Higher order wave-particle duality

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The complementarity of single-photon's particle-like and wave-like behaviors can be described by the inequality $D^2 + V^2 \leq 1$, with D being the path distinguishability and V being the fringe visibility. In this paper, we generalize this duality relation to multi-photon case, where two new concepts, higher order distinguishability and higher order fringe visibility, are introduced to quantify the higher order particle-like and wave-like behaviors of multi-photons.

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I. INTRODUCTION

The complementarity principle, developed and introduced by Bohr in 1927 [1], is fundamentally important in the quantum theory, which predicts that a quantum system may exhibit different properties based on different measurement schemes. As the most typical example of complementarity, wave-particle duality has attracted much attention since the early days of the quantum theory [2]. By defining the particle-like knowledge of an object governed by quantum mechanics as the distinguishability (D) of its passage in a two-path interferometer, and the wave-like knowledge as the visibility (V) of the interference pattern behind the interferometer, a tradeoff relation between an object's particle-like and wave-like behaviors can be established [3–7],

$$D^2 + V^2 \leq 1, \tag{1}$$

where the equal sign holds for pure state of single-particles. This duality relation, already confirmed in many experiments [8], is valid even when the choice of measuring apparatus is delayed after the single-particle's entrance into the interferometer [9]. Such a delayed-choice gedanken experiment has been realized in experiments at single-photon level by Roch group [10]. The scheme of quantum eraser [11], with the experimental realization reported in Ref.[12], provides another clear way to demonstrate the exclusive relation between an object's particle-like and wave-like behaviors. Recently, a new optical device named quantum beam splitter (QBS) is theoretically proposed in Ref. [13, 14], and the wave-particle morphing behaviors of single-photons in this quantum device is becoming a hot topic [15].

In this paper, we generalize the discussion on the duality of single-photons and investigate the higher order duality relations of multi-photons, no matter what kind of state, pure or mixed, is prepared for the multi-photons.

The organization of the paper is as follows: In section II we introduce two new concepts, higher order distinguishability and visibility. In section III, we derive an inequality for higher order duality. In section IV, we present a physical interpretation on the higher order distinguishability and visibility and in section V we present a measurement scheme for the higher order visibility.

II. HIGHER ORDER DISTINGUISHABILITY AND VISIBILITY

Before the concept of higher order duality is introduced, we first recall the definitions of the particle-like information and the wave-like information for single-photons. By feeding the interferometer with single-photons, the particle-like information is usually quantified as the distinguishability (D) of single-photons' passage along the two paths inside the interferometer (see Fig. 1). Thus we can use an operator [16],

$$\hat{D} \equiv \frac{a_1^\dagger a_1 - a_2^\dagger a_2}{\langle a_1^\dagger a_1 \rangle + \langle a_2^\dagger a_2 \rangle}, \tag{2}$$

to describe the measurement of the particle-like information mentioned above. Here $a_1^\dagger$ (a_1) and $a_2^\dagger$ (a_2) denote the creation (annihilation) operators of the modes in path 1 and 2, and the denominator $\langle a_1^\dagger a_1 \rangle + \langle a_2^\dagger a_2 \rangle$ is for normalization. The wave-like information is defined as the visibility (V) of the interference pattern after the single-photons pass through the A interferometer, whose measurement is in accord with the following operator,

$$\hat{V} \equiv \frac{a_1^\dagger a_2 \mathrm{e}^{\mathrm{i}\phi} + a_2^\dagger a_1 \mathrm{e}^{-\mathrm{i}\phi}}{\langle a_1^\dagger a_1 \rangle + \langle a_2^\dagger a_2 \rangle} \tag{3}$$

By describing the distinguishability and visibility in terms of operators, the particle-like and wave-like information of single-photons are then the modules of the two expectation values, i.e., $D = |\langle \hat{D} \rangle|$ and $V = |\langle \hat{V}' \rangle|_{\text{max by }\phi}$, where the phase parameter ϕ, controlled by the phase shifter in the interferometer, should be appropriately chosen to maximize the expectation value of the operator (3).

The terms $a_1^\dagger a_1$ ($a_2^\dagger a_2$) in Eq. (2) and $a_1^\dagger a_2$ in Eq. (3) are just the first order auto-correlation of the field in path 1 (2) and the first order coherence between the fields in the two paths [17], respectively. Therefore the distinguishability and visibility defined in Eqs. (2) and (3) can be regarded as the difference and coherence between the first order correlation function of the two fields in the two paths 1 and 2.

Based on this viewpoint, we now introduce the concepts of kth-order distinguishability,

$$\hat{D}_k \equiv \frac{(a_1^\dagger)^k a_1^k - (a_2^\dagger)^k a_2^k}{\langle (a_1^\dagger)^k a_1^k\rangle + \langle (a_2^\dagger)^k a_2^k\rangle}, \tag{4a}$$

and kth-order visibility,

$$\hat{V}_k \equiv \frac{(a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}}{\langle (a_1^\dagger)^k a_1^k\rangle + \langle (a_2^\dagger)^k a_2^k\rangle}, \tag{4b}$$

the denominator $\langle (a_1^\dagger)^k a_1^k\rangle + \langle (a_2^\dagger)^k a_2^k\rangle$ in both equations is again for normalization. Here we have used kth-order auto-correlation $(a_1^\dagger)^k a_1^k$ ($(a_2^\dagger)^k a_2^k$) and kth-order coherence $(a_1^\dagger)^k a_2^k$ to replace the first order auto-correlation and the first order coherence used in Eqs. (2) and (3), which can now be regarded as the special case of the definitions in (4) by setting $k = 1$. Similar to the above treatment on the first order distinguishability and visibility, the kth-order particle-like information and kth-order wave-like information are just the modules of the corresponding expectation values, i.e., $D_k = |\langle \hat{D}_k\rangle|$ and $V_k = |\langle \hat{V}_k\rangle|_{\text{max by }\phi}$. Here the phase parameter ϕ should be appropriately chosen to maximize the visibility V_k, whose measurement will be introduced in section V in more details.

III. INEQUALITY FOR HIGHER ORDER DUALITY

Now we define the kth-order duality as the sum of the squared kth-order particle-like information and kth-order wave-like information, which is,

$$D_k^2 + V_k^2 = \left(\frac{\langle (a_1^\dagger)^k a_1^k\rangle + \langle (a_2^\dagger)^k a_2^k\rangle}{\langle (a_1^\dagger)^k a_1^k\rangle + \langle (a_2^\dagger)^k a_2^k\rangle}\right)^2 + 4\frac{\left|\langle (a_1^\dagger)^k a_2^k\rangle\right|^2 - \langle (a_1^\dagger)^k a_1^k\rangle\langle (a_2^\dagger)^k a_2^k\rangle}{\left(\langle (a_1^\dagger)^k a_1^k\rangle + \langle (a_2^\dagger)^k a_2^k\rangle\right)^2}. \tag{5}$$

The Cauchy-Schwarz inequality predicts

$$\left|\langle (a_1^\dagger)^k a_2^k\rangle\right|^2 \leq \langle (a_1^\dagger)^k a_1^k\rangle\langle (a_2^\dagger)^k a_2^k\rangle. \tag{6}$$

Therefore, the second term in Eq. (5) is negative or zero. This result leads to the inequality for higher order duality,

$$D_k^2 + V_k^2 \leq 1, \tag{7}$$

which is the main conclusion in this paper. Although this inequality has a similar formula to Eq. (1), it undoubtedly carries more information about the wave-particle duality and helps deepen our understanding. In fact, we have generalized the duality relation from single-photon fields to multi-photon fields. In a typical duality experiment, if the interferometer (see Fig. 1) is fed with multi-photons, besides single-photons, according to our conclusion, the fields in the two paths have to obey not only the first order duality relation (1), but also the higher order duality relation (7). Since n-photon component in a field only contributes the kth-order correlation function and the kth-order coherence with $k \leq n$, and takes no effect for the k'th-order correlation function or coherence if $k' > n$, all higher than nth-order duality information does not exist (sums up to zero) for the case that at most n-photon component is found in the field. As a consequence, for a state with n-photons there exist n inequalities, $D_k^2 + V_k^2 \leq 1$, with $k = 1, 2, \ldots, n$. That is why we only need to consider the first order distinguishability (2) and the first order visibility (3) in the duality experiments with single-photons.

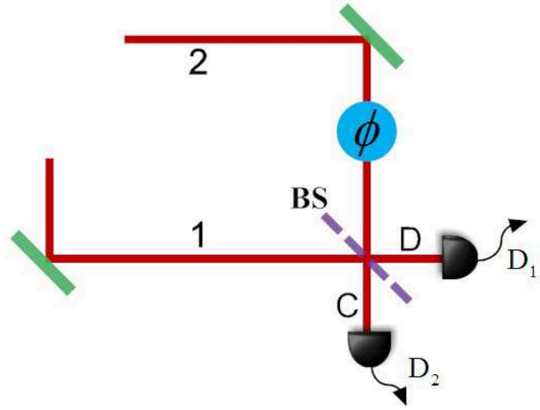

FIG. 1: The distinguishability and visibility are measured in the open (removing the beam splitter BS) and closed (employing the beam splitter BS) interferometer, respectively.

From Eq. (5), it is easy to find that the equality sign in the higher order duality relation (7) will be satisfied under the condition $|\langle (a_1^\dagger)^k a_2^k\rangle|^2 = \langle (a_1^\dagger)^k a_1^k\rangle\langle (a_2^\dagger)^k a_2^k\rangle$. For example, the kth-order duality, if it exists, is saturated for k-photon pure states, which is a generalization of the first order duality relation $D_1^2 + V_1^2 = 1$ for the single-photons in a pure state. The condition on which the kth-order duality (7) achieves its maximum value unity is usually very complicated if n-photon component with $n > k$ is involved in the field, and $|\langle (a_1^\dagger)^k a_2^k\rangle|^2 = \langle (a_1^\dagger)^k a_1^k\rangle\langle (a_2^\dagger)^k a_2^k\rangle$ is at present the only test equation we can obtain.

IV. PHYSICAL INTERPRETATION

The kth-order auto-correlation $(a_i^\dagger)^k a_i^k$ (i=1,2) used in the definition (4a) is equal to,

$$(a_i^\dagger)^k a_i^k = \prod_{j=0}^{k-1}(\hat{n}_i^\dagger - j), \tag{8}$$

with the number operator $\hat{n}_i = a_i^\dagger a_i$. Imposing this operator onto a number state $|n_i>$, we have, $(a_i^\dagger)^k a_i^k |n_i>= \left(\begin{array}{c} n \\ k \end{array}\right)|n_i>$, with the binomial coefficient $\left(\begin{array}{c} n \\ k \end{array}\right) = \frac{n!}{k!(n-k)!}$. Thus $\langle (a_i^\dagger)^k a_i^k\rangle$ can be regarded as the combination number of picking out k photons, disregarding order, in path i, no matter what state is prepared for the optical field in this path. The kth order distinguishability D_k then has a very clear physical meaning, i.e., the normalized difference between the k-combinations of the photons in paths 1 and 2.

In the basis $|0_1 0_2>, |0_1 1_2>, \cdots, |n_1 n_2>$, a general quantum state for the photons in the interferometer can be described by a density matrix,

$$\rho = \left(\begin{array}{ccc} \rho_{11} & \cdots & \rho_{1(n+1)^2} \\ \vdots & \ddots & \vdots \\ \rho_{(n+1)^2 1} & \cdots & \rho_{(n+1)^2(n+1)^2} \end{array}\right) \tag{9}$$

Under this quantum state, we can directly write down the expectation value of the kth order distinguishability,

$$\langle D_k\rangle = \frac{\sum_{i=k}^{n}\sum_{j=0}^{n}\left(\rho_{p_{i,j}p_{i,j}} - \rho_{q_{i,j}q_{i,j}}\right)\left(\begin{array}{c} i \\ k \end{array}\right)}{\sum_{i=k}^{n}\sum_{j=0}^{n}\left(\rho_{p_{i,j}p_{i,j}} + \rho_{q_{i,j}q_{i,j}}\right)\left(\begin{array}{c} i \\ k \end{array}\right)}, \tag{10}$$

with $p_{i,j} = i*n+i+j+1$ and $q_{i,j} = j*n+i+j+1$. Here we see that only the diagonal elements contribute to the distinguishability.

Just as we already mentioned, the visibility in a duality experiment is actually related to the coherence between the two paths. Thus the visibility is determined by the off diagonal elements of the density matrix for the photons in an interferometer. For example, for the operator $(a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}$ used in the definition of the kth order visibility (4b), the off diagonal elements $< m_1' m_2''|\rho|(m'+k)_1(m''-k)_2 >$ and $< m_1' m_2''|\rho|(m'-k)_1(m''+k)_2 >$ with $m', m'' \in [0,n]$ have contributions. The diagonal elements play no role in the evaluation of the visibility. However, for a density matrix, what we can directly measure in experiments are just the diagonal elements, usually represented as the photon counting or higher order coincidence counting. So we have to turn the information carried by the off diagonal elements to diagonal elements. That is why a $50:50$ beam splitter is to be employed at the output of the interferometer for the measurement of the visibility. For more details on the measurement of higher order visibility, please see section V.

Now we can conclude that the distribution of the photons in an interferometer projected on the Fock states, i.e., the diagonal elements of the density matrix in the Fock state basis, determines the photons' particle information, and the wave information relies on the off diagonal elements, no matter what order distinguishability and visibility is considered.

V. MEASUREMENT SCHEME FOR THE VISIBILITY

Compared with the measurement of the distinguishability D_k, which is directly defined by the auto-correlation, the measurement of the fringe visibility V_k, which depends on higher order coherence, is more complicated. For $k=1$, there exist a straight forward way to measure V_1. Suppose the beam splitter in Fig. 1 is active and the modes after the beam splitter are denoted with C and D, then the annihilation operators c and d of the modes in these two paths are connected to the annihilation operators a_1 and a_2 of the modes in paths 1 and 2 through the relations, $c = \frac{1}{\sqrt{2}}(a_1 + a_2 e^{i\phi})$ and $d = \frac{1}{\sqrt{2}}(a_1 - a_2 e^{i\phi})$, respectively, where the phase difference ϕ between the two paths can be controlled in experiments by a phase shifter (see Fig. 1). The counting difference between the two detectors D_1 and D_2 (see Fig. 1) is equal to,

$$\langle c^\dagger c - d^\dagger d\rangle_\phi = \langle a_1^\dagger a_2 \mathrm{e}^{\mathrm{i}\phi} + a_2^\dagger a_1 \mathrm{e}^{-\mathrm{i}\phi}\rangle, \tag{11}$$

which is just the first order visibility defined in Eq. (3).

The second order correlation function, needed for V_2, can also be measured in experiments based on current technology [18]. However, the measurement of the higher order visibility V_k is complicated.

For a general description, we now suppose the two detectors, D_1 and D_2, in Fig. 1 are ideal ones, so that all Fock states with arbitrary photon number can be directly detected and distinguished. For the case of $k=2$, we take the sum of the expectation value of the both detectors and obtain,

$$\begin{aligned}\langle (c^\dagger)^2 c^2 + (d^\dagger)^2 d^2\rangle_\phi = \frac{1}{2}\langle (a_1^\dagger)^2 a_1^2 + (a_2^\dagger)^2 a_2^2 + 4a_1^\dagger a_2^\dagger a_1 a_2 \\ + (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}\phi} + (a_2^\dagger)^2 a_1^2 \mathrm{e}^{-2\mathrm{i}\phi}\rangle.\end{aligned} \tag{12}$$

Further retarding the phase shift ϕ between the two path 1 and 2 by the value $\pi/2$, we obtain another similar relation,

$$\begin{aligned}\langle (c^\dagger)^2 c^2 + (d^\dagger)^2 d^2\rangle_{\phi+\pi/2} = \frac{1}{2}\langle (a_1^\dagger)^2 a_1^2 + (a_2^\dagger)^2 a_2^2 + 4a_1^\dagger a_2^\dagger a_1 a_2 \\ - (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}\phi} - (a_2^\dagger)^2 a_1^2 \mathrm{e}^{-2\mathrm{i}\phi}\rangle.\end{aligned} \tag{13}$$

In the following, we use two symbols $R_{k,\phi}^{\pm}$, whose values are in principle obtainable in experiments, to replace the expectation values $\langle (c^\dagger)^k c^k \pm (d^\dagger)^k d^k\rangle_\phi$. Thus the equality (11) can be rewritten as $\langle a_1^\dagger a_2 \mathrm{e}^{\mathrm{i}\phi} + a_2^\dagger a_1 \mathrm{e}^{-\mathrm{i}\phi}\rangle = R^-_{1,\phi}$, and the quantity $\langle (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}\phi} + (a_2^\dagger)^2 a_1^2 \mathrm{e}^{-2\mathrm{i}\phi}\rangle$, involved in both equalities (12) and (13), can be described by

$$\langle (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}\phi} + (a_2^\dagger)^2 a_1^2 \mathrm{e}^{-2\mathrm{i}\phi}\rangle = R^+_{2,\phi} - R^+_{2,\phi+\pi/2}. \quad (14)$$

The maximum value of $\langle (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}\phi} + (a_2^\dagger)^2 a_1^2 \mathrm{e}^{-2\mathrm{i}\phi}\rangle$ over the phase factor ϕ, required in the quantification of the second order visibility V_2, can then be evaluated by,

$$|\langle (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}\phi} + (a_2^\dagger)^2 a_1^2 \mathrm{e}^{-2\mathrm{i}\phi}\rangle|^2_{\text{max by }\phi} = \left(R^+_{2,\phi'} - R^+_{2,\phi'+\pi/2}\right)^2 + \left(R^+_{2,\phi'-\pi/4} - R^+_{2,\phi'+\pi/4}\right)^2, \quad (15)$$

where $(R^+_{2,\phi'} - R^+_{2,\phi'+\pi/2})$ is the real part of the vector $2\langle (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}\phi'}\rangle$ (see Eq. (14)), and $(R^+_{2,\phi'-\pi/4} - R^+_{2,\phi'+\pi/4})$ is the real part of the vector $2\langle (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}(\phi'-\pi/4)}\rangle$, which is equal to the imaginary part of the vector $2\langle (a_1^\dagger)^2 a_2^2 \mathrm{e}^{2\mathrm{i}\phi'}\rangle$. The absolute value of this quantity is mathematically equivalent to the unnormalized visibility V_2, due to the relation $|\langle (a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}\rangle|_{\text{max by }\phi} = 2|\langle (a_1^\dagger)^k a_2^k\rangle|$. The phase ϕ' can be arbitrarily chosen, because the modulus of a vector should remain invariant under the rotation of the coordinate system.

In general, the term $\langle (a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}\rangle$ can be determined by adding and subtracting $\langle (c^\dagger)^k c^k \pm (d^\dagger)^k d^k\rangle_\phi$ for k different values of ϕ. For example, for odd number of k, we have

$$\langle (a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}\rangle = \frac{2^{k-1}}{k}\sum_{m=0}^{k-1} R^-_{k,\phi+2m\pi/k}. \quad (16)$$

Accordingly, the maximum value of $\langle (a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}\rangle$ over the phase factor ϕ, used for the kth order visibility V_k, can be evaluated by,

$$|\langle (a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}\rangle|^2_{\text{max by }\phi} = \left(\frac{2^{k-1}}{k}\right)^2 \times \left[\left(\sum_{m=0}^{k-1} R^-_{k,\phi'+2m\pi/k}\right)^2 + \left(\sum_{m=0}^{k-1} R^-_{k,\phi'-\pi/(2k)+2m\pi/k}\right)^2\right], \quad (17)$$

where the phase ϕ' can be arbitrarily chosen.

For even number of k, we have

$$\langle (a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}\rangle = \frac{2^{k-1}}{k}\sum_{m=0}^{k-1} (-1)^m R^+_{k,\phi+m\pi/k}. \quad (18)$$

The maximum value of $\langle (a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}\rangle$ over the phase factor ϕ, used for the kth order visibility V_k, can be evaluated by,

$$|\langle (a_1^\dagger)^k a_2^k \mathrm{e}^{\mathrm{i}k\phi} + (a_2^\dagger)^k a_1^k \mathrm{e}^{-\mathrm{i}k\phi}\rangle|^2_{\text{max by }\phi} = \left(\frac{2^{k-1}}{k}\right)^2 \times \left[\left(\sum_{m=0}^{k-1} (-1)^m R^+_{k,\phi'+m\pi/k}\right)^2 + \left(\sum_{m=0}^{k-1} (-1)^m R^+_{k,\phi'-\pi/(2k)+m\pi/k}\right)^2\right], \quad (19)$$

with an arbitrary phase factor ϕ'.

VI. CONCLUSIONS

The distinguishability of photons' passage in the interferometer and the visibility of the interference pattern after the interferometer can be regarded as the first order particle-like information and the first order wave-like information. By introducing the concepts of higher order distinguishability and visibility for multi-photons, which are related to higher order auto-correlation and coherence between the fields in the two paths of the interferometer, we generalize the wave-particle duality relation from the first order case to higher order case. We believe it to be a useful tool for analyzing the duality experiments with the input of multi-photons, or even a classical light. If we do the duality experiment by using different light sources, the same results may be obtained if only the first order duality is considered. However, we believe it will exhibit different results for higher order duality information. The concept of higher order duality may provide us more information about the duality experiments, especially with the input of multi-photons, and accordingly helps us deepen the understanding on the wave-particle duality.

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