Asymptotic normalization coefficient of ^{8}B from breakup reactions and the S$_{17}$ astrophysical factor

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We show that asymptotic normalization coefficients can be extracted from one nucleon breakup reactions of loosely bound nuclei at 30-300 MeV/u. In particular, the breakup of ^{8}B is described in terms of an extended Glauber model. The ^{8}B ANC extracted from breakup data at several energies and on different targets, $C^2_{tot} = 0.450 \pm 0.039$ fm^{-1}, leads to the astrophysical factor $S_{17}(0) = 17.4 \pm 1.5$ eV·b for the key reaction for solar neutrino production ^{7}Be(p,γ)^{8}B. The procedure described provides an indirect method to determine reaction rates of astrophysical interest with beams of loosely bound radioactive nuclei.

^{8}B, produced in the ^{7}Be(p,γ)^{8}B reaction, is the major source of the high-energy neutrinos observed by solar neutrino detectors [1]. Results of recent direct and indirect determinations of the astrophysical factor $S_{17}(0)$ (see [2–6] and references therein) do not fully agree, but span the full adopted range, 19^{+4}_{-2} eV b [7]. The asymptotic normalization coefficient (ANC) approach [8] is based on the fact that the ^{7}Be(p,γ)^{8}B reaction is highly peripheral. Thus, what one needs most is a precise quantitative description of the tail or periphery of the ^{7}Be + $p \leftrightarrow$ ^{8}B overlap function, which is given by the corresponding ANC, rather than of the full many-body wave function inside the core. The advantage of this approach is that ANCs may be determined from other peripheral nuclear reactions that have cross sections orders of magnitude larger than the radiative capture reaction of interest. This technique has been applied to determine S_{17} from measurements of two (^{7}Be,^{8}B) proton transfer reactions [6], after being tested to give results to better than 9% [9] for ^{16}O(p,γ)^{17}F.

In the present paper we show that one can extract asymptotic normalization coefficients for the ^{8}B ground state wave function from data on one-proton removal (or breakup) reactions at energies of 30-300 MeV/u [10–13]. Improvements in the nuclear reaction model used make a quantitative description possible for the energy range and the ANC extracted reliable. Then, this ANC can be used to obtain an independent determination of the astrophysical factor $S_{17}(0)$.

Recent advances in experimental techniques permit the selection of the final state of the remaining core in breakup reactions at large energies. Typically an exotic nucleus $B = (Ap)$, where B is a bound state of the core A and the proton p, is produced by fragmentation from a primary beam, separated and used to bombard a secondary target. After the breakup occurs, the breakup cross section and the parallel momentum distribution of the core A are measured. By detecting the final state of the core, e.g. using coincident gamma rays, one can determine important spectroscopic information about the ground state of the exotic projectile [14,15]. The measured momentum distribution of the core can be related to the momentum distribution of the bound nucleon.

In all reactions where the core survives (either transfer or one-nucleon breakup) the matrix elements include the overlap integral $I^B_{Ap}(\vec{r})$ for the nuclei A, p, and B, obtained after the integration over the internal coordinates of fully antisymmetric wave functions, with $\vec{r}$ the vector connecting the center of mass of nucleus A with p [16,8]. The overlap integrals are not normalized to unity, but to the spectroscopic factors S_{nlj}. At asymptotic distances where nuclear forces are vanishingly small, $r > R_N$, the overlap integrals behave as

$$I^B_{Aplj}(r) \to S^{1/2}_{nlj}\varphi_{nlj}(r) \to C^B_{Aplj}\frac{W_{-\eta,l+1/2}(2kr)}{r}. \quad (1)$$

Here C^B_{Aplj} is the asymptotic normalization coefficient defining the amplitude of the tail of the overlap integral, W is the Whittaker function, k is the wave number, and η is the Sommerfeld parameter for the bound state (Ap). The asymptotic normalization coefficients C^B_{Aplj} can be extracted from any peripheral observables that are measured experimentally.

For the reaction model calculations we assume that the ground state of the projectile (J^π) can be approximated by a superposition of configurations of the form $[\Psi_c^{\pi_c} \otimes nlj]^{J^\pi}$, where $\Psi_c^{\pi_c}$ denote the core states and nlj are the quantum numbers for the single particle wave function $\varphi_{nlj}(r)$ in a spherical mean field potential. These single particle states are normalized to unity and have the asymptotic behavior given by Eq. 1, with the single particle asymptotic normalization coefficients b_{nlj}. When more than one configuration contributes to a selected core state, the total cross section for one-nucleon breakup is written as an incoherent superposition of single particle cross sections:

$$\sigma_{-1p} = \sum S(c,nlj)\sigma_{sp}(nlj). \quad (2)$$

A similar relation holds for the momentum distribution. Typically the nucleon is not measured, therefore the calculated cross sections $\sigma_{sp}(nlj)$ contain a stripping term (the loosely bound nucleon is absorbed by the target and the core is scattered and detected), a diffraction dissociation term (the nucleon is scattered away by the target, the core is scattered by the target and is detected) and a Coulomb dissociation term:

$$\sigma_{sp} = \int_0^\infty 2\pi b db (P_{str}(b) + P_{diff}(b)) + \sigma_{Coul} \quad (3)$$

In previous analyses of breakup reactions, a structure for the projectile (the spectroscopic factors $S(c,nlj)$) was assumed and agreement between the calculated and experimental values was considered a validation both of the assumed nuclear structure and of the reaction model calculations used [11,17,18]. In the present case we use an extended Glauber model, in the eikonal approximation with non-eikonal correction terms up to the second order [10,19]. Realistic nucleon-target and core-target S-matrix elements are used in the evaluation of the impact parameter dependent probabilities. We find that the largest contributions to the cross sections $\sigma_{sp}(nlj)$ come from large impact parameters and therefore the phenomena are peripheral. It then follows that we can express the results in terms of the asymptotic normalization coefficients and reverse the process: we can use the experimental results to extract the ANCs.

Calculations were done for several ^{8}B breakup reactions for which data exist in the literature. The model is similar to that developed by Bertsch et al. [20,21]. It has been tested before on 23 different reactions in the *p-sd* shell [10,22]. The loosely bound proton and the core moving on an eikonal trajectory interact independently with the target nucleus, an assumption valid at these energies. For the proton-target interaction we used that of Jeukenne, Lejeune and Mahaux (JLM) [23], in the updated version of Bauge et al. [24]. For the target-core nucleus-nucleus interaction we use the double folding procedure described in [25]: the same JLM interaction is folded with Hartree-Fock nuclear matter distributions of the core and of the target. Subsequently the double folding potentials were renormalized to reproduce a variety of elastic scattering data for light nuclei. We found there that the real part needed a substantial renormalization at about 10 MeV/u (N_V=0.366), but the imaginary part did not (N_W=1.00). We have checked the procedure on a much wider set of data from literature at higher energies, and found a similar conclusion for the imaginary part, while the renormalization of the real part approaches unity around 50 MeV/u. Therefore, in the present calculations we adopted the procedure of [25], with the JLM(1) interaction, and N_W=1.00. The S-matrix calculations that enter the first two terms of Eq. 3 are expected to depend primarily on the imaginary part of the interaction. Indeed, calculations with large variations of the renormalization of the real potential (N_V=0.366 and N_V=0.80) give the same cross sections. The integrated Coulomb term is treated in a perturbative method that retains the dipole and quadrupole terms, equivalent with that of Ref. [26], but using radial matrix elements calculated with realistic Woods-Saxon radial wave functions.

The impact parameter dependence of the first two terms in Eq. 3 is plotted in Fig. 1 for the case of the breakup of ^{8}B on a Si target at 38 MeV/u [10]. Clearly, both terms are dominated by the periphery but have contributions from small impact parameters. The Coulomb term is even more peripheral, due to its long range. To investigate the influence of the nuclear interior on the extracted ANC, different single particle wave functions were used for the outer proton to calculate the breakup cross section in the same reaction model. We chose a range of radii and diffusenesses ($R = 2.20 - 2.60$ fm and $a = 0.50 - 0.70$ fm) for the Woods-Saxon potentials and repeated the calculations. A correct spin-orbit term was included, and in all cases we adjusted the depth of the potential to reproduce the proton separation energy $S_p = 137$ keV. The radial behavior of $1p_{1/2}$ and $1p_{3/2}$ orbitals is identical at large distances and, for a given (R,a), differs at small radii by much less than the variation associated with the choice of Woods-Saxon potential. Thus, for simplicity, only the $1p_{3/2}$ component was included, and we rewrite Eq. 2 as:

$$\sigma_{-1p} = (S_{p_{3/2}} + S_{p_{1/2}})\sigma_{sp} = (C^2_{p_{3/2}} + C^2_{p_{1/2}})\sigma_{sp}/b_p^2, \quad (4)$$

where b_p is the ANC of the normalized $1p_j$ radial single particle wave function. The experimental value for the breakup cross section is 222±15 mb. The calculated cross section varies from 226 mb to 326 mb with the choice of the single particle wave function used (i.e. (R,a) the radius and diffuseness parameters of the Woods-Saxon potential used), which is equivalent to 44% variation if a spectroscopic factor $S_{tot} = S_{p_{3/2}} + S_{p_{1/2}}$ is extracted using the first part of Eq. 4. However if the square of the ANC $C^2_{tot} = C^2_{p_{3/2}} + C^2_{p_{1/2}}$ is extracted instead, the result is very stable, as shown in Fig. 2 (where for convenience the results are plotted against the single particle ANC b_p calculated for each geometry assumed). The variation in the ANC over the full range considered here is <±3%. A flat curve for C^2 in Fig. 2 would be a signature of a purely peripheral reaction; the small slope reflects the participation of the interior of the nucleus. Similar calculations were done for the same target and two other energies (35 and 28 MeV/u), and for other targets (^{12}C, Sn and ^{208}Pb) at other energies: 40, 142 and 285 MeV/u [12,11]. Pictures similar to Fig. 2 are obtained in each and every case. These experiments have not determined the yield to the ^{7}Be first excited state. A Coulomb breakup experiment carried out at 50 MeV/u on a Pb target [14] found it to be small, about 5% of the total. From this we estimated the ANC for the core excitation part in the wave function, then calculated its contribution to the one-proton removal cross section on each target and subtracted it (e.g. 7.5% for the Si target). The corrected ANCs extracted are presented in Table I. With the exception of two data points on ^{12}C, there is good agreement among the ANCs which come from data over a wide range of incident energies and both low and high Z targets. In order to extract an average ANC, we have done an unweighted average of the individual measurements. Using all of the data points, this results in $C^2_{tot} = 0.450(30)$ fm^{-1} where the uncertainty is the standard error of the mean. If the two ^{12}C data points (4 and 5 in the table) at 40 and 142 MeV/u are removed from our average (note that they fail a simple test of the expected energy dependence), we find $C^2_{tot} = 0.456(14)$ fm^{-1}. Several correlated uncertainties must be added: 4% for the renormalization coefficients used for the optical model parameters, 3% for the variation in C^2 as a function of b_p and 2% for the uncertainty in the excited state contribution. Including these we find $C^2_{tot} = 0.450(39)$ fm^{-1} for the full data set and $C^2_{tot} = 0.456(28)$ fm^{-1} when the two ^{12}C data points are removed. The values are similar and, without additional information, we shall adopt the first.

Using a wave function with the asymptotic normalization as extracted above, we have calculated the distribution of the parallel momentum of the core measured in the breakup of ^{8}B at 41 MeV/u on a ^{9}Be target [13]. The result is compared in Fig. 3 with the experimental data. A very good description is obtained. Notably, unlike the black disk model [27] calculation which describes the width of the parallel momentum distribution [28], the extended Glauber model calculation also matches the large momentum tails due to the nuclear interior. This gives further credibility to our calculations and the entire approach.

The value extracted here from the breakup of ^{8}B at 30-300 MeV/u agrees very well with the one extracted from transfer reactions at 12 MeV/u [6]. We can use the ANC extracted to evaluate the astrophysical S factor for the reaction ^{7}Be(p,γ)^{8}B at very low energies, following the procedure of Ref. [8]. Using the value from breakup we find $S_{17}(0) = 17.4 \pm 1.5$ eV·b. We can also use the ANC extracted here to determine the rms radius for the ^{8}B proton halo, following the procedure of [28]. We find a $r_h = 4.20 \pm 0.21$ fm.

In conclusion, reliable spectroscopic information can be extracted from one-nucleon breakup reactions of loosely bound nuclei at energies around and above the Fermi energy. However, we have shown that, despite a more transparent meaning of the spectroscopic factors, the values obtained are not unambiguous, and a better quantitative description is achieved in terms of the asymptotic normalization coefficients. In turn, these can be used to calculate any observables that are dominated by the periphery of the nucleus, notably rms radii for halos and astrophysical S-factors. Calculations using an extended Glauber model for the breakup data of ^{8}B on a wide range of targets and energies lead to an unambiguous value for the ANC and an astrophysical factor $S_{17}(0)$ in very good agreement with the values from recent determinations from direct measurements [2] and with those using indirect methods [5,6]. New measurements for the elastic scattering of ^{8}B, a more accurate determination of the breakup cross sections (eventually separating the stripping and diffraction dissociation components) and a precise determination of the core excitation contribution, can increase the reliability of the ANC extracted. The validity of the procedure is wider than for the ^{8}B case discussed above. In addition to peripherality, ensured more or less for the halo nuclei, the requirements are good absolute values for the breakup cross sections, with the identification of the final state of the core, and reliable cross section calculations. The method can be used to extract valuable information for nuclear astrophysics. Very difficult or even impossible direct measurements that would involve bombarding short lived targets with very low energy protons can be replaced or supplemented by indirect methods seeking the relevant ANCs, rather than complete knowledge of the ground state wave function of these exotic nuclei. In addition, the indirect ANC method is subject to different systematic errors than the direct measurements, and therefore redundance of the results is very much welcome, particularly for critical astrophysical S-factors, such as that for the ^{7}Be(p,γ)^{8}B reaction.

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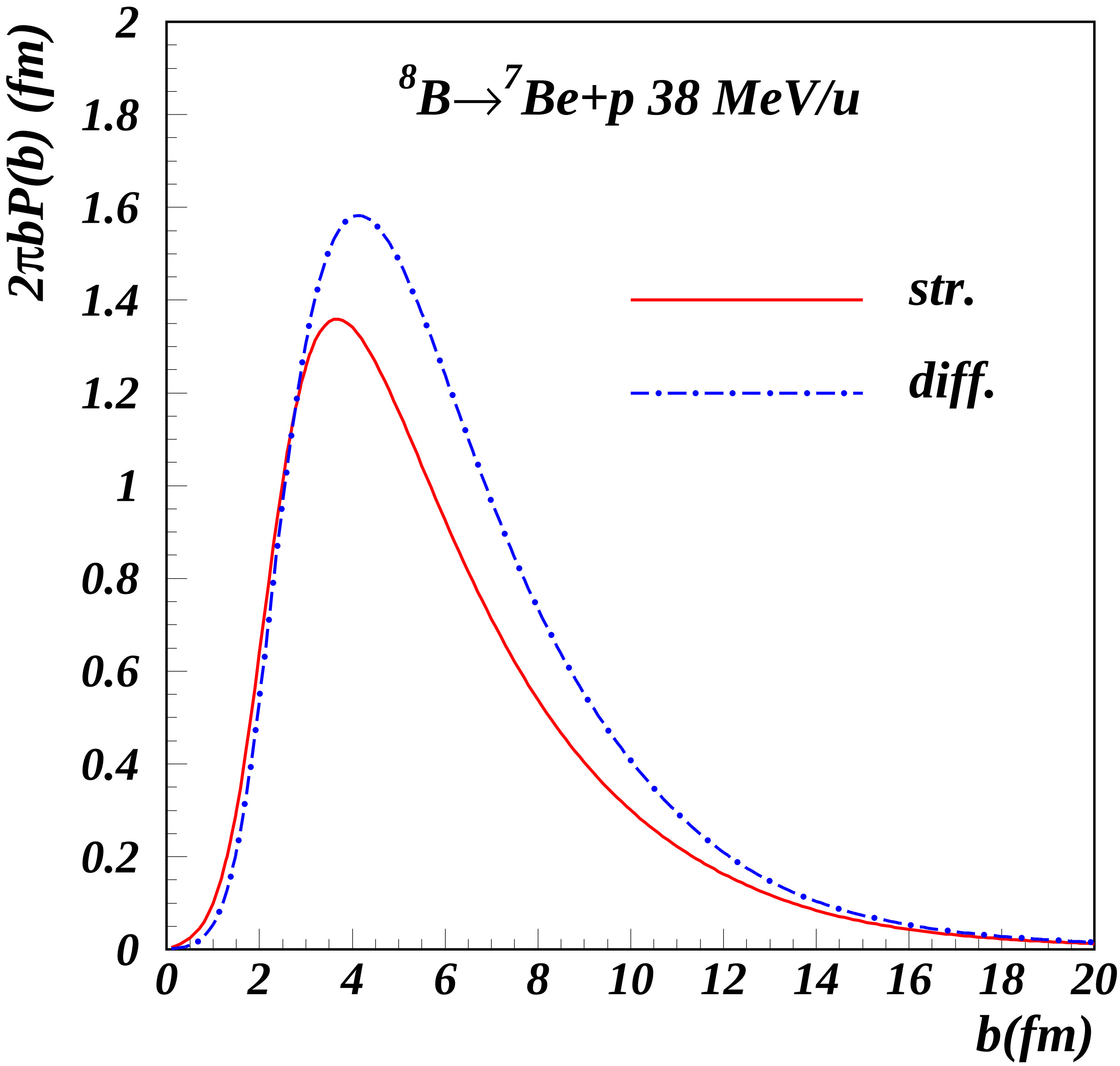
FIG. 1. The stripping and diffraction dissociation parts of the breakup probability as a function of the impact parameter.

FIG. 2. Comparison of the spectroscopic factors S_{tot}(dots) and of the ANC C^2_{tot} (triangles) extracted from the ^{8}B breakup data on Si at 38 MeV/u [10], for different parameters of the single particle Woods-Saxon potentials.

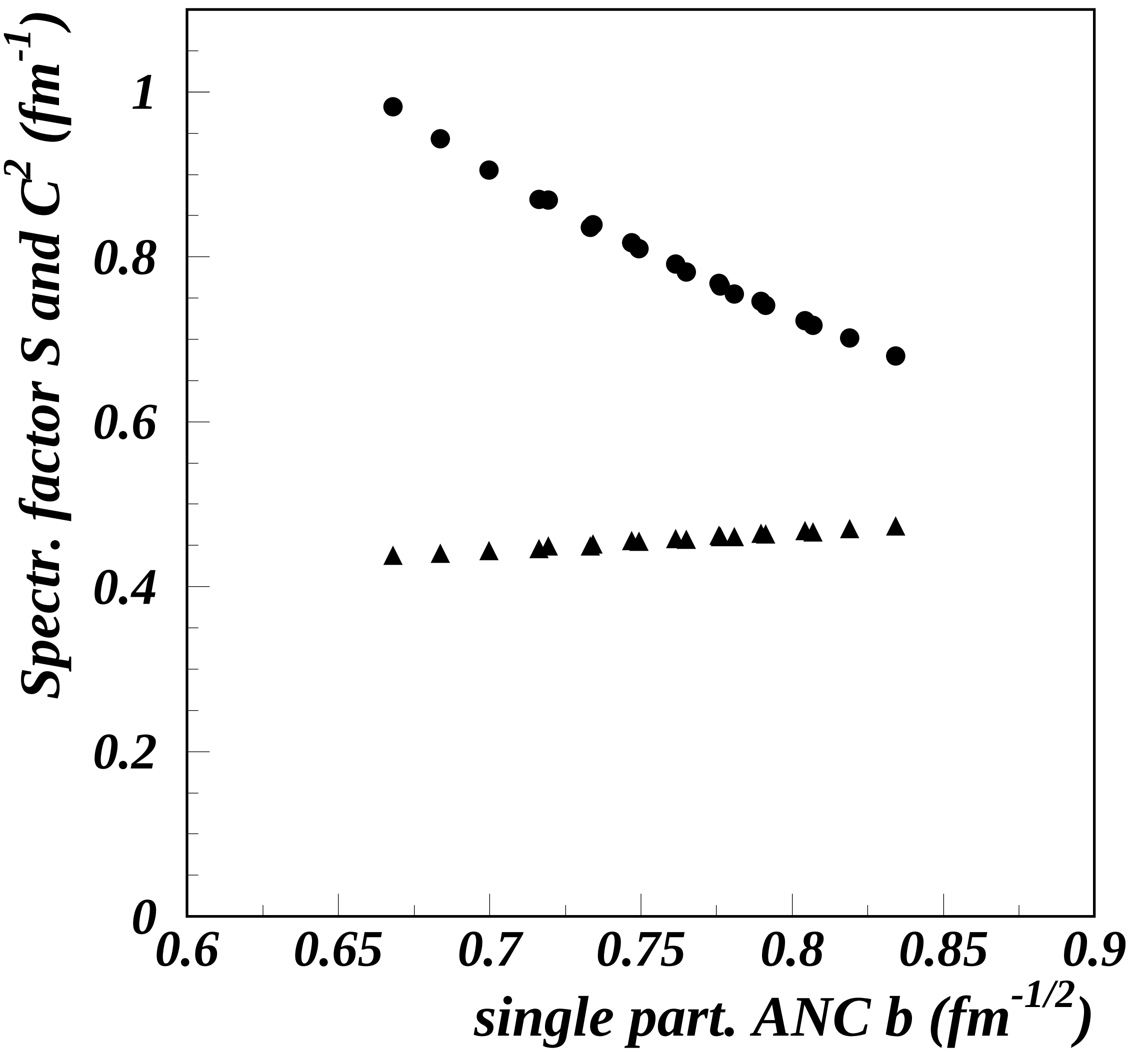
FIG. 3. Calculated parallel momentum distributions for core-like fragments in a breakup reaction of ^{8}B on a ^{9}Be target at 41 MeV/u, are compared with experimental data [13]. The curve labeled "intr" is the result of a calculation with the Serber model in the transparent limit, that labeled "disk" with the black disk model.

TABLE I. Summary of the ANC extracted from different ^{8}B breakup reactions.

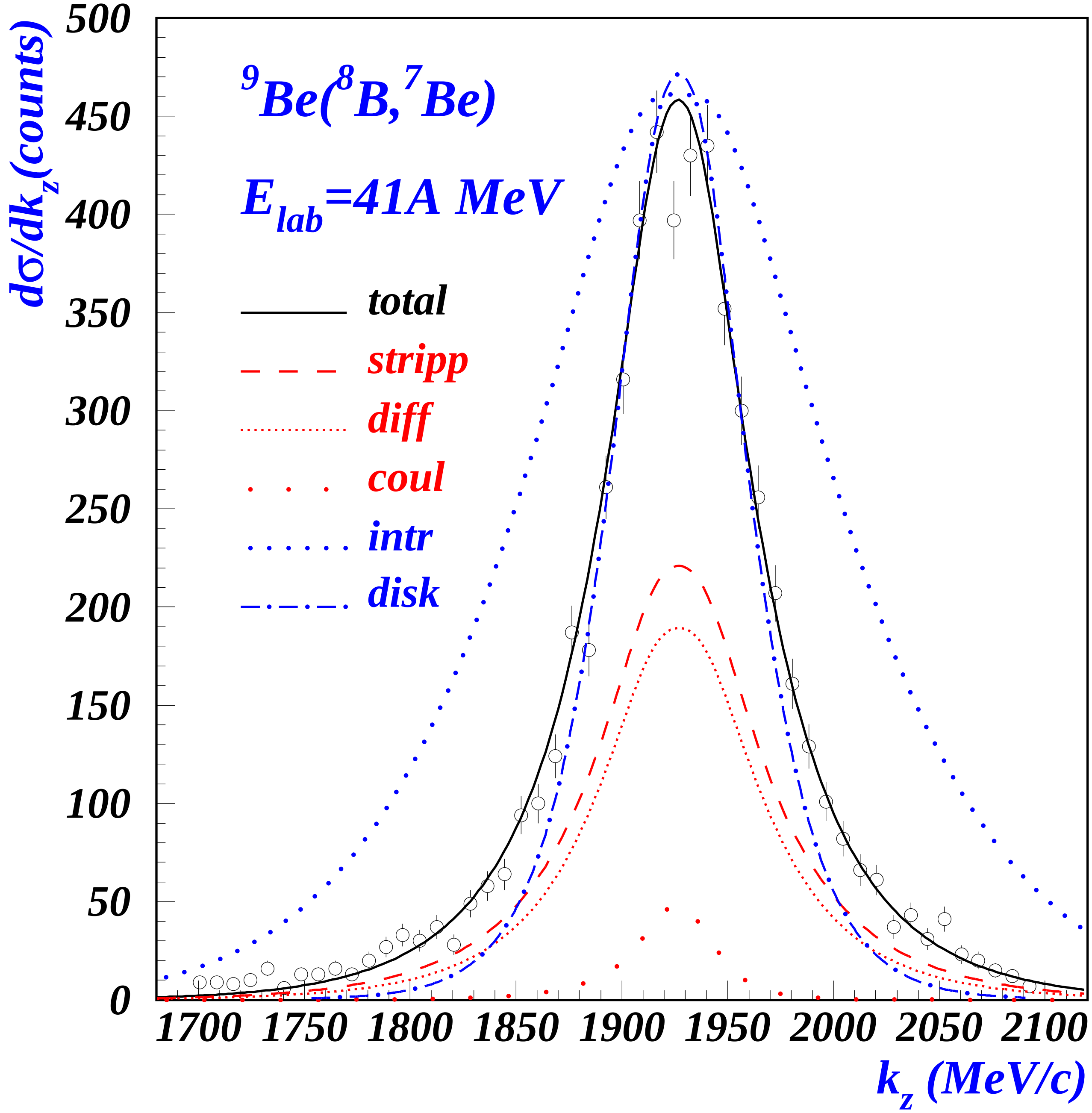
Target	E/A [MeV/u]	exp. c.s. [mb]	Ref.	C^2_{tot} [fm^{-1}]
^{28}Si	28	244(15)	[10]	0.435
	35	225(15)	[10]	0.420
	38	222(15)	[10]	0.423
^{12}C	40	80(15)	[12]	0.250
	142	109(1)	[11]	0.597
	285	89(2)	[11]	0.482
Sn	142	502(6)	[11]	0.547
	285	332(6)	[11]	0.464
^{208}Pb	142	744(9)	[11]	0.421
	285	542(9)	[11]	0.460
aver				0.450(30)



$^8B \rightarrow ^7Be+p$ 38 MeV/u
str.
diff.
$2\pi bP(b)$ (fm)
b(fm)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
2
4
6
8
10
12
14
16
18
20



Spectr. factor S and C² (fm⁻¹)
1
0.8
0.6
0.4
0.2
0
0.6
0.65
0.7
0.75
0.8
0.85
0.9
single part. ANC b (fm⁻¹/²)



$d\sigma/dk_z$(counts)
$^9Be(^8B,^7Be)$
E_{lab}=41A MeV
total
stripp
diff
coul
intr
disk
500
450
400
350
300
250
200
150
100
50
0
1700
1750
1800
1850
1900
1950
2000
2050
2100
k_z (MeV/c)