CTP-TAMU-34/01

UU-01-08

hep-th/0112100

7D Bosonic Higher Spin Gauge Theory: Symmetry Algebra and Linearized Constraints

E. Sezgin

Center for Theoretical Physics, Texas A&M University, College Station, TX 77843, USA

P. Sundell

Department for Theoretical Physics, Uppsala University, Box 803, S-75108, Uppsala, Sweden

Abstract

We construct the minimal bosonic higher spin extension of the 7D AdS algebra $SO(6,2)$, which we call $hs(8^*)$. The generators, which have spin $s = 1, 3, 5, ...$, are realized as monomials in Grassmann even spinor oscillators. Irreducibility, in the form of tracelessness, is achieved by modding out an infinite dimensional ideal containing the traces. In this a key role is played by the tree bilinear traces which form an $SU(2)_K$ algebra. We show that gauging of $hs(8^*)$ yields a spectrum of physical fields with spin $s = 0, 2, 4, ...$ which make up a UIR of $hs(8^*)$ isomorphic to the symmetric tensor product of two 6D scalar doubletons. The scalar doubleton is the unique $SU(2)_K$ invariant 6D doubleton. The spin $s \geq 2$ sector comes from an $hs(8^*)$-valued one-form which also contains the auxiliary gauge fields required for writing the curvature constraints in covariant form. The physical spin $s = 0$ field arises in a separate zero-form in a 'quasi-adjoint' representation of $hs(8^*)$. This zero-form also contains the spin $s \geq 2$ Weyl tensors, i.e. the curvatures which are non-vanishing on-shell. We suggest that the $hs(8^*)$ gauge theory describes the minimal bosonic, massless truncation of M theory on $AdS_7 \times S^4$ in an unbroken phase where the holographic dual is given by N free $(2,0)$ tensor multiplets for large N.

1 Introduction

A higher spin (HS) gauge theory is a general covariant field theory with an additional infinite set of local symmetries. The full set of symmetries is based on a rigid HS algebra, which typically extends the spacetime (super) AdS group. The first step in the construction of a HS gauge theory is therefore to identify this algebra and find its representations. Extensive results along these lines already exist in the literature for $D \leq 5$ [1, 2, 3, 4, 5, 6, 7, 8]. In this paper we construct the minimal bosonic HS algebra in $D = 7$ and its linearized massless field equations. Altogether these results show that the HS theories in diverse dimensions have many features in common, which points to some underlying universal principle.

As will be discussed in more detail in [9], HS gauge theories in diverse dimensions that include *massive* higher spin fields, are the anti-holographic duals of free conformal field theories in the limit of large number of free fields [10, 11, 6, 12]. HS field equations do not seem to admit any truncation to ordinary (super)gravity, because lower spin fields appear as sources of HS fields. This is consistent with the fact that the stress-energy tensor of a free CFT does not form a closed OPE algebra, as has been illustrated in the case of the 3d supersingleton [13] (though the conserved charges of course form the finite dimensional conformal group which closes). Thus, if one wishes to give an anti-holographic description of a renormalization group flow in the vicinity of a (free) CFT with conserved HS currents one should use HS gauge theory instead of ordinary (super)gravity. In particular, the HS theory discussed in this paper is the minimal bosonic truncation of a supersymmetric HS theory which is [9] is proposed to contain solutions describing the flow from the strongly coupled A_{N-1} fixed point [14] to the free theory of N $(2,0)$ tensors.

Our construction of $hs(8^*)$ is given in terms of a basic Grassmann even oscillator y_α which is a Dirac spinor of $SO(6,1)$. The generators of $hs(8^*)$ are traceless monomials in the oscillators. The trace is taken with the symmetric charge conjugation matrix. Importantly, the tracelessness condition is imposed as a coset condition, namely $hs(8^*) = \mathcal{L}/\mathcal{I}$ where $\mathcal{L}$ is a certain Lie algebra which contains both traceless generators and generators with non-zero trace, and $\mathcal{I} \subset \mathcal{L}$ is an ideal containing all the generators with non-zero trace.

In fact, the above construction is similar to the one of $hs(2,2)$ in $D = 5$ [6, 7], which is the minimal bosonic HS extension of $SO(4,2)$. The ideal $\mathcal{I}$ occurring in that case contains the trace $K \sim \bar{y}y$. The spectrum of the 5D theory is given by the product of two scalar doubletons, which have vanishing K charge. This yields massless fields in $D = 5$ with spin $s = 0, 2, 4, ...$ The spin $s = 0$ field and the on-shell curvatures (Weyl tensors) are collectively described in HS theory by a master field Φ which is a zero-form. As a consequence of modding out $\mathcal{I}$ the master field Φ is dressed up by 'K^2-expansions' which are crucial for getting the correct AdS lowest weights [6, 7].

In $D = 7$, the ideal $\mathcal{I}$ is generated by the traces, $K_3 \sim \bar{y}y$, $K_- \sim yy$ and $K_+ \sim \bar{y}\bar{y}$, which form an $SU(2)_K$ algebra (note that the charge conjugation matrix is symmetric in $D = 7$ whereas it is antisymmetric in $D = 5$). Massless fields in AdS_7 arise in tensor products of two 6d conformal tensors known as doubletons [15][1]. The spectrum of the theory is given by the product of two

[1]In the $\mathcal{N} = 2$ superextension of the 7D theory the superdoubleton squares belong to an isolated series [16, 17].

scalar doubletons. The scalar doubleton is the only one with vanishing $SU(2)_K$ charge (in the $(2,0)$ superextension of the theory this scalar is one of the five scalars in the tensor multiplet). As in five dimensions, this yields massless fields of spin $s = 0, 2, 4, ...$ We will show that the gauging of $hs(8^*)$ in $D = 7$ yields a matching set of gauge fields. In particular, we verify that for each spin s the algebra generators are in one-to-one correspondence with the previously known set of physical and auxiliary spin s gauge fields given in terms of a particular set of spin s Lorentz tensors [18, 19]. Moreover, modding out the ideal $\mathcal{I}$ the 7D master scalar field Φ is dressed up with K^2-expansions, where now $K^2 = K_I K_I$. It is gratifying that these expansions indeed lead to the correct critical mass-terms in the linearized equations for ϕ and the Weyl tensors such that the AdS energies assume the correct massless values.

This paper is organized as follows. In Section 2 we describe the doubleton representations of the AdS_7 group and compute explicitly the massless 7D field content from the product of two scalar doubletons. In Section 3 we construct the bosonic HS algebra $hs(8^*)$. In Section 4 we gauge $hs(8^*)$ by introducing a master gauge field in the adjoint representation and a master scalar field in a quasi-adjoint representation of $hs(8^*)$, and in particular we compute the K^2-dressing. In Section 5 we write the linearized curvature constraints and show that they yield the correct spectrum of massless fields. Our results are summarized and further discussed in Section 6. Appendix A contains the details of the calculation of the mass terms occurring in the linearized field equations. Appendix B contains the harmonic analysis on $\mathrm{AdS}_7 \times S^4$ needed in the computation of the AdS energies.

2 $SO(6,2)$ Representation Theory

Physical representations of $SO(6,2)$ can be obtained from a set of Grassmann even oscillator y_α $(\alpha = 1, .., 8)$, which is a $SO(6,1)$ Dirac spinor, obeying the oscillator algebra[2]

$$y_\alpha \star \bar{y}_\beta - \bar{y}_\beta \star y_\alpha = 2C_{\alpha\beta} \ , \tag{2.1}$$

where $\star$ denotes the product of the oscillator algebra. We also define a Weyl ordered product as follows:

$$y_\alpha \bar{y}_\beta = \bar{y}_\beta y_a = y_\alpha \star \bar{y}_\beta - C_{\alpha\beta} \ . \tag{2.2}$$

The Weyl ordered product extends in a straightforward fashion to contraction rules between arbitrary polynomials of the oscillators similar to those used in expanding products of Dirac matrices. For example,

$$(\bar{y}_\alpha y_\beta) \star (\bar{y}_\gamma y_\delta) = \bar{y}_\alpha \bar{y}_\gamma y_\beta y_\delta + C_{\beta\gamma} \bar{y}_\alpha y_\delta - C_{\alpha\delta} \bar{y}_\gamma y_\beta - C_{\beta\gamma} C_{\alpha\delta} \ . \tag{2.3}$$

On the other hand, the massless fields of the $D = 5$, $\mathcal{N} = 4$ higher spin theory [8], which arise in the square of a certain superdoubleton, saturate the bound of a continuous series [16, 17]. Thus the 5D theory can be Higgsed in a continuous fashion [9].

[2] We use mostly positive signature and Dirac matrices $(\Gamma^a)_\alpha{}^\beta$ obeying $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$ $(a = 0, 1, 2, 3, 4, 5, 7)$. The charge conjugation matrix $C_{\alpha\beta}$ is symmetric and real and $(\Gamma^a C)_{\alpha\beta}$ are anti-symmetric. The Dirac conjugate of y_α is defined by $\bar{y}_\alpha = (y^\dagger i \Gamma^0 C)_\alpha$.

The $SO(6,2)$ generators are given by $(A = a, 8)$:

$$M_{AB} = \tfrac{1}{2}\bar{y} \star \Sigma_{AB} y = \tfrac{1}{2}\bar{y}\Sigma_{AB} y \ , \quad \Sigma_{a8} = \frac{i}{2}\Gamma_a \ , \quad \Sigma_{ab} = \frac{i}{2}\Gamma_{ab} \ , \tag{2.4}$$

$$[M_{AB}, M_{CD}] = i\eta_{BC} M_{AD} + 3 \text{ more} \ , \quad (M_{AB})^\dagger = M_{AB} \ . \tag{2.5}$$

The spin group $Spin(6,2)$ is a non-compact version of $SO(8;\mathbf{C})$, which we denote by $SO(8^*)$. The 7D spin group $Spin(6,1)$ is a subgroup of $SO(8^*)$.

The maximal compact subgroup $SO(6,2)$ is given by $L^0 = SO(6) \times U(1)_E$ where $SO(6)$ is the group of spatial rotations and $U(1)_E$ is generated by the the AdS energy $E = M_{08}$. There is a three-grading $SO(6,2) = L^- \oplus L^0 \oplus L^+$, where $L^\pm$ contain non-compact energy-lowering and energy-raising operators $M^\pm$ such that $[E, M^\pm] = \pm M^\pm$. The three-grading also requires $[L^+, L^+] = [L^-, L^-] = 0$, $[L^+, L^-] = L^0$ and $[L^\pm, L^0] = L^\pm$. The reality conditions are $(L^0)^\dagger = L^0$ and $(L^+)^\dagger = L^-$. Positive energy representations of $SO(6,2)$ are weight spaces $D(E_0; n_1, n_2, n_3)$ labeled by the lowest energy E_0 and $SO(6)$ highest weight labels (n_1, n_2, n_3) obeying $n_1 \geq n_2 \geq |n_3|$. In case of integer spin these determine an $SO(6)$ Young tableaux with n_1 boxes in the first row, n_2 boxes in the second row and $|n_3|$ boxes in the third row. If $n_3 \neq 0$ the Young tableaux can be self-dual or anti-self-dual, which corresponds to $n_3 > 0$ and $n_3 < 0$, respectively. The $SO(6)$ Young tableaux can be converted into an $SU(4)$ Young tableaux by contracting it with $SO(6)$ Dirac matrices. A column with N boxes ($N = 1, 2, 3$) stacked on top of each other is contracted with the rank N Dirac matrix. Their symmetry properties are as follows: $(\sigma^R)_{ij}$ ($R = 1, ..., 6$, $i, j = 1, ..., 4$) is anti-symmetric, $(\sigma^{RST})_{ij}$ and $(\sigma^{RST})^{ij}$ are symmetric and with definite self-duality properties, and $(\sigma^{RS})_i{}^j$ belongs to $4 \times \bar{4}$. In the case of half-integer spin all three highest weight labels are half-integers determining a σ-traceless tensor-spinor whose tensor structure is determined as above by the highest weight labels $(n_1 - \frac{1}{2}, n_2 - \frac{1}{2}, n_3 - \epsilon(n_3)\frac{1}{2})$, where $\epsilon(n_3)$ is the sign of n_3. For example, $(1,0,0)$ denotes the (real) 6-plet, $(1,1,\pm 1)$ denotes the self-dual and anti-self-dual (complex) 10-plets, $(\frac{1}{2}, \frac{1}{2}, \pm\frac{1}{2})$ denotes the chiral $SU(4)$ spinors and $(\frac{3}{2}, \frac{1}{2}, \pm\frac{1}{2})$ has the $(1,0,0)$ tensor structure and denotes a chiral and σ-traceless vector-spinor.

Our next aim is to describe the UIRs of $SO(6,2)$ which arise in the oscillator construction [15]. A convenient choice of Dirac matrices is

$$C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \quad \Gamma^0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \ . \tag{2.6}$$

The Dirac spinor oscillator y_α obeying (2.1) then split into two sets of $U(4)$-covariant oscillators $(i = 1, ..., 4)$:

$$y_\alpha = \sqrt{2}(a^i, b_j) \ , \quad (a_i)^\dagger = a^i \ , \quad (b_i)^\dagger = b^i \ , \tag{2.7}$$

$$[a_i, a^j]_* = \delta_i^j \ , \quad [b_i, b^j]_* = \delta_i^j \ . \tag{2.8}$$

The $U(4)$-covariant oscillators have a unitary representation in the Fock space built on the vacuum state $|0\rangle$ defined by

$$a_i|0\rangle = b_i|0\rangle = 0 \ . \tag{2.9}$$

The identification (2.7) implies that the Fock space is a unitary representation space of $SO(6,2)$. Other unitary representations of $SO(6,2)$ can then be obtained by considering tensor products of several copies of the Fock space.

The Fock space and its tensor products decompose into physical UIR's of $SO(6,2)$. The three-grading $L^- \oplus L^0 \oplus L^+$ takes the following form in the (single) Fock space representation:

$$\begin{aligned} L^+ \quad &: \quad L^{ij} = a^{[i} \star b^{j]} \ , & (2.10)\\ L^0 \quad &: \quad L^i_j = a^i \star a_j + b^i \star b_j + 4\delta^i_j \ , & (2.11)\\ L^- \quad &: \quad L_{ij} = a_{[i} \star b_{j]} \ . & (2.12) \end{aligned}$$

The energy operator, which is the trace-part of L^i_j, is bounded from below and contains a constant contribution when it is written in normal ordered form:

$$E = \frac{i}{4}\bar{y} \star \Gamma^0 y = \frac{1}{4} y^\dagger \star y = \tfrac{1}{2}(a^i \star a_i + b^i \star b_i) + 2 = \tfrac{1}{2}(N_a + N_b) + 2 \ . \tag{2.13}$$

Since $L^0 = U(4) \simeq SU(4) \times U(1)_E$ where $SU(4)$ is the diagonal sum of $SU(4)_a$ and $SU(4)_b$, it is possible to form lowest weight states (lws) carrying the same L^0 weight by exchanging a^i and b^i oscillators. To be more precise, the lws carry an extra label of the $SU(2)_K$ group under which (a^i, b^i), $i = 1, ..., 4$ transform as doublets. In fact, the $SU(2)_K$ is generated by:

$$\begin{aligned} K_+ \quad &= \quad \frac{i}{4}\bar{y}\bar{y} = -ib^i a_i \ , & (2.14)\\ K_- \quad &= \quad \frac{i}{4} yy = i a^i b_i \ , & (2.15)\\ K_3 \quad &= \quad \frac{1}{4}\bar{y}y = \frac{1}{4}\bar{y} \star y + 2 = \tfrac{1}{2}(N_b - N_a) \ . & (2.16) \end{aligned}$$

For computational purposes it is convenient to write ($I = 1, 2, 3$)

$$K_I = \frac{1}{8} y^{\underline{A}} (\sigma_I)_{\underline{AB}} y^{\underline{B}} \ , \quad y_{\underline{A}} = y^{\underline{B}} \Omega_{\underline{BA}} = (y_\alpha, \bar{y}_\alpha) \ , \tag{2.17}$$

where $(\sigma_I)_{\underline{AB}}$ are symmetric Pauli matrices tensored with the $SO(8^*)$ charge conjugation matrix and y_A obeys

$$y_{\underline{A}} \star y_{\underline{B}} = y_{\underline{A}} y_{\underline{B}} + \Omega_{\underline{AB}} \ , \quad \Omega_{\underline{AB}} = \begin{pmatrix} 0 & C_{\alpha\beta} \\ -C_{\alpha\beta} & 0 \end{pmatrix} \ . \tag{2.18}$$

By construction $[SO(6,2), SU(2)_K]_\star = 0$. Thus the Fock space and its tensor products decompose into $(2j+1)$-plets of identical $SO(6,2)$ weight spaces, which we denote by $D^{(j)}(E_0; n_1, n_2, n_3)$.

The Fock space $\mathcal{F}$ of a single set of oscillators decomposes into $SO(6,2)\times SU(2)_K$ weight spaces, known as doubletons, as follows [15]:

$$\mathcal{F}=\sum_{s=0,\frac{1}{2},1,\frac{3}{2},\ldots} D^{(s)}(E_0=s+2;s,s,\pm s)\ . \tag{2.19}$$

The lws with K_3-eigenvalue m is given by

$$a^{(i_1}\cdots a^{i_{2s-2m}}b^{i_{2s-2m+1}}\cdots b^{i_{2s})}|0\rangle\ . \tag{2.20}$$

The doubletons describe off-shell conformal tensors in six dimensions[3] with spin s and scaling dimension E_0. In particular, the $SU(2)_K$-singlet describes a scalar field with scaling dimension 2.

The P-fold tensor product $\mathcal{F}^{\otimes P}$ of the oscillator Fock space $\mathcal{F}$ can be described by adding a flavor-index $(r,s=1,...,N)$:

$$[a_i(r),a^j(s)]=\delta_{rs}\delta_i^j\ ,\quad [b_i(r),b^j(s)]=\delta_{rs}\delta_i^j\ . \tag{2.21}$$

The representation of $SO(6,2)\times SU(2)_K$ on the tensor product is given by:

$$L_{ij}=\sum_r L_{ij}(r)\ ,\quad L^i{}_j=\sum_r L^i{}_j(r)\ ,\quad L^{ij}=\sum_r L^{ij}(r)\ , \tag{2.22}$$

$$K_I=\sum_r K_I(r)\ . \tag{2.23}$$

In particular, the total energy operator is given by:

$$E=2P+\frac{1}{2}\sum_r\left[N_a(r)+N_b(r)\right]\ . \tag{2.24}$$

Thus the tensor product decomposes into weight spaces with lowest energy $E_0\geq s+2P$ (the lws with $E_0>s+2P$ are obtained by anti-symmetrizing oscillators carrying different flavor indices). In particular, $P=2$ yields massless fields in AdS_7 [15][4].

3 The Massless Spectrum of The Theory

As discussed in the Introduction, the minimal bosonic 7D HS theory is conjectured to be the anti-holographic dual of the 6d theory of N free scalar fields φ^i , i.e. N copies of the scalar

[3]The compact $SO(6)$ labels are related to the non-compact $SO(5,1)$ labels, i.e. the six-dimensional spin labels, and the energy E_0 to the six-dimensional scaling dimension by a non-unitary rotation [20].

[4]These fields also satisfy the masslessness criteria defined in [21]. In [21] certain UIRs of $SO(6,2)$ that cannot be obtained by multiplying any number of 6d doubletons. However, the unitarity bounds of $Osp(8^*|4)$ seem to exclude this possibility [22, 23].

doubleton, in the limit of large N [12, 9]. The composite operators of this theory couple to (non-normalizable) bulk modes of the HS theory. In particular, the bilinear operators are the spin $s=0$ operator $\varphi^i\varphi^i$ and a set of conserved, symmetric and traceless tensors of spin $s=2,4,6,...$ [24]. These tensors are in one-to-one correspondence with the massless representations in the symmetric tensor product of two 6d scalar doubletons. The anti-symmetric tensor product contains massless representations which correspond to descendants. The massless spectrum of our 7D theory is therefore given by:

$$\mathcal{S}=[D(2;0,0,0)\otimes D(2;0,0,0)]_S\ . \tag{3.1}$$

In order to decompose $D(2;0,0,0)\otimes D(2;0,0,0)$ under $SO(6,2)$ we compute the lws with energy $E_0=4+s$ for $s=0,1,2,....$ To this end, we expand a general state $|\psi\rangle\in D(2;0,0,0)\otimes D(2;0,0,0)$ with that energy as follows:

$$|\psi\rangle=\sum_{\mu=0}^{s}|\psi^{(\mu)}\rangle=\sum_{\mu=0}^{s}\psi^{(\mu)}{}_{i_1j_1,...,i_\mu j_\mu;k_1l_1,...,k_{s-\mu}l_{s-\mu}}L^{i_1j_1}(1)\cdots L^{i_\mu j_\mu}(1)L^{k_1l_1}(2)\cdots L^{k_{s-\mu}l_{s-\mu}}(2)|0\rangle\ . \tag{3.2}$$

The quantities $L^{i_1j_1}(1)\cdots L^{i_\mu j_\mu}(1)$ and $L^{k_1l_1}(2)\cdots L^{k_{s-\mu}l_{s-\mu}}(2)$ are irreducible under $SU(4)$. Their $SO(6)$ highest weight labels are $(\mu,0,0)$ and $(s-\mu,0,0)$, respectively. Thus the $SU(4)$ tensors $\psi^{(0)}$ and $\psi^{(n)}$ are irreducible, and given by the $SU(4)$ Young tableaux with two rows of length s. This $SU(4)$ irrep, which we shall denote by R_s, has spin given by s and can be converted to a real, symmetric rank s $SO(6)$ tensor by contracting it with s (anti-symmetric) $SO(6)$ Dirac matrices. Its $SO(6)$ highest weight labels are $(s,0,0)$. The remaining $\psi^{(\mu)}$, $0<\mu<s$, are reducible and decompose into R_s plus a set of various other irreps, $\{R^{(\mu)}\}$ say (where each irrep occur once and only once), which we write as

$$\psi^{(\mu)}=\psi^{(\mu)}(R_s)+\sum_{R^{(\mu)}}\psi^{(\mu)}(R^{(\mu)})\ ,\quad 0<\mu<s\ . \tag{3.3}$$

The state $|\psi\rangle$ is a lws provided that

$$L_{ij}|\psi\rangle\equiv(L_{ij}(1)+L_{ij}(2))|\psi\rangle\equiv\sum_{\nu=0}^{s-1}|\chi^{(\nu)}\rangle=0\ , \tag{3.4}$$

where $|\chi^{(\nu)}\rangle$ contains ν factors of $L^+(1)$ and $s-1-\nu$ factors of $L^+(2)$. Thus

$$|\chi^{(\nu)}\rangle=0\ ,\quad \nu=0,\dots,s-1\ . \tag{3.5}$$

The state $|\chi^{(\nu)}\rangle$ is a linear combination of contributions from $\psi^{(\nu)}$ and $\psi^{(\nu+1)}$. From $|\chi^{(0)}\rangle=0$ and the fact that the contributions from the various irreps are linearly independent it follows that

$$\psi^{(1)}(R_s)=-s^2\psi^{(0)}\ ,\quad \psi^{(1)}(R^{(1)})=0\ . \tag{3.6}$$

From $|\chi^{(1)}\rangle = 0$ it then follows that

$$\psi^{(2)}(R_s) = -\frac{(s-1)^2}{4}\psi^{(1)}(R_s) = \binom{s}{2}^2 \psi^{(0)} \ , \quad \psi^{(2)}(R^{(2)}) = 0 \ . \tag{3.7}$$

Iterating this procedure we find that there is precisely one lws with energy $E_0 = 4 + s$, which belongs to the $SU(4)$ irrep R_s described above. Collecting the above results, we find the following explicit expression for this state:

$$|E_0; s, 0, 0\rangle = \sum_{\mu=0}^{s} (-1)^\mu \binom{s}{\mu}^2 L^{i_1 j_1}(1)\cdots L^{i_\mu j_\mu}(1) L^{i_{\mu+1} j_{\mu+1}}(2)\cdots L^{i_s j_s}(2)|0\rangle \ , \tag{3.8}$$

where separate symmetrization of the i and j indices is assumed. The lws with even spin belong to the symmetric tensor product and those with odd spin to the anti-symmetric product. In summary:

$$[D(2;0,0,0)\otimes D(2;0,0,0)]_{\rm S} \ = \ \sum_{s=0,2,4,...} D(E_0 = s+4; s, 0, 0) \ , \tag{3.9}$$

$$[D(2;0,0,0)\otimes D(2;0,0,0)]_{\rm A} \ = \ \sum_{s=1,3,5,...} D(E_0 = s+4; s, 0, 0) \ . \tag{3.10}$$

We have thus computed the massless spectrum of our minimal bosonic HS theory, which hence consists of massless states with spin $s = 0, 2, 4, ...$ and vanishing $SU(2)_K$ charge.

4 The Higher Spin Algebra $hs(8^*)$

Our next task is to determine the HS symmetry algebra, $hs(8^*)$. From the boundary point of view, $hs(8^*)$ is the algebra of charges of the set of conserved currents built from the $s \geq 2$ symmetric traceless tensors [24]. These rigid symmetries of the free CFT induce local symmetries in the anti-holographic bulk theory, including general covariance. However, instead of determining $hs(8^*)$ from the current algebra we construct it directly in terms of the oscillators. We do this by making use of the properties of the spectrum derived in the previous section, and the knowledge of which gauge fields are required for writing the covariant constraints for a massless field of given spin $s \geq 2$ in a linearization around AdS$_7$ [18, 19]. In fact, the ‘canonical’ set of spin s gauge fields which contains physical as well as auxiliary fields is in one-to-one correspondence with the above mentioned set of conserved currents of that spin [24].

Thus, the methodology we adopt is to impose constraints on general oscillator expansions. Essentially we impose three types of constraints: 1) we project out all monomials except those which are of degree $4\ell + 2$, where $\ell = 0, 1, 2, ...$ is a level index; 2) we impose neutrality under $SU(2)_K$; and 3) we mod out an ideal which contains all the traces[5].

[5]The last step leads to an algebra which cannot be represented on doubletons that carry non-zero $SU(2)_K$ charge.

We begin by taking $\mathcal{A}$ to be the space of arbitrary polynomials $f(y, \bar{y})$ in the oscillators, which is an associative algebra with a $\star$-product defined by the Weyl ordering (2.2). An element of $\mathcal{A}$ can therefore be expanded in terms of Weyl ordered monomials in the basic oscillator y_α and its Dirac conjugate $\bar{y}_\alpha$ with complex coefficients which are multispinors. We shall use the following normalization convention (which we give here for a single monomial):

$$f(y, \bar{y}) = \frac{1}{m!\, n!}\, \bar{y}^{\alpha_1} \cdots \bar{y}^{\alpha_m} y^{\beta_1} \cdots y^{\beta_n} f_{\alpha_1 \ldots \alpha_m, \beta_1 \ldots \beta_n} \ . \tag{4.1}$$

We next define a linear anti-automorphism τ of $\mathcal{A}$ as follows:

$$\tau(f(y, \bar{y})) = f(iy, i\bar{y}) \ , \tag{4.2}$$

$$\tau(f_1 \star f_2) = \tau(f_2) \star \tau(f_1) \ . \tag{4.3}$$

A linear anti-automorphisms of an associative algebra can be used to define a Lie subalgebra. In our case we define the following Lie subalgebra of $\mathcal{A}$:

$$\mathcal{L} = \{P \in \mathcal{A} : \tau(P) = P^\dagger = -P \ , \quad [K_I, P]_\star = 0\} \ , \tag{4.4}$$

where K_I are the $SU(2)_K$ generators defined in (2.17). The Lie bracket of $\mathcal{L}$ is

$$[P_1, P_2]_\star = P_1 \star P_2 - P_2 \star P_1 \ . \tag{4.5}$$

The generators of $\mathcal{L}$ have expansions with multispinor coefficients which in general have non-zero trace parts. In order to impose tracelessness we define a new ordering of elements in $\mathcal{A}$ which amounts to factoring out the trace parts explicitly [6]. An element $P \in \mathcal{L}$ is thus expanded as:

$$P = \sum_{n=0}^{\infty} P_{(n)}^{I_1 \ldots I_n}(y, \bar{y}) \star K_{I_1} \star \cdots \star K_{I_n} \ , \tag{4.6}$$

where $P_{(n)}^{I_1 \ldots I_n}(y, \bar{y})$ has an expansion in terms of traceless, Weyl ordered multispinors and the $SU(2)_K$ indices $I_1 \ldots I_n$ are symmetric. The conditions on $P_{(n)}^{I_1 \ldots I_n}$ imply that

$$K_I \star P_{(n)}^{I_1 \ldots I_n} = K_I P_{(n)}^{I_1 \ldots I_n} - \frac{in}{2} \epsilon_{IK}{}^{(I_1} P_{(n)}^{I_2 \ldots I_n)K} \ . \tag{4.7}$$

The expansion (4.6) leads to the following decomposition of $\mathcal{L}$:

$$\mathcal{L} = \mathcal{L}_{(0)} \oplus \mathcal{L}_{(1)} \oplus \mathcal{L}_{(2)} \oplus \cdots = \mathcal{L}_{(0)} + \mathcal{I} \ , \quad \mathcal{I} = \mathcal{L}_{(1)} \oplus \mathcal{L}_{(2)} \oplus \cdots \ , \tag{4.8}$$

where $\mathcal{L}_{(n)}$ represents the n'th term in (4.6). If we were to change the prescription for ordering the $SU(2)_K$ generators in (4.6) this would only affect the multispinors in $\mathcal{I}$. Thus the traceless multispinors in $\mathcal{L}_{(0)}$ are uniquely defined by (4.6).

The space $\mathcal{L}_{(0)}$ of traceless generators decomposes into levels labeled by $\ell = 0, 1, 2, \ldots$ consisting of elements of the form

$$P = \frac{1}{(n!)^2}\; P_{\alpha_1\ldots\alpha_n,\beta_1\ldots\beta_n}\bar{y}^{\alpha_1}\cdots\bar{y}^{\alpha_n}y^{\beta_1}\cdots y^{\beta_n}\ ,\quad n=2\ell+1\ , \tag{4.9}$$

where the multispinor $P_{\alpha_1\ldots\alpha_n,\beta_1\ldots\beta_n}$ belongs to an (irreducible) Young tableaux of $Spin(6,2)\simeq SO(8^*)$ with two rows of equal length n. In order to see this we first note that from the condition

$$[K_3,P]_\star = 0 \tag{4.10}$$

it follows that P must have an equal number of y and $\bar{y}$ oscillators. The conditions

$$[K_\pm,P]_\star = 0 \tag{4.11}$$

then set to zero the $SO(8^*)$ irreps in P described by Young tableaux which have more boxes in the first row than in the second row.

The reality condition $P^\dagger = -P$ implies that the real dimension of (4.9) equals that of the corresponding $SO(8)$ Young tableaux. The Young tableaux is reducible under the 7D spin group $Spin(6,1)\subset SO(8^*)$. It decomposes into irreps labeled by (tensorial) $SO(6,1)$ Young tableaux with two rows of which the first one has n boxes and the second one m boxes, where $0\le m\le n$. The counting works out simply because $SO(8^*)$ decomposes under $Spin(6,1)$ in the same way as $SO(6,2)$ decomposes under $SO(6,1)$. The conversion from the $SO(8^*)$ irrep to the set of $SO(6,1)$ irreps can be explicitly done by contracting the $SO(8^*)$ Young tableaux with m second rank Dirac matrices $(\Gamma^{ab})_{\alpha\beta}$ and n first rank Dirac matrices $(\Gamma^c)_{\alpha\beta}$, where the $SO(6,1)$ indices belong to the $SO(6,1)$ Young tableaux. We express the resulting generators as:

$$\begin{aligned}\frac{1}{(n!)^2}P^{(m,n-m)}_{\alpha_1\ldots\alpha_n,\beta_1\ldots\beta_n}\bar{y}^{\alpha_1}\cdots\bar{y}^{\alpha_n}y^{\beta_1}\cdots y^{\beta_n} \quad &= \quad P^{a_1\ldots a_m\,c_1\ldots c_{n-m}}_{b_1\ldots b_m}M^{b_1}_{a_1}\cdots M^{b_m}_{a_m}P_{c_1}\cdots P_{c_{n-m}}\ ,\\ n=2\ell+1\ ,\quad 0\le m\le n\ .\end{aligned} \tag{4.12}$$

Upon gauging, these generators lead to the canonical set of gauge fields with spin $s=2\ell+2$ ($\ell=0,1,2,\ldots$) as discussed earlier. In particular, the zeroth level consists of the $SO(6,2)$ generators P_a and M_{ab} corresponding to the gravitational gauge fields.

The space $\mathcal{I}$ defined in (4.8) forms an ideal:

$$[\mathcal{L},\mathcal{I}]_* \subset \mathcal{I}\ . \tag{4.13}$$

Moreover, there is a redundancy such that content of $\mathcal{L}_{(0)}$ is reproduced in the $SU(2)_K$ trace part of $\mathcal{L}_{(n)}$ for $n=2,4,\ldots$. For example, the generators $P\delta^{IJ}\star K_I\star K_J$ give rise to one copy of $\mathcal{L}_{(0)}$, and so on. We are therefore led to defining the HS algebra by

$$hs(8^*) = \mathcal{L}/\mathcal{I}\ . \tag{4.14}$$

The elements of $hs(8^*)$ are thus equivalence classes $[P]$ of elements in $\mathcal{L}$ defined by

$$[P] = \{P' \in \mathcal{L} \mid P' - P \in \mathcal{I}\} \ . \tag{4.15}$$

The Lie bracket of $[P_1]$ and $[P_2]$ is given by

$$[[P_1],[P_2]] = [[P_1,P_2]_*] \ . \tag{4.16}$$

In order to examine the representation theory of the HS algebra $hs(8^*)$ we observe the following grading of $hs(8^*)$:

$$hs(8^*) \;=\; \sum_{n\in\mathbf{Z}} L^{(n)} \ , \qquad [E, L^{(n)}]_\star = nL^{(n)} \ , \tag{4.17}$$

$$[L^{(n)}, L^{(m)}]_\star \;=\; L^{(m+n)} \ , \qquad (L^{(n)})^\dagger = L^{(-n)} \ . \tag{4.18}$$

Moreover, from (4.12) it follows that $hs(8^*)$ is a subalgebra of the universal enveloping algebra $Env(SO(6,2))$:

$$hs(8^*) = Env(SO(6,2))/\mathcal{R} \ , \tag{4.19}$$

where $\mathcal{R}$ is an ideal generated by various polynomials in $Env(SO(6,2))$ which vanish by Fierz identities that arise when the single oscillator realization of $SO(6,2)$ is used. By a choice of ordering we can thus write

$$L^{(n)} = \Big\{(L^+)^p \star (L^0)^{\star\ q} \star (L^-)^r : p - r = n;\ p+q+r = 2\ell+1,\ \ell = 0,1,2,...\Big\}/\mathcal{R} \ , \tag{4.20}$$

where $L^- \oplus L^0 \oplus L^+$ is the three-grading of $SO(6,2)$, ℓ is the level index and we are using the non-commutative $\star$-product. From (4.20) it follows that $L^{(0)} \sim L^0 + (L^0)^3 + \cdots$ when acting on an $SO(6,2)$ lowest weight state. A physical representation of $hs(8^*)$ is thus a lowest weight representation $\widehat{D}(E_0; m_1, m_2, m_3)$ based on an $SO(6,2)$ lowest weight state $|E_0; m_1, m_2, m_3\rangle$ obeying the additional conditions

$$L^{(-n)}|\Omega\rangle = 0 \ , \quad n = 1,2,3,... \ . \tag{4.21}$$

Since the single oscillator vacuum state $|0\rangle$ obeys (4.21), the scalar doubleton, which is the fundamental $SO(6,2)$ UIR, is also the fundamental UIR of $hs(8^*)$:

$$\widehat{D}(2;0,0,0) = D(2;0,0,0) \ .$$

Tensor products of the scalar doubleton also form $hs(8^*)$ representations. If $P \in hs(8^*)$ then the representation of P on the N-fold tensor product $(D(2;0,0,0))^{\otimes N}$ is given by $P = \sum_{r=1}^N P_r$, where P_r acts on the rth factor in the tensor product. Thus, in particular, the massless spectrum $\mathcal{S}$ given in (3.1) is an $hs(8^*)$ representation. The $SO(6,2)$ lowest weight state $|4;0,0,0\rangle = |0\rangle \in \mathcal{S}$ is an $hs(8^*)$ lowest weight. There are no other $hs(8^*)$ lws in $\mathcal{S}$. To see this we first note that any $hs(8^*)$ lws must also be an $SO(6,2)$ lws. It thus suffices to show that for each lws in (3.8) with $s = 2, 4, 6, ...$ there exists at least one energy-lowering operator in $hs(8^*)$ which does not annihilate this state. To exhibit such an operator we first write (3.8) in the following schematic form ($s = 2, 4, 6, ...$):

$$|s+2; s, 0, 0\rangle = \Big((L_1^+)^s + (L_1^+)^{s-1} L_2^+ + \cdots + (L_2^+)^s\Big) |0\rangle \ . \tag{4.22}$$

Acting on $|s+2; s, 0, 0\rangle$ with the element $L^+ \star (L^-)^s \in L^{(-s+1)}$ (the level of this state is given by $\ell = s/2$) yields

$$\Big[L^+ \star (L^-)^s\Big] |s+2; s, 0, 0\rangle = \Big[L_1^+ \star (L_1^-)^s + L_2^+ \star (L_2^-)^s\Big] |\psi\rangle = (L_1^+ + L_2^+)|0\rangle \neq 0 \ . \tag{4.23}$$

It follows that the massless spectrum $\mathcal{S}$ defined in (3.1) is an irreducible $hs(8^*)$ multiplet:

$$\mathcal{S} = \widehat{D}(4; 0, 0, 0) \ . \tag{4.24}$$

5 Gauging $hs(8^*)$

In order to gauge $hs(8^*)$ we introduce an $hs(8^*)$-valued one-form $[A]$ and a zero-form Φ obeying the conditions:

$$\tau(A) = A^\dagger = -A \ , \quad [K_I, A]_\star = 0 \ , \tag{5.1}$$

$$\tau(\Phi) = \Phi^\dagger = \pi(\Phi) \ , \quad K_I \star \Phi = \Phi \star K_I = 0 \ . \tag{5.2}$$

Here τ is the anti-automorphism defined in (4.2-4.3) and π is an automorphism acting on $SU(2)_K$-invariant elements $f \in \mathcal{A}$ as follows:

$$\pi(f^{(m,n)}) = (-1)^n f^{(m,n)} \ , \quad \pi(f_1 \star f_2) = \pi(f_1) \star \pi(f_2) \ , \tag{5.3}$$

where m and n are related to the $SO(6,1)$ highest weight labels as:

$$m_1 = m + n \ , \quad m_2 = m \ , \quad m_3 = 0 \ . \tag{5.4}$$

Here we have used the fact that any $SU(2)_K$ invariant element $f \in \mathcal{A}$ can be expanded in terms of such $SO(6,1)$ irreps as explained in the previous section; see the analysis following (4.9). In order to show that π is an automorphism, we make use of the fact that if $f_1, f_2 \in \mathcal{A}$ are $SU(2)_K$ invariant then also $f_1 \star f_2$ is $SU(2)_K$ invariant. Thus $f_1 \star f_2$ can also be expanded in terms of $SO(6,1)$ Young tableaux with $m_3 = 0$. Using the basic rules for $SO(6,1)$ tensor products one can then verify that π is an automorphism.

The conditions on Φ defines a representation of $hs(8^*)$ which we call quasi-adjoint. It is essential to introduce this representation to accommodate the physical scalar field and the spin

$s \geq 2$ Weyl tensors as well as all the derivatives of these fields. The curvature and covariant derivative are defined by

$$
\begin{aligned}
F_{[A]} &= [F_A] = [dA + A \star A] \ , \quad D_{[A]}\Phi = d\Phi + A \star \Phi - \Phi \star \pi(A) \ , &(5.5)\\
\delta_{[\epsilon]}[A] &= [d\epsilon + [A, \epsilon]_\star] \ , \quad \delta_{[\epsilon]}\Phi = \epsilon \star \Phi - \Phi \star \pi(\epsilon) \ . &(5.6)
\end{aligned}
$$

The conditions on Φ implies that $D_{[A]}\Phi$ and $\delta_{[\epsilon]}\Phi$ are also quasi-adjoint elements and that they do not depend on the choice of A and ϵ. The role of the π automorphism is to distinguish between the translations-like and rotations-like generators in the HS algebra; for example[6]:

$$
\pi(P_a) = -P_a \ , \quad \pi(M_{ab}) = M_{ab} \ . \tag{5.7}
$$

Hence, if Ω denotes an $SO(3,2)$ valued connection, we find that $D_\Omega \Phi = \nabla \Phi + dx^\mu \{P_\mu, \Phi\}_\star$, where ∇ is the Lorentz covariant derivative. As will be shown in in the next section, this means that the whole one-form $D_\Omega\Phi$ can be constrained without having the consequences of setting Φ to constant.

We next solve the conditions on Φ. By subtracting the two K_I-conditions in (5.2) we obtain $[K_I, \Phi]_\star = 0$. Thus we can choose to expand Φ as follows:

$$
\Phi = \sum_{n=0}^{\infty} \Phi_{(n)} f_{(n)} = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi_{(n)}^{I_1 \dots I_n}(y, \bar{y}) K_{I_1} \cdots K_{I_n} f_{(n)}(K^2) \ , \tag{5.8}
$$

where $\Phi_{(n)}^{I_1 \dots I_n}(y, \bar{y})$ consists of traceless multispinors which are also traceless in their internal $SU(2)_K$ indices, i.e.

$$
\begin{aligned}
K_I \star \Phi_{(n)}^{I_1 \dots I_n} &= K_I \Phi_{(n)}^{I_1 \dots I_n} - \frac{in}{2} \epsilon_{IK}{}^{(I_1} \Phi_{(n)}{}^{I_2 \dots I_n)K} \ , &(5.9)\\
\delta_{JK} \Phi_{(n)}^{JKI_1 \dots I_{n-2}} &= 0 \ , \quad n > 1 \ . &(5.10)
\end{aligned}
$$

The quantities $f_{(n)}(K^2)$ are analytical functions of $K^2 = K_I K_I$, which we determine below. Note that had we chosen to use $\star$-products of $SU(2)_K$ generators in (5.8), instead of classical products, the K_I-condition would only admit the trivial solution $\Phi = 0$.

It thus remains to impose $K_I \star \Phi = 0$. After some algebra, where it is convenient to use (2.17) and (2.18), we find

[6]For $D = 4, 5$, the map π can also be defined as a linear transformation of the oscillators. This does not seem to be possible in $D = 7$, since $(\Gamma_a C)_{\alpha\beta}$ and $(\Gamma_{ab} C)_{\alpha\beta}$ have the same symmetry. It is worth noting that there is an alternative way of implementing the π map by extending the oscillator algebra $\mathcal{A}$ with an inner Kleinian operator $\kappa = (-1)^{2K_3}$. The associative algebra $\tilde{\mathcal{A}} = \mathcal{A} \oplus (\kappa\mathcal{A})$ has the π map $\pi(f(y, \bar{y}; \kappa) = f(y, \bar{y}; -\kappa)$, and the AdS generators can be taken to be $\tilde{P}_a = \kappa P_a$ and $\tilde{M}_{ab} = M_{ab}$. More generally, $hs(8^*)$ is generated by $\kappa^{m+\sigma} P^{(m, 2\ell+1-m)}$ $(m = 0, \dots, 2\ell+1)$ for $\sigma = 1$. Taking $\tilde{\Phi}$ to obey (5.2) and imposing the curvature constraints (6.1-6.2), which are now expansions in κ, one finds that $\sigma = 0$ gives rise to an entire set of auxiliary gauge fields. Thus the physical field content remains the same in this framework.

$$K_I \star \Big(\Phi_{(n)} f_{(n)}\Big) = K_I \Phi_{(n)} D[f_{(n)}] - 4\Big(\mid \Phi_{(n)} \mid +4\Big) \frac{\partial \Phi_{(n)}}{\partial K_I} f_{(n)} \ , \tag{5.11}$$

where D is the second order differential operator

$$D[f_{(n)}(z)] = f_{(n)}(z) - \frac{1}{8}(7+ \mid \Phi_{(n)} \mid) f'_{(n)}(z) - \frac{1}{4} z f''_{(n)}(z) \ , \tag{5.12}$$

and $\mid \Phi_{(n)} \mid$ denotes the number of irreducible spinor indices in the multispinor defining $\Phi_{(n)}$:

$$y^{\underline{A}} \partial_{\underline{A}} \Phi^{I_1 \dots I_n}(y, \bar{y}) = \mid \Phi_{(n)} \mid \Phi^{I_1 \dots I_n}(y, \bar{y}) \ . \tag{5.13}$$

From $K_I \star \Phi = 0$ it follows that[7]

$$f_{(n)}(K^2) = \begin{cases} F(\mid \Phi_{(0)} \mid ; K^2) & n = 0 \\ 0 & n > 0 \end{cases} \tag{5.14}$$

where the function $F(w; z)$ is given by

$$F(w; z) = \sum_{n=0}^{\infty} \frac{(4z)^n}{n!} \frac{\Gamma(\frac{1}{2}(w+7))}{\Gamma(\frac{1}{2}(w+7+2n))} \ . \tag{5.15}$$

From $[K_I, \Phi_{(0)}]_\star = 0$, and following the analysis below (4.9), it follows that the y and $\bar{y}$ expansion of $\Phi_{(0)}(y, \bar{y})$ gives rise to traceless multispinors $\Phi_{(0)\alpha_1 \dots \alpha_s, \beta_1 \dots \beta_s}$ that belong to spin s $SO(8^*)$ Young tableaux that have two rows of equal length s. Taking into account also the condition $\tau(\Phi) = \pi(\Phi)$ we find (from now on we drop the subscript (0))

$$\begin{aligned} \Phi(y, \bar{y}) &= \sum_{\substack{m = 0, 2, 4, \dots \\ n = 0, 1, 2, \dots\dots}} \Phi^{(m,n;0)}(y, \bar{y}) F(2s; K^2) && (5.16) \\ &= \sum_{\substack{m = 0, 2, 4, \dots \\ n = 0, 1, 2, \dots\dots}} \sum_{k=0}^{\infty} \Phi^{(m,n;k)}(y, \bar{y}) \ , && (5.17) \end{aligned}$$

where the y and $\bar{y}$ expansion of $\Phi^{(m,n;0)}(y, \bar{y})$ yields a traceless multispinor $\Phi^{(m,n;0)}_{\alpha_1 \dots \alpha_s, \beta_1 \dots \beta_s}$ which is equivalent to a spin s $SO(6,1)$ Young tableaux with $s = m + n$ boxes in the first row and m boxes in the second row. The multispinor $\Phi^{(m,n;k)}_{\alpha_1 \dots \alpha_{s+k}, \beta_1 \dots \beta_{s+k}}$ $(k = 1, 2, 3, \dots)$ which by definition contain k traces times an $SO(6,1)$ Young tableaux with $s = m + n$ boxes in the first row and m boxes in the second row, is given by (5.15) and (5.17). For example

[7] A solution to $K_I \star \Phi = \Phi \star K_I = 0$ can also be constructed formally as $\Phi = \Pi \star \Phi$, where Π is the projector of the single oscillator Fock space (2.19) onto the spin $s = 0$ doubleton, which is the $SU(2)_K$-singlet.

$$
\begin{aligned}
\Phi^{(m,n;1)}_{\alpha_1\dots\alpha_{s+1},\beta_1\dots\beta_{s+1}} &= 0 \ , \\
\Phi^{(m,n;2)}_{\alpha_1\dots\alpha_{s+2},\beta_1\dots\beta_{s+2}} &= \frac{(s+2)^2(s+1)^2}{2(7+2s)}\Phi^{(m,n;0)}_{\alpha_1\dots\alpha_s,\beta_1\dots\beta_s} \\
&\quad \times(C_{\alpha_{s+1}\beta_{s+1}}C_{\alpha_{s+2}\beta_{s+2}}-C_{\alpha_{s+2}\alpha_{s+2}}C_{\beta_{s+1}\beta_{s+2}}) \ ,
\end{aligned} \tag{5.18}
$$

where separate symmetrization in the α and β indices is assumed.

The spin $s=0$ sector of Φ is a single real scalar field ϕ with the following K^2-dressing:

$$
\phi(1+\frac{8}{7}K^2+\cdots) \ , \tag{5.19}
$$

where the coefficient 8/7 is read off from (5.15). The spin $s=1$, i.e. $|\Phi|=2$, sector contains an $SO(6,1)$ vector ϕ_a. In the spin $s=2$ sector, i.e. for $\mid \Phi \mid = 4$ we find a symmetric traceless tensor ϕ_{ab} and a traceless tensor $C_{ab,cd}=C_{cd,ab}$ obeying $C_{[ab,c]d}=0$. These tensors match the on-shell second order derivatives of a scalar and a graviton, respectively. This pattern extends to higher spins such that the tensorial content of Φ is isomorphic to the derivatives of a spin $s=0$ field and a set of spin $s=2,4,6,...$ Weyl tensors (obeying Klein-Gordon equations and Bianchi identities for $s\geq 2$).

6 The Linearized Curvature Constraints

In analogy with the 5D case [6], we propose the following linearized curvature constraints ($s=2,4,6,...$):

$$
F^{\rm lin}_{\alpha_1\dots\alpha_{s-1},\beta_1\dots\beta_{s-1}} = e^a\wedge e^b(\Gamma_{ab})^{\gamma\delta}\Phi^{(s,0;0)}_{\gamma\alpha_1\dots\alpha_n,\delta\beta_1\dots\beta_n} \ , \tag{6.1}
$$

$$
D_\Omega\Phi = 0 \ , \tag{6.2}
$$

where Ω is the AdS background

$$
\Omega=i(P^a e_a+\tfrac{1}{2}M^{ab}\omega_{ab}) \ , \quad d\Omega+\Omega\wedge\star\Omega=0 \ , \tag{6.3}
$$

and $F^{\rm lin}$ is the linearized curvature:

$$
F^{\rm lin}=dA+\Omega\star A+A\star\Omega \ . \tag{6.4}
$$

These constraints are invariant under the linearized form of the gauge transformations (5.6):

$$
\delta_\epsilon A=d\epsilon+[\Omega,\epsilon]_\star \ , \quad \delta_\epsilon\Phi=\epsilon\star\Phi-\Phi\star\pi(\epsilon) \ . \tag{6.5}
$$

Consistency of the constraints requires integrability, i.e. $d^2=0$. The integrability of the scalar constraint (6.2) follows immediately from the flatness of Ω. The integrability of the curvature constraint (6.1) is equivalent to the Bianchi identity $dF^{\rm lin}+[\Omega,F^{\rm lin}]_\star=0$, which takes the following form in components:

$$
\begin{aligned}
&\nabla_{[\mu}F^{\rm lin}_{\nu\rho],\alpha_1\dots\alpha_{s-1},\beta_1\dots\beta_{s-1}}\\
&+\tfrac{(s-1)^2}{2}\Big((\Gamma_{[\mu})_{\alpha_1}{}^{\gamma}F^{\rm lin}_{\nu\rho]\,\alpha_2\dots\alpha_{s-1}\gamma,\beta_1\dots\beta_{s-1}}-(\Gamma_{[\mu})_{\beta_1}{}^{\gamma}F^{\rm lin}_{\nu\rho]\,\alpha_1\dots\alpha_{s-1},\beta_2\dots\beta_{s-1}\gamma}\Big)=0\ ,
\end{aligned}
\tag{6.6}
$$

where separate symmetrization in α and β indices is assumed and ∇_μ is the Lorentz covariant derivative In order to verify this we use the component form of the scalar constraint (6.2) which reads:

$$
\nabla_\mu\Phi_{\alpha_1\dots\alpha_n,\beta\dots\beta_n}+\tfrac{1}{2}(\Gamma_\mu)^{\gamma\delta}\Phi_{\gamma\alpha_1\dots\alpha_n,\delta\beta\dots\beta_n}+\tfrac{n^2}{2}(\Gamma_\mu)_{\alpha_1\beta_1}\Phi_{\gamma\alpha_2\dots\alpha_n,\delta\beta_2\dots\beta_n}=0\ , \tag{6.7}
$$

where again separate symmetrization in α and β indices is assumed. Inserting the curvature constraint (6.1) into the Bianchi identity (6.6) and making use of (6.7) we find that (6.6) decomposes into two irreducible parts. In the notation defined in (4.12) these are given by an $(s,0)$ part and an $(s-1,1)$ part (by construction these equations have no trace parts). These are satisfied due to the following Fierz identities ($s\geq 2$):

$$
\begin{aligned}
(\Gamma_{a[b})^{\alpha\beta}(\Gamma_{cd]})^{\gamma\delta}\Phi^{(s,0;0)}_{\alpha\gamma\epsilon_1\dots\epsilon_{s-2},\beta\delta\phi_1\dots\phi_{s-2}} &= 0\ , &(6.8)\\
(\Gamma_{[a})^{\alpha\beta}(\Gamma_{bc]})^{\gamma\delta}\Phi^{(s-1,1;0)}_{\alpha\gamma\epsilon_1\dots\epsilon_{s-2},\beta\delta\phi_1\dots\phi_{s-2}} &= 0\ . &(6.9)
\end{aligned}
$$

In order to verify these it is important to use the symmetries of $\Phi^{(s,0;0)}$ and $\Phi^{(s-1,1;0)}$ as implied by the properties of their $SO(6,1)$ Young tableaux. To implement these properties it is convenient to use expansions similar to (A.4).

The scalar constraint (6.2) implies that the independent fields in Φ are given by $\Phi^{(s,0;0)}$ ($s=0,2,4,\dots$) and that the remaining components are derivatives of these fields; schematically $\Phi^{(s,n;0)}\sim\nabla^n\Phi^{(s,0;0)}$. The constraint also implies the following Klein-Gordon equations (the derivation of the mass-term for arbitrary s is given in Appendix A):

$$
(\nabla^2-m^2)\Phi^{(s,0;0)}=0\ ,\quad m^2=-8-2s\ . \tag{6.10}
$$

Using the harmonic analysis in Appendix B, we find the following lowest energy of $\Phi^{(s,0;0)}$:

$$
E_0=s+4\ . \tag{6.11}
$$

As discussed in Section 2, this is the correct value for a massless spin s field. Thus the independent field content of the quasi-adjoint representation Φ is isomorphic to the spectrum $\mathcal{S}$ in (3.1). We remark that the spectrum $\mathcal{S}$ is a massless $hs(8^*)$ multiplet. The global $hs(8^*)$ transformations on Φ are realized in terms of gauge transformations (5.6) with rigid parameters ϵ obeying the Killing equation

$$
D_\Omega\epsilon=0\ . \tag{6.12}
$$

The K^2-expansion found in the previous section play a crucial role in obtaining the correct critical mass-value in (6.10). Let us demonstrate this in the case of spin $s=0$. We examine the leading equations in the spin $s=0$ sector of (6.2):

$$\begin{aligned} \partial_\mu \phi &= -\frac{1}{2}(\Gamma_\mu)^{\alpha\beta}\Phi_{\alpha,\beta} \ , & (6.13)\\ \nabla_\mu \phi_{\alpha,\beta} &= -\frac{1}{2}(\Gamma_\mu)^{\gamma\delta}\Phi_{\alpha\gamma,\beta\delta} - \frac{1}{2}(\Gamma_\mu)_{\alpha\beta}\phi \ . & (6.14)\end{aligned}$$

It follows that

$$\nabla^\mu \partial_\mu \phi = \frac{1}{4}\Big((\Gamma^\mu)^{\alpha\gamma}(\Gamma_\mu)^{\beta\delta}\Phi_{\alpha\beta,\gamma\delta} + (\Gamma^\mu)^{\alpha\beta}(\Gamma_\mu)_{\alpha\beta}\phi\Big) \ . \tag{6.15}$$

The second term on the right hand side contributes to m^2 by -14. The multispinor in the first term can be decomposed into $SO(6,1)$ irreps as follows:

$$\Phi_{\alpha\beta,\gamma\delta} = \Phi^{(2,0;0)}_{\alpha\beta,\gamma\delta} + \Phi^{(1,1;0)}_{\alpha\beta,\gamma\delta} + \Phi^{(0,2;0)}_{\alpha\beta,\gamma\delta} + \Phi^{(0,0;2)}_{\alpha\beta,\gamma\delta} \ . \tag{6.16}$$

The contributions to m^2 from the first three terms vanish, due to the Fierz identity

$$(\Gamma^\mu)^{\alpha\gamma}(\Gamma_\mu)^{\beta\delta}\Phi^{(m,n;0)}_{\alpha\beta,\gamma\delta} = 0 \ , \quad m+n=2 \ . \tag{6.17}$$

The $SO(6,1)$ singlet $\Phi^{(0,0;2)}_{\alpha\beta,\gamma\delta}$ yields a non-zero contribution to m^2 given by

$$\frac{1}{4}(\Gamma^\mu)^{\alpha\gamma}(\Gamma_\mu)^{\beta\delta}\ \frac{2}{7}\frac{1}{4}\left(2C_{\alpha\gamma}C_{\beta\delta} + 2C_{\alpha\delta}C_{\beta\gamma} - 4C_{\alpha\beta}C_{\gamma\delta}\right) = 6 \ . \tag{6.18}$$

As a result we find that the spin $s=0$ field obeys

$$(\nabla^\mu\partial_\mu - m^2)\phi = 0 \ , \quad m^2 = -14+6 = -8 \ , \tag{6.19}$$

which leads to the lowest weight energy $E_0 = 4$.

The curvature constraint (6.1), when written in $SO(6,1)$ tensorial basis, is of the canonical form [18, 19]. Thus the gauge fields consist of $(s = 2,4,6,...)$

$$\begin{aligned} \text{generalized vielbeins} \ &: \ A^{(0,s-1)}_{\mu\,\alpha_1\ldots\alpha_{s-1},\beta_1\ldots\beta_{s-1}} & (6.20)\\ \text{auxiliary gauge fields} \ &: \ A^{(1,s-2)}_{\mu\,\alpha_1\ldots\alpha_{s-1},\beta_1\ldots\beta_{s-1}}, \ldots\ldots, A^{(s-1,0)}_{\mu\,\alpha_1\ldots\alpha_{s-1},\beta_1\ldots\beta_{s-1}} & (6.21)\end{aligned}$$

The auxiliary gauge fields can be solved for in terms of derivatives of the generalized vielbeins. The linearized field equations, which are second order equations, are obtained by solving for $A^{(1,s-2)}_\mu$ from $F^{(0,s-1)}_{\mu\nu} = 0$ and substituting into a certain projection of the constraint on $F^{(1,s-2)}_{\mu\nu}$. Upon fixing a Lorentz-like gauge, one finds that the physical fields carry the representations $D(s+4; s,0,0)$.

In summary, as a result of the constraints (6.1) and (6.2) the independent set of fields in the theory are

$$\left\{ \phi, \ A_\mu{}^a, \ A_\mu{}^{abc}, \ A_\mu{}^{abcde}, \ \ldots\ldots \ \right\} . \tag{6.22}$$

which obey second order field equations for massless fields carrying the representations in the spectrum $\mathcal{S}$ given in (3.1) and (3.9).

7 Conclusions

We have constructed a minimal bosonic HS extension of the AdS$_7$ group $SO(6,2)$, which we call $hs(8^*)$, consisting of generators of spin $s = 1, 3, 5, ...$ We have realized this symmetry in a 7D HS gauge theory with massless fields of spin $s = 0, 2, 4, ...,$ which are given in (6.22). The $s \geq 2$ fields are contained in a $hs(8^*)$ valued one-form and the spin $s = 0$ field, the Weyl tensors and their derivatives are assembled in a zero-form which transforms in a quasi adjoint representation of $hs(8^*)$. The spectrum of physical fields form a UIR of $hs(8^*)$ which is isomorphic to the symmetric product of two scalar doubletons. The gauge fields (6.20-6.21) and the spin $s = 0$ field correspond to the full set of conserved bilinear currents [24] and the quadratic 'mass' term $\varphi^i\varphi^i$, respectively, of a 6d theory of free scalar fields φ^i $(i = 1, ..., N)$ which carry the scalar doubleton representation.

The internal $SU(2)_K$ algebra given in (2.17) plays a key role for embedding $hs(8^*)$ and its quasi-adjoint representation in an associative oscillator algebra. Irreducibility is achieved by imposing the $SU(2)_K$ invariance conditions on the master fields as in (5.1-5.2), choosing the ordering prescriptions (4.6) and (5.8) and modding out the ideal $\mathcal{I}$ defined in (4.8). The $SU(2)_K$ invariance implies that a 6d realization of $hs(8^*)$ in terms of doubletons must necessarily be given in terms of the scalar doubleton (which is part of the tensor multiplet in the case of $(2,0)$ supersymmetry).

The structure of the above 7D HS theory suggests that it describes the massless sector of a $hs(8^*)$ gauge theory which includes massive fields and whose holographic dual is the 6d scalar field theory in the limit of large N [12, 9]. The scalar doubleton theory is the minimal bosonic truncation of the theory of N free $(2,0)$ tensor multiplets. We expect that the latter theory has an anti-holographic dual for large N which is a 7D gauge theory based on a superextension of $hs(8^*)$ with $\mathcal{N} = 2$ supersymmetry. Once we have computed the interactions in the 7D theory, these ideas can be tested explicitly by comparing an 6d n-point functions to the corresponding amplitude of the bulk theory. Since both sides are weakly coupled when N is large [12, 9] this provides an explicit example of a directly verifiable AdS/CFT correspondence, similar to the one proposed in [10, 11, 6] for $D = 5$.

The results obtained this far on HS gauge theories in diverse dimensions point to underlying universal features. In particular:

- They are gauge theories of HS algebras that are infinite dimensional extensions of the finite dimensional AdS group. These algebras are based on oscillator realizations, or equivalently

AdS group enveloping algebras modded by certain ideals.

- The massless sector of a HS gauge theory is a UIR of the HS algebra given by the symmetric tensor product of two ultra-short multiplets, known as singletons or doubletons, which decomposes into an infinite tower of AdS supermultiplets where the first level is the supergravity multiplet.

- Their massless field content is given by an adjoint gauge field A_μ and a quasi-adjoint zero-form Φ.

- Their background independent field equations follow from a universal set of curvature constraints.

These properties and other arguments which will be presented elsewhere [9] suggest that HS gauge theories have holographic duals given by various free, large N conformal field theories and that finite N corrections are encoded into the bulk theory in a universal background independent 'quantization' scheme. This would describe an unbroken phase of Type IIB string/M theory.

Clearly much remains to be done to develop HS gauge theories further. The supersymmetric extension of the linearized 7D HS gauge theory presented here, and the linearized 6D HS gauge theory based on the HS extension of the AdS_6 superalgebra F_4 should be straightforward. As for the interactions, they are known fully in $D = 4$ [1] and some cubic couplings have been computed in the bosonic 5D HS theory [7]. Those in $D = 4$ are given in a closed form but considerable amount of work remains to be done to exhibit their structure explicitly. The construction of the full interactions in $D > 4$ HS gauge theories also remains an open problem though we do no expect any fundamental obstacle in achieving this. In testing ideas of higher spin AdS/CFT correspondence, it is also important to incorporate the massive HS multiplets which arise in higher than second order tensor product of singletons/doubletons [15]. The coupling of massless and massive HS multiplets is therefore an important open problem.

Another open problem is to understand the role played by the $U(1)_K$ and $SU(2)_K$ charged higher spin doubletons in $d = 4$ and $d = 6$, respectively. They may ultimately be necessary in the description of an unbroken phase of M theory. In any event, it seems likely that their inclusion will lead to a HS conformal field theory in the boundary and corresponding generalized HS gauge theory in the bulk based on a HS algebra whose maximal finite dimensional subalgebra is the symplectic extension of the AdS algebra [26].

Acknowledgements

P.S. is thankful to U. Danielsson and F. Kristiansen for discussions. This research project has been supported in part by NSF Grant PHY-0070964.

A Calculation of Mass Terms

In order to derive the linearized field equation obeyed by the spin s Weyl tensor $\Phi^{(s,0;0)}_{\alpha_1\dots\alpha_s,\beta_1\dots\beta_s}$ we start from the following components of the scalar constraint (6.7):

$$\begin{aligned}
\nabla_\mu \Phi_{\alpha_1\dots\alpha_s,\beta_1\dots\beta_s} &= -\frac{1}{2}(\Gamma_\mu)^{\gamma\delta}\Phi_{\gamma\alpha_1\dots\alpha_s,\delta\beta_1\dots\beta_s} - \frac{s^2}{2}(\Gamma_\mu)_{\alpha_1\beta_1}\Phi_{\alpha_2\dots\alpha_s,\beta_2\dots\beta_s} \ , \qquad \text{(A.1)}\\
\nabla_\mu \Phi_{\alpha_1\dots\alpha_{s+1},\beta_1\dots\beta_{s+1}} &= -\frac{1}{2}(\Gamma_\mu)^{\epsilon\phi}\Phi_{\epsilon\alpha_1\dots\alpha_{s+1},\phi\beta_1\dots\beta_{s+1}} - \frac{(s+1)^2}{2}(\Gamma_\mu)_{\alpha_1\beta_1}\Phi_{\alpha_2\dots\alpha_{s+1},\beta_2\dots\beta_{s+1}} \ .
\end{aligned}$$

Here and in the remainder of this section we assume separate symmetrization of α and β indices. Combining the two equations and restricting to the $(s,0;0)$ sector we find:

$$\begin{aligned}
\nabla^2\Phi^{(s,0;0)}_{\alpha(s),\beta(s)} &= \tfrac{1}{4}(\Gamma^\mu)^{\gamma\delta}\Big[(\Gamma_\mu)_{\gamma\delta}\Phi^{(s,0;0)}_{\alpha(s),\beta(s)} + s\ (\Gamma_\mu)_{\gamma\beta_1}\Phi^{(s,0;0)}_{\alpha(s),\delta\beta(s-1)}\\
&\quad + s\ (\Gamma_\mu)_{\alpha_1\delta}\Phi^{(s,0;0)}_{\gamma\alpha(s-1),\beta(s)} + \tfrac{1}{4}(\Gamma_\mu)^{\epsilon\phi}\Phi^{(s,0;2)}_{\gamma\epsilon\alpha(s),\delta\phi\beta(s)}\Big] \qquad \text{(A.2)}\\
&\equiv m^2\Phi^{(s,0;0)}_{\alpha(s),\beta(s)} \ ,
\end{aligned}$$

where $\alpha(s)=\alpha_1\dots\alpha_s$ and idem β. The contribution to m^2 from the first three terms on the right hand side is readily found to be

$$-14-\frac{7}{2}s \ . \qquad \text{(A.3)}$$

The calculation of the contribution to m^2 from the last term in (A.2) is more elaborate. We need to use (5.18) and break up the overlapping symmetrizations. In doing so it is convenient to go over to the $SO(6,1)$ tensorial basis using

$$\Phi^{(s,0;0)}_{\alpha(s),\beta(s)} = (\Gamma^{a_1}{}_{b_1})_{\alpha_1\beta_1}\cdots(\Gamma^{a_s}{}_{b_s})_{\alpha_s\beta_s}\Phi^{b_1\dots b_s}_{a_1\dots a_s} \ , \qquad \text{(A.4)}$$

where $\Phi^{a_1\dots a_s}_{b_1\dots b_s}$ belongs to the $SO(6,1)$ Young tableaux with two rows of length s. Making also use of the Fierz identities

$$\begin{aligned}
(\Gamma^a{}_{b\mu})_{\alpha_1\beta_1}(\Gamma^c{}_d{}^\mu)_{\alpha_2\beta_2}\Phi^{bd}_{ac} &= -\Phi^{(2,0;0)}_{\alpha(2),\beta(2)} \ , \qquad \text{(A.5)}\\
(\Gamma^a{}_{b\mu})_{\alpha_1\alpha_2}(\Gamma^c{}_d{}^\mu)_{\beta_1\beta_2}\Phi^{bd}_{ac} &= -4\Phi^{(2,0;0)}_{\alpha(2),\beta(2)} \ , \qquad \text{(A.6)}
\end{aligned}$$

where the right hand sides are defined as in (A.4) and extra indices on Φ have been suppressed, we find that the contribution to m^2 from the last term in (A.2) is given by

$$\frac{1}{8(7+2s)}(24s^2+180s+336) = 6+\frac{3}{2}s \ . \qquad \text{(A.7)}$$

Adding the two contributions in (A.3) and (A.7) we find

$$m^2 = -8 - 2s \ . \tag{A.8}$$

B Harmonic Analysis

To determine the $SO(6,2)$ content of the spectrum, we shall follow the technique used in [25] which is based on the analytic continuation of AdS$_7$ to S^7, and consequently the group $SO(6,2)$ to $SO(8)$. The Casimir eigenvalues for an $SO(6,2)$ representation $D(E_0; n_1, n_2, n_3)$, where $n_1 \geq n_2 \geq |n_3|$ are $SO(6)$ highest weight labels, and an $SO(8)$ representation with highest weight labels $\ell_1 \geq \ell_2 \geq \ell_3 \geq |\ell_4|$, are given by

$$\begin{aligned} C_2[SO(6,2)] &= E_0(E_0-6) + n_1(n_1+4) + n_2(n_2+2) + n_3^2 \\ C_2[SO(8)] &= \ell_1(\ell_1+6) + \ell_2(\ell_2+4) + \ell_3(\ell_3+2) + \ell_4^2 \ . \end{aligned} \tag{B.1}$$

The continuation from AdS$_7$ to S^7 requires the identification:

$$\nabla^2|_{\mathrm{AdS}_7} \to -\nabla^2|_{S^7} \ , \quad \ell_1 = -E_0 \ , \quad \ell_{2,3,4} = n_{1,2,3} \ . \tag{B.2}$$

A tensor $T_{(m_1 m_2 m_3)}$ on S^7 in an irrep R of $SO(7)$ with highest weight labels $m_1 \geq m_2 \geq m_3 \geq 0$ can be expanded as

$$T_{(m_1 m_2 m_3)}(x) = \sum_{\substack{(\ell_1\ell_2\ell_3\ell_4) \\ p}} T_p^{(\ell_1\ell_2\ell_3\ell_4)} D^{(\ell_1\ell_2\ell_3\ell_4)}_{(m_1 m_2 m_3),p}(L_x^{-1}) \ , \tag{B.3}$$

where L_x is a coset representative of a point $x \in S^7$ and $(\ell_1\ell_2\ell_3\ell_4)$ label all $SO(8)$ representations satisfying the embedding condition

$$\ell_1 \geq m_1 \geq \ell_2 \geq m_2 \geq \ell_3 \geq m_3 \geq |\ell_4| \ . \tag{B.4}$$

The Laplacian acting on T yields

$$-\nabla^2|_{S^7} D^{(\ell_1\ell_2\ell_3\ell_4)}_{(m_1 m_2 m_3),p} = \Big(C_2[SO(8)] - C_2[SO(7)] \Big) D^{(\ell_1\ell_2\ell_3\ell_4)}_{(m_1 m_2 m_3),p} \ , \tag{B.5}$$

where the $SO(7)$ Casimir is given by

$$C_2[SO(7)] = m_1(m_1+5) + m_2(m_2+3) + m_3(m_3+1) \ . \tag{B.6}$$

Using the notation introduced in (5.17), the $SO(7)$ highest weight labels of $\Phi^{(m,n)}_{\alpha_1\ldots\alpha_{2s}}$ $(m+n=s)$ are

$$m_1 = m + n \ , \quad m_2 = m \ , \quad m_3 = 0 \ . \tag{B.7}$$

The embedding condition (B.4) implies

$$\ell_1 \geq s \ , \quad \ell_2 = s \ , \quad \ell_3 = 0, 1, ..., s \ , \quad \ell_4 = 0 \ . \tag{B.8}$$

The on-shell conditions on $\Phi^{(s,0;0)}$, which follow from the $(s, 0; 0)$ part of (A.1), remove the $SO(6,2)$ irreps with $\ell_3 = 0, 1, ..., s-1$. Recalling (A.8) we thus find that the lowest weight energy of $\Phi^{(s,0;0)}_{\alpha(s),\beta(s)}$ is given by

$$E_0 = 3 + \sqrt{9 + s(s+5) + s(s+3) - s(s+4) - 8 - 2s} = 4 + s \ . \tag{B.9}$$

Thus the physical field content in $\Phi^{(s,0;0)}_{\alpha(s),\beta(s)}$ is the massless spin s field carrying the representation $D(s+4; s, 0, 0)$.

References

[1] M.A. Vasiliev, *Consistent equations for interacting gauge fields of all spins in* $3+1$ *dimensions*, Phys. Lett. **B243**(1990) 378.

[2] S.E. Konstein and M.A. Vasiliev, *Extended higher-spin superalgebras and their massless representations*, Nucl. Phys. **B331** (1990) 475.

[3] E. Sezgin and P. Sundell, *Higher spin N=8 supergravity*, JHEP 9811 (1998) 016, `hep-th/9805125`.

[4] E. Sezgin and P. Sundell, *Higher spin N=8 supergravity in* AdS_4, hep-th/9903020.

[5] M.A. Vasiliev, *Higher spin gauge theories: star-product and AdS space*, hep-th/9910096.

[6] E. Sezgin and P. Sundell, *Doubletons and 5D higher spin gauge theory*, hep-th/0105001.

[7] M.A. Vasiliev, *Cubic interactions of bosonic higher spin gauge fields in* AdS_5, hep-th/0106200.

[8] E. Sezgin and P. Sundell, *Towards massless higher spin extension of D=5, N=8 gauged supergravity*, JHEP 0109 (2001) 025, hep-th/0107186.

[9] E. Sezgin and P. Sundell, to appear.

[10] E. Witten, private communications.

[11] B. Sundborg, *Stringy gravity, interacting tensionless strings and massless higher spins*, hep-th/0103247.

[12] E. Witten, talk given at J.H. Schwarz' 60th Birthday Conference, Cal Tech, Nov 2-3, 2001.

[13] E. Bergshoeff, E. Sezgin and Y. Tanii, *Stress-tensor commutators and Schwinger terms in singleton theories*, Int. J. Mod. Phys.**A5** (1990) 3599.

[14] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Large N field theories, string theory and gravity*, Phys. Rep. **323** (2000) 183, hep-th/9905111.

[15] M. Gunaydin and S. Takemae, *Unitary supermultiplets of* $OSp(8^*|4)$ *and the* AdS_7/CFT_6 *duality*, Nucl. Phys. **B578** (2000) 405, hep-th/9910110.

[16] B. Eden, S. Ferrara and E. Sokatchev, $(2,0)$ *Superconformal OPEs in* $D=6$*, selection rules and non-renormalization theorems*, JHEP 0111 (2001) 020, hep-th/0107084.

[17] S. Ferrara and E. Sokatchev, *Universal properties of superconformal OPEs for 1/2 BPS operators in* $3 \leq D \leq 6$, hep-th/0110174.

[18] V.E. Lopatin and M.A. Vasiliev, *Free massless bosonic fields of arbitrary spin in d-dimensional de Sitter space*, Mod. Phys. Lett. **A3** (1987) 257.

[19] R.R. Metsaev, *Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space*, hep-th/9810231.

[20] S. Fernando, M. Gunaydin and S. Takemae, *Supercoherent states of* $OSp(8^*|2N)$*, conformal superfields and the* AdS_7/CFT_6 *duality*, hep-th/0106161.

[21] S. Ferrara and C. Fronsdal, *Conformal fields in higher dimensions*, hep-th/0006009.

[22] S. Minwalla, *Restrictions imposed by superconformal invariance on quantum field theories*, Adv. Theor. Math. Phys. **2** (1998) 781, hep-th/9712074.

[23] S. Ferrara and E. Sokatchev, *Superconformal interpretation of BPS states in AdS geometries*, Int. J. Theor. Phys. **40** (2001) 935, hep-th/0005151.

[24] S.E.Konstein, M.A.Vasiliev and V.N.Zaikin7, *Conformal higher spin currents in any dimension and AdS/CFT correspondence*, JHEP **0012** (2000) 018, hep-th/0010239.

[25] 30) E. Sezgin, *The spectrum of the eleven dimensional supergravity compactified on the round seven sphere*, Phys. Lett. **B138** (1984) 57.

[26] M.A. Vasiliev, *Conformal higher spin symmetries of 4d massless supermultiplets and osp(L,2M) invariant equations in generalized (super)space*, hep-th/0106149.