using the helicity formalism. The relevant channels concern $Z, Higgs$ (s-channel) and sfermion \tilde{f} (t and u channel) exchange, see figure 1. For every helicity amplitude we need calculate the following matrix elements involving the spinors of the initial, $\chi_{\lambda_1}(p_1), \chi_{\lambda_2}(p_2)$, and final states, $f_{\lambda_3}(p_3), \bar{f}_{\lambda_4}(p_4)$, which are denoted by

²For more details see for instance [32] and references therein.

$\langle 0 | \Gamma | \chi_{\lambda_1}(p_1) \chi_{\lambda_2}(p_2) \rangle$, $\langle f_{\lambda_3}(p_3) \bar{f}_{\lambda_4}(p_4) | \Gamma | 0 \rangle$, $\langle f_{\lambda_3}(p_3) | \Gamma | \chi_{\lambda_{1,2}}(p_{1,2}) \rangle$, and $\langle \bar{f}_{\lambda_4}(p_4) | \Gamma | \chi_{\lambda_{1,2}}(p_{1,2}) \rangle$. These are the helicity dependent fundamental building blocks encountered in the vertices of the s, t and u - channel Feynman diagrams. In these Γ stands for $\Gamma = \{I, \gamma^\mu, \gamma_5, \gamma^\mu \gamma_5\}$ depending on the case under consideration. For lack of space we do not present their analytic expressions.

Each of these blocks is expressed in terms of the helicity spinors described previously. These are functions of the angle θ , the scattering angle in the center of mass frame, assuming that the two annihilated neutralinos move along the z - axis, the azimuthal angle ϕ and the center of mass energy squared s . In the center of mass frame the magnitudes of the incoming and outgoing particle 3-momenta are,

$$p_{1,2}(s) = \left(\frac{s}{4} - m_{\tilde{\chi}}^2\right)^{1/2}, \quad p_{3,4}(s) = \left(\frac{s}{4} - m_f^2\right)^{1/2}.$$

and the Mandelstam variables t and u as functions of s and θ are given by,

$$\begin{aligned} t &= m_{\tilde{\chi}}^2 + m_f^2 - \frac{s}{2} + 2p_1(s)p_3(s) \cos \theta \\ u &= m_{\tilde{\chi}}^2 + m_f^2 - \frac{s}{2} - 2p_1(s)p_3(s) \cos \theta \end{aligned}.$$

In this formalism the unpolarized total cross section for the annihilation of the two neutralinos into a fermion - antifermion pair, in the center of mass frame, is given by

$$\sigma(s) = \frac{p_3}{64\pi p_1 s} \sum_J \sum_{\lambda_3 \lambda_4; \lambda_1 \lambda_2} (2J+1) |\mathcal{M}_{\lambda_3 \lambda_4; \lambda_1 \lambda_2}^{(J)}|^2 \quad (8)$$

and hence no integration over the scattering angle is required once the helicity amplitudes $\mathcal{M}_{\lambda_3 \lambda_4; \lambda_1 \lambda_2}^{(J)}$ are known. Loss of accuracy occurs by truncating the partial wave series to a maximum value J_{max} of the total angular momentum. However as stated in the introduction the series converges fast in the regime of energies which are relevant for the calculation of the relic density. In this paper we use a value of $J_{max} = 3$ which is good enough to obtain accurate results and at the same time a fast code which returns the cross section and its thermal average.

Using the machinery outlined before one can express any s, t and u - channel scattering amplitude as functions of s and $\cos \theta$ and bring them in the form given by Eq. (6). This will provide us with the helicity amplitudes appearing in Eq. (8). The analytic expressions for the s, t and u - channel helicity amplitudes are listed below.

2.1 s-channel contributions

The s - channel are relatively easy to handle. Since the exchanged particle carries spin of either $s = 1$, for the Z - boson, or $s = 0$ for the Higgses, the partial wave series terminates

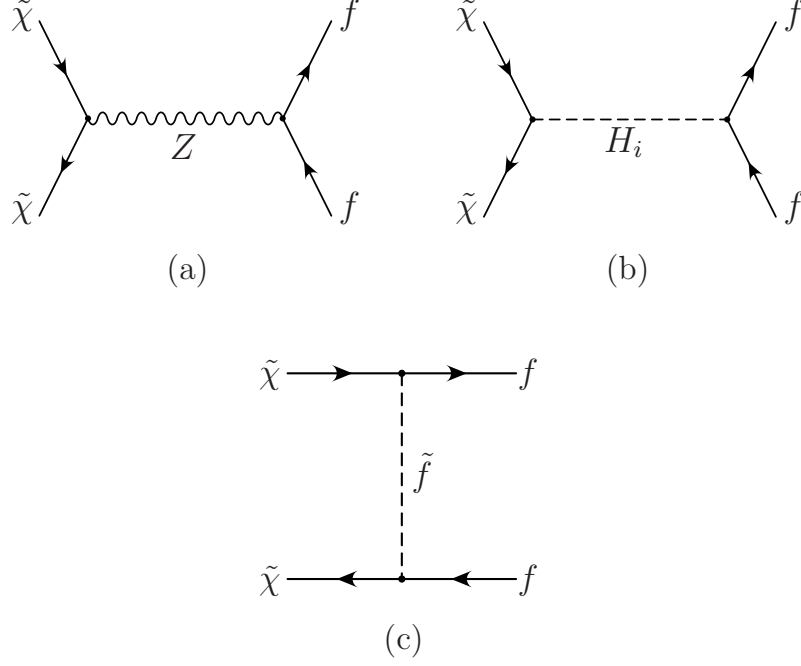


Figure 1: Graphs that contribute to the process $\tilde{\chi} \tilde{\chi} \rightarrow f \bar{f}$. H_i in the second graph denote the Higgs mass eigenstates

at $J \leq 1$.

2.1.1 Z-exchange

The reduced matrix element for the Z - exchange, as it follows from the Feynman rules using the helicity blocks described in the previous section, is

$$\mathcal{M}_{\lambda_3 \lambda_4; \lambda_1 \lambda_2}^{(J), Z} = - \frac{4m_{\tilde{\chi}} m_f}{s - M_Z^2 + iM_Z \Gamma_Z} N_{\lambda_1 \lambda_2}^{\mu 5} P_{\mu\nu} (V^f F_{\lambda_3 \lambda_4}^\nu + A^f F_{\lambda_3 \lambda_4}^{\nu 5}) A_{\tilde{\chi} \tilde{\chi}} \quad (9)$$

where the vertices $N_{\lambda_1 \lambda_2}^{\mu 5}$, $F_{\lambda_3 \lambda_4}^\nu$ and $F_{\lambda_3 \lambda_4}^{\nu 5}$ can be expressed in terms of the following matrix elements,

$$\begin{aligned} N_{\lambda_1 \lambda_2}^{\mu 5} &= \langle 0 | \gamma^\mu \gamma_5 | \chi_{\lambda_1}(p_1) \chi_{\lambda_2}(p_2) \rangle \\ F_{\lambda_3 \lambda_4}^\nu &= \langle f_{\lambda_3}(p_3) \bar{f}_{\lambda_4}(p_4) | \gamma^\mu | 0 \rangle \\ F_{\lambda_3 \lambda_4}^{\nu 5} &= \langle f_{\lambda_3}(p_3) \bar{f}_{\lambda_4}(p_4) | \gamma^\mu \gamma_5 | 0 \rangle \quad . \end{aligned}$$

$A_{\tilde{\chi} \tilde{\chi}}$ is the coupling of the Z- boson to the neutralinos and V^f, A^f , are the vector and axial couplings of the Z- boson to a fermion - antifermion pair labeled by f .

The numerator of the Z propagator, in the unitary gauge, is $P_{\mu\nu} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{M_Z^2}$ where $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. Since the four-momentum of the exchanged Z- boson in the

center of mass frame is $k_\mu = (\sqrt{s}, \mathbf{0})$ the tensor $P_{\mu\nu}$ receives the following form

$$P_{\mu\nu} = \begin{cases} -1 + \frac{s}{M_Z^2} & , \quad \mu = \nu = 0 \\ 0 & , \quad \mu = 0, \nu = i \\ -g_{ij} & , \quad \mu = i, \nu = j \end{cases} .$$

where $i, j = 1, 2, 3$.

After evaluating (9) , we find that a non-zero contribution emerges only when the helicities of initial and final states are combined to give $J = 0$ or 1 as expected. In fact

$$\mathcal{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J=0,1),Z} = \frac{-A_{\tilde{\chi}\tilde{\chi}}}{s - M_Z^2 + iM_Z\Gamma_Z} K_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J=0,1),Z}. \quad (10)$$

with

$$\begin{aligned} K_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J=0),Z} &= (-1)^{\lambda_3-\lambda_1} \delta_{\mu 0} \delta_{\mu' 0} (4m_{\tilde{\chi}} m_f) \left(-1 + \frac{s}{M_Z^2}\right) A^f \\ K_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J=1),Z} &= \frac{1}{3} (\delta_{\mu 1} - \delta_{\mu-1}) \sqrt{s - 4m_{\tilde{\chi}}^2} \\ &\quad \left[\delta_{\mu' 0} (2m_f) \sqrt{2} V^f + 2 (\delta_{\mu' 1} + \delta_{\mu' -1}) (\sqrt{s} V^f + \mu' \sqrt{s - 4m_f^2} A^f) \right] \end{aligned} \quad (11)$$

2.1.2 Higgs exchange

Using the technique outlined previously one can obtain the corresponding amplitudes for the s - channel annihilation process through the Higgs mass eigenstates $H_i, i = 1, 2, 3$

$$\mathcal{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J),H_i} = - \frac{1}{s - M_{H_i}^2 + iM_{H_i}\Gamma_{H_i}} K_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J),H_i} \quad (12)$$

In this equation , the superscript H_i denotes the exchanged Higgs and the non - vanishing $K_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J),H_i}$ are those corresponding to $J = 0$ only, since Higgses carry zero spin. These are given by

$$\begin{aligned} K_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J=0),H_i} &= \\ \delta_{\mu 0} \delta_{\mu' 0} &\left(g_S^{H_i} \sqrt{s - 4m_{\tilde{\chi}}^2} + g_A^{H_i} \sqrt{s} (-1)^{\lambda_1 - \frac{1}{2}} \right) \left(g_S^{f,i} \sqrt{s - 4m_f^2} + g_A^{f,i} \sqrt{s} (-1)^{\lambda_3 + \frac{1}{2}} \right) \end{aligned} \quad (13)$$

In these , $g_S^{H_i}, g_A^{H_i}$ denote the scalar(S) and the pseudoscalar (A) couplings of the neutralino LSP $\tilde{\chi}$ to the Higgs H_i , while $g_S^{f,i}, g_A^{f,i}$ are the corresponding couplings of the same Higgs to a fermion pair $f\bar{f}$,

$$\frac{1}{2} \tilde{\chi} (g_S^{H_i} + g_A^{H_i} \gamma_5) \tilde{\chi} H_i + \bar{f} (g_S^{f,i} + g_A^{f,i} \gamma_5) f H_i$$

Note that in our final results we should include a minus sign in the s - channel amplitudes to account for the relative (-1) difference from the corresponding t -channel amplitudes.

Note that we have allowed for the more general coupling of Higgses to fermions and the lightest of the neutralinos so that our formulae are applicable when supersymmetric CP -violating phases are present in the A and μ parameters which in turn cause mixing of the CP -even and CP -odd Higgs eigenstates.

2.2 t & u -channel : sfermions exchange

The t and u amplitudes are rather difficult to evaluate since the exchanged sfermion propagators depend on the scattering angle θ . The propagators can be expanded using the formula

$$\frac{1}{1-ax} = \sum_{n=0}^{\infty} \Pi_n(a) P_n(x) \quad (14)$$

where $P_n(x)$ is the first Legendre function of order n and its argument is $x = \cos\theta$. In this equation $a \leq 1$ and

$$\Pi_n(a) = \frac{2n+1}{2} \int_{-1}^{+1} \frac{P_n(\cos\theta)}{1-a\cos\theta} d(\cos\theta). \quad (15)$$

The function $\Pi_n(a)$ appearing in the above formula is actually $(2n+1) Q_n(1/a)/a$ where Q_n is the second Legendre function. Multiplying by the corresponding vertex contributions, the t and u channels can be cast in the form (6). Instead of presenting separately the contributions of the t and u channels we find it more convenient to present their sum. Note that a relative (-1) sign between the t and u channels must be included which has been duly taken into account in the following expressions. We find that their sum gives rise to the following partial wave amplitudes

$$\mathcal{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J),\tilde{f}} = \sum_{i=1,2} \frac{(2J+1)^{-1}}{F_i(s)} \begin{pmatrix} g_S^i & g_A^i \end{pmatrix} \hat{F}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J),\tilde{f}_i} \begin{pmatrix} g_S^{i*} \\ -g_A^{i*} \end{pmatrix} \quad (16)$$

\tilde{f}_i . The constants g_S^i, g_A^i are the scalar and pseudoscalar neutralino-fermion-sfermion couplings defined as

$$\tilde{\chi}(g_S^i + g_A^i \gamma_5) \tilde{f}_i^\dagger f + h.c.$$

while the quantity $F_i(s)$ is given by

$$F_i(s) = \frac{s}{2} - \left(m_{\tilde{\chi}}^2 + m_f^2 - m_{\tilde{f}_i}^2 \right)$$

The matrices $\hat{F}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J),\tilde{f}_i}$ depend on the energy $E = \sqrt{s}/2$ through

$$\begin{aligned}\hat{A}_{\pm} &= \sqrt{(E + m_f)(E + m_{\tilde{\chi}})} \pm \sqrt{(E - m_f)(E - m_{\tilde{\chi}})} \\ \hat{B}_{\pm} &= \sqrt{(E + m_f)(E - m_{\tilde{\chi}})} \pm \sqrt{(E - m_f)(E + m_{\tilde{\chi}})}\end{aligned}$$

and also on the functions $\Pi_J(a_i)$ whose arguments are $a_i \equiv \frac{2p_1 p_3}{F_i(s)} = \frac{\sqrt{(s - 4m_f^2)(s - 4m_{\tilde{\chi}}^2)}}{2F_i(s)}$.

For large energies a_i approaches unity and Π_J approaches a logarithmic singularity. In table [1] the matrices $\hat{F}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J),\tilde{f}}$ are presented for all helicity combinations of the incoming LSP's and the outgoing fermions.

3 Handling the thermal average and the relic density

The primarily task in calculating the relic density is to find the thermal integral ³

$$\langle \sigma v_{rel} \rangle (T) = \frac{1}{2m_{\tilde{\chi}}^4 T} \frac{1}{(K_2(m_{\tilde{\chi}}/T))^2} \int_{4m_{\tilde{\chi}}^2}^{\infty} ds p W(s) K_1(\sqrt{s}/T). \quad (17)$$

The momentum p is related to the CM energy and $m_{\tilde{\chi}}$ through $s = 4(p^2 + m_{\tilde{\chi}}^2)$. Scaling the temperature in terms of $m_{\tilde{\chi}}$ we can use as thermal variable $x = \frac{T}{m_{\tilde{\chi}}}$. Introducing a new variable y defined by

$$y = \frac{1}{x} \left(\frac{\sqrt{s}}{m_{\tilde{\chi}}} - 2 \right), \quad (18)$$

Eq. (17) can be cast in the form

$$\langle \sigma v_{rel} \rangle (x) = \frac{1}{2m_{\tilde{\chi}}^2} \frac{1}{(K_2(1/x))^2} \int_0^{\infty} dy (xy + 2) \sqrt{xy(xy + 4)} W(y) K_1(y + 2/x). \quad (19)$$

In this equation $W(y)$ is defined to be $W(s)$ with s replaced by its value from Eq. (18), that is $\sqrt{s} = m_{\tilde{\chi}}(xy + 2)$. Before continuing we should perhaps comment on how the singular behavior of the Bessel functions encountered in (17), or same (19), are evaded in our numerical code. This problem is not related to the partial wave expansion we employ to calculate the cross sections but it is due solely to the behavior of the Bessel functions appearing outside and within the integrands in the above relations. In fact the presence of the $K_2(1/x)$ in the denominator of Eq. (17) is potentially dangerous, since for $x < x_c$, with x_c a small number of order 0.003 or so, the Bessel function underflows

³The $W(s)$ that used here is related to the cross section by $\sigma(s) = W(s)/p\sqrt{s}$ (see Ref. [19]).

$K_2 \approx 0$, resulting to numerical overflow. However, the asymptotic expansion of the Bessel functions for large arguments is known to be

$$K_n(z) \xrightarrow{z>c} \sqrt{\frac{\pi}{2z}} e^{-z} P_n(z), \quad (20)$$

where $c = 1/x_c$ and

$$P_n(z) = 1 + \frac{4n^2 - 1}{1! 8z} + \frac{(4n^2 - 1)(4n^2 - 3^2)}{2! (8z)^2} + \dots \quad (21)$$

Using Eq. (20) one can obtain a form of Eq. (19) which is free of such overflows in the offending region $x < x_c$

$$\langle \sigma v_{rel} \rangle(x) \xrightarrow{x < x_c} \frac{1}{m_{\tilde{\chi}}^2} \sqrt{\frac{1}{2\pi}} \frac{1}{P_2(1/x)^2} \int_0^\infty dy \sqrt{y(xy+2)(xy+4)} W(y) e^{-y} P_1(y+2/x). \quad (22)$$

For $x > x_c$ the K_2 does not underflow but the Bessel function K_1 within the integral does in the integration region in which its argument exceeds $1/x_c$. In this integration region and in order to avoid such a numerical underflow we approximate K_1 by Eq. (20) within the integral. These results can be applied for an accurate numerical evaluation of the integral of Eq. (19) valid for any x .

In obtaining the LSP relic density one has to solve the Boltzmann transport equation numerically or use an approximate solution which however is quite accurate. Regarding the first approach the details of how one solves the Boltzmann equation are described by Edsjo and Gondolo and by Gelmini and Gondolo in reference [27]. Also an efficient method, reminiscent of the WKB approximation, for solving Boltzmann equation is presented in [23]. The details concerning the second approach, which is mostly used in the literature, are given in [1, 33], or by Griest et al. in the second of references in [21]. According to it the freeze - out temperature T_f and the corresponding point x_f is determined by

$$x_f = \left[\ln \left(\frac{0.038 g_{eff} M_{Planck} m_{\tilde{\chi}} \langle \sigma v_{rel} \rangle x_f^{1/2}}{g_*^{1/2}} \right) \right]^{-1}. \quad (23)$$

In this equation g_{eff} are the effective energy degrees of freedom and $g_*^{1/2}$ is the total number of effective relativistic degrees of freedom related to the entropy degrees of freedom h and g_{eff} by

$$g_*^{1/2} = \left(h + \frac{m_{\tilde{\chi}}}{3} \frac{dh}{dT} \right) \frac{1}{\sqrt{g_{eff}}}. \quad (24)$$

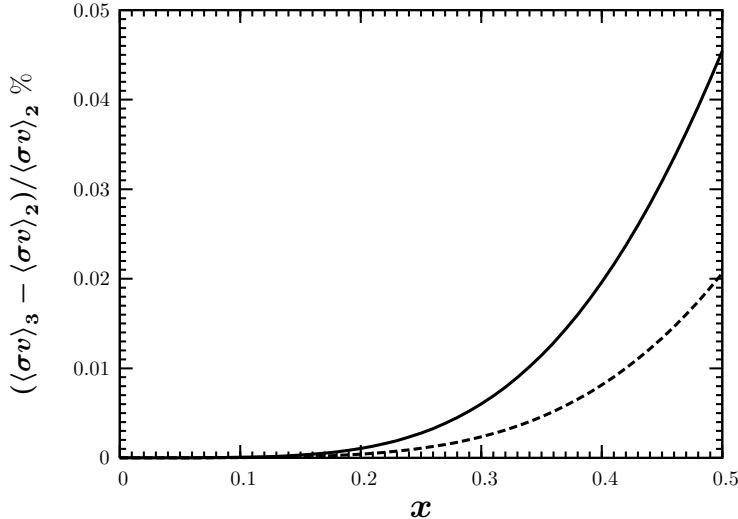


Figure 2: The ratio $(\langle\sigma v_{rel}\rangle_3 - \langle\sigma v_{rel}\rangle_2)/\langle\sigma v_{rel}\rangle_2$, in %, as a function of x . The subscripts refer to the value of J_{max} in each case. The figure corresponds to values $A_0 = 0$, $m_0 = 1.2$ TeV, $\tan\beta = 55$ and values of $m_{1/2}$ equal to 900 GeV (solid line) and 800 GeV (dashed line).

Eq. (23) can be iteratively solved to yield the freeze - out value x_f and from this by integrating $\langle\sigma v_{rel}\rangle(x)$ one can have the integral

$$J_* = \int_0^{x_f} \langle\sigma v_{rel}\rangle(x) dx . \quad (25)$$

The LSP relic density is then given by the expression

$$\Omega_{\tilde{\chi}} h^2 = \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{g_*^{1/2} M_{Planck} J_*} . \quad (26)$$

The value of the freeze - out point x_f is roughly $\approx 1/20$ and thus the values of $\langle\sigma v_{rel}\rangle$ needed for the calculation of (26) refer to points $x \leq x_f$ as is obvious from (25).

In order to estimate the error induced by truncating the partial wave series to values $J \leq J_{max}$ in figure 2 we display the ratio $(\langle\sigma v_{rel}\rangle_3 - \langle\sigma v_{rel}\rangle_2)/\langle\sigma v_{rel}\rangle_2$, where the subscripts refer to the value of J_{max} in each case. The plot of the figure corresponds to values $A_0 = 0$, $m_0 = 1.2$ TeV, $\tan\beta = 55$ and values of $m_{1/2}$ equal to 900 GeV (solid line) and 800 GeV (dashed line). Both cases refer to $\mu > 0$. The figure clearly shows that this ratio is small $\simeq 10^{-4}$ when x lies between 0 and 0.5, which includes the range of integration in Eq. 25, indicating that the partial wave series saturates the exact result if one truncates at a value $J_{max} = 2$. This is a general feature valid in the entire region of

the parameter space. In our numerical analyses concerning this work the value of J_{max} , which is actually input in our code, is set to $J_{max} = 3$. Higher values will yield more precise results at the expense of slowing down our numerical code.

It should be pointed out that the partial wave expansion we employ in terms of the total angular momentum J differs from the expansion in terms of the orbital angular momentum L . Since $L + S$ is even for the initial state of the two Majorana LSPs, expanding up to $J = 2$ includes not only the $L = 1$ terms (p-waves) but in addition all amplitudes characterized by $L = 2$. This is much improvement over the existing orbital angular momentum schemes which are truncated to $L = 1$ equivalent to keeping terms up to $\mathcal{O}(v_{rel}^2)$ in the relative velocity of the two LSPs. For this reason the series in terms of J yields very accurate results if already truncated at $J_{max} = 2$, as shown in figure 2.

4 Neutralino abundance with CP -violation at large $\tan \beta$

We apply the above described helicity amplitude technique to calculate the neutralino relic density at the large $\tan \beta$ regime, where it is known that in the CP -conserving case the contribution for the neutralino pair annihilation cross-section through the pseudo-scalar Higgs boson A exchange plays an important role in obtaining cosmologically acceptable relic densities far from the coannihilation and focus point regions. Since in this work we are mainly interested in the aforementioned mechanism for the annihilation of the LSP's, other production channels, e.g. gauge, Higgs bosons etc, do not contribute significantly. Hence we consider the neutralino LSP pair annihilation to fermion pairs at the relativistic level, including all finite width contributions. The code we employ, which is based on the partial wave analysis we outlined in the previous chapters, is pretty fast and can cope with the Higgs resonances which play an important role in obtaining an LSP relic abundance in accord with the recent cosmological data. As already stated in the previous section the value of the maximum allowed angular momentum J_{max} is input but a value of $J_{max} = 3$ is sufficient in obtaining a very good accuracy.

In this work we have focused our interest in the so-called “funnel” region of the parameter space, where one obtains cosmological accepted relic density through annihilation to Higgs boson resonances. In principle one expects that relatively sizable CP -violating effect can also occur in the focus point region, where the LSP carries a non-zero Higgsino component. This results to an enhancement of the LSP pair annihilation cross section to massless fermions, through the Z -boson exchange. Although we will not present numeri-

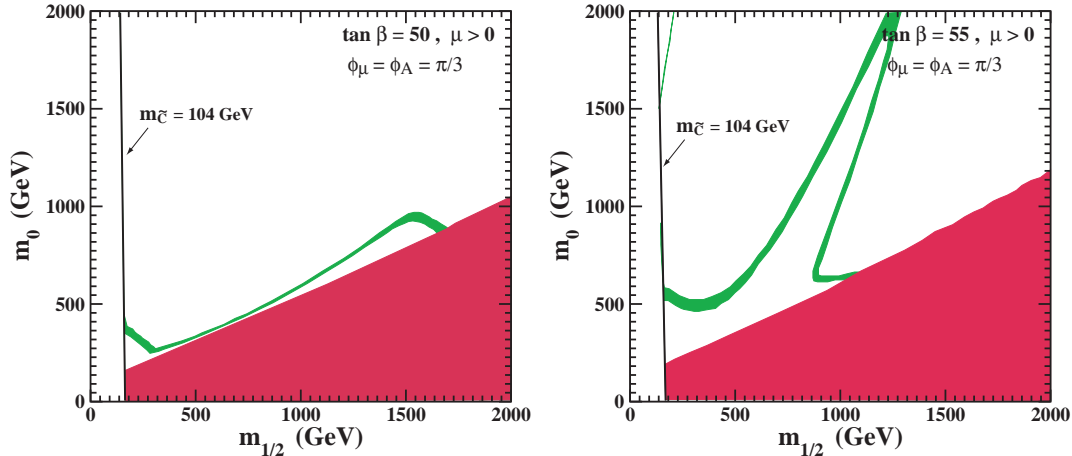


Figure 3: The neutralino relic density, in the $(m_0, m_{1/2})$ plane, for $\tan \beta = 50$ and 55 .

cal results in the focus point region, we have checked that there the CP -violating effects are almost comparable with those around the Higgs pole. Furthermore, the latest CDF and D0 data [34] suggest a somewhat higher value for the top quark mass $m_t = 178$ GeV, pushing the focus point region up to considerably high values of m_0 .

The method we develop is capable of accommodating CP -violating sources residing in the μ and the trilinear A parameters as well as in other parameters, as for instance in the gaugino mass parameters, which are important when we depart from the minimal schemes. In this work we consider the constrained scenario and hence only the μ and A phases are allowed. In figure 3 and for nonvanishing phases equal to $\phi_\mu, \phi_A = \pi/3$ at the weak scale we present in the $m_0, m_{1/2}$ plane the allowed domains (in green), which are consistent with the recent cosmological data from WMAP [2] $\Omega_{\tilde{\chi}} h_0^2 = 0.1126_{-0.0181}^{+0.0161}$. The cases shown are for values of $\tan \beta$ equal to 50 and 55 respectively. The shaded area (in magenta) at the bottom, in both panels, is excluded since in this the stau is the LSP. In figure 4 we display the relic density for fixed m_0, A_0 as a function of $m_{1/2}$ for $\tan \beta = 55$ and for nonvanishing phases $\phi_\mu, \phi_A = \pi/3$ of μ and A . For comparison the case with $\phi_\mu, \phi_A = 0$ is also shown. In the left and right panel of this figure the cases $A_0 = 0$ and $A_0 = -500$ GeV are presented. We should remark that in our analysis we have relaxed the EDM constraints [35, 36] in order to locate possible regions of the parameter space which yield substantial deviations from the results of the CP -conserving case.

From figure 4 one sees that no significant change is observed and the results for the CP -conserving and CP -violating cases are almost identical. This holds for other values

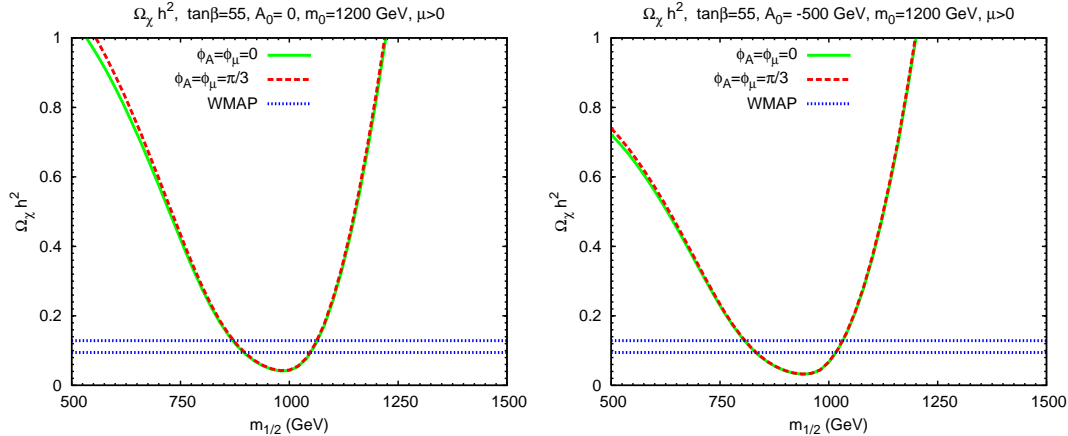


Figure 4: The relic density $\Omega_{\tilde{\chi}} h_0^2$ for fixed m_0 , as a function of $m_{1/2}$ for values of the phases and the remaining parameters shown in the figure. Only the $\mu > 0$ cases are shown. For comparison the corresponding values when the phases are zero are displayed (solid line). The horizontal dotted lines mark the cosmologically allowed region from the WMAP data.

of m_0, A_0 and phases as well showing that the effect of the phases plays no important role to the relic density. Phrased in another way, despite the fact that the CP -even and CP -odd Higgs eigenstates get mixed when CP is violated, the Higgs exchange is still the dominant mechanism in obtaining small values of the relic density, as long as we are away from the coannihilation and focus point regions. The mixing of Higgses [37, 38] induced by the phases of μ and A has little effect and this is in agreement with the findings of Ref. [39]. Thus the CP -conserving case results are not destabilized by switching on the phases of ϕ_μ, ϕ_A parameters and the conclusions reached in previous analyses [5, 12, 15, 16] hold true.

Significant changes in the relic density can occur if one allows for the existence of phases in the gaugino mass parameters mainly because the latter affect the value of the bottom quark mass as shown in [39]. In particular, the supersymmetric QCD corrections to the bottom quark mass depend on the phase ξ_3 of the gluino mass M_3 at the GUT scale. Even if we ignore the phases ϕ_μ, ϕ_A as well as the phases of the gaugino masses M_1, M_2 and the angle specifying the misalignment of the vev's of the Higgs fields, the effect of ξ_3 is quite important and can drastically change the picture. In the discussion that follows we have only allowed for a nonvanishing ξ_3 setting all other phases to zero. With only ξ_3 present the electroweak sector is not directly affected by it, up to the one loop order. At this order the phase ξ_3 affects the electroweak sector implicitly through changes in

the bottom Yukawa coupling as described earlier. In particular the Higgs sector is the same as in the CP -conserving case and the 1-loop expressions for Higgs masses still hold. It is known that the mass of the pseudoscalar Higgs A is quite sensitive to the size of low energy threshold supersymmetric corrections to the bottom Yukawa coupling [16, 39]. Consequently ξ_3 influences the position of the A -Higgs pole, $M_A \simeq 2m_{\tilde{\chi}}$, where the rapid neutralino pair annihilation yields cosmologically acceptable relic densities. In order to see the effect of the phase of M_3 on the relic density, in fig. 5 we plot the values of the pseudoscalar mass M_A , the value of the mass of the annihilated neutralino pair $2m_{\tilde{\chi}}$ and the relic density as functions of the phase ξ_3 for two characteristic sets of input parameters shown on the left and right panel. As already stated we have only switched on the phase ξ_3 . On the left panel it is observed that although the value of $\tan\beta$ is not quite high the approach to the pole, $M_A \simeq 2m_{\tilde{\chi}}$, takes place for values $\xi_3 \simeq 3\pi/4$ resulting to rapid $\tilde{\chi}$ annihilation through A -Higgs boson exchange and small relic densities in accord the WMAP data ($\Omega_{\tilde{\chi}} h_0^2 \simeq 0.1$). On the right panel and for different input values we observe that the relic density becomes significantly smaller for values $\xi \simeq \pi$, but not compatible with the WMAP bound. In both case the plots look symmetric upon $\xi_3 \rightarrow -\xi_3$. This is due to the fact that the SQCD corrections are insensitive to the sign of ξ_3 if the other phases are put to zero.

In the left panel of figure 6 we plot the neutralino relic density for various values of ξ_3 and the same input parameters $m_0 = 1200$ GeV, $A_0 = 0$, $\tan\beta = 55$, $\mu > 0$. One can see that for $\xi_3 = 0$, CP -conserving case, the minimum of the $\Omega_{\tilde{\chi}} h_0^2$ appears at $m_{1/2} = 1000$ GeV, while for $\xi_3 = \pi/6$ this occurs at a lower value $m_{1/2} = 750$ GeV. This means that as ξ_3 increases, the funnel of the right panel of figure 3 is widening up and moving to the left allowing for smaller values of $m_{1/2}$ and hence lighter supersymmetric mass spectrum in general. This is clearly seen in the right panel of figure 6 which should be compared to the right panel of figure 3⁴.

5 Conclusions and Outlook

We have presented an improved partial wave expansion technique for calculating the neutralino pair annihilation to a fermion pair, based on the helicity amplitude method. In the partial wave treatment the amplitudes carrying different total angular momenta are

⁴In figure 3 the phases ϕ_μ, ϕ_A have been taken equal to $\pi/3$ and not vanishing as in figure 6. However when ξ_3 is zero the effect of ϕ_μ, ϕ_A is negligible and the figure 3 is the same with the corresponding one in which ϕ_μ, ϕ_A are put to zero.

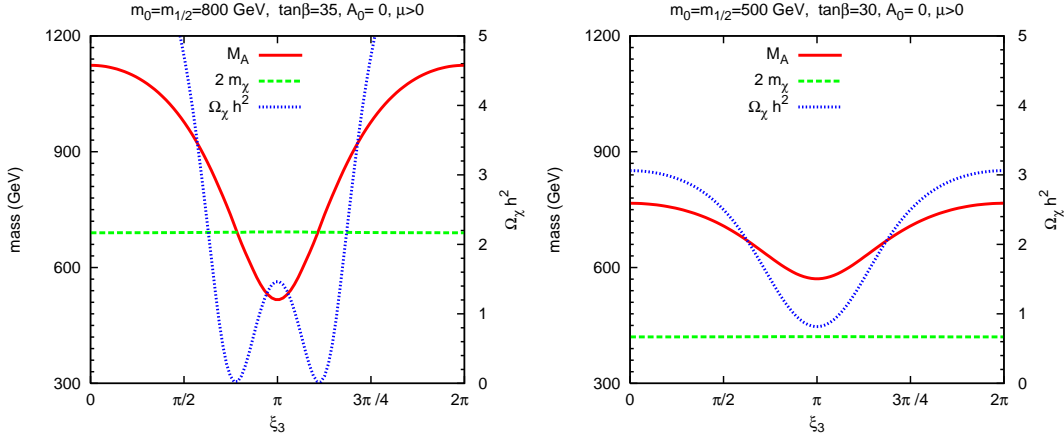


Figure 5: The mass M_A (solid line), $m_{\tilde{\chi}}$ (dashed line) and the neutralino relic density (dotted line) as functions of ξ_3 when all other phases are set to zero. In the left (right) panel $m_0 = m_{1/2} = 800$ GeV, $\tan\beta = 35$ ($m_0 = m_{1/2} = 500$ GeV, $\tan\beta = 30$), and $A_0 = 0, \mu > 0$.

summed incoherently in the total cross section, avoiding numerous interference terms, resulting to a better theoretical and numerical control of the neutralino annihilation process. Also the non relativistic expansion is avoided obtaining a more accurate numerical treatment of the thermally averaged cross section especially near poles some of which, as those of Higgses, are significant in obtaining small LSP relic densities as recent cosmological data suggest. The scheme we employ allows for contributions of partial waves of any J , offering a better approximation of the partial wave series. The methods existing in the literature are usually truncated to include terms up to p - waves in the orbital angular momentum L of the initial state of the annihilating LSP's corresponding to values of $J \leq 2$. Our scheme is formulated in such a way as to allow easy handling of CP -violations residing in the superpotential Higgs mixing parameter μ and the trilinear A soft couplings as well as in the presence of additional CP -violating phases occurring in nonminimal extensions of the MSSM.

Using this method we include non - vanishing CP -violating phases ϕ_μ, ϕ_A to the μ and A parameters of the CMSSM. We study the effect of these phases at the large $\tan\beta$ regime, where it is known that the exchanges of Higgses is one of the dominant mechanisms to reduce the values of the LSP relic density to cosmologically acceptable levels if CP is conserved. We found that even if one relaxes the EDM constraints the effect of these phases is small, in the entire region of the parameter space away from the coannihilation and focus point regions, and the values of the neutralino relic density obtained are almost

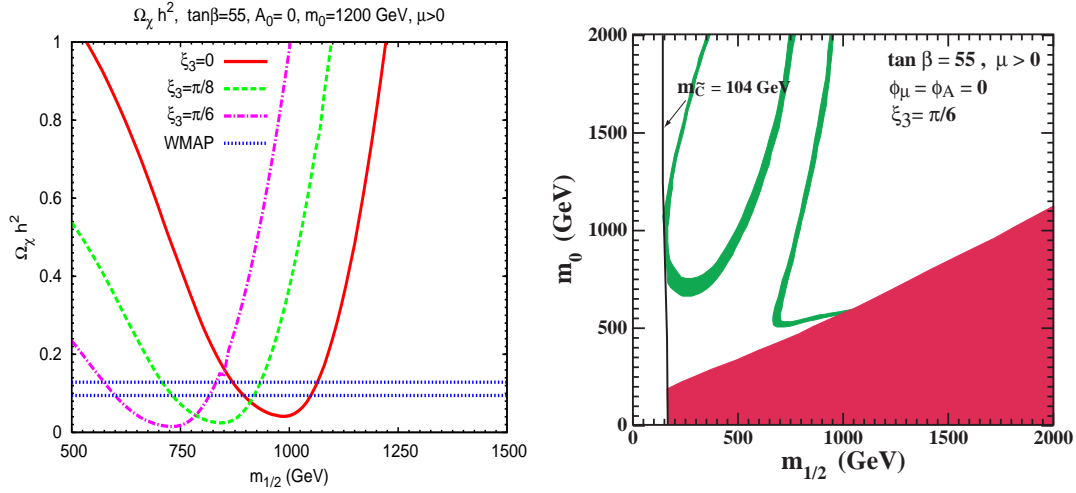


Figure 6: (Left panel) The neutralino relic density as a function of $m_{1/2}$, for $m_0 = 1200$ GeV, $A_0 = 0$, $\tan \beta = 55$, $\mu > 0$, for $\xi_3 = 0$, $\pi/8$ and $\pi/6$ (solid, dashed and dashed-dotted lines). The WMAP favored region is marked by the horizontal dotted lines. (Right panel) The neutralino relic density, in the $(m_0, m_{1/2})$ plane, for $\tan \beta = 55$ and ξ_3 equal to $\pi/6$.

the same as in the CP -conserving case. Therefore the Higgs exchange mechanism is still one of the dominant mechanism in obtaining small values for the relic density and the CP -conserving case results are not upset by switching on the phases ϕ_μ, ϕ_A . Significant changes in the relic density can occur if one allows for the existence of phases in the gaugino mass parameters as shown elsewhere [39, 40] which can only happen if one departs from the minimal scenario.

The method of partial waves presented here will be generalized to cover the subdominant processes of annihilation of the neutralino LSP's to channels other than that of a fermion pair and to include also the coannihilation processes which are important in particular regions of the parameter space. The results of such an analysis will be presented in a forthcoming publication.

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Table 1

$\lambda_3\lambda_4\lambda_1\lambda_2$	$\mu\mu'$	$\hat{F}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^{(J),\bar{f}}$
$++++$ $+- --$ $-- ++$ $----$	$\mu = 0$ $\mu' = 0$	J even $\hat{M}_{\lambda_1}^{(\lambda_3\lambda_4)} \left[\Pi_J + (-1)^{\lambda_3-\lambda_1} \left(\Pi_{J-1} \frac{J}{2J-1} + \Pi_{J+1} \frac{J+1}{2J+3} \right) \right]$
$+- ++$ $+- --$ $- + ++$ $- + --$	$\mu = 0$ $\mu' \neq 0$	J even $\mu' \hat{M}_{\lambda_1}^{(\lambda_3\lambda_4)} \left[\sqrt{J(J+1)} \left(\frac{\Pi_{J-1}}{2J-1} - \frac{\Pi_{J+1}}{2J+3} \right) \right]$
$+++-$ $--+-$ $+- --$ $----$	$\mu \neq 0, \mu' = 0$	$J \geq 1$ $\frac{\sqrt{J(J+1)}}{2} \left[-\hat{M}_2^{(\lambda_3\lambda_4)} + (-1)^J \hat{M}_3^{(\lambda_3\lambda_4)} \right] \left(\frac{\Pi_{J-1}}{2J-1} - \frac{\Pi_{J+1}}{2J+3} \right)$ <hr style="border-top: 1px dashed black;"/> $(-1)^J \otimes (\text{previous})$
$+- +-$ $- + +-$ $+- --$ $- + --$	$\mu \neq 0, \mu' \neq 0$	$J \geq 1$ $-\text{sign}(\mu') \frac{1}{2} \left[-\hat{M}_2^{(\lambda_3\lambda_4)} + (-1)^J \hat{M}_3^{(\lambda_3\lambda_4)} \right] \Pi_J +$ $+\frac{1}{2} \left[\hat{M}_2^{(\lambda_3\lambda_4)} + (-1)^J \hat{M}_3^{(\lambda_3\lambda_4)} \right] \left(\Pi_{J-1} \frac{J+1}{2J-1} + \Pi_{J+1} \frac{J}{2J+3} \right)$ <hr style="border-top: 1px dashed black;"/> $(-1)^J \otimes (\text{previous})$

Table 1: The matrices \hat{F} appearing in Eq. (16). The matrices \hat{M} are given in table 2. The \pm in the helicity column stands for $\pm\frac{1}{2}$.

Table 2

	\hat{M}
$\mu = 0, \mu' = 0$	$\hat{M}_+^{++} = \begin{pmatrix} -\hat{A}_-^2 & -\hat{A}_- \hat{B}_- \\ -\hat{A}_- \hat{B}_- & -\hat{B}_-^2 \end{pmatrix}$ $\hat{M}_-^{++} = \begin{pmatrix} \hat{A}_+^2 & -\hat{A}_+ \hat{B}_+ \\ -\hat{A}_+ \hat{B}_+ & \hat{B}_+^2 \end{pmatrix}$ $\hat{M}_+^{--} = \begin{pmatrix} \hat{A}_+^2 & \hat{A}_+ \hat{B}_+ \\ \hat{A}_+ \hat{B}_+ & \hat{B}_+^2 \end{pmatrix}$ $\hat{M}_-^{--} = \begin{pmatrix} -\hat{A}_-^2 & \hat{A}_- \hat{B}_- \\ \hat{A}_- \hat{B}_- & -\hat{B}_-^2 \end{pmatrix}$
$\mu = 0, \mu' \neq 0$	$\hat{M}_+^{+-} = \begin{pmatrix} -\hat{A}_- \hat{A}_+ & -\hat{A}_- \hat{B}_+ \\ -\hat{A}_+ \hat{B}_- & -\hat{B}_- \hat{B}_+ \end{pmatrix}$ $\hat{M}_-^{+-} = \begin{pmatrix} -\hat{A}_- \hat{A}_+ & \hat{A}_+ \hat{B}_- \\ \hat{A}_- \hat{B}_+ & -\hat{B}_- \hat{B}_+ \end{pmatrix}$ $\hat{M}_+^{-+} = \begin{pmatrix} \hat{A}_- \hat{A}_+ & \hat{A}_+ \hat{B}_- \\ \hat{A}_- \hat{B}_+ & \hat{B}_- \hat{B}_+ \end{pmatrix}$ $\hat{M}_-^{-+} = \begin{pmatrix} \hat{A}_- \hat{A}_+ & -\hat{A}_- \hat{B}_+ \\ -\hat{A}_+ \hat{B}_- & \hat{B}_- \hat{B}_+ \end{pmatrix}$
$\mu \neq 0, \mu' = 0$	$\hat{M}_2^{++} = \begin{pmatrix} \hat{A}_- \hat{A}_+ & -\hat{A}_- \hat{B}_+ \\ \hat{A}_+ \hat{B}_- & -\hat{B}_- \hat{B}_+ \end{pmatrix}$ $\hat{M}_3^{++} = \begin{pmatrix} -\hat{A}_- \hat{A}_+ & -\hat{A}_+ \hat{B}_- \\ \hat{A}_- \hat{B}_+ & \hat{B}_- \hat{B}_+ \end{pmatrix}$ $\hat{M}_2^{--} = \begin{pmatrix} \hat{A}_- \hat{A}_+ & -\hat{A}_+ \hat{B}_- \\ \hat{A}_- \hat{B}_+ & -\hat{B}_- \hat{B}_+ \end{pmatrix}$ $\hat{M}_3^{--} = \begin{pmatrix} -\hat{A}_- \hat{A}_+ & -\hat{A}_- \hat{B}_+ \\ \hat{A}_+ \hat{B}_- & \hat{B}_- \hat{B}_+ \end{pmatrix}$
$\mu \neq 0, \mu' \neq 0$	$\hat{M}_2^{+-} = \begin{pmatrix} -\hat{A}_-^2 & \hat{A}_- \hat{B}_- \\ -\hat{A}_- \hat{B}_- & \hat{B}_-^2 \end{pmatrix}$ $\hat{M}_3^{+-} = \begin{pmatrix} -\hat{A}_+^2 & -\hat{A}_+ \hat{B}_+ \\ \hat{A}_+ \hat{B}_+ & \hat{B}_+^2 \end{pmatrix}$ $\hat{M}_2^{-+} = \begin{pmatrix} -\hat{A}_+^2 & \hat{A}_+ \hat{B}_+ \\ -\hat{A}_+ \hat{B}_+ & \hat{B}_+^2 \end{pmatrix}$ $\hat{M}_3^{-+} = \begin{pmatrix} -\hat{A}_-^2 & -\hat{A}_- \hat{B}_- \\ \hat{A}_- \hat{B}_- & \hat{B}_-^2 \end{pmatrix}$

Table 2: The matrices \hat{M} appearing in table 1.

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