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$SU(7)$ Unification of $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$

Csaba Balázs[1], Tianjun Li[2,3], Fei Wang[1] and Jin Min Yang[2]

[1] *School of Physics, Monash University, Melbourne Victoria 3800, Australia*
[2] *Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China*
[3] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA*

ABSTRACT: We propose the SUSY $SU(7)$ unification of the $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ model. Such unification scenario has rich symmetry breaking chains in a five-dimensional orbifold. We study in detail the SUSY $SU(7)$ symmetry breaking into $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ by boundary conditions in a Randall- Sundrum background and its AdS/CFT interpretation. We find that successful gauge coupling unification can be achieved in our scenario. Gauge unification favors low left-right and unification scales with tree-level $\sin^2\theta_W = 0.15$. We use the AdS/CFT dual of the conformal supersymmetry breaking scenario to break the remaining $\mathcal{N} = 1$ supersymmetry. We employ AdS/CFT to reproduce the NSVZ formula and obtain the structure of the Seiberg duality in the strong coupling region for $\frac{3}{2}N_c < N_F < 3N_C$. We show that supersymmetry is indeed broken in the conformal supersymmetry breaking scenario with a vanishing singlet vacuum expectation value.

Contents

1. Introduction

The standard model (SM) of electroweak interactions, based on the spontaneously broken $SU(2)_L \times U(1)_Y$ gauge symmetry, has been extremely successful in describing phenomena below the weak scale. However, the SM leaves some theoretical and aesthetical questions unanswered, two of which are the origin of parity violation and the smallness of neutrino masses. Both of these questions can be addressed in the left-right model based on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry [1]. The supersymmetric extension of this model [2] is especially intriguing since it automatically preserves R-parity. This can lead to a low energy theory without baryon number violating interactions after R-parity is spontaneously broken. However, in such left-right models parity invariance and the equality of the $SU(2)_L$ and $SU(2)_R$ gauge couplings is ad hoc and has to be imposed by hand. Only in Grand Unified Theories (GUTs) [3, 4] can the equality of the two $SU(2)$ gauge couplings be naturally guaranteed through gauge coupling unification.

Novel attempts for the unification of the left-right symmetries have been proposed in the literature, such as the $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ [5, 6, 7] or $SU(3)_C \times SU(4)_W$ [8]. In these attempts, the equality of the left-right gauge couplings and the parity in the left-right model are understood by partial unification. In this work, we propose to embed the $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ partially unified model into an $SU(7)$ GUT. Unfortunately, the doublet-triplet splitting problem exists in various GUT models. An elegant solution to this is to invoke a higher dimensional space-time and to break the GUT symmetry by boundary conditions such as orbifold projection. Orbifold GUT models for $SU(5)$ were proposed in [9, 10, 11] and widely studied thereafter in [12, 13, 14, 15, 16, 17, 18, 19, 20]. The embedding of the supersymmetric GUT group into the Randall-Sundrum (RS) model [21] with a warped extra dimension is especially interesting since it has a four-dimensional (4D) conformal field theory (CFT) interpretation [22, 23]. By assigning different symmetry breaking boundary conditions to the two fixed point, the five-dimensional (5D) theory is interpreted to be the dual of a 4D technicolor-like theory or a composite gauge symmetry model.

It is desirable to introduce supersymmetry in warped space-time[24] because we can not only stabilize the gauge hierarchy by supersymmetry but also set the supersymmetry broken scale by warping. It is well known that supersymmetry (SUSY) can be broken by selecting proper boundary conditions in the high dimensional theory. For example, 5D $\mathcal{N} = 1$ supersymmetry, which amounts to $\mathcal{N} = 2$ supersymmetry in 4D, can be broken to 4D $\mathcal{N} = 1$ supersymmetry by orbifold projection. Various mechanisms can be used to break the remaining $\mathcal{N} = 1$ supersymmetry. One intriguing possibility is the recently proposed conformal supersymmetry breaking mechanism [25, 26] in vector-like gauge theories which can be embedded into a semi-direct gauge mediation model. Such a semi-direct gauge mediation model can be very predictive having only one free parameter. It is interesting to recast it in a warped extra dimension via the Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [27], and use it to break the remaining supersymmetry.

This paper is organized as follows. In Section 2, as a warm up, we discuss the SUSY $SU(7)$ orbifold GUT model and its symmetry breaking chains in a flat extra dimension.

In Section 3, we present the SUSY $SU(7)$ GUT model with a warped extra dimension and its 4D CFT dual interpretation. In Section 4 we consider gauge coupling unification in the RS background. In Section 5 we discuss the AdS/CFT dual of the semi-direct gauge mediation model in the conformal window in vector-like gauge theories. Section 6 contains our conclusions.

2. SUSY $SU(7)$ Unification in a Flat Extra Dimension

We consider $\mathcal{M}_4 \times S^1/Z_2$, the 5D space-time comprising of the Minkowski space $\mathcal{M}_4$ with coordinates x_μ and the orbifold S^1/Z_2 with coordinate $y \equiv x_5$. The orbifold S^1/Z_2 is obtained from S^1 by moduling the equivalent classes

$$Z_5 : \; y \to -y \; . \tag{2.1}$$

There are two inequivalent 3-branes located at $y = 0$ and $y = \pi R$, denoted by O and O', respectively.

The 5D $\mathcal{N} = 1$ supersymmetric gauge theory has 8 real supercharges, corresponding to $\mathcal{N} = 2$ SUSY in 4D. The vector multiplet contains a vector boson A_M ($M = 0, 1, 2, 3, 5$), two Weyl gauginos $\lambda_{1,2}$, and a real scalar σ. From the 4D $\mathcal{N} = 1$ point of view, it contains a vector multiplet $V(A_\mu, \lambda_1)$ and a chiral multiplet $\Sigma((\sigma + iA_5)/\sqrt{2}, \lambda_2)$ which transform in the adjoint representation of the gauge group. The 5D hypermultiplet contains two complex scalars ϕ and ϕ^c, a Dirac fermion Ψ, and can be decomposed into two 4D chiral mupltiplets $\Phi(\phi, \psi \equiv \Psi_R)$ and $\Phi^c(\phi^c, \psi^c \equiv \Psi_L)$, which are conjugates of each other under gauge transformations. The general action for the gauge fields and their couplings to the bulk hypermultiplet Φ is [28, 29]

$$\begin{aligned}
S = & \int d^5x \frac{1}{kg^2} \mathrm{Tr}\left[\frac{1}{4}\int d^2\theta \left(W^\alpha W_\alpha + \mathrm{H.C.}\right)\right. \\
& \left. + \int d^4\theta \left((\sqrt{2}\partial_5 + \bar{\Sigma})e^{-V}(-\sqrt{2}\partial_5 + \Sigma)e^V + \partial_5 e^{-V}\partial_5 e^V\right)\right] \\
& + \int d^5x \left[\int d^4\theta \left(\Phi^c e^V \bar{\Phi}^c + \bar{\Phi} e^{-V} \Phi\right)\right. \\
& \left. + \int d^2\theta \left(\Phi^c(\partial_5 - \frac{1}{\sqrt{2}}\Sigma)\Phi + \mathrm{H.C.}\right)\right] \; .
\end{aligned} \tag{2.2}$$

We introduce the following orbifold projections

$$Z_5 : x_5 \to -x_5 \; , \qquad T_5 : x_5 \to x_5 + 2\pi R_5 \; , \tag{2.3}$$

and use them to impose the following boundary conditions on vector and hypermultiplets in terms of the fundamental representation

$$V(-x_5) = Z_5 V(x_5) Z_5 \; , \quad \Sigma_5(-x_5) = -Z_5 \Sigma_5(x_5) Z_5 \; , \tag{2.4}$$

$$\Phi(-x_5) = \eta_\Phi Z_5 \Phi(x_5) \; , \quad \Phi^c(-x_5) = -\eta_\Phi Z_5 \Phi(x_5) \; , \tag{2.5}$$

and

$$V(x_5 + 2\pi R_5) = T_5 V(x_5) T_5 \ , \quad \Sigma_5(x_5 + 2\pi R_5) = T_5 \Sigma_5(x_5) T_5 \ , \tag{2.6}$$

$$\Phi(x_5 + 2\pi R_5) = \zeta_\Phi T_5 \Phi(x_5) \ , \quad \Phi^c(x_5 + 2\pi R) = \zeta_\Phi T_5 \Phi(x_5) \ , \tag{2.7}$$

with $\eta_\Phi = \pm 1$, and $\zeta_\Phi = \pm 1$. The 5D $\mathcal{N} = 1$ supersymmetry, which corresponds to 4D $\mathcal{N} = 2$ SUSY, reduces to 4D $\mathcal{N} = 1$ supersymmetry after the Z_5 projection.

It is well known that we can have different gauge symmetries at the two fixed points by assigning different boundary conditions. We can rewrite (Z_5, T_5) in terms of (Z_5, Z_6) by introducing the transformation

$$Z_6 = T_5 Z_5 \ , \tag{2.8}$$

which gives

$$Z_6 : y + \pi R \to -y + \pi R \ . \tag{2.9}$$

Then the massless zero modes can preserve different gauge symmetries which are obtained by assigning proper (Z_5, Z_6) boundary conditions to the two fixed points.

In our setup, as a warm up, we consider a $SU(7)$ gauge symmetry in the 5D bulk of $\mathcal{M}_4 \times S^1/Z^2$. This implies the following different symmetry breaking possibilities.

- Case I:

$$Z_5 = I_{4,-3} \ , \quad Z_6 = I_{1,-6} \ , \tag{2.10}$$

  where $I_{a,-b}$ denotes the diagonal matrix with the first a entries 1 and the last b entries -1. These boundary conditions break the $SU(7)$ gauge symmetry down to $SU(4)_W \times SU(3)_C \times U(1)_{B-L}$ at the fixed point $y = 0$, to $SU(6) \times U(1) \times U(1)_X$ at the fixed point $y = \pi R_5$, and preserve $SU(3)_C \times SU(3)_L \times U(1) \times U(1)_X$ in the low energy 4D theory.

- Case II:

$$Z_5 = I_{4,-3} \ , \quad Z_6 = I_{2,-5} \ , \tag{2.11}$$

  which break the gauge symmetry $SU(7)$ to $SU(4)_w \times SU(3)_C \times U(1)_{B-L}$ at the fixed point $y = 0$, to $SU(5) \times SU(2) \times U(1)_X$ at the fixed point $y = \pi R_5$, and preserve $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_X$ in the low energy 4D theory.

- Case III:

$$Z_5 = I_{4,-3} \ , \quad Z_6 = I_{3,-4} \ , \tag{2.12}$$

  which break $SU(7)$ to $SU(4)_w \times SU(3)_C \times U(1)_{B-L}$ at the fixed point $y = 0$, to $SU(3) \times SU(4)_c \times U(1)_X$ at the fixed point $y = \pi R_5$, and preserve $SU(3)_C \times SU(3)_L \times U(1) \times U(1)_X$ in the low energy 4D theory.

- Case IV:

$$Z_5 = I_{1,-6} \ , \quad Z_6 = I_{2,-5} \ , \tag{2.13}$$

which break $SU(7)$ to $SU(6)\times U(1)$ at the fixed point $y=0$, to $SU(5)\times SU(2)\times U(1)_X$ at the fixed point $y=\pi R_5$ which preserves $SU(5)\times U(1)\times U(1)_X$ in the low energy 4D theory.

- Case V:

$$Z_5 = I_{1,-6} \ , \quad Z_6 = I_{4,-3} \ , \tag{2.14}$$

which break $SU(7)$ to $SU(6)\times U(1)$ at the fixed point $y=0$, to $SU(3)_C \times SU(4) \times U(1)_X$ at the fixed point $y=\pi R_5$, and preserve $SU(3)_C\times SU(3)_L \times U(1)$ in the low energy 4D theory.

We will not discuss these various symmetry breaking chains in detail, we simply note that several interesting low energy theories can be embedded into a 5D $SU(7)$ gauge theory. To construct a realistic theory, we must also introduce the proper matter content. The simplest possibility to introduce matter in this scenario is to localize it at the fixed point branes and fitting it into multiplets of the corresponding gauge symmetry preserved in the given brane. Bulk fermions which are $SU(7)$ invariant are possible in case Z_5 or Z_6 is trivial. For most general boundary conditions, bulk fermions do not always lead to realistic low energy matter content. In our case, the motivation for the $SU(7)$ gauge symmetry is the unification of $SU(3)_C \times SU(4)_w \times U(1)_{B-L}$. Thus, we will discuss in detail this symmetry breaking chain and the matter content of this scenario.

The compactification of gauge symmetry in flat and warped extra dimensions share many common features. Consequently, we will concentrate on the orbifold breaking of $SU(7)$ in a warped extra dimension and discuss its AdS/CFT interpretation. The flat extra dimension results can be obtained by taking the AdS curvature radius to infinity.

3. SUSY $SU(7)$ Unification in Warped Extra Dimension

We consider the AdS_5 space warped on S^1/Z_2 with $SU(7)$ bulk gauge symmetry. The AdS metric can be written as

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \ , \tag{3.1}$$

where $\sigma = k|y|$, $1/k$ is the AdS curvature radius, and y is the coordinate in the extra dimension with the range $0 \le y \le \pi R$. Here we assume that the warp factor $e^{-k\pi R}$ scales ultra-violet (UV) masses to TeV.

As noted in [30], in AdS space the different fields within the same supersymmetric multiplets acquire different masses. The action for bulk vector multiplets (V_M, λ^i, Σ) and hypermultiplets (H^i, Ψ) can be written as [31, 32, 33]

$$S_5 = -\frac{1}{2}\int d^4x \int dy \sqrt{-g} \ \ \left[\frac{1}{2g_5^2}F_{MN}^2 + (\partial_M \Sigma)^2 + i\bar{\lambda}^i\gamma^M D_M\lambda^i + m_\Sigma^2\Sigma^2 \ ,\right.$$

$$
\begin{aligned}
&+ \ \ im_\lambda \bar{\lambda}^i(\sigma_3)^{ij}\lambda^j + \left|\partial_M H^i\right|^2 + i\bar{\Psi}\gamma^M D_M\Psi + m^2_{H^i}|H^i|^2 \ , \\
&+ \ \ \left. im_\Psi \bar{\Psi}\Psi \right] , \qquad (3.2)
\end{aligned}
$$

with supersymmetry preserving mass terms for vector multiplets

$$m^2_\Sigma = -4k^2 + 2\sigma'' \ , \qquad (3.3)$$

$$m_\lambda = \frac{1}{2}\sigma' \ , \qquad (3.4)$$

and for hypermultiplets

$$m^2_{H^{1,2}} = (c^2 \pm c - \frac{15}{4})k^2 + \left(\frac{3}{2} \mp c\right)\sigma'' \ , \qquad (3.5)$$

$$m_\Psi = c\sigma' \ . \qquad (3.6)$$

We introduce the generic notation

$$m^2_\phi = ak^2 + b\sigma'' \ , \qquad (3.7)$$

$$m_\psi = c\sigma' \ , \qquad (3.8)$$

for the AdS mass terms of bosons (ϕ) and fermions (ψ) with

$$\sigma' = \frac{d\sigma}{dy} = k\epsilon(y) \ , \qquad (3.9)$$

$$\sigma'' = 2k\left[\delta(y) - \delta(y - \pi R)\right] \ , \qquad (3.10)$$

where the step function is defined as $\epsilon(y) = +1$ (-1) for positive (negative) y. With this notation, we can parametrize the bulk mass terms for vector multiplets as

$$a = -4 \ , \quad b = 2 \ , \quad c = \frac{1}{2} \ , \qquad (3.11)$$

and for hypermultiplets as

$$a = c^2 \pm c - \frac{15}{4} \ , \quad b = \frac{3}{2} \mp c \ . \qquad (3.12)$$

The parameter c controls the zero mode wave function profiles [32, 34]. When $c > 1/2$, the massless modes will be localized towards the $y = 0$ (UV) brane. The larger the value of c the stronger is the localization. On the other hand, when $c < 1/2$, the zero modes will be localized towards the $y = \pi R$ (IR) boundary. Kaluza-Klein (KK) modes localized near the IR brane, according to the AdS/CFT dictionary, correspond dominantly to CFT bound states.

The $SU(7)$ gauge symmetry can be broken into $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ by the Higgs mechanism or by boundary conditions. The spontaneous breaking of $SU(7)$ will lead to the doublet-triplet (D-T) splitting problem. Thus, it is advantageous to consider the breaking of the gauge symmetry via boundary conditions which elegantly eliminates the

D-T splitting problem. We chose the following boundary conditions in terms of Z_5 and Z_6 parity

$$Z_5 = (+1\ ,+1\ ,+1\ ,-1\ ,-1\ ,-1\ ,-1\ )\ , \tag{3.13}$$
$$Z_6 = (+1\ ,+1\ ,+1\ ,+1\ ,+1\ ,+1\ ,+1\ )\ , \tag{3.14}$$

which break $SU(7)$ to $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ at the fixed point $y = 0$. The parity assignments of $SU(7)$ vector supermultiplets in terms of (Z_5, Z_6) are

$$\begin{aligned} V^g(\mathbf{48}) &= V^{++}_{(\mathbf{8},\mathbf{1})_0} \oplus V^{++}_{(\mathbf{1},\mathbf{15})_0} \oplus V^{++}_{(\mathbf{1},\mathbf{1})_0} \oplus V^{-+}_{(\mathbf{3},\bar{\mathbf{4}})_{7/3}} \oplus V^{-+}_{(\bar{\mathbf{3}},\mathbf{4})_{-7/3}}\ , \\ \Sigma^g(\mathbf{48}) &= \Sigma^{--}_{(\mathbf{8},\mathbf{1})_0} \oplus \Sigma^{--}_{(\mathbf{1},\mathbf{15})_0} \oplus \Sigma^{--}_{(\mathbf{1},\mathbf{1})_0} \oplus \Sigma^{+-}_{(\mathbf{3},\bar{\mathbf{4}})_{7/3}} \oplus \Sigma^{+-}_{(\bar{\mathbf{3}},\mathbf{4})_{-7/3}}\ , \end{aligned} \tag{3.15}$$

where the lower indices show the $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ quantum numbers. After KK decomposition, only the $\mathcal{N} = 1$ SUSY $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ components of the vector multiplet V^g have zero modes. Kaluza-Klein modes which have warped masses of order $M_{UV} e^{-k\pi R} \sim$ TeV are localized towards the symmetry preserving IR brane, so they are approximately $SU(7)$ symmetric.

It is also possible to break the gauge symmetry on the $y = \pi R$ brane (by interchanging Z_5 and Z_6) which will have a different 4D CFT dual description in contrast to the previous case. If $SU(7)$ breaks to $SU(3) \times SU(4)_W \times U(1)_{B-L}$ on the IR brane, the dual description is a technicolor-like theory in which $SU(7)$ is broken to $SU(3) \times SU(4)_W \times U(1)_{B-L}$ by strong dynamics at the TeV scale. In case the gauge symmetry is broken on the UV brane, the dual descriptions is a theory with $SU(7)$ global symmetry and a weakly interacting $SU(3) \times SU(4)_W \times U(1)_{B-L}$ gauge group at the UV scale. The IR brane with a spontaneously broken conformal symmetry respects the $SU(7)$ gauge group. In this scenario, the $SU(7)$ gauge symmetry is composite(emergent) which is similar to the rishon model [43]. In order to reproduce the correct Weinberg angle $\sin^2\theta_W$, it is in general not advantageous to break the GUT symmetry on the IR brane because, from the AdS/CFT correspondence, the running of the gauge couplings is $SU(7)$ invariant at the TeV scale.

It is possible to strictly localize matter on the UV brane that preserves the $SU(3) \times SU(4)_W \times U(1)_{B-L}$ gauge symmetry. However, in this case we will not get a prediction of the weak mixing angle because we lack the absolute normalization factor of the hypercharge of the SM particles.[1] Thus it is preferable to place matter in $SU(7)$ multiplets into the 5D bulk so it can be approximately localized towards the UV brane by introducing bulk mass terms. In this case, the $U(1)_{B-L}$ charges of matter are quantized according to $SU(7)$ multiplet assignments and we could understand the observed electric charge quantization of the Universe.

We arrange quark supermultiplets into $\mathbf{28}, \overline{\mathbf{28}}$ symmetric $SU(7)$ representations and lepton multiplets into $\mathbf{7}, \bar{\mathbf{7}}$ representations

$$QX(\mathbf{28})_a = \begin{pmatrix} (\mathbf{6},\mathbf{1})_{8/3} & (\mathbf{3},\mathbf{4})_{1/3} \\ (\mathbf{3},\mathbf{4})_{1/3} & (\mathbf{1},\mathbf{10})_{-2} \end{pmatrix}\ , \tag{3.16}$$

[1]The relative normalization within the matter sector is determined by anomaly cancellation requirements.

$$\overline{QX}(\overline{\mathbf{28}})_a = \begin{pmatrix} (\bar{\mathbf{6}},\mathbf{1})_{-8/3} & (\bar{\mathbf{3}},\bar{\mathbf{4}})_{-1/3} \\ (\bar{\mathbf{3}},\bar{\mathbf{4}})_{-1/3} & (\mathbf{1},\overline{\mathbf{10}})_{2} \end{pmatrix} , \tag{3.17}$$

$$LX(\mathbf{7})_a = (\mathbf{3},\mathbf{1})_{\mathbf{4/3}} \oplus (\mathbf{1},\mathbf{4})_{-\mathbf{1}} , \tag{3.18}$$

$$\overline{LX}(\bar{\mathbf{7}})_a = (\bar{\mathbf{3}},\mathbf{1})_{-\mathbf{4/3}} \oplus (\mathbf{1},\bar{\mathbf{4}})_{\mathbf{1}} , \tag{3.19}$$

with the subscript a being the family index.

To obtain zero modes for chiral quark and lepton multiplets, we assign the following (Z_5, Z_6) parities to them

$$QX(\mathbf{28})_a = (\mathbf{6},\mathbf{1})^{-,+}_{\mathbf{8/3}} \oplus (\mathbf{1},\mathbf{10})^{-,+}_{-\mathbf{2}} \oplus (\mathbf{3},\mathbf{4})^{+,+}_{\mathbf{1/3}} , \tag{3.20}$$

$$\overline{QX}(\overline{\mathbf{28}})_a = (\bar{\mathbf{6}},\mathbf{1})^{-,+}_{-\mathbf{8/3}} \oplus (\mathbf{1},\overline{\mathbf{10}})^{-,+}_{\mathbf{2}} \oplus (\bar{\mathbf{3}},\bar{\mathbf{4}})^{+,+}_{-\mathbf{1/3}} , \tag{3.21}$$

$$LX(\mathbf{7})_a = (\mathbf{3},\mathbf{1})^{-,+}_{\mathbf{4/3}} \oplus (\mathbf{1},\mathbf{4})^{+,+}_{-\mathbf{1}} , \tag{3.22}$$

$$\overline{LX}(\bar{\mathbf{7}})_a = (\bar{\mathbf{3}},\mathbf{1})^{-,+}_{-\mathbf{4/3}} \oplus (\mathbf{1},\bar{\mathbf{4}})^{+,+}_{\mathbf{1}} . \tag{3.23}$$

Parity assignments for the conjugate fields Φ^c are opposite to those for Φ.

Because of the unification of $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ into $SU(7)$, we can determine the normalization of the $U(1)_{B-L}$ charge based on the the matter sector charge assignments in the fundamental representation of $SU(7)$

$$Q_{B-L} = (\ 4/3,\ 4/3,\ 4/3, -1, -1, -1, -1) . \tag{3.24}$$

Then from the relation of the gauge couplings

$$g_{B-L}\frac{Q_{B-L}}{2} = g_7 T^{B-L} , \tag{3.25}$$

we obtain

$$g_{B-L} = \sqrt{\frac{3}{14}} g_7 . \tag{3.26}$$

Here we normalize the $SU(7)$ generator as

$$Tr(T^a T^b) = \frac{1}{2}\delta^{ab} . \tag{3.27}$$

Thus the tree-level weak mixing angle can be predicted to be

$$\sin^2\theta_W \equiv \frac{g_Y^2}{g_Y^2 + g_L^2} = \frac{3}{20} = 0.15 . \tag{3.28}$$

In previous SUSY $SU(7)$ unification scenario with quark contents fitting in $\mathbf{28}, \overline{\mathbf{28}}$ dimensional representations, there is no ordinary proton decay problem related to heavy gauge boson exchanges (D-type operators) and dimension-five operators which can be seen from the charge assignments of the $SU(7)$ matter multiplets. Contributions from dimension four operators of the form $\lambda_{ijk}(QX)_i(\overline{LX})_j(\overline{LX})_k + \tilde{\lambda}_{ijk}(\overline{QX})_i(LX)_j(LX)_k$ can be forbidden by R-parity. However, it is also possible to fit the quark sectors in $\mathbf{21}, \overline{\mathbf{21}}$ dimensional representations of SU(7) instead of $\mathbf{28},\overline{\mathbf{28}}$ dimensional representations. Then dangerous IR-brane localized dimension five F-type operators of the form

$$\mathcal{L} = \int d^2\theta \frac{1}{M}[\lambda_{1ijkl}(QX)_i(QX)_j(QX)_k(LX)_l + \lambda_{2ijkl}(\overline{QX})_i(\overline{QX})_j(\overline{QX})_k(\overline{LX})_l],$$

can be introduced. Such dimension five operators can arise from a diagram involving the coupling of matter to the $\mathbf{35},\overline{\mathbf{35}}$ Higgs multiplet and the insertion of μ-term like mass terms for such Higgs fields. If matter is localized towards the IR brane, then the suppression scale M is of order TeV and this results in rapid proton decay. Since the profile of zero modes for bulk matter with $c \gtrsim 1/2$ is

$$\phi_+^{(0)} \sim e^{-(c-\frac{1}{2})ky} \ , \tag{3.29}$$

we could assign bulk mass terms to matter with $c \gtrsim 1/2$ to suppress the decay rates. Then for an IR brane localized dimension five operator, we require

$$\frac{1}{(\mathrm{TeV})} e^{-\sum\limits_i (c_i-\frac{1}{2})k\pi R} \lesssim \frac{10^{-8}}{M_{Pl}} \approx \frac{e^{-3k\pi R/2}}{(\mathrm{TeV})} \ , \tag{3.30}$$

which satisfies proton decay bounds [35]. However such requirements will lead to difficulty in giving natural Yukawa couplings. So we consider only the case with quark sector fitting in $\mathbf{28}$ and $\overline{\mathbf{28}}$ dimensional representations.

There are several ways to introduce Yukawa couplings. Orbifold GUTs are well known to solve the D-T splitting problem by assigning appropriate boundary conditions to bulk Higgs fields. Thus, it is also possible to introduce bulk Higgs fields in our scenario. Since

$$\mathbf{28} \otimes \overline{\mathbf{28}} = \mathbf{1} \oplus \mathbf{48} \oplus \mathbf{735} \ , \tag{3.31}$$

$$\mathbf{7} \otimes \overline{\mathbf{7}} = \mathbf{1} \oplus \mathbf{48} \ , \tag{3.32}$$

$$\overline{\mathbf{7}} \otimes \overline{\mathbf{7}} = \overline{\mathbf{21}} \oplus \overline{\mathbf{28}} \ , \tag{3.33}$$

we can introduce bulk Higgses $\Sigma, \tilde{\Sigma}$ in the $SU(7)$ adjoint representation $\mathbf{48}$, Δ_1, Δ_2 in $SU(7)$ symmetric representations $\mathbf{28},\overline{\mathbf{28}}$, and an $SU(7)$ singlet Higgs S to construct $SU(7)$ gauge invariant Yukawa couplings. We impose the following boundary conditions on the bulk Higgs fields

$$\Sigma, \tilde{\Sigma}(\mathbf{48}) = (\mathbf{8},\mathbf{1})_0^{-,+} \oplus (\mathbf{1},\mathbf{15})_0^{+,+} \oplus (\mathbf{1},\mathbf{1})_0^{-,+} \oplus (\mathbf{3},\overline{\mathbf{4}})_{7/3}^{-,+} \oplus (\overline{\mathbf{3}},\mathbf{4})_{-7/3}^{-,+} \ , \tag{3.34}$$

$$\Delta_1(\mathbf{28}) = (\mathbf{6},\mathbf{1})_{\mathbf{8/3}}^{-,+} \oplus (\mathbf{1},\mathbf{10})_{-\mathbf{2}}^{+,+} \oplus (\mathbf{3},\mathbf{4})_{\mathbf{1/3}}^{-,+} \ , \tag{3.35}$$

$$\Delta_2(\overline{\mathbf{28}}) = (\overline{\mathbf{6}},\mathbf{1})_{-\mathbf{8/3}}^{-,+} \oplus (\mathbf{1},\overline{\mathbf{10}})_{\mathbf{2}}^{+,+} \oplus (\overline{\mathbf{3}},\overline{\mathbf{4}})_{-\mathbf{1/3}}^{-,+} \ , \tag{3.36}$$

$$S(\mathbf{1}) = (\mathbf{1},\mathbf{1})_0^{+,+} \ . \tag{3.37}$$

In the orbifold projection above, we choose the most general boundary conditions [36, 37, 38] to eliminate unwanted zero modes. The results can be obtained from naive orbifolding by introducing the relevant heavy brane mass terms (on the UV brane) to change the Neumann boundary conditions to Dirichlet ones. Then the surviving zero modes give the

Higgs content required in a 4D SUSY $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ theory[2]. Bulk Yukawa couplings can be introduced as

$$S_{5D} = \int d^4x \int dy \sqrt{-g} \int d^2\theta \sum_{i=1,2,3} \Big(\tilde{y}^{QX}_{1ij} (QX)^i \Sigma (\overline{QX})^j + \tilde{y}^{LX}_{1ij} (LX)^i \Sigma (\overline{LX})^j$$
$$+ \tilde{y}^{QX}_{2ij} (QX)^i S (\overline{QX})^j + \tilde{y}^{LX}_{2ij} (LX)^i S (\overline{LX})^j + \tilde{y}^{LX}_{3ij} (\overline{LX})^i \Delta_1 (\overline{LX})^j$$
$$+ \tilde{y}^{LX}_{4ij} (LX)^i \Delta_2 (LX)^j \Big) \ . \tag{3.38}$$

Then at low energies, after the heavy KK modes are projected out, the effective 4D Yukawa couplings are

$$S_{4D} = \int d^4x \int d^2\theta \sum_{i=1,2,3} \Big(y^Q_{1ij} Q^i_L \Sigma_1 (Q^c_L)^j + y^L_{1ij} L^i_L \Sigma_1 (L^c_L)^j + y^Q_{2ij} Q^i_L S (Q^c_L)^j$$
$$+ y^L_{2ij} L^i_L S (L^c_L)^j + y^{N^c}_{ij} (L^c_L)^i \Delta (L^c_L)^j + y^N_{ij} (L_L)^i \overline{\Delta} (L_L)^j \Big) \ . \tag{3.39}$$

Here we denote the $SU(3) \times SU(4)_W \times U(1)_{B-L}$ multiplet $(\mathbf{3}, \mathbf{4})_{\mathbf{1/3}}$ by Q_L, the $(\bar{\mathbf{3}}, \bar{\mathbf{4}})_{\mathbf{-1/3}}$ by Q^c_L, the $(\mathbf{1}, \mathbf{4})_{\mathbf{-1}}$ by L_L, the $(\mathbf{1}, \bar{\mathbf{4}})_{\mathbf{1}}$ by L^c_L, the $(\mathbf{1}, \mathbf{15})_0$ by Σ_1, the $(\mathbf{1}, \mathbf{10})_2$ by Δ, the $(\mathbf{1}, \overline{\mathbf{10}})_{-2}$ by $\overline{\Delta}$, and the $(\mathbf{1}, \mathbf{1})_{\mathbf{0}}$ by S. As indicated in Ref. [6], such Yukawa interactions are necessary in a 4D theory to give acceptable low energy spectra. The SM fermionic masses and mixing hierarchy, which is related to the coefficients of the 4D Yukawa couplings, can be understood from the wave function profile overlaps [32, 44]. The profile of the bulk Higgs fields can also be determined from their bulk mass terms. In our scenario, bulk Higgses other than the **35** multiplet are not responsible for proton decay and can be localized anywhere (such as towards the IR brane to generate enough hierarchy). For simplicity, we can set the zero modes of bulk Higgs profiles to be flat with the mass terms $c_\Sigma = c_{\tilde{\Sigma}} = c_S = c_{\Delta_i} = c_H = 1/2$. The low energy Yukawa coupling coefficients that appeared in previous expressions are of order

$$y_{ij} \sim 4\pi \tilde{y}^{QX,LX}_{ij} \sqrt{k} \left(\prod_i \sqrt{\frac{1-2c_i}{e^{-2(c_i - \frac{1}{2})k\pi R} - 1}} \right) e^{-(c_{Xi} + c_{\bar{X}i} + c_H - \frac{3}{2})k\pi R} \ , \tag{3.40}$$

which can generate the required mass hierarchy and CKM mixing by the Froggatt-Nielson mechanism [52]. We can, for example, chose

$$c^{QX}_1 = c^{\overline{QX}}_1 = \frac{1}{2} + \frac{1}{8}, \quad c^{LX}_1 = c^{\overline{LX}}_1 = \frac{1}{2} + \frac{1}{16} \ , \tag{3.41}$$
$$c^{QX}_2 = c^{\overline{QX}}_2 = \frac{1}{2} + \frac{1}{16}, \quad c^{LX}_2 = c^{\overline{LX}}_2 = \frac{1}{2} + \frac{1}{32} \ , \tag{3.42}$$
$$c^{QX}_3 = c^{\overline{QX}}_3 = \frac{1}{2}, \qquad c^{LX}_3 = c^{\overline{LX}}_3 = \frac{1}{2} \ . \tag{3.43}$$

Here we use the fact that $e^{-k\pi R} \simeq (\mathrm{TeV})/k \simeq \mathcal{O}(10^{-16})$ and $\tilde{y}^{QX,LX}_{ij} \sqrt{k} \sim \mathcal{O}(1)$. Besides, it is obvious from the charge assignment in the matter sector that there are no unwanted mass relations in our scenario, such as $m_\mu : m_e = m_s : m_d$, that appear in an $SU(5)$ GUT.

[2] We can also eliminate the bulk singlet Higgs field S and choose the boundary conditions so that the $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ singlet comes from (projections of) bulk Higgs hypermultiplets $\Sigma(\mathbf{48})$. An additional $\Sigma_2(\mathbf{1}, \mathbf{15})_0$ from $\tilde{\Sigma}$ is required to break $SU(4)_W \times U(1)_{B-L}$ to $SU(2)_L \times SU(2)_R \times U(1)_Z \times U(1)_{B-L}$.

4. Gauge Coupling Unification in SUSY SU(7) Unification

The Lagrangian relevant for the low energy gauge interactions has the following form

$$S = \int d^4x \int_0^{\pi R} dy \sqrt{-g} \left[-\frac{1}{4g_5^2} F^{aMN} F^a_{MN} - \delta(y) \frac{1}{4g_0^2} F^a_{\mu\nu} F^{a\mu\nu} - \delta(y - \pi R) \frac{1}{4g_\pi^2} F^a_{\mu\nu} F^{a\mu\nu} \right] ,$$

where g_5 is the dimensionful gauge coupling in the 5D bulk, g_0 and g_π are the relevant gauge couplings on the $y = 0$ and $y = \pi R$ brane, respectively. The brane kinetic terms are necessary counter terms for loop corrections of the gauge field propagator. In AdS space there are several tree-level mass scales which are related as

$$\mu \ll M_{KK} \simeq \frac{\pi k}{e^{\pi k R} - 1} \ll \frac{1}{R} \ll k \simeq M_* \ . \tag{4.1}$$

Thus, the 4D tree-level gauge couplings can be written as [22]

$$\frac{1}{g_a^2}(\mu) = \frac{\pi R}{g_5^2} + \frac{1}{g_0^2} + \frac{1}{g_\pi^2} + \frac{1}{8\pi^2} \tilde{\Delta}(\mu, Q) \ , \tag{4.2}$$

where the first three terms contain the tree-level gauge couplings, and $\tilde{\Delta}(\mu, Q)$ represents the one-loop corrections. The explicit dependence on the subtraction scale cancels that of the running boundary couplings in such a way that the quantity $g_a^2(\mu)$ is independent of the renormalization scale. We assume that the bulk and brane gauge groups become strongly coupled at the 5D Planck scale $M_{5D} = \Lambda$ with

$$\frac{1}{g_0^2}(\Lambda) \approx \frac{1}{g_\pi^2}(\Lambda e^{-k\pi R}) \approx \frac{1}{16\pi^2} \ , \qquad \frac{\pi R}{g_5^2}(\Lambda) \approx \mathcal{O}(1) \ . \tag{4.3}$$

The GUT breaking effects at the fixed points are very small compared to bulk GUT symmetry preserving effects. Thus, we can split the contributions to the gauge couplings into symmetry preserving and symmetry breaking pieces

$$\begin{aligned}
\frac{1}{g_a^2}(\mu) &= \frac{\pi R}{g_5^2}(\Lambda) + \frac{1}{g_0^2}(\Lambda) + \frac{1}{g_\pi^2}(\Lambda e^{-k\pi R}) + \frac{1}{8\pi^2} \left[\tilde{\Delta}(\mu, \Lambda) + b_0^a \ln \frac{\Lambda}{\mu} + b_\pi^a \ln \frac{\Lambda e^{-k\pi R}}{\mu} \right] , \\
&\simeq (\text{SU(7) symmetric}) + \frac{1}{8\pi^2} \left[\tilde{\Delta}(\mu, \Lambda) + b_0^a \ln \frac{\Lambda}{\mu} + b_\pi^a \ln \frac{\Lambda e^{-k\pi R}}{\mu} \right] , \\
&\equiv (\text{SU(7) symmetric}) + \frac{1}{8\pi^2} \Delta(\mu, \Lambda) \ .
\end{aligned} \tag{4.4}$$

The general expression for $\Delta(\mu, \Lambda)$ was calculated in [54]. The contributions from the vector multiplets are

$$\begin{pmatrix} \Delta_{U(1)_{B-L}} \\ \Delta_{SU(3)_C} \\ \Delta_{SU(4)_W} \end{pmatrix}_V = (\text{SU(7) symmetric}) + \begin{pmatrix} 0 \\ -9 \\ -12 \end{pmatrix} \ln\left(\frac{k}{\mu}\right) \ , \tag{4.5}$$

with

$$T(V_{++}) = (\ 0,\ 3,\ 4) \tag{4.6}$$

$$T(V_{-+}) = (\ 7,\ 4,\ 3) \ , \tag{4.7}$$

for $U(1)_{B-L}$, $SU(3)_C$, and $SU(4)_W$, respectively. Here we normalize the $U(1)_{B-L}$ gauge coupling according to $g_{B-L}^2 = 3g_7^2/14$.

We can use the facts $c_i^{QX}, c_i^{LX} \geq 1/2$ to simplify the matter contributions to

$$\Delta_M(\mu,k) = T(H_{++})\left[\ln\left(\frac{k}{\mu}\right) - c_H \ln\left(\frac{k}{T}\right)\right] + c_H T(H_{+-})\ln\left(\frac{k}{T}\right) \\ - c_H T(H_{-+})\ln\left(\frac{k}{T}\right) + T(H_{--})\left[\ln\left(\frac{k}{\mu}\right) - (1+c_H)\ln\left(\frac{k}{T}\right)\right] . \quad (4.8)$$

Thus the contributions from the bulk matter hypermultiplets are

$$\begin{pmatrix} \Delta_{U(1)_{B-L}} \\ \Delta_{SU(3)_C} \\ \Delta_{SU(4)_W} \end{pmatrix}_H = (\text{SU(7) symmetric}) + \begin{pmatrix} \frac{12}{7} \\ 12 \\ 12 \end{pmatrix} \ln\left(\frac{k}{\mu}\right) , \quad (4.9)$$

with

$$T(H_{++})|^m_{H+H^c} = \left(\frac{12}{7},\ 12,\ 12\right) , \quad (4.10)$$

$$T(H_{-+})|^m_{H+H^c} = \left(\frac{198}{7},\ 18,\ 18\right) . \quad (4.11)$$

The contributions from the bulk Higgs hypermultiplets include two **48** dimensional representations, **28** and $\overline{\mathbf{28}}$ dimensional representations and possible one singlet.[3] The contributions from the bulk Higgs hypermultiplets are

$$\begin{pmatrix} \Delta_{U(1)_{B-L}} \\ \Delta_{SU(3)_C} \\ \Delta_{SU(4)_W} \end{pmatrix}_M = (\text{SU(7) symmetric}) + \begin{pmatrix} \frac{30}{7} \\ 0 \\ 14 \end{pmatrix} \ln\left(\frac{k}{\mu}\right) , \quad (4.16)$$

with

$$T(H_{++})|^h_{H+H^c} = \left(\frac{30}{7},\ 0,\ 14\right) , \quad (4.17)$$

$$T(H_{-+})|^h_{H+H^c} = \left(\frac{131}{7},\ 23,\ 9\right) . \quad (4.18)$$

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[3]We can also add Higgs fields in the **35** and $\overline{\mathbf{35}}$ representations with flat profiles and impose the following boundary conditions

$$\mathbf{35} = (\mathbf{1},\mathbf{1})_4^{-,+} \oplus (\mathbf{1},\bar{\mathbf{4}})_{-3}^{+,+} \oplus (\bar{\mathbf{3}},\mathbf{4})_{5/3}^{+,+} \oplus (\mathbf{3},\mathbf{6})_{-2/3}^{-,+} , \quad (4.12)$$

$$\overline{\mathbf{35}} = (\mathbf{1},\mathbf{1})_{-4}^{-,+} \oplus (\mathbf{1},\mathbf{4})_{3}^{+,+} \oplus (\mathbf{3},\bar{\mathbf{4}})_{-5/3}^{+,+} \oplus (\bar{\mathbf{3}},\bar{\mathbf{6}})_{2/3}^{-,+} . \quad (4.13)$$

Then the beta functions receive additional contributions:

$$T(H_{++})|^h_{H+H^c} = \left(\frac{52}{7},\ 4,\ 4\right) , \quad (4.14)$$

$$T(H_{-+})|^h_{H+H^c} = \left(\frac{18}{7},\ 6,\ 6\right) . \quad (4.15)$$

Thus, the total contribution to the RGE running of the three gauge couplings are

$$\frac{1}{g_a^2} = (\text{SU}(7)\ \text{symmetric}) + \frac{1}{8\pi^2}\Delta_a \ , \tag{4.19}$$

with

$$\begin{pmatrix} \Delta_{U(1)_{B-L}} \\ \Delta_{SU(3)_C} \\ \Delta_{SU(4)_W} \end{pmatrix} = (\text{SU}(7)\ \text{symmetric}) + \begin{pmatrix} 6 \\ 3 \\ 14 \end{pmatrix} \ln\left(\frac{k}{\mu}\right) \ . \tag{4.20}$$

We summarize the supermultiplets in SUSY SU(7) GUT model that contribute to running of the three gauge couplings upon $M_{\tilde{U}}$ as follows:

- Gauge: $V^g(\mathbf{48}), \mathbf{\Sigma^g}(\mathbf{48})$.
- Matter: $QX_a(\mathbf{28}), \overline{QX}_a(\overline{\mathbf{28}}), LX(\mathbf{7}), \overline{LX}_a(\overline{\mathbf{7}})$ $(a = 1, 2, 3)$.
- Higgs: $\Sigma(\mathbf{48}), \tilde{\Sigma}(\mathbf{48}), \Delta_1(\mathbf{28}), \Delta_2(\mathbf{28})$.

We can also consider the following symmetry breaking chain for the partial unification $SU(4)_W \times U(1)_{B-L}$:

$$SU(4)_W \times U(1)_{B-L} \to SU(2)_L \times SU(2)_R \times U(1)_Z \times U(1)_{B-L} \to SU(2)_L \times U(1)_Y \ .$$

Detailed discussions on this symmetry breaking chain can be found in our previous work [6].

Assuming that the left-right scale, which is typically the $SU(2)_R$ gauge boson mass scale M_R, is higher than that of the soft SUSY mass parameters M_S, the RG running of the gauge couplings below the $SU(4)_W \times U(1)_{B-L}$ partial unification scale $M_{\tilde{U}}$ is calculated as follows.

- For $M_Z < E < M_S$, the $U(1)_Y, SU(2)_L$, and $SU(3)_C$ beta-functions are given by the two Higgs-doublet extension of the SM

$$(b_1, b_2, b_3) = (\ 7, -3, -7) \ . \tag{4.21}$$

- For $M_S < E < M_R$, the $U(1)_Y, SU(2)_L$, and $SU(3)_C$ beta-functions are given by

$$(b_1, b_2, b_3) = (12,\ 2, -3) \ . \tag{4.22}$$

- For $M_R < E < M_{\tilde{U}}$, the $\sqrt{2}U(1)_Z, \sqrt{\frac{14}{3}}U(1)_{B-L}, SU(2)_L = SU(2)_R$, and $SU(3)_C$ beta functions are given by

$$(b_0, b_1, b_2, b_3) = (22,\ 6,\ 16,\ 3) \ . \tag{4.23}$$

In our calculation the mirror fermions are fitted into $SU(4)_W$ multiplets and acquire masses of order M_R. The $\tilde{\Sigma}(\mathbf{15})$ Higgs fields decouple at scales below $M_{\tilde{U}}$ [6]. We can calculate the $SU(7)$ unification scale when we know the $SU(4)_W \times U(1)_{B-L}$ partial unification scale

$M_{\tilde{U}}$, which can be determined from the coupling of $U(1)_Z$ at M_R. Here we simply set $M_{\tilde{U}}$ as a free parameter. At the weak scale our inputs are [39]

$$M_Z = 91.1876 \pm 0.0021 \ , \tag{4.24}$$

$$\sin^2\theta_W(M_Z) = 0.2312 \pm 0.0002 \ , \tag{4.25}$$

$$\alpha_{em}^{-1}(M_Z) = 127.906 \pm 0.019 \ , \tag{4.26}$$

$$\alpha_3(M_z) = 0.1187 \pm 0.0020 \, , \tag{4.27}$$

which fix the numerical values of the standard $U(1)_Y$ and $SU(2)_L$ couplings at the weak scale

$$\alpha_1(M_Z) = \frac{\alpha_{em}(M_Z)}{\cos^2\theta_W} = (98.3341)^{-1} \ , \tag{4.28}$$

$$\alpha_2(M_Z) = \frac{\alpha_{em}(M_Z)}{\sin^2\theta_W} = (29.5718)^{-1} \ . \tag{4.29}$$

The RGE running of the gauge couplings reads

$$\frac{d\ \alpha_i}{d\ln E} = \frac{b_i}{2\pi}\alpha_i^2 \ , \tag{4.30}$$

where E is the energy scale and b_i are the beta functions. Our numerical results (See fig.1) show that successful unification of the three gauge couplings is only possible for small $M_R \lesssim 500$ GeV and relatively high $M_{\tilde{U}}$. For example, if we choose $M_S = 200$ GeV, $M_R = 400$ GeV and $M_{\tilde{U}} = 2.0 \times 10^6$ GeV, we obtain successful $SU(7)$ unification at

$$M_U = 9.0 \times 10^6 \text{ GeV} \ , \tag{4.31}$$

and

$$\frac{1}{g_U^2} \simeq 4.65 \ . \tag{4.32}$$

Such low energy M_R may be disfavored by electro-weak[40] and flavor precision bounds[41]. In general, with additional matter and Higgs contents (for example, additional bulk $\mathbf{7}, \overline{\mathbf{7}}$ messenger fields), the low M_R requirement for gauge coupling unifications can be relaxed. Besides, symmetry breaking of $SU(3) \times SU(4)_W \times U(1)_{B-L}$ will lead to non-minimal left-right model[6]. Thus relatively low M_R can be consistent with flavor precision bounds[41]. The choice of $M_{\tilde{U}} = 2.0 \times 10^6$ generates a hierarchy between the weak scale and the partial unification scale. Lacking the knowledge of $U(1)_Z$ gauge coupling strength upon SUSY left-right scale, the mild hierarchy between partial unification scale $M_{\tilde{U}}$ and SUSY left-right scale can be the consequences of logarithm running of the various gauge couplings.

It follows from the AdS/CFT correspondence that the $SU(7)$ unification in RS model are also a successful 4D unification. Our $SU(7)$ model is vector-like and thus anomaly free. The 5D theory in the bulk is also anomaly free because the theory on the UV (which is a $SU(3) \times SU(4)_W \times U(1)_{B-L}$ theory [6]) and IR branes is non-anomalous [42].

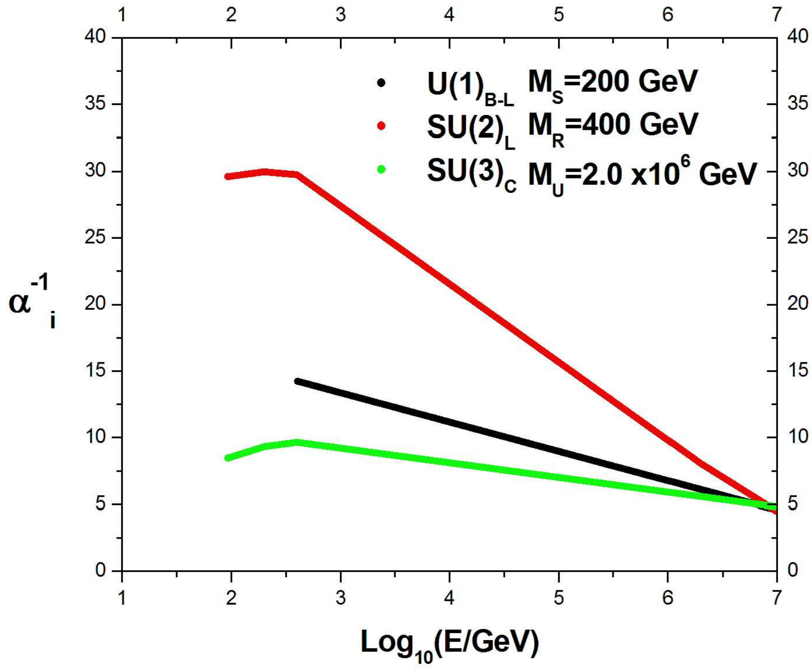

Figure 1: One loop relative running of the three gauge coupling in SUSY SU(7) GUT model. Here $SU(4)_W$ gauge coupling (upon $M_{\tilde{U}}$) is identified with $SU(2)_L$ gauge coupling. The $U(1)_{B-L}$ gauge coupling strength at the left-right scale M_R is determined by $U(1)_Y$ and $SU(2)_L$ gauge coupling. Due to the discontinuity between $U(1)_{B-L}$ and $U(1)_Y$ gauge coupling at M_R, we do not show the $U(1)_Y$ running below M_R in this figure.

5. Supersymmetry Breaking and Semi-direct Gauge Mediation

The orbifold projection reduces the 5D $\mathcal{N}=1$ supersymmetry, which amounts to 4D $\mathcal{N}=2$ SUSY, to 4D $\mathcal{N}=1$ supersymmetry. We need to break the remaining $\mathcal{N}=1$ supersymmetry to reproduce the SM matter and gauge content. One interesting possibility is to use the predictive conformal supersymmetry breaking proposed for vector-like gauge theories [25, 26]. Conformal supersymmetry breaking in a vector-like theory can be embedded into a semi-direct gauge mediation model [53] by identifying a subgroup of the flavor group to be the unifying group of the SM.

5.1 Supersymmetry Breaking in the Conformal Window

The setup of conformal supersymmetry breaking in a vector-like theory involves an $\mathcal{N}=1$ $SU(N_c)$ gauge theory with $N_Q < N_c$ quarks $Q_i, \tilde{Q}_i$ $(i=1,\cdots,N_Q)$ in fundamental and anti-fundamental representations, and $N_Q \times N_Q$ gauge singlets S_i^j. Messenger fields $P_a, \tilde{P}_a$ $(a=1,\cdots,N_P)$ with mass m are also introduced to promote the model to a superconformal theory. The total number of flavors satisfies $3N_C/2 < N_Q+N_P < 3N_c$. The superpotential reads

$$W = \lambda Tr(SQ\tilde{Q}) + mP\tilde{P} \ , \tag{5.1}$$

with $Tr(SQ\tilde{Q}) = S_i^j Q^i \tilde{Q}_j$.

When the mass parameter m can be neglected, the theory has a infrared fixed point. When S_i^j develop vacuum expectation values (VEVs), Q_i and $\tilde{Q}_i$ can be integrated out. Because $N_Q < N_C$, the theory has a runaway vacuum when all quark fields are integrated out. Such runaway vacuum can be stabilized by quantum corrections to the Kähler potential and leads to dynamical supersymmetry breaking. The conformal gauge mediation model is especially predictive because m is its only free parameter.

5.2 The AdS/CFT Dual of Seiberg Duality in the Conformal Region and Semi-Direct Gauge Mediation

The AdS/CFT correspondence [27] indicates that the compactification of Type IIB string theory on $AdS_5 \times S^5$ is dual to $\mathcal{N} = 4$ super Yang-Mills theory. The duality implies a relation between the AdS radius R and $g_{YM}^2 N = g_s N$: $R^4 = 4\pi g_s N l_s^4$, in string units l_s. The source of an operator in the CFT sides correspond to the boundary value of a bulk field in gravity side. The generating function of the conformal theory is identified with the gravitational action in terms of ϕ_0:

$$\left\langle \exp\left(-\int d^4x \phi_0 \mathcal{O}\right)\right\rangle_{CFT} = \exp(-\Gamma[\phi_0]) \ . \tag{5.2}$$

The AdS/CFT correspondence can be extended to tell us that any 5D gravitational theory on AdS_5 is holographically dual to some strongly coupled, possibly large N, 4D CFT [47, 48]. The metric of an AdS_5 slice can be written as

$$ds^2 = \frac{L^2}{z^2}\left(g_{\mu\nu} dx^\mu dx_\nu + dz^2\right) \ , \tag{5.3}$$

which is related to RS metric [21] by

$$z = Le^{y/L} \ , \tag{5.4}$$

with $L = 1/k$ being the AdS radius. According to the AdS/CFT dictionary [34, 49, 51], the RG scale μ is related to the fifth coordinate by $\mu = 1/z$.

We introduce the bulk gauge symmetry $SU(N_F) \times SU(N_P) \times SU(N_Q) \times U(1)_R$ with $SU(N_P) \times SU(N_Q) \times U(1)_R$ being the global symmetry of the 4D theory. The gauge symmetry $SU(N_F)$ is broken into $SU(N_C) \times SU(N_F - N_C)$ at the boundary. The matter content is N_F chiral multiplets in fundamental and N_F chiral multiplets in anti-fundamental $SU(N_F)$ representations. We require the boundary conditions to yield N_F chiral multiplets in both the fundamental and anti-fundamental representations of $SU(N_C)$ at the UV brane, and N_F chiral multiplets in both the fundamental and anti-fundamental representations of $SU(N_F - N_C)$ at the IR brane. These boundary conditions can be realized by choosing the projection modes at $y = 0$ and $y = \pi R$ as

$$P_1(y=0) = P_2(y=\pi R) = (\underbrace{1, \cdots, 1}_{N_c}, \underbrace{-1, \cdots, -1}_{N_F - N_c}) \ . \tag{5.5}$$

Thus, in terms of $SU(N_c) \times SU(N_F - N_C)$ quantum numbers, the field parities and projections are

$$\begin{aligned} Q(N_F) &= (N_C, 1)_{++} \oplus (1, N_F - N_C)_{--} \ , & (5.6)\\ Q^c(\overline{N_F}) &= (\overline{N_C}, 1)_{--} \oplus (1, \overline{N_F - N_C})_{++} \ , & (5.7)\\ P(N_F) &= (N_C, 1)_{++} \oplus (1, N_F - N_C)_{--} \ , & (5.8)\\ P^c(\overline{N_F}) &= (\overline{N_C}, 1)_{--} \oplus (1, \overline{N_F - N_C})_{++} \ , & (5.9)\\ S(1) &= (1,1)_{++} \ , \quad S^c(1,1) = (1,1)_{--} \ . & (5.10) \end{aligned}$$

For $Q(N_F)$ and $P(N_F)$ with $c \gg 1/2$, the $(N_C, 1)$ multiplets (denoted by Q, P) are fully localized to the UV brane while the $(1, \overline{N_F - N_C})$ multiplets (denoted by q, p corresponding to Q^c, P^c respectively) are strictly localized to the IR brane. The bulk zero modes localized towards the UV brane correspond to elementary fields. So, in the conformal supersymmetry breaking setting, we have the fundamental fields $Q_i, \tilde{Q}_i, P, \tilde{P}$ and we can introduce their interactions on the UV brane

$$W|_{UV} = \lambda Tr(SQ\tilde{Q}) + mP\tilde{P} \ . \tag{5.11}$$

The presence of the additional gauge symmetry $SU(N_F)$ is required by anomaly matching of $SU(N_C)$ and $SU(N_F - N_C)$ in the Seiberg duality. Anomaly matching in the Seiberg duality is equivalent to anomaly inflow of the Chern-Simmons terms of the 5D bulk, which gives opposite contributions on the two boundaries [58].

According to the setup of the conformal supersymmetry breaking scenario, we require the theory to enter a superconformal region when we can neglect the masses of P and $\tilde{P}$. To ensure that the theory is superconformal in a certain energy interval, and to be predictive, we need to determine the exact gauge beta functions. In the 5D picture, we can determine the beta-functions by calculating the variation of the gauge couplings with respect to the fifth dimensional coordinate. The gauge couplings are obtained by calculating the correlation functions of the conserved currents. Then from the 5D gauge coupling running [54], we can obtain the dependence on the fifth dimension by replacing $k\pi R$ with $\ln(z/L) = -\ln(\mu L)$. This way, we obtain the following leading contributions

$$\begin{aligned} \frac{1}{g_a^2} = &-\frac{\ln(\mu L)}{kg_5^2} + \frac{\ln(\mu L)}{8\pi^2}\left[\frac{3}{2}T_a(V_{++}) + \frac{3}{2}T_a(V_{+-}) - \frac{3}{2}T_a(V_{-+}) - \frac{3}{2}T_a(V_{--})\right] \\ &-\frac{\ln(\mu L)}{8\pi^2}\left[(1-c_H)T_a(H_{++}) + c_H T_a(H_{+-}) - c_H T_a(H_{-+})\right. \\ &\left.+(1+c_H)T_a(H_{--})\right] \ . \end{aligned} \tag{5.12}$$

To determine the bulk couplings, we consider the $SU(N_F)$ gauge symmetry on the UV brane, IR brane and in the bulk. Then, by matching the beta function in the dual description

$$b = \frac{8\pi^2}{kg_5^2} - \frac{3}{2}N_F = -3N_F \ , \tag{5.13}$$

we obtain

$$\frac{8\pi^2}{kg_5^2} = -\frac{3}{2}N_F \ . \tag{5.14}$$

In our case with a bulk gauge group $SU(N_F)$ and a gauge group $SU(N_c)$ on the UV and IR branes, for the $SU(N_c)$ gauge couplings we have

$$T(V_{++}) = N_c \ , \quad T(V_{--}) = N_F - N_c \ , \quad T_a(H_{++}) = \frac{1}{2} \ . \tag{5.15}$$

The leading contributions are[4]

$$b_a = \frac{8\pi^2}{kg_5^2} - \frac{3}{2}N_c + \frac{3}{2}(N_F - N_c) + (1 - C_P)N_P + (1 - C_Q)N_Q \tag{5.16}$$

$$= -3N_c + (1 - C_P)N_P + (1 - C_Q)N_Q \ . \tag{5.17}$$

The sub-leading contributions to the gauge couplings depend on $\ln\ln\mu$ and correct the beta functions with

$$\delta b_a = -\frac{1}{\ln(\mu L)}T_a(V_{++}) \approx \frac{g_a^2 N_c b_a}{8\pi^2} \ . \tag{5.18}$$

This expression is valid at two-loop level which we can see reproducing the NSVZ formula [56]

$$\frac{dg^2}{d\ln\mu} = -\frac{g^4}{8\pi^2}\frac{3T(Ad) - \sum_j T(r_j)(1-\gamma_j)}{1 - \frac{T(Ad)g^2}{8\pi^2}} \ , \tag{5.19}$$

by identifying

$$\gamma_P = C_P \ , \quad \gamma_Q = C_Q \ . \tag{5.20}$$

Via the AdS/CFT correspondence, the bulk mass is related to the conformal dimension of the operator $\mathcal{O}$ that couples to p-forms [46]

$$(\Delta + p)(\Delta + p - 4) = m^2 \ . \tag{5.21}$$

Since the anomalous dimensions γ_P and γ_Q are determined by the superconformal invariance of the boundary, we can obtain the bulk mass terms for the P and Q hypermultiplets. We can obtain the scaling dimension of the 4D superconformal theory via the R-symmetry charge assignments

$$\Delta = \frac{3}{2}R_{sc} \ . \tag{5.22}$$

The $U(1)_R$ symmetry of the superconformal theory on the UV brane is determined by the a-maximization technique [55], with a defined by t'Hooft anomalies of the superconformal R-charge

$$a = \frac{3}{32}(3TrR^3 - TrR) \ , \tag{5.23}$$

[4]The matter contributions are valid for $c_{++} > 1/2$. For $c_{++} \leq 1/2$, $1 - c_P$ in front of N_P is replaced by c_P.

and the R-charge being the combination of an arbitrarily chosen R-charge R_0 and other U(1) charges

$$R = R_0 + \sum_i c_i Q_i \ . \tag{5.24}$$

This value is the same as the one obtained in [57]. For example with $N_c = 4, N_Q = 3, N_P = 5$, it is [57]

$$\Delta_s = 1.48 \ , \quad \Delta_Q = 0.765 \ . \tag{5.25}$$

In the 4D picture the RG fixed points require $\gamma_S^* + 2\gamma_Q^* = 0$ because of the superconformal nature of the theory.

From the AdS/CFT point of view the spontaneous breaking of the CFT originates from the IR brane. In the limit $c_Q \gg 1/2$ ($c_{\tilde{Q}} \ll -1/2$) the q and $\tilde{q}$ fields are localized to the IR brane, which means that they are composites in the strongly interacting CFT. The UV brane interaction can be promoted to a bulk Yukawa coupling between bulk hypermultiplets S and $Q, \tilde{Q}$

$$S = \int d^4x dy \sqrt{-g} \int d^2\theta \lambda_b S \tilde{Q} Q \ , \tag{5.26}$$

which, after projection, will give the IR brane coupling

$$S = \int d^4x dy \sqrt{-g} \int d^2\theta \delta(y - \pi R) \tilde{\lambda} S \tilde{q} q \ . \tag{5.27}$$

Thus, we can anticipate interactions of the form $\tilde{\lambda} S \tilde{q} q$ in the IR brane. If q and $\tilde{q}$ are not strongly localized, they are mixtures of composite and elementary particles. The coupling of S to $q, \tilde{q}$ will also lead to a coupling between S and CFT operators $\mathcal{O}$ at the boundary. This can also be seen if we completely localize q and $\tilde{q}$. The hypermultiplet S at the UV boundary is a source of conformal operators. With $c = 1/2$ for S the mixing of CFT states ($SU(N_F - N_c)$ singlets) and S is marginal.[5] According to the AdS/CFT interpretation[6], they correspond to the Seiberg dual superpotential with the coupling of the form

$$W = \tilde{\lambda} S \tilde{q} q + \omega S \mathcal{O} \ . \tag{5.28}$$

The coefficients $\tilde{\lambda}$ and ω can be determined by the AdS/CFT correspondence via two-point correlation functions. We simply match to the standard Seiberg dual result giving

$$\tilde{\lambda} = \frac{1}{\mu} \ , \qquad \omega = \lambda \ . \tag{5.29}$$

Here μ can be defined in the context of SQCD, where the beta function coefficients for the magnetic ($\tilde{b}$) and electric (b) theories and their respective dynamical transmutation scales $\tilde{\Lambda}$ and Λ are related as

$$\Lambda^b \bar{\tilde{\Lambda}}^{\tilde{b}} = (-1)^{F-N} \mu^{b+\tilde{b}} \ . \tag{5.30}$$

[5] The mixing is important for $|c| \leq 1/2$ but marginal for $c = 1/2$.

[6] From the AdS/CFT dictionary [51] we can see that the operator $\mathcal{O}$ is dynamical appearing in the low energy superpotential.

In the dual description the fields related to P and $\tilde{P}$ are integrated out after the RGE running from energies z_{UV}^{-1} to z_{IR}^{-1} if the mass parameter satisfies $z_{UV}^{-1} > m > z_{IR}^{-1}$. Thus we anticipate that $\tilde{p}$ and p does not appear as massless fields on the IR brane. This can also be understood by observing that adding only the UV mass terms spoils the zero mode solutions. So the original zero modes $\tilde{p}$ and p, which are localized towards the IR brane, are no longer massless and will not appear in the dual superpotential. This AdS/CFT interpretation of the Seiberg duality is valid in the IR region for $3/2N_C < N_F < 3N_C$ which are strongly coupled. If the mass parameter is small, $m < z_{IR}^{-1}$, then it appears as a small perturbation on the UV brane. We then can promote the mass parameter m to a bulk field L, with $L(z_0) = m$, and introduce bulk Yukawa couplings between L and the $\tilde{P}, P$ hypermultiplets. Similarly to the case of $\tilde{Q}$ and Q, the dual description on the IR brane has the form[7]

$$W \sim \frac{1}{\mu}\left[S\tilde{q}q + L\tilde{p}p\right] + \tilde{\omega}\langle\mathcal{O}_1\rangle L + \lambda S\mathcal{O} \ , \tag{5.31}$$

$$\sim \frac{1}{\mu}\left[S\tilde{q}q + L\tilde{p}p\right] + mL + \lambda S\mathcal{O} \ , \tag{5.32}$$

with the coefficients, again, determined by matching to the Seiberg duality. Here we require the conformal symmetry is spontaneously broken by $\langle\mathcal{O}_1\rangle \neq 0$. After integrating out the fields S and $\mathcal{O}$ such that

$$S = 0 \ , \quad \mathcal{O} = -\frac{1}{\mu}\tilde{q}q \ , \tag{5.33}$$

we can see that the F-term of L

$$-F_L^\dagger = m + \frac{1}{\mu}\tilde{p}p \ , \tag{5.34}$$

is non-vanishing (by rank conditions [53]) which indicates that SUSY is broken. It was pointed out in [53] that SUSY breaking by F-term VEVs of L can cause some problems, such as a low energy Landau pole and vanishing gaugino masses if we identify the flavor symmetry with the SM gauge group. Thus it is preferable to study the case with $z_{UV}^{-1} > m > z_{IR}^{-1}$ where we can integrate out the fields related to P and $\tilde{P}$. Neglecting the additional contributions from $\tilde{P}$ and P, the 5D action is [59]

$$\begin{aligned}\mathcal{L} = & \int d^4\theta \frac{1}{2}(T+T^\dagger)e^{-(T+T^\dagger)\sigma}\left(S^\dagger e^{-V}S + S^c e^V S^{c\dagger} + (S \leftrightarrow Q, \tilde{Q})\right) \\ & + \int d^2\theta e^{-3T\sigma} S^c \left[\partial_5 - \frac{1}{\sqrt{2}}\chi - (\frac{3}{2} - c)T\sigma'\right] S + h.c. + (S \leftrightarrow Q, \tilde{Q}) \\ & + W_0\delta(y) + e^{-3T\sigma}W_{\pi R}\delta(y - \pi R) \ ,\end{aligned} \tag{5.35}$$

where T is the radion supermultiplet

$$T = R + iB_5 + \theta\Psi_R^5 + \theta^2 F_{\tilde{S}} \ , \tag{5.36}$$

[7] In the presence of P and $\tilde{P}$ there are also terms of the form $(K\tilde{q}p + M\tilde{p}q)/\mu$, which is similar to the case of S with $\lambda = 0$.

B_5 is the fifth component of the graviphoton, Ψ^5_R is the fifth component of the right-handed gravitino, and $F_{\tilde{S}}$ is a complex auxiliary field. After the lowest component of the radion acquires a VEV, we can re-scale the fields

$$(S\ ,\ S^c) \to \frac{e^{k|y|}}{\sqrt{R}}(S\ ,\ S^c)\ . \tag{5.37}$$

Neglecting the gauge sector, for the F-terms of S and S^c we have

$$\begin{aligned} -F_S^\dagger &= \frac{e^{-k|y|}}{R}\left[-\partial_5 + (\frac{1}{2}+c_S)k\epsilon(y)\right]S^c + \frac{e^{-k|y|}}{R}\lambda_b\tilde{Q}Q \\ &+ \delta(y)\lambda\tilde{Q}Q + \delta(y-\pi R)e^{-2k\pi R}\left(\frac{1}{\mu}\tilde{q}q + \lambda\mathcal{O}\right)\ , \end{aligned} \tag{5.38}$$

$$-F_{S^c}^\dagger = \frac{e^{-k|y|}}{R}\left[\partial_5 - (\frac{1}{2}-c_S)k\epsilon(y)\right]S\ , \tag{5.39}$$

while for the Q and $\tilde{Q}$ fields

$$\begin{aligned} -F_Q^\dagger &= \frac{e^{-k|y|}}{R}\left[-\partial_5 + (\frac{1}{2}+c_Q)k\epsilon(y)\right]Q^c + \frac{e^{-k|y|}}{R}\lambda_b S\tilde{Q} \\ &+ \delta(y)\lambda S\tilde{Q} + \delta(y-\pi R)e^{-2k\pi R}\frac{1}{\mu}S\tilde{q}\ , \end{aligned} \tag{5.40}$$

$$-F_{Q^c}^\dagger = \frac{e^{-k|y|}}{R}\left[\partial_5 - (\frac{1}{2}-c_Q)k\epsilon(y)\right]Q\ . \tag{5.41}$$

The solutions for S, Q, and $\tilde{Q}$ are

$$S(y) = C_S e^{(\frac{1}{2}-c_S)k|y|}\ , \tag{5.42}$$

$$Q(y) = C_Q e^{(\frac{1}{2}-c_Q)k|y|}\ , \tag{5.43}$$

$$\tilde{Q}(y) = C_{\tilde{Q}} e^{(\frac{1}{2}-c_{\tilde{Q}})k|y|}\ , \tag{5.44}$$

with the boundary conditions

$$C_S = S\ ,\quad C_Q = Q\ ,\quad Qe^{(\frac{1}{2}-c_Q)k\pi R} = q\ , \tag{5.45}$$

and $c_S = 1/2$ for S. Substituting the previous expressions into the flatness conditions we can see that, except for the boundary terms, the solutions for S^c and Q^c are

$$S^c(y) = \frac{\lambda_b C_Q C_{\tilde{Q}}}{k(c_Q + c_{\tilde{Q}})}\epsilon(y)e^{(\frac{1}{2}-c_S-c_Q-c_{\tilde{Q}})k|y|}\ , \tag{5.46}$$

$$Q^c(y) = \frac{\lambda_b C_S C_{\tilde{Q}}}{k(c_S + c_{\tilde{Q}})}\epsilon(y)e^{(\frac{1}{2}-c_S-c_Q-c_{\tilde{Q}})k|y|}\ . \tag{5.47}$$

The boundary conditions determine the SUSY relations

$$S^c(y=0) = \lambda\tilde{Q}Q\ ,\quad S^c(y=\pi R) = \frac{1}{\mu}\tilde{q}q + \lambda\mathcal{O} \tag{5.48}$$

$$Q^c(y=0) = \lambda S\tilde{Q}\ ,\quad Q^c(y=\pi R) = \frac{1}{\mu}S\tilde{q}\ . \tag{5.49}$$

Substituting back into the previous solutions, we find that the $F_S^\dagger$ and $F_Q^\dagger$ flatness conditions cannot be satisfied at the same time. So supersymmetry is broken in this scenario. This conclusion agrees with the conjecture of [26] for a vanishing S VEV. The non-vanishing F-term VEV of S, which has an R-charge $2N_c/N_Q - 2 \neq 0$, breaks the R-symmetry spontaneously. Thus, gaugino masses are not prohibited. Sfermion masses can be generated by the operator which arises from integrating out the messengers P and $\tilde{P}$

$$\Delta K \sim -\left(\frac{g_{SM}^2}{16\pi^2}\right)^2 \int d^4\theta \frac{c_1}{m^2} Tr(S^\dagger S)(\Phi^\dagger \Phi) \ , \tag{5.50}$$

which gives

$$m_{\tilde{f}}^2 \sim \left(\tfrac{g_{SM}^2}{16\pi^2}\right)^2 \tfrac{c_1}{m^2}(F_S^\dagger F_S) \ . \tag{5.51}$$

Gaugino masses can be generated by an anti-instanton induced operator [26]

$$c_2 \int d^4\theta \left(\frac{1}{16\pi^2}\right) \frac{(\Lambda_L^\dagger)^{2N_c+1}}{m^{4N_c+2}} Tr(S^\dagger S)\det(\bar{D}^2 S^\dagger) W_a W^a \tag{5.52}$$

where $\Lambda_L^\dagger$ is the holomorphic dynamical scale below the thresholds of P and $\tilde{P}$. The gaugino masses

$$m_{gaugino} = c_2 \left(\frac{g_{SM}^2}{16\pi^2}\right) \frac{(\Lambda_L^\dagger)^{2N_c+1}}{m^{4N_c+2}} (F_S^\dagger F_S)(F_S^\dagger)^{N_Q} \ , \tag{5.53}$$

are not too small because the gauge couplings are large [26].

6. Conclusion

In this paper, we propose the SUSY $SU(7)$ unification of the $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ model. Such unification scenario has rich symmetry breaking chains in a five-dimensional orbifold. We study in detail the SUSY $SU(7)$ symmetry breaking into $SU(3)_C \times SU(4)_W \times U(1)_{B-L}$ by boundary conditions in a Randall- Sundrum background and its AdS/CFT interpretation. We find that successful gauge coupling unification can be achieved in our scenario. Gauge unification favors low left-right and unification scales with tree-level $\sin^2\theta_W = 0.15$. We use the AdS/CFT dual of the conformal supersymmetry breaking scenario to break the remaining $\mathcal{N} = 1$ supersymmetry. We employ AdS/CFT to reproduce the NSVZ formula and obtain the structure of the Seiberg duality in the strong coupling region for $\frac{3}{2}N_c < N_F < 3N_C$. We show that supersymmetry is indeed broken in the conformal supersymmetry breaking scenario with a vanishing singlet vacuum expectation value.

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