Effects of symmetry energy on two-nucleon correlation functions in heavy-ion collisions induced by neutron-rich nuclei

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Using an isospin-dependent transport model, we study the effects of nuclear symmetry energy on two-nucleon correlation functions in heavy ion collisions induced by neutron-rich nuclei. We find that the density dependence of the nuclear symmetry energy affects significantly the nucleon emission times in these collisions, leading to larger values of two-nucleon correlation functions for a symmetry energy that has a stronger density dependence. Two-nucleon correlation functions are thus useful tools for extracting information about the nuclear symmetry energy from heavy ion collisions.

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The nuclear symmetry energy is a measure of the difference between the binding energy of an asymmetric nuclear matter and that of a nuclear matter with equal numbers of protons and neutrons. Knowledge on the density-dependence of the nuclear symmetry energy is essential for understanding not only the structure of radioactive nuclei [1, 2, 3, 4] but also many important issues in astrophysics [5, 6, 7, 8, 9]. For example, the nuclear symmetry energy plays an important role in understanding the nucleosynthesis during the pre-supernova evolution of massive stars, the mechanisms for supernova explosions, the cooling rate of protoneutron stars and the associated neutrino fluxes, and the kaon condensation as well as the hadron to quark-gluon plasma phase transitions in neutron stars [5, 6]. However, our knowledge of nuclear symmetry energy remains limited in spite of extensive theoretical studies [7, 10, 11]. On the other hand, radioactive beams, particularly the very energetic ones to be available at the planned Rare Isotope Accelerator (RIA) and the new accelerator facility at the German Heavy Ion Accelerator Center (GSI), provide a great opportunity to study the density dependence of the nuclear symmetry energy. Significant progress has already been achieved in identifying the observables that are sensitive to the symmetry energy. It was proposed that information about the nuclear symmetry energy can be extracted from measurements of the neutron-skins of radioactive nuclei via their total reaction cross sections [1] and those of stable heavy nuclei via parity-violating electron scatterings [3]. Promising probes to the nuclear symmetry energy have also been found in heavy-ion collisions induced by neutron-rich nuclei, and these include the pre-equilibrium neutron/proton ratio [12], the isospin fractionation [13, 14, 15, 16, 17], the isoscaling in multifragmentation [18], the proton differential elliptic flow [19] and the neutron-proton differential transverse flow [20] as well as the π^- to π^+ ratio [21].

Since two-particle correlation functions, through final state interactions and quantum statistical effects, have been shown to be a sensitive probe to the space-time distributions of emitted particles in heavy-ion collisions [22], it is of interest to investigate if they can also be used to study the density dependence of the nuclear symmetry energy. In this Letter, we show that the emission times of neutrons and protons are indeed sensitive to the nuclear symmetry energy. A stronger density dependence in the nuclear symmetry energy leads to an earlier and more correlated emission of pre-equilibrium neutrons and protons. Consequently, strengths of the correlation functions for nucleon pairs with high total momenta, especially for neutron-proton pairs with low relative momenta, are larger for a stiffer symmetry energy. Measurements of two-nucleon correlation functions in heavy ion collisions thus provides another possible tool for extracting useful information about the nuclear symmetry energy.

Our study is based on an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model (e.g., Refs. [12, 20, 21, 23, 24]). In this model, the initial positions of protons and neutrons in the colliding nuclei are determined according to their density distributions predicted by the relativistic mean-field (RMF) theory. Their initial momenta are then taken to have a uniform distribution inside the neutron or proton Fermi sphere with its Fermi momentum determined from the local density using the Thomas-Fermi approximation. For the isoscalar potential, we use as default the Skyrme potential with an incompressibility $K_0 = 380$ MeV. This potential has been shown to approximately reproduce the transverse flow data in heavy-ion collisions, although the latter are best described by a momentum-dependent soft potential with $K_0 = 210$ MeV [25, 26]. The isospin effects are included through the isospin-dependent total and differential nucleon-nucleon cross sections, the different Pauli blockings for protons and neutrons, the symmetry potential, and the Coulomb potential for protons. For nucleon-nucleon cross sections, we use as default the experimental values in free space, in which the neutron-proton cross section is about a factor of 3 larger than the neutron-neutron or proton-proton cross section. For a review of the IBUU model, we refer the reader to Ref. [27].

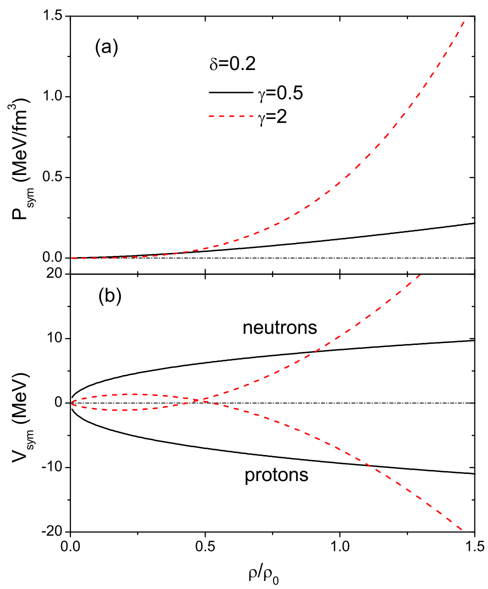

FIG. 1: The symmetry pressure (upper panel) and potential (lower panel) in nuclear matter with an isospin asymmetry $\delta = 0.2$ for the soft (solid curves) and stiff (dashed curves) symmetry energies.

The energy per nucleon in an asymmetric nuclear matter is usually expressed as

$$E(\rho,\delta) = E(\rho,\delta=0) + E_{\rm sym}(\rho)\delta^2 + \mathcal{O}(\delta^4), \qquad (1)$$

where $\rho = \rho_n + \rho_p$ is the baryon density; $\delta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is the relative neutron excess; and $E(\rho, \delta = 0)$ is the energy per particle in a symmetric nuclear matter, while $E_{\rm sym}(\rho)$ is the nuclear symmetry energy. For the latter, we adopt the parameterization used in Ref. [28] for studying the properties of neutron stars, i.e.,

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) \cdot u^\gamma, \qquad (2)$$

where $u \equiv \rho/\rho_0$ is the reduced density and $E_{\rm sym}(\rho_0) = 35$ MeV is the bulk symmetry energy at normal nuclear matter density $\rho_0 = 0.16$ fm^{-3}. In the following, we consider the two cases of $\gamma = 0.5$ (soft) and 2 (stiff) to explore the large range of $E_{\rm sym}(\rho)$ predicted by different theoretical models [7]. The pressure due to the symmetry energy, given by $P_{\rm sym} = (\rho^2 \partial E_{\rm sym}/\partial\rho)\delta^2$, in asymmetric nuclear matter with an isospin asymmetry $\delta = 0.2$ are shown in the upper panel of Fig. 1 for the two symmetry energies. It is seen that the stiff symmetry energy leads to a larger pressure in asymmetric nuclear matter than that from the soft symmetry energy at the same density.

The symmetry potential acting on a nucleon derived from the above nuclear symmetry energy is [19]

$$\begin{aligned} V_{\rm sym} &= \pm 2[E_{\rm sym}(\rho_0)u^\gamma - 12.7u^{2/3}]\delta \\ &\quad + [E_{\rm sym}(\rho_0)(\gamma-1)u^\gamma + 4.2u^{\frac{2}{3}}]\delta^2, \end{aligned} \qquad (3)$$

where "+" and "-" are for neutrons and protons, respectively. In the lower panel of Fig. 1, the symmetry potentials for protons and neutrons in an isospin asymmetric nuclear matter with $\delta = 0.2$ are shown. The symmetry potential is seen to be attractive for protons and repulsive for neutrons and increases with density, except for the stiff symmetry potential at densities below $0.5\rho_0$.

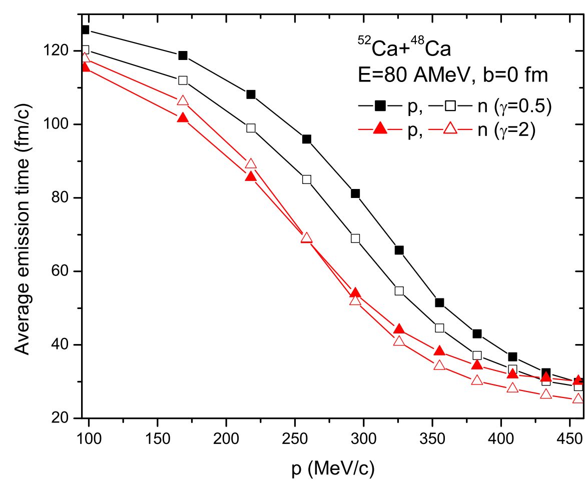

FIG. 2: The average emission times of protons or neutrons as functions of their momenta for soft (solid curves) and stiff (dashed curves) symmetry energies.

As an example, we study in this letter central collisions of ^{52}Ca + ^{48}Ca at E=80 MeV/nucleon. This reaction system with an isospin asymmetry $\delta = 0.2$ can be studied at RIA. Nucleons are considered as being emitted when their local densities are less than $\rho_0/8$ and subsequent interactions with the mean field do not cause their recapture into regions of higher density. Other emission criteria, such as taking the nucleon emission time as its last collision time in the IBUU model, do not change our conclusions. Shown in Fig. 2 are the average emission times of protons and neutrons as functions of their momenta. It is seen that the average emission time of nucleons with a given momentum is earlier for the stiff symmetry energy than for the soft one, as nucleon emissions are mainly governed by the pressure of the excited matter created during the collisions [19, 29], which is larger for the stiffer symmetry energy than for the soft one. Moreover, there is a significant delay in the emission of protons when the symmetry energy is soft. This is due to the fact that the symmetry potential is generally repulsive for neutrons and attractive for protons, and their magnitudes at low densities, where most nucleons are emitted, are larger for the soft symmetry energy than for the stiff symmetry potential as seen in Fig. 1. The relative emission times of neutrons and protons in the case of the stiff symmetry energy depends on the density at which they are emitted, as the stiff symmetry potential changes sign when the nuclear density is below $0.5\rho_0$. It is interesting to note that high momentum nucleons are emitted during the early pre-equilibrium stage of the collision when the density is relatively high, while low momentum ones are mainly emitted when the system is close to equilibrium and the density is low.

It is well known that the space-time distributions of emitted particles can be extracted from the two-particle correlation functions; see, e.g., Refs. [22, 30, 31], for reviews. The two-proton correlation function has been studied most extensively. Recently, experimental data on two-neutron and neutron-proton correlation functions have also become available, and this has made it possible to deduce the relative emission times of neutrons and protons [32]. Theoretically, the two-nucleon correlation function can be evaluated in the standard Koonin-Pratt formalism [33, 34] by convoluting the emission function $g(\mathbf{p}, x)$, i.e., the probability for emitting a particle with momentum $\mathbf{p}$ from space-time point $x = (\mathbf{r}, t)$, with the relative wave function of the two particles, i.e.,

$$C(\mathbf{P}, \mathbf{q}) = \frac{\int d^4x_1 d^4x_2 g(\mathbf{P}/2, x_1) g(\mathbf{P}/2, x_2) \left|\phi(\mathbf{q}, \mathbf{r})\right|^2}{\int d^4x_1 g(\mathbf{P}/2, x_1) \int d^4x_2 g(\mathbf{P}/2, x_2)}. \tag{4}$$

In the above, $\mathbf{P}(= \mathbf{p_1} + \mathbf{p_2})$ and $\mathbf{q}(=\frac{1}{2}(\mathbf{p_1} - \mathbf{p_2}))$ are, respectively, the total and relative momenta of the particle pair; and $\phi(\mathbf{q}, \mathbf{r})$ is their relative wave function with $\mathbf{r}$ being their relative position, i.e., $\mathbf{r} = (\mathbf{r}_2 - \mathbf{r}_1) - \frac{1}{2}(\mathbf{v_1} + \mathbf{v_2})(t_2 - t_1)$.

Using the program Correlation After Burner [35], which takes into account final-state nucleon-nucleon interactions, we have evaluated the two-nucleon correlations from the emission function given by the IBUU model. Shown in Fig. 3 are the two-nucleon correlation functions gated on the total momentum (P) of nucleon pairs from central collisions of ^{52}Ca + ^{48}Ca at E=80 MeV/nucleon. The left and right panels are for $P < 300$ MeV/c and $P > 500$ MeV/c, respectively. For both neutron-neutron and neutron-proton correlation functions, they peak at $q \approx 0$ MeV/c. The proton-proton correlation function is, on the other hand, peaked at about $q = 20$ MeV/c due to the strong final-state s-wave attraction but is suppressed at $q = 0$ as a result of Coulomb repulsion and wave-function antisymmetrization between the two protons. Since the emission times of low momentum nucleons are not affected much by the different symmetry energies used in the IBUU model as shown in Fig. 2, the two-nucleon correlation functions are thus similar for the stiff and soft symmetry energies. On the other hand, the emission times of high momentum nucleons, which are dominated by those with momenta

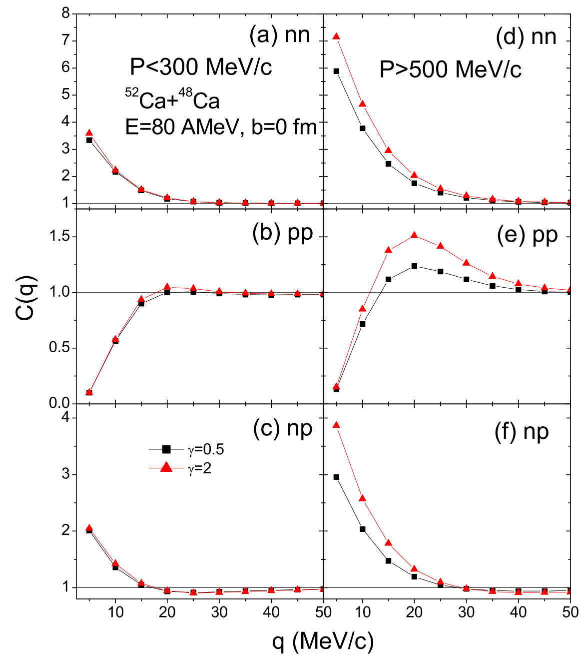

FIG. 3: Two-nucleon correlation functions gated on the total momentum of nucleon pairs. Left panels are for $P < 300$ MeV/c while right panels are for $P > 500$ MeV/c.

near 250 MeV/c, differ appreciably for the two symmetry energies considered here. The correlation functions of these nucleon pairs thus show a strong dependence on symmetry energy. Gating on nucleon pairs with high total momentum thus allows one to select those nucleons that have stronger correlations as a result of small spatial separations at emissions. From Fig. 3, it is seen that the correlation functions for neutron-neutron and neutron-proton pairs with high total momentum but low relative momentum of $q = 5$ MeV/c are, respectively, about 20% and 30% higher for the stiff symmetry energy than for the soft symmetry energy, while the correlation function for proton-proton pairs with high total momentum and relative momentum of $q = 20$ MeV/c is about 20% higher for the stiff symmetry energy than for the soft symmetry energy. The neutron-proton correlation function thus has the largest sensitivity to nuclear symmetry energy. As shown in Fig. 2 and discussed earlier, the relative emission times of neutrons and protons is sensitive to nuclear symmetry energy. Since the pre-equilibrium neutrons and protons are emitted almost simultaneously in the case of the stiff symmetry energy, they are strongly correlated, leading thus to a larger value for the neutron-proton correlation function. On the other hand, protons and neutrons are less correlated in the case of the soft symmetry energy as proton emissions are delayed relative to neutron emissions. As a result, the soft symmetry energy gives a smaller value for the neutron-proton correlation function. Furthermore, with the stiff symmetry energy, neutrons as well as protons have smaller spatial separations at emissions as they are emitted earlier during the collision. Therefore, neutrons and also protons are more correlated among themselves for the stiff symmetry energy than for the soft one. Our results thus clearly demonstrate that correlation functions of nucleon pairs with high total momentum can indeed reveal sensitively the effect of the nuclear symmetry energy on the space-time distributions of nucleons at emissions.

We have also studied the dependence of two-nucleon correlation functions on the incompressibility K_0 and nucleon-nucleon cross sections. We find that changing the value of K_0 from 380 to 210 MeV or using the in-medium nucleon-nucleon cross sections predicted by the Dirac-Brueckner approach based on the Bonn A potential [36], which has a smaller magnitude and weaker isospin dependence than the free ones but a strong density dependence, changes both the neutron-proton and proton-proton correlation functions by about 5%, similar to those found in Ref. [22]. The weak dependence of the two-nucleon correlation functions on the stiffness of isoscalar energy is due to the reduced difference in the pressure of the excited matter as a result of different maximum densities reached in the collision, with the stiff one giving a lower density than the soft one. Details on these results as well as the dependence of the symmetry energy effects studied here on the impact parameter, incident energy, and masses of the colliding system will be reported elsewhere [37].

In conclusion, we have studied the two-nucleon correlation functions in intermediate-energy heavy-ion collisions induced by neutron-rich nuclei in the framework of an isospin-dependent transport model. We find that the nuclear symmetry energy affects significantly the emission times of neutrons and protons. A stiffer symmetry energy leads to an earlier and nearly simultaneous emission of pre-equilibrium neutrons and protons. For the soft symmetry energy, nucleon emissions are delayed compared to that for the stiff symmetry energy, and the delay is longer for protons than for neutrons. As a result, the correlation functions for nucleon pairs with high total momenta, especially for neutron-proton pairs with low relative momenta, are stronger for a stiffer symmetry energy. Studies of two-nucleon correlation functions in heavy ion collisions thus provides a possible tool for extracting useful information about the density dependence of the nuclear symmetry energy.

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