

Domain Wall Depinning in Random Media by AC Fields

A. Glatz,¹ T. Nattermann,^{1,2} and V. Pokrovsky^{3,4}

¹*Institut für Theoretische Physik, Universität zu Köln, Zùlpicher Str. 77, D-50937 Köln, Germany*

²*LPMMH, Ecole Supérieure de Physique et de Chimie Industrielles, 75231 Paris Cedex 05, France*

³*Department of Physics, Texas A&M University, College Station, Texas 77843-4242*

⁴*Landau Institute for Theoretical Physics, Chernogolovka, Moscow District, 142432, Russia*

(Dated: June 6, 2018)

The viscous motion of an interface driven by an *ac external field* of frequency ω_0 in a random medium is considered here for the first time. The velocity exhibits a smeared depinning transition showing a *double hysteresis* which is absent in the adiabatic case $\omega_0 \rightarrow 0$. Using scaling arguments and an approximate renormalization group calculation we explain the main characteristics of the hysteresis loop. In the low frequency limit these can be expressed in terms of the depinning threshold and the critical exponents of the adiabatic case.

PACS numbers: 75.60.-d, 74.60.Ge

The driven viscous motion of an interface in a medium with random pinning forces is one of the paradigms of condensed matter physics [1, 2, 3]. This problem arises, e.g., in the domain wall motion of magnetically or structurally ordered systems with impurities [4] or when an interface between two immiscible fluids is pushed through a porous medium [5]. Closely related problems are the motion of a vortex line in an impure superconductor [6], of a dislocation line in a solid [7] or driven charge density waves [8]. For a constant external driving force this problem has been considered close to the zero temperature critical depinning threshold [9, 10, 11, 12] and in the creep region [7, 13]. More recently, the results of this approach have been used to study hysteresis effects in magnets subjected to an external force changing *adiabatically* in time [14, 15]. It is the aim of this paper to develop a description of pinning phenomena in an *ac-field* in the weak pinning limit. As a main result we find that the zero temperature depinning transition is smeared and shows a pronounced *velocity hysteresis*. The latter has to be distinguished from the hysteresis of the magnetization which persists also in the adiabatic case [14, 15]. Using numerical simulations, scaling arguments and an approximate renormalization group (RG) calculation we derive scaling laws for the velocity hysteresis at zero temperature. Also, we discuss the influence of thermal fluctuations briefly. Despite the fact, that we use the terminology of driven interfaces, the results apply correspondingly also to all the other systems mentioned above.

Model and zero frequency critical depinning.— We focus on a simple realization of the problem, the motion of a D -dimensional interface profile $z(\mathbf{x}, t)$ obeying the following equation of motion [16]

$$\frac{1}{\gamma} \frac{\partial z}{\partial t} = \Gamma \nabla^2 z + h_0 \cos \omega_0 t + g(\mathbf{x}, z). \quad (1)$$

γ and Γ denote the mobility and the stiffness constant of the interface, respectively, and $h(t) = h_0 \sin \omega_0 t$ is the *ac* driving force. The random force $g(\mathbf{x}, z)$ is as-

sumed to be Gaussian distributed with $\langle g \rangle = 0$ and $\langle g(\mathbf{x}, z)g(\mathbf{x}', z') \rangle = \delta^D(\mathbf{x} - \mathbf{x}')\Delta_0(z - z')$. We further assume $\Delta_0(z) = \Delta_0(-z)$ to be a monotonically decreasing function of z for $z > 0$ which decays to zero over a finite distance l . Under these conditions the relation between applied force $h(t)$ and the average velocity $\langle \dot{z} \rangle = v$ shows in the steady state the *inversion symmetry* $h \rightarrow -h$, $v \rightarrow -v$ (cf. Fig. 1). We therefore restrict ourselves to the region $h > 0$ in the further discussion.

In [9, 10] eq. (1) was considered in the adiabatic limit $\omega_0 \rightarrow 0$. In this case the interface undergoes a second order depinning transition at $h_0 = h_P$ where the velocity v vanishes as a power law $v \sim (h_0 - h_P)^\beta$ for $h_0 \searrow h_P$, $\beta \leq 1$. At $h_0 = h_P$ the interface is self-similar with a roughness exponent ζ , $0 \leq \zeta < 1$, and the dynamics is superdiffusive with a dynamical exponent z , $1 \leq z \leq 2$. The critical exponents were calculated up to order $\epsilon = 4 - D$ in [9, 10] and recently to order ϵ^2 in [12], and are related by the scaling laws $\beta = \nu(z - \zeta)$ and $\nu = 1/(2 - \zeta)$ [9], where ν denotes the correlation length exponent: $\xi_0 \sim |h_0 - h_P|^{-\nu}$. For $h_0 \nearrow h_P$ the divergence of ξ_0 is related to the increasing size of avalanches.

In the case of an *ac-drive* the behavior of the system is governed by the two dimensionless quantities h_0/h_P and ω_0/ω_P , where $\omega_P = \gamma h_P/l$. In this paper we mainly focus on the most interesting case $0 < \omega_0 \ll \omega_P$ and $h_0 > h_P$ since this is the region where universality is expected to hold. As illustrated by the numerical solution of eq. (1) for $D = 1$ at finite ω_0 , the sharp depinning transition is replaced by a velocity hysteresis, which has clockwise rotation: the velocity reaches zero at $h(t) = \pm h_c$ for decreasing and increasing field, respectively (see Fig. 1). For $h_0 \gg h_P$ a second weak hysteresis is found in the region $h > h_c \approx h_P$ which has anticlockwise rotation.

Scaling considerations.— First we consider the relevant length scales of the problem, beginning with $\omega_0 = 0$. Comparison of the curvature and the random force term on r.h.s. of (1) shows, that weak random forces accumulate only on the Larkin scale $L_P \approx ((\Gamma l)^2/\Delta_0(0))^{1/(4-D)}$

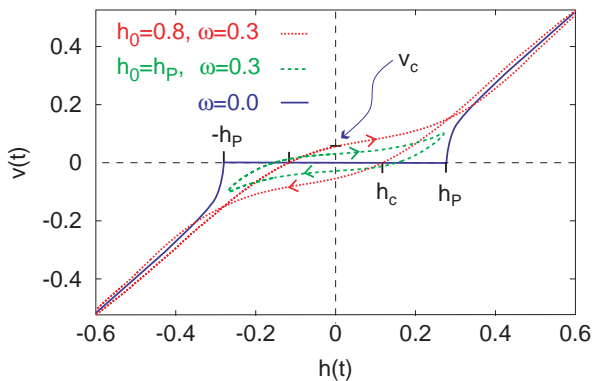


FIG. 1: Numerical solution of eq. (1) for a 1D interface in the dc (solid line) and ac case (dotted line) for $\omega = \omega_0/\omega_P = 0.3$ and $h_0 > h_P \approx 0.27$. x is discretized in $N = 1000$ sites and g random in $[-0.5, 0.5]$. The solution is averaged over typically 100 disorder (g -) configurations. The arrows show the direction of the hysteresis, for $|h(t)| < h_P$ it is clockwise and anticlockwise for $|h(t)| > h_P$. The dashed line shows the hysteresis in the case $h_0 \approx h_P$.

to a value comparable to the curvature force. On scales $L < L_P$ the curvature force density ΓL^{-2} is larger than the pinning force density, the interface is essentially flat and hence there is no pinning. For $L > L_P$ pinning force densities exceed the curvature forces, the interface becomes rough and adapts to the spatial distribution of pinning forces. The largest pinning force density then results from $L \approx L_P$ from which one estimates the depinning threshold $h_P \approx l\Gamma L_P^{-2}$. On scales $L \gg L_P$ perturbation theory breaks down. The RG calculation performed in [9, 10, 11, 12] resulted in a scale dependent mobility and renormalized pinning forces. A finite frequency ω_0 of the driving force acts as an infrared cut-off for the propagation of perturbations, resulting from the local action of pinning centers on the interface. As follows from (1) (with $\gamma \rightarrow \gamma(L/L_P)^{2-z}$ for $L > L_P$ [9]) these perturbations can propagate up to a length scale $L_\omega = L_P(\gamma\Gamma/\omega_0 L_P^2)^{1/z} \equiv L_P(\omega_P/\omega_0)^{1/z}$ [17]: (i) If $L_\omega < L_P$, i.e., $\omega_0 > \omega_P$, z has to be replaced by 2. During one cycle of the ac drive, perturbations resulting from local pinning centers affect the interface configuration only up to scale L_ω , such that the resulting curvature force is always larger than the pinning force – there is no pinning anymore and the velocity hysteresis disappears. (ii) In the opposite case $L_\omega > L_P$, i.e., $\omega_0 < \omega_P$, the pinning forces can compensate the curvature forces at length scales larger than L_P . As a result of the adaption of the interface to the disorder pinning forces are renormalized. This renormalization is truncated at L_ω . In the following we will argue, that, contrary to the adiabatic limit $\omega_0 \rightarrow 0$, there is no depinning transition if $\omega_0 > 0$. Indeed, a necessary condition for the existence of a sharp transition in the adiabatic case was the requirement, that the fluctuations of the depinning threshold in a corre-

lated volume $\delta h_P \approx h_P(L_P/\xi_0)^{(D+\zeta)/2}$ are smaller than $(h - h_P)$, i.e., $(D + \zeta)\nu \geq 2$ [9]. For $\omega_0 > 0$ the correlated volume has a maximal size L_ω and hence the fluctuations δh_P are given by

$$\frac{\delta h_P}{h_P} \approx \left(\frac{L_P}{L_\omega}\right)^{(D+\zeta)/2} = \left(\frac{\omega_0}{\omega_P}\right)^{(D+\zeta)/(2z)}. \quad (2)$$

Thus, different parts of the interface see different de-

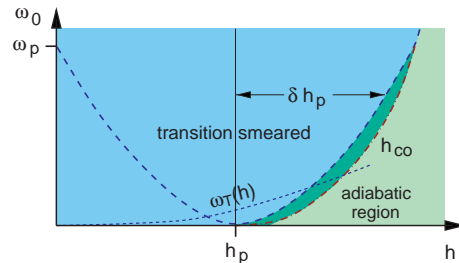


FIG. 2: Schematic frequency-field diagram for the depinning in an ac external field (with $h_0 > h_P$): For $0 < \omega_0 \ll \omega_P$ the depinning transition is smeared but traces of the $\omega_0 = 0$ transition are seen in the frequency dependency of the velocity at $h = h_P$. This feature disappears for $\omega_0 \gg \omega_P$.

pinning thresholds – the depinning transition is *smeared*. δh_P has to be considered as a lower bound for this smearing. A full understanding of the velocity hysteresis requires the consideration of the coupling between the different L_ω -segments of the interface, which we will do further below. When approaching the depinning transition from sufficiently large fields, $h_0 \gg h_P$ ($\omega_0 \ll \omega_P$), one first observes the critical behavior of the adiabatic case as long as $\xi_0 \ll L_\omega$. The equality $\xi_0 \approx L_\omega$ defines a field h_{c0} signaling a cross-over to an *inner* critical region where singularities are truncated by L_ω . Note that $h_{c0} - h_P = h_P(\omega_0/\omega_P)^{1/(\nu z)} \geq \delta h_P$ (cf. Fig. 2). It is then obvious to make the following scaling Ansatz for the mean interface velocity ($h_0 > h_P$, $v_P = \omega_P l$)

$$v(h(t)) \approx v_P \left(\frac{\omega_0}{\omega_P}\right)^{\frac{\beta}{z}} \phi_{\pm} \left[\left(\frac{h}{h_P} - 1\right) \left(\frac{\omega_P}{\omega_0}\right)^{\frac{1}{\nu z}} \right]. \quad (3)$$

Here the subscript \pm refers to the cases of $\dot{h} \gtrless 0$, respectively, and $\phi_{\pm}[x \rightarrow \infty] \sim x^{\beta}$ (for $h - h_P \gg h_P$ the classical exponent $\beta = 1$ applies). For $|x| \ll 1$, ϕ_{\pm} approaches a constant c_{\pm} . Function ϕ_{-} changes sign at a critical value $h_c(\omega_0) \approx h_P(1 - c_{-}(\omega_0/\omega_P)^{1/(\nu z)})$. Direct numerical solution of eq. (1) in $D = 1$ is in good agreement with this prediction as shown in Fig. 3.

Renormalized perturbation theory.— To consider the coupling between different L_ω -segments we treat model (1) in perturbation theory. After going over to a co-moving frame one obtains in lowest non-trivial order in g the following equation for the velocity $v = \dot{z}_0(t)$, with

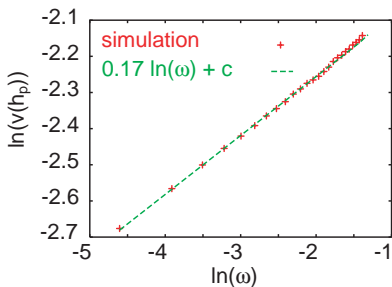


FIG. 3: Numerical results for $v(h_P)$, using eq. (1), as a function of $\omega = \omega_0/\omega_P$ in $D = 1$. The dashed line shows the prediction of eq. (4) with $\beta/\nu z = 0.17$, in good agreement with the value 0.19 found in [12].

$z_0(t) \equiv \langle z(\mathbf{x}, t) \rangle$ [16]:

$$\frac{1}{\gamma} v(t) = h(t) + \int_0^\infty dt' \int_{\mathbf{p}} \langle \tilde{g}'_{\mathbf{p}}(z_0(t)) \tilde{g}_{-\mathbf{p}}(z_0(t-t')) \rangle \times \\ \times \gamma e^{-\Gamma \mathbf{p}^2 t'} \equiv h(t) + r_0(t). \quad (4)$$

Here $\int_{\mathbf{p}} = \int \frac{d^D \mathbf{p}}{(2\pi)^D}$ and $\tilde{g}_{\mathbf{p}}(z) = \int d^D x e^{i\mathbf{p}\mathbf{x}} g(\mathbf{x}, z)$. Replacing the pair correlator of the random forces by $\Delta_0(z_0(t, t'))$ where $z_0(t, t') = \int_{t-t'}^t dt'' v(t'')$ we get

$$r_0(t) \sim \int_0^\infty dt' (1 + \omega_P t')^{-D/2} \Delta'_0(z(t, t')). \quad (5)$$

which results in corrections to the driving force and to the mobility which are in general non-local in time. If we assume that $\Delta_0(z)$ is an analytic function of its argument, it is easy to show that for $\omega_0 \gg \omega_P$, $r_0(t)$ is of the order $(\omega_P/\omega_0)^2$ and hence small. In this parameter region the pinning potential merely slows down the motion of the wall in agreement with the result of our scaling considerations. In the opposite case, $\omega_0 \ll \omega_P$, perturbation theory breaks down. To treat this frequency region it is instructive to consider first the case of a *dc drive*, $h(t) \equiv h_0$, where the velocity is constant and hence $z(t, t') = vt'$. The t' -integral (5) leads to a correction of the mobility which diverges as $(l/v)^{(4-D)/2}$ for $v \rightarrow 0$ and $D < 4$. This divergence could be removed by a RG treatment developed in [9, 10]. As a result of the elimination of the Fourier components $z_{\mathbf{p}'}$ with $|\mathbf{p}'| < |\mathbf{p}| < L_P^{-1}$ from eq. (1), γ and $\Delta_0(z)$ are replaced there by the renormalized quantities

$$\gamma(p) \simeq \gamma(pL_P)^{-2+z}, \quad (6)$$

$$\Delta_p(z) \approx K_D^{-1} (\Gamma l / L_P^\zeta)^2 p^{4-D-2\zeta} \Delta^*(z(pL_P)^\zeta / l). \quad (7)$$

$\Delta^*(x)$ exhibits a cusp-like singularity at $x = 0$ which develops on scales larger than L_P . In particular, $\Delta^*(x) \approx 1 - \sqrt{\epsilon - 2\zeta}|x| + (\epsilon - \zeta)x^2/6 + \mathcal{O}(|x|^3)$ for $|x| \ll 1$ [9, 10] and $\Delta^* e^{-\Delta^*} = e^{-1-x^2/6}$ for $x \gg 1$ [18]. In this way one generates a *renormalized* equation of motion which

serves as starting point for a *convergent* perturbative expansion. The replacements (6), (7) are valid for momenta $\xi^{-1} < p < L_P^{-1}$ where ξ denotes here the correlation length ξ_0 of the zero frequency depinning transition. In the spirit of the RG treatment fluctuations on scales larger than ξ can be neglected since they are uncorrelated. To get the lowest order corrections in the convergent expansion one has to replace in eq. (4) the bare quantities by the renormalized ones. This leads in the limit $v \rightarrow 0$ to $r_0 = h_P \Delta^{*'}(0^+) / (2 - \zeta) \equiv -\tilde{h}_P$ which is the RG result for the threshold value \tilde{h}_P . Replacing in addition γ by $\gamma(\xi^{-1})$ on the l.h.s. of eq. (4), one obtains the correct result for the critical behavior of the velocity: $v \approx \gamma(\xi/L_P)^{2-z} (h - \tilde{h}_P) \approx v_P ((h - \tilde{h}_P)/\tilde{h}_P)^\beta$.

In the case of an *ac drive* the velocity $v(t)$ is periodic with $2\pi/\omega_0$. In each cycle of $h(t)$ the velocity goes through a region of small values, in which perturbation theory gives a contribution to γ^{-1} proportional to $(l/v(t))^{\frac{4-D}{2}}$ as long as the period is large compared to $l/v(t)$. The cutoff t_c of the t' -integration is given by the approximate relation $t_c^{-1} \approx \omega_0 + v(t)/l$. These contributions are still large at $\omega_0, v(t) \rightarrow 0$. This breakdown of perturbation theory can be overcome by using the RG results discussed above on intermediate length scales as in the dc case. Such a procedure is justified for momenta in the range $L_P^{-1} > |\mathbf{p}| \gg \xi^{-1}$, where ξ is the minimum of ξ_0 and L_ω . At these scales the interface is still at criticality. Since ξ_0 depends via $h(t)$ on time and eq. (4) includes retardation effects, a time dependent cutoff complicates the problem. Therefore we will restrict our consideration to the inner critical region where $\xi \approx L_\omega$, i.e., $|h_P - h| < h_P(\omega_0/\omega_P)^{1/\nu z}$. The renormalized effective equation of motion follows from (4) with the replacements (6) and (7) [19]:

$$\frac{v(t)}{\gamma(L_\omega^{-1})} = h(t) + h_P \omega_P \int_0^\infty dt' \int_{\tilde{L}_\omega^{-1}}^1 d\tilde{p} \tilde{p}^{1+z-\zeta} \times \\ e^{-\omega_P \tilde{p}^2 t'} \Delta^{*'} \left(\int_{t-t'}^t dt'' v(t'') \tilde{p}^\zeta / l \right), \quad (8)$$

with $\tilde{L}_\omega = (\omega_P/\omega_0)^{1/z}$ and $\tilde{p} = pL_P$. With that form of the equation of motion, we can explain the hysteresis appearing for $|h| < h_P$ (cf. Fig. 1) more detailed. First we consider $\dot{h} < 0$: At h_c the sign of the velocity changes although the driving force is still positive. This can be understood as follows: until time $t = t_c$, with $h(t_c) = h_c$, the velocity was positive during half a period, hence the argument of the $\Delta^{*'}$ function is positive. Therefore the second term of the r.h.s of eq. (8) is negative and cancels the positive driving force. To solve eq. (8) analytically we consider a parameter region where the argument of $\Delta^{*'}$ is small compared to unity, i.e., $\Delta^{*'}(x) \approx \Delta^{*'}(0^+) \text{sgn}(\int_{t-t'}^t dt'' v(t''))$. One can show a posteriori that this condition is satisfied if $h_0 = \mathcal{O}(h_P)$. With this approximation the momentum integral in (8)

can be calculated, and we get:

$$\frac{v(t)}{\gamma \tilde{L}_\omega^{2-z}} \approx h(t) - \frac{\tilde{h}_P}{\nu z} \left[S(t, \omega_P) - \tilde{L}_\omega^{-\frac{1}{\nu}} S(t, \omega_0) \right] \quad (9)$$

Here $S(t, \omega) \equiv \int_0^\infty d\tau \tau^{-\delta} \tilde{\Gamma}_\delta(\tau) \text{sgn} z_0(t, \tau/\omega)$, $\delta = 1/(\nu z) + 1$ and $\tilde{\Gamma}_\delta(\tau) \equiv \Gamma_\delta(0) - \Gamma_\delta(\tau)$, where $\Gamma_\delta(\tau) = \int_\tau^\infty dt t^{\delta-1} e^{-t}$. $z_0(t, \tau/\omega)$ changes its sign at $t = t_0 + n\pi/\omega_0$, $n \in \mathbb{Z}$. The dominating part to $S(t, \omega)$ comes from $\tau < \mathcal{O}(1)$. To solve this integral equation for $h \geq 0$ and $\omega_0 \ll \omega_P$, we note, that the sign of $z_0(t, \tau/\omega_P)$ is always positive for $\tau < 1$ and hence $S(t, \omega_P) \approx \nu z$. For $t \lesssim t_c$ also $z_0(t, \tau/\omega_0) > 0$ for the dominating small τ region of the τ -integration in $S(t, \omega_0)$. This leads to $\phi_-(x) \approx c_- + x$, $c_- \approx S(t_c, \omega_0)/\nu z$. By decreasing t , $S(t, \omega_0)$ is diminished since regions with negative $z_0(t, \tau/\omega_0)$ contribute increasingly, which in turn explains the second weak hysteresis observed in Fig. 1 [20]. Next we consider the region $t \gtrsim t_c$, i.e., $h < h_c$, $v < 0$. By increasing t , $S(t, \omega_0)$ is reduced with respect to $S(t_c, \omega_0)$ which leads to a positive curvature of $v(h)$ for $h < 0$. Although that region is beyond the scope of our RG calculation, since retardation effects require to consider the avalanche motion in the region $h < h_c$, it is then tempting to conclude $|v(h=0, h < 0)| = \mathcal{O}(\omega_0/\omega_P)^{\beta/\nu z}$. For large negative values of h , $v(h)$ has to reach again the result of the adiabatic limit. Together with the inversion symmetry this explains the inner hysteresis.

Thermal fluctuations.— Finally we consider the influence of thermal fluctuations on the force - velocity relation, restricting ourselves to the low temperature region $T \ll T_P = \Gamma l^2 L_P^D$, T_P is a typical pinning energy. (i) In the adiabatic limit and for $|h_0 - h_P| \ll h_P$, the velocity obeys the scaling relation $v(h, T) = (h - h_P)^\beta \psi[(h - h_P)^\theta/T]$ where θ is a new exponent which depends on the shape of the potential at the scale L_P [21]. For $\omega_0 > 0$ one can extend the scaling relation (3) to a second scaling field $\frac{h-h_P}{h_P} \left(\frac{T_P}{T}\right)^{1/\theta}$ and one finds in particular for $h \approx h_P$

$$v(h_P, T) \approx v_P \left(\frac{\omega_0}{\omega_P}\right)^{\frac{\beta}{\nu z}} \tilde{\phi}_\pm \left[\left(\frac{T}{T_P}\right)^{\frac{1}{\theta}} \left(\frac{\omega_P}{\omega_0}\right)^{\frac{1}{\nu z}} \right], \quad (10)$$

with $\tilde{\phi}_\pm[x \rightarrow \infty] \sim x^\beta$ and $\tilde{\phi}_\pm[x \rightarrow 0] \sim \tilde{c}_\pm$. The thermal smearing of the zero frequency depinning transition is still seen at finite ω_0 as long as $\omega_0 < \omega_T(h_P) \approx \omega_P \left(\frac{T}{T_P}\right)^{\nu z/\theta}$. On the other hand for small fields, $h \ll h_P$, the domain wall shows creep behavior $v(h, T) \approx v_P e^{-\frac{T_P}{T} \left(\frac{h_P}{h}\right)^\mu}$, $\mu = \frac{2\zeta+D-2}{2-\zeta}$, where ζ denoted the equilibrium roughness exponent [7]. (ii) It was shown in [15], that the creep law is valid also at finite frequencies as long as $\omega_0 \ll \omega_T(h) \approx \omega_P e^{-\frac{T_P}{T} \left(\frac{h_P}{h}\right)^\mu}$, $h \ll h_P$. For $\omega_0 > \omega_T(h)$ and $h_0 \ll h_P$ thermal effect are inessential. Thus, in the region $\omega_0 \ll \omega_T(h)$ (Fig. 2) the force

- velocity relation is that of the adiabatic case at finite temperature.

To conclude, we have shown, that the sharp depinning transition of an interface driven by a dc field is smeared showing a pronounced velocity hysteresis, when the external drive is oscillating. The size of the hysteresis is described by the power laws eqs. (3) and (10) which are supported by an approximate renormalization group analysis and a numerical simulation. The case $h_0 < h_P$ will be the subject of forthcoming studies.

We thank T. Emig, B. Rosenow, S. Scheidl, and in particular S. Stepanow for fruitful discussions. V. P. acknowledges support by NSF under the grant DMR 0072115 and DOE under the grant DE-FG03-96ER45598, and A.G. and T.N. by Sonderforschungsbereich 608.

-
- [1] D.S. Fisher, Phys. Rep. **301**, 113 (1998).
 - [2] M. Kardar, Phys. Rep. **301**, 85 (1998).
 - [3] J.P. Sethna, K.A. Dahmen, and C.R. Myers, Nature **410**, 242 (2001).
 - [4] see, e.g., P.A. Young (ed.) *Spin Glasses and Random Fields*, World Scientific, Singapore 1999.
 - [5] M.A. Rubio, C.A. Edwards, A. Dougherty, and J.P. Golub, Phys. Rev. Lett. **63**, 1685 (1989).
 - [6] G. Blatter, M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin, and V.M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994); T. Nattermann and S. Scheidl, Adv. Phys. **49**, 607 (2000).
 - [7] L.B. Ioffe and V.M. Vinokur, J. Phys. C **20**, 6149 (1987).
 - [8] R.E. Thorne, Physics Today **49**, 42 (May 1996).
 - [9] T. Nattermann, S. Stepanow, L.-H. Tang, and H. Leschhorn, J. Phys. II France **2**, 1483 (1992).
 - [10] O. Narayan and D. S. Fisher, Phys. Rev. B **48**, 7030 (1993).
 - [11] D. Ertas and M. Kardar, Phys. Rev. E **49**, R2532 (1994).
 - [12] P. Chauve, P. Le Doussal and K. Wiese, Phys. Rev. Lett. **86**, 1785 (2001) and cond-mat/0205108.
 - [13] T. Nattermann, Europhys. Lett. **4**, 1241 (1986); S. Lemerle et al., Phys. Rev. Lett. **80**, 849 (1998).
 - [14] I.F. Lyuksyutov, T. Nattermann, and V. Pokrovsky, Phys. Rev. B **59**, 4260 (1999).
 - [15] T. Nattermann, V. Pokrovsky, and V.M. Vinokur, Phys. Rev. Lett. **87**, 197005 (2001).
 - [16] M.V. Feigel'man, Sov. Phys. JETP **58**, 1076 (1983). We neglected an inertial term $\rho \frac{\partial^2 z}{\partial t^2}$ which is justified as long as $\gamma \omega_0 \rho \ll 1$. A velocity hysteresis due to inertial effects has been considered by J.M. Schwarz and D.S. Fisher, Phys. Rev. Lett. **87**, 096107 (2001).
 - [17] The consideration of higher harmonics in the interface motion results in the existence of additional length scales $L_{n\omega}$ which are however of the same scale as L_ω .
 - [18] D.S. Fisher, Phys. Rev. Lett. **56**, 1964 (1985).
 - [19] We neglect here contributions from momenta larger than L_P^{-1} which are expected to have a small effect if $\omega_0 \ll \omega_P$.
 - [20] A similar hysteresis loop has been seen experimentally in type-II superconductors, see, e.g., V. Metlushko et al., cond-mat/9804121 (1998).
 - [21] A.A. Middleton, Phys. Rev. Lett. **68**, 670 (1992).