Test of scaling theory in two dimensions in the presence of valley splitting and intervalley scattering in Si-MOSFETs.

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We show that once the effects of valley splitting and intervalley scattering are incorporated, renormalization group theory consistently describes the metallic phase in silicon metal-oxide-semiconductor field-effect transistors down to the lowest accessible temperatures.

The two-parameter scaling theory of quantum diffusion in two dimensions [1, 2] has been remarkably successful in describing the properties around the metal-insulator transition (MIT) in electron systems confined to silicon inversion layers (MOSFETs) [3, 4, 5]. The theory is based on the scaling hypothesis that both the resistivity and the electron-electron scattering amplitudes become scale dependent in a diffusive system due to the singular long ranged nature of the diffusive propagators, $\mathcal{D}(q,\omega) = 1/(Dq^2+\omega)$, in a disordered medium [6, 7]. The predicted scale dependencies calculated using renormalization group (RG) theory [1] were recently verified experimentally in Ref. [3] without any fitting parameters. Since the theory considered the valleys to be degenerate and distinct, the experiments were limited to temperatures larger than the characteristic valley splitting and intervalley scattering rate ($T \gtrsim 500\,\mathrm{mK}$). The effects of scaling are, however, significant at low temperatures and it is therefore important to test the scaling hypothesis at much lower temperatures. We show that when the RG theory is extended to include valley splitting and intervalley scattering [8] the scaling properties in the metallic phase can be described quantitatively down to the lowest reliably accessible temperatures, $T \approx 200\,\mathrm{mK}$.

The evolution with scale (temperature) of the two-parameters, namely, the resistance, ρ, and the electron-electron interaction strength, γ_2, in the spin-triplet channel were discussed in detail for $\rho \lesssim 1$ (in units of $\pi h/e^2$) in terms of RG theory in Ref. [1]. (In Fermi-liquid notation, γ_2 is related to the amplitude F_0^a as $\gamma_2 = -F_0^a/(1+F_0^a)$.) The theory predicts that, while γ_2 increases monotonically as the temperature is reduced, ρ behaves non-monotonically, changing from insulating behavior ($d\rho/dT < 0$) at high temperatures to metallic behavior ($d\rho/dT > 0$) at low temperatures, with the crossover occurring when γ_2 attains the value $\gamma_2^* = 0.45$. Although the maximum value $\rho_{\max}$ occurs at a crossover temperature $T = T_{\max}$, both of which are sample specific and hence non-universal, the two-parameter scaling theory predicts that the behaviors of $\rho(T)/\rho_{\max}$ and $\gamma_2(T)$ are universal when plotted as functions of $\xi = \rho_{\max}\ln(T_{\max}/T)$. The above predictions, including the value of γ_2^*, were verified experimentally in Refs. [1, 3] in the temperature range where the two valleys may be considered to be degenerate and distinct.

For n-(001) silicon inversion layer the conduction band has two almost degenerate valleys located close to the X-points in the Brillouin zone. While the sharpness of the interface of the inversion layer leads to the splitting, Δ_v, of the two valley bands, the atomic scale irregularities found at the interface gives rise to a finite intervalley scattering rate, $\hbar/\tau_\perp$ [9]. The singularity of the diffusion modes, especially those in the valley-triplet sector, are cut-off at low frequencies as a result [8, 10]. Hence, the specific form of the RG equations, which is sensitive only to the number of singular modes, depends on if k_BT is greater than or less than the scales Δ_v or/and $\hbar/\tau_\perp$.

The relevant RG equations for the different temperature ranges may be combined as follows [8]:

$$\frac{d\rho}{d\xi} = \rho^2\left[1-(4K-1)\left(\frac{\gamma_2+1}{\gamma_2}\log(1+\gamma_2)-1\right)\right] \quad (1a)$$

$$\frac{d\gamma_2}{d\xi} = \rho\frac{(1+\gamma_2)^2}{2} \quad (1b)$$

The parameter K accounts for the number of singular diffusion modes in each temperature range. For temperatures $T \gtrsim T_v$ and $T_\perp$, where $k_BT_v = \Delta_v$ and $k_BT_\perp = \hbar/\tau_\perp$, the two bands are effectively degenerate and distinct; the constant K in this case is proportional to the square of the number of valleys, n_v, i.e., $K = n_v^2 = 4$ ($n_v = 2$ for silicon). In the temperature range $T_\perp \lesssim T \lesssim T_v$, the two bands remain distinct but are split and hence each valley contributes independently to $\Delta\sigma(b)$, i.e., $K = n_v = 2$. At still lower temperatures $T \lesssim T_\perp$, intervalley scattering mixes the two valleys to effectively produce a single valley so that $K = 1$.

A few important clarifications regarding the use of Eq. (1) are discussed below. First, for the case $K = 2$,

when the bands are split but distinct, it has been shown that using a single amplitude γ_2 to describe the interaction in all the seven $(4K-1)$ modes is an approximation that is valid only if the temperature range $T_\perp \lesssim T \lesssim T_v$ is not too wide [8]. In general, when the bands are split certain amplitudes evolve differently from γ_2, thereby necessitating the need to go beyond the two-parameter scaling description [8, 11]. The deviation is large when the RG evolution is allowed to proceed to exponentially large scales or $T \ll T_v$. In our case, however, since $T_\perp$, which effectively mixes the two bands, is only a fraction smaller than T_v, the deviation of the amplitudes is quickly limited by $T_\perp$. We therefore assume that all the amplitudes remain degenerate and contribute equally to ρ, which amounts to taking $K = 2$ in Eq. (1).

The second point concerns the weak-localization (WL) contribution [12] to Eq. (1). It is seen experimentally that the phase breaking rate, $\hbar/\tau_\varphi$, saturates at low electron densities ($n \lesssim 10^{11}\,\mathrm{cm}^{-2}$) for $T \lesssim 500\,\mathrm{K}$. Correspondingly, a strong suppression of the WL correction is also observed in this regime [13]. These observations are consistent with our results, as is discussed later. We have therefore neglected the weak-localization contribution in Eq. (1) when analyzing the cases $K = 2$ and 1 (these are the relevant cases at low temperatures).

In Ref. [3] it was shown that γ_2 may be determined experimentally by exploiting the b^2 dependence of the magnetoconductance $\Delta\sigma(b) \equiv \Delta\sigma(B,T) = \sigma(B,T) - \sigma(0,T)$ in a weak parallel magnetic field $b = g\mu_B B/k_B T \lesssim 1$. In the weak field limit $\Delta\sigma(b)$ is given as [14, 15]

$$\Delta\sigma(b) = -0.091\frac{e^2}{\pi h} K\gamma_2\,(\gamma_2+1)\,b^2 \qquad (2)$$

Hence the slope of $\Delta\sigma(b^2)$ provides a direct measure of γ_2, given of course that K is known.

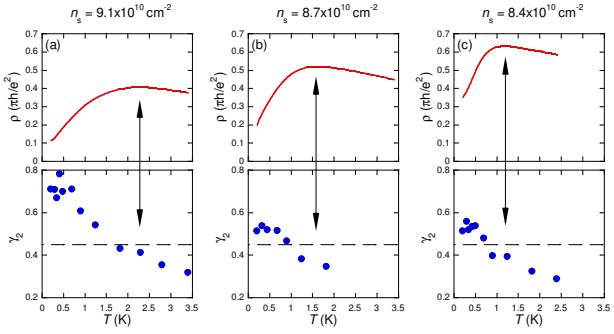

FIG. 1: Upper panels: $\rho(T)$ traces (in units of $\pi h/e^2$) for three different electron densities, $n_s = 9.87, 9.58$ and $9.14 \times 10^{10}\,\mathrm{cm}^{-2}$. Lower panels: Extracted values of $\gamma_2(T)$ using Eq. 2 using $K = 4$, for the same electron densities. (See Ref. [3] for further details.) The dashed lines are positioned at the critical value $\gamma_2^* = 0.45$. Note that the maximum in $\rho(T)$ occurs when γ_2 attains approximately the value γ_2^*.

In the upper panels in Fig. 1, we plot $\rho(T)$ at zero magnetic field for three different electron densities. They show a characteristic non-monotonic behavior as predicted in (1). In the lower panels in Fig. 1 we plot the extracted values of γ_2 using Eq. (2) with $K = 4$, i.e., assuming that the valleys are degenerate and distinct. The dashed horizontal line marks the point $\gamma_2 \approx 0.45$ approximately where $\rho(T)$ attains its maximum value in remarkable agreement with Eq. (1). (At these temperatures quantum coherence is relevant and its contribution to weak localization, $d\rho/d\xi = n_v\rho^2$, is to be added to Eq. (1a).) The results of the comparison between theory and experiment are presented in Fig. 2. The solid squares (■) are the experimental data points for $n_s = 9.1 \times 10^{10}\,\mathrm{cm}^{-2}$, reproduced here from Fig. 1(a). The solid lines are the predicted theoretical curves for $\rho(T)$ and $\gamma_2(T)$ with the parameters $K = 4$, $\rho_{max} = 0.4$ and $T_{max} = 2.3\,\mathrm{K}$. (Here, T_{max} is the temperature at which $\rho(T)$ attains its maximum value, $\rho(T_{max}) = \rho_{max}$.) The remarkable agreement between theory and experiment is especially striking given that the theory has no adjustable parameters.

At temperatures below 0.5 K, the experimentally extracted values of $\gamma_2(T)$ in Fig. 2(b) seem to saturate with further decrease in T. We believe that the saturation is an artifact of the analysis related to our assumptions that both the valley splitting and the intervalley scattering are negligible at the lowest temperatures. As noted earlier, the large number of valley modes $K = n_v^2$ reduces to just $K = n_v$ for temperature $T_\perp \lesssim T \lesssim T_v$ and to just $K = 1$ for $T \lesssim T_\perp$. In the following, we recalculate $\gamma_2(T)$ taking these considerations into account.

The experimentally extracted values of γ_2, using $K = 2$ and $K = 1$, are shown in Fig. 2(b) as diamonds (red ♦) and stars (blue ★), respectively. The procedure used to extract these values are the same as that used for $K = 4$, namely, by fitting the $\sigma(b^2)$ traces in Fig. 1(b) to Eq. (2) using the appropriate K values. We find very favorable agreement with theory (solid line) if the crossover scales are chosen such that $T_v \approx 0.5\,\mathrm{K}$ and $T_\perp \approx 0.2\,\mathrm{K}$. (Note that for these temperatures the WL corrections have not been included in Eq. (1) for the reasons discussed earlier.) These values are in good agreement with earlier estimates of T_v [16] and $T_\perp$ [17] obtained at higher densities employing different methods. We checked by direct calculation using Eq. (1) that the theoretical values of ρ and γ_2 are not affected significantly when crossing these scales, provided that the WL corrections are not included below $T \lesssim 500\mathrm{K}$. Deviations from the solution for $K = 4$ taking $K = 2$ and $K = 1$ are shown in Fig. 2 as long (red) and short (blue) dashed lines, respectively. As can be seen, the deviations are insignificant (almost indiscernible) down to $T = 0.2\,\mathrm{K}$.

By comparing with experiments we have extended the test of the scaling equations (1) down to the lowest reliably measurable temperatures $T \approx 0.2\,\mathrm{K}$. Concerning still lower temperatures, i.e., lower than $T = 0.2\,\mathrm{K}$, the theory predicts (not shown here) that while $\rho(T)$ satu-

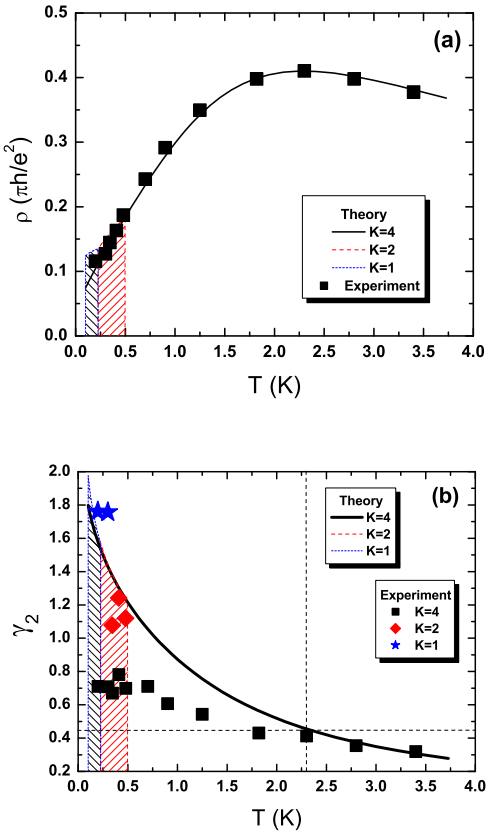

FIG. 2: The result of the comparison between theory (lines) and experiment (symbols) for ρ and γ_2 are presented in (a) and (b), respectively. The parameter $K = 4$ corresponds to the case when the two valleys ($n_v = 2$) are degenerate, i.e., $T > T_v$, where $T_v \approx 0.5\,\mathrm{K}$ is the estimated valley splitting. $K = 2$ corresponds to the temperature range $T \lesssim T_v$, and $K = 1$ corresponds to the region $T \lesssim T_\perp \approx 0.2\,\mathrm{K}$ where the intervalley scattering mixes the two valleys to give one valley.

rates and then begins to drop again at ultra low temperatures ($T \lesssim 50\,\mathrm{mK}$), $\gamma_2(T)$ rises fast monotonically for $K = 1$. Further tests of these predictions are in progress.

To conclude, we have shown that if valley splitting and intervalley scattering are incorporated into the RG theory, the latter quantitatively describes the metallic phase down to the lowest readily accessible temperatures. The extracted values of intervalley scattering time and valley splitting are in good agreement with those previously obtained at higher densities using different methods.

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