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On the Evaluation of Thermal Corrections to False Vacuum Decay Rates

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Abstract

We examine the computation of the nucleation barrier used in the expression for false vacuum decay rates in finite temperature field theory. By a detailed analysis of the determinantal prefactor, we show that the correct bounce solution used in the computation of the nucleation barrier should not include loop corrections coming from the scalar field undergoing decay. Temperature corrections to the bounce appear from loop contributions from other fields coupled to the scalar field. We compute the nucleation barrier for a model of scalar fields coupled to fermions, and compare our results to the expression commonly used in the literature. We find that, for large enough self-couplings, the inclusion of scalar loops in the expression of the nucleation barrier leads to an underestimate of the decay rate in the neighborhood of the critical temperature.

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I. INTRODUCTION

For the past decade or so, the study of first-order phase transitions in cosmology has been the focus of much interest due to their possible relevance to the physics of the early Universe. Some well-known examples are inflationary models [1], the quark-hadron transition [2] and, more recently, the generation of the cosmological baryon asymmetry in the electroweak phase transition [3].

In a first-order phase transition, the initial metastable phase decays to the stable phase by the nucleation of bubbles larger than a critical size. This decay may be triggered by either quantum or thermal fluctuations, depending on how the ambient temperature compares to the nucleation barrier [4]. Within a cosmological context, the cooling is provided by the expansion of the Universe; the long-wavelength modes of the order parameter responsible for the symmetry breaking transition are coupled to the "environment", which is assumed to be in local thermal equilibrium at some temperature T. (Here, we are mainly concerned with "late" transitions, for which the typical fluctuation time-scales are much shorter than the expansion rate.)

Of great relevance to the understanding of the evolution of the phase transition is the determination of the bubble nucleation rate per unit volume. This is a well-known problem in classical statistical mechanics, with an long history [4]. Phenomenological field-theoretic treatments were developed by Cahn and Hilliard [5], and by Langer [6] in the context of a time-dependent coarse-grained Ginzburg-Landau model. Classical homogeneous nucleation theory within a field theoretic context has been recently shown by numerical experiments to successfully predict the nucleation barrier [7]. In the case of zero-temperature quantum field theory, the study of metastable vacuum decay was initiated with the work of Voloshin, Kobzarev, and Okun [8], and was put onto firm theoretical ground by Coleman and Callan in the late seventies [9]. Finite temperature corrections to the vacuum decay rate were first considered by Linde [10], who argued that temperature corrections to the nucleation rate are obtained recalling that finite temperature field theory (at sufficiently high temperatures) in $d+1$-dimensions is equivalent to d-dimensional euclidean quantum field theory with $\hbar$ substituted by T. Thus, in $d+1$ dimensions, the nucleation rate is proportional to $\exp[-S_E^d(\varphi_b)/T]$, where $S_E^d(\varphi_b)$ is the d-dimensional euclidean action evaluated at its extremum (specifically, a saddle point), the critical bubble, or bounce, $\varphi_b(r)$. The usual expression for the nucleation rate per unit volume used in the literature is [10]

$$\Gamma = T\left(\frac{S_E^3(\varphi_b(r,T),T)}{2\pi T}\right)^{\frac{3}{2}}\left\{\frac{\det[-\nabla^2+V''_{\rm eff}(\varphi_f,T)]}{\det'[-\nabla^2+V''_{\rm eff}(\varphi_b(r,T),T)]}\right\}^{\frac{1}{2}}\exp\left[-\frac{S_E^3(\varphi_b(r,T),T)}{T}\right] , \tag{1.1}$$

where φ_f is the value of the field ϕ at the metastable minimum, and the prime in the determinant in the denominator is a reminder that one should omit the zero and negative eigenvalues, associated with the translation symmetry of the bubble and with its instability (being a saddle point configuration), respectively. $V_{\rm eff}(\phi,T)$ is the one-loop approximation to the finite temperature effective potential, and $V''_{\rm eff}(\varphi,T)=\frac{\partial^2 V_{\rm eff}(\phi,T)}{\partial\phi^2}|_{\phi=\varphi}$.

There are three important points here. The first is that in order to estimate the determinantal prefactor (the "equilibrium" part of the prefactor; there is a dynamic factor which can not be obtained by using equilibrium arguments) one usually proceeds by invoking dimensional arguments to approximate it by a term of order T_C^4 (T_C is the critical temperature). How good is this approximation? Clearly, in most cases it is impossible to evaluate the determinants exactly. But can one obtain a better approximation than the simple use of dimensional arguments? The second, and most important, point is that the critical bubble configuration $\varphi_b(r,T)$ used to evaluate the nucleation barrier, denoted above by $S_E^3(\varphi_b(r,T))$, was obtained from an effective potential which includes corrections coming from scalar loops. Hence the temperature dependence in $\varphi(r,T)$. We will show here that this procedure is not in general justified and is only a good approximation if the corrections from scalar loops are negligible. Finally, in the expression for the temperature corrected barrier in Eq. (1.1), $S_E^3(\varphi_b(r,T),T)$, one uses the *temperature corrected* effective potential, $V_{\rm eff}(\varphi,T)$ as opposed to the tree level potential. Thus, it is claimed that $S_E^3(\varphi_b(r,T),T)$ is equivalent to the free energy of the temperature dependent bounce, given by

$$S_E^3(\varphi_b(r,T),T)=\int d^3x\left[\frac{1}{2}\left(\nabla\varphi_b(r,T)\right)^2+V_{\rm eff}(\varphi_b(r,T),T)\right] . \tag{1.2}$$

As far as we know, apart from the work of Affleck in the context of finite temperature quantum mechanics [11], this point has never been properly addressed in the literature. How did the temperature corrected potential appear in the exponent? Is the exponentiation of the massless modes sufficient? In fact, most of the work done on cosmological phase transitions in which temperature effects are important (including some by the present authors) simply invokes Linde's results. Given the many applications of finite temperature vacuum decay in cosmology, we feel that this important question should not be left unscrutinized. This concern has also been expressed in recent works by Csernai and Kapusta [12], and by Buchmüller, Helbig, and Walliser [13]. Both works attempted to improve on Linde's results, by generalizations of Langer's work. Csernai and Kapusta obtained an expression for the dynamical prefactor by using a relativistic hydrodynamic approach, while Buchmüller *et al.* obtained an approximate expression for the decay rate in scalar electrodynamics (and more recently, with Z. Fodor, in the standard electroweak model) by integrating out the electromagnetic degrees of freedom from the partition function. However, a more detailed analysis of the nucleation barrier and how it compares to the usual result is still lacking.

In this paper we address the three points raised above. We will be mostly interested in the regime in which thermal fluctuations are much larger than quantum fluctuations. This way we avoid the question of how to match continuously the two regimes, although we believe this to be a very important question [14]. (See also Refs. [11] and [15].) The hope is that in most situations of interest the transition will be dominated by one or the other regime. By a saddle-point evaluation of the partition function in the case of a self-interacting scalar field, it is possible to show that the temperature corrections to the nucleation barrier can be interpreted as entropic contributions due to stable vibrational modes on the *tree-level* bounce configuration, $\varphi_b(r)$. In other words, the first corrections to the energy of the critical bubble configuration come from temperature induced stable fluctuations on the bubble, which will modify its volume and surface energies. We will show that these corrections are given by the temperature corrected effective action evaluated at the tree-level bounce $\varphi_b(r)$. In expression (1.1), the bounce is obtained from the effective potential which includes scalar loops. The difference between the two nucleation barriers will be important whenever scalar loops are not negligible. We will show that they become particularly important within the so-called thin-wall limit, that is, in the vicinity of the critical temperature for the transition. This is perfectly consistent with the fact that large entropic corrections are expected near the critical temperature. We will obtain this result by a perturbative evaluation of the determinantal prefactor. In principle, the determinantal prefactor can be evaluated in two ways. Clearly, the computation can be done directly if we know the eigenvalues related to a given bubble configuration. This method is not very useful in practice, since we in general do not know the eigenvalues. (Unless, of course, we obtain them numerically.) Writing down explicitly the eingenvalue equations, and using a thin-wall approximation to the bubble configuration, we show how the temperature corrections to the nucleation barrier originate from fluctuations about the critical bubble configuration. Even though the thin-wall approximation is not very useful in realistic situations, there is no reason to believe that thicker wall bubbles will behave any differently. (Unless the transition becomes too weak, in which case nucleation of critical bubbles may not be the relevant mechanism for the transition [17].) The second approach we use to evaluate the prefactor relies on a perturbative expansion of the determinants. Within first-order, it is again possible to show how the prefactor accounts for the temperature corrections to the nucleation barrier.

The paper is organized as follows. In Section 2 we briefly review Langer's formalism for obtaining nucleation rates, adapted to field theory at finite temperatures. That is, we obtain the partition function for the metastable phase plus a nucleating fluctuation by a saddle-point evaluation of the functional integral. In Section 3 we show how the determinantal prefactor can account for the finite temperature corrections to the nucleation barrier. For simplicity, the calculation is performed in the context of the thin-wall approximation for the bubble profile, although in principle one could obtain results for any configuration. In Sections 2 and 3, for the sake of clarity, the discussion is somewhat oversimplified. We *assume* that the system is initially in a metastable state and study only the scalar degrees of freedom in the problem. This situation is not unrealistic, as it can be reproduced in numerical simulations of vacuum decay [7]. In Section 4 we study a model of a scalar field coupled to fermions. This example is particularly interesting as it illustrates how a thermal state evolves into a metastable state due to radiative corrections, very much like in the standard electroweak model. (Recall that in a cosmological context the cooling is provided by the expansion of the Universe.) A related problem has been recently studied by E. Weinberg, in the context of massless scalar models for which symmetry breaking occurs due to radiative corrections. In the regime dominated by quantum fluctuations, Weinberg showed how radiative corrections induce a metastable state and how it is possible to evaluate its decay rate [18]. We show how to obtain an effective partition function for the scalar field by integrating out the fermionic modes, and proceed to obtain the decay rate. We then compare our results for the nucleation barrier to the results obtained using Eq. (1.1). Conclusions are presented in Section 5 and two Appendices are included to clarify a few technical points.

II. FINITE TEMPERATURE DECAY RATE: GENERAL FORMALISM

Consider a scalar field model with four-dimensional euclidean action

$$S_{\rm E} = \int d^4x_E \ \mathcal{L}_E \ , \tag{2.1}$$

where $\mathcal{L}_E$ is the euclidean lagrangian density given by

$$\mathcal{L}_E = \frac{1}{2}(\partial_\mu\phi)^2 + V(\phi) \ , \tag{2.2}$$

and the potential in (2.2) has a metastable minimum at $\phi = \varphi_f$ and a stable minimum at $\phi = \varphi_t$, as shown in Fig. 1. [We will only consider potentials with two minima here.]

Let us *assume* that the system is prepared initially in the metastable phase, without worrying for the moment about how this is done. (See Section 4.) The metastable phase will decay into the stable phase by the nucleation of bubbles larger than a critical size. (For a review see, e.g., [4].) As is well-known, in order to study the decay of the false vacuum at finite temperature we impose the periodic boundary condition (anti-periodic for fermions) $\phi(0,\mathbf{x}) = \phi(\beta\hbar,\mathbf{x})$, so that the euclidean action becomes [16]

$$S_E[\phi] = \int_0^\beta d\tau \int d^3x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] . \tag{2.3}$$

The partition function of the system is given by a functional integral over all possible field configurations weighted by their euclidean action,

$$Z = \int D\phi e^{-S_E(\phi)} . \tag{2.4}$$

Following Langer [6], we describe the nucleation of bubbles of the stable phase inside the metastable phase under the assumption that a dilute gas approximation for these droplets is valid. Unless the transition is weakly first-order, this should be a very good approximation to describe the early stages of the transition, when bubble collisions and other complicated kinetic effects can be neglected. The critical configuration is an extremum of the euclidean action,

$$\frac{\delta S_E(\phi)}{\delta \phi}|_{\phi=\varphi_b} = 0 , \tag{2.5}$$

being thus a solution of the equation of motion,

$$(\frac{\partial^2}{\partial \tau^2} + \nabla^2)\phi = V'(\phi) , \tag{2.6}$$

with boundary conditions, $\lim_{\tau \to \pm\infty} \phi(\tau, \vec{x}) = \varphi_f$, and $\lim_{|\vec{x}| \to \infty} = \varphi_f$. Coleman, Glaser, and Martin [19], have shown that the configuration with minimum energy, *i.e.*, the minimum of all the maxima, will have $O(4)$-symmetry. As argued by Linde [10], for sufficiently high temperatures the problem becomes effectively three-dimensional, and the saddle point will be given by the $O(3)$-symmetric, or static, solution of

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = V'(\phi) , \tag{2.7}$$

with boundary conditions, $\lim_{r \to +\infty} \phi = \varphi_f$ and $\frac{d\phi}{dr}|_{r=0} = 0$. Note that the potential in (2.7) is the *tree-level* potential. For future reference we note that when the false vacuum energy density $[\Delta V \equiv V(\varphi_f) - V(\varphi_t)]$ is much smaller than the barrier height [V_h, see Fig. 1], the bubble radius R is much larger than the wall thickness $\Delta R \sim m^{-1}$, where m is a typical mass scale in the problem. In this case, the solution to Eq. (2.7) can be estimated by the so-called thin-wall approximation and is given by

$$\varphi_b(r) = \begin{cases} \varphi_t, & 0 < r < R - \Delta R \\ \varphi_{wall}(r), & R - \Delta R < r < R + \Delta R \\ \varphi_f, & r > R + \Delta R \end{cases} \tag{2.8}$$

which describes a bubble of radius R of the stable phase φ_t embedded in the metastable phase φ_f. $\varphi_{wall}(r)$ describes the bubble wall separating the two phases.

The main advantage of Langer's approach is that in a dilute gas of bubbles one can infer the thermodynamics of the system from the knowledge of the partition function of a single bubble. We can write the partition function for the system with a bubble of the stable vacuum inside the metastable vacuum as

$$Z = Z(\varphi_f) + Z(\varphi_b)\,, \tag{2.9}$$

where $Z(\varphi_f)$ and $Z(\varphi_b)$ are the partition functions of the system for the vacuum field configuration φ_f and for the bubble field configuration φ_b, respectively [20]. The generalization of (2.9) for several bubbles is given by

$$\begin{aligned} Z &\simeq Z(\varphi_f) + Z(\varphi_f)\left[\frac{Z(\varphi_b)}{Z(\varphi_f)}\right] + Z(\varphi_f)\frac{1}{2!}\left[\frac{Z(\varphi_b)}{Z(\varphi_f)}\right]^2 + \ldots \\ &\simeq Z(\varphi_f)\exp\left[\frac{Z(\varphi_b)}{Z(\varphi_f)}\right]\,. \end{aligned} \tag{2.10}$$

The proof of (2.10) can be found in the work of Arnold and McLerran [21], who studied the properties of a dilute gas of sphalerons. They expressed the multiple-sphaleron configurations as the superposition of many single sphalerons, with partition function approximated as above.

The partition functions in (2.10) can be evaluated by the saddle-point method, expanding the lagrangian field in (2.4) as $\phi(\vec{x},\tau) \to \varphi_b(\vec{x}) + \eta(\vec{x},\tau)$ for $Z(\varphi_b)$ and $\phi(\vec{x},\tau) \to \varphi_f + \zeta(\vec{x},\tau)$ for $Z(\varphi_f)$. $\eta(\vec{x},\tau)$ and $\zeta(\vec{x},\tau)$ are small perturbations around the classical field configurations $\varphi_b(\vec{x})$ and φ_f, respectively. These perturbations around each configuration bring the temperature corrections to the nucleation barrier into the problem, as we shall see. Up to 1–loop order one keeps the quadratic terms in the fluctuations $\eta(\vec{x},\tau)$ and $\zeta(\vec{x},\tau)$ in the expansion of the scalar field in the partition function (2.4). This way one can write the following expressions for $Z(\varphi_b)$ and $Z(\varphi_f)$, respectively,

$$Z(\varphi_b) \overset{1-loop\, order}{\simeq} e^{-S_E(\varphi_b)} \int D\eta \exp\left\{ -\int_0^\beta d\tau \int d^3x \frac{1}{2}\eta \left[-\Box_E + V''(\varphi_b)\right] \eta \right\} \tag{2.11}$$

and

$$Z(\varphi_f) \overset{1-loop\, order}{\simeq} e^{-S_E(\varphi_f)} \int D\zeta \exp\left\{ -\int_0^\beta d\tau \int d^3x \frac{1}{2}\zeta \left[-\Box_E + V''(\varphi_f)\right] \zeta \right\} , \tag{2.12}$$

where $V''(\varphi) = \frac{d^2V(\phi)}{d\phi^2}|_{\phi=\varphi}$ and $\Box_E = \frac{\partial^2}{\partial\tau^2} + \vec{\nabla}^2$.

Performing the functional Gaussian integrals in (2.11) and (2.12) one gets the following expression for the ratio between the partition functions, $\frac{Z(\varphi_b)}{Z(\varphi_f)}$, appearing in (2.10):

$$\frac{Z(\varphi_b)}{Z(\varphi_f)} \overset{1-loop\, order}{\simeq} \left[\frac{\det(-\Box_E + V''(\varphi_b))_\beta}{\det(-\Box_E + V''(\varphi_f))_\beta}\right]^{-\frac{1}{2}} e^{-\Delta S} , \tag{2.13}$$

where $[\det(M)_\beta]^{-\frac{1}{2}} \equiv \int D\eta \exp\left\{ -\int_0^\beta d\tau \int d^3x \frac{1}{2}\eta[M]\eta \right\}$ and $\Delta S = S_E(\varphi_b) - S_E(\varphi_f)$ is the difference between the euclidean actions for the field configurations φ_b and φ_f. Note that $S_E(\varphi)$, and hence ΔS, *does not include any temperature corrections.*

From (2.10) and (2.13), the free energy of the system, $\mathcal{F} = -\beta^{-1} \ln Z$ can be written as

$$\mathcal{F} = -T \left[\frac{\det(-\Box_E + V''(\varphi_b))_\beta}{\det(-\Box_E + V''(\varphi_f))_\beta}\right]^{-\frac{1}{2}} e^{-\Delta S} . \tag{2.14}$$

As is well-known, the determinantal prefactor evaluated for the bounce configuration has one negative eigenvalue, signalling the presence of a metastable state, and also three zero eigenvalues related with the translational invariance of the bubble in three-dimensional space. Because of the negative eigenvalue, the free energy $\mathcal{F}$ is imaginary. As shown by Langer [6] (see also Ref. [11]), the decay rate is proportional to the imaginary part of $\mathcal{F}$

$$\mathcal{R} = \frac{|E_-|}{\pi T} \mathrm{Im}\mathcal{F} , \tag{2.15}$$

where $|E_-|$ is the single negative eigenvalue. In general it depends on non-equilibrium aspects of the dynamics, such as the coupling strength to the thermal bath.

III. EVALUATION OF THE DETERMINANTS

In this Section we compute the ratio of the determinants appearing in the decay rate, and show how it provides a finite temperature correction to the nucleation barrier.

First recall that for static field configurations ΔS is given by

$$\Delta S = \beta \int d^3x \left[\mathcal{L}_E(\varphi_b) - \mathcal{L}_E(\varphi_f)\right] = \frac{\Delta E}{T} \;, \tag{3.1}$$

where ΔE is simply the nucleation *energy* barrier, that is, the energy of a critical nucleation within the metastable phase. For example, in the thin-wall approximation of Eq. (2.8), ΔE is

$$\Delta E = -\frac{4\pi R^3}{3}\Delta V + 4\pi R^2 \sigma_0 \;, \tag{3.2}$$

where σ_0 is the tree-level surface tension of the bubble wall (*i.e.* with no corrections due to fluctuations around the bubble wall field configuration φ_{wall})

$$\sigma_0 \simeq \int_{-\Delta R}^{+\Delta R} dr \left[\mathcal{L}_E(\varphi_{wall}) - \mathcal{L}_E(\varphi_f)\right] \;. \tag{3.3}$$

Using (3.1) in Eqs. (2.14) and (2.15) we can write the decay rate as

$$\mathcal{R} = -\frac{|E_-|}{\pi} \mathrm{Im} \left[\frac{\det(-\Box_E + V''(\varphi_b))_\beta}{\det(-\Box_E + V''(\varphi_f))_\beta}\right]^{-\frac{1}{2}} \exp\left(-\frac{\Delta E}{T}\right) \;. \tag{3.4}$$

The determinantal prefactor in (3.4) will provide the temperature corrections to the nucleation barrier. This should come as no surprise, given that the determinant is obtained from integrating over thermally induced fluctuations about the tree-level bubble configuration. (Recall that we are only considering the regime in which thermal fluctuations are much larger than quantum fluctuations.) Once the negative and zero eigenvalues are taken care of, the positive eigenvalues are easily associated with entropic contributions to the activation energy due to stable deformations of the bubble's shape, as in classical nucleation theory.

We now proceed to show how to incorporate temperature corrections to the nucleation barrier. This can be done without an explicit evaluation of the eigenvalues of the operators in the determinants. As we show next, all that we need is to separate consistently the positive eigenvalues from the negative and zero eigenvalues, and then show how the former can be exponentiated. In principle, the computation of the determinantal prefactor in (2.14) can be performed by two different methods. The first involves obtaining directly (analytically, or more realistically, numerically) the eigenvalues for the determinants in Eq. (2.14). The second method consists in developing a consistent perturbative expansion for the ratio of the determinants. We now examine both these possibilities.

A. Evaluating the Determinantal Prefactor: Eigenvalue Equations

Consider the eigenvalue equations for the differential operators that appear in the determinantal prefactor,

$$[-\Box_E + V''(\varphi_f)]\,\Phi_f(i) = \varepsilon_f^2(i)\Phi_f(i) \tag{3.5}$$

and

$$[-\Box_E + V''(\varphi_b)]\,\Phi_b(i) = \varepsilon_b^2(i)\Phi_b(i)\ . \tag{3.6}$$

In momentum space one writes, $\varepsilon^2 = \omega_n^2 + E^2$, where $\omega_n = \frac{2\pi n}{\beta}$, $n = 0, \pm 1, \pm 2, \ldots$, for bosons (for fermion fields $\omega_n = \frac{(2n+1)\pi}{\beta}$). From (3.5) and (3.6) one can write the determinant ratio in (2.14) as

$$\begin{aligned}\left[\frac{\det(-\Box_E + V''(\varphi_f))_\beta}{\det(-\Box_E + V''(\varphi_b))_\beta}\right]^{\frac{1}{2}} &= \exp\left\{\frac{1}{2}\ln\left[\frac{\det(-\Box_E + V''(\varphi_f))_\beta}{\det(-\Box_E + V''(\varphi_b))_\beta}\right]\right\} = \\ &= \exp\left\{\frac{1}{2}\ln\left[\frac{\prod_{n=-\infty}^{+\infty}\prod_i\left(\omega_n^2 + E_f^2(i)\right)}{\prod_{n=-\infty}^{+\infty}\prod_j\left(\omega_n^2 + E_b^2(j)\right)}\right]\right\}\ .\end{aligned} \tag{3.7}$$

Using the identity,

$$\prod_{n=1}^{+\infty}\left(1 + \frac{z^2}{n^2}\right) = \frac{\sinh(\pi z)}{\pi z} \tag{3.8}$$

and taking into account that we have in the denominator of (3.7) one negative and three zero eigenvalues, one obtains (for details see Appendix A)

$$\begin{aligned}\left[\frac{\det(-\Box_E + V''(\varphi_f))_\beta}{\det(-\Box_E + V''(\varphi_b))_\beta}\right]^{\frac{1}{2}} &= \mathcal{V}\frac{T^4}{i|E_-|}\frac{\beta\frac{|E_-|}{2}}{\sin\left(\beta\frac{|E_-|}{2}\right)}\left[\frac{\Delta E}{2\pi T}\right]^{\frac{3}{2}} \\ &\times \exp\left\{\sum_i\left[\frac{\beta}{2}E_f(i) + \ln\left(1 - e^{-\beta E_f(i)}\right)\right]\right. \\ &\left. - \sum_j{}'\left[\frac{\beta}{2}E_b(j) + \ln\left(1 - e^{-\beta E_b(j)}\right)\right]\right\}\ .\end{aligned} \tag{3.9}$$

The factor $\mathcal{V}\left[\frac{\Delta E}{2\pi T}\right]^{\frac{3}{2}}$ in the right hand side of Eq. (3.9) comes from the contribution of the zero eigenvalues, which can be handled as in ref. [9], through the use of collective coordinates corresponding to the position of the bubble. $\mathcal{V}$ is the volume of three space, while the prime in $\sum_j$ is a reminder that we have excluded the negative and the three zero eigenvalues from the sum. Thus, the argument of the exponential incorporates only the contributions from the stable vibrational modes on the bubble, very much like in Langer's result [6].

Substituting Eq. (3.9) in (3.4) one obtains the following expression for the nucleation rate per unit volume, $\Gamma \equiv \mathcal{R}/\mathcal{V}$, as defined in Eq. (2.15):

$$\Gamma = \mathcal{A} T^4 \exp\left[-\frac{\Delta F(T)}{T}\right] , \tag{3.10}$$

where we have denoted by $\mathcal{A}$ the dimensionless factor

$$\mathcal{A} = \frac{1}{\pi} \frac{\frac{|E_-|}{2T}}{\sin\left(\frac{|E_-|}{2T}\right)} \left[\frac{\Delta E}{2\pi T}\right]^{\frac{3}{2}} . \tag{3.11}$$

Note that the zero-mode contribution in the prefactor depends on the energy barrier of the critical nucleation, and not on its free energy as in Eq. (1.1). In Eq. (3.10) we have incorporated the exponential contribution from the prefactor into the definition of the temperature corrected nucleation barrier, which we call $\Delta F(T)$. Within the thin-wall approximation we can write

$$\Delta F(T) = -\frac{4\pi R^3}{3} \Delta V_{\rm eff}(T) + 4\pi R^2 \sigma(T) , \tag{3.12}$$

where

$$\begin{aligned} \Delta V_{\rm eff}(T) = V(\varphi_f) - V(\varphi_t) + T \int \frac{d^3k}{(2\pi)^3} \ln\left[1 - e^{-\beta\sqrt{\vec{k}^2 + m^2(\varphi_f)}}\right] - \\ - T \int \frac{d^3k}{(2\pi)^3} \ln\left[1 - e^{-\beta\sqrt{\vec{k}^2 + m^2(\varphi_t)}}\right] \end{aligned} \tag{3.13}$$

and

$$\begin{aligned} \sigma(T) = \sigma_0 + \frac{T}{4\pi R^2} \Bigg\{ {\sum_j}' \left[\frac{\beta}{2} E_{wall}(j) + \ln\left(1 - e^{-\beta E_{wall}(j)}\right)\right] - \\ - \sum_i \left[\frac{\beta}{2} E_f(i) + \ln\left(1 - e^{-\beta E_f(i)}\right)\right] \Bigg\} . \end{aligned} \tag{3.14}$$

In Eq. (3.13) we have substituted the discrete sums by integrals over momenta. [For field configurations φ_f and φ_t, we have the continuum eigenvalues, $E_f^2 = \vec{k}^2 + m^2(\varphi_f)$ and $E_t^2 = \vec{k}^2 + m^2(\varphi_t)$, respectively, with $m^2(\varphi_f) = \frac{d^2V(\phi)}{d\phi^2}|_{\phi=\varphi_f}$ and $m^2(\varphi_t) = \frac{d^2V(\phi)}{d\phi^2}|_{\phi=\varphi_t}$.] We have omitted the usual zero-temperature ultraviolet divergent terms, $\int d^3k\sqrt{\vec{k}^2+m^2(\varphi)}$, from Eq. (3.13) since they can always be handled by the introduction of the usual counterterms [22].

In Eq. (3.14) $E_{wall}(j)$ are the eigenvalues related with the bubble wall field configuration φ_{wall}. Thus, within the thin-wall approximation, the problem is reduced to the computation of these eigenvalues for a field configuration describing the bubble wall, a non-trivial task. It is possible, however, that the temperature corrections to the surface density are negligible. For example, in the context of the QCD transition, the surface density is obtained from lattice calculations, and is shown not to be very sensitive to temperature [12].

It is easy to see from Eq. (3.13) that $\Delta V_{\rm eff}(T)$ is the usual 1-loop approximation to the finite temperature false vacuum energy density [22]. The second term in the right hand side of (3.14) comes from the finite temperature 1-loop contribution to the surface tension σ_0, due to thermal fluctuations on the bubble wall. Thus, by exponentiating the contribution from the stable modes, the activation energy for the critical bubble becomes indeed an activation free energy. Note however, that contrary to Eq. (1.1), the free-energy functional is evaluated for the tree-level bounce. In Section 4 we will compare the results obtained with the two approaches.

B. Evaluating the Determinantal Prefactor: Perturbative Expansion

A second approach to the computation of the determinantal prefactor in (3.4) consists in developing a perturbative expansion for it. First we write the determinantal ratio as

$$\left[\frac{\det(-\Box_E+V''(\varphi_f))_\beta}{\det'(-\Box_E+V''(\varphi_b))_\beta}\right]^{\frac{1}{2}} = \exp\left\{\frac{1}{2}{\rm Tr}\ \ln\left[-\Box_E+V''(\varphi_f)\right]_\beta - \right. \left. -\frac{1}{2}{\rm Tr}'\ \ln\left[-\Box_E+V''(\varphi_b)\right]_\beta\right\}\ , \qquad (3.15)$$

where we have used in (3.15) the identity $\ln\det\hat{M} = {\rm Tr}\ \ln\hat{M}$ and the prime in both sides denote that the negative and the zero modes have been omitted. (They are treated as in previous Section.)

We rewrite (3.15) as

$$\left[\frac{\det(-\Box_E + V''(\varphi_f))_\beta}{\det'(-\Box_E + V''(\varphi_b))_\beta}\right]^{\frac{1}{2}} = \exp\left\{-\frac{1}{2}\mathrm{Tr}\ \ln\Big[1 + G_\beta(\varphi_f)\,[V''(\varphi_b) - V''(\varphi_f)]\Big]\right\}\ , \tag{3.16}$$

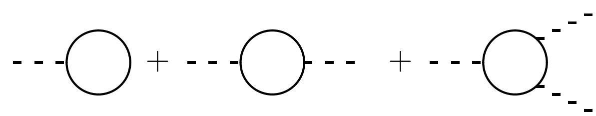
where

$$G_\beta(\varphi_f) = \frac{1}{-\Box_E + m^2(\varphi_f)} \tag{3.17}$$

is just the propagator for the scalar field ϕ, with $m^2(\varphi_f) = V''(\varphi_f)$.

Expanding the logarithm in (3.16) in powers of $G_\beta(\varphi_f)\,[V''(\varphi_b) - V''(\varphi_f)]$, we obtain the graphic representation,

$\mathrm{Tr}\ \ln\{1 + G_\beta(\varphi_f)\,[V''(\varphi_b) - V''(\varphi_f)]\} =$



$+\ \ldots\ ,$ (3.18)

where the dashed lines correspond to the background "field" $[V''(\varphi_b) - V''(\varphi_f)]$ and the internal lines denote the propagator $G_\beta(\varphi_f)$. The expression (3.18) can be written as

$$\begin{aligned}\mathrm{Tr}\ \ln\{1 + G_\beta(\varphi_f)\,[V''(\varphi_b) - V''(\varphi_f)]\} &= \sum_{m=1}^{+\infty}\frac{(-1)^{m+1}}{m}\int d^3x\,[V''(\varphi_b) - V''(\varphi_f)]^m \times \\ &\times \sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\left[\omega_n^2 + \vec{k}^2 + m^2(\varphi_f)\right]^m}\ .\end{aligned} \tag{3.19}$$

The sum in m can be performed and one obtains

$$\mathrm{Tr}\ \ln\{1 + G_\beta(\varphi_f)\,[V''(\varphi_b) - V''(\varphi_f)]\} = \int d^3x \sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\ln\left[1 + \frac{V''(\varphi_b) - V''(\varphi_f)}{\omega_n^2 + \vec{k}^2 + m^2(\varphi_f)}\right]\ . \tag{3.20}$$

This expression can be further simplified by means of the identity, ($E_b^2(\vec{k}) \equiv \vec{k} + m^2(\varphi_b)$, and $E_f^2(\vec{k}) \equiv \vec{k} + m^2(\varphi_f)$)

$$\begin{aligned}\sum_{n=-\infty}^{+\infty}\ln\left[\frac{\omega_n^2 + E_b^2(\vec{k})}{\omega_n^2 + E_f^2(\vec{k})}\right] &= \beta E_b(\vec{k}) + 2\ln\left(1 - e^{-\beta E_b(\vec{k})}\right) - \\ &- \beta E_f(\vec{k}) - 2\ln\left(1 - e^{-\beta E_f(\vec{k})}\right)\ .\end{aligned} \tag{3.21}$$

The terms proportional to β can be renormalized. We are left with the familiar temperature corrected effective potential (we neglect terms coming from zero temperature quantum corrections) [22],

$$V_{\rm eff}(\varphi,T) = V(\varphi) + T\int \frac{d^3k}{(2\pi)^3}\ln\left(1 - e^{-\beta\sqrt{\vec{k}^2+m^2(\varphi)}}\right)\ . \qquad (3.22)$$

Within the thin-wall limit, using (3.21) into (3.20), and substituting into Eq. (3.16), we obtain, from Eq. (3.4) the temperature corrected barrier,

$$\Delta F(T) = -\frac{4\pi R^3}{3}\Delta V_{\rm eff}(T) + 4\pi R^2\sigma(T)\ , \qquad (3.23)$$

where

$$\Delta V_{\rm eff}(T) = V(\varphi_f) - V(\varphi_t) - \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3}\ln\left[\frac{1 - e^{-\beta E_t(\vec{k})}}{1 - e^{-\beta E_f(\vec{k})}}\right] \qquad (3.24)$$

and

$$\sigma(T) = \sigma_0 + \frac{1}{4\pi R^2}\int d^3x \frac{1}{2\beta}\sum_{n=-\infty}^{+\infty}\int \frac{d^3k}{(2\pi)^3}\ln\left[1 + \frac{V''(\varphi_{wall}) - V''(\varphi_f)}{\omega_n^2 + \vec{k}^2 + m^2(\varphi_f)}\right]\ . \qquad (3.25)$$

Expression (3.24) for the temperature corrected false vacuum energy density can be easily identified with $\Delta V_{\rm eff}(T)$, as defined in Eq. (3.13). Also, expression (3.25) for $\sigma(T)$ can be identified with the temperature corrected surface tension of the bubble wall, by writing it as

$$\sigma(T) = \frac{1}{4\pi R^2}\left[F(\varphi_{wall},T) - F(\varphi_f,T)\right]\ , \qquad (3.26)$$

with

$$F(\varphi,T) = -\frac{1}{\beta}\ln Z(\varphi)\ , \qquad (3.27)$$

where $F(\varphi,T)$ is the free energy for the field configuration φ. $\sigma(T)$, as given by (3.26), is then the free energy difference (per unit area of the bubble) between the two configurations (φ_{wall} and φ_f), which defines the surface tension. As shown in Ref. [23], the expressions for the free energies in (3.26), at 1-loop order give (3.25).

IV. EFFECTS OF INTERACTIONS ON THE DECAY RATE

In the previous two Sections we have obtained the temperature effects on the nucleation barrier by *assuming* that the system becomes trapped in a metastable phase as the temperature drops below the critical temperature T_C. This assumption is not very realistic, and was addopted so that we could stress the main points of the calculation without having to worry about the effects of interactions of the order parameter with other fields. However, in models of interest within a cosmological context, such as the standard electroweak model and some of its extensions, the Universe becomes trapped in a metastable phase due to the interactions of the Higgs field (or effective scalar order parameter) with other massless fields, such as gauge bosons or fermions. In fact, it is possible to formulate this statement in terms of conditions that a general effective potential must satisfy, if it is to develop a metastable phase below T_C [24]. Basically, the condition states that the mass gap between the symmetric and broken-symmetric phases must be large enough so that massive fluctuations away from the symmetric minimum are suppressed. Since the mass gap is given in terms of the (temperature dependent) vacuum expectation value of the Higgs field and of its coupling to other fields, the condition states that the transition cannot be too weakly first order.

The question then is what effective potential should be considered when calculating the decay rate. As recently shown by E. Weinberg for the zero-temperature Coleman-Weinberg models (which exhibit symmetry breaking only due to radiative corrections), the effective potential relevant in the calculation of the bounce solution is obtained by integrating over all fields *but* the scalar field; the radiative corrections coming form these fields modify the vacuum structure of the model, making metastability possible. In this Section, we argue that the same procedure must be followed when calculating the nucleation rate at finite temperature. We first review, in general terms, how to take into account the interactions of the scalar field with other fields in the calculation of the decay rate. (For details see, Ref. [18].) Then we apply our method to an example involving fermions, explicitly comparing our results to those obtained from Eq. (1.1).

A. General Formalism

Let us consider a system described by a scalar field ϕ and a set of fields ξ (bosonic or fermionic fields) which are coupled to ϕ. The partition function of the system is given by

$$Z = \int D\phi D\xi e^{-S_E(\phi,\xi)} , \tag{4.1}$$

where $S_E(\phi, \xi)$ is the euclidean action of the system and the functional integration is carried over field configurations subject to periodic boundary conditions, $\phi(\vec{x}, 0) = \phi(\vec{x}, \beta)$, for bosons or antiperiodic, $\psi(\vec{x}, 0) = -\psi(\vec{x}, \beta)$, for fermion fields.

If one integrates out the ξ fields in (4.1), the partition function can be written as

$$Z = \int D\phi e^{-W^\beta(\phi)} , \tag{4.2}$$

where

$$W^\beta(\phi) = -\ln \int D\xi e^{-S_E(\phi,\xi)} . \tag{4.3}$$

$W^\beta(\phi)$ can be viewed as an effective action for the scalar field ϕ, where only ξ-loop terms are included. Note that these ξ-loop terms introduce finite temperature corrections in (4.3). $S_E(\phi, \xi)$ in (4.1) and (4.3) includes renormalization counter-terms and, if one or more of the ξ-fields is a gauge field, $S_E(\phi, \xi)$ also includes the gauge fixings and the corresponding ghost terms.

The procedure is now, in principle, straightforward. We evaluate the partition function in Eq. (4.2) semiclassically by expanding the effective action $W^\beta(\phi)$ around its extremum configuration, which will be the bounce. Note that the bounce will include the radiative corrections coming from the fields that couple to ϕ, *but not from ϕ itself.* The determinantal prefactor can be evaluated as before, by considering the negative and zero eigenvalues separately from the positive eigenvalues. However, as pointed out by Weinberg [18], $W^\beta(\phi)$ cannot always be obtained in closed form, being in general a nonlocal functional. He proposes to resolve this difficulty by considering a local action $W_0(\phi)$ which is close enough to the original one. From a derivative expansion of $W^\beta(\phi)$, $W_0(\phi)$ is found to be

$$W_0(\phi) = \int d^4x_E \, \left[\frac{1}{2}(\partial_\mu\phi)^2 + \hat{V}_{1-loop}(\phi)\right] , \tag{4.4}$$

where $\hat{V}_{1-loop}(\phi)$ includes the tree level potential $V(\phi)$ and the 1-loop contributions coming *only* from the ξ-field integration. The bounce solution can then be obtained from (4.4). $W_0(\phi)$ should be a good approximation to $W^\beta(\phi)$ as long as the typical interaction length scale set by the field(s) ξ is shorter than the scale for the nonlocality of $W^\beta(\phi)$.

In what follows we will apply the above formalism to a specific example of a scalar field coupled to fermions. For large enough Yukawa couplings, this model has been shown to satisfy the conditions for metastability specified in Ref. 24.

B. Application: Real scalar field coupled to fermion fields

Consider a model of a real scalar field ϕ coupled to fermion fields with a lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) - g\phi\bar{\psi}\psi + i\bar{\psi}\,\partial\!\!\!/\psi\,, \tag{4.5}$$

with $V(\phi)$ given by

$$V(\phi) = \frac{\lambda}{4!}\phi^4 - \frac{\alpha\mu}{3!}\phi^3 + \frac{\mu^2}{2}\phi^2\,, \tag{4.6}$$

where λ and α are positive, dimensionless constants and $\mu^2 > 0$ is a $(\text{mass})^2$ parameter. A large enough coupling to fermions guarantees that a metastable phase is possible as the system is cooled below T_C; the high temperature minimum of the 1-loop effective potential, $\langle\varphi\rangle_T \simeq \frac{\alpha}{\lambda+4g^2}\mu$, lies to the left of the maximum of the potential at $T = T_C$, and thermal fluctuations away from the symmetric minimum are suppressed. Numerical values for the couplings satisfying these conditions can be found in Ref. 24.

The partition function for this model is given by

$$Z = \int D\phi D\psi D\bar{\psi} e^{-\int_0^\beta d\tau \int d^3x \mathcal{L}(\phi,\psi,\bar{\psi})}\,. \tag{4.7}$$

As the fermion fields appear quadratically in (4.7), they can be integrated out, giving

$$Z = \int D\phi e^{-W_0(\phi)}\,, \tag{4.8}$$

where

$$W_0(\phi) = \int_0^\beta d\tau \int d^3x \left[\frac{1}{2}(\partial_\tau\phi)^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + V(\phi)\right] - \mathrm{Tr}\ln(-\,\partial\!\!\!/ - ig\phi)_\beta\,, \tag{4.9}$$

and

$$\mathrm{Tr}\ln(-\not\partial - ig\phi)_\beta = \ln\det(-\not\partial - ig\phi)_\beta \,. \tag{4.10}$$

Thus, $W_0(\phi)$ can be written as

$$W_0(\phi) = \int_0^\beta d\tau \int d^3x \left[\frac{1}{2}(\partial_\tau\phi)^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \hat{V}_\psi(\phi, T)\right] \,, \tag{4.11}$$

where, for sufficiently smooth fields (see Appendix B), the effective potential obtained after integrating over the fermions and renormalizing is (we will drop all zero temperature quantum corrections)

$$\hat{V}_\psi(\phi, T) = V(\phi) - 4T\int \frac{d^3k}{(2\pi)^3} \ln\left(1 + e^{-\beta\sqrt{\vec{k}^2 + g^2\phi^2}}\right) \,, \tag{4.12}$$

The temperature dependent term accounts for finite temperature corrections coming from fermion loops. $\hat{V}_\psi(\phi, T)$ is the potential we should use to compute the bounce. Note that, neglecting the $T = 0$ quantum corrections, the high-temperature limit of $\hat{V}_\psi(\phi, T)$ is approximately

$$\hat{V}_\psi(\phi, T) \simeq V(\phi) + \frac{T^2}{12} g^2\phi^2 \,, \tag{4.13}$$

so that for $(T/\mu)^2 > (9\alpha^2/\lambda - 24)/4g^2$, a condition which is easily satisfied for reasonable values of the couplings, the high-temperature minimum is $\langle\varphi\rangle_T \simeq 0$.

Once we have the action $W_0(\phi)$, the bounce is obtained as a solution of

$$\frac{\delta W_0(\phi)}{\delta\phi}|_{\phi=\varphi_b} = 0 \,. \tag{4.14}$$

Thus, for a static, spherically symmetric configuration, the bounce configuration $\varphi_b(r)$ will be a solution of

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \hat{V}'_\psi(\phi, T) \,, \tag{4.15}$$

with boundary conditions, $\lim_{r\to\infty}\varphi_b(r) = \varphi_f \simeq 0$ and $\frac{d\varphi_b}{dr}|_{r=0} = 0$. (From now on φ_f and φ_t should be understood as being the minima of (4.12) and not the minima of the tree-level potential $V(\phi)$.)

The procedure is now identical to that of Sections 2 and 3. Having a bounce solution we can evaluate the partition function written in Eq. (4.8) semiclassically, exactly as was done in Eqs. (2.11) and (2.12), by expanding around φ_f and φ_b. We then obtain, from Eqs. (2.14) and (2.15), the nucleation rate,

$$\mathcal{R} = -\frac{|E_-|}{\pi}\text{Im}\left[\frac{\det(-\Box_E + m_\beta^2(\varphi_b))_\beta}{\det(-\Box_E + m_\beta^2(\varphi_f))_\beta}\right]^{-\frac{1}{2}} e^{-\Delta W_0} , \tag{4.16}$$

where $m_\beta^2(\varphi) = \frac{d^2\hat{V}_\psi(\phi)}{d\phi^2}|_{\phi=\varphi}$, with $\hat{V}_\psi(\phi)$ obtained above. ΔW_0 is given by

$$\Delta W_0 = W_0(\varphi_b) - W_0(\varphi_f) , \tag{4.17}$$

where $W_0(\phi)$ was defined in Eq. (4.11).

In order to proceed, we must rewrite the determinantal prefactor explicitly isolating the negative and zero modes from the positive modes. This is done following the same steps of Section 3, although now we must handle the fermionic contribution to the determinants. The details of the perturbative expansion for the fermionic determinantal prefactor are given in Appendix B. We can then write the nucleation rate per unit volume as

$$\Gamma = \frac{T^4}{\pi}\frac{\frac{|E_-|}{2T}}{\sin\left(\frac{|E_-|}{2T}\right)}\left[\frac{\Delta W_0}{2\pi}\right]^{\frac{3}{2}} \exp\left[-\frac{\Delta F(T)}{T}\right] , \tag{4.18}$$

where $\Delta F(T)$, the bubble activation free energy in the 1-loop approximation, is given by

$$\Delta F(T) = \int d^3x \left\{\frac{1}{2}(\nabla\varphi_b)^2 + \hat{V}_\psi(\varphi_b) - \hat{V}_\psi(\varphi_f) + \frac{1}{2\beta}\sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\ln\left[1 + \frac{m_\beta^2(\varphi_b) - m_\beta^2(\varphi_f)}{\omega_n^2 + \vec{k}^2 + m_\beta^2(\varphi_f)}\right]\right\} . \tag{4.19}$$

As usual, the sum over n can be performed and we get,

$$\Delta F(T) = \int d^3x \left[\frac{1}{2}(\nabla\varphi_b)^2 + \hat{V}_{\text{eff}}(\varphi_b, T) - \hat{V}_{\text{eff}}(\varphi_f, T)\right] , \tag{4.20}$$

where the effective potential $\hat{V}_{\text{eff}}(\phi, T)$, is given by (neglecting zero temperature quantum corrections)

$$\hat{V}_{\text{eff}}(\phi, T) = V(\phi) - 4T\int\frac{d^3k}{(2\pi)^3}\ln\left(1 + e^{-\beta\sqrt{\vec{k}^2+g^2\phi^2}}\right) + \\ + T\int\frac{d^3k}{(2\pi)^3}\ln\left(1 - e^{-\beta\sqrt{\vec{k}^2+m_\beta^2(\phi)}}\right) , \tag{4.21}$$

where $V(\phi)$ is the tree level potential (4.6) and the mass term appearing in the scalar loop contribution is $m_\beta^2(\phi) \simeq V''(\phi) + \frac{T^2}{6} g^2$ in leading order in the fermion loops. It is instructive to contrast this result with that obtained for self-coupled scalars, Eq. (3.22). The coupling to fermions modifies the scalar mass propagating in the loops. This effect naturally improves the infrared behavior of the theory, and can be of importance in weak first-order transitions. We will say more about this later. Note also a crucial difference between this expression for the free energy barrier and the expression for the free energy barrier in Eq. (1.1): Here, the bounce is obtained with the effective potential that does not include the corrections coming from scalar loops. The corrections from scalar loops which appear in the last term of Eq. (4.21) are thermally induced fluctuations about the bounce solution computed with $\hat{V}_\psi(\phi, T)$. In the usual expression for the nucleation barrier the bounce is obtained from the full effective potential including the scalar loops. The two expressions are definetely not equivalent, even though for small scalar self-couplings the differences are negligible. In order to illustrate the differences let us look at a specific example.

In Fig. 2 we contrast the two approaches by comparing the nucleation barriers as a function of the temperature for a fixed set of coupling constants. The barriers in the figure were obtained by a numerical integration of the bounce equation including the relevant loops according to each approach. For clarity let us call the nucleation barrier obtained in the usual approach, *i.e.*, by including the scalar loops in the bounce calculation, the scalar barrier. The nucleation barrier obtained without including the scalar loops we call the fermionic barrier. In Fig. 2 we take $\lambda = 1.0$, $\alpha = 2.0$, and $g^2 = 0.5$. Since λ controls the strength of the scalar corrections, we expect the differences between the two barriers to be noticeable. We find that this is indeed the case, noting that as we approach the critical temperature (that is, as we move closer to the thin-wall limit) the differences between the two barriers increase, with the scalar barrier *always larger* than the fermionic barrier. This is precisely what one expects if the scalar corrections are entropic corrections to the nucleation barrier. Thus, the nucleation barrier used in expression Eq. (1.1) is overestimated for large enough scalar corrections.

Finally, we point out two additional differences between the results. First note that the contribution from the zero modes to the prefactor depends on $\Delta W_0(\phi)$, as opposed to $\Delta F(T)$. This could be important for weak transitions in which the prefactor may play a relevant rôle. Most importantly, the expression for $\hat{V}_{\rm eff}(\phi, T)$, Eq. (4.21), differs from the usual 1-loop finite temperature effective potential by the mass term for the scalar field loops, $m_\beta^2(\phi) = \hat{V}_\psi''(\phi)$. Since we have used the stationary points of $W_0(\phi)$, Eq. (4.11), as opposed to the stationary points of $S_E(\phi, \psi)$, as the effective "background" fields in the saddle-point evaluation of the partition function, the scalar field propagator carries the finite temperature mass $m_\beta^2(\phi)$. The propagator is dressed by the quantum corrections due to fermion loops. In the usual 1-loop finite temperature effective potential, the stationary points are obtained from the tree level action, with mass term for scalar loops, $m_0^2(\phi) = V''(\phi)$. This results in the usual negative mass terms related to the change in convexity of the effective potential between the inflection points, and, in the case of very shallow potentials, in bad infrared behavior near φ_f. The incorporation of the fermionic corrections to the scalar propagator, which is demanded by our method of calculation atenuates these problems. In the example above, the scalar mass gets dressed by fermionic loops, being given by $m_\beta^2(\phi) \simeq V''(\phi) + \frac{1}{6}g^2T^2$, where $V(\phi)$ is the tree-level potential (4.6). The temperature term in m_β works as the infrared regulator for small values of $m_0^2(\phi) = V''(\phi)$. This result is independent of the particular model studied. Similar conclusions have been obtained in Ref. [13] for scalar electrodynamics.

V. CONCLUSIONS

In this paper we examined in some detail the computation of false vacuum decay rates at finite temperatures in the regime in which quantum fluctuations are negligibly small compared to thermal fluctuations. We have shown that temperature corrections to the nucleation barrier can be obtained from a saddle-point evaluation of the partition function in a dilute gas approximation. In fact, the temperature corrections are simply due to the positive eigenvalues from stable fluctuations around the critical bubble. That is, they are the entropic contributions due to thermally induced deformations on the bubble.

Even though this result has been known in classical statistical mechanics for more than two decades [6], we believe that a consistent treatment within field theory is still lacking. Although we left many questions unanswered, we hope to have clarified some of the issues involved in the calculation of finite-temperature decay rates. Of particular importance is the fact that the bounce is *not* obtained from the full 1-loop corrected effective potential, but from the potential excluding the scalar loops. Thus, for a self-interacting scalar, the bounce is obtained from the tree-level potential. The full finite temperature potential appears in the exponent only after properly accounting for the positive eigenvalues of the determinantal prefactor. That is, the scalar contributions account for entropic corrections to the nucleation barrier. We obtained a temperature corrected nucleation barrier which can differ from the usual result. We showed this to be particularly true for sufficiently large scalar self-couplings in the vicinity of the critical temperature for the transition.

Also, we found that the interaction with other fields gives rise to a potential which is better behaved in the infrared. (See also Ref. [13].) This result is the finite-temperature equivalent to what E. Weinberg found for the zero-temperature case, once the integration over the other fields is performed [18].

The reader may be wondering if our results will have any consequences to current work on the electroweak phase transition. The answer depends on the Higgs mass. For a sufficiently light Higgs it is consistent to neglect the contribution from scalar loops to the effective potential. In this case, the usual estimate for the nucleation barrier is a valid approximation. However, the situation may change for a heavier Higgs. Given that the experimental lower bound on the Higgs mass is now above 60 GeV, we believe it worthwhile to study this question in more detail, keeping in mind that the transition becomes weaker as the Higgs mass increases.

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Figure Captions

Figure 1: A typical asymmetric double-well potential.

Figure 2: A comparison of the nucleation barrier as a function of temperature, in units of mass parameter μ, obtained by including (stars) and excluding (dots) scalar loops in the computation of the bounce. The parameters in the tree-level potential are $\lambda = 1.0,\ \alpha = 2.0,\ g^2 = 0.5$.

Figure 3: A comparison of the terms $g^2\phi^2(r)$ and $g\ d\phi(r)/dr$ appearing in Appendix B. The parameters in the tree-level potential are $\lambda = 1,\ \alpha = 0.56,\ g^2 = 0.5$.

APPENDIX A:

Let us show that the determinant ratio of Eq. (3.7), denoted here as R, gives (3.9). Separating the negative and zero eingevalues in the denominator of Eq. (3.7), one can write

$$R = \exp\left\{\frac{1}{2}\ln\left[\frac{\prod_{n=-\infty}^{+\infty}\prod_i \left(\omega_n^2 + E_f^2(i)\right)}{\prod_{n=-\infty}^{+\infty}(\omega_n^2+E_-^2)(\omega_n^2+E_0^2)^3\prod_j'\left(\omega_n^2+E_b^2(j)\right)}\right]\right\}\ , \tag{A1}$$

where the prime in $\prod_j$ means that the negative eigenvalue, E_-^2, and the three zero eigenvalues, E_0^2, are now excluded from the product. The term for $n=0$ in $(\omega_n^2+E_0^2)$, can be handled as in Ref. [9], resulting in the factor $\mathcal{V}\left[\frac{\Delta E}{2\pi T}\right]^{\frac{3}{2}}$ in Eq. (3.9). Separating the $n=0$ modes both in the numerator and the denominator of (A1), and using the identity (3.8), we get,

$$R=\mathcal{V}\left[\frac{\Delta E}{2\pi T}\right]^{\frac{3}{2}}\exp\left\{\left(-4+\sum_i-\sum_j{}'\right)\ln\prod_{n=1}^{+\infty}\omega_n^2-\ln\left(E_-^2\right)^{1/2}-\ln\left[\frac{\sin(\frac{\beta}{2}|E_-|)}{\frac{\beta}{2}|E_-|}\right]+\right.$$
$$+\left(\sum_j{}'-\sum_i\right)\ln\beta+\sum_i\left[\frac{\beta}{2}E_f(i)+\ln\left(1-e^{-\beta E_f(i)}\right)\right]+$$
$$\left.-\sum_j{}'\left[\frac{\beta}{2}E_b(j)+\ln\left(1-e^{-\beta E_b(j)}\right)\right]\right\}\ . \qquad \text{(A2)}$$

In the above expression we used that the negative eigenvalue can be written as $(E_-^2)^{\frac{1}{2}}=i|E_-|$. Remembering that $\sum_j'$ has four eigenvalues less than $\sum_i$, we can write

$$R=\mathcal{V}\left[\frac{\Delta E}{2\pi T}\right]^{\frac{3}{2}}\exp\left\{-4\ln\beta-\ln\left(E_-^2\right)^{1/2}-\ln\left[\frac{\sin(\frac{\beta}{2}|E_-|)}{\frac{\beta}{2}|E_-|}\right]+\right.$$
$$\left.+\sum_i\left[\frac{\beta}{2}E_f(i)+\ln\left(1-e^{-\beta E_f(i)}\right)\right]-\sum_j{}'\left[\frac{\beta}{2}E_b(j)+\ln\left(1-e^{-\beta E_b(j)}\right)\right]\right\} \qquad \text{(A3)}$$

which reduces to Eq. (3.9).

APPENDIX B:

In (4.16), the exponential term ΔW_0 can be written as (from (4.17) and (4.9))

$$\Delta W_0=[S_E(\varphi_b)-\mathrm{Tr}\ln(-\not\partial-ig\varphi_b)_\beta]-[S_E(\varphi_f)-\mathrm{Tr}\ln(-\not\partial-ig\varphi_f)_\beta]\ , \qquad \text{(B1)}$$

where $S_E(\phi)=\int_0^\beta d\tau\int d^3x\left[\frac{1}{2}(\partial_\mu\phi)^2+V(\phi)\right]$, is the classical action for the scalar field as in (4.5). The exponential in (4.16) can be written then as

$$e^{-\Delta W_0}=\left[\frac{\det(-\not\partial-ig\varphi_b)_\beta}{\det(-\not\partial-ig\varphi_f)_\beta}\right]e^{-\Delta S}\ , \qquad \text{(B2)}$$

with $\Delta S=S_E(\varphi_b)-S_E(\varphi_f)$.

If one uses the identity (which follows from charge-conjugation invariance):

$$[\det(-\not\partial - ig\varphi)]^2 = \det(-\not\partial - ig\varphi).\det(-\not\partial + ig\varphi) =$$
$$= \det\left[(-\Box_E + g^2\varphi^2)1_{4\times 4} - ig\gamma_E^\mu\partial_\mu\varphi\right] \ , \qquad \text{(B3)}$$

where $1_{4\times 4}$ is the 4×4 unit matrix, then, for $\varphi = \varphi_f$, one gets

$$\det(-\not\partial - ig\varphi_f)_\beta = \left[\det(-\Box_E + g^2\varphi_f^2)_\beta\right]^{\frac{1}{2}} \ . \qquad \text{(B4)}$$

For the determinant involving the spherically symmetric bounce φ_b, the Dirac matrix γ_E^μ in (B3) is radial and one can write

$$\gamma_r = i\begin{pmatrix} 1_{2\times 2} & 0 \\ 0 & -1_{2\times 2} \end{pmatrix} \ , \qquad \text{(B5)}$$

where $1_{2\times 2}$ denotes a 2×2 unit matrix. Then, for $\varphi_b = \varphi_b(r)$, $\det(-\not\partial - ig\varphi_b(r))_\beta$ can be written as

$$\det(-\not\partial - ig\varphi_b(r))_\beta = \det\hat\Omega^{(+)}(\varphi_b).\det\hat\Omega^{(-)}(\varphi_b) \ , \qquad \text{(B6)}$$

where

$$\hat\Omega^{(\pm)}(\varphi_b) = -\Box_E + g^2\varphi_b^2 \pm g\frac{\partial\varphi_b}{\partial r} \ . \qquad \text{(B7)}$$

Therefore, the determinants in (B2) can be written as (using that $\ln\det\hat M = \mathrm{Tr}\ln\hat M$)

$$\frac{\det(-\not\partial - ig\varphi_b)_\beta}{\det(-\not\partial - ig\varphi_f)_\beta} = \exp\left\{\mathrm{Tr}\ln(-\not\partial - ig\varphi_b)_\beta - \mathrm{Tr}\ln(-\not\partial - ig\varphi_f)_\beta\right\} =$$
$$= \exp\left\{\mathrm{Tr}\ln\left[-\Box_E + g^2\varphi_b^2 + g\frac{\partial\varphi_b}{\partial r}\right]_\beta + \mathrm{Tr}\ln\left[-\Box_E + g^2\varphi_b^2 - g\frac{\partial\varphi_b}{\partial r}\right]_\beta - \right.$$
$$\left. - \ 2\mathrm{Tr}\ln\left[-\Box_E + g^2\varphi_f^2\right]_\beta\right\} \ . \qquad \text{(B8)}$$

As in (3.16), one can write (B8) as

$$\frac{\det(-\not\partial - ig\varphi_b)_\beta}{\det(-\not\partial - ig\varphi_f)_\beta} = \exp\left\{\mathrm{Tr}\ln\left[1 + S_\beta(\varphi_f)\left[g^2(\varphi_b^2 - \varphi_f^2) + g\frac{\partial\varphi_b}{\partial r}\right]\right] + \right.$$
$$\left. + \ \mathrm{Tr}\ln\left[1 + S_\beta(\varphi_f)\left[g^2(\varphi_b^2 - \varphi_f^2) - g\frac{\partial\varphi_b}{\partial r}\right]\right]\right\} \ , \qquad \text{(B9)}$$

where, in analogy to Eq. (3.16), we introduce the propagator

$$S_\beta(\phi_f) = \frac{1}{-\Box_E + g^2\varphi_f^2} \tag{B10}$$

The argument of the exponent in the rhs of (B9) can be written as a series, analogously to Eq. (3.19):

$$\mathrm{Tr}\ln\left[1 + S_\beta(\varphi_f)\left[g^2(\varphi_b^2 - \varphi_f^2) \pm g\frac{\partial\varphi_b}{\partial r}\right]\right] = \sum_{m=1}^{+\infty}\frac{(-1)^{m+1}}{m}\int d^3x\left[g^2(\varphi_b^2 - \varphi_f^2) \pm g\frac{\partial\varphi_b}{\partial r}\right]^m \times$$
$$\times \sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\left[\bar{\omega}_n^2 + \vec{k}^2 + g^2\varphi_f^2\right]^m}\,, \tag{B11}$$

where $\bar{\omega}_n = \frac{(2n+1)\pi}{\beta}$. As before, (B11) can be expressed as a graphic expansion similar to (3.18), with the propagators $G_\beta(\varphi_f)$ replaced now by $S_\beta(\varphi_f)$ and the external lines given by $g^2(\varphi_b^2 - \varphi_f^2) + g\frac{\partial\varphi_b}{\partial r}$ or $g^2(\varphi_b^2 - \varphi_f^2) - g\frac{\partial\varphi_b}{\partial r}$.

The determinant factor in (4.16), coming from the functional integration of the scalar field, can be evaluated by the same methods of Sec. 3. In (4.16), the determinant term $\det[-\Box_E + m_\beta^2(\varphi_b)]_\beta$, with $m_\beta^2(\varphi_b) = \hat{V}''_\psi(\varphi_b)$, has a negative eigenvalue, E_-^2, associated with the instability of the critical bubble, and the three zero eigenvalues, associated with the translational invariance of the bubble. These eigenvalues can be handled as usual, giving the preexponential term in (4.18). The part of the determinant involving the positive eigenvalues can be written as an expansion exactly as in (3.19),

$$\left[\frac{\det'(-\Box_E + \hat{V}''_\psi(\varphi_b))_\beta}{\det(-\Box_E + \hat{V}''_\psi(\varphi_f))_\beta}\right]^{-\frac{1}{2}} = \exp\left\{-\frac{1}{2}\mathrm{Tr}\ \ln\Big[1 + \hat{G}_\beta(\varphi_f)\left[\hat{V}''_\psi(\varphi_b) - \hat{V}''_\psi(\varphi_f)\right]\Big]\right\}\,, \tag{B12}$$

with $\hat{G}_\beta(\varphi_f) = \frac{1}{-\Box_E + m_\beta^2(\varphi_f)}$ and

$$\mathrm{Tr}\ \ln\left\{1 + \hat{G}_\beta(\varphi_f)\left[\hat{V}''_\psi(\varphi_b) - \hat{V}''_\psi(\varphi_f)\right]\right\} = \sum_{m=1}^{+\infty}\frac{(-1)^{m+1}}{m}\int d^3x\left[\hat{V}''_\psi(\varphi_b) - \hat{V}''_\psi(\varphi_f)\right]^m \times$$
$$\times \sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\left[\omega_n^2 + \vec{k}^2 + m_\beta^2(\varphi_f)\right]^m}\,. \tag{B13}$$

The sum in m in both (B11) and (B13) can be performed as in Eq. (3.19)). Therefore, from Eqs. (B2), (B11) and (B13), we can write the relevant part of Eq. (4.16) as

$$\left[\frac{\det(-\Box_E+\hat{V}''_\psi(\varphi_b))_\beta}{\det(-\Box_E+\hat{V}''_\psi(\varphi_f))_\beta}\right]^{-\frac{1}{2}} e^{-\Delta W_0} = \left[\frac{\det(-\Box_E+\hat{V}''_\psi(\varphi_b))_\beta}{\det(-\Box_E+\hat{V}''_\psi(\varphi_f))_\beta}\right]^{-\frac{1}{2}} \frac{\det(-\not\partial - ig\varphi_b)_\beta}{\det(-\not\partial - ig\varphi_f)_\beta}\; e^{-\Delta S} =$$

$$= \mathcal{V}\frac{T^4}{i|E_-|}\frac{\beta\frac{|E_-|}{2}}{\sin\left(\beta\frac{|E_-|}{2}\right)}\left[\frac{\Delta W_0}{2\pi}\right]^{\frac{3}{2}} \exp\left\{-\Delta S + \int d^3x \sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\left[\ln\left(1+\frac{g^2(\varphi_b^2-\varphi_f^2)+g\frac{\partial\varphi_b}{\partial r}}{\bar{\omega}_n^2+\vec{k}^2+g^2\varphi_f^2}\right)+\right.\right.$$

$$\left.\left. + \ln\left(1+\frac{g^2(\varphi_b^2-\varphi_f^2)-g\frac{\partial\varphi_b}{\partial r}}{\bar{\omega}_n^2+\vec{k}^2+g^2\varphi_f^2}\right) - \frac{1}{2}\ln\left(1+\frac{m_\beta^2(\varphi_b)-m_\beta^2(\varphi_f^2)}{\omega_n^2+\vec{k}^2+m_\beta^2(\varphi_f^2)}\right)\right]\right\}, \tag{B14}$$

where ΔW_0 is given by

$$\Delta W_0 = \Delta S - \int d^3x \sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\left[\ln\left(1+\frac{g^2(\varphi_b^2-\varphi_f^2)+g\frac{\partial\varphi_b}{\partial r}}{\bar{\omega}_n^2+\vec{k}^2+g^2\varphi_f^2}\right)+\right.$$

$$\left. + \ln\left(1+\frac{g^2(\varphi_b^2-\varphi_f^2)-g\frac{\partial\varphi_b}{\partial r}}{\bar{\omega}_n^2+\vec{k}^2+g^2\varphi_f^2}\right)\right]. \tag{B15}$$

Apart from the derivative terms $\partial\varphi_b/\partial r$, the momentum integral reproduces the finite temperature corrections to the the tree-level potential appearing in ΔS. When we wrote the expression for $\Delta F(T)$ in Eq. (4.19), these terms were not included in the effective potential $\hat{V}_{\rm eff}(\phi, T)$. There are two reasons for negleting this term. First, due to the graphic expansion we used for the determinants, it is easy to see that at least at the tadpole level, their contribution cancels. Since the tadpole gives the dominant temperature contribution to the potential, terms that depend on $\partial\varphi_b/\partial r$ will be sub-dominant. Second, it is possible to explicitly compare the terms $g^2\varphi_b^2$ and $g\partial\varphi_b/\partial r$, by obtaining $\varphi_b(r)$ numerically. We have performed this comparison for the same set of parameters used in Figs. 2 and 3, and convinced ourselves that the derivative term will indeed be sub-dominant. A typical example is shown in Fig. 4. Thus, neglecting the term $\partial\varphi_b/\partial r$, we can use Eqs. (B14) and (B15) to obtain the expression for $\Delta F(T)$ in (4.19).

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