

ACT-45  
CERN-TH.6229/91  
CTP-TAMU-66/91

## On the connection between Quantum Mechanics and the geometry of two-dimensional strings

**John Ellis** and **N.E. Mavromatos**

Theory Division, CERN, CH-1211, Geneva 23, Switzerland.  
and

**D.V. Nanopoulos**

Center for Theoretical Physics, Dept. of Physics,  
Texas A & M University, College Station, TX 77843-4242, USA  
and

Astroparticle Physics Group  
Houston Advanced Research Center (HARC),  
The Woodlands, TX 77381, USA

and  
Theory Division, CERN, CH-1211, Geneva 23, Switzerland.

### Abstract

On the basis of an area-preserving symmetry in the phase space of a one-dimensional matrix model - believed to describe two-dimensional string theory in a black-hole background which also allows for space-time foam - we give a geometric interpretation of the fact that two-dimensional stringy black holes are consistent with conventional quantum mechanics due to the infinite gauged 'W-hair' property that characterises them.

ACT-45  
CERN-TH.6229/91  
CTP-TAMU-66/91  
September 1991



# 1 Introduction and Summary

In a recent paper [1] we identified the mechanism by which we believe that quantum mechanics is reconciled with general relativity, in the context of string theory in a two-dimensional target space-time. This identification was based on studies of  $c = 1$  matrix models [2] that solve, in a certain limit, two-dimensional quantum gravity with matter coupled to a Liouville mode. The latter can be seen to correspond to coset  $\frac{SU(1,1)}{U(1)}$  Wess-Zumino models [3, 4]. The crucial observation, due to Witten [4], is that such models can be interpreted as two-dimensional target space stringy  $\sigma$ -models in a black hole background metric. It is then argued that the  $c = 1$  matrix model is the last stage of the black hole evaporation.

The  $c = 1$  matrix model can be represented as a theory of essentially *free fermions*, and as such it possesses an infinity of conserved currents of high spin [5]. After bosonisation [6] such conservation laws have been seen to give rise to an infinity of (target space) conserved charges [7], that are known to form a Cartan subalgebra of a  $W_\infty$ -algebra <sup>1</sup>[8].

In singular space-times, symmetries do not imply the existence of conserved quantities [10, 11], unless the symmetries are *gauged* by coupling the corresponding currents to long-range fields [12]. In ref. [1] we argued that this is just what happens in the case of two-dimensional black holes, thus ensuring the maintenance of quantum coherence, which in local point-like theories has been argued to be violated due to black hole evaporation [13]. In other words we identified the infinite set of hair, associated with a  $w_\infty$ -algebra of the target space, which we call ‘W-hair’, that reconciles quantum mechanics with non-perturbative quantum gravity as described by two-dimensional string theory. We illustrated this identification in ref. [1] by exhibiting the conserved current that couples to one of the ‘massive’ topological ‘Q-graviton’ states that constitutes the last phase of black hole evaporation. Such a discrete string state is the first of an infinite tower of stringy states with standard ghost number and definite energy and momentum, which in two target space-time dimensions constitute the only remnants of the higher-spin string modes [14]. It should be remembered that the ‘tachyon’, which is actually massless in this case, is the only fully-fledged field of the theory. We conjectured that all of these massive modes are associated with stringy *gauge* symmetries in target space, which lead to

---

<sup>1</sup>To avoid confusion we should mention that the  $W_\infty$ -symmetries we are talking about are the  $w_\infty$ -symmetries in the notation of Pope *et al.* [8], which admit central extension only in the usual Virasoro sector, generated by spin-2 fields. In this sense the algebra may be considered as the classical limit of the centrally extended (beyond Virasoro sectors)  $W_\infty$ -algebras discovered by Pope, Romans and Shen [8]. It should be noted that at present a geometric interpretation exists only for  $w_\infty$ -algebras, as being a subgroup of area-preserving diffeomorphisms on a certain two-manifold [9]. It is mainly this kind of extended conformal algebras that we shall be concerned with in this work, and from now on we stick to the  $w$ -notation, although in the relevant literature usually there is no notational distinction [7].

the infinity of conserved charges of the matrix model, viewed as the last stage of black hole evaporation which respects the conservation laws.

Subsequently, and independently, Moore and Seiberg [15] confirmed the association of the infinite set of conserved charges in the  $c = 1$  system with the discrete topological modes of the two-dimensional string, by showing that for the particular values of energy and momentum that characterised the discrete high-spin modes one can find field transformations that leave the action of the matrix model (in the fermion formalism) *invariant*. They showed that these transformations generate  $w_{\infty+1}$ -symmetries for the target space-time. In particular they found a larger infinite set of conserved charges, as compared to the results of [7], which obey the commutation relations of a  $w_{\infty+1}$ -algebra. The charges of [7] constitute a commuting subset (Cartan subalgebra) of this algebra. In this way Moore and Seiberg confirmed the existence of ‘W-hair’ characterising the black hole solution of the associated  $\sigma$ -model theory. For completeness we review their results in the fermion formalism of the  $c = 1$  matrix model.

The action takes the form

$$S = \frac{1}{4} \int dt \left[ \left( \frac{d\psi(t)}{dt} \right)^2 - \psi(t)^2 \right] \quad (1)$$

where  $t$  is a *Euclidean time* and will be identified with the target space time in the effective field theory (after a Wick rotation). Symmetries of the action are found by redefining the fermion variables  $\psi(t)$  according to [15],

$$\delta^{s,l} \psi(t) = i \frac{\partial}{\partial p} [H^{s,l}] e^{ilt} \quad (2)$$

where  $p = \frac{d\psi(t)}{dt}$ ,  $s$  is a positive integer,  $l = s, s - 2, \dots - s$ , and  $H^{s,l} = (-ip - \psi)^{\frac{s+l}{2}} (-ip + \psi)^{\frac{s-l}{2}}$ . For these particular values of  $l$  the variation of the action (1) vanishes. It can be shown from the analysis of [15] that these values precisely correspond to the extra discrete modes that we mentioned above. Such modes may be seen to correspond formally to what one would call ‘boundary operators’ in matrix models [16], which however for *the particular* values of momentum  $l$  given above become ‘bulk’ operators corresponding to the discrete massive high-spin states of the two-dimensional string theory obtained as a certain continuum limit of the matrix model. This infinite family of symmetries is generated by conserved charges of the form  $O^{s,l} = H^{s,l} e^{ilt}$  which are shown to obey the commutation relation of a  $w_{\infty+1}$  algebra,

$$[O^{s,l}, O^{s',l'}] = (s'l - s'l') O^{s+s'-2, l+l'} \quad (3)$$

The charges discussed in [7] the beginning correspond to the commuting subgroup  $l = 0$ . Notice the explicit time dependence of the charges for  $l \neq 0$ . This is analogous to the case of the generators of Lorentz boosts in conventional field theories <sup>2</sup>. In

---

<sup>2</sup>We thank L. Alvarez-Gaumé for an illuminating discussion on this point.

view of the conjectured equivalence of the matrix model and  $\sigma$ -model formalism one expects that upon bosonising the theory, by taking into account the extra operators, one would recover this set of symmetries as target space-time symmetries of the corresponding (1+1)-dimensional  $\sigma$ -model. In this way the conserved charges could be expressed as functionals of the target-space backgrounds integrated over the ‘space’ (Liouville coordinate) of the two-dimensional string [1]. In this framework the symmetries of the matrix model could be viewed as *exact* (anomaly-free) target-space *gauge* symmetries of the  $\sigma$ -model, formulated in higher genera.

There are two ways in which such symmetries are realised in the  $\sigma$ -model approach to string theory. In one of them, the symmetries correspond to redefinitions,  $\delta X$ , of the  $\sigma$ -model target space coordinates  $X$  that leave the Fradkin-Tseytlin (FT) functional invariant [17]. In this way the symmetries are dissociated (at least formally) from the conformally invariant critical point, given that the FT generating functional can be defined away from the fixed point [18]. In the alternative approach [19], conformal invariance of the underlying world-sheet field theory is essential. In this approach the symmetries are associated with a family of string vacua. A target space stringy symmetry in this case would imply the existence of infinitesimal (or finite) conformal field theory operators that generate isomorphisms between conformal field theories with target space fields  $\{g^i\}$  and  $\{g^i + \delta g^i\}$  [19]. The formal criteria for such a symmetry can be summarised as follows [19],

(i) Any infinitesimal world-sheet operator  $h$  generates an isomorphism between conformal field theories with stress tensors  $T$  and  $T + i[h, T]$ <sup>3</sup>.

(ii) Two conformal field theories with stress tensors that differ by an operator  $\chi_{(1,1)}$  of conformal spin (1,1) correspond to backgrounds that differ by an infinitesimal change appropriate to  $\chi_{(1,1)}$ . This follows from the assumption that in a  $\sigma$ -model language one considers as physical (background) space-time fields the ones that are in one-to-one correspondence with primary fields of dimension (1, 1). However this restriction does not prevent other operators of the theory from being important in determining the structure of space-time symmetries, as we shall discuss shortly.

(iii) Symmetries of the target space of the string are, therefore, generated by currents  $J$  of conformal spin (1,0) or (0,1) (this is so because in that case the charge  $\int d\sigma J \equiv h$  has the property that  $i[h, T] = \chi_{(1,1)}$  [19]).

It is understood that appropriate extensions to finite transformations are expected to hold, but the situation is not clear. Discrete string symmetries might form a

---

<sup>3</sup>We use Euclidean notation with  $T_{zz} \equiv T, T_{\bar{z}\bar{z}} \equiv \bar{T}$ , and  $T$  is the total stress tensor including the ghost contributions. It is related to the Hamiltonian  $H_g$  of the  $\sigma$ -model by  $\int d\sigma(T + \bar{T})$ , with  $\sigma$  a spatial world-sheet coordinate.

different category, although the belief is that they also can be incorporated in the second scheme.

It is not clear at present whether the two approaches are equivalent. Certainly there are examples where a change of variables in the FT functional induces non-trivial changes in the background fields, but it is not yet known whether this includes the full set of stringy symmetries <sup>4</sup>. We conjecture [1] that in the particular case of two-dimensional strings, the  $w_{\infty+1}$ -symmetry of the target space can be represented as a change in  $\sigma$ -model fields. We base this conjecture mainly on intuition at this stage, but also on some preliminary results in simplified systems characterised by  $W_{\infty}$ -symmetries <sup>5</sup>. Our intuition is based on the so-called  $W$ -gravity systems, which are known to possess extended conformal symmetries [21]. For instance, the action of a two-dimensional free complex scalar field,  $\int d^2z \partial\phi\bar{\partial}\phi^*$ , is known to possess  $W_{\infty}$  as well  $w_{\infty}$ -symmetries, which are represented as appropriate changes of the field  $\phi$  [21]. A complex scalar field may be considered as parametrising a complexified two-dimensional target space, and the above theory is nothing other than string theory in a flat space-time background <sup>6</sup>. In this picture the world-sheet symmetry is realised as a transformation of the spacetime coordinates [17] which leaves the flat background invariant, and in this sense it is trivial from a target space-time point of view <sup>7</sup>. Once non-trivial backgrounds (field interactions) are introduced one expects that generalised coordinate transformations will induce changes in these backgrounds, and in this way a world-sheet symmetry is elevated into a target space one. We argue below that this is precisely the case with the particular background corresponding to matrix models.

In any case, irrespectively of the form of the symmetries in the  $\sigma$ -model language, their existence would lead to Ward identities of amplitudes mixing various string

---

<sup>4</sup>For Bosonic strings, the higher gauge symmetries corresponding to massive multiplets up to rank four are known to be obtained from appropriate transformations of the target-space coordinates  $X^M \rightarrow X^M + \zeta_{NP}^M(X)\partial X^N\bar{\partial}X^P$ ,  $\partial$  being a world-sheet derivative [20].

<sup>5</sup>We thank E. Kiritsis and I. Bakas for interesting discussions on their preliminary results on this issue.

<sup>6</sup>It goes without saying that this correspondence is only formal since string theory on flat backgrounds cannot be formulated consistently in two target-space dimensions. To this end one should employ non-trivial backgrounds.

<sup>7</sup>There is an important difference from the usual critical string case, in that the target space coordinate transformations contain an explicit dependence on the world-sheet coordinates which appears in the respective parameter. For example the flat background  $W_{\infty}$  symmetry corresponds to changes of the field  $\phi$  of the form [21]  $\delta_{k_l}\phi = \sum_{k=0}^l \alpha_k^i \partial^k(k_l \partial^{l-k+1}\phi)$ , where the parameters  $k_l$  are *holomorphic* functions of the world-sheet coordinate  $z$ . It is interesting to note [22] that by ‘fermionising’ the complex boson problem, i.e. by defining a pair of fermions  $\eta$  and  $\chi$  such that  $\phi \equiv: \eta e^{-\sigma} :$  and  $\phi^* \equiv: \partial\chi e^{\sigma} :$  (where  $: \dots :$  denotes the appropriate normal ordering), one can obtain non-linear realisations of  $W_{\infty}$  and  $W_{\infty+1}$ . By further bosonisation of fermions [22] one can then obtain - after appropriate truncation - realisations of the *classical*  $w_{\infty}$ -symmetries in terms of two real scalars, which may be (formally) closer to the stringy target-space interpretation argued above.

levels [17, 1]. They would correspond to higher-spin massive string states<sup>8</sup>. The existence of such operators and the corresponding Ward identities in the matrix model [15] certainly points in this direction, but more work needs to be done, especially in higher genera, before rigorous results are derived.

Recently Witten [26] has managed to exhibit a larger symmetry for the  $c = 1$  matrix model by making use of new discrete states of non-standard ghost number, which were known in the previous literature [27], but not extensively discussed. The basic feature is the existence of modes of ghost number zero and conformal spin zero that form a ring, the so-called ‘ground ring’. In terms of matrix models this ring helps in understanding the symmetry of the system, which is nothing other than a phase-space area-preserving transformation that leaves the Fermi surface invariant. It is important that the world-sheet currents that generate this particular Lie algebra of symmetries are operators of non-standard ghost number but of conformal spin  $(1, 0)$  and/or  $(0, 1)$ . Given that in the discussion we gave above on ‘hidden’ symmetries in string theory the ghost number seems to play no essential role, we have an understanding of this symmetry as an ordinary stringy symmetry of target space-time of a two-dimensional  $\sigma$ -model. The fact that the symmetry is known in a closed form is due to the low dimensionality of target space. The symmetry is bigger than that discussed previously [7, 1, 15], but the extra symmetries are quite trivial as far as the matrix model is concerned. However, they are important in the conformal field theory framework. As we mentioned in the beginning of the article, it has been known for some time [9] that  $w$ -algebras are associated with specific area-preserving diffeomorphisms for fields of high spin, but the connection with string theory had not been clear.

In the body of this note we develop a physical interpretation of this remarkable observation in the context of the symplectic geometry of Hamiltonian dynamics. Specifically, it ensures that the matrix model’s time evolution respects Liouville’s theorem, which is a necessary condition for canonical quantisation. Comparing with a generalised evolution equation for the density matrix proposed previously [10] to accommodate the breakdown of quantum coherence conjectured by Hawking [13], we demonstrate the vanishing of the extra term that otherwise would allow pure states to evolve into mixed ones. The basic feature is that the extra terms violate

---

<sup>8</sup>In view of this mixing of levels of different mass one would expect that in higher-dimensional string theories such symmetries would be spontaneously broken by the background. From the analysis of [19] it is evident that the order parameter of such a breaking is the metric for space-time. An interesting result is obtained by noting that the symmetries are not restored for any value of this field. This has been associated in [19] with the symmetric phase of the string where an S-matrix cannot be constructed [23]. In the particular case of two-dimensional string, spontaneous breaking cannot occur due to Coleman’s theorem [24]. In contrast, one might encounter a Kosterlitz-Thouless type of symmetry breaking. The topological nature of the space-time graviton mode in this case [14], and the fact that such excitations characterise the final stage of black hole evaporation [1, 25], might then prove useful in understanding the nature of the string vacuum characterising the matrix model system.

the exact stringy symmetry of the model, which is expressed as the transformation preserving the phase-space volume element <sup>9</sup>. This geometric view-point on the restoration of quantum coherence in two-dimensional strings probably opens new ways to attack similar problems in higher-dimensional string theories. We also recall that the area-preserving diffeomorphisms of the matrix model leave invariant the Fermi surface [26]. An analogous result in four-dimensional field theory would reflect the existence of a large string symmetry interrelating vacua with the same value of the cosmological constant  $\Lambda$ . Assuming the existence of a supersymmetric vacuum with zero cosmological constant,  $\Lambda = 0$ , the spontaneous breakdown of this large string symmetry would leave string in some asymmetric vacuum with  $\Lambda = 0$ .

## 2 The matrix model, black holes and space-time foam

However, there is a non-trivial conjecture that one has to make about the nature of the matrix model in order to apply the above arguments. The conjecture concerns the role of the black hole as well as space-time foam in this context. The matrix model is believed to be the last stage of black hole evaporation [4]. Black hole solutions have been discussed so far in the context of solving  $\beta$ -function conditions for  $\sigma$ -models formulated on genus zero, while the matrix model is considered as a stringy regularisation of world-sheet quantum gravity where the summation over genera is done exactly. It is quite plausible that higher genus corrections do not destroy the black hole character of the tree level conditions, and this will be assumed in the following. The black hole solution is asymptotic (at spatial infinity) to a space-time which is exactly that of a matrix model. The symmetries of the matrix model in this way become asymptotic symmetries of space-time *in the presence of a black hole* that can be assumed to evaporate somehow. Given that under time evolution the phase space of the matrix model, which is essentially the phase space of  $c = 1$  matter interacting with the black hole, is preserved as a result of the stringy symmetry, one has an elegant geometric interpretation of the maintenance of quantum coherence in the system.

Is that the end of the story? In other words, is the classical black hole background and its evaporation the only source of loss of coherence in the system? One might naively answer positively, given that after the black hole evaporates one recovers flat space-time and the problem of defining an S-matrix that factorises in the case of matrix model seems quite natural and somehow trivial. However, the essential

---

<sup>9</sup>It should be stressed that this symmetry is a property of two-dimensional *strings*, associated with the infinity of modes characterising various string multiplets, which does not exist in conventional local field theories.



point that will concern us below lies in the fact that, if one takes the dynamics of space-time seriously in the case of  $c = 1$  theories, there are additional complications even after the black hole evaporation, that can be seen on a *microscopic* scale. The problems come from the observation that a background-independent formalism of the Das-Jevicki action [6] is possible, which implies that if one considers quantum fluctuations around any background, then creation and annihilation of *virtual* black holes may occur, and as a result space-time foam will appear.

Let us be more precise. First we argue how one can formally achieve a background-independent formulation. From the collective field theory approach to the matrix model [6, 28], it becomes evident the latter yields a *string field theory* of tachyons that propagate in a *flat* space-time configuration, where the dilaton field is linear in the the spatial coordinate and all the rest of the higher-spin string modes are vanishing. The reason for having a tachyon field theory and not simply a classical background is simple. One starts from the *partition function* of the lattice model and then makes a change of variables (which is a *canonical* transformation that leaves invariant the classical phase-space volume element) following [28] to pass to a target space field formalism. In this way, the collective field coordinate is integrated over in the path integral. This coordinate can be seen to correspond to the tachyon field [6, 29]. There is no dynamics in the matrix model for the high-spin string modes, by construction. From classical symmetry considerations [7], however, one can anticipate a background-independent formalism, which will enable one to integrate over the space-time fields and obtain a *quantum* string field theory. To this end, one has first to covariantise the Das-Jevicki action. That this is possible follows directly from the fact that the amplitudes of the Das-Jevicki theory are nothing other than string theory amplitudes for the tachyon field in a two-dimensional target space [14]. The insertion of graviton-dilaton states with generic momenta in tachyonic amplitudes can be seen to yield vanishing results, which can be interpreted [1] as arising from certain stringy Ward identities expressing general covariance of the theory. In this way general covariance is viewed as one of the the infinite gauge symmetries of the two-dimensional string associated with massive high-spin levels [17]. It is straightforward [1], following the dictionary outlined in [6, 29], to express the Das-Jevicki lagrangian in terms of a tachyon field propagating in non-trivial (generic) graviton and dilaton backgrounds. The result is an effective action, involving an infinite number of derivatives, describing the interaction of the tachyon with these fields. The infinite derivative order is immediately seen from the fact that the Das-Jevicki lagrangian [6] for the collective field  $\phi(x, t)$  contains, in the scaling limit, kinetic terms of the form

$$\int dx \frac{1}{2} \left[ \frac{(\partial_x^{-1} \frac{\partial \phi}{\partial t})^2}{\phi} + L(\phi) + O(\phi^3) \right] \quad (4)$$

where the interaction terms  $L(\phi)$  depend on the details of the matrix model potential. For a study of the critical properties of the model it suffices to consider inverted harmonic oscillator potentials [6, 29]. In this case,  $L(\phi) = (\mu_F + \frac{1}{2}x^2)\phi$ , where  $\mu_F$  is

the chemical potential of the matrix model [6]. It should be noted that  $\mu_F$  is essential for the non-perturbative (field-theoretic) quantum corrections we are interested in, although it may be ignored when one considers only classical tachyon backgrounds [29]. To get a covariant form, one may redefine the collective field by looking at fluctuations around static classical minima,  $v_0(x) = \frac{1}{\pi}\sqrt{2\mu_F + x^2}$ , of the effective potential corresponding to (4). Thus we write  $\phi = v_0(x) + \frac{1}{v_0(x)}\eta(x, t)$ , change variables from  $x$  to a space-like coordinate  $q$  (the Liouville mode<sup>10</sup>) defined by,  $q = \int^x \frac{dx}{v_0}$ , and define a tachyon field  $T$  by  $\eta \equiv \partial_q T(t, q)$ . One immediately observes that the kinetic terms of (4) when *combined* with the potential terms  $L(\phi) = \frac{1}{\phi}L(\phi^2)$  yield (after a Wick rotation in time  $t$ ) terms of the form

$$\int dq dt \frac{(\partial_t T)^2 - (\partial_q T)^2 + O[v_0(q; \mu_F), \partial_q T(t, q)]}{1 + \frac{1}{v_0^2(q; \mu_F)} \partial_q T} \quad (5)$$

where for the sake of simplicity we denoted collectively the interaction terms by  $O[\dots]$ . It is immediately seen that that the covariantisation is obtained naively by setting  $2q \rightarrow \Phi$  (a dilaton field),  $\partial_q T \rightarrow \frac{1}{2}G_{\mu\nu}\nabla_\mu\Phi\nabla_\nu T$ , and  $(\partial_t T)^2 - (\partial_q T)^2 \rightarrow \nabla_\mu T \nabla_\nu T G^{\mu\nu}$ . The standard  $\sigma$ -model form is obtained by redefining  $T' = e^{-\Phi}T$ , which induces the non trivial dilaton factors in the integration measure  $[4, 1] \int d^2y e^{-\Phi} \sqrt{G} \dots$ . There are ambiguities in such a covariantisation since the matrix model is formulated on a *flat* target space-time and therefore curvature terms, which determine space-time dynamics, are vanishing. Moreover, one cannot really distinguish between constant factors in the action and purely dilaton terms. To resolve such questions one has to start from a stringy point of view [1, 14]. If one is interested only in classical space-time dynamics for the high-spin modes (which in two dimensions have definite energy and momentum), the  $\sigma$ -model point of view seems sufficient. The latter can be re-expressed, at least as far as the graviton-dilaton multiplet is concerned

---

<sup>10</sup>It should be noted at this stage that it is not at all obvious that we can establish an exact (formal) correspondence of the spatial coordinate  $q$  appearing in the Das-Jevicki action with the Liouville mode of the continuum model. In view of the results of [15] this appears to be true only for low energies, while in general there is a non-local integral transform that connects the two parameters, which makes the connection of the two theories technically obscured. Indeed, it can be readily seen that in the low-energy limit the equations of motion obtained from the action (5) are equivalent to the lowest order ( $\alpha'$ )  $\sigma$ -model  $\beta$ -function conditions. However the situation is not at all clear with the higher-order corrections, since exact results (on higher genera) from the  $\sigma$ -model are not yet available. One should hope that when taking into account the field redefinition ambiguities in the  $\sigma$ -model formulation and the relevant subtleties [15] of the connection of the Liouville mode and the spatial coordinate of the Das-Jevicki action, the connection of the two formalisms might be elucidated. This immediately raises the question of which theory is more convenient (or even correct) to describe the dynamics of the  $c = 1$  theory. The point of view we took in [1], and which we follow here, is that the stringy approach is more fundamental. After all, it is essentially this approach that gave rise to black-hole solutions [4, 25]. The connection of the conformal field theory with the matrix model, and therefore the Das-Jevicki formalism, is then elucidated further in [26] by exploiting the extra structure of the conformal field theory, that cannot be seen in the matrix model approach. It is probably this extra structure [30] that explains a discrepancy between the works of Gross and Klebanov [31], on the triviality of the S-matrix of  $c = 1$  matrix model theory, and that of Polyakov [14]. It is in view of this connection that the background-independent formalism of the Das-Jevicki string field theory is useful.

[4], as a  $\frac{SU(1,1)}{U(1)}$  Wess-Zumino model. However, to get a consistent string theory one should also include the rest of the string modes that couple to the tachyons [1]. That such couplings exist becomes clear from the  $\sigma$ -model formalism, where the stringy gauge symmetry transformations generated by these modes mix the various string levels [17, 19, 1]. These symmetries exist already at the conformal field theory level, i.e. a first-quantised string formulated on flat space-times, imposing non-trivial constraints (Ward identities) on string S-matrix elements [17, 1]. Such constraints can be interpreted as arising from symmetries of the target space-time associated with the rest of the spin modes that are vanishing in the fixed background in which we are working. To find the precise form of the relevant terms is formally a complicated task, and at present there is no closed form known for the collective field theory to incorporate these modes, but in principle it exists. Nevertheless, following the above methods of covariantisation, one can show explicitly [1] that the first two lowest-lying conserved charges of the effective theory associated with the string levels zero and one (fermion number and the spin-two current related to the energy of the black hole) are surface terms, and as such characterise the asymptotic black hole state and are, therefore, quite insensitive to what is going on inside the horizon. One can anticipate [1] a similar result for the *entire* set of the conserved charges of the matrix model, associated with other string modes. In this way one has an infinity of independent charges that are exactly conserved even during the Hawking process of black hole evaporation. They constitute the gauged ‘W-hair’ of the black hole [1], which we argued to be responsible for the restoration of quantum coherence, since they are remnants of the black hole after the evaporation.

When gravity and higher-spin fields are included, hopefully through an appropriate extension of the Das-Jevicki approach [6], one should integrate over all possible configurations of these fields. Although these modes are discrete string states of definite energy and momentum, as can be readily seen in an expansion around *flat* target space-time [14, 1], in general they may give rise to various field configurations, which can only be seen if one starts from a background-independent string field theory and not expanding around a *given* background, as in the  $\sigma$ -model approach. In such a procedure one should include topology-changing configurations (in Euclidean formulations), which are in any case included in the set of allowed classical solutions of the  $\sigma$ -model  $\beta$ -function conditions. It is therefore natural to assume that the non-perturbative solution to the matrix model includes the creation and annihilation of virtual *microscopic* black holes. In this interpretation Witten’s black hole, which is a solution of the beta function conditions of the  $\sigma$ -model at genus zero, plays the role of what Hawking would call a *macroscopic* black hole [13]. The  $\sigma$ -model (formulated in higher genera) would involve quantum fluctuations of the space-time metric which lead to the formation of virtual black holes and microscopic horizons. The latter survive the evaporation of the macroscopic black hole, and thus characterise the asymptotic quantum field theory, which in this case may be interpreted as describing the interaction of  $c = 1$  matter with a space-time foam whose role is similar to that of a quantum-mechanical reservoir.

Hence, in principle, the modification of the quantum mechanical evolution suggested in [10] could apply. However, this might not be the case due to stringy symmetries, as we shall argue below. There is also a nice geometric interpretation of these symmetries, which also illuminates the relation between matrix model and string theory in fixed backgrounds.

### 3 The symplectic geometry of Quantum Coherence

We now proceed to demonstrate the geometric origin of the argued restoration of quantum coherence in the two-dimensional string. To understand better the problem and its solution in the matrix model framework it is first useful to recall some facts about the possible modifications of quantum mechanics as a result of singular space-time metrics [10].

Consider a quantum mechanical problem described by a Hamiltonian  $H(p, q)$  and a density matrix  $\rho(p, q, t)$ . In conventional quantum mechanics, the time evolution of the density matrix is given by

$$\frac{\partial \rho}{\partial t} = i[\rho, H] \quad (6)$$

where  $[\cdot, \cdot]$  denotes the conventional commutator. Eq. (6) is nothing other than the quantum-mechanical analogue of Liouville's theorem for the conservation of the classical probability density under time evolution in classical mechanics. There are two features that lead to such a classical conservation. For our purposes it is instructive to review them briefly.

The *first* is the fact that the subsequent motion of the classical system (defined by a point in phase space) is completely determined by its location in phase space at a particular time. This implies that the number of systems inside the infinitesimal volume element surrounding a given point remains constant under time evolution. A system initially inside the volume can never get out, since at the moment of crossing the border it has the same location as one of the boundary points and therefore both points would be forced to travel together from that moment. The *second* important point is that time evolution is a canonical transformation, and as such leaves the infinitesimal volume element in phase space invariant. Thus, the density of phase-space points remains constant during time evolution.

Eq. (6) might be modified if singular space-time metric backgrounds were taken into account, due to the Hawking evaporation phenomena that accompany them

[10]. Already in ref. [13] Hawking had proposed that once non-perturbative metric fluctuations were taken into account, the usual S-matrix relation between initial and final states should be replaced by a general linear relation

$$\rho_{out} = \mathcal{S}\rho_{in} \quad (7)$$

where  $\mathcal{S}$  cannot in general be factorised into the product of  $S$  and  $S^\dagger$ , as in conventional quantum field theory. It was suggested in ref. [10] that the corresponding modification of eq. (6) would be

$$\frac{\partial \rho}{\partial t} = i[\rho, H] + \hat{\mathcal{H}}\rho \quad (8)$$

where the extra part represents an abstract operator, whose typical effects are suppressed by powers of the Planck scale [10]. The immediate interpretation of this modified quantum mechanics would be that gravity induced non-invariances of the phase space volume of the system. The existence of microscopic event horizons might cause similarities with the situation in *open* systems, where there is a flow of information to and/or from the system, and this simply induces a failure of Liouville's theorem for the open system. Indeed, eq. (8) is similar in form to the markovian master equation that controls the evolution of a reduced quantum mechanical system in interaction with a reservoir [32]. The important difference [10] is that in this scenario for quantum gravity the role of the reservoir is played by the space-time foam, which however is intrinsically unobservable. Therefore the modification (8) is essential, and not an artefact of a voluntary choice to observe only a reduced part of the world.

Of the two features stated above that characterise the motion of an isolated classical Hamiltonian system, the one that is violated in the case of open systems is the second, namely the invariance of the phase space volume element under the time evolution, due to the non-existence of unitary Hamiltonian motion. This can be understood by the fact that in the case of the evolution of the open system there is flow of information through the boundaries. This is effectively described by the non-invariance of the phase-space differential volume. As a result, the equation for the probability density, and evidently the density matrix evolution, is modified by the extra term  $\hat{\mathcal{H}}$  in (8), whose precise form depends on the details of the interaction of the subsystem in question with the reservoir. We do not know how such a modification due to space-time foam would be avoided within the context of conventional local quantum field theory.

However, in the particular case of two-dimensional string black holes (matrix models) the conservation of the phase space volume is guaranteed in the quantum system by a *symmetry* of the theory, as we shall now discuss, and as such the above modification of quantum mechanics is not present. This is the promised geometric interpretation of the restoration of quantum coherence argued in [1] on the basis of the infinite gauged 'W-hair' property of the two-dimensional string black hole.

Following Witten [26], we consider the matrix model Hamiltonian which has the standard inverted harmonic oscillator form,

$$H(p, q) = \frac{1}{2}(p^2 - q^2) \quad (9)$$

The action density of the model in form language is given by the two-form  $\omega = dp \wedge dq - (pdp - qdq) \wedge dt$ , where  $t$  is a time parameter. Solving the model means to find an appropriate change of variables in  $(p, q)$  space (as functions of  $t$ ) so that the equations of motion  $\frac{dp}{dt} = -\frac{\partial H}{\partial q}$  and  $\frac{dq}{dt} = \frac{\partial H}{\partial p}$  look trivial. The transformation is readily obtained [26]

$$\begin{aligned} p' &= p \cosh t - q \sinh t \\ q' &= -p \sinh t + q \cosh t \end{aligned} \quad (10)$$

Indeed, the equations of motion in the  $p' - q'$  frame are  $\frac{dp'}{dt} = 0$  and  $\frac{dq'}{dt} = 0$ . In the  $(p', q')$ -system of phase space coordinates the action density itself becomes the phase space volume element  $\omega = dp' \wedge dq'$ . It is therefore evident that the hamiltonian vector fields, which by definition leave the  $dp' \wedge dq'$  element invariant, are *symmetries* of the matrix model. Such fields are easy to construct [26],

$$\bar{V} = \frac{\partial g(p', q')}{\partial q'} \frac{\partial}{\partial p'} - \frac{\partial g(p', q')}{\partial p'} \frac{\partial}{\partial q'} + u(p', q', t) \frac{\partial}{\partial t} \quad (11)$$

In terms of the original  $p - q$  system, the above vector fields generate symmetry transformations for the matrix model. In particular,  $g(p', q')$  is associated with shifts in the  $q$ -coordinate, while  $u(p', q', t)$  represents changes in the time coordinate  $t$ . It is clear that the time evolution of the quantum system in the  $p' - q'$  plane becomes in this way an exact symmetry of the conformal field theory<sup>11</sup>. Hence, the classical result of the invariance of the phase space volume element is promoted in this particular case into a symmetry of the problem. As a consequence, the Liouville-von Neuman evolution of the density matrix in ordinary quantum mechanics is *guaranteed* by the symmetries of the theory. From the above-described equivalence of the matrix model and the Das-Jevicki tachyon field theory, one can anticipate that the phase space of the latter will remain constant under time evolution, for stringy symmetry reasons. Witten's analysis [26] shows that this evolution corresponds to a symmetry of the conformal field theory which is associated with states of given energy and momentum characterised by non-standard ghost number adjacent to the one characterising the states discussed in refs. [14, 1, 15]. The extension of these symmetries to non-trivial backgrounds, which incorporate black holes, is non-trivial to check, especially in view of the non-existence, as yet, of a satisfactory string field theory formalism that resums higher genera in the continuum. However, in view of the above-described background independent formalism of the Das-Jevicki action and its appropriate extension to include high-spin string modes, one should

---

<sup>11</sup>Actually the only restriction in that case is that the function  $u$  does not have an explicit dependence on time.

anticipate that such an extension is possible. In this case, taking into account the above interpretation of the role of space-time foam in this context, and the connection of the asymptotic states of the black hole to the matrix model, one has an elegant geometric interpretation of the restoration of quantum coherence in the two-dimensional string theory. The basic feature is the existence of a stringy symmetry, associated with various string excitations, which characterises the ground state of the system. As a result, the extra term in (8) that would spoil conventional quantum mechanics is forced to vanish, as it is incompatible with the symmetries of the theory. This happens due to the fact that the symmetry in question finds a nice interpretation as a phase-space volume-preserving diffeomorphism which is precisely the property violated in the modified evolution equation (8). The existence of the ‘W-hair’ [1], that characterises the asymptotic states of the black hole (matrix model), certainly offers field-theoretic support to these arguments.

The area-preserving diffeomorphisms in phase space are a feature of the  $w_\infty$  algebras [9] discussed at the beginning of this article. It should be stressed that the  $w_\infty$ -symmetry that characterises the standard ghost number discrete states of high spin is included in the above symmetry. As we mentioned above, from the matrix model point of view the extra symmetries are trivial and do not lead to extra conserved charges. One should probably anticipate that the charges discussed previously, associated with the standard ghost number string states, are the only ones that characterise the conformal field theory and therefore the black hole solutions. However, it is the entire spectrum of string states, including the non-standard ghost number ones, that are really responsible for the maintenance of quantum coherence and the restoration of the relation between symmetries and conservation laws. *Hence it seems that it is the particularity of ‘W-hair’, which is area-preserving and characterises the two-dimensional black hole, that is relevant for the reconciliation of quantum mechanics and general relativity, and that just having an infinity of conserved charges may not suffice.*

In view of this result, one might speculate that if the above scenario is to be carried through in higher dimensional space-times, it is necessary that there exist analogous infinite-dimensional symmetries associated with area-preserving diffeomorphisms in some phase space variables. One might need something like the  $w_\infty$ -algebras that characterise four-dimensional self-dual Euclidean Einstein spaces [33]. There, it is known that four-dimensional self-dual Einstein gravity follows as the large N-limit of a two-dimensional Wess-Zumino  $SU(N)$   $\sigma$ -model. Infinite-dimensional symmetries of Einstein’s equations are shown to form a  $w_\infty$ -algebra. The self-duality of the model may not be essential [33] for the existence of infinite dimensional integrability, and one might hope that non-self-dual four-dimensional gravity models obtained from string theories with higher-dimensional target spaces might possess extended conformal symmetries of the type discussed in this note.

## 4 Remarks on the cosmological constant problem

After these speculative remarks about the possible extension of our ideas to higher-dimensional space-times, we conclude with some comments on the cosmological constant in the light of the above insights into matrix models. Witten [26] has observed that the infinite set of string symmetries that preserve the two-dimensional area of phase space of the matrix model also leave invariant the Fermi surface

$$F \equiv \left\{ \frac{1}{2}(p^2 - q^2) \right\} = 0 \tag{12}$$

If this property of zero vacuum energy also could be lifted to realistic string models with an effective field theory in four-dimensional target space-time, it could explain why the cosmological constant  $\Lambda = 0$ . We presume that such theories would have a large number of consistent vacua, among which would be a supersymmetric one with  $\Lambda = 0$ . As in the case of a conventional finite-dimensional field theoretical symmetry, this vacuum might be invariant under the infinite-dimensional string symmetry<sup>12</sup>. However, there is an equally generic possibility that this is just one of a degenerate set of string vacua which are transformed into each other by the string  $W$ -symmetries. The theory must choose between these by breaking the string  $W$ -symmetry spontaneously, analogously to the Goldstone or Higgs mechanism in local field theory. A key role is played in this idea by supersymmetry, which ensures that the set of  $\Lambda = 0$  vacua is at least non-empty. If the set is indeed degenerate, this should be apparent in the effective four-dimensional field theory, which could well exhibit a set of candidate vacua that have ‘small’ values of the cosmological constant, at least in some approximation. Relevant examples of such effective field theories are the flat-potential no-scale supergravity models [34], which have a continuum of degenerate tree level vacua that break supersymmetry, and appear naturally in string theories [35]. It would indeed be remarkable if the two fundamental problems of the restoration of quantum coherence and the vanishing of the cosmological constant could be solved within the framework of a single string symmetry.

The relation of these ideas to those of [36, 37, 38] is not clear, since the semiclassical approach used there is not justified for two-dimensional gravity which has a dimensionless coupling [39]. However, the two-dimensional target space models offer hope that non-perturbative effects may guarantee  $\Lambda = 0$  even beyond the semiclassical approximation, which has in any case never been derived even in the four-dimensional case. The approach of refs. [36, 37, 38] required some hierarchy between the Planck scale ( $M_P$ ) and the sizes of the wormholes considered, and would ‘iron out’ the cosmological constant only if it was ‘pre-cooked’ to be small compared with  $M_P^4$ , as in generic models with approximate supersymmetry and no-scale supergravity models in particular [34]. Thus our ideas are not obviously contradictory with those of refs. [36, 37, 38].

---

<sup>12</sup>We note parenthetically that the two-dimensional cosmological constant operator is central within the string symmetry algebra.



## **Acknowledgements**

We thank C. Bachas, I. Bakas, E. Kiritsis, N.D. Vlachos and S. Yankielowicz for discussions. The work of D.V.N. is partially supported by DOE grant DE-FG05-91-ER-40633.

## References

- [1] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, CERN-TH.6147/91, CTP-TAMU-41/91, to appear in Phys. Lett. B.
- [2] D.J. Gross and A.A. Migdal, Phys. Rev. Lett. 64 (1990), 717;  
M. Douglas and S. Shenker, Nucl. Phys. B335 (1990), 635;  
E. Brezin and V. Kazakov, Phys. Lett. 236B (1990), 144.
- [3] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988), 393.
- [4] E. Witten, Phys. Rev. D44 (1991), 314.
- [5] A. Sengupta and S. Wadia, Int. Jour. Mod. Phys. A6 (1991), 1961;  
D.J. Gross and I.R. Klebanov, Nucl. Phys. B352 (1991), 671.
- [6] S.R. Das and A. Jevicki, Mod. Phys. Lett. A5 (1990), 1639.
- [7] J. Avan and A. Jevicki, Phys. Lett. B266 (1991), 35.
- [8] I. Bakas and E. Kiritsis, Nucl. Phys. B343 (1990), 185; Progr. Theor. Phys. Suppl. 102 (1990), 15.  
C.N. Pope, L.J. Romans and X. Shen, Phys. Lett. B236 (1990), 173; Nucl. Phys. B339 (1990), 191.
- [9] I. Bakas, Phys. Lett. B228 (1989), 57;  
C.N. Pope, L.J. Romans and X. Shen, *A brief history of  $W_\infty$* , in *Strings '90*, World Scientific (1991).
- [10] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984), 381.
- [11] D.J. Gross, Nucl. Phys. B236 (1984), 349.
- [12] T.J. Allen, M.J. Bowick and A. Lahiri, Phys Lett. 237B (1990), 47.
- [13] S. Hawking, Comm. Math. Phys. 43 (1975), 199; *ibid* 87 (1982), 395.
- [14] A.M. Polyakov, Mod. Phys. Lett. A6 (1991), 635.
- [15] G. Moore and N. Seiberg, Rutgers University preprint RU-91-29, YCTP-P19-91 (1991).
- [16] G. Moore, N. Seiberg and M. Staudacher, Rutgers/Yale preprint RU-91-11, YCTP-P11-91, Nucl. Phys. B in press.

- [17] G. Veneziano, Phys. Lett. B167 (1986), 388.  
J. Maharana and G. Veneziano, Nucl. Phys. B283 (1987), 126.
- [18] A.B. Zamolodchikov, Sov. J. Nucl. Phys. 44 (1987), 529; *ibid* 46 (1987), 1090.
- [19] M. Evans and B. Ovrut, Phys. Rev. D39 (1989), 3016; *ibid* D41 (1990), 3149.
- [20] G. Veneziano, unpublished;  
J. Maharana, N.E. Mavromatos and G. Veneziano, unpublished.
- [21] E. Bergshoeff, C.N. Pope, L.J. Romans, E. Sezgin and X. Shen, Mod. Phys. Lett. A5 (1990) 1957;  
See also Pope *et al.* in ref. [9].
- [22] X. Shen and X.J. Wang, Texas AM-University preprint CTP TAMU-59/91 (1991).
- [23] D. Gross, Phys. Rev. Lett. 60 (1988), 1229.;  
E. Witten, Institute for Advanced Studies preprint, IASSNS-HEP-88/55, based on talk given in the Royal Society meeting on *String Theory*, London, December 1988.
- [24] S. Coleman, Comm. Math. Phys. 31 (1973), 259.
- [25] G. Mandal, A. Sengupta and S. Wadia, Institute of Advanced Studies preprint, Princeton IASSNS-10-91 (1991).
- [26] E. Witten, Institute for Advanced Studies preprint, IASSNS-HEP-91/51.
- [27] B. Lian and G. Zuckerman, Phys. Lett. B254 (1991), 417; *ibid* B266 (1991), 21.
- [28] A. Jevicki and B. Sakita, Nucl. Phys. B165 (1980), 511; *ibid* B185 (1981), 89.
- [29] J. Polchinski, Texas-Austin University preprint UTTG-91-06 (1991), Nucl. Phys. B in press.
- [30] D. Minic and Z. Yang, Austin-Texas University preprint UTTG-23-91 (1991).
- [31] D.J. Gross and I. Klebanov, Nucl. Phys. B359 (1991), 3.
- [32] V. Gorini, A. Frigerio, M. Verri, A. Kossakowski and E.C. Sudarshan, Rep. Math. Phys. 13 (1978), 149.
- [33] Q-Han Park, Phys. Lett. B236 (1989), 429; *ibid* B238 (1990), 287; see also E. Witten, in *Strings '90*, World Scientific (1991).

- [34] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, Phys. Lett. B133 (1983), 61;  
J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B241 (1984), 406 and Nucl. Phys. B247 (1984), 373.
- [35] E. Witten, Phys. Lett. B155 (1985), 151.
- [36] S. Coleman, Nucl. Phys. B307 (1988), 867; *ibid* B310 (1988), 643.
- [37] E.B. Baum, On topology in Euclidean Gravity, Princeton University preprint, 82-0504 (1985).
- [38] S. Hawking, Phys. Lett. B195 (1987), 337; Nucl. Phys. B335 (1990), 155.
- [39] J. Polchinski, Nucl. Phys. B324 (1989), 123.