CERN-TH.6309/91
ACT-53
CTP-TAMU-90/91

On the Evaporation of Black Holes in String Theory

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Abstract

We show that, in string theory, the quantum evaporation and decay of black holes in two-dimensional target space is related to imaginary parts in higher-genus string amplitudes. These arise from the regularisation of modular infinities due to the sum over world-sheet configurations, that are known to express the instabilities of massive string states in general, and are not thermal in character. The absence of such imaginary parts in the matrix-model limit confirms that the latter constitutes the final stage of the evaporation process, at least in perturbation theory. Our arguments appear to be quite generic, related only to the summation over world-sheet surfaces, and hence should also apply to higher-dimensional target spaces.

CERN-TH.6309/91
ACT-53
CTP-TAMU-90/91
November 1991

1 Introduction and Summary

Recently a lot of attention has been paid to black-hole solutions of two-dimensional string theories [1, 2, 3], which are discretized by certain matrix models describing $c = 1$ conformal matter coupled to the Liouville mode. This connection is possible thanks to the discovery of non-critical strings [4, 5], that has opened the way for studying non-trivial string dynamics. These string theories are described by non-compact coset Wess-Zumino models on an arbitrary world-sheet [2, 4]. They provide a powerful theoretical laboratory for seeing how ideas about black hole physics fare in the context of string theory. In particular, the existence of an infinite set of 'W-hair' sufficient to maintain quantum coherence has been conjectured and demonstrated [6, 7]. Several authors [8] have now extended the black-hole construction to higher-dimensional target spaces by gauging more complicated groups,offering eventually the hope of understanding black holes in four-dimensional target space.

So far all known black-hole solutions are *static*, which restricts their physical significance. One expects that physical black holes are produced by collapsing matter, and that quantum effects make them evaporate [9, 10]. Therefore, static solutions cannot be the whole story. The main purpose of this note to argue that the static character of the black hole is a feature only of the classical string tree level, and that formulating the theory on *summed-up* higher-genus world-sheets leads to quantum instabilities, that correspond to evaporation and decay of the black hole, and do not have a finite-temperature interpretation. These instabilities manifest themselves as imaginary parts in string correlation functions arising from the regularisation of modular infinities [11]. These imaginary parts are absent for the discrete states in the corresponding $c = 1$ matrix model, confirming (at least within string perturbation theory) their interpretation as the end-points of black hole evaporation, and strengthening their role as guardian angels of quantum coherence [6, 7]. We shall concentrate on the two-dimensional black-hole case although our arguments appear to be quite generic to the divergences emerging from world-surface summations, and can be applied to higher-dimensional cases as well.

The outline of the paper is as follows: in section 2 we review briefly the origin of the imaginary parts in correlation functions in ordinary string theories. In section 3 we discuss some aspects of the two-dimensional strings that will be useful in our discussion. Section 4 is devoted to a discussion of string propagation in a Minkowskian black-hole background on the torus. We construct the corrections to the tree-level effective action coming from summing up genus-zero and -one world-sheet surfaces, and show explicitly the existence of imaginary mass-shifts for the black-hole solution. The latter are interpreted as a signal for evaporation induced by quantum effcts in target space-time. In section 5 we discuss the interpretation of the Euclidean (thermal) black hole, and in section 6 we briefly discuss higher-dimensional target-space black holes. Finally, we present some conclusions and discuss prospects for future progress in section 7.

2 Critical String Theory in Higher Genera

There is a rather extensive literature on this issue [12, 13, 14, 15]. Here we shall recall only the parts that are relevant for our purposes, namely the regularisation of modular infinities arising from summing over world-sheet tori in the one-string-loop approximation. The relevant analysis has been done first by Marcus [11], whose method will be followed here.

Consider an N-point tachyon amplitude in closed bosonic string theory. Unitarity requires factorisation in the sense that the amplitude exhibits poles whenever a particular combination of external momenta approaches an 'on-shell' value for any of the intermediate string modes. The formal origin of the amplitude poles can be traced back to the well-known operator product singularity, which occurs when two (or more) of the vertex operators approach each other.

Using the factorisation formula,

$$\Gamma(k_1, ..., k_n) \to \sum_{states} \Gamma_L(k_1, ..., k_l, -k) \frac{1}{k^2 + m^2 - i\epsilon} \Gamma_R(k, k_{l+1}, ...k_n) \tag{1}$$

and combining the tree and torus amplitudes, one can deduce a mass renormalisation for the intermediate massive string modes, which was discussed by Weinberg [16]. The relevant mass shift is given by the two-point function on the torus:

$$\delta m^2 = -\Gamma(k, -k) \tag{2}$$

The latter expression in general diverges when one sums over tori, as required in a string theory formulation. Such a procedure involves integration over the Teichmuller parameters $\tau = \tau_1 + i\tau_2$ which describe the various tori (e.g. τ_2 is related to the area of the torus). The modular infinities arise from the region of the τ_2 integration where $\tau_2 \to \infty$, i.e. large tori, and are, therefore, considered by many as 'infrared' infinities. As Marcus [11] observed, the relevant part of the divergences encountered in generic string amplitudes is similar in structure to the 'tachyon' infinity of the bosonic string. This means that even in the superstring case, where the tachyons are absent, there will be infinities of the form

$$\int_{M>R} \frac{d^2\tau}{\tau_2^2} \tau_2^{-12} \frac{1}{|\Delta_{12}|^2} \tag{3}$$

where the integration is over the part of the fundamental region which is above the cut-off R. The final results expressing regularised divergences turn out, as expected, to be independent of R. The rest of the τ-integration over $M' < R$ yields modular-convergent results that we shall not be concerned with in this note. The quantity Δ_{12} is a cusp form of weight 12, related to the Dedekind η-function by $\eta(\tau) = q^{\frac{1}{24}} \Pi_{n=1}^{\infty}(1 - q^n) \equiv \Delta_{12}^{\frac{1}{24}}$, in standard notation with $q = exp(i2\pi\tau)$. In the limit where $\tau_2 \to \infty$, $\Delta_{12} \to exp(-2\pi\tau_2)$. It is therefore evident that the modular-divergent part has the generic form

$$\int_{M>R} \frac{d^2\tau}{\tau_2^2} \tau_2^{-12} e^{4\pi\tau_2} \tag{4}$$

It should be stressed again that this type of divergence, although it is similar in form to the ones induced by tachyons in bosonic strings, exists independently of tachyons in the theory. This will be relevant for the case of two-dimensional strings, where the tachyons are *massless*. Divergences of the form (4) exist in that case as well.

Marcus [11] applied analytic continuation to regularise these modular divergences. He used the integral

$$\int_R^\infty \frac{dx}{x} x^{-\beta} e^{-\alpha x} \tag{5}$$

which was first evaluated at $\alpha > 0$, and then he analytically continued it to complex values of α. The result is,

$$\int_R^\infty \frac{dx}{x} x^{-\beta} e^{(\alpha + i\epsilon)x} = [\alpha e^{-i(\pi-\epsilon)}]^\beta \int_{-\alpha R - i\epsilon}^\infty \frac{dx}{x} x^{-\beta} e^{-x} \tag{6}$$

The imaginary part of the integral is independent of R, as expected, and constitutes the only remnant of the divergence [11]:

$$Im \int_R^\infty \frac{dx}{x} x^{-\beta} e^{(\alpha + i\epsilon)x} = \frac{\pi}{\Gamma(1+\beta)} \alpha^\beta \tag{7}$$

In this way , in ordinary string theories one evaluates the imaginary part of the one-loop induced cosmological constant of the bosonic string, expressing the *instability* of the false tachyonic vacuum.

Imaginary parts, similar to the one appearing in the cosmological constant, also appear in higher-point functions of massive string modes, and reflect decay of the massive states with life-times determined by $\Gamma = -Im(\delta m)$, where δm is the mass shift of the state in question.

In closed bosonic strings, imaginary parts also appear in the two- and three-point functions of the (massless) graviton-dilaton multiplet, whose mass is now shifted due to the induced cosmological constant term. This graviton mass shift is consistent with general covariance [17]. Such lowest-order computations of string amplitudes contain information about the string effective action, to lowest order in target-space derivatives, or equivalently to first order in the Regge slope parameter α'. The one-loop amplitudes, involving an integration over tori, yield in this way a cosmological constant term and a shift in the Einstein term of the effective string action [12, 14].

Standard computational techniques yield for the one-string-loop corrected effective action the following form:

$$S_{eff} = \int d^D x \sqrt{G} e^{\Phi} [(1 - \frac{3}{4\pi} C_{torus} \quad \int_M \quad d^2\tau \tau_2^{-14} \frac{1}{|\Delta_{12}|^2} E(\tau, 1)) R + C_{torus} \quad \int_M \quad d^2\tau \tau_2^{-14} \frac{1}{|\Delta_{12}|^2} + ...] \tag{8}$$

where C_{torus} is a positive constant [14]. The modular τ-integration is over the fundamental region and the function $E(\tau, 1)$ is given by the formula [14]

$$E(\tau, 1) = lim_{\varepsilon \to 0} \frac{1}{\varepsilon} + 2\pi(\gamma - \frac{1}{2} ln\tau_2 - \frac{1}{12} ln|\Delta_{12}|) \tag{9}$$

in ζ-function regularisation, where γ is Euler's constant. The logarithmic $\frac{1}{\varepsilon}$ divergence in (9) is 'absorbed' in the dilaton tadpole, and corresponds to higher-genus corrections to tree-level beta-function equations (local infinities) [15]. This is a rather general rule [11]. Logarithmic divergences are the ones that cannot be regularised by analytic continuation. In addition to these divergences there are τ_2 modular infinities arising from the region of τ_2 integration where $\tau_2 \to \infty$.The latter are the types of divergences that are going to interest us in this note. They can be regularised by analytic continuation in the way outlined in (5), yielding imaginary parts of the form (7). Indeed, the leading τ_2 divergences in the cosmological constant and Einstein parts of the action (8) are similar in form, up to irrelevant proportionality constants,[1] and are of the type (5). The regulated expressions have imaginary parts which in ordinary bosonic string theories are attributed to the tachyonic contributions in the string tadpole graphs. However, as Marcus [11] noticed, such divergences exist also in superstring theories where they express the decay of massive string states. For instance, N-point superstring amplitudes involving massless modes as external states contain similar divergences expressing the decay rates of the exchanged states.

3 Aspects of Two-dimensional Bosonic Strings

The consistent formulation of closed bosonic string theory in two target-space dimensions is possible, provided one allows for non-trivial backgrounds. It is sufficient to have a non-zero dilaton background which is linear in the 'spatial' target space coordinate, associated with the Liouville mode [4, 5, 18]. The only propagating field in this string theory is the scalar 'tachyon' mode - which is however massless in two dimensions. The rest of the string modes are 'topological' in the sense that they make non-trivial contributions to amplitudes only for particular values of the energy and momenta [19]. There is a simple reason for this. Consider, for example, the first

[1]The leading divergence in the $E(\tau, 1)$ function comes from $-ln|\Delta_{12}|$ which behaves like τ_2 for large τ_2.

'massive' string multiplet that consists of the graviton-dilaton modes. The massive character of the multiplet is due to the cosmological constant terms in the action, as a result of the non-critical dimensionality of space-time. The graviton is massive with mass proportional to $Q = \sqrt{\frac{25-c}{3}} = 2\sqrt{2}$ $(c = 1)$. The conformal invariance conditions on a flat space read [20, 6]

$$\begin{aligned} q^{\mu}(q_{\mu} + Q_{\mu}) &= 0 \\ (q^{\mu} + Q^{\mu})h_{\mu\nu}(q) &= 0 \\ q^{\mu}\tilde{h}_{\mu\nu}(q + Q) &= 0 \end{aligned} \tag{10}$$

where $h_{\mu\nu}$ is the polarisation tensor for the graviton-dilaton multiplet, $Q^{\mu} = (Q, 0)$ and the tilde denotes Q-conjugation in the sense of [19] [2]. In two-dimensional target space, the above relations imply the decoupling of all the gravitons whose momenta are different from 0 or $-Q$,since they are longitudinal and hence can be gauged away. However, for the discrete values 0 or $-Q$ there is a discontinuity in the degrees of freedom, and extra modes become relevant. Due to the definite energy and momentum they possess, their propagators cannot be defined, since the latter involve analytic continuations off-mass-shell. It is in this sense that these modes are considered 'semi-topological' or 'of co-dimension two' [19]. The same is true for all the higher string modes. Each one of them is associated with a stringy *gauge* symmetry, leading to Ward identities satisified by the relevant string amplitudes defined on-shell [22, 6] [3]. Such Ward identities imply the gauging away of any massive mode whose momentum is off mass-shell, and this is the reason why in two dimensions the only remnants of the higher string states are *discrete* semi-topological modes [4].

The impossibility of applying analytic continuation to the mass-shell of the discrete massive states of the two-dimensional string implies the absence of any modular infinities in closed string loops associated with these modes. If this were not the case, then according to the Marcus analysis [11] it would be impossible to regulate such divergences either by absorbing them in tree-level σ-model coupling constants,

[2] In the original works on stringy representations of Liouville theory [21], people gave arguments for disregarding Q-conjugate states. However, it turns out that these states have physical significance. For instance the Q-gravitons constitute the last stage of black hole evaporation [6, 1], as we discuss below.

[3] In this sense two-dimensional target-space general covariance is considered as the gauge symmetry associated with the first excited string multiplet (graviton-dilaton). In the two-dimensional example, general covariance is expressed via Ward-identities of the form [22, 6] $(q^{\mu}+Q^{\mu}) < V^{G}_{\mu\nu}(q)\Pi_{j=1,\dots,N}V^{T}(k_j) >= 0$, for a closed string amplitude involving, say, one graviton and N tachyons.

[4] It should be noticed that the presence of these extra modes is necessitated by the same arguments of unitarity that require the factorisation of the convnentional S-matrix in string theory. A similar factorisation, but now involving the exchange of co-dimension two states, occurs in the two-dimensional string case [19, 23], thereby explaining the extra poles in tachyon scattering amplitudes of the $c = 1$ theory [19].

or by analytic continuation, and therefore the theory would be sick. Fortunately this is not the case. It can be shown, when considering tachyon amplitudes on the torus, that the only propagating field in the loop is again the tachyon whilst the massive states yield non-zero but finite contributions [19]. The latter result has also been confirmed in the Das-Jevicki [24] string field theory approach to the $c = 1$ matrix model, where it has been shown that the extra discrete states yield non-zero but *real* contributions to the scattering amplitudes of the massless propagating fields (tachyons) of the theory [25].

4 Minkowskian Two-dimensional Black Holes and the Summation over Riemann Surfaces

The previous Ward-identity arguments about the decoupling of higher string states except at *discrete* values of their momenta do not apply if some of the modes' polarisation tensors exhibit *singularities*, as is the case of target space-time black holes [1, 2]. In that case, the familiar Einstein terms in the effective action for graviton-dilaton modes appear, and one finds the black-hole solutions by the usual variational principles that one applies to dynamical graviton fields in ordinary point-like theories. It will be useful, for subsequent purposes, to recall some of the basic properties of the static black hole solutions in 2D string theory.

Such objects are found as solutions of the beta function equations for a bosonic σ-model to lowest order in the Regge slope parameter α'. It was Witten's observation [2] that such constructions on an arbitrary Riemann surface arise from appropriate gauging of a Wess-Zumino coset model [5] on $\frac{SL(2,R)}{U(1)}$ with the correspondence of $k-2$ to $\frac{1}{4\pi\alpha'}$, where k is the level parameter of the Wess-Zumino term.The lowest order (in α') solutions of the σ-model correspond therefore to the large-k limit of the group-theoretic model. The effective action at tree level for graviton-dilaton backgrounds reads

$$\int d^2x\sqrt{G}e^{\phi}(R+\Lambda+O[(\nabla\phi)^2]+...) \tag{11}$$

where for our discussion we ignore matter (tachyon) parts. In two-dimensional target spaces the non-trivial solutions of the equations of motion obtained from (11) are of black-hole type [1, 2], leading to singular metrics in a certain coordinate system, although, as usual, such singularities can be eliminated by going to an appropriate coordinate system. The black hole solutions are static and can be thought of as the classical final state of gravitational collapse of two-dimensional matter [26]. The mass of the black hole is essentially determined by an arbitrary constant α, which expresses a shift in the dilaton field. Indeed, if we make the change $\phi \to \phi + \alpha$, the mass of the black hole turns out to be [2] $M_{bh} = \sqrt{\frac{2}{k-2}}e^{\alpha}$. The family of black

[5] The Minkowskian black hole is obtained by gauging a non-compact subgroup of $SL(2,R)$.

hole solutions leads to invariant line elements in space-time given by the following expression [1]

$$ds^2 = (1 - Me^{Q\rho})dt^2 - \frac{1}{1 - Me^{Q\rho}}d\rho^2 \qquad (12)$$

where ρ is the space coordinate (Liouville mode). In the limiting case $M \to 0$, the black hole solution (12) becomes identical to a Q-graviton perturbation, which can, at least in this sense, be identified with the last stage of black hole evaporation. In addition to the graviton mode, all the rest of the higher massive string modes are also excited. This underpins the interpretation of the continuum version of the $c = 1$ matrix model as the final stage of the black-hole evaporation, as conjectured by Witten [2].

It is the purpose of this section to discuss the origin of the evaporation proccess. In ordinary local gravity theories the evaporation of a black hole is a *quantum* phenomenon which must therefore be associated with loop corrections [9, 10]. If a similar mechanism operates in our case, one should expect to see the evaporation process when one performs the sum over world-sheet genera, that represents the string analogue of the loop corrections to the gravity action. This is precisely what happens in our case, as we shall argue below. We shall demonstrate our arguments by restricting ourselves to the torus case, which is sufficient for our purpose.

There is also a formal reason for this. In a stringy formulation of two-dimensional quantum gravity, the Liouville field is usually considered as a free field whose space is unrestricted. However, since this mode is associated with the covariant short-distance cut-off in the theory, it is natural to think of it as being bounded from below at a value defining the cut-off, α, in a flat two-dimensional world-sheet [27]. Representing the covariant cut-off, then, as $e^{\rho}\alpha$, the ρ integration extends from 0 to ∞ [20]. These 'boundaries' in Liouville space have important consequences for Liouville energy non-conservation in string amplitudes, except in the torus case [20]. To see this in a simplified way, let us complexify the Liouville field by going to the $i\rho$-formalism, and therefore considering complex two-dimensional surfaces. Due to boundaries in the integration over the zero-modes of ρ, the result in string amplitudes is not a delta-function conservation of the energy associated with the coordinate ρ (in a Fourier expansion of the backgrounds) but rather resonant forms $\frac{1}{s}$ [19], where $s = \sum_i \varepsilon(k_i) + Q(g-1)$, with $\beta^{\mu} \equiv (\epsilon(k), k)$ being the two-vectors representing conformal charges in the Liouville and matter sectors, and the sum is over states in the relevant string amplitude. The residues of these resonances are the string amplitudes we are considering [19]. In any other topology except that of the torus, the Liouville energy conservation law is modified by the 'charge at infinity' Q [4, 19]. At genus one one there is *exact* energy conservation despite the presence of Q. This makes the contribution of this particular topology somewhat special. This also implies that the regularisation of the associated modular infinities coming from this topology could not be cancelled by higher genera.

After these parenthetic remarks we are now in a position to discuss the evaporation of the static black hole solution (12) induced by quantum effects in the torus case. Energy conservation, even in the in Liouville sector, implies that the torus computation can be considered formally identical to the one in critical strings outlined in previous sections. The only point that deserves attention concerns the role of the target-space dimensionality. In the case of closed bosonic strings living in non-critical dimensions D of target space-time the (modular invariant) torus partition function is given by [4]

$$Z_D = (2\tau_2)^{\frac{2-D}{2}}(\eta(\tau)\overline{\eta}(\tau))^{2-D} \tag{13}$$

Naively one expects no modular τ_2 infinities in $D = 2$, and indeed this is the case in matrix model backgrounds [29]. However, when computing correlation functions in Liouville theory with a world-sheet cosmological constant it seems necessary to continue analytically the matter central charge [28], which in turn implies a formal continuation away from the $D = 2$ value. Upon such a procedure, which could be viewed as the analogue of target-space dimensional regularisation, loop corrections to the string effective action acquire, in the limit $D \to 2^+$, *finite* imaginary parts from the regularisation of τ_2-modular infinities that appear in the case $D > 2$, as becomes clear from (13). Similarly to the critical string theory, the imaginary parts of the corrections to the Einstein term of the two-dimensional string are given by the $\alpha \to 0$ and $\beta \to 0$ limit of (7): the result is π. The imaginary parts of the torus correction to the (tree-level) cosmological constant, on the other hand, are given by (7) upon setting $\alpha = \varepsilon \to 0$, $\beta = 1$: the result is $\varepsilon\frac{\pi}{2}$ [6]. Hence, in the case of two-dimensional strings there are no imaginary parts in the one-string-loop corrected cosmological constant. This reflects the stability of the two-dimensional flat-space tachyonic vacuum, which is *massless*.

The important point is that the proportionality constant in front of the Einstein term can be identified with the mass of the black hole [2]. If a represents a constant shift in the dilaton, then, by computing the stress-tensor of the graviton-dilaton system (in the absence of matter) one can determine the (conserved) energy, i.e. mass, of the black hole as

$$M_{bh} = \sqrt{\frac{2}{k-2}}e^a \tag{14}$$

Therefore, the torus contribution is to shift the mass of the tree-level black hole by an amount related to an infinite integral over the moduli space of genus-one surfaces. The latter, as we have mentioned, has both logarithmic divergences, that can be absorbed in a renormalisation of the dilaton field at tree-level à la Fischler and

[6] It should be noted that this is the form of the imaginary parts in the correction to the Einstein term arising from regularisation of subleading τ_2 divergences. If there are singularities in the curvature terms, as is the case of the Minkowskian black hole solution in the asymptotic region where the dilaton approaches $-\infty$ [2], then the above analysis implies the existence of additional *finite* imaginary parts in the torus correction to the Einstein term in the effective action.

Susskind [15], and modular τ_2 infinities whose analytic continuation and regularisation yield imaginary parts in the black-hole mass shifts [7]. The situation, therefore, is similar in nature to what happens in critical string theories, where massive string states acquire complex mass-shifts in higher genera, reflecting their decay with a life-time inversely proportional in magnitude to the imaginary part of the pertinent mass-shift. In our case it is the Minkowski black-hole state that is *unstable* due to quantum effects, although classically, at the tree string-level, it is a stable background configuration. It is in this sense that we exhibit the evaporation of the two-dimensional quantum black hole. In this point of view, evaporation is expressed as an *instability* of the black hole vacuum with respect to stringy quantum corrections. It should be stressed that imaginary parts arising from the regularisation of modular τ_2-infinities appear in any target-space dimensionality $D \geq 2$, which gives a sort of universality to this decay mechanism. This mechanism for evaporation is purely stringy and has no counterpart in local gravity theories. The imaginary parts that express the instability arise from the regularisation of large-area tori, and therefore rely on the concept of an underlying world-sheet structure, i.e. string theory. This is to be contrasted with local point-like theories of gravity, where such phenomena do not occur. One could still refer to this type of process as 'Hawking radiation', due to the fact that it is triggered by *quantum* effects, but is not thermal as in the local field theory case.We cannot, however, yet exclude the possibility that thermal instabilities might arise in our picture in higher-dimensional target-spaces.

We can now answer questions concerning the manner of the black hole evaporation, as well as its final stage. As argued in [6, 7] there is an infinity of gauge conservation laws that accompany black hole solutions in two-dimensional strings, which express stringy gauge symmetries associated with the infinite tower of string states. These laws imply the existence of *conserved* charges that constitute the 'hair' of black holes. These quantum numbers, being expressible as total spatial derivatives, remain conserved during the evaporation process, thereby restricting the modes of black-hole radiation. It can be shown that the gauge group associated with these charges contains classical w_∞-symmetries [30] which preserve the phase-space area (two-dimensional volume) of the matrix model [31]. There is no loss of quantum coherence due to the evaporation process, for the reasons explained in [7]. From the observation that the vanishing mass limit of the black-hole solution describes the Q-graviton and the other discrete topological states of the two-dimensional string, as well as the fact that the contribution of the latter to scattering amplitudes determining the matrix-model target-space effective action contains *no imaginary* parts, we conclude that the continuum version of the $c = 1$ matrix model constitutes, at least in string perturbation theory, the final stage of black-hole evaporation.

[7] In string perturbation theory the real part of the one-string-loop correction to the Einstein-term cannot reverse the positive sign of the tree-level coefficient, so the combined result of the tree and one-loop string level computations can still be represented as an exponential of a shifted dilaton field.

Unfortunately, it is not known how to perform in the continuum language the sum over genera in closed string cases [32]. The matrix model approach for $c < 1$ looks helpful, but the situation concerning $c = 1$ matrix models is still unclear. However, in our case we have shown that higher-genus corrections make black hole solutions unstable *even in perturbation theory*, thereby implying their decay (evaporation). At least as far as two-dimensional continuum string theory is concerned, the perturbation theory result seems to indicate that the flat-space linear dilaton background solution of the latter is the final point of the evaporation. It is of course possible that non-perturbative effects lead to a different end-point, but such a possibility goes beyond the scope of this analysis.

A final comment we would like to make in this section concerns the possibility of regarding the static (classical) black hole solution as the result of some sort of gravitational collapse. This would be useful in considering the two-dimensional black hole as a laboratory for the study of higher-dimensional physically interesting cases, where such phenomena occur. In two dimensions the concept of collapsing matter is not obvious. The matter-stress tensor obtained naively from the Einstein tensor vanishes, since in two-dimensional target spaces $R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R$ vanishes identically. However, in string effective theories there are non-trivial dilaton terms that accompany the Einstein curvature term in the effective action. Their presence make the matter (tachyon) stress tensor non-zero even classically [6, 33]. Shifting the dilaton field by a constant defines a family of objects characterised by various masses. Thus collapsing matter could lead to a black-hole. By matching - in the boundary of the dust (matter) - the static black hole solutions studied in [2, 3], with the solutions obtained from non-zero matter-stress tensors, it can be shown [26] that the former correspond to the final state of such a collapse, at the classical level.

5 Euclidean Black Holes

Euclidean black holes are described by coset models in which the gauged subgroup of the $SL(2, R)$ is *compact.* They can be thought of as being obtained from the euclideanisation of the Minkowski time [2]. From the point of view of Liouville theory, Euclidean black holes correspond to a two-dimensional Liouville theory coupled to matter described by a field compactified on a circle [29, 34]. In such models the partition function on the torus is known to possess no modular τ_2 infinities, since the pertinent η-function factors cancel between Liouville and matter sectors [29]. In physical terms, this means that such black holes are static thermal objects which are in constant interaction with a heat bath, and therefore one does not expect them to lose any mass [3]. We cannot yet exclude the possibility that in higher-dimensional target spaces, there might be thermal instabilities of such objects in higher genera, due to the non-cancellation of η-factors that generate modular infinities. However, there is no example known so far that supports this idea. For instance, in certain string theories, like the open $D = 4$ string [35], it is known that below the Hagedorn temperature, although there exist thermal mass shifts which do not occur in the zero-temperature formalism [36], nevertheless they are modular finite. It might therefore be that there exists some sort of cancellation of 'infrared' infinities in thermal (static) string configurations, at least in certain cases [37]. However such an analysis falls beyond the scope of the present work.

Before closing this section it is worth mentioning another difference between the Euclidean and the Minkowski framework within the context of two-dimensional strings. In models with a *compactified* matter boson field (as is also the case of $c < 1$ matrix models) the form of the Ward identities that express certain stringy symmetries is modified with respect to the uncompactified case. The higher string modes do not decouple any longer due to the discrete matter momenta. Consider, for instance, the graviton-dilaton Ward identity expressing general covariance of the theory. In the compactified case it reads [22, 19]

$$(q^\mu + Q^\mu) < V^G_{\mu\nu}(q)\Pi_{j=1,\ldots,N}V^T(k_j) >= \sum_{j=1,\ldots,N} k_{j\nu} < V^T(k_j + q)\Pi_{i\neq j}V^T(k_i) > \quad (15)$$

The right hand-side part is a contact term and vanishes 'on-shell' for the uncompactified case due to analyticity properties (the well-known [38] 'cancelled propagator argument'). This is no longer true in the compact case due to the discrete momenta in the matter sector [19]. This can be interpreted formally as the breakdown of general covariance which is to be expected in a thermal theory. This is one of the main differences between Minkowskian and Euclidean formalisms of the $c = 1$ string theory.

6 Higher-Dimensional Target Spaces

Analogous descriptions of higher-dimensional singular space-times seem possible. Support for this was recently given in a number of works [8, 39] where it was shown that by gauging more complicated coset Wess-Zumino models (even supersymmetric ones [39]) on arbitrary Riemann surfaces, one can get interesting singular field configurations for higher dimensional target spaces, among which one finds black holes [40] and black strings [41]. In view of the arguments presented in this work, the summation over Riemann surfaces is expected to lead to instabilities in the Minkowski formalism. That this is indeed the case can be demonstrated explicitly by looking at the four-dimensional black hole of ref. [40] and the three-dimensional black string of [41]. Let us start from the latter.

The existence of the antisymmetric tensor field $B_{\mu\nu}$ (which is gauged away in two dimensions) leads to a conventional *axionic* charge Q for the black hole, in addition to its mass M. For completeness, we briefly outline the construction [41]. One adds a free boson field z to the Wess-Zumino action, which is equivalent to considering a group $G = SL(2, R) \times R$. Then one gauges an appropriate one-dimensional subgroup generated by that of the two-dimensional case [2] together with a translation in the boson field z. This leads to two arbitrary parameters in the problem, the Wess-Zumino level parameter k, and an extra parameter, λ, which is associated with the translation of z. The effective action describing gravitational dynamics contains again a non-trivial dilaton conformal factor acompanying the Einstein term. In non-singular curvature cases in dimensions higher than two, one can absorb these factors in a conformal rescaling of the graviton. However, we choose not to do it in singular cases such as black strings. The reason is that the arbitrariness in shifting the dilaton field by a constant α determines a family of solutions characterised by various values of Q and M [41],

$$\begin{aligned} Q &= e^{\alpha}\sqrt{\frac{2\lambda(1+\lambda)}{k}} \\ M &= e^{\alpha}(1+\lambda)\sqrt{\frac{2}{k}} \end{aligned} \tag{16}$$

The summation over higher genera will produce imaginary parts in α and therefore one has complex shifts for the black hole mass and axion charge, which are again interpreted as signal for evaporation. The *quantum* three-dimensional black string will therefore evaporate parts of its mass and axion charge. The exact conservation of these quantum numbers (due to their being total space derivatives) implies that the evaporated parts of the charges have to be carried away by particles emitted from the black hole.

In a similar way, one can study four-dimensional black hole solutions [40]. These may be obtained by twisted products of one Euclidean and one Minkowskian two-dimensional black hole. The antisymmetric tensor vanishes in this case, but the four-dimensional metric is off-diagonal. Again the role of the dilaton field is essential in defining families of solutions, and our arguments on the instabilities induced by higher genera apply as in the black string case. One can even construct direct higher-dimensional black holes by gauging more complicated Wess-Zumino models, involving antisymmetric tensor fields [42]. In such a case there is axionic charge on the black hole solutions. From standard field-theoretic arguments [43] one expects the three-dimensional black hole not to evaporate all of its axion charge. Similarly to the two-dimensional case, however, one expects quantum coherence to be restored, not because of the axion charge alone, but because of an infinity of hair provided by the higher string modes. In two dimensions this type of hair is phase-space area-preserving [7, 31]. In view of the general description of black holes by Wess-Zumino models, and the relation of the latter in certain cases to such area-preserving (w-type) symmetries [44] we conjecture that the existence of a phase-space volume-element-preserving symmetry is a general feature of such a Wess-Zumino theory, and thus coherence is maintained during the evaporation process. This fits in with the observation [45] that the Hawking temperature of an evaporating black hole is reduced as the amount of hair characterising the black hole is increased. This leads one to expect that a black hole with infinite hair evaporates at zero temperature [8]. However, this conjecture remains to be investigated further in future work.

Another comment concerns the extension of these ideas to supersymmetric cases. Supersymmetric Wess-Zumino models have been considered [46] with the result that the effective two-dimensional theory describes supersymmetric σ-models in black hole backgrounds. In space-time supersymmetric backgrounds there are non-renormalisation theorems [13] that prevent the Einstein terms from receiving corrections in higher genera. This is equivalent to the vanishing of the dilaton tadpoles in supersymmetric string theories. One might naively think that the breaking of space-time supersymmetry by the black hole background seems to be essential in allowing imaginary parts in the higher-genus corrections to the mass of the black hole, and hence leading to evaporation in a way similar to the bosonic case. However, in view of Marcus's analysis [11], instabilities of the kind discussed in the present work also appear in superstring theories, expressing in general the decays of massive states. An explicit example of such a situation is the type-I open superstring [47]. Thus we expect that a similar mechanism of non-thermal black hole decay will also operate in superstrings and heterotic strings.

7 Conclusions and Prospects

Let us now summarise the view of black hole quantum physics that has emerged from this analysis and our previous papers [6, 7]. We have identified an infinite set of gauge w-symmetries that are sufficient to maintain quantum coherence for two-dimensional black holes [6], and characterize an infinite set of 'topological states' that constitute the final states of black hole evolution described by a matrix model [48]. These w-symmetries have the geometrical interpretation [31] of preserving the phase-space volume-element of the matrix model, and thereby exclude [7] the general form of non-quantum-mechanical, non-Hamiltonian modification of the evolution equation for the density matrix [49], which would otherwise have caused all quantum-mechanical systems to appear 'open' as a conjectured consequence of microscopic space-time topology change [50]. In this paper we have shown that the evaporation of two-dimensional Minkowskian black holes can be understood as a quantum instability appearing in higher genera, analogously to the normal decays of massive string states [11]. This mechanism for Minkowskian black hole decay does not have a direct thermal interpretation, and, moreover, the two-dimensional Euclidean finite-temperature black hole solution [3] is *static* and does not exhibit this decay instability [29].

It is now appropriate to speculate on the possible extension of these results to four-dimensional black holes. As yet, there is no general characterisation of four-

[8]This is consistent with the fact that the two-dimensional black hole under consideration resembles an extreme Reissner-Nordstrom type [2].

dimensional stringy black holes at the conformal field theory level, but some partial results are becoming available [8]. These generalize, and often incorporate, the non-compact coset Wess-Zumino models describing the two-dimensional solutions. As such, we expect them to include and extend the w-symmetries that save quantum coherence in two dimensions. A very large set of gauge symmetries is in fact known to exist in generic four-dimensional string models [22], possibly in correspondence to the number of massive string states and providing an amount of 'hair' sufficient to quench the entropy usually associated with four-dimensional black holes [9]. It certainly seems that the intrinsically stringy non-thermal higher-genus black hole decay mechanism identified in this paper could carry over to four dimensions, although we cannot yet exclude the existence of additional thermal instabilities. It seems that the string answer to the conundrum of reconciling quantum mechanics with general relativity is at hand.

Acknowledgements

We would like to acknowledge useful discussions with J.L. Miramontes and J. Sanchez-Guillèn. The work of D.V.N. is partially supported by DOE grant DE-FG05-91-ER-40633.

[9]For another approach, see [51].

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