Current in narrow channels of anisotropic superconductors

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We argue that in channels cut out of anisotropic single crystal superconductors and narrow on the scale of London penetration depth, the persistent current must cause the transverse phase difference provided the current does not point in any of the principal crystal directions. The difference is proportional to the current value and depends on the anisotropy parameter, on the current direction relative to the crystal, and on the transverse channel dimension. An experimental set up to measure the transverse phase is proposed.

In isotropic superconductors the supercurrent density is proportional to the gradient of the gauge invariant phase $\nabla\varphi = \nabla\chi + 2\pi\mathbf{A}/\phi_0$, where χ is the phase of the order parameter $\psi = |\psi|e^{i\chi}$, $\phi_0 = \pi\hbar c/|e|$ is the flux quantum, and $\mathbf{A}$ is the vector potential:

$$\mathbf{j} = \frac{2e\hbar}{M}|\psi|^2\,\nabla\varphi\,;\qquad(1)$$

M is the carrier mass. If the cross-section of a superconducting channel has small dimensions compared with the London penetration depth λ, both $\mathbf{j}$ and $\nabla\varphi$ have the only nonzero components along the channel. For straight channels with the long dimension along x, $\partial_y\varphi = \partial_z\varphi = 0$, i.e., the phase is constant in the transverse directions.

This, however, is not the case for *anisotropic* superconductors, where

$$j_i = 2e\hbar\, M_{ik}^{-1}|\psi|^2\,\partial_k\varphi\,.\qquad(2)$$

Here, M_{ik} is the mass tensor, and summation is implied over repeated indices. It is convenient to normalize the masses: $m_{ik} = M_{ik}/(M_a M_b M_c)^{1/3} = M_{ik}/\overline{M}$. Then the eigenvalues of m_{ik} are related by $m_a m_b m_c = 1$. In the uniaxial case which we consider for simplicity, $m_a^2 m_c = 1$; the masses then are expressed in terms of a single parameter, the anisotropy ratio $\gamma^2 = m_c/m_a$: $m_a = \gamma^{-2/3}$, $m_c = \gamma^{4/3}$.

We invert Eq. (2) to obtain

$$\partial_i\varphi = \frac{\overline{M}}{2e\hbar|\psi|^2}\,m_{ik}\,j_k\,,\qquad(3)$$

which shows that the phase gradient $\nabla\varphi$ and the current $\mathbf{j}$ are not parallel unless both of them point in a principal crystal direction.

Consider a channel of a rectangular cross-section and denote by W and d its width in the y direction and the thickness in the z direction. To further simplify the problem, we take z as one of principal crystal directions, say

b. The axes c and a are then situated in the xy plane as shown in Fig. 1. Denoting by θ the misalignment angle between the c axis and x, we readily obtain:

$$
\begin{aligned}
m_{xx} &= \gamma^{-2/3}(\sin^2\theta + \gamma^2\cos^2\theta)\,,\\
m_{yy} &= \gamma^{-2/3}(\cos^2\theta + \gamma^2\sin^2\theta)\,,\\
m_{xy} &= \gamma^{-2/3}(\gamma^2 - 1)\sin\theta\cos\theta\,,
\end{aligned}\qquad(4)
$$

whereas $m_{zz} = m_b = \gamma^{-2/3}$ and $m_{zx} = m_{zy} = 0$.
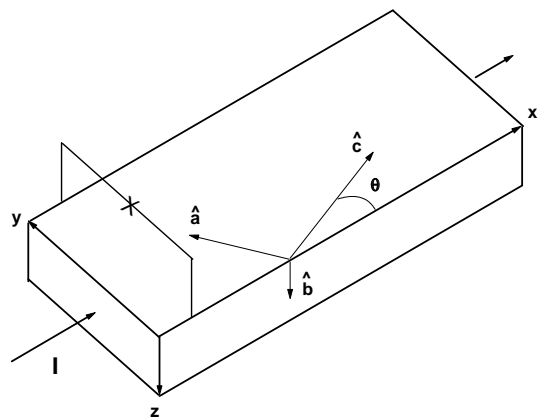

FIG. 1: The current I in the superconducting channel of the width W ($0 < y < W$) and of the thickness d. The crystal axis b coincides with z. The cross denotes the Josephson junction.

Let a small supercurrent I be fed into the channel along x. Then, Eq. (3) yields the only non-zero transverse component of $\nabla\varphi$:

$$\partial_y\varphi = \frac{\overline{M}}{2e\hbar|\psi|^2}\,m_{yx}\,\frac{I}{Wd}\,.\qquad(5)$$

Therefore, the side faces of the channel at $y = 0, W$

should possess the phase difference of

$$\Delta\varphi = \frac{\overline{M}}{2e\hbar|\psi|^2}\, m_{xy}\, \frac{I}{d} \qquad (6)$$

for any fixed x.

In principle, this phase difference can be recorded by attaching a superconducting wire between the points $\{x,0\}$ and $\{x,W\}$. If the wire carries the Josephson junction, the current in the wire is

$$I_w = I_0 \sin\left(\frac{\overline{M}\, m_{xy}}{2e\hbar|\psi|^2 d}\, I\right), \qquad (7)$$

where I_0 is the maximum Josephson current. Thus, the current I_w in the wire oscillates as a function of the driving current I in the channel with the period

$$\Delta I = \frac{4\pi e\hbar|\psi|^2 d}{\overline{M}\, m_{xy}} \quad . \qquad (8)$$

Near the critical temperature T_c, the domain for which the above formulas are written, the equilibrium order parameter ψ in clean materials is related to the carrier density n_e: $|\psi|^2 \approx n_e\tau$, $\tau = 1 - T/T_c$. This estimate holds provided $I \ll I_{dp}$ where $I_{dp} \approx (c\phi_0/16\pi^2\lambda^2\xi)Wd$ is the depairing current.

To find out conditions under which the current oscillations can be seen near T_c, we require that both I and ΔI are small relative to I_{dp}. Taking $W \sim \lambda(T) \approx \lambda_0/\sqrt{\tau} \approx \overline{M}c^2/4\pi e^2 n_e\sqrt{\tau}$ and $\xi \approx \xi_0/\sqrt{\tau}$, we obtain:

$$m_{xy} \gg 16\pi/\kappa\,, \qquad (9)$$

where $\kappa = \lambda_0/\xi_0$. Since $m_{xy} \sim m_c = \gamma^{4/3}$, this inequality can be satisfied for strongly anisotropic materials with large Ginzburg-Landau parameter κ.

At low T's one can use the London expression for the current density instead of Eq. (2):

$$j_i = -\frac{c\phi_0}{4\pi^2\lambda^2}\, m_{ik}^{-1}\left(\nabla\chi + \frac{2\pi}{\phi_0}\, \mathbf{A}\right)_k, \qquad (10)$$

where $\lambda = (\lambda_a^2\lambda_c)^{1/3}$. We, therefore, can replace in the above formulas $2e\hbar|\psi|^2/\overline{M}$ with $c\phi_0/4\pi^2\lambda^2$. In particular, we have

$$\Delta I = \frac{c\phi_0 d}{2\pi\lambda^2\, m_{xy}} \quad . \qquad (11)$$

We then obtain a condition $m_{xy} \gg 8\pi/\kappa$ similar to (9) in the whole temperature domain; if this condition is not satisfied, the window of driving currents I for the oscillations of I_w to occur becomes narrow or disappears.

Thus, the period ΔI is proportional to the channel thickness d and to τ, the temperature distance from T_c. Taking for an estimate realistic values of $\lambda = 2000\,\text{Å}$, $d = 1000\,\text{Å}$, and $\gamma = 10$, we obtain $\Delta I \approx 1\,\text{mA}$ for low temperatures; the period shrinks on approaching T_c.

It is worth noting that for the effect to be observable, the wire contacts must be small relative to W. Otherwise the variation of the phase along the channel will destroy the quantum coherence. On the other hand, if the point contacts are fixed at different values of x, an additional phase difference

$$(\Delta\varphi)_x = (\Delta\varphi)_y\, \frac{m_{xx}}{m_{xy}}\, \frac{\Delta x}{W} \qquad (12)$$

enters the argument of the sine in Eq. (7); here $\Delta x = |x_2 - x_1|$ and $x_{1,2}$ are the contacts positions. This contribution changes the period ΔI of the Josephson current oscillations, but the very fact of periodic dependence of I_w on I persists.

If the driving current I changes with time, so does the phase difference $\Delta\varphi$. Then, the junction in the wire is subject to a voltage

$$V = \frac{\hbar}{2e}\partial_t\Delta\varphi = \frac{2\pi\lambda^2}{c^2 d}\, m_{xy}\, \partial_t I \sim \frac{2\pi\lambda^2}{c^2 d}\, m_{xy}\, \omega I\,, \quad (13)$$

where ω is the driving frequency. For the parameters used above and $\omega \sim 10^9\,\text{Hz}$, the voltage may reach values of a few μV; this estimate improves if thinner samples and higher temperatures are used: $V \propto 1/\tau d$.

Writing the driving ac current as $I = I_a\sin\omega t$, we readily obtain:

$$\begin{aligned} I_w &= I_0\sin(\varphi_a\sin\omega t) \\ &= 2I_0\sum_{n=0}^{\infty} J_{2n+1}(\varphi_a)\,\sin(2n+1)\omega t \qquad (14) \end{aligned}$$

where J's are the Bessel functions and

$$\varphi_a = \frac{4\pi^2\lambda^2 m_{xy}}{c\phi_0 d}\, I_a \quad . \qquad (15)$$

For small amplitudes I_a of the driving current, one can keep only the term $n = 0$ in the series (14): $I_w = I_0\varphi_a\sin\omega t$, i.e., the junction response is linear. Since the voltage in this case is $V = (\phi_0\varphi_a/2\pi c)\omega\cos\omega t$, one can say that the junction is loaded with an inductance $L = c\phi_0/2\pi I_0$. This simple interpretation does not hold if the condition $\varphi_a \ll 1$ is violated; the junction then should generate odd harmonics.

We have considered a particular case of a misalignment between the supercurrent direction and the crystal. Of course, situations different from that of Fig. 1 can be readily treated. Our aim is to turn the community attention to the "transverse" phase difference which must accompany supercurrents as long as they are not parallel to principal crystal directions. We believe that effects related to the transverse phase in anisotropic materials and their complexity are not exhausted by simple examples we describe. In particular, we did not consider details of the electric field penetration into the channel (implicitly assuming the penetration length

$\ell_E \gg W$). Given richness of time dependent Josephson phenomena, one may envisage a host of time dependent effects related to the transverse phase. It should also be stressed that phenomena we describe, happen on the scale of the London λ, which might be large as compared to the coherence length ξ, the scale relevant to the well-studied phase slips.