KUL-TF-03-06 MIFP-03-08 ULB-TH/03-11

April 2003 **hep-th/0304022**

Interpolating from AdS$_{D-2} \times S^2$ to AdS$_D$

S. Cucu *[1], H. Lü †[2] and J.F. Vázquez-Poritz ‡[3]

*Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven,
Celestijnenlaan 200D B-3001 Leuven, Belgium

†George P. and Cynthia W. Mitchell Institute for Fundamental Physics,
Texas A& M University, College Station, TX 77843-4242, USA

‡Physique Théorique et Mathématique, Université Libre de Bruxelles,
Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium

ABSTRACT

We investigate a large class of supersymmetric magnetic brane solutions supported by $U(1)$ gauge fields in AdS gauged supergravities. We obtain first-order equations in terms of a superpotential. In particular, we find systems which interpolate between AdS$_{D-2} \times \Omega^2$ (where $\Omega^2 = S^2$ or H^2) at the horizon and AdS$_D$-type geometry in the asymptotic region, for $4 \le D \le 7$. The boundary geometry of the AdS$_D$-type metric is Minkowski$_{D-3} \times \Omega^2$. This provides smooth supergravity solutions for which the boundary of the AdS spacetime compactifies spontaneously. These solutions indicate the existence of a large class of superconformal field theories in diverse dimensions whose renormalization group flow runs from the UV to the IR fixed-point. We show that the same set of first-order equations also admits solutions which are asymptotically AdS$_{D-2} \times \Omega^2$ but singular at small distance. This implies that the stationary AdS$_{D-2} \times \Omega^2$ solutions typically lie on the inflection points of the modulus space.

[1] Research supported in part by the Federal Office for Scientific, Technical and Cultural Affairs through the Interuniversity Attraction Pole P5/27 and the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime.

[2] Research supported in part by DOE grant DE-FG03-95ER40917.

[3] Research supported in part by the Francqui Foundation (Belgium), the Actions de Recherche Concertées of the Direction de la Recherche Scientifique - Communauté Francaise de Belgique, IISN-Belgium (convention 4.4505.86) and by a "Pole d'Attraction Interuniversitaire."

Contents

1 Introduction

Anti-de Sitter (AdS) spacetime has a natural boundary which, under the Poincaré patch, is a Minkowski spacetime. This provides a simple but non-trivial example with which the holographic principle can be tested through the AdS/CFT correspondence [1, 2, 3]. Since AdS spacetime can exist in higher-dimensions ($D_{\rm max} = 7$ in supergravities), it is worth studying the possibility of spontaneous compactification of the AdS boundary. Any Ricci-flat perturbation on the boundary is consistent with the equations of motion of pure Einstein gravity with a negative cosmological constant. It is of interest to obtain solutions in gauged supergravities whose asymptotic geometry is described by the metric

$$d\hat{s}_D^2 = e^{2kz}\, ds_{D-1}^2 + dz^2\,, \qquad (1.1)$$

where the metric ds_{D-1}^2 is not Ricci-flat. This metric cannot be Einstein for general z, since the corresponding Ricci-tensor is given by

$$\begin{aligned} \hat{R}_{ab} &= e^{-2kz}\, R_{ab} - (D-1)\, k^2\, \eta_{ab} \\ \hat{R}_{zz} &= -(D-1)\, k^2\,. \end{aligned} \qquad (1.2)$$

However, at the boundary with $z \to \infty$, the spacetime is Einstein even for non-vanishing R_{ab}, provided that it is smooth. In this paper, we consider boundary metrics of the type

$$ds_{D-1}^2 = dx^\mu\, dx_\mu + \lambda^{-2}\, d\Omega_2^2\,, \qquad (1.3)$$

where $d\Omega_2^2$ is the metric of a two-sphere S^2, hyperbolic two-plane H^2 or two-torus T^2. We will refer to the metric (1.1), with the boundary given by (1.3), as the AdS-type metric.

Brane solutions whose boundary metric is given by (1.3) have been studied in various dimensions [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. They are all supported by a set of 2-form field strengths, which are proportional to the volume of Ω^2, together with a set of scalars and a superpotential. Thus, these solutions can be viewed as the magnetic duals of the AdS black hole solutions obtained in [15, 16, 17, 18, 19, 20]. Smooth and supersymmetric solutions of this type have hitherto been limited to H^2, with the exception of a smeared NS5-brane on S^2 [10].

The case of S^2, with the radius given by λ^{-1}, provides a simple procedure for wrapping extra dimensions. This enables one to make contact between D-dimensional gauged supergravities and $(D-3)$-dimensional quantum field theories. In fact, investigations into the possibility that our $D=4$ world arises as a spontaneous compactification of a $D=6$ theory on Minkowski$_4 \times S^2$ has a long history . In 1953, Pauli attempted, in one of his many unpublished works, to obtain an $SU(2)$ Yang-Mills field from such a compactification of pure six-dimensional Einstein gravity [21]. Such a solution was later obtained in [22], where additional gauge and Higgs fields, together with a scalar potential, were added. A six-dimensional supergravity theory with such a solution was constructed in [23]. However, the string or M-theory origin of this supergravity theory remained to be understood. In this paper, we have obtained brane solutions in $D=7$ gauged supergravity whose boundary is precisely the metric (1.3). This suggests that the effective action at the boundary (with an UV cut-off for inducing gravity) of our brane solution is precisely the aforementioned six-dimensional gauged supergravity.

We use a simple method to obtain first-order equations which describe such solutions and obtain smooth solutions for which $d\Omega_2^2$ is the metric for an S^2, as well as an H^2 and T^2. We begin by performing a Kaluza-Klein reduction on S^2, keeping only the singlet of the group action $SO(3)$. The resulting $(D-2)$-dimensional theory consists of the metric and a set of scalars with a scalar potential. We find that there exists a superpotential, which enables us to obtain the first-order equations for a domain wall solution. Lifting this system back to D dimensions yields a magnetic $(D-3)$-brane.

With this method, we obtain the first-order equations for a large class of solutions, which include previously-known solutions as well as new ones. Although we have not found the most general analytical solutions, their qualitative structures can easily be investigated. In particular, numerical analysis shows that there is a large class of solutions which smoothly interpolate between $\mathrm{AdS}_{D-2} \times \Omega^2$ geometry in the horizon and AdS_D-type geometry in the asymptotic region, where Ω^2 can be S^2 or H^2 and $4 \le D \le 7$. There are also solutions which have the asymptotic geometry $\mathrm{AdS}_{D-2} \times \Omega^2$ and are singular at small distance. The stationary solutions of $\mathrm{AdS}_{D-2} \times \Omega^2$ with constant scalars and their M-theory and string theory interpretation have already been reported in a letter [24].

This paper is organized as the follows. In the next section, we re-derive magnetic brane solutions in D-dimensional AdS Einstein-Maxwell gravity by using the $(D-2)$-dimensional superpotential. In sections 3 and 4, we use the superpotential method to find single-charge and two-equal-charge branes in D-dimensional AdS gauged supergravities, respectively. In sections 5-8, we consider general multi-charge branes in various dimensions. Analytical solutions are found for the single-charge branes, whereas numerical analysis is used when there are multiple charges. The ten and eleven dimensional origins of our solutions are described in section 9. Conclusions are presented in section 10.

2 Branes in AdS Einstein-Maxwell gravity

Let us consider a D-dimensional theory with the Lagrangian

$$\hat{e}^{-1}\,\hat{\mathcal{L}} = \hat{R} - \frac{1}{2n!}\,\hat{F}^2_{(n)} + \Lambda\,, \tag{2.1}$$

where $\hat{e} = \sqrt{-\hat{g}}$, $\hat{F}_{(n)} = d\hat{A}_{(n-1)}$ and the cosmological constant is given by $-\Lambda$. We perform a dimensional reduction on an n-dimensional space with the ansatz

$$\begin{aligned} d\hat{s}^2_D &= e^{2\alpha\,\varphi}\,ds^2_{D-n} + \lambda^{-2}\,e^{2\beta\,\varphi}\,d\Omega^2_n\,,\\ \hat{F}_{(n)} &= \epsilon\,m\,\lambda^{-n}\,\Omega_{(n)}\,, \end{aligned} \tag{2.2}$$

where

$$\alpha = -\sqrt{\frac{n}{2(D-2)(D-n-2)}}\,,\qquad \beta = -\frac{(D-n-2)\,\alpha}{n}\,. \tag{2.3}$$

The reduction is consistent and the resulting $(D-n)$-dimensional Lagrangian becomes

$$e^{-1}\,\mathcal{L} = R - \tfrac{1}{2}(\partial\varphi)^2 - V(\varphi)\,, \tag{2.4}$$

where the scalar potential is given by [25]

$$V = \tfrac{1}{2}\epsilon^2 m^2 e^{2(D-n-1)\,\alpha\,\varphi} - \epsilon\,n(n-1)\,\lambda^2\,e^{\frac{2(D-2)}{n}\alpha\,\varphi} - \Lambda\,e^{2\alpha\,\varphi}\,. \tag{2.5}$$

The constant $\epsilon = 1$, -1 or 0, corresponding to $d\Omega^2_n$ being the metric for a unit n-sphere, hyperbolic n-plane or n-torus, respectively. We would like to express the scalar potential (2.5) in terms of a superpotential W. Namely,

$$V = (\frac{\partial W}{\partial\varphi})^2 - \frac{D-n-1}{2(D-n-2)}\,W^2\,. \tag{2.6}$$

We find that this to be possible only for $n=2$, in which case the superpotential is given by

$$W=\sqrt{\frac{D-2}{D-3}}(\epsilon\, m\, e^{(D-3)\,\alpha\,\varphi}+2\lambda^2\, m^{-1}\, e^{\alpha\,\varphi})\,, \tag{2.7}$$

provided that the constraint $\Lambda=\frac{2(D-1)\,\lambda^4}{(D-3)\,m^2}$ is satisfied. Note that, in the case of $\epsilon=0$ which corresponds to a 2-torus, the 2-form field strength vanishes. In fact, in this case we find that, if we do turn on $F_{(2)}$, then there is a superpotential only in eleven dimensions, which is given by

$$W=\frac{1}{2\sqrt{2}}\Big(3m\, e^{-\frac{8}{3\sqrt{7}}\,\varphi}+\Lambda\, m^{-1}\, e^{\frac{2}{\sqrt{7}}\,\varphi}\Big)\,. \tag{2.8}$$

Though a curious feature, since there is no vector field in the eleven-dimensional sector of M-theory we shall not further consider this case.

We can now construct a domain wall solution in $(D-2)$ dimensions, with the metric ansatz

$$ds_{D-2}^2=e^{2A}\, dx^\mu\, dx_\mu+dy^2\,, \tag{2.9}$$

where A and φ are assumed to depend only on the transverse coordinate y. The equations of motion are given by

$$\begin{aligned}
&\varphi''+(D-3)\,A'\,\varphi'=\frac{\partial V}{\partial\varphi}\,,\qquad A''+(D-3)\,A'^2=-\frac{V}{D-4}\,,\\
&\tfrac{1}{2}\varphi'^2-(D-3)\,(D-4)\,A'^2=V\,,
\end{aligned} \tag{2.10}$$

where a prime denotes a derivative with respect to y. Since the scalar potential can be expressed in terms of a superpotential, these second-order equations can be solved by the first-order equations

$$\varphi'=\sqrt{2}\,\frac{\partial W}{\partial\varphi}\,,\qquad A'=-\frac{1}{\sqrt{2}\,(D-4)}\,W\,. \tag{2.11}$$

We lift these equations of motion back to D-dimensions, expressed in terms of the coordinate ρ such that the solution takes the form

$$\begin{aligned}
ds_D^2 &= e^{2u}\, dx^\mu\, dx_\mu+e^{2v}\,\lambda^{-2}\, d\Omega_2^2+d\rho^2\,,\\
F_{(2)} &= \epsilon m\,\lambda^{-2}\,\Omega_{(2)}\,.
\end{aligned} \tag{2.12}$$

The first-order equations for the functions u and v are given by

$$\frac{du}{d\rho}=-\frac{2\lambda^2-\epsilon\, m^2\, e^{-2v}}{m\,\sqrt{2(D-2)(D-3)}}\,,\qquad \frac{dv}{d\rho}=-\frac{2\lambda^2+(D-3)\,\epsilon\, m^2\, e^{-2v}}{m\,\sqrt{2(D-2)(D-3)}}\,, \tag{2.13}$$

which have the solution

$$e^{2v} = e^{-\frac{4\lambda^2}{m\sqrt{2(D-2)(D-3)}}\,\rho} - \frac{(D-3)m^2\epsilon}{2\lambda^2}\,, \quad u = -\sqrt{\frac{2(D-2)}{(D-3)^3}}\frac{\lambda^2}{m}\,\rho - \frac{v}{D-3}\,. \qquad (2.14)$$

With the new coordinate $r = \lambda^{-1}\,e^{v}$, the solution can be expressed as

$$\begin{aligned} ds_D^2 &= (\lambda\,r)^2 H^{\frac{D-2}{D-3}} dx_\mu^2 + \frac{dr^2}{(\lambda\,r)^2 H^2} + r^2 d\Omega_2^2\,, \\ F_{(2)} &= \epsilon\,m\,\lambda^{-2}\Omega_{(2)} = \epsilon\sqrt{\tfrac{2(D-1)}{D-3}\,\Lambda}\,\Omega_{(2)}, \end{aligned} \qquad (2.15)$$

where $H = 1 + \frac{\epsilon\,(D-1)}{\Lambda r^2}$. It is interesting to note that, once the cosmological constant is fixed, there is no free parameter associated with the $F_{(2)}$ field strength. This is rather different from the standard brane solution, in which the charge of a supporting field strength is typically an arbitrary integration constant. In the present case, it is uniquely determined by a supersymmetric condition, which translates into the condition presented below (2.7). For vanishing ϵ, the above solution is purely gravitational with locally AdS_D geometry.

The second derivative of the function u with respect to the co-moving coordinate ρ is given by

$$\frac{d^2u}{d\rho^2} = \frac{2\epsilon\,H}{(D-2)(D-3)\,r^2}\,. \qquad (2.16)$$

We find that, for $\epsilon = 0$, $d^2u/d\rho^2 = 0$. For $\epsilon = -1$, we have $d^2u/d\rho^2 \le 0$. In both cases, the solution is regular everywhere. On the other hand, if $\epsilon = 1$, then $d^2u/d\rho^2 > 0$ and the solution is singular. Thus, there seems to be a clear correlation between the singularity structure and the sign of $d^2u/d\rho^2$. This is rather different from supersymemtric domain wall solutions where $d^2u/d\rho^2 = -(dW/d\phi)^2$ is always non-positive regardless the singularity structure [26, 27].

These solutions are among those that have already been found in [12] by solving the D-dimensional equations of motion directly [1], and the corresponding Killing spinors have been determined in [13]. The cases of the $D = 5$ magnetic string and the $D = 4$ "cosmic monopole" can be embedded in $N = 2$ gauged supergravity [4, 6, 8].

[1]To recover the notation of [12], let $\lambda = \ell$ and $m = \sqrt{\frac{2(D-2)}{D-3}}q\,\ell^2$.

3 Single-charge branes in AdS gauged supergravities

The above analysis is only applicable in gauged supergravities in $D=4$ and $D=5$, since in gauged supergravities in $D\ge 6$, the 2-form field strengths always have a dilaton coupling. In general, a 2-form field strength couples to a dilatonic scalar in the following fashion:

$$\mathcal{L}=-\tfrac{1}{4}\,e^{-a\,\phi}\,F_{(2)}^2\,, \tag{3.1}$$

where the constant a can be parameterized as [28]

$$a^2=\Delta-\frac{2(D-3)}{D-2}\,. \tag{3.2}$$

Here, Δ takes the values $4/N$ with $N=1,2,3,4$. In [29, 30], the $N=1$ solitons are considered as building blocks, while the $N=2,3,4$ solitons are considered as threshold binding states of these basic building blocks, since the field strength associated with $N>1$ can be viewed as linear combinations of those with $N=1$. In this section and the next, we consider magnetic brane solutions in AdS gauged supergravities supported by a field strength whose dilaton coupling is characterized as $\Delta=4$ and 2. Hence, they can be viewed as single-charge and two-equal-charge branes, respectively.

The relevant Lagrangian can be expressed as

$$\hat{e}^{-1}\,\hat{\mathcal{L}}=\hat{R}-\tfrac{1}{2}(\partial\phi)^2-\frac{1}{4}e^{-a_1\,\phi}\,\hat{F}_{(2)}^2-\hat{V}(\phi)\,, \tag{3.3}$$

where a_1 is given by a in (3.2). The scalar potential can be expressed in terms of a superpotential

$$\hat{V}=(\frac{\partial\hat{W}}{\partial\phi})^2-\frac{D-1}{2(D-2)}\,\hat{W}^2\,, \tag{3.4}$$

and $\hat{W}$ is given by

$$\hat{W}=2g\left(\frac{1}{a_1}\,e^{\frac{1}{2}a_1\,\phi}-\frac{1}{a_2}\,e^{\frac{1}{2}a_2\,\phi}\right). \tag{3.5}$$

The value of a_2, which can be determined by examining the gauged supergravities in $D=7,6$ and 5, is given by

$$a_1\,a_2=-\frac{2(D-3)}{D-2}\,. \tag{3.6}$$

We can reduce the theory on $d\Omega_2^2$ with the ansatz

$$\begin{aligned} ds_D^2 &= e^{2\alpha\,\varphi}\, ds_{D-2}^2 + \lambda^{-2}\, e^{2\beta\,\varphi}\, d\Omega_2^2\,,\\ F_{(2)} &= \epsilon\, m\, \lambda^{-2}\, \Omega_{(2)}\,, \qquad \alpha = -\frac{1}{\sqrt{(D-2)(D-4)}}\,, \qquad \beta = -\tfrac{1}{2}(D-4)\,\alpha\,. \end{aligned} \tag{3.7}$$

The $(D-2)$-Lagrangian is given by

$$\begin{aligned} e^{-1}\,\mathcal{L} &= R - \tfrac{1}{2}\partial\phi^2 - \tfrac{1}{2}(\partial\varphi)^2 - V\,,\\ V &= \tfrac{1}{2}\epsilon^2 m^2\, e^{-a_1\,\phi + 2(D-3)\,\alpha\,\varphi} - 2\epsilon\,\lambda^2\, e^{(D-2)\,\alpha\,\varphi} + \hat{V}\, e^{2\,\alpha\,\varphi}\,. \end{aligned} \tag{3.8}$$

As in the previous discussion, $\epsilon = 1$ for S^2 and $\epsilon = -1$ for H^2. Now it amounts to finding a superpotential W such that

$$V = (\frac{\partial W}{\partial\phi})^2 + (\frac{\partial W}{\partial\varphi})^2 - \frac{D-3}{2(D-4)}\, W^2. \tag{3.9}$$

The superpotential exists, provided that a_1 and a_2 satisfy (3.6) and that λ, g and m satisfy

$$(2\lambda^4 - m^2\, g^2)\,(D-2)\, a_1^2 - 2(D-3)\, m^2\, g^2 = 0\,. \tag{3.10}$$

The superpotential is given by

$$W = \frac{2a_1\,(D-2)\,\lambda^2\,\epsilon}{g\,(2(D-3) + a_1^2\,(D-2))}\, e^{-\frac{1}{2}a_1\,\phi + (D-3)\,\alpha\,\varphi} + e^{\alpha\,\varphi}\,\hat{W}\,. \tag{3.11}$$

It is worth noting that the existence of the superpotential only depends on the condition (3.6), rather than on any specific value of a_1.

In the case of a single-charge brane, $\Delta = 4$ and hence $a_1^2 = \frac{2(D-1)}{D-2}$. The constraint (3.10) becomes

$$\lambda^2 = \sqrt{2}\, a_1^{-1}\, m\, g\,, \tag{3.12}$$

and the superpotential reduces to

$$W = \tfrac{1}{\sqrt{2}}\,\epsilon\, m\, e^{-\frac{1}{2}a_1\,\phi + (D-3)\,\alpha\,\varphi} + \, e^{\alpha\,\varphi}\,\hat{W}\,. \tag{3.13}$$

The first-order equations for the $(D-2)$-dimensional system are therefore given by

$$\varphi' = \sqrt{2}\,\frac{\partial W}{\partial\varphi}\,, \qquad \phi' = \sqrt{2}\,\frac{\partial W}{\partial\phi}\,, \qquad A' = -\frac{1}{\sqrt{2}\,(D-4)}\, W\,. \tag{3.14}$$

Having obtained the first-order equations for the $D-2$-dimensional domain walls, it is of interest to lift these equations to D dimensions so that they correspond to magnetic branes. The brane solutions have the structure

$$\begin{aligned} ds_D^2 &= e^{2u}\, dx^\mu\, dx^\nu\, \eta_{\mu\nu} + e^{2v}\, \lambda^{-2}\, d\Omega_2^2 + d\rho^2\,, \\ F_{(2)} &= \epsilon\, m\, \lambda^{-2}\, \Omega_{(2)} = \frac{\epsilon\, a_1}{\sqrt{2}\, g}\, \Omega_{(2)}\,, \end{aligned} \tag{3.15}$$

where the functions u and v and the dilaton ϕ depend only on the coordinate ρ. They satisfy the first-order equations

$$\begin{aligned} \frac{d\phi}{d\rho} &= \sqrt{2}\Big(-\tfrac{1}{2\sqrt{2}}\epsilon\, m\, a_1\, e^{-\frac{1}{2}a_1\,\phi - 2v} + \frac{d\hat{W}}{d\phi}\Big)\,, \\ \frac{dv}{d\rho} &= -\frac{1}{\sqrt{2}\,(D-2)}\,\Big(\tfrac{1}{\sqrt{2}}\epsilon\, m\,(D-3)\, e^{-\frac{1}{2}a_1\,\phi - 2v} + \hat{W}\Big)\,, \\ \frac{du}{d\rho} &= \frac{1}{\sqrt{2}\,(D-2)}\,\Big(\tfrac{1}{\sqrt{2}}\epsilon\, m\, e^{-\frac{1}{2}a_1\,\phi - 2v} - \hat{W}\Big)\,. \end{aligned} \tag{3.16}$$

Note that the (ϕ, v) fields form a closed system. (3.16) can be solved explicitly by making the coordinate transformation $d\rho = \mathrm{e}^{-\frac{a_2}{2}\phi}\, dy$. Defining

$$F \equiv \mathrm{e}^{\frac{1}{2}(a_2 - a_1)\phi}\,, \quad G \equiv \mathrm{e}^{\frac{1}{2}(a_1 + a_2)\phi + 2v}\,, \tag{3.17}$$

we find that the first two equations in (3.16) yield

$$G' + \epsilon\, m + \gamma\, G = 0\,, \tag{3.18}$$

where $\gamma = \frac{4\sqrt{2}\, g}{(D-3)a_1}$. The solution is

$$G = \mathrm{e}^{-\gamma\, y} - \frac{\epsilon\, m}{\gamma}\,. \tag{3.19}$$

We have absorbed a trivial integration constant by a constant shift in the coordinate y. The dilaton equation of motion gives

$$\frac{1}{a_1 - a_2}\frac{F'}{F} = \frac{\epsilon\, m\, a_1}{4G} + \frac{g}{\sqrt{2}}(1 - F^{-1})\,. \tag{3.20}$$

Plugging in G we find that, for $D \neq 5$,

$$F = \frac{\mathrm{e}^{-\gamma\, y} - \frac{D-3}{D-5}\frac{\epsilon\, m}{\gamma} + c_1\, \mathrm{e}^{\frac{1}{2}(D-5)\,\gamma\, y}}{G}\,. \tag{3.21}$$

This yields

$$e^{\frac{1}{2}(a_1-a_2)\phi} = \frac{e^{-\gamma\, y} - \frac{\epsilon\, m}{\gamma}}{e^{-\gamma\, y} - \left(\frac{D-3}{D-5}\right)\frac{\epsilon\, m}{\gamma} + c_1\, e^{\frac{1}{2}(D-5)\,\gamma\, y}},$$

$$e^{2u} = c_2\, e^{-\gamma\, y} e^{-\frac{1}{2}(a_1+a_2)\phi}\,, \quad e^{2v} = (e^{-\gamma\, y} - \frac{\epsilon\, m}{\gamma}) e^{-\frac{1}{2}(a_1+a_2)\phi}\,. \tag{3.22}$$

The metric of the solution can be expressed as

$$ds_D^2 = (g\, r)^2\, H^{-\frac{1}{D-2}} \Big(dx_\mu^2 + (1 - \frac{\epsilon\, m}{\gamma\,(g\, r)^2})\, \lambda^{-2}\, d\Omega_2^2\Big) + H^{\frac{D-3}{D-2}}\, \frac{4dr^2}{\gamma^2\, r^2}\,, \tag{3.23}$$

where

$$H = \frac{1 - \frac{\epsilon\, m}{\gamma\,(g\, r)^2}}{1 - \frac{D-3}{D-5}\,\frac{\epsilon\, m}{\gamma\,(g\, r)^2} + \frac{c_1}{(g\, r)^{D-3}}}\,, \tag{3.24}$$

and the dilaton is given by

$$e^{\frac{1}{2}(a_1-a_2)\phi} = H, \tag{3.25}$$

with the new coordinate $r = g^{-1}\, e^{-\gamma\, y/2}$. This solution is valid for $D \neq 5$; we shall present the $D = 5$ solution momentarily. To understand the solution better, it is instructive to write the function $H = \widetilde{H}/W$, where the definitions of $\widetilde{H}$ and W can be straightforwardly read off from (3.24). Then the metric can be expressed as

$$ds_D^2 = \widetilde{H}^{-\frac{1}{D-2}}\, W^{\frac{1}{D-2}}\, \Big[(g\, r)^2\, (dx^\mu\, dx_\mu + \widetilde{H}\, \lambda^{-2}\, d\Omega_2^2) + \widetilde{H}\, W^{-1}\, \frac{4dr^2}{\gamma^2\, r^2}\Big]\,. \tag{3.26}$$

Thus, the solution can be viewed as an intersection of a domain wall, characterized by the function H, and a $(D-3)$-brane, characterized by the function $\widetilde{H}$.

Clearly, the asymptotic infinite region of the metric is $r \to \infty$, in which case $H \to 1$ and the metric behaves as

$$ds_D^2 = (g\, r)^2\, (dx^\mu\, dx^\mu + \lambda^{-2}\, d\Omega_2^2) + \frac{dr^2}{\gamma^2\, r^2}\,. \tag{3.27}$$

If $\epsilon = 0$, in which case $d\Omega_2^2$ is the metric of a 2-torus, then the above metric describes locally AdS$_D$ spacetime. For $\epsilon = \pm 1$, the above metric can be viewed as a domain wall wrapped on Ω^2.

The full solution with $\epsilon = 0$, also obtained in [31], describes a domain wall. This can be viewed as a distribution of branes from the ten or eleven-dimensional point of view. Hence, this solution corresponds to the Coulomb branch of the superconformal Yang-Mills theory on the boundary [31, 32, 33].

For $\epsilon = -1$, the metric is regular everywhere provided that the constant $c_1 = 0$. The corresponding metric interpolates between $\mathrm{AdS}_{D-2} \times H^2$ at small distance to the AdS_D-type metric (3.27) at large distance. When c_1 is non-vanishing, the c_1 term can dominate at small distance and hence the metric can become singular.

For $\epsilon = 1$, the solution is always singular at small distance. The singularity can be naked if H vanishes at the singularity but can coincide with the horizon if H is divergent at the singularity. The nature of the singularity depends on the specific value of the constant c_1.

Let us now look at the examples of $D = 7, 6, 5$ and 4 in detail.

3.1 3-brane in $D = 7$

This solution was also obtained in [9]. In this case, the dilaton and the function H are given by

$$
\mathrm{e}^{\sqrt{\frac{5}{3}}\phi} = H = \frac{1 - \frac{\epsilon\, m}{\gamma\,(g\,r)^2}}{1 - \frac{2\epsilon\, m}{\gamma\,(g\,r)^2} + \frac{c_1}{(g\,r)^4}}\,. \tag{3.28}
$$

For $\epsilon = -1$ and $c_1 = 0$, the metric is regular everywhere, which approaches $\mathrm{AdS}_5 \times H^2$ at small distance. For $c_1 \neq 0$, the metric has a singularity which is naked if $c_1 > 0$ and which coincides with a horizon if $c_1 < 0$. The structure of the singularity is qualitatively the same as the domain wall singularity corresponding to $\epsilon = 0$.

If we set $c_1 = 0$ and rescale the coordinates as the following:

$$
g\,r \to k\,g\,r\,, \qquad x^\mu \to \tfrac{1}{k}\,x^\mu\,, \tag{3.29}
$$

then by sending $k \to 0$ we obtain the geometry $\mathrm{AdS}_5 \times H^2$ with a constant dilaton. If we instead take $c_1 = k^2\,\tilde{c}_1\,m/\gamma$ then, after sending $k \to 0$, we obtain the solution

$$
\begin{aligned}
ds_7^2 &= \frac{(g\,r)^2}{H^{1/5}}\,dx^\mu\,dx_\mu + \frac{m}{\gamma\,\lambda^2 H^{1/5}}\,d\Omega_2^2 + H^{4/5}\,\frac{4dr^2}{\gamma^2\,r^2}\,,\\
H^{-1} &= 2 + \frac{\tilde{c}_1}{(g\,r)^2}\,.
\end{aligned} \tag{3.30}
$$

When $r \to \infty$, the metric becomes $\mathrm{AdS}_5 \times H^2$, where AdS_5 approaches the boundary. Depending on the sign of $\tilde{c}_1$, the metric approaches a singularity at $r = 0$ or at some finite value $r = r_0$. In both cases, the singularity coincides with a horizon. However, for $\tilde{c}_1 > 0$ the metric components of H^2 diverge while, for $\tilde{c}_1 < 0$, they vanish.

This is rather different from our previous solution (3.28) with $c_1 = 0$, in that the structure of $\mathrm{AdS}_5 \times H^2$ occurs at the asymptotic region instead of at the horizon. It is rather interesting that the $\mathrm{AdS}_5 \times H^2$ can be in both the IR region, as in (3.28), and in the UV region, as in (3.30). This indicates that the stationary solution $\mathrm{AdS}_5 \times H^2$ lies on the inflection point of the modulus space.

For $\epsilon = 1$, the function H has a simpler form when $c_1 = c_1^*$, where $c_1^* = m^2/\gamma^2$. In this case,

$$H = \Big(1 - \frac{m}{\gamma\,(g\,r)^2}\Big)^{-1} . \tag{3.31}$$

Thus, the singularity at small distance coincides with the horizon. Clearly, for $c_1 \le c_1^*$, the denominator of H in (3.28) approaches zero first and hence the singularity coincides with the horizon. On the other hand, for $c_1 > c_1^*$ the numerator of H in (3.28) approaches zero first and hence the singularity is naked.

3.2 Membrane in $D = 6$

In this case, the dilaton and the function H are given by

$$\mathrm{e}^{\sqrt{\frac{8}{5}}\phi} = H = \frac{1 - \frac{\epsilon\,m}{\gamma\,(g\,r)^2}}{1 - \frac{3\epsilon\,m}{\gamma\,(g\,r)^2} + \frac{c_1}{(g\,r)^3}} . \tag{3.32}$$

The singularity structure is similar to that of $D = 7$. For $\epsilon = -1$ and $c_1 = 0$, we recover a smooth solution that goes to $\mathrm{AdS}_4 \times H^2$ in the IR limit. For $c_1 > 0$, the denominator of H in (3.32) is strictly positive and we have a naked singularity. On the other hand, for $c_1 < 0$ the singularity is located on the horizon.

If we set $c_1 = 0$ and perform the rescaling (3.29), then for $k \to 0$ we obtain the geometry $\mathrm{AdS}_4 \times H^2$ with a constant dilaton. If we instead take $c_1 = k\,\tilde{c}_1\,m/\gamma$, then after sending $k \to 0$ we obtain the solution

$$\begin{aligned} ds_6^2 &= \frac{(g\,r)^2}{H^{1/4}}\,dx^\mu\,dx_\mu + \frac{m}{\gamma\,\lambda^2 H^{1/4}}\,d\Omega_2^2 + H^{3/4}\,\frac{4dr^2}{\gamma^2\,r^2} , \\ H^{-1} &= 3 + \frac{\tilde{c}_1}{g\,r} , \end{aligned} \tag{3.33}$$

which approaches $\mathrm{AdS}_4 \times H^2$ in the asymptotic region, where AdS_4 approaches the boundary. The short-distance behavior is dependent on $\tilde{c}_1$. More precisely, while the singularity coincides with the horizon for positive and negative $\tilde{c}_1$, for $\tilde{c}_1 > 0$ the length parameter of H^2 diverges at the singularity. On the other hand, for $\tilde{c}_1 < 0$ it vanishes. Thus, as in the case of $D=7$, the $\mathrm{AdS}_4\times H^2$ solution appears in both the IR and UV regimes.

Finally, the case of $\epsilon=1$ can be described in terms of the parameter $c_1^*=2(m/\gamma)^{3/2}$. For $c_1>c_1^*$ there is a naked singularity while, for $c_1\le c_1^*$, the singularity coincides with a horizon.

3.3 String in $D=5$

One can find the string solution in a manner analogous to the previous cases, with the coordinate transformation $d\rho=\mathrm{e}^{\frac{\phi}{\sqrt{6}}}\,dy$. The resulting metric has the form (3.26) for $D=5$ but with the dilaton and the function H given by

$$\mathrm{e}^{\sqrt{\frac{3}{2}}\phi}=H=\frac{1-\frac{\epsilon\,m}{\sqrt{3}\,g^2r}}{1+(c_1-\frac{\epsilon\,m}{\sqrt{3}\,g}\log(g\,r))\,(g\,r)^{-1}}\,, \tag{3.34}$$

where $g\,r=\mathrm{e}^{-\sqrt{3}g\,y}$. This solution was obtained and analyzed in [9]. Here, we make a further observation for $\epsilon=-1$ that if we make the rescaling (3.29), accompanied by $c_1=(\tilde{c}_1-\log k)m/(\sqrt{3}g)$, then after sending $k\to 0$ the function H becomes $H^{-1}=\tilde{c}_1+\log(g\,r)$ and the metric (3.23) becomes

$$ds_5^2=\frac{(g\,r)^2}{H^{1/3}}\,dx^\mu\,dx_\mu+\frac{m}{\gamma\,\lambda^2\,H^{1/3}}\,d\Omega_2^2+H^{2/3}\,\frac{4dr^2}{\gamma^2\,r^2}\,. \tag{3.35}$$

3.4 Black hole in $D=4$

In this case we have

$$\mathrm{e}^{\sqrt{\frac{4}{3}}\phi}=H=\frac{1-\frac{\epsilon\,m}{\gamma\,(g\,r)^2}}{1+\frac{c_1}{g\,r}+\frac{\epsilon\,m}{\gamma\,(g\,r)^2}}\,. \tag{3.36}$$

The metric is clearly singular regardless of the sign of ϵ. For $\epsilon=-1$, the singularity always coincides with the horizon. For $\epsilon=1$, the singularity is naked if $c_1>c_1^*\equiv -2(m/\gamma)^{1/2}$ but coincides with the horizon if $c_1\le c_1^*$.

For $\epsilon=-1$, we can make the rescaling (3.29) accompanied by $c_1=m\,\tilde{c}_1/(k\,\gamma)$. Then, after sending $k\to 0$, the function $H^{-1}=\tilde{c}_1\,g\,r-1$ and the metric (3.23) becomes

$$ds_4^2=-\frac{(g\,r)^2}{H^{1/2}}\,dt^2+\frac{m}{\gamma\,\lambda^2\,H^{1/2}}\,d\Omega_2^2+H^{1/2}\,\frac{4dr^2}{\gamma^2\,r^2}\,. \tag{3.37}$$

Thus, our solution is more general than the previously-known AdS black hole [19, 20].

4 Two-equal-charge branes in AdS gauged supergravities

For the case of a brane with two equal charges, $\Delta = 2$ and hence $a_1^2 = 2/(D-2)$. The constraint (3.10) becomes

$$\lambda^2 = a_1^{-1}\, m\, g\,, \tag{4.1}$$

and the superpotential (3.11) becomes

$$W = \epsilon\, m\, e^{-\frac{1}{2}a_1\,\phi + (D-3)\,\alpha\,\varphi} + e^{\alpha\,\varphi}\,\hat{W}\,. \tag{4.2}$$

The first-order equations for the $(D-2)$-dimensional system are therefore given by (3.14). Having obtained the first-order equations for the domain walls, it is of interest to lift these equations back to give rise to the first-order equations for the magnetic branes, whose structure is given by (3.15). The higher-dimensional equations of motion are given by

$$\begin{aligned}
\frac{d\phi}{d\rho} &= \sqrt{2}\Big(-\tfrac{1}{2}\epsilon\, m\, a_1\, e^{-\frac{1}{2}a_1\,\phi - 2v} + \frac{d\hat{W}}{d\phi}\Big)\,,\\
\frac{dv}{d\rho} &= -\frac{1}{\sqrt{2}\,(D-2)}\Big(\epsilon\, m\,(D-3)\, e^{-\frac{1}{2}a_1\,\phi - 2v} + \hat{W}\Big)\,,\\
\frac{du}{d\rho} &= \frac{1}{\sqrt{2}\,(D-2)}\Big(\epsilon\, m\, e^{-\frac{1}{2}a_1\,\phi - 2v} - \hat{W}\Big)\,.
\end{aligned} \tag{4.3}$$

Note that the (ϕ, v) fields form a closed system. We can find exact solutions of this system only in particular cases. In the case of $D = 4$, the system can be solved exactly but the form of the solution is not very illuminating [11]. However, a simple partial solution for $D = 4$ is given by

$$\phi - 2v = 4g\,y\,, \qquad \tfrac{d\rho}{dy} = \mathrm{e}^{-\phi/2}. \tag{4.4}$$

4.1 Domain wall solution

For $\epsilon = 0$, and therefore vanishing flux, the first-order equations (4.3) can be solved straightforwardly by making the coordinate transformation

$$d\rho = dx\, \mathrm{e}^{\frac{D-4}{4}a_1\,\phi}\,. \tag{4.5}$$

After applying a second coordinate transformation, $(g\,r)^{\frac{D-3}{2}} = \sinh(-\frac{g}{\sqrt{2}a_1}x)$, the solution can be expressed as

$$ds_D^2 = (g\,r)^2 H^{\frac{2}{(D-3)(D-2)}}(dx_\mu^2 + \lambda^{-2}d\Omega_2^2) + \frac{dr^2}{(g\,r)^2 H^{\frac{2}{D-2}}},$$
$$e^\phi = H^{a_1}, \tag{4.6}$$

where $H = 1+1/(g\,r)^{D-3}$ and r has absorbed a constant factor. In the above, we have used the convention that $g < 0$. This is a domain wall solution which was obtained in [31].

4.2 Asymptotic boundary region

For $\epsilon = \pm 1$, we do not find analytical solutions. Here, we shall present the asymptotic behavior at the boundary of the metric. Making the coordinate transformation $d\rho = dy\, e^u$, we express the metric (3.15) as

$$ds_D^2 = e^{2u}(dx^\mu dx^\nu \eta_{\mu\nu} + dy^2) + e^{2v}\lambda^{-2}d\Omega_2^2. \tag{4.7}$$

When $y \to 0$ the boundary conditions are $u(y) \sim v(y) \sim -\log y$, and $\phi \to 0$. The solution of (4.3) can be solved near $y = 0$ as an expansion. For $D \neq 5$, the result is

$$\begin{aligned}
e^{2u} &= \frac{(D-3)^2}{(D-2)\,g^2\,y^2}\left(1 + \frac{2\,\epsilon\,m\,g}{3c^2\,(D-3)\,\sqrt{2(D-2)}}\,y^2 + \cdots\right), \\
e^{2v} &= \frac{(D-3)^2\,c^2}{(D-2)\,g^2\,y^2}\left(1 - \frac{\epsilon\,m\,g\,(3D-8)}{3c^2\,(D-3)\,\sqrt{2(D-2)}}\,y^2 + \cdots\right), \\
e^{\phi} &= 1 + \frac{2\epsilon\,m\,g}{c^2\,(D-3)(D-5)}\,y^2 + \cdots,
\end{aligned} \tag{4.8}$$

where c is an arbitrary integration constant which measures the relative scale size of $d\Omega_2^2$ and $dx^\mu\,dx_\mu$. When $D = 5$, the solution of (4.3) is given by the expansion

$$\begin{aligned}
e^{2u} &= \frac{4}{3g^2\,y^2}\,(1 + \frac{\epsilon\,m\,g}{3\sqrt{6}\,c^2}\,y^2 + \cdots), \\
e^{2v} &= \frac{4c^2}{3g^2\,y^2}\,(1 - \frac{7\epsilon\,m\,g}{6\sqrt{6}\,c^2}\,y^2 + \cdots), \\
\phi &= -\tfrac{1}{2}c^{-2}\,\epsilon\,m\,g\,y^2\,\log(y)\,\cdots.
\end{aligned} \tag{4.9}$$

In this case, the field ϕ is dual to an operator of dimension $\Delta = 2$ [9]. The solutions of the two-dimensional wave equation go as r^2 and $r^2\log r$. Notice that ϕ has the second solution only for $D=5$, which is the non-normalizable mode associated with the operator dual to the field ϕ turned on.

Having obtained the boundary solution, it is of interest to examine how the solution flows into the bulk region. In the case of $\epsilon=1$, since there is no $\mathrm{AdS}_{D-2}\times S^2$ fixed-point in the system, the solution can only flow into a singularity in the bulk. For the case of $\epsilon=-1$, there is a $\mathrm{AdS}_{D-2}\times H^2$ fixed-point and it is natural to expect that the solution flows into this fixed-point in the bulk. We shall examine this in the following.

4.3 $\mathrm{AdS}_{D-2}\times H^2$

For $\epsilon=-1$, there are fixed-point solutions with constant v and ϕ for $D\ge 5$; they are given by

$$
\begin{aligned}
\mathrm{e}^{-2v} &= \frac{2g}{m\,a_1\,(D-4)}\Big(\frac{D-4}{D-3}\Big)^{a_1^2}\,,\qquad \mathrm{e}^{\phi}=\Big(\frac{D-4}{D-3}\Big)^{a_1}\,,\\
u &= -\frac{\sqrt{2}g}{a_1\,(D-4)}\Big(\frac{D-4}{D-3}\Big)^{a_1^2/2}\rho\,,
\end{aligned}
\tag{4.10}
$$

which have the geometry $\mathrm{AdS}_{D-2}\times H^2$. For $D=5$, this was found in [9]. For $\epsilon=1$, such a solution would be complex.

4.4 Interpolating from $\mathrm{AdS}_{D-2}\times H^2$ to AdS_D

It is of interest to study whether these fixed-point solutions lie in the IR or UV region. If $\mathrm{AdS}_{D-2}\times H^2$ lies in the IR region, then it can smoothly flow to the UV region of the AdS_D-type solution (4.9). On the other hand, if $\mathrm{AdS}_{D-2}\times H^2$ lies in the UV region, then it must instead flow into a singularity.

We study this issue by examining the flow of the solution using the Taylor expansion method. If we assume that there exists a solution which is $\mathrm{AdS}_{D-2}\times H^2$ at the asymptotic boundary, then we can Taylor expand to find the next order in the solution slightly away from the boundary. We find the following:

$$
\begin{aligned}
e^{2u} &= \frac{(D-4)^{\frac{2(D-3)}{D-2}}\,(D-3)^{\frac{2}{D-2}}}{(D-2)\,g^2\,y^2}\,(1+c\,y^n+\cdots)\,,\\
e^{2v} &= \frac{m\,(D-4)^{\frac{D-4}{D-2}}\,(D-3)^{\frac{2}{D-2}}}{\sqrt{2(D-2)}\,g}\,(1-\tfrac12 c\,(D-4)\,y^n+\cdots)\,,
\end{aligned}
$$

$$e^{\frac{\phi}{\sqrt{2(D-2)}}} = \Big(\frac{D-4}{D-3}\Big)^{\frac{1}{D-2}}\Big(1 + \tfrac{1}{8}c\,(D^2-3D-2+(D-2)\sqrt{D^2-2D+7})\,y^n+\cdots\Big)\,, \quad (4.11)$$

where $n=\frac{1}{2}(D-5+\sqrt{D^2-2D-7})$ and the coordinate y is defined by $d\rho = e^u\,dy$. The constant c is a free parameter determining how fast the solution flows away from the AdS$_{D-2}$ boundary. The solution will flow into a singularity in the IR region.

Now, we will instead assume that there exists a solution with AdS$_{D-2}\times H^2$ on its horizon. Then, expanding away from the horizon yields

$$\begin{aligned}
e^{2u} &= (D-4)^{-\frac{2(D-3)}{D-2}}\,(D-3)^{\frac{2}{D-2}}\,(D-2)\,g^2\,r^2\,(1+c\,r^n+\cdots)\,,\\
e^{2v} &= \frac{m\,(D-4)^{\frac{D-4}{D-2}}\,(D-3)^{\frac{2}{D-2}}}{\sqrt{2(D-2)}\,g}\,(1-\tfrac{1}{2}c\,(2D-7+\sqrt{D^2-2D-7})\,r^n+\cdots)\,,\\
e^{\frac{\phi}{\sqrt{2(D-2)}}} &= \Big(\frac{D-4}{D-3}\Big)^{\frac{1}{D-2}}\Big(1 + \tfrac{1}{8}c\,(D^2-5D+(D-4)\sqrt{D^2-2D+7})\,r^n+\cdots\Big)\,, \qquad (4.12)
\end{aligned}$$

where $n=\frac{1}{2}(5-D+\sqrt{D^2-2D-7})$ and the coordinate r is defined by $d\rho = e^{-u}\,dr$. The constant c is again a free parameter determining how fast the solution flows away from the AdS$_{D-2}$ horizon. In fact, the solution smoothly flows to the AdS$_D$-type boundary solution (4.9). We verify this numerically by using (4.12) as our initial data. For example, Fig. 1 shows the functions e^u, e^v and $e^{\phi/\sqrt{10}}$ for $D=7$. We see that the dilaton, although not constant, is finite. The functions e^u and e^v become linearly dependent on the coordinate when the coordinate increases as governed by (4.9) with $y\to 1/y$.

To conclude, we find that for two-equal-charge branes there are two types of solutions. The first type has AdS$_{D-2}\times H^2$ in the UV boundary region but is singular in the IR region. The second type interpolates between the AdS$_D$-type solution in the UV region (4.9) and the AdS$_{D-2}\times H^2$ solution in the IR region.

5 Two-charge 3-brane in $D=7$

The maximal gauged supergravity in $D=7$ has $SO(5)$ gauge fields. We consider the truncation to the two diagonal $U(1)$ subsector. The relevant $D=7$ Lagrangian is

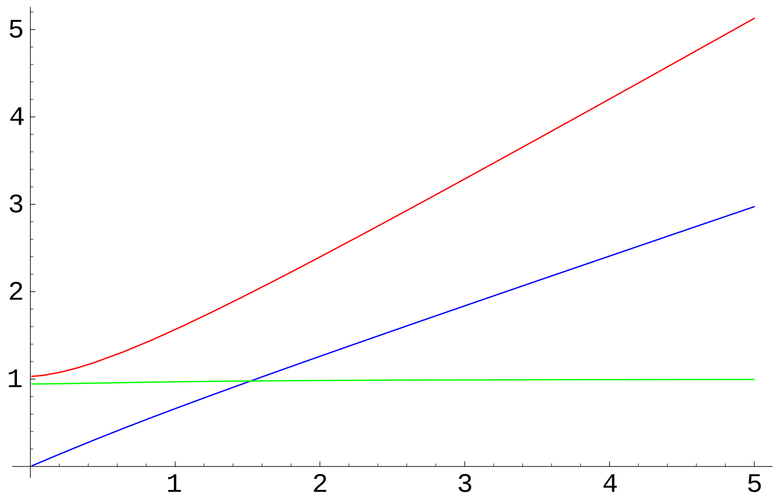

Figure 1: e^u (blue), e^v (red) and ϕ (green) for a smooth solution that runs from $\mathrm{AdS}_5 \times H^2$ at the horizon to an AdS_7-type geometry in the asymptotic region.

given by

$$\hat{e}^{-1}\mathcal{L}_7 = \hat{R} - \tfrac{1}{2}(\partial\phi_1)^2 - \tfrac{1}{2}(\partial\phi_2)^2 - \hat{V} - \tfrac{1}{4}\sum_{i=1}^{2} X_i^{-2}\,(\hat{F}^i_{(2)})^2\,, \tag{5.1}$$

where $X_i = e^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = (\sqrt{2}\,, \sqrt{\tfrac{2}{5}})\,, \qquad \vec{a}_2 = (-\sqrt{2}\,, \sqrt{\tfrac{2}{5}})\,. \tag{5.2}$$

The scalar potential $\hat{V}$ is given by [20]

$$\hat{V} = g^2\,(-4X_1\,X_2 - 2X_0\,X_1 - 2X_0\,X_2 + \tfrac{1}{2}X_0^2)\,, \tag{5.3}$$

where $X_0 = (X_1\,X_2)^{-2}$. The potential can be expressed in terms of a superpotential given by

$$\hat{W} = \frac{g}{\sqrt{2}}\,(X_0 + 2X_1 + 2X_2)\,. \tag{5.4}$$

We can consistently reduce the theory on the S^2 with the same metric ansatz as before but specializing to $D = 7$. The ansatz for the two $U(1)$ 2-form field strengths is given by

$$F^i_{(2)} = \epsilon\, m_i\,\lambda^{-2}\,\Omega_{(2)} \equiv \tfrac{1}{2}\epsilon\, q_i\, g^{-1}\,\Omega_{(2)}\,. \tag{5.5}$$

We obtain the $D = 5$ scalar potential

$$V = \tfrac{1}{2}\epsilon^2\,(\sum_{i=1}^{2} m_i^2\,X_i^{-2})\,e^{8\alpha\,\varphi} - 2\epsilon\lambda^2\,e^{5\alpha\,\varphi} + \hat{V}\,e^{2\alpha\,\varphi}\,. \tag{5.6}$$

It is straightforward to obtain the corresponding superpotential, which is given by

$$W = \frac{\epsilon}{\sqrt{2}}(\sum_{i=1}^{2} m_i\, X_i^{-1})\, e^{4\alpha\,\varphi} + \hat{W}\, e^{\alpha\,\varphi}\,, \tag{5.7}$$

provided that the constraint

$$\lambda^2 = (m_1 + m_2)\, g \tag{5.8}$$

is satisfied. Thus, the charge parameters q_i have the constraint $q_1 + q_2 = 2$. The first-order equations of the five-dimensional domain wall are given by

$$\phi_i' = \sqrt{2}\,\frac{\partial W}{\partial \phi_i}\,, \qquad \varphi' = \sqrt{2}\,\frac{\partial W}{\partial \varphi}\,, \qquad A' = -\frac{1}{3\sqrt{2}}\,W\,. \tag{5.9}$$

Lifting the equations back to $D = 7$ yields the equations of motion for the 3-brane:

$$\begin{aligned}
\frac{d\vec{\phi}}{d\rho} &= \sqrt{2}\Big(-\frac{\epsilon}{2\sqrt{2}}\,(m_1\,\vec{a}_1\,X_1^{-1} + m_2\,\vec{a}_2\,X_2^{-1})\,e^{-2v} + \frac{d\hat{W}}{d\vec{\phi}}\Big)\,,\\
\frac{dv}{d\rho} &= -\frac{1}{5\sqrt{2}}\Big(2\sqrt{2}\,\epsilon\,(m_1\,X_1^{-1} + m_2\,X_2^{-1})\,e^{-2v} + \hat{W}\Big)\,,\\
\frac{du}{d\rho} &= \frac{1}{5\sqrt{2}}\Big(\frac{\epsilon}{\sqrt{2}}\,(m_1\,X_1^{-1} + m_2\,X_2^{-1})\,e^{-2v} - \hat{W}\Big)\,.
\end{aligned} \tag{5.10}$$

In the case of $m_2 = 0$, it is consistent to set $\phi_1 = \sqrt{5}\phi_2$, and the system (5.10) reduces to the $D = 7$ single-charge system discussed in section 3.1. After taking $g \to \sqrt{5/6}\,g$ and $\phi_2 \to \phi/\sqrt{6}$, the exact solution is given by (3.26) and (3.28). This solution was obtained in [9]. For $m_1 = m_2$, the system reduces to the $D = 7$ two-equal-charge system discussed in section 3.2. For general m_i, we do not find analytical solutions and so it is instructive to study the solution using numerical approach.

5.1 General asymptotic region

We begin by determining the asymptotic behavior in the AdS$_7$-type boundary. As discussed in the previous section, we can express the metric in the form (4.7) by making the coordinate transformation $d\rho = dy\, e^u$. We have the boundary conditions $u(y) \sim v(y) \sim -\log y$, and $\phi \to 0$ for small y. The leading terms in the Taylor expansion of the solution of (5.10) are given by

$$e^{2u} = \frac{4}{g^2\,y^2}\,(1 + \frac{\epsilon\,(m_1 + m_2)\,g}{30c^2}\,y^2 + \cdots)\,,$$

$$
\begin{aligned}
e^{2v} &= \frac{4c^2}{g^2\,y^2}\,(1-\frac{13\epsilon\,(m_1+m_2)\,g}{60c^2}\,y^2+\cdots)\,,\\
e^{\frac{\phi_1}{\sqrt{2}}} &= 1+\frac{\epsilon\,(m_1-m_2)\,g}{8c^2}\,y^2+\cdots\,,\\
e^{\frac{\phi_2}{\sqrt{10}}} &= 1+\frac{\epsilon\,(m_1+m_2)\,g}{40c^2}\,y^2+\cdots\,,
\end{aligned}
\tag{5.11}
$$

where c is an arbitrary integration constant. The normalizable solutions for ϕ_1 and ϕ_2 imply that the dual operators are not turned on. Next, we examine how these solutions flow from the above boundary into the bulk.

5.2 AdS$_5 \times H^2$ and AdS$_5 \times S^2$

For $\epsilon = \pm 1$, solutions to the equations (5.10) with constant v and $\vec\phi$ are given by

$$
\begin{aligned}
\mathrm{e}^{\sqrt{2}\phi_1} &= \frac{m_2-m_1\pm\sqrt{(m_2-m_1)^2+m_1\,m_2}}{m_2}\,,\quad \mathrm{e}^{-\sqrt{\frac{5}{2}}\phi_2}=\frac{4}{3}\cosh(\phi_1/\sqrt{2})\,,\\
\mathrm{e}^{-2v} &= -\frac{g\,\mathrm{e}^{-\frac{3}{\sqrt{10}}\phi_2}}{\epsilon\,(m_1\,\mathrm{e}^{-\frac{\phi_1}{\sqrt{2}}}+m_2\,\mathrm{e}^{\frac{\phi_1}{\sqrt{2}}})}\,,\quad u=-\frac{g}{2}\mathrm{e}^{-\frac{4}{\sqrt{10}}\phi_2}\rho\equiv-\frac{2}{R_{\rm ads}}\,\rho\,.
\end{aligned}
\tag{5.12}
$$

This solution is discussed in detail in [24]. Note that the AdS radius $R_{\rm ads}$ depends on two variables, namely g and the ratial of the charge parameter m_1/m_2. This is because the charges of this system are constrained by $q_1+q_2=2$, as discussed earlier. It is invariant under the simultaneous interchanges $m_1 \leftrightarrow m_2$ and $\phi_1 \leftrightarrow -\phi_1$. The reality conditions of the solution constrain the constants m_i and g, as well as the choice of $\pm$ in the solution. Let us first consider the case $\epsilon = -1$, corresponding to $d\Omega_2^2$ as the metric of a unit (non-compact) hyperbolic 2-plane. In this case, the reality of the solution implies that $m_1\,m_2 \ge 0$. This includes the choices of $m_1 = 0$ (or $m_2 = 0$) and $m_1 = m_2$, which were discussed in [9]. The first case gives rise to $\mathcal{N} = 4$ supersymmetry in $D = 5$, while the second case gives rise to $\mathcal{N} = 2$ supersymmetry.

Now let us look at the choice of $\epsilon = +1$, corresponding to $d\Omega_2^2$ as the metric of S^2. In this case, the reality conditions for (5.12) imply that $m_1\,m_2 < 0$. The condition (5.8) implies further that $m_1 \ne -m_2$. Therefore, the AdS$_5 \times S^2$ solution can only have $\mathcal{N} = 2$ supersymmetry but cannot arise from the pure $D = 7$ minimal gauged supergravity.

If we define a charge parameter $q = 2m_1/(m_1+m_2)$, then the condition for having

S^2 versus H^2 can be summarized as the following:

$$\begin{aligned} q \in [0,2] \quad &\Longrightarrow \quad H^2\,, \\ q \in (-\infty, 0) \text{ or } (2,\infty) \quad &\Longrightarrow \quad S^2\,. \end{aligned} \tag{5.13}$$

The solution was lifted to $D = 11$ in [24], and the corresponding metric describes a warped product of AdS_5 with a six-dimensional space which can be viewed as S^4 bundle over H^2 or S^2. In the latter case, the solution provides a concrete example of a supersymmetric and smooth compactification of M-theory to AdS_5 [24].

5.3 Interpolating from $\mathrm{AdS}_5 \times H^2$ to AdS_7

As discussed above, $\mathrm{AdS}_5 \times H^2$ arises for $m_1\, m_2 \geq 0$ and $\epsilon = -1$. The special cases of a single non-vanishing m_i or $m_1 = m_2$ have been discussed in sections 3 and 4, respectively . We now consider the general case of m_i with $m_1\, m_2 > 0$. As in the previous cases, $\mathrm{AdS}_5 \times H^2$ can occur at either the boundary or the horizon of the AdS_5. In order to demonstrate this, we use a special case of $m_1 = 16$, $m_2 = 6$ and a positive sign in (5.12). For simplicity, we also set $g = 1$. As a boundary solution, the Taylor expansion of the metric components and the scalars goes as

$$\begin{aligned} e^{2u} &= \tfrac{18}{5}\,(\tfrac{18}{125})^{1/5}\, y^{-2}\,(1 + c\,y^n + \cdots)\,, \\ e^{2v} &= 6(450)^{1/5}\,(1 - \frac{3}{2} c\,y^n + \cdots)\,, \\ e^{\frac{\phi_1}{\sqrt{2}}} &= \sqrt{\tfrac{2}{3}}\,(1 - \frac{75c\,(n+1)}{2(25n-68)}\,y^n + \cdots)\,, \\ e^{\frac{\phi_2}{\sqrt{10}}} &= (\tfrac{27}{50})^{1/10}\,(1 - \frac{c\,(n+1)}{2(n-4)}\,y^n + \cdots)\,, \end{aligned} \tag{5.14}$$

where the coordinate y is defined to be $d\rho = e^u\, dy$ and c is an arbitrary integration constant. The constant n can take two values: $n = 2$ or $n = 1 + \frac{1}{5}\sqrt{193}$. Clearly, this solution will flow into a singularity in the bulk.

On the other hand, as a solution near the horizon, we find that the Taylor expansion of the next to leading order is given by

$$\begin{aligned} e^{2u} &= \tfrac{18}{5}\,(\tfrac{18}{125})^{1/5}\, r^2\,(1 + c\,r^n + \cdots)\,, \\ e^{2v} &= 6(450)^{1/5}\,(1 - \frac{3(2895 + 209\sqrt{193})\,c}{2(805 + 59\sqrt{193})}\,r^n + \cdots)\,, \end{aligned}$$

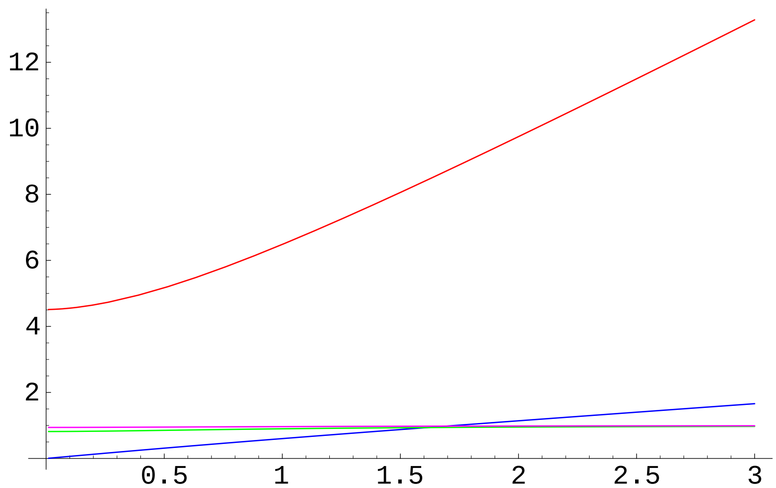

Figure 2: Plots of e^u (blue), e^v (red), $e^{\phi_1/\sqrt{2}}$ (green) and $e^{\phi_2/\sqrt{10}}$ (purple) in a smooth solution that runs from $\mathrm{AdS}_{(5)} \times H^2$ at the horizon to the AdS_7-type geometry in the asymptotic region. Note that the two scalar curves are almost identical on this scale. $m_1 = 16, m_2 = 6$, $g = 1$, and $c = -0.1$.

$$
\begin{aligned}
e^{\frac{\phi_1}{\sqrt{2}}} &= \sqrt{\tfrac{2}{3}}\,(1 - \frac{15\sqrt{193}\,c}{2(43+5\sqrt{193})}\,r^n + \cdots)\,,\\
e^{\frac{\phi_2}{\sqrt{10}}} &= (\tfrac{27}{50})^{1/10}\,(1 - \frac{c\sqrt{193}}{2(15+\sqrt{193})}\,r^n + \cdots)\,,
\end{aligned}
\tag{5.15}
$$

where $n = \frac{1}{5}\sqrt{193} - 1$ and c is an arbitrary integration constant. The coordinate r is defined by $d\rho = e^{-u}\,dr$. We can use the above as the initial data to obtain a numerical solution. Plots of the functions $e^u, e^v, e^{\phi_1/\sqrt{2}}$ and $e^{\phi_2/\sqrt{10}}$ are presented in Fig. 2, which clearly shows that the solution runs smoothly from $\mathrm{AdS}_5 \times H^2$ at $r \to 0$ to the AdS_7-type solution (5.12) at $r \to \infty$.

5.4 Interpolating from $\mathrm{AdS}_5 \times S^2$ to AdS_7

The $\mathrm{AdS}_5 \times S^2$ fixed-point arises for $m_1 m_2 < 0$ and $\epsilon = +1$. The Taylor expansion analysis indicates that the solution can occur either in the boundary or the horizon of the AdS_5. For a concrete example, we choose $m_1 = 5$, $m_2 = -3$, $g = 1$ and a negative sign in (5.12). The boundary solution behaves as

$$
\begin{aligned}
e^{2u} &= (\tfrac{25}{8})^{2/5}\,y^{-2}\,(1 + c\,y^n + \cdots)\,, \qquad e^{2v} = (\tfrac{625}{2})^{1/5}\,(1 - \tfrac{3}{2}c\,y^n + \cdots)\,,\\
e^{\frac{\phi_1}{\sqrt{2}}} &= \sqrt{5}\,(1 + \frac{5(n+1)}{n-10}c\,y^n + \cdots)\,,
\end{aligned}
$$

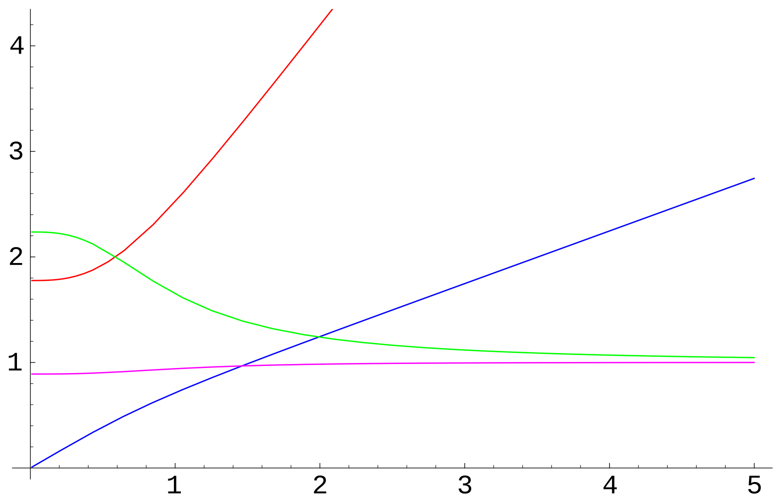

Figure 3: e^u (blue), e^v (red), $e^{\phi_1/\sqrt{2}}$ (green) and $e^{\phi_2/\sqrt{10}}$ (purple) for a smooth solution that runs from $\mathrm{AdS}_{(5)}\times S^2$ at horizon to the AdS_7-type geometry in the asymptotic region. $m_1=5, m_2=-3$, $g=1$, and $c=-0.2$.

$$e^{\frac{\phi_2}{\sqrt{10}}} = (\tfrac{5}{16})^{1/10}\,(1-\frac{n+1}{2(n-4)}c\,y^n+\cdots)\,, \tag{5.16}$$

where $n=2$ or $n=1+\sqrt{15}$. The constant c is an arbitrary integration constant. Clearly, the flow of this solution from the boundary leads to a singularity in the bulk.

As a horizon, $\mathrm{AdS}_5\times S^2$ has the behavior

$$\begin{aligned}
e^{2u} &= (\tfrac{64}{625})^{1/5}\,r^2\,(1+c\,r^n+\cdots)\,,\\
e^{2v} &= (\tfrac{625}{2})^{1/5}\,(1-\frac{3(15+4\sqrt{15})\,c}{2(7+2\sqrt{15})}\,r^n+\cdots)\,,\\
e^{\frac{\phi_1}{\sqrt{2}}} &= \sqrt{5}\,(1+\frac{5\sqrt{15}\,c}{9+\sqrt{15}}\,r^n+\cdots)\,,\\
e^{\frac{\phi_2}{\sqrt{10}}} &= (\tfrac{5}{16})^{1/10}\,(1-\frac{\sqrt{15}\,c}{2(3+\sqrt{15})}\,r^n+\cdots)\,,
\end{aligned} \tag{5.17}$$

where $n=-1+\sqrt{15}$, c is an integration constant and r is defined by $d\rho=e^{-u}\,dr$. We can use this solution as our initial data for numerical analysis. The result is plotted in Fig. 3.

The plot clearly demonstrates that there is a smooth solution which interpolates between $\mathrm{AdS}\times S^2$ at $r\to 0$ and the AdS_7-type solution (5.12) at $r\to\infty$. This solution provides a concrete framework with which to study the M-theory dual to a certain six-dimensional theory whose IR dynamics is given by a $D = 4$, $\mathcal{N} = 1$ superconformal field theory.

6 Two-charge membrane in $D = 6$

The scalar potential in gauged supergravity with two $U(1)$ isometries was obtained in [31]. From this, we deduce that the relevant Lagrangian involving the two $U(1)$ vector fields is given by

$$\hat{e}^{-1}\mathcal{L}_6 = \hat{R} - \tfrac{1}{2}(\partial\phi_1)^2 - \tfrac{1}{2}(\partial\phi_2)^2 - \hat{V} - \tfrac{1}{4}\sum_{i=1}^{2} X_i^{-2}\,(\hat{F}^i_{(2)})^2\,, \tag{6.1}$$

where $X_i = e^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = (\sqrt{2}\,, \frac{1}{\sqrt{2}})\,, \qquad \vec{a}_2 = (-\sqrt{2}\,, \frac{1}{\sqrt{2}})\,. \tag{6.2}$$

The scalar potential is given by

$$\hat{V} = \tfrac{4}{9}g^2\,(X_0^2 - 9X_1\,X_2 - 6X_0\,X_1 - 6X_0\,X_2)\,, \tag{6.3}$$

where $X_0 = (X_1\,X_2)^{-3/2}$. As in the previous case, the scalar potential can be expressed in terms of a superpotential $\hat{W}$, given by

$$\hat{W} = \frac{g}{\sqrt{2}}\,(\tfrac{4}{3}X_0 + 2X_1 + 2X_2)\,. \tag{6.4}$$

We consistently reduce the theory on the S^2 or H^2 with the same metric ansatz as before but specializing to $D = 6$. The ansatz for the two $U(1)$ field strengths is given by $F^i_{(2)} = \epsilon m_i\,\lambda^{-2}\,\Omega_{(2)} \equiv \frac{1}{2}\epsilon\,q_i\,g^{-1}\,\Omega_{(2)}$. We obtain the $D = 4$ scalar potential

$$V = \tfrac{1}{2}\epsilon^2\,(\sum_{i=1}^{2} m_i^2\,X_i^{-2})\,e^{6\alpha\,\varphi} - 2\epsilon\lambda^2\,e^{4\alpha\,\varphi} + \hat{V}\,e^{2\alpha\,\varphi}\,. \tag{6.5}$$

It is straightforward to obtain the corresponding superpotential, given by

$$W = \frac{\epsilon}{\sqrt{2}}(\sum_{i=1}^{2} m_i\,X_i^{-1})\,e^{3\alpha\,\varphi} + \hat{W}\,e^{\alpha\,\varphi}\,. \tag{6.6}$$

As in the $D = 7$ case, the constraint $\lambda^2 = (m_1 + m_2)\,g$ must be satisfied, implying $q_1 + q_2 = 2$. The first-order equations of the four-dimensional domain-wall has the same form as (5.9), but with $A' = -W/(2\sqrt{2})$. Lifting the equations back to $D = 6$ yields

$$
\begin{aligned}
\frac{d\vec{\phi}}{d\rho} &= \sqrt{2}\Big(-\frac{\epsilon}{2\sqrt{2}}\,(m_1\,\vec{a}_1\,X_1^{-1} + m_2\,\vec{a}_2\,X_2^{-1})\,e^{-2v} + \frac{d\hat{W}}{d\vec{\phi}}\Big)\,,\\
\frac{dv}{d\rho} &= -\frac{1}{4\sqrt{2}}\Big(\frac{3}{\sqrt{2}}\,\epsilon\,(m_1\,X_1^{-1} + m_2\,X_2^{-1})\,e^{-2v} + \hat{W}\Big)\,,\\
\frac{du}{d\rho} &= \frac{1}{4\sqrt{2}}\Big(\frac{\epsilon}{\sqrt{2}}\,(m_1\,X_1^{-1} + m_2\,X_2^{-1})\,e^{-2v} - \hat{W}\Big)\,. \qquad (6.7)
\end{aligned}
$$

In the case of $m_2 = 0$, it is consistent to set $\phi_1 = 2\phi_2$, and the system reduces to the $D = 6$ system discussed in section 3.1. After taking $g \to \sqrt{4/5}\,g$ and $\phi_2 \to \phi/\sqrt{5}$, the exact solution is given by (3.26) and (3.32). For general m_i, we do not find analytical solutions and so it is instructive to study the system using the numerical approach.

6.1 Asymptotic boundary region

We can express the metric in the form (4.7) by making the coordinate transformation $d\rho = dy\,\mathrm{e}^{u}$. We have the boundary conditions $u(y) \sim v(y) \sim -\log y$ and $\phi \to 0$ for small y. The leading terms in the Taylor expansion of the solution of (6.7) are given by

$$
\begin{aligned}
e^{2u} &= \frac{9}{4g^2\,y^2}\,(1 + \frac{\epsilon\,(m_1 + m_2)\,g}{18c^2}\,y^2 + \cdots)\,,\\
e^{2v} &= \frac{9c^2}{4g^2\,y^2}\,(1 - \frac{5\epsilon\,(m_1 + m_2)\,g}{18c^2}\,y^2 + \cdots)\,,\\
e^{\frac{\phi_1}{\sqrt{2}}} &= 1 + \frac{\epsilon\,(m_1 - m_2)\,g}{3c^2}\,y^2 + \cdots\,, \qquad e^{\frac{\phi_2}{\sqrt{8}}} = 1 + \frac{\epsilon\,(m_1 + m_2)\,g}{12c^2}\,y^2 + \cdots\,, \qquad (6.8)
\end{aligned}
$$

where c is an arbitrary integration constant. As in the case of $D = 7$, the normalizable solutions for ϕ_1 and ϕ_2 imply that the dual operators are not turned on. Next, we examine how these solutions flow from the above boundary into the bulk.

6.2 AdS$_4 \times H^2$ and AdS$_4 \times S^2$

For $\epsilon = \pm 1$, solutions to the equations (6.7) with constant v and $\vec{\phi}$ are given by

$$
\mathrm{e}^{\sqrt{2}\phi_1} = \frac{3}{2}\,\frac{m_2 - m_1 \pm \sqrt{(m_2 - m_1)^2 + \frac{4}{9}m_1\,m_2}}{m_2}\,, \quad \mathrm{e}^{-\sqrt{2}\phi_2} = \frac{3}{2}\cosh(\frac{\phi_1}{\sqrt{2}})\,,
$$

$$\mathrm{e}^{-2v} = -\frac{4g\,\mathrm{e}^{-\frac{\phi_2}{\sqrt{2}}}}{3\epsilon\,(m_1\,\mathrm{e}^{-\frac{\phi_1}{\sqrt{2}}} + m_2\,\mathrm{e}^{\frac{\phi_1}{\sqrt{2}}})}\,, \quad u = -\frac{2}{3}g\,\mathrm{e}^{-\frac{3}{\sqrt{8}}\phi_2}\rho \equiv -\frac{2}{R_{\rm ads}}\,\rho, \tag{6.9}$$

which have the geometry $\mathrm{AdS}_4 \times H^2$ or $\mathrm{AdS}_4 \times S^2$ for $\epsilon = -1$ or $+1$, respectively. These solutions were discussed in detail in [24] and are similar to one found in [10]. As in the $D = 7$ result, we can define a charge parameter $q = \frac{2m_1}{m_1+m_2}$. We have H^2 or S^2 depending on the following conditions:

$$\begin{aligned} q \in [0,2] \quad &\Longrightarrow \quad H^2\,, \\ q \in (-\infty,0) \text{ or } (2,\infty) \quad &\Longrightarrow \quad S^2\,. \end{aligned} \tag{6.10}$$

When $q = 0$ or $q = 2$, the system has $\mathcal{N} = 4$ supersymmetry. Otherwise, it has $\mathcal{N} = 2$ supersymmetry. Note that the AdS_4 radius $R_{\rm ads}$ depends on only g and m_1/m_2.

These solutions have been lifted back to $D = 10$ massive supergravity in [24]. The $D = 10$ metric describes a warped product of Ads_4 with an internal metric composed of an S^4 bundle over S^2 or H^2.

6.3 Interpolating from $\mathrm{AdS}_4 \times H^2$ to AdS_6

As we discussed above, $\mathrm{AdS}_4 \times H^2$ arises for $m_1\,m_2 \geq 0$ and $\epsilon = -1$. The special cases of one non-vanishing m_i and $m_1 = m_2$ have been discussed in sections 3 and 4, respectively . We will now consider the general case of m_i with $m_1\,m_2 > 0$. As in the previous cases, $\mathrm{AdS}_4 \times H^2$ can occur as an asymptotic geometry in one solution and a horizon geometry in another. In order to demonstrate this explicitly, we take $m_1 = 5$, $m_2 = 2$ and a positive sign in (6.9). For simplicity, we also set $g = 1$. As an asymptotic boundary solution, the Taylor expansions of the metric components and the scalars go as

$$\begin{aligned} e^{2u} &= (\tfrac{128}{81})^{1/4}\,y^{-2}\,(1 + c\,y^n + \cdots)\,, \\ e^{2v} &= (648)^{1/4}\,(1 - c\,y^n + \cdots)\,, \\ e^{\frac{\phi_1}{\sqrt{2}}} &= \tfrac{1}{\sqrt{2}}\,(1 - \tfrac{12(n+1)}{9n-17}c\,y^n + \cdots)\,, \\ e^{\frac{\phi_2}{\sqrt{8}}} &= (\tfrac{32}{81})^{1/8}\,(1 - \tfrac{(3n+5)(6n-7)}{2(9n-17)}\,c\,y^n + \cdots)\,, \end{aligned} \tag{6.11}$$

where the coordinate y is defined to be $d\rho = e^u\,dy$ and c is an arbitrary integration constant. The constant n can take two values: $n = 1$ or $n = \frac{1}{6}(3 + \sqrt{185})$. Clearly, this solution will flow into a singularity in the bulk.

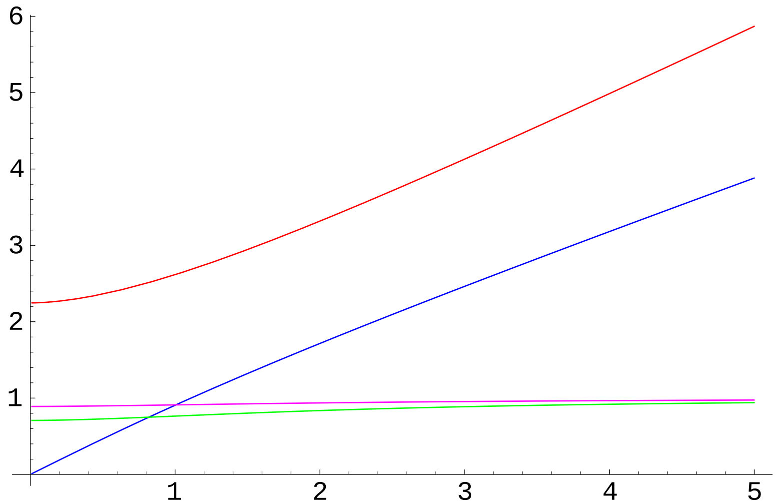

Figure 4: e^u (blue), e^v (red), $e^{\phi_1/\sqrt{2}}$ (green) and $e^{\phi_2/\sqrt{8}}$ (purple) for a smooth solution running from $\mathrm{AdS}_4 \times H^2$ at the horizon to an asymptotic AdS_7-type geometry. $m_1 = 5, m_2 = -3$, $g = 1$, and $c = -0.05$.

On the other hand, for the solution with $\mathrm{AdS}_4 \times H^2$ at the horizon, we find the Taylor expansion of the next leading order to be given by

$$\begin{aligned}
e^{2u} &= \left(\tfrac{81}{128}\right)^{1/4} r^2 \left(1 + c\, r^2 + \cdots\right), \\
e^{2v} &= (648)^{1/4} \left(1 - \tfrac{25n-69}{7n-19}\, c\, r^2 + \cdots\right), \\
e^{\frac{\phi_1}{\sqrt{2}}} &= \tfrac{1}{\sqrt{2}} \left(1 - \tfrac{12(n+1)}{9n-17}\, c\, r^2 + \cdots\right), \\
e^{\frac{\phi_2}{\sqrt{8}}} &= \left(\tfrac{32}{81}\right)^{1/8} \left(1 - \tfrac{(n+1)\, c}{2(n+3)}\, c\, r^2 + \cdots\right),
\end{aligned} \tag{6.12}$$

where $n = \frac{1}{6}(-3+\sqrt{185})$ and c is an integration constant. The coordinate r is defined by $d\rho = e^{-u}\, dr$. This solution runs smoothly to large distance, where it asymptotes to the AdS_6-type solution (6.8). This can be demonstrated with numerical calculation, using the above Taylor expansions as the initial data. In Fig. 4, the smooth functions e^u, e^v, $e^{\phi_1/\sqrt{2}}$ and $e^{\phi_2/\sqrt{8}}$ are plotted.

6.4 Interpolating from $\mathrm{AdS}_4 \times S^2$ to AdS_6

The $\mathrm{AdS}_4 \times S^2$ fixed-point arises for $m_1\, m_2 < 0$ and $\epsilon = +1$. The Taylor expansion analysis indicates that there are solutions with $\mathrm{AdS}_4 \times S^2$ in the asymptotic boundary as well as at the near-horizon geometry. For a concrete example, we choose $m_1 = 7$,

$m_2=-5$, $g=1$ and a negative sign in (6.9). The boundary solution behaves as

$$
\begin{aligned}
e^{2u} &= \tfrac{9}{4}(\tfrac{7}{36})^{3/4}\,y^{-2}\,(1+c\,y^n+\cdots)\,,\\
e^{2v} &= 3(\tfrac{343}{36})^{1/4}\,(1-c\,y^n+\cdots)\,,\\
e^{\frac{\phi_1}{\sqrt{2}}} &= \tfrac{1}{\sqrt{7}}\,(1-\tfrac{6(n+1)}{2n-11}c\,y^n+\cdots)\,,\\
e^{\frac{\phi_2}{\sqrt{8}}} &= (\tfrac{7}{36})^{1/8}\,(1-\tfrac{n+1}{2(n-3)}c\,y^n+\cdots)\,,
\end{aligned}
\tag{6.13}
$$

where $n=1$ or $n=\frac{1}{2}(1+\sqrt{35})$, and c is an arbitrary integration constant. Clearly, the solution runs from this asymptotic region into a singularity in the bulk.

In the solution whose near-horizon geometry is $\mathrm{AdS}_4\times S^2$, the Taylor expansion is given by

$$
\begin{aligned}
e^{2u} &= (\tfrac{16384}{3087})^{1/4}\,(1+c\,r^2+\cdots)\,,\\
e^{2v} &= (\tfrac{3087}{4})^(1/4)\,(1-\tfrac{1}{13}(19+2\sqrt{35})\,c\,r^2+\cdots)\,,\\
e^{\frac{\phi_1}{\sqrt{2}}} &= \sqrt{7}\,(1+\tfrac{3}{65}(-25+9\sqrt{35})\,c\,r^2+\cdots)\,,\\
e^{\frac{\phi_2}{\sqrt{8}}} &= (\tfrac{7}{36})^{1/8}\,(1+\tfrac{1}{2}(-3+2\sqrt{7/5})\,c\,r^2+\cdots)\,,
\end{aligned}
\tag{6.14}
$$

where c is an integration constant and r is defined by $d\rho=e^{-u}\,dr$. We can use the above Taylor expansion as the initial conditions for numerical calculation. The plots of the above functions are presented in Fig. 5, which clearly show that the solution runs smoothly to the asymptotic AdS_6-type solution given by (6.8).

7 Three-charge string in $D=5$

Let us now consider the minimal gauged supergravity in $D=5$ coupled to two vector multiplets. The Lagrangian is given by

$$
e^{-1}\mathcal{L}_5=\hat{R}-\tfrac{1}{2}(\partial\phi_1)^2-\tfrac{1}{2}(\partial\phi_2)^2-\frac{1}{4}\sum_{i=1}^{3}X_i^{-2}(\hat{F}^i_{(2)})^2-\hat{V}+e^{-1}\,\tfrac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda}\,\hat{F}^1_{\mu\nu}\,\hat{F}^2_{\rho\sigma}\,\hat{A}^3_\lambda\,, \tag{7.1}
$$

with the scalar potential

$$
\hat{V}=-4g^2\sum_{i=1}^{3}X_i^{-1}\,. \tag{7.2}
$$

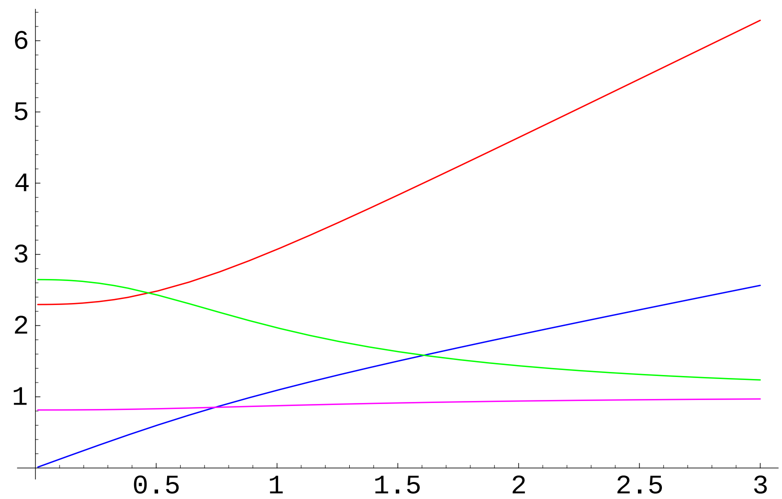

Figure 5: e^u (blue), e^v (red), $e^{\phi_1/\sqrt{2}}$ (green) and $e^{\phi_2/\sqrt{8}}$ (purple) for a smooth solution running from $\mathrm{AdS}_4 \times S^2$ at the horizon to an asymptotic AdS_6-type geometry. $m_1 = 7, m_2 = -5$, $g = 1$, and $c = -0.2$.

The quantities X_i are given by

$$X_i = e^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}\,,$$
$$\vec{a}_1 = (\sqrt{2}\,, \frac{2}{\sqrt{6}})\,, \qquad \vec{a}_2 = (-\sqrt{2}\,, \frac{2}{\sqrt{6}})\,, \qquad \vec{a}_3 = (0, -\frac{4}{\sqrt{6}})\,. \tag{7.3}$$

The scalar potential $\hat{V}$ in (7.2) can be also expressed in terms of the superpotential $\hat{W}$, given by

$$\hat{W} = \sqrt{2}\,g\sum_i X_i\,. \tag{7.4}$$

We now reduce the theory on S^2 with the previous metric ansatz specializing on $D = 5$. The ansatz for the three $U(1)$ 2-form field strengths is given by

$$F^i_{(2)} = \epsilon\, m_i\,\lambda^{-2}\,\Omega_{(2)} \equiv \tfrac{1}{2}\epsilon\, q_i\, g^{-1}\,\Omega_{(2)}\,. \tag{7.5}$$

The resulting $D = 3$ scalar potential is

$$V = \tfrac{1}{2}\epsilon^2(\sum_i m_i^2\, X_i^{-2})\, e^{4\alpha\varphi} - 2\epsilon\lambda^2\, e^{3\alpha\varphi} + \hat{V}\, e^{2\alpha\varphi}\,, \tag{7.6}$$

and the corresponding superpotential is

$$W = \tfrac{\epsilon}{\sqrt{2}}(\sum_i m_i\, X_i^{-1})\, e^{2\alpha\varphi} + \hat{W}\, e^{\alpha\varphi}\,, \tag{7.7}$$

provided that the following constraint is satisfied:

$$\lambda^2 = g \sum_i m_i \,. \tag{7.8}$$

Thus, the charge parameters q_i satisfy $q_1 + q_2 + q_3 = 2$. The first-order equations for the three-dimensional system have the same form as (5.9) but now with $A' = -\frac{1}{\sqrt{2}}\, W$. Lifting the equations, we obtain the first-order equations describing the three-charge magnetic string in $D = 5$:

$$\begin{aligned}
\frac{d\vec{\phi}}{d\rho} &= \sqrt{2}\Big(-\frac{\epsilon}{2\sqrt{2}}\,(m_1\,\vec{a}_1\,X_1^{-1} + m_2\,\vec{a}_2\,X_2^{-1} + m_3\,\vec{a}_3\,X_3^{-1})\,e^{-2v} + \frac{d\hat{W}}{d\vec{\phi}}\Big)\,, \\
\frac{dv}{d\rho} &= -\frac{1}{3\sqrt{2}}\Big(\sqrt{2}\,\epsilon\,(m_1\,X_1^{-1} + m_2\,X_2^{-1} + m_3\,X_3^{-1})\,e^{-2v} + \hat{W}\Big)\,, \\
\frac{du}{d\rho} &= \frac{1}{3\sqrt{2}}\Big(\frac{\epsilon}{\sqrt{2}}\,(m_1\,X_1^{-1} + m_2\,X_2^{-1} + m_3\,X_3^{-1})\,e^{-2v} - \hat{W}\Big)\,.
\end{aligned} \tag{7.9}$$

A 3-charge string with constant scalars was found in [7]. On the other hand, for $m_1 = m_2 = m_3$, it is consistent that ϕ_1 and ϕ_2 vanish and this system reduces to the $D = 5$ system of section 2, with the solution given by (2.15) [12]. For $m_1 = m_2$ we can consistently set $\phi_1 = 0$ while keeping ϕ_2. In particular, if $m_1 = m_2 = 0$, this system reduces to the $D = 5$ system of section 3.1. After taking $g \to \sqrt{3/4}\, g$, an exact solution is given by the formulae (3.26) and (3.34).

For $m_1 = m_2 = m_3$, it has recently been found that one can consistently set $\phi_1 = \sqrt{3}\phi_2$, with an exact solution given by [14]

$$\begin{aligned}
ds_5^2 &= H^{-2/3}\Big[(g\,r)^2(\mathrm{e}^{\frac{\epsilon\,m}{2g^3\,r^2}}\,dx_\mu^2 + H^2\lambda^{-2}d\Omega_2^2) + H^2\frac{dr^2}{(g\,r)^2}\Big]\,, \\
\mathrm{e}^{-\sqrt{6}\phi_2} &= H\,,
\end{aligned} \tag{7.10}$$

where we made the coordinate transformation

$$\frac{d\rho}{dr} = -\frac{\mathrm{e}^v}{(g\,r)^2}, \tag{7.11}$$

and where

$$H = 1 - \frac{\epsilon\,m}{2g^3\,r^2}. \tag{7.12}$$

For general m_i, we do not find analytical solutions and so it is instructive to study the system with a numerical approach.

7.1 Asymptotic boundary region

We can express the metric in the form (4.7) by making the coordinate transformation $d\rho = dy\, \mathrm{e}^u$. The boundary conditions are $u(y) \sim v(y) \sim -\log y$ and $\phi \to 0$ for small y. The leading terms of the Taylor expansion of the solution of (7.9) are given by

$$\begin{aligned}
e^{2u} &= \frac{1}{g^2\, y^2}\,(1 + \frac{\epsilon\,(m_1+m_2+m_3)\,g}{9c^2}\,y^2 + \cdots)\,, \\
e^{2v} &= \frac{c^2}{g^2\, y^2}\,(1 - \frac{7\epsilon\,(m_1+m_2+m_3)\,g}{18c^2}\,y^2 + \cdots)\,, \\
e^{\frac{\phi_1}{\sqrt{2}}} &= 1 - \frac{\epsilon\,(m_1-m_2)\,g}{2c^2}\,y^2\log y + \cdots\,, \\
e^{\frac{\phi_2}{\sqrt{6}}} &= 1 - \frac{\epsilon\,(m_1+m_2-2m_3)\,g}{6c^2}\,y^2\log y + \cdots\,,
\end{aligned} \tag{7.13}$$

where c is an arbitrary integration constant. The non-normalizable solutions for ϕ_1 and ϕ_2 imply that the dual operators are turned on, provided that the values of m_i are such that the fields are non-vanishing [9]. Next, we examine how these solutions flow from the above boundary into the bulk.

7.2 $\mathrm{AdS}_3 \times H^2$ and $\mathrm{AdS}_3 \times S^2$

For $\epsilon = \pm 1$, the fixed-point solution is given by

$$\begin{aligned}
e^{\sqrt{2}\,\phi_1} &= \frac{m_1}{m_2}\Big(\frac{m_3+m_2-m_1}{m_3-m_2+m_1}\Big)\,, \qquad e^{\sqrt{6}\,\phi_2} = \frac{m_1\,m_2\,(m_3^2-(m_1-m_2)^2)}{m_3^2\,(m_1+m_2-m_3)^2}\,, \\
e^{-2v} &= -\epsilon\, g\,\Big(\frac{(m_1+m_2-m_3)(m_3^2-(m_1-m_2)^2)}{m_1^2\,m_2^2\,m_3^2}\Big)^{\frac{1}{3}}\,, \\
u &= -g\, e^{\frac{\phi_2}{\sqrt{6}}}\Big(\cosh(\phi_1/\sqrt{2}) + \tfrac{1}{2}e^{-\sqrt{\frac{3}{2}}\,\phi_2}\Big)\,\rho \equiv \frac{-2}{R_{\mathrm{ads}}}\,\rho\,.
\end{aligned} \tag{7.14}$$

This solution was discussed in detail in [24]. The reality condition of the solution implies that when three vectors with the magnitudes $|m_i|$ can form a triangle, $d\Omega_2^2$ should be the H^2 metric. On the other hand, when they cannot form a triangle, the metric should be that of S^2.[2] If any of the m_i vanish, there is no fixed-point solution, except when one vanishes with the remaining two being equal. The AdS_3 radius depends on g and two of the three charge parameters.

[2] $\mathrm{AdS}_3 \times S^2$ solutions were also recently found in [14] in a different construction.

7.3 Interpolating from $\mathrm{AdS}_3 \times H^2$ to AdS_5

As in the previous cases, $\mathrm{AdS}_3 \times H^2$ can occur either as an asymptotic geometry or on the horizon. In order to demonstrate this, we take $m_1 = m_2 = 2$ and $m_3 = 3$. For this choice of charge parameters, we can set $\phi_1 = 0$. For simplicity, we also set $g = 1$. As an asymptotic boundary geometry, the Taylor expansions of the metric components and the scalars are

$$\begin{aligned} e^{2u} &= \tfrac{8\,2^{1/3}}{25}\, y^{-2}(1 + c\, y^n + \cdots)\,, \\ e^{2v} &= 2\,2^{1/3}\,(1 - \tfrac{1}{2} c\, y^n + \cdots)\,, \\ e^{\frac{\phi_2}{\sqrt{6}}} &= 2^{1/3}\,(1 + -\tfrac{1}{16}(11 + 3\sqrt{17})\, y^n + \cdots)\,, \end{aligned} \tag{7.15}$$

where the coordinate y is defined to be $d\rho = e^u\, dy$ and c is an arbitrary integration constant. The constant n can take two values: $n = \frac{1}{5}(-3 + \sqrt{17})$. Clearly, this solution will flow into a singularity in the bulk.

On the other hand, for the solution with $\mathrm{AdS}_3 \times H^2$ at its horizon, we find the Taylor expansion of the next leading order to be

$$\begin{aligned} e^{2u} &= \tfrac{25}{8\,2^{1/3}}\, r^2\,(1 + c\, r^2 + \cdots)\,, \\ e^{2v} &= 2\,2^{1/3}\,(1 - \tfrac{1}{26}(33 + 10\sqrt{17})\, c\, r^2 + \cdots)\,, \\ e^{\frac{\phi_2}{\sqrt{6}}} &= 2^{1/3}\,(1 + \tfrac{1}{208}\,(147 - 11\sqrt{17})\, r^2 + \cdots)\,, \end{aligned} \tag{7.16}$$

where c is an integration constant and r is defined by $d\rho = e^{-u}\, dr$. We can use this as the initial conditions for the numerical calculation. The resulting plots are presented in Fig. 6), which show that the solution runs smoothly from $\mathrm{AdS}_3 \times H^2$ at the horizon to the AdS_5-type asymptotic behavior given by (7.14). The 3-charge string with constant scalars also interpolates between $\mathrm{AdS}_3 \times H^2$ and the AdS_5-type geometry [7].

7.4 Interpolating from $\mathrm{AdS}_3 \times S^2$ to AdS_5

$\mathrm{AdS}_3 \times S^2$ can also occur as an asymptotic geometry for one solution or as a horizon geometry for another. As a concrete example, let us consider $m_1 = m_2 = -1$ and $m_3 = 4$, for which case we have $\phi_1 = 0$. For simplicity, we set $g = 1$. The boundary

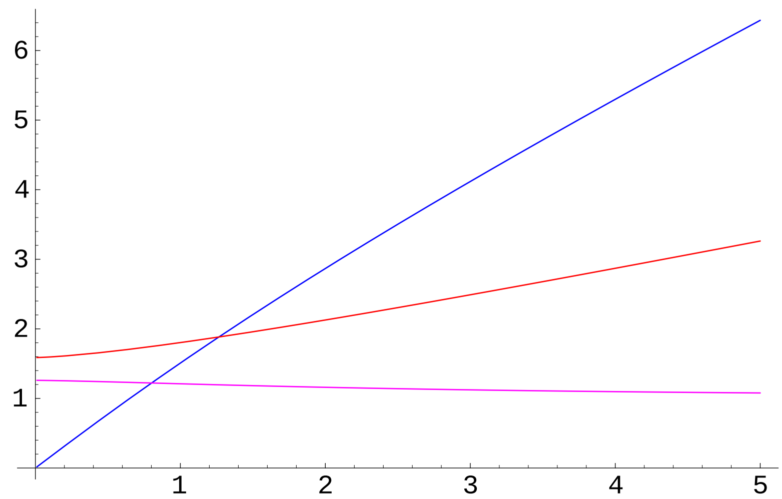

Figure 6: e^u (blue), e^v (red) and $e^{\phi_2/\sqrt{6}}$ (purple) for a smooth solution running from $\mathrm{AdS}_3 \times H^2$ at the horizon to an asymptotic AdS_5-type geometry. $m_1 = m_2 = 2$, $m_3 = 3$, $g = 1$, and $c = -0.05$.

solution behaves as

$$\begin{aligned} e^{2u} &= (\tfrac{9}{1024})^{1/3}\, y^{-2}\,(1 + c\,y^n + \cdots)\,, \\ e^{2v} &= 6^{-1/3}\,(1 - \tfrac{1}{2}c\,y^n + \cdots)\,, \\ e^{\frac{\phi_2}{\sqrt{6}}} &= 6^{-1/3}\,(1 + \tfrac{1}{20}(11 + 3\sqrt{19})\,c\,y^n + \cdots)\,, \end{aligned} \tag{7.17}$$

where $n = \frac{1}{2}(1 + \sqrt{19})$ and c is an integration constant. Clearly, the solution will encounter a singularity when it runs away from this asymptotic region.

For the solution with $\mathrm{AdS}_3 \times S^2$ at its horizon, the near-horizon behavior is

$$\begin{aligned} e^{2u} &= (\tfrac{1024}{9})^{1/3}\,(1 + c\,r^n + \cdots)\,, \\ e^{2v} &= 6^{-1/3}\,(1 - \tfrac{1}{10}(11 + 2\sqrt{19})\,c\,r^n + \cdots)\,, \\ e^{\frac{\phi_2}{\sqrt{6}}} &= 6^{-1/3}\,(1 + \tfrac{1}{100}(7 - 11\sqrt{19})\,c\,r^n + \cdots)\,, \end{aligned} \tag{7.18}$$

where $n = \frac{1}{2}(-1 + \sqrt{19})$ and c is an integration constant. We can use this as the initial conditions for a numerical calculation and the results are plotted in Fig. 7. This shows that the solution runs smoothly from $\mathrm{AdS}_3 \times S^2$ at the horizon to the AdS_5-type asymptotic geometry given by (7.14).

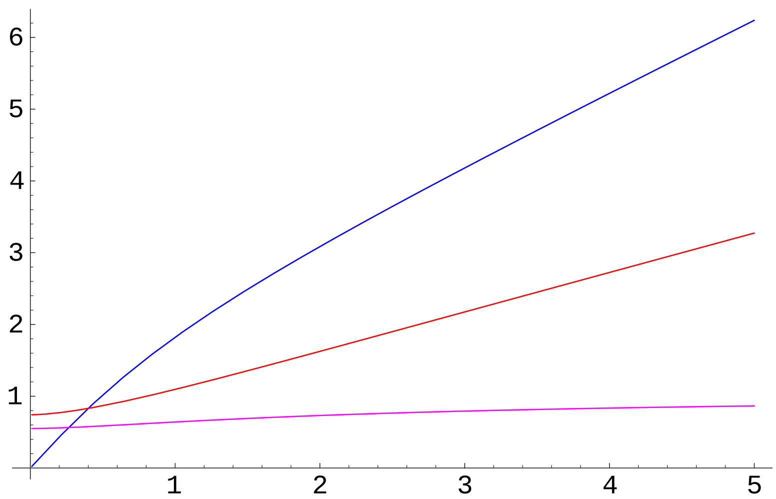

Figure 7: e^u (blue), e^v (red) and $e^{\phi_2/\sqrt{6}}$ (purple) for a smooth solution running from $\mathrm{AdS}_3 \times S^2$ at the horizon to an asymptotic AdS_5-type geometry. $m_1 = m_2 = -1$, $m_3 = 4$, $g = 1$, and $c = -0.3$.

8 Four-charge black hole in $D = 4$

Let us now consider the $U(1)^4$ gauged $N = 2$ supergravity in four dimensions. The Lagrangian is given by

$$e^{-1}\mathcal{L}_4 = \hat{R} - \tfrac{1}{2}(\partial\phi_1)^2 - \tfrac{1}{2}(\partial\phi_2)^2 - \tfrac{1}{2}(\partial\phi_3)^2 \frac{1}{4}\sum_{i=1}^{4} X_i^{-2}(\hat{F}^i_{(2)})^2 - \hat{V}\,, \tag{8.1}$$

with the scalar potential

$$\hat{V} = -4g^2 \sum_{i<j} X_i\, X_j\,. \tag{8.2}$$

The quantities X_i are given by

$$X_i = e^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}\,,$$
$$\vec{a}_1 = (1,1,1), \quad \vec{a}_2 = (1,-1,-1), \quad \vec{a}_3 = (-1,1,-1), \quad \vec{a}_4 = (-1,-1,1). \tag{8.3}$$

The scalar potential $\hat{V}$ in (8.2) can be also expressed in terms of the superpotential $\hat{W}$, where

$$\hat{W} = \sqrt{2}\, g \sum_{i=1}^{4} X_i\,. \tag{8.4}$$

We use the metric ansatz (2.12) and take the four $U(1)$ 2-form field strengths to be

$$F^i_{(2)} = \epsilon m_i\, \lambda^{-2}\, \Omega_{(2)} \equiv \tfrac{1}{2}\epsilon\, q_i\, g^{-1}\, \Omega_{(2)}\,. \tag{8.5}$$

A solution will be supersymmetric provided that the following constraint is satisfied:

$$\lambda^2 = g \sum_i m_i \,, \tag{8.6}$$

and hence $q_1 + q_2 + q_3 + q_4 = 2$. The equations of motion are given by

$$\begin{aligned}
\frac{d\vec{\phi}}{d\rho} &= \sqrt{2}\Big(-\frac{\epsilon}{2\sqrt{2}} \sum_{i=1}^{4} m_i\, \vec{a}_i\, X_i^{-1}\, e^{-2v} + \frac{d\hat{W}}{d\vec{\phi}} \Big) \,, \\
\frac{dv}{d\rho} &= -\frac{1}{2\sqrt{2}} \Big(\frac{\epsilon}{\sqrt{2}} \sum_{i=1}^{4} m_i\, X_i^{-1}\, e^{-2v} + \hat{W} \Big) \,, \\
\frac{du}{d\rho} &= \frac{1}{2\sqrt{2}} \Big(\frac{\epsilon}{\sqrt{2}} \sum_{i=1}^{4} m_i\, X_i^{-1}\, e^{-2v} - \hat{W} \Big) \,.
\end{aligned} \tag{8.7}$$

For $m_1 = m_2 = m_3 = m_4$, it is consistent that ϕ_1 and ϕ_2 vanish and this system reduces to the $D = 4$ system of section 2, with the solution given by (2.15) [12]. For general m_i, we do not find analytical solutions and it is instructive to study the system using the numerical approach.

8.1 Asymptotic boundary region

We can express the metric in the form (4.7) by making the coordinate transformation $d\rho = dy\, \mathrm{e}^u$. We have the boundary conditions $u(y) \sim v(y) \sim -\log y$ and $\phi \to 0$ for small y. The leading terms of the Taylor expansion of the solution of (8.7) are given by

$$\begin{aligned}
e^{2u} &= \frac{1}{4g^2\, y^2}\, (1 + \frac{\epsilon \sum_{i=1}^{4} m_i\, g}{3c^2}\, y^2 + \cdots) \,, \\
e^{2v} &= \frac{c^2}{4g^2\, y^2}\, (1 - \frac{2\epsilon \sum_{i=1}^{4} m_i\, g}{3c^2}\, y^2 + \cdots) \,, \\
e^{\frac{1}{2}\vec{\phi}} &= 1 - \frac{\epsilon \sum_{i=1}^{4} \vec{a}_i\, m_i\, g}{2c^2}\, y^2 + \cdots \,,
\end{aligned} \tag{8.8}$$

where c is an arbitrary integration constant. As in the cases of $D = 7$ and 6, the normalizable solutions for ϕ_1 and ϕ_2 imply that the dual operators are not turned on. Next, we examine how these solutions flow from the above boundary into the bulk.

8.2 $\mathbf{AdS}_2 \times H^2$ and $\mathbf{AdS}_2 \times S^2$

For $\epsilon = \pm 1$, we have not obtained the general solution for arbitrary m_i. However, a class of special solutions results from setting $m_2 = m_3 = m_4$ [11]. This enables one to consistently set $\phi_1 = \phi_2 = \phi_3 \equiv \phi$. For this truncation, fixed-point solutions for $\epsilon = \pm 1$ are given by

$$\begin{aligned} \mathrm{e}^{2\phi} &= \frac{3m_2 - m_1 \pm \sqrt{(m_1 - m_2)(m_1 - 9m_2)}}{2m_2}\,, \qquad \mathrm{e}^{-2v} = 4g\,\epsilon\,\frac{\sinh\phi}{m_1\,\mathrm{e}^{-2\phi} - m_2}\,, \\ u &= \tfrac{1}{2}g\left(\frac{2(m_1\,\mathrm{e}^{-\frac{3}{2}\phi} + 3m_2\,\mathrm{e}^{\frac{1}{2}\phi})}{m_2 - m_1\,\mathrm{e}^{-2\phi}}\sinh\phi + \mathrm{e}^{\frac{3}{2}\phi} + 3\mathrm{e}^{-\frac{1}{2}\phi}\right)\rho = \frac{2}{R_{\rm ads}}\,\rho\,. \end{aligned} \tag{8.9}$$

This solution was discussed in detail in [24, 11]. The reality condition of the solution implies that for $\epsilon = -1$, corresponding to H^2, we must have either $m_2 > 0$ and $0 < m_1 \le m_2$ or $m_2 < 0$ and $m_2 \le m_1 < -3m_2$. For $\epsilon = 1$, corresponding to S^2, we must have $m_2 \le 0$ and $m_1 > -3m_2$. In general, the AdS_2 radius depends on g and three of the four charge parameters.

8.3 Interpolating from $\mathbf{AdS}_2 \times H^2$ to $\mathbf{AdS}_4$

As in the previous cases, $\mathrm{AdS}_2 \times H^2$ can occur at the asymptotic boundary of one solution and the horizon geometry of another. This can be demonstrated using Taylor expansion. For a concrete example, let us consider $m_1 = 2$ and $m_2 = m_3 = m_4 = 3$. As shown in the previous subsection, we have $\phi_i = \phi$. Near the asymptotic boundary, the solution behaves as

$$e^{2u} = \tfrac{\sqrt{2}}{25}\,y^{-2}\,(1 + c\,y^{2/5} + \cdots)\,, \quad e^{2v} = \sqrt{2}\,(1 + \cdots)\,, \quad e^{\frac{1}{2}\phi} = 2^{1/4}\,(1 - \tfrac{7}{6}\,c\,y^{2/5} + \cdots). \tag{8.10}$$

Clearly this solution will run into a singularity in the bulk.

On the other hand, for the solution with the $\mathrm{AdS}_2 \times H^2$ horizon, the near-horizon behavior is given by

$$e^{2u} = \tfrac{25}{\sqrt{2}}\,r^2\,(1 + c\,r + \cdots)\,, \quad e^{2v} = \sqrt{2}\,(1 - \tfrac{35}{19}\,c\,r + \cdots)\,, \quad e^{\frac{1}{2}\phi} = 2^{1/4}\,(1 + \tfrac{5}{38}\,c\,r + \cdots)\,, \tag{8.11}$$

where c is an integration constant. Using this as initial data for numerical calculation, we plot the result in Fig. 8. Thus, we see that the solution runs from $\mathrm{AdS}_2 \times H^2$ at the horizon to the AdS_4-like geometry (8.8) in the asymptotic region. A black hole solution which interpolates between $\mathrm{AdS}_2 \times H^2$ and the AdS_4-type geometry was first found in [5].

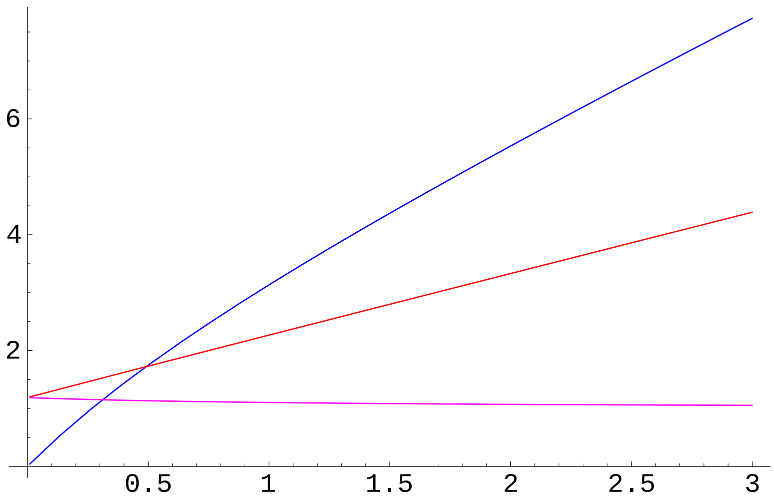

Figure 8: e^u (blue), e^v (red), and $e^{\phi_2/2}$ (purple) for a smooth solution running from $\mathrm{AdS}_2 \times H^2$ at the horizon to an asymptotic AdS_4-type geometry. $m_1 = 2$, $m_2 = m_3 = m_4 = 3$, $g = 1$, and $c = -0.5$.

8.4 Interpolating from $\mathbf{AdS}_2 \times S^2$ to $\mathbf{AdS}_4$

Analogously, $\mathrm{AdS}_2 \times S^2$ can also either occur at the horizon region of a solution and at the asymptotic region of another. As an example, let us consider $m_1 = 7$ and $m_2 = m_3 = m_4 = -2$, which implies that $\phi_i = \phi$. For the solution with $\mathrm{AdS}_2 \times S^2$ in its asymptotic region, the behavior can be studied using Taylor expansion:

$$e^{2u} = \tfrac{\sqrt{7}}{100}\,(1 + c\,r^{6/5} + \cdots)\,, \quad e^{2v} = \tfrac{\sqrt{7}}{4}\,(1 + \cdots)\,, \quad e^{\frac{1}{2}\phi} = 7^{1/4}\,(1 - \tfrac{11}{18}\,c\,r^{6/5} + \cdots)\,, \tag{8.12}$$

where c is an arbitrary integration constant. Clearly, the flow of this solution from the boundary leads to a singularity in the bulk.

For the solution with $\mathrm{AdS}_2 \times S^2$ at its horizon, the near-horizon behavior is given by

$$e^{2u} = \tfrac{100}{\sqrt{7}}\,r^2\,(1 + c\,r + \cdots)\,, \quad e^{2v} = \tfrac{\sqrt{7}}{4}(1 - \tfrac{55}{41}\,c\,r + \cdots)\,, \quad e^{\frac{1}{2}\phi} = 7^{1/4}\,(1 + \frac{15}{82}\,c\,r + \cdots)\,, \tag{8.13}$$

where c is an integration constant. Using this as initial data, we perform numerical calculation. The plots are presented in Fig. 9. From the plots, we see that the solution runs smoothly from the $\mathrm{AdS}_2 \times S^2$ horizon to the AdS_4-like asymptotic region given in (8.8).

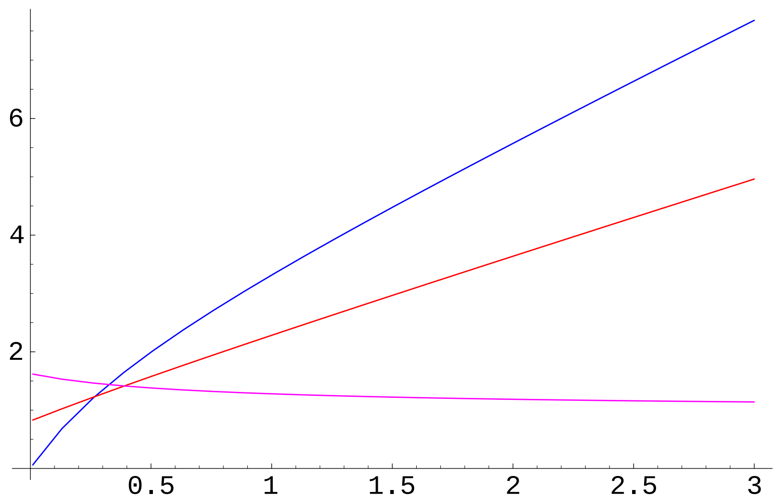

Figure 9: e^u (blue), e^v (red), and $e^{\phi_2/2}$ (purple) for a smooth solution running from $\mathrm{AdS}_2 \times S^2$ at the horizon to an asymptotic AdS_4-type geometry. $m_1 = 7$, $m_2 = m_3 = m_4 = -2$, $g = 1$, and $c = -1.5$.

9 String and M-theory origins

Gauged supergravities in $D = 4, 5, 6$ and 7 can be obtained from consistent sphere reductions of M-theory, type IIB or massive type IIA supergravities. Thus, it is straightforward to lift our solutions back to higher dimensions and study their properties. A particularly interesting class of solutions which we obtained are those that smoothly interpolate between $\mathrm{AdS}_{D-2} \times \Omega_2$ at the horizon and AdS_D-type spacetime in the asymptotic region. When lifted to higher dimensions, the asymptotic region becomes a direct product of AdS_D with a relevant sphere [3]. On the other hand, the horizon region becomes a warped product of AdS_{D-2} with an internal metric that is an S^p bundle over S^2 for $p = 4, 5$ and 7. We shall look at these cases in detail. Some of the results have been reported in a recent letter [24].

9.1 $D = 7$ solutions embedded in M-theory

It is straightforward to lift the $\mathrm{AdS}_5 \times S^2$ and $\mathrm{AdS}_5 \times H^2$ solutions given by (5.12) to $D = 11$ by using the ansatz obtained in [20]. Since the solutions for general m_i are

[3]Except in the case of massive type IIA supergravity, for which the AdS_6 embedding involves a singular warp factor [34, 35].

rather complicated to present, we only consider a representative example of $\mathrm{AdS}_5 \times S^2$ with $m_1 = 5g$ and $m_2 = -3g$. The M-theory metric is given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{1}{3}} \Big[ds_{\mathrm{AdS}_5}^2 + \frac{1}{g^2\, c} \Big\{ \frac{1}{4c} (d\theta^2 + \sin^2\theta\, d\varphi^2) \\ &\quad + \frac{1}{\Delta} \Big(\tfrac{1}{4}\, d\mu_0^2 + \tfrac{1}{5} (d\mu_1^2 + \mu_1^2\, (d\phi_1 - \tfrac{5}{2} \cos\theta\, d\varphi)^2) \\ &\quad + d\mu_2^2 + \mu_2^2\, (d\phi_2 + \tfrac{3}{2} \cos\theta\, d\varphi)^2 \Big) \Big\} \Big] \,, \end{aligned} \tag{9.1}$$

where $c = 10^{-2/5}$ and μ_i are spherical coordinates which satisfy $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. The warp factor Δ, which is not to be confused with the dilaton coupling parameter in section 3, is given by

$$\Delta = c\, (4\mu_0^2 + 5\mu_1^2 + \mu_2^2) > 0 \,. \tag{9.2}$$

The AdS_5 metric is given by

$$ds_{\mathrm{AdS}_5}^2 = e^{-\frac{2\rho}{R}}\, dx^\mu\, dx_\mu + d\rho^2 \,, \tag{9.3}$$

where the AdS radius is $R = \frac{1}{2c\, g}$. Note that, for all lifted solutions, g is defined as in sections 4-7.

The case of $m_1 = m_2$ provides an example of $\mathrm{AdS}_5 \times H^2$ which is easily embedded in M-theory. The corresponding eleven-dimensional metric is given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{1}{3}} \Big[ds_{\mathrm{AdS}_5}^2 + (\tfrac{3}{4})^{3/5} \frac{1}{g^2} \Big\{ d\theta^2 + \sin^2\theta\, d\varphi^2 + d\psi_1^2 \\ &\quad + (\tfrac{3}{4})^{-4/5} \tfrac{1}{\Delta} \cos^2\psi_1 \Big(d\psi_2^2 + \cos^2\!\psi_2\, (d\phi_1 - \tfrac{1}{2} \cosh\theta\, d\varphi)^2 \\ &\quad + \sin^2\!\psi_2\, (d\phi_2 - \tfrac{1}{2} \cosh\theta\, d\varphi)^2 \Big) \Big\} \Big] \,. \end{aligned} \tag{9.4}$$

The warp factor Δ is given by

$$\Delta = (\tfrac{3}{4})^{-4/5} \sin^2\psi_1 + (\tfrac{3}{4})^{1/5} \cos^2\psi_1 > 0 \,, \tag{9.5}$$

and the AdS radius is $R = (\frac{3}{4})^{4/5}\, \frac{2}{g}$.

In the above geometries, the internal space of the $D = 11$ metric can be viewed as an S^4 bundle over S^2 or H^2, with two diagonal $U(1)$ bundles [4]. In general, the

[4]Supersymmetric AdS solutions of M-theory with internal spaces which can be viewed as S^4 bundles over various spaces have also been discussed in [36, 37, 38].

internal metric can be labelled by the two diagonal monopole charges $(q_1, q_2) = (\frac{2m_1}{m_1+m_2}, \frac{2m_2}{m_1+m2})$. In the specific example of $\mathrm{AdS}_5 \times S^2$ above, $(q_1, q_2) = (5, -3)$ and the solution is smooth everywhere. For general (m_1, m_2), the metric does not have a power-law singularity. However, it could have a conical orbifold singularity, which is absent only if (q_1, q_2) are integers. Since q_i satisfy the constraint $q_1 + q_2 = 2$, it follows that they are either both even or both odd integers. In the even case, the bundle is topologically trivial, whilst it is twisted for the odd case. The internal space can be regarded as having a generalized holonomy group [39], since it is not Ricci flat and involves a form field.

The solution preserves $\mathcal{N} = 2$ supersymmetry; it is a supergravity dual to an $\mathcal{N} = 1$, $D = 4$ superconformal field theory on the boundary of AdS_5. This provides a concrete framework to study $\mathrm{AdS}_5/\mathrm{CFT}_4$ from the point of view of M-theory.

The seven-dimensional single-charge 3-brane given by (3.23) and (3.28) can also be lifted to M-theory. The corresponding eleven-dimensional metric is given by

$$ds_{11}^2 = \Delta^{1/3} ds_7^2 + \frac{1}{g^2 \Delta^{2/3}} \Big[H^{-1/5} \Delta\, d\theta^2 + H^{2/5} \cos^2\theta\, d\tilde{\Omega}_2^2 + H^{-3/5} \sin^2\theta\, (d\phi_1 + A_{(1)})^2 \Big], \tag{9.6}$$

where

$$\Delta = H^{3/5} \sin^2\theta + H^{-2/5} \cos^2\theta > 0\,, \qquad dA_{(1)} = \Omega_{(2)}\,. \tag{9.7}$$

9.2 $D = 6$ solutions embedded in massive IIA theory

Using the reduction ansatz in [31, 35], it is straightforward to lift the $\mathrm{AdS}_4 \times S^2$ and $\mathrm{AdS}_4 \times H^2$ solutions given by (6.9) up to $D = 10$ massive IIA supergravity. The metric is given by

$$\begin{aligned} ds_{10}^2 \;\; = \;\; & \mu_0^{\frac{1}{12}}\, X_0^{\frac{1}{8}}\, (X1\, X_2)^{\frac{1}{4}}\, \Delta^{\frac{3}{8}} \Big[ds_6^2 + g^{-2}\, \Delta^{-1} \Big(X_0^{-1}\, d\mu_0^2 \\ & + X_1^{-1}\, (d\mu_1^2 + \mu_1^2\, (d\varphi_1 + g\, A_{(1)}^2)^2) + X_2^{-1}\, (d\mu_2^2 + \mu_2^2\, (d\varphi_2 + g\, A_{(2)}^1)^2) \Big) \Big]\,, \end{aligned} \tag{9.8}$$

where $\Delta = \sum_{\alpha=0}^{2} X_\alpha\, \mu_\alpha^2 > 0$ and $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. Thus, the $D = 10$ metric is a warped product of AdS_4 with an internal six-metric, which is an S^4 bundle over S^2 or H^2, depending on the charge parameter p according to the rule (6.10).

As an example of a supersymmetric, though singular, compactification of AdS_4 from massive IIA theory, we can take $m_1 = 7g$, $m_2 = -5g$ and a negative sign in

(6.9). This gives $X_0 = 6c$, $X_1 = 7c$ and $X_2 = c$, where $c = 6^{-1/4}\, 7^{-3/8}$. Also, $A^1_{(1)} = -\frac{7}{2g}\cos\theta\, d\varphi$, $A^2_{(1)} = \frac{5}{2g}\cos\theta\, d\varphi$ and the radius of AdS$_4$ is given by $R = 1/(4c\, g)$. On the other hand, $m_1 = m_2$ provides an embedding of AdS$_4 \times H^2$ in massive IIA theory.

The six-dimensional single-charge membrane given by (3.23) and (3.32) can also be lifted to massive IIA theory. The corresponding ten-dimensional metric is given by

$$
\begin{aligned}
ds_{10}^2 = (\cos\theta\cos\psi)^{1/12} H^{1/64}\Big(\Delta^{3/8} ds_6^2 + \frac{1}{g^2\Delta^{5/8}}\Big[H^{-1/4}\Delta\, d\theta^2 + \\
H^{3/8}\cos^2\theta\, d\tilde{\Omega}_2^2 + H^{-5/8}\sin^2\theta\,(d\phi_1 + A_{(1)})^2\Big]\Big),
\end{aligned}
\tag{9.9}
$$

where

$$
\Delta = H^{5/8}\sin^2\theta + H^{-3/8}\cos^2\theta > 0\,, \qquad dA_{(1)} = \Omega_{(2)}\,. \tag{9.10}
$$

Note that the overall warping factor depends on two internal coordinates.

9.3 $D = 5$ solutions embedded in IIB theory

The AdS$_3 \times S^2$ and AdS$_3 \times H^2$ solutions given by (7.14) can be lifted to ten-dimensional IIB theory with the reduction ansatz obtained in [20]. Since the solution with general m_i is complicated to present, we consider a simpler case with $m_2 = m_1$. The ten-dimensional metric is

$$
\begin{aligned}
ds_{10}^2 &= \sqrt{\Delta}\Big\{ds^2_{\mathrm{AdS}_3} + \epsilon\, g^{-2}\big(\frac{m_1}{m_3 - 2m_1}\big)^{1/3}\,(\tfrac12 q_1\, d\Omega_2^2 + d\theta^2) \\
&\quad + g^{-2}\,\Delta^{-1}\Big[c^{-1/3}\,\cos^2\theta\,\Big(d\psi^2 + \sin^2\psi\,(d\varphi_1 + \tfrac12 q_1\, A_{(1)})^2 \\
&\quad + \cos^2\psi\,(d\varphi_2 + \tfrac12 q_1\, A_{(1)})^2\Big) + c^{2/3}\,\sin^2\theta\,(d\varphi_3 + \tfrac12 q_3\, A_{(1)})^2\Big]\Big\}\,,
\end{aligned}
\tag{9.11}
$$

where

$$
\begin{aligned}
&c = \Big|\frac{m_1}{2m_1 - m_3}\Big|\,, \qquad \Delta = c^{1/3}\,\cos^2\theta + c^{-2/3}\sin^2\theta > 0\,, \qquad dA_{(1)} = \Omega_{(2)}\,, \\
&ds^2_{\mathrm{AdS}_3} = e^{-\frac{2\rho}{R}}\,(-dt^2 + dx^2) + d\rho^2\,, \qquad R = \Big|\frac{2m_1}{g\,(4m_1 - m_3)\, c^{1/3}}\Big|\,.
\end{aligned}
\tag{9.12}
$$

We have introduced the charge parameters $q_i = 2m_i/(m_1 + m_2 + m_3)$, and hence they satisfy the constraint $q_1 + q_2 + q_3 = 2$. In the above solution, if $|m_3| < 2|m_1|$, we should have $\epsilon = -1$, corresponding to H^2; if $|m_3| > 2|m_1|$, we should have $\epsilon = 1$, corresponding to S^2. In general, the internal metric is an S^5 bundle over S^2 or H^2, depending the values of the q_i according to the above rules.

It is especially simple to lift the five-dimensional equal-three-charge string given by (2.15), due to the absence of scalars. The corresponding ten-dimensional metric is given by

$$ds_{10}^2 = ds_5^2 + \frac{1}{g^2}\Big[d\theta^2 + \cos^2\theta\Big(d\psi^2 + \sin^2\psi(d\phi_1 + \frac{1}{3}A_{(1)})^2 + \cos^2\psi(d\phi_2 + \frac{1}{3}A_{(1)})^2\Big) + \sin^2\theta(d\phi_3 + \frac{1}{3}A_{(1)})^2\Big], \tag{9.13}$$

where $dA_{(1)} = \Omega_{(2)}$.

For the five-dimensional three-equal-charged string with a nontrivial dilaton, given by (7.10), the corresponding ten-dimensional metric is

$$ds_{10}^2 = \sqrt{\Delta}ds_5^2 + \frac{1}{g^2\sqrt{\Delta}}\Big[\Delta H^{1/3}d\theta^2 + H^{2/3}\sin^2\theta(d\phi_1 + \frac{1}{3}A_{(1)})^2 + H^{-1/3}\cos^2\theta\Big(d\psi^2 + \sin^2\psi(d\phi_2 + \frac{1}{3}A_{(1)})^2 + \cos^2\psi(d\phi_3 + \frac{1}{3}A_{(1)})^2\Big)\Big], \tag{9.14}$$

where

$$\Delta = H^{1/3}\cos^2\theta + H^{-2/3}\sin^2\theta > 0\,, \qquad dA_{(1)} = \Omega_{(2)}\,. \tag{9.15}$$

For five-dimensional single-charge string given by (3.23) and (3.34), the corresponding metric is

$$ds_{10}^2 = \sqrt{\Delta}ds_5^2 + \frac{1}{g^2\sqrt{\Delta}}\Big[H^{1/3}\Delta\, d\theta^2 + H^{-1/3}\cos^2\theta\, d\Omega_3^2 + H^{2/3}\sin^2\theta(d\phi_3 + A_{(1)})^2\Big], \tag{9.16}$$

where Δ and $A_{(1)}$ are given by (9.15).

9.4 $D = 4$ solutions embedded in M-theory

We use the reduction ansatz obtained in [20] to lift the $\mathrm{AdS}_2 \times S^2$ and $\mathrm{AdS}_2 \times H^2$ solutions given by (8.9) back to $D = 11$, with the metric

$$\begin{aligned} ds_{11}^2 &= \Delta^{2/3}\Big\{ds_{\mathrm{AdS}_2}^2 + \frac{\mathrm{e}^{2v}}{(m_1+3m_2)g}d\Omega_2^2 \\ &\quad + \frac{1}{g^2\,\Delta}\Big[\mathrm{e}^{-\frac{3}{2}\phi}\Big(d\mu_1^2 + \mu_1^2(d\phi_1 + \frac{\epsilon\, m_1}{m_1+3m_2}A_{(1)})^2\Big) \\ &\quad + \mathrm{e}^{\frac{1}{2}\phi}\sum_{i=1}^{3}\Big(d\mu_i^2 + \mu_i^2(d\phi_i + \frac{\epsilon\, m_2}{m_1+3m_2}A_{(1)})^2\Big)\Big]\Big\}\,, \end{aligned} \tag{9.17}$$

where

$$\begin{aligned}
\Delta &= (\mathrm{e}^{\frac{3}{2}\phi} - \mathrm{e}^{-\frac{1}{2}\phi})\mu_1^2 + \mathrm{e}^{-\frac{1}{2}\phi} > 0\,, \quad dA_{(1)} = \Omega_{(2)}\,, \quad ds^2_{\mathrm{AdS}_2} = -e^{-\frac{2\rho}{R}}\, dt^2 + d\rho^2\,,\\
R &= \frac{2}{g}\Big[\frac{2(m_1\,\mathrm{e}^{-\frac{3}{2}\phi} + 3m_2\,\mathrm{e}^{\frac{1}{2}\phi})}{m_2 - m_1\,\mathrm{e}^{-2\phi}}\sinh\phi + \mathrm{e}^{\frac{3}{2}\phi} + 3\mathrm{e}^{-\frac{1}{2}\phi}\Big]^{-1}\,. \qquad (9.18)
\end{aligned}$$

In general, the nine-dimensional internal metric is an S^7 bundle over S^2 or H^2, depending the values of the m_i. This is an especially interesting example of a space with generalized holonomy group, since nine-dimensional Ricci-flat manifolds do not have an irreducible special holonomy group.

The four-dimensional single-charge black hole given by (3.26) and (3.36) can also be embedded in M-theory, with the corresponding eleven-dimensional metric

$$\begin{aligned}
ds^2_{11} = \Delta^{2/3} ds_4^2 + \frac{H^{1/4}}{g^2\Delta^{1/3}}\Big[\Big(1 + (H^{-1/2} - 1)\sin^2\theta\cos^2\varphi\Big)d\theta^2 +\\
\cos^2\theta\,\Big(1 + (H^{-1/2} - 1)\sin^2\varphi\Big)d\varphi^2 + \frac{1}{2}(H^{-1/2} - 1)\sin(2\theta)\sin(2\varphi)\,d\theta\,d\varphi +\\
H^{-1/2}\cos^2\theta\cos^2\varphi\, d\Omega_3^2 + \sin^2\theta\,(d\phi_1 + A_{(1)})^2 + \cos^2\theta\sin^2\varphi\, d\phi_2^2\Big], \qquad (9.19)
\end{aligned}$$

where

$$\Delta = H^{-1/4}(\sin^2\theta + \cos^2\theta\sin^2\varphi) + H^{1/4}\cos^2\theta\cos^2\varphi > 0\,, \quad dA_{(1)} = \Omega_{(2)}\,. \qquad (9.20)$$

10 Conclusions

We have investigated a large class of supersymmetric magnetic brane solutions supported by $U(1)$ gauge fields in gauged supergravities of dimensions $4 \le D \le 7$. We have obtained first-order equations by using a superpotential approach. These equations admit stationary $\mathrm{AdS}_{D-2} \times \Omega^2$ solutions, where Ω^2 can be S^2 or H^2 depending on the values of the $U(1)$-charges. The $U(1)$-charges q_i satisfy the condition

$$q_1 + q_2 + \cdots + q_n = 2\,. \qquad (10.1)$$

For $D = 7$ and 6, a maximum of two $U(1)$ charges are allowed and the rule for having S^2 or H^2 can be clearly summarized by (5.13). For $D = 5$ and 4, the maximum number of charges can be 3 and 4, respectively, and the complete rule remains to be specified, although we have obtained explicit examples of S^2 and H^2. Such constraints show that magnetic branes in AdS gauged supergravity are rather different from those in ungauged supergravities since, for the latter case, charges are unconstrained integration constants. The radius of AdS_{D-2} depends on the D-dimensional cosmological constant and $(n-1)$ charge parameters.

The fraction of supersymmetry preserved by our brane solutions is the same as for those in ungauged supergravities, and depends on the number of $U(1)$ field strengths involved. Embedded in maximal supersymmetric theories, the preserved fraction of the supersymmetry is 2^{-N}, for $N = 1$, 2, or 3 field strengths. In the case of $N = 4$, the introduction of the fourth field strength does not break further supersymmetry and hence the preserved fraction is 1/8.

Our first-order equations support two types of solutions that run from the aforementioned stationary ones. The first type has a geometry which approaches $\mathrm{AdS}_{D-2}\times \Omega^2$ in the asymptotic region and the solution runs into a singularity at small distance. The second type, which is more interesting, has a near-horizon geometry of $\mathrm{AdS}_{D-2} \times \Omega^2$ and smoothly runs to an AdS_D-type geometry (1.1) in the asymptotic region. The non-constant scalar fields are bounded. This implies the existence of a large class of conformal field theories in diverse dimensions whose renormalization group flows from the UV conformal fixed point to the IR conformal fixed point. The fact that, from the same set of equations, $\mathrm{AdS}_{D-2}\times\Omega^2$ arises as the horizon geometry in one type of solution and the asymptotic geometry in another, suggests that the stationary solution lies at the inflection point of the modulus space. Thus, a phase transition can occur, determining whether the system runs into a singularity or an AdS_D-type metric.

The asymptotic AdS_D-type geometry of the second type of solution has a boundary of $\mathrm{Minkowski}_{D-3} \times \Omega^2$. The case of $D = 7$ arises as a vacuum solution of a six-dimensional gauged supergravity constructed in [23]. This vacuum solution is of particular interest since the $SU(2)$ Yang-Mills field can be obtained from the internal S^2 without introducing a non-vanishing cosmological constant. Similar possibilities have also been observed for S^3 and brane-world Kaluza-Klein reduction of string theory [40]. However, unlike these present solutions, the reduction discussed in [40] is non-supersymmetric and singular. Of course, the asymptotic AdS_D-type geometry

(1.1) is not an exact solution everywhere, since it runs to $\mathrm{AdS}_5 \times S^2$ in the bulk. We do not expect that there exists a consistent Kaluza-Klein reduction of $D = 7$ gauged supergravity to the $D = 6$ theory of [23]. Rather, the latter theory should be viewed as the effective theory only at the boundary of our solutions. Since there are also solutions that smoothly run from $\mathrm{AdS}_5 \times H^2$ to an AdS_7-type geometry with the boundary $\mathrm{Minkowski}_4 \times H^2$, we expect that there exists an effective gauged supergravity theory in $D = 6$ that admits such a vacuum solution.

ACKNOWLEDGMENTS

We are grateful to Gary Gibbons, Chris Pope and Ergin Sezgin for useful discussions.

References

[1] J.M. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. 2 231 (1998); Int. J. Theor. Phys. **38** 1113 (1999), hep-th/9711200.

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, Phys. Lett. **B428** 105 (1998), hep-th/9802109.

[3] E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. **2** 253 (1998), hep-th/9802150.

[4] L.J. Romans, Nucl. Phys. **B383** 395 1992.

[5] M.M. Caldarelli and D. Klemm, *Supersymmetry of anti-de Sitter black holes*, Nucl. Phys. **B545** 434 (1999), hep-th/9808097.

[6] A.H. Chamseddine and W.A. Sabra, *Magnetic strings in five dimensional gauged supergravity theories*, Phys. Lett. **B477** 329 (2000), hep-th/9911195.

[7] D. Klemm and W.A. Sabra, *Supersymmetry of black strings in $D = 5$ gauged supergravities*, Phys. Rev. **D62** 024003 (2000), hep-th/0001131.

[8] A.H. Chamseddine and W.A. Sabra, *Magnetic and dyonic black holes in $D = 4$ gauged supergravity*, Phys. Lett. **B485** 301 (2000), hep-th/0003213.

[9] J. Maldacena and C. Nunez, *Supergravity description of field theories on curved manifolds and a no go theorem*, Int. J. Mod. Phys. **A16** 822 (2001), hep-th/0007018.

[10] C. Nunez, I.Y. Park, M. Schvellinger and T.A. Tran, *Supergravity duals of gauge theories from F(4) gauged supergravity in six dimensions*, JHEP **0104** 025 (2001), hep-th/0103080.

[11] J.P. Gauntlett, N. Kim, S. Pakis and D. Waldram, *Membranes wrapped on holomorphic curves*, Phys. Rev. **D65** 026003 (2002), hep-th/0105250.

[12] W.A. Sabra, *Magnetic branes in d-dimensional AdS Einstein-Maxwell gravity*, Phys. Lett. **B545** 175 (2002), hep-th/0207128.

[13] W.A. Sabra, *Curved branes in AdS Einstein-Maxwell gravity and killing spinors*, Phys. Lett. **B552** 247 (2003), hep-th/0210265.

[14] S.L. Cacciatori, D. Klemm and W.A. Sabra, *Supersymmetric domain walls and strings in $D = 5$ gauged supergravity coupled to vector multiplets*, hep-th/0302218.

[15] K. Behrndt, A.H. Chamseddine and W.A. Sabra, *BPS black holes in $N = 2$ five dimensional AdS supergravity*, Phys. Lett. **B442** 97 (1998), hep-th/9807187.

[16] D. Birmingham, *Topological black holes in anti-de Sitter space*, Class. Quant. Grav. **16** 1197 (1999), hep-th/9808032.

[17] D. Klemm, *BPS black holes in gauged $N = 4$, $D = 4$ supergravity*, Nucl. Phys. **B545** 461 (1999), hep-th/9810090.

[18] K. Behrndt, M. Cvetič and W.A. Sabra, *Non-extreme black holes of five dimensional $N = 2$ AdS supergravity*, Nucl. Phys. **B553** 317 (1999), hep-th/9810227.

[19] M.J. Duff and J.T. Liu, *Anti-de Sitter black holes in gauged $N = 8$ supergravity*, Nucl. Phys. **B554** 237 (1999), hep-th/9901149.

[20] M. Cvetič, M.J. Duff, P. Hoxha, James T. Liu, H. Lü, J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati, Tuan A. Tran, *Embedding AdS black holes in ten and eleven dimensions*, Nucl. Phys. **B558** 96 (1999), hep-th/9903214.

[21] W. Pauli, Wissenschaftlichter Briefwedisel, Vol. IV, Part II (1999), Springer-Verlag, edited by K.V. Meyenn.

[22] E. Cremmer and J. Scherk, *Spontaneous Compactification Of Extra Space Dimensions*, Nucl. Phys. **B118** 61 (1977).

[23] H. Nishino and E. Sezgin, *The Complete $N = 2$, $D = 6$ Supergravity With Matter And Yang-Mills Couplings,* Nucl. Phys. **B278**, 353 (1986).

[24] S. Cucu, H. Lü and J.F. Vázquez-Poritz, *A supersymmetric and Smooth Compactification of M-theory to AdS_5*, hep-th/0303211.

[25] M.S. Bremer, M.J. Duff, H. Lü, C.N. Pope and K.S. Stelle, *Instanton cosmology and domain walls from M-theory and string theory*, Nucl. Phys. **B543** 321 (1999), hep-th/9807051.

[26] S.S. Gubser, *Curvature singularities: the good, the bad and the naked*, Adv. Theor. Math. Phys. **4** 679 (2002), hep-th/0002160.

[27] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Renormalization group flows from holography– supersymmetry and a c-theorem*, Adv. Theor. Math. Phys. **3** 363 (1999), hep-th/9904017.

[28] H. Lü, C.N. Pope, E. Sezgin and K.S. Stelle, *Stainless super p-branes*, Nucl. Phys. **B456** 669 (1995), hep-th/9508042.

[29] J. Rahmfeld, *Extremal Black Holes as Bound States*, Phys. Lett. **B372** 198 (1996), hep-th/9512089.

[30] H. Lü, C.N. Pope, T.A. Tran and K.W. Xu, *Classification of p-branes, NUTs, waves and intersections*, Nucl. Phys. **B511** 98 (1998), hep-th/9708055.

[31] M. Cvetič, S.S. Gubser, H. Lü and C.N. Pope, *Symmetric potentials of gauged supergravities in diverse dimensions and Coulomb branch of gauge theories*, Phys. Rev. **D62** 086003 (2000), hep-th/9909121.

[32] P. Kraus, F. Larsen and S.P. Trivedi, *The Coulomb branch of gauge theory from rotating branes*, JHEP **9903** 003 (1999), hep-th/9811120.

[33] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Continuous distributions of D3-branes and gauged supergravity*, JHEP **0007** 038 (2000), hep-th/9906194.

[34] A. Brandhuber and Y. Oz, *The D4-D8 brane system and five dimensional fixed points*, Phys. Lett. **B460**, 307 (1999), hep-th/9905148.

[35] M. Cvetič, H. Lü and C.N. Pope, *Gauged six-dimensional supergravity from massive type IIA*, Phys. Rev. Lett. **83** 5226 (1999), hep-th/9906221.

[36] B.S. Acharya, J.P. Gauntlett and N. Kim, *Fivebranes wrapped on associative three-cycles*, Phys. Rev. D **63** 106003 (2001), hep-th/0011190.

[37] J.P. Gauntlett, N. Kim and D. Waldram, *M-fivebranes wrapped on supersymmetric cycles*, Phys. Rev. D **63** 126001 (2001), hep-th/0012195.

[38] J.P. Gauntlett and N. Kim, *M-fivebranes wrapped on supersymmetric cycles. II*, Phys. Rev. D **65** 086003 (2002), hep-th/0109039.

[39] M.J. Duff and J.T. Liu, *Hidden Spacetime Symmetries and Generalized Holonomy in M-theory*, hep-th/0303140.

[40] J.T. Liu, H. Lü and C.N. Pope, *The radion mode in consistent brane-world reductions*, hep-th/0212037.