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From AdS Black Holes to Supersymmetric Flux-branes

H. Lü^{†*12}, C.N. Pope^{†1} and J.F. Vázquez-Poritz^{‡3}

[†]*George P. and Cynthia W. Mitchell Institute for Fundamental Physics,
 Texas A&M University, College Station, TX 77843-4242, USA*

^{*}*Interdisciplinary Center for Theoretical Study
 University of Science and Technology of China, Hefei, Anhui 230026*

[‡]*Physique Théorique et Mathématique, Université Libre de Bruxelles,
 Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium*

ABSTRACT

We show that AdS black hole solutions admit an analytical continuation to become magnetic flux-branes. Although a BPS AdS black hole generally has a naked singularity, the BPS flux-brane can be regular everywhere with an appropriate choice of $U(1)$ -charges. This flux-brane interpolates from $\text{AdS}_{D-2} \times H^2$ at small distance to an asymptotic AdS_D -type metric with an $\text{AdS}_{D-2} \times S^1$ boundary. We also obtain a smooth cosmological solution of de Sitter Einstein-Maxwell gravity which flows from $dS_2 \times S^{D-2}$ in the infinite past to a dS_D -type metric, with an $S^{D-2} \times S^1$ boundary, in the infinite future.

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1 Introduction

According to the AdS/CFT correspondence [1–3], solutions in gauged supergravities can provide dual pictures of certain quantum field theories. Supersymmetric and regular solutions are of particular interest, since then the supergravity approximation can be trusted. However, in both non-gauged and gauged supergravities, solutions with these properties are rare. In the latter case, one expects that the geometry of the solution should be asymptotic to AdS spacetime at large distance. Thus, the regularity of the solution depends on its small-distance behavior. The possible short-distance regular geometries are clearly limited. One possibility is AdS_D , with a different cosmological constant from the large-distance AdS_D . Such a solution, which is called a domain wall, is typically supported by a scalar potential with two fixed points. This type of potential is quite rare. An example in $D = 5$ gauged supergravity was given in [4]. A classification of $D = 4$ domain walls was given in [5, 6]. Another possibility is that the small-distance geometry is $\text{AdS}_{D-n} \times S^n$ or $\text{AdS}_{D-n} \times H^n$. Supersymmetric solutions with H^2 have been constructed in [7–16], and recent examples with S^2 were given in [15, 16]¹. These solutions are regular everywhere, since they interpolate from $\text{AdS}_{D-2} \times S^2$ or $\text{AdS}_{D-2} \times H^2$ at small distance to an asymptotic AdS_D form.

These solutions are supported by a set of $U(1)$ gauge fields carrying magnetic charges. Their electric duals, namely the AdS black holes, were constructed earlier [21–27]. By contrast, the electric solutions are typically singular. In this paper, we observe that there exists an analytical continuation in which the electric BPS AdS black hole becomes a supersymmetric magnetic flux-brane. Unlike the AdS black hole, the flux-brane is regular everywhere, interpolating between $\text{AdS}_{D-2} \times H^2$ at small distance to AdS_D with a boundary of $\text{AdS}_{D-1} \times S^1$ in the large-distance asymptotic region.

Thus, we see that the same $\text{AdS}_{D-2} \times H^2$ at small distance can interpolate to two different AdS_D -type spacetimes at large distance. For the magnetic ($D - 3$)-branes constructed in [7–16], the boundary of the AdS_D -type spacetime is $M_{D-3} \times H^2$, whilst for the flux-branes presently constructed, the boundary is $\text{AdS}_{D-1} \times S^1$.

¹Solutions with $n > 2$, for H^n mostly, were constructed in [17–20].

To understand why these two possibilities exist, let us write down the metric of $\text{AdS}_{D-2} \times H^2$, given by

$$ds_D^2 = d\rho^2 + e^{2\gamma\rho} dx^2 + dz^2 + e^{2\tilde{\gamma}z} (-dt^2 + dx^i dx^i). \quad (1.1)$$

There are two directions which can act as a “radial”-type coordinate, namely ρ and z , providing two different possible interpolation coordinates. Previous works have established that there can be an interpolation between the above $\text{AdS}_{D-2} \times H^2$ metric at the horizon and an AdS_D -type metric in the asymptotic region:²

$$ds_D^2 = dz^2 + e^{2z} (-dt^2 + dx^i dx^i + d\rho^2 + e^{2\gamma'\rho} dx^2). \quad (1.2)$$

In other words, asymptotically the geometry of AdS_D type, with a $\text{Minkowski}_{D-3} \times H^2$ boundary. In this case, the interpolation coordinate is z . Alternatively, we can use the other radial coordinate for interpolation, namely ρ . In this case, the natural guess for the asymptotic behavior must again be an AdS_D -type metric but with the boundary $\text{AdS}_{D-2} \times S^1$. The metric is of the form

$$ds_D^2 = d\rho^2 + e^{2\rho} (dx^2 + dz^2 + e^{2\tilde{\gamma}'z} (-dt^2 + dx^i dx^i)). \quad (1.3)$$

We find that such a solution indeed exists and is closely related to the AdS black hole. This can be seen by looking at the metric ansatz for these solutions:

$$ds_D^2 = a^2 dx^2 + d\rho^2 + c^2 ds_{\text{AdS}_{D-2}}^2, \quad (1.4)$$

which is simply the Wick rotation of the AdS black hole ansatz:

$$ds_D^2 = -\tilde{a}^2 dt^2 + d\rho^2 + \tilde{c}^2 d\Omega_{D-2}^2. \quad (1.5)$$

The resulting magnetic solution is called a flux-brane because the supporting $U(1)$ field strength is proportional to the volume of the whole transverse space spanned by the radial coordinate r and the angular coordinate x .

The fact that the magnetic flux-brane can be obtained from an analytical continuation of the standard AdS black hole solution is not necessarily surprising. In

²By an AdS_D -type metric, we mean one with the same general structure as AdS_D described in Poincaré coordinates, except that the constant-radius sections carry a curved metric, rather than a Minkowski metric, as, for example, in (1.2).

fact, it was through an analytic continuation of this kind that flux-branes were first constructed, in [28]. It was observed some time ago in [29] that a particular limit of a Reissner-Nordström black hole can be analytically continued to the Melvin Universe [30]. The latter solution is known as a flux tube [31], a four-dimensional antecedent of the more recently-known flux-branes. The recently introduced S-branes [32] and flux-branes [33] were constructed some time ago, in [34, 35] and [36], respectively. In fact, they are obtained by making the same kind of ansatz as that for standard p -branes. Solutions with a rather general metric ansatz of the form

$$ds^2 = -e^{2U} dt^2 + e^{2A} d\bar{s}_q^2 + e^{2B} ds_{\tilde{q}}^2, \quad (1.6)$$

were analyzed in detail in [34]. In the above metric, U , A and B are functions of t , and $d\bar{s}_q^2$ and $ds_{\tilde{q}}^2$ denote metrics on maximally-symmetric spaces of positive, negative or zero curvature, with dimensions q and \tilde{q} , respectively. It was shown in [36] that, with the appropriate analytical continuation, the ansatz (1.6) can give rise to p -branes, as well as additional solutions which have since been called S-branes and flux-branes.

These S-brane and flux-brane solutions are non-supersymmetric. Analytical continuations of supersymmetric p -branes were obtained in [37]. Again, this is not mysterious, since first-order equations associated with supersymmetry can be viewed as the “square-root” of the second-order equations of motion. Thus, there can be two different types of first-order equations, providing two different solutions which are connected by analytical continuation. Examples of this were given in [37].

In the present work, it is pleasantly surprising to find that the resulting flux-branes in gauged supergravity are both supersymmetric and regular. Although flux-branes in standard ungauged supergravities are typically regular, since the flux contribution vanishes at small distance and hence cannot cause a singularity, they are generally non-supersymmetric. Not only are our gauged supergravity solutions regular and supersymmetric, they have the additional property that the flux-brane world-volume is AdS instead of Minkowski spacetime.

The paper is organized as follows. In section 2, we consider as a toy model D -dimensional anti-de Sitter Einstein-Maxwell gravity. We give a detailed presentation of the construction of a non-extremal AdS charged black hole solution using a superpotential approach. Surprisingly, the extremality condition does not coincide with

the BPS bound required by supersymmetry. While the supersymmetric AdS black hole is singular, a supersymmetric and regular magnetic flux-brane solution can be obtained from analytical continuation. In section 3, we apply the same analytical continuation in order to obtain supersymmetric and regular magnetic flux-branes in AdS gauged supergravities of dimensions $D = 7, 6, 5$ and 4 . In section 4, we find the $D = 5$ first-order equations that follow by requiring supersymmetry. We show that the flux-branes solve these equations, and hence that they are supersymmetric. At first sight, the charge parameters obey a complicated constraint in order to have small-distance regularity. However, for particular cases, a more appropriate definition of charge renders the constraint identical to a simple charge constraint obeyed by previously-known magnetic brane solutions in AdS gauged supergravity. The equivalence of the charge constraint may hold more generally. In section 5, we use analytical continuation to obtain smooth cosmological solutions of de Sitter Einstein-Maxwell gravity. The solution interpolates from $dS_2 \times S^{D-2}$ in the infinite past to a dS_D -type spacetime, with an $S^{D-2} \times S^1$ boundary, in the infinite future. The conclusions are given in section 6.

2 AdS Einstein-Maxwell gravity: A toy model

2.1 Superpotential and general solution

Let us consider D -dimensional AdS Einstein-Maxwell theory with the Lagrangian

$$\hat{e}^{-1} \hat{\mathcal{L}} = \hat{R} - \frac{1}{4} \hat{F}_{(2)}^2 + (D-1)(D-2)g^2. \quad (2.1)$$

The equations of motion for the AdS black hole ansatz of the form (1.5) with $*F_{(2)} = \lambda \Omega_{(D-2)}$ can be summarized in terms of a Hamiltonian $H = T + V$, where $T = \frac{1}{2} \sum g_{\alpha\beta} \partial\varphi^\alpha \partial\varphi^\beta$, $\vec{\varphi} = (\log c, \log a)$ and

$$g_{\alpha\beta} = \begin{pmatrix} 2(D-2)(D-3) & 2(D-2) \\ 2(D-2) & 0 \end{pmatrix}. \quad (2.2)$$

U is given by

$$U = a^2 (-(D-1)(D-2)g^2 c^{2(D-2)} - \epsilon (D-2)(D-3) e^{2(D-3)} + \frac{1}{2}\lambda^2), \quad (2.3)$$

where $\epsilon = \pm 1$ or 0 according to whether the $d\Omega_{D-2}^2$ has positive, negative or zero curvature. We find that U can be expressed in terms of a superpotential W as

$$U = -\frac{1}{2}g^{\alpha\beta} \frac{\partial W}{\partial\varphi^\alpha} \frac{\partial W}{\partial\varphi^\beta}, \quad (2.4)$$

where

$$W = a \left[4(D-2)^2 g^2 c^{2(D-2)} + 4\epsilon (D-2)^2 c^{2(D-3)} + \frac{2(D-2)\lambda^2}{D-3} - \alpha c^{D-3} \right]^{\frac{1}{2}}, \quad (2.5)$$

and α is an arbitrary constant. From this superpotential, we can read off the first-order equations

$$\begin{aligned} \dot{c} &= \frac{W}{2(D-2)a c^{D-3}}, \\ \dot{a} &= \frac{a^2 \left(8(D-2)^2 g^2 c^{2D} - 4(D-2)\lambda^2 c^4 + \alpha(D-3)c^{D+1} \right)}{4(D-2)c^{D+2}W}. \end{aligned} \quad (2.6)$$

The above equations can be solved straightforwardly, by making a coordinate transformation $d\rho = 2(D-2)a c^{D-3} W^{-1} dr$, giving

$$\begin{aligned} ds_D^2 &= -H dt^2 + H^{-1} dr^2 + r^2 d\Omega_{D-2}^2, \\ *F_{(2)} &= \lambda \Omega_{(D-2)}, \\ H &= g^2 r^2 + \epsilon - \frac{M}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}, \end{aligned} \quad (2.7)$$

where the mass M and the charge parameter Q are given by

$$M = \frac{\alpha}{4(D-2)^2}, \quad Q = \frac{\lambda}{\sqrt{2(D-2)(D-3)}}. \quad (2.8)$$

This is the non-extremal AdS charged black hole solution [21–26]. It is rather surprising that this general non-extremal solution arises from a first-order system *via* a superpotential construction.

2.2 Supersymmetry Versus Extremality

AdS Einstein-Maxwell gravity can be embedded in a supersymmetric theory for $D = 4$ and $D = 5$. We shall show later that the supersymmetry of the AdS black holes requires the BPS bound $M = 2Q$, as well as $\epsilon = 1$. In this case, the corresponding superpotential is given by

$$W = 2(D-2)a \sqrt{g^2 c^{2(D-2)} + (c^{D-3} + Q)^2}. \quad (2.9)$$

The solution is then

$$H = g^2 r^2 + \left(1 - \frac{Q}{r^{D-3}}\right)^2. \quad (2.10)$$

Clearly, the solution has a naked singularity at small distance. For vanishing g , $M = 2Q$ is precisely the extremality bound for the black hole. However, in general, the extremality bound depends on g . We may define a black hole that is extremal by requiring that there exists a horizon, and furthermore, that the near-horizon geometry is not of the form R^2 . Thus, let us assume that there is a surface located at r_0 , which satisfies

$$H(r_0) = 0, \quad H'(r_0) = 0. \quad (2.11)$$

The first equation implies that $r = r_0$ is a horizon, while the second implies that the geometry is not of the form R^2 , and hence has zero temperature. In fact, the near-horizon geometry is precisely $\text{AdS}_2 \times \Omega^{D-2}$. The solution is given by

$$\begin{aligned} ds_D^2 &= -e^{2\gamma\rho} dt^2 + d\rho^2 + c^2 d\Omega_{D-2}^2, \\ *F_{(2)} &= \lambda \Omega_{(D-2)}, \end{aligned} \quad (2.12)$$

where c is a non-vanishing constant. The equations of motion imply that

$$\begin{aligned} (D-1)g^2 + \frac{(D-3)\epsilon}{c^2} &= \frac{(D-3)Q^2}{c^{2(D-2)}}, \\ \gamma^2 &= (D-1)g^2 + \frac{(D-3)^2 Q^2}{c^{2(D-2)}}. \end{aligned} \quad (2.13)$$

Clearly, there exist solutions for $\epsilon = 1, -1$ and 0 . The mass of the solution is given by

$$M = \left(\epsilon + g^2 c^2 + \frac{Q^2}{c^{2(D-3)}}\right) c^{D-3}. \quad (2.14)$$

Note that in the case of $g = 0$ and $\epsilon = 1$, the above condition reduces to $M = 2Q$.

2.3 Supersymmetric and regular flux-branes

The fact that the extremality condition does not coincide with the BPS condition for the AdS black hole is rather disturbing. One may ask whether the supersymmetric solution is necessarily singular, or whether on the other hand a smooth extension can be found. The answer turns out to be the latter, and the smooth extension can be obtained by analytic continuation. All we need do is to perform Wick rotations such

that the t coordinate becomes spatial and $d\Omega_{D-2}^2$ becomes AdS, dS or Minkowski spacetime. Since the supersymmetric black hole exists only for $\epsilon = 1$, then in the Wick-rotated solution we must have $\epsilon = -1$, corresponding to AdS_{D-2} . The solution is given by³

$$\begin{aligned} ds_D^2 &= H dx^2 + H^{-1} dr^2 + r^2 ds_{\text{AdS}_{D-2}}^2, \\ H &= g^2 r^2 - \left(1 - \frac{Q}{r^{D-3}}\right)^2. \end{aligned} \quad (2.15)$$

Clearly, if Q and g are related by

$$g^2 Q^{\frac{2}{D-3}} = (D-3)^2 (D-2)^{-\frac{2(D-2)}{D-3}}, \quad (2.16)$$

then we have

$$H(r_0) = 0, \quad H'(r_0) = 0, \quad (2.17)$$

with $r_0^{D-3} = (D-2)Q$. Thus, the solution interpolates from $\text{AdS}_{D-2} \times H^2$ at small distance ($\rho \rightarrow -\infty$), given by

$$ds_D^2 = d\rho^2 + e^{2\sqrt{D-2}g\rho} dx^2 + r_0^2 ds_{\text{AdS}_{D-2}}^2, \quad (2.18)$$

to an AdS_D -type geometry, with the boundary $\text{AdS}_{D-2} \times S^1$, in the asymptotic region ($\rho \rightarrow \infty$), given by

$$ds_D^2 = d\rho^2 + e^{2g\rho} \left(dx^2 + ds_{\text{AdS}_{D-2}}^2\right). \quad (2.19)$$

The above solution is called a flux-brane, since the 2-form field strength $F_{(2)}$ is proportional to the volume of the whole transverse space spanned by the radial coordinate r and the angular coordinate x . Since there is no time component in the field strength, the solution is magnetic.

Magnetic branes which interpolate from $\text{AdS}_{D-2} \times H^2$ in the horizon to an asymptotic AdS_D -type geometry with the boundary $\text{Minkowski}_{D-3} \times H^2$ have also been found [12, 15, 16]. Therefore, $\text{AdS}_{D-2} \times H^2$ can flow to an AdS_D -type geometry with two possible boundaries; either $\text{Minkowski}_{D-3} \times H^2$ or $\text{AdS}_{D-2} \times S^1$.

³For general solutions, $H = \epsilon + g^2 r^2 - \frac{M}{r^{D-3}} - \frac{Q^2}{r^{2(D-3)}}$.

3 Regular supersymmetric flux $(D - 3)$ -branes

In the previous section, we have shown that the AdS Reissner-Nordström black hole in D -dimensional AdS Einstein-Maxwell gravity admits an analytical continuation to become a magnetic flux $(D - 3)$ -brane. In four and five dimensions, both solutions have a BPS limit in which they are supersymmetric, and can be embedded in a supersymmetric theory. We now apply the same analytical continuation in order to obtain multiple-charge magnetic flux-branes in gauged supergravities of diverse dimensions.

3.1 $D = 7$

The bosonic sector of seven-dimensional gauged supergravity, with a truncation of the gauge fields to the $U(1) \times U(1)$ subgroup that is sufficient for constructing 2-charge black holes, is described by the Lagrangian

$$\mathcal{L}_7 = R * \mathbf{1} - \frac{1}{2} * d\vec{\phi} \wedge d\vec{\phi} - \frac{1}{2} \sum_{i=1}^2 X_i^{-2} * F_{(2)}^i \wedge F_{(2)}^i - V * \mathbf{1}, \quad (3.1)$$

where

$$\begin{aligned} V &= \frac{1}{2} g^2 (X_1^{-4} X_2^{-4} - 8 X_1 X_2 - 4 X_1^{-1} X_2^{-2} - 4 X_1^{-2} X_2^{-1}), \\ X_i &= e^{-\frac{1}{2} \vec{a}_i \cdot \vec{\phi}}, \quad \vec{a}_1 = (\sqrt{2}, \sqrt{\frac{2}{5}}), \quad \vec{a}_2 = (-\sqrt{2}, \sqrt{\frac{2}{5}}). \end{aligned} \quad (3.2)$$

The two-charge AdS₇ black hole in the maximal gauged $D = 7$ supergravity was obtained in [26]. The solution is given by

$$\begin{aligned} ds_7^2 &= -(H_1 H_2)^{-4/5} f dt^2 + (H_1 H_2)^{1/5} (f^{-1} dr^2 + r^2 d\Omega_{5,k}^2), \\ f &= k - \frac{\mu}{r^4} + \frac{1}{4} g^2 r^2 H_1 H_2, \quad X_i = (H_1 H_2)^{2/5} H_i^{-1}, \\ A_{(1)}^i &= \sqrt{k} \coth \beta_i (1 - H_i^{-1}) dt, \quad H_i = 1 + \frac{\mu \sinh^2 \beta_i}{r^4}, \end{aligned} \quad (3.3)$$

where $d\Omega_{5,k}^2$ is the metric on a unit S^5 , T^5 or H^5 , according to whether $k = 1, 0$ or -1 . In particular, we are interested in the case $k = 1$, since only then does there exist an extremal limit with $\mu \rightarrow 0$ while keeping $\ell_i^4 = \mu \sinh^2 \beta_i$ fixed, such that the $A_{(1)}^i$ do not vanish. The metric and the X_i in the extremal solution have the same form

given in (3.3), but with $f = 1 + \frac{1}{4}g^2 r^2 H_1 H_2$ and $H_i = 1 + \frac{\ell_i^4}{r^4}$. The gauge fields are given by $A_{(1)}^i = (1 - H_i^{-1}) dt$. We now perform the following analytical continuation:⁴

$$r \rightarrow ir, \quad t \rightarrow x, \quad f \rightarrow -f, \quad d\Omega_5^2 \rightarrow -ds_{\text{AdS}_5}^2. \quad (3.4)$$

The extremal AdS₇ black hole solution becomes an extremal magnetic flux-brane, given by

$$\begin{aligned} ds_7^2 &= (H_1 H_2)^{-4/5} f dx^2 + (H_1 H_2)^{1/5} (f^{-1} dr^2 + r^2 ds_{\text{AdS}_5}^2), \\ f &= \frac{1}{4}g^2 r^2 H_1 H_2 - 1, \quad X_i = H_i^{-1} (H_1 H_2)^{2/5}, \\ A_{(1)}^i &= (1 - H_i^{-1}) dx, \quad H_i = 1 + \frac{\ell_i^4}{r^4}. \end{aligned} \quad (3.5)$$

Note that t automatically becomes spacelike without a Wick rotation. On the other hand, although we have sent r to ir , the coordinate r remains spatial. The metric approaches an AdS₇-type metric with an AdS₅ × S¹ boundary in the asymptotic region $r \rightarrow \infty$, given by

$$ds_7^2 = r^2 \left(\frac{1}{4}g^2 dx^2 + ds_{\text{AdS}_5}^2 \right) + \frac{4dr^2}{g^2 r^2}. \quad (3.6)$$

In general, the metric is singular at small distance. However, for appropriate choices of the charge parameters ℓ_i , it becomes AdS₅ × H² at small distance, and hence gives rise to a smooth solution interpolating from AdS₅ × H² at small distance to the AdS₇-type metric in the asymptotic region. Clearly, the condition for such a solution is that there exist an r_0 such that

$$f(r_0) = 0, \quad f'(r_0) = 0. \quad (3.7)$$

Eliminating r_0 in the above two equations provides a relation between ℓ_i and g for which the metric becomes AdS₅ × H² as r approaches r_0 . For the general solution (3.5), the constraints (3.7) become

$$g^2 (r_0^4 + \ell_1^4) (r_0^4 + \ell_2^4) - 4r_0^6 = 0, \quad r_0^8 - (\ell_1^4 + \ell_2^4) r_0^4 - 3\ell_1^4 \ell_2^4 = 0. \quad (3.8)$$

We shall now consider special cases. For $\ell_1 = \ell$ and $\ell_2 = 0$, we have

$$r_0 = \ell, \quad g^2 \ell^2 = 2, \quad (3.9)$$

⁴It is also possible to perform an analytical continuation such that the sphere transforms into de Sitter spacetime. However, in that case there is no extremal limit, since $k = -1$ and hence $\coth \beta_i \rightarrow \cos \beta_i$.

and the corresponding function f is given by

$$f = \frac{(r^2 - \ell^2)^2}{2\ell^2 r^2}. \quad (3.10)$$

Thus, the metric clearly approaches $\text{AdS}_5 \times H^2$ near r_0 . Another simple case is when $\ell_1 = \ell_2 = \ell$, for which we have

$$\begin{aligned} r_0 &= 3^{1/4} \ell, & g^2 \ell^2 &= \frac{3}{4} \sqrt{3}, \\ f &= \frac{(r^2 - r_0^2)^2 [(3r^2 + r_0^2)^2 - 4r_0^2 r^2]}{16r_0^2 r^6}. \end{aligned} \quad (3.11)$$

The geometry again approaches $\text{AdS}_5 \times H^2$ as $r \rightarrow r_0$.

For generic ℓ_i , we find after some algebra that the requirement of an asymptotic $\text{AdS}_5 \times H^2$ form is achieved when the parameters satisfy the real solution of the equation

$$g^8 (\ell_1^4 - \ell_2^4)^4 - 4g^4 (\ell_1^4 + \ell_2^4)(\ell_1^8 - 34\ell_1^4 \ell_2^4 + \ell_2^8) - 432\ell_1^4 \ell_2^4 = 0. \quad (3.12)$$

It is straightforward to lift the general solution (3.5) to $D = 11$ by using the reduction ansatz obtained in [26]. The M-theory metric is given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{1}{3}} \left[r^2 ds_{\text{AdS}_5}^2 + f^{-1} dr^2 + (H_1 H_2)^{-1} f dx^2 \right. \\ &\quad \left. + \frac{1}{g^2 \Delta} \left\{ d\mu_0^2 + \sum_{i=1}^2 H_i \left(d\mu_i^2 + \mu_i^2 (d\phi_i + g(1 - H_i^{-1}) dx)^2 \right) \right\} \right], \end{aligned} \quad (3.13)$$

where μ_i are spherical coordinates which satisfy $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. The warp factor Δ is given by ⁵

$$\Delta = H_1 H_2 \mu_0^2 + H_2 \mu_1^2 + H_1 \mu_2^2. \quad (3.14)$$

Since $\Delta > 0$ and $r \geq r_0 > 0$, the metric (3.13) has no power-law singularity.

The metric (3.13) is a warped product of AdS_5 with an internal metric which can be regarded as having a reduced generalized holonomy group, since it is not Ricci-flat and involves a form field. As r approaches r_0 , the internal space can be viewed as an S^4 bundle over H^2 , with two diagonal $U(1)$ fibres⁶.

⁵ $\Delta = (H_1 H_2)^{\frac{3}{5}} \tilde{\Delta}$, where $\tilde{\Delta}$ is the warp factor defined in [26].

⁶Supersymmetric AdS solutions of M-theory with internal spaces that can be viewed as S^n bundles over various spaces have also been discussed in [10, 15–20, 38].

3.2 $D = 6$

With appropriate matter fields, the bosonic sector of six-dimensional gauged supergravity can be truncated to a $U(1) \times U(1)$ gauge subgroup, which is sufficient for constructing 2-charge black holes. The corresponding Lagrangian is given by

$$\mathcal{L}_6 = R * \mathbf{1} - \frac{1}{2} * d\vec{\phi} \wedge d\vec{\phi} - \frac{1}{2} \sum_{i=1}^2 X_i^{-2} * F_{(2)}^i \wedge F_{(2)}^i - V * \mathbf{1}, \quad (3.15)$$

where

$$\begin{aligned} V &= \frac{4}{9} g^2 (X_0^2 - 9X_1 X_2 - 6X_0 X_1 - 6X_0 X_2), \\ X_i &= e^{-\frac{1}{2} \vec{a}_i \cdot \vec{\phi}}, \quad \vec{a}_1 = (\sqrt{2}, \frac{1}{\sqrt{2}}), \quad \vec{a}_2 = (-\sqrt{2}, \frac{1}{\sqrt{2}}), \end{aligned} \quad (3.16)$$

and $X_0 \equiv (X_1 X_2)^{-3/2}$.

The AdS₆ black hole was obtained in [39]. Using the same convention and notation of [39] and [40], we can follow the same strategy and obtain the extremal $D = 6$ flux-branes, given by

$$\begin{aligned} ds_6^2 &= (H_1 H_2)^{-3/4} f dx^2 + (H_1 H_2)^{1/4} (f^{-1} dr^2 + r^2 ds_{\text{AdS}_4}^2), \\ A_{(1)}^i &= (1 - H_i^{-1}) dx, \quad X_i = (H_1 H_2)^{3/8} H_i^{-1}, \\ f &= \frac{2}{9} g^2 r^2 H_1 H_2 - 1, \quad H_i = 1 + \frac{\ell_i^3}{r^3}. \end{aligned} \quad (3.17)$$

Analogous to the $D = 7$ example, the solution becomes AdS₆-type with the boundary of AdS₄ \times S^1 . At small distance, the solution can be regular if there exists an r_0 which satisfies the constraint (3.7).

Let us consider two simple cases. For $\ell_1 = \ell$ and $\ell_2 = 0$, we find that

$$r_0^3 = \frac{1}{2} \ell^3, \quad 2g^2 \ell^2 = 3 \cdot 2^{2/3}, \quad f = \frac{(r - r_0)^2 (r + 2r_0)}{2r_0^2 r}. \quad (3.18)$$

A second case is when $\ell_1 = \ell_2 = \ell$, for which we find that

$$\begin{aligned} r_0^3 &= 2\ell^3, \quad g^2 \ell^2 = 2^{1/3}, \\ f &= \frac{(r - r_0)^2 (2r + r_0) (2r^3 + 3r_0 r^2 + r_0^3)}{9r_0^2 r^4}, \end{aligned} \quad (3.19)$$

For both cases, the metric approaches AdS₄ \times H^2 as r approaches r_0 .

For generic ℓ_i , the constraint is given by

$$g^{12} (\ell_1^3 - \ell_2^3)^6 - 27g^6 (\ell_1^{12} + \ell_2^{12} - 282\ell_1^6 \ell_2^6 - 76\ell_1^3 \ell_2^3 (\ell_1^6 + \ell_2^6)) - 23328\ell_1^3 \ell_2^3 = 0. \quad (3.20)$$

It is straightforward to lift the general solution (3.17) to $D = 10$ by using the reduction ansatz obtained in [39, 40]. The ten-dimensional metric is given by

$$\begin{aligned}
ds_{10}^2 &= \mu_0^{\frac{1}{12}} \Delta^{\frac{3}{8}} \left[r^2 ds_{\text{AdS}_4}^2 + f^{-1} dr^2 + (H_1 H_2)^{-1} f dx^2 \right. \\
&\quad \left. + \frac{1}{g^2 \Delta} \left\{ d\mu_0^2 + H_1 \left(d\mu_1^2 + \mu_1^2 (d\phi_1 + g(1 - H_1^{-1}) dx)^2 \right) \right. \right. \\
&\quad \left. \left. + H_2 \left(d\mu_2^2 + \mu_2^2 (d\phi_2 + g(1 - H_2^{-1}) dx)^2 \right) \right\} \right], \tag{3.21}
\end{aligned}$$

where μ_i are spherical coordinates which satisfy $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. The warp factor Δ is given by

$$\Delta = H_1 H_2 \mu_0^2 + H_2 \mu_1^2 + H_1 \mu_2^2. \tag{3.22}$$

Note that the metric (3.21) is singular, due to the overall angular factor.

3.3 $D = 5$

In the conventions and notation of [26], the relevant bosonic sector of five-dimensional gauged supergravity, truncated to the $U(1)^3$ subgroup of $SO(6)$, is described by the Lagrangian

$$\mathcal{L}_5 = R * \mathbf{1} - \frac{1}{2} * d\vec{\phi} \wedge d\vec{\phi} - \frac{1}{2} \sum_{i=1}^3 X_i^{-2} * F_{(2)}^i \wedge F_{(2)}^i + \frac{1}{6} \epsilon_{ijk} F_{(2)}^i \wedge F_{(2)}^j \wedge A_{(1)}^k - V * \mathbf{1}, \tag{3.23}$$

where

$$\begin{aligned}
V &= -4g^2 \sum_{i=1}^3 X_i^{-1}, \tag{3.24} \\
X_i &= e^{-\frac{1}{2} \vec{a}_i \cdot \vec{\phi}}, \quad \vec{a}_1 = \left(\frac{2}{\sqrt{6}}, \sqrt{2} \right), \quad \vec{a}_2 = \left(\frac{2}{\sqrt{6}}, -\sqrt{2} \right), \quad \vec{a}_3 = \left(-\frac{4}{\sqrt{6}}, 0 \right).
\end{aligned}$$

The three-charge AdS₅ black hole was obtained in [21, 24]. Following the same procedure as we used previously, we obtain AdS₅ flux-branes given by

$$\begin{aligned}
ds_5^2 &= (H_1 H_2 H_3)^{-2/3} f dx^2 + (H_1 H_2 H_3)^{1/3} (f^{-1} dr^2 + r^2 ds_{\text{AdS}_3}^2), \\
X_i &= H_i^{-1} (H_1 H_2 H_3)^{1/3}, \quad A_{(1)}^i = (1 - H_i^{-1}) dx, \\
f &= g^2 r^2 (H_1 H_2 H_3) - 1, \quad H_i = 1 + \frac{\ell_i^2}{r^2}. \tag{3.25}
\end{aligned}$$

Asymptotically, the metric becomes AdS₅-type, with an AdS₃ \times S^1 boundary. In general, the solution is singular at small distance. However, we are interested in the

possibility of finding choices of ℓ_i such that the small-distance behavior approaches $\text{AdS}_3 \times H^2$. This can be achieved by satisfying the condition (3.7). The elimination of r_0 in the two equations gives rise to a constraint on ℓ_i and g .

If there is only one non-vanishing ℓ_i , then it can be easily seen that the constraint (3.7) cannot be satisfied. Hence in this case, there can be no $\text{AdS}_3 \times H^2$ limit. This was also observed in [15, 16]. Now let us consider $\ell_3 = 0$, with non-vanishing ℓ_1 and ℓ_2 . We find that

$$r_0^2 = \ell_1 \ell_2, \quad g(\ell_1 + \ell_2) = 1, \quad f = \frac{(r^2 - r_0^2)^2}{r^2(\ell_1 + \ell_2)^2}. \quad (3.26)$$

Another simple case arises when $\ell_i = \ell$ for all i , in which case

$$r_0^2 = 2\ell^2, \quad g^2 \ell^2 = \frac{4}{27}, \quad f = \frac{(r^2 - r_0^2)^2 (8r^2 + r_0^2)}{27r_0^2 r^4}. \quad (3.27)$$

For both cases, the metric approaches $\text{AdS}_3 \times H^2$ as r approaches r_0 .

For generic ℓ_i , the constraint is given by

$$\begin{aligned} & g^6(\ell_1^2 - \ell_2^2)^2(\ell_2^2 - \ell_3^2)^2(\ell_3^2 - \ell_1^2)^2 + g^4 \left(-2\ell_1^2 \ell_2^2 (\ell_1^4 + \ell_2^4) + \text{cyclic on } 1,2,3 \right. \\ & \left. + 4\ell_1^2 \ell_2^2 \ell_3^2 (2\ell_1^4 + 2\ell_2^4 + 2\ell_3^4 - \ell_1^2 \ell_2^2 - \ell_2^2 \ell_3^2 - \ell_3^2 \ell_1^2) \right) \\ & \left. + g^2 \left(\ell_1^4 \ell_2^4 + \ell_2^4 \ell_3^4 + \ell_3^4 \ell_1^4 - 10\ell_1^2 \ell_2^2 \ell_3^2 (\ell_1^2 + \ell_2^2 + \ell_3^2) \right) + 4\ell_1^2 \ell_2^2 \ell_3^2 = 0. \end{aligned} \quad (3.28)$$

It is straightforward to lift the general solution (3.25) to $D = 10$ by using the reduction ansatz obtained in [26]. The ten-dimensional metric is given by

$$\begin{aligned} ds_{10}^2 &= \sqrt{\Delta} \left[r^2 ds_{\text{AdS}_3}^2 + f^{-1} dr^2 + (H_1 H_2 H_3)^{-1} f dx^2 \right. \\ & \left. + \frac{1}{g^2 \Delta} \sum_{i=1}^3 H_i \left(d\mu_i^2 + \mu_i^2 (d\phi_i + g(1 - H_i^{-1}) dx)^2 \right) \right], \end{aligned} \quad (3.29)$$

where μ_i are spherical coordinates which satisfy $\sum_i \mu_i^2 = 1$. The warp factor Δ is given by

$$\Delta = H_1 H_2 H_3 \sum_{i=1}^3 H_i^{-1} \mu_i^2. \quad (3.30)$$

Since $\Delta > 0$ and $r \geq r_0 > 0$, the metric (3.29) has no power-law singularity.

3.4 $D = 4$

In the conventions and notation of [26], the relevant bosonic sector of the $U(1)^4$ truncation of gauged $SO(8)$ four-dimensional supergravity is described by the Lagrangian

$$\mathcal{L}_4 = R * \mathbb{1} - \frac{1}{2} * d\vec{\phi} \wedge d\vec{\phi} - \frac{1}{2} \sum_{i=1}^4 X_i^{-2} * F_{(2)}^i \wedge F_{(2)}^i - V * \mathbb{1}, \quad (3.31)$$

where

$$\begin{aligned} V &= -2g^2 \sum_{i,j} X_i X_j, & X_i &= e^{-\frac{1}{2} \vec{a}_i \cdot \vec{\phi}}, \\ \vec{a}_1 &= (1, 1, 1), & \vec{a}_2 &= (1, -1, -1), & \vec{a}_3 &= (-1, 1, -1), & \vec{a}_4 &= (-1, -1, 1). \end{aligned} \quad (3.32)$$

The four-charge AdS black hole in $D = 4$ maximal gauged supergravity was obtained in [25]. Following the same procedure we discussed earlier, we can obtain magnetic flux-branes, given by

$$\begin{aligned} ds_4^2 &= (H_1 H_2 H_3 H_4)^{-1/2} f dx^2 + (H_1 H_2 H_3 H_4)^{1/2} (f^{-1} dr^2 + r^2 ds_{\text{AdS}_2}^2), \\ A_{(1)}^i &= (1 - H_i^{-1}) dx, & X_i &= H_i^{-1} (H_1 H_2 H_3 H_4)^{1/4}, \\ f &= 4g^2 r^2 (H_1 H_2 H_3 H_4) - 1, & H_i &= 1 + \frac{\ell_i}{r}. \end{aligned} \quad (3.33)$$

Asymptotically, the metric approaches AdS₄-type, with an AdS₂ × S¹ boundary. With an appropriate choice of the parameters ℓ_i , the metric approaches AdS₂ × H² at small distance. The constraint for general ℓ_i is given by (3.7). Let us look at a few examples. First, if more than one ℓ_i vanishes, the solution does not admit an AdS₂ × H² limit. For the simple choice $\ell_3 = \ell_1$ and $\ell_4 = \ell_2$, we have

$$\begin{aligned} r_0^2 &= \ell_1 \ell_2, & 2g(\sqrt{\ell_1} + \sqrt{\ell_2})^2 &= 1, \\ f &= \frac{(r - r_0)^2 \left((r - r_0)^2 + 2r(\sqrt{\ell_1} + \sqrt{\ell_2})^2 \right)}{(\sqrt{\ell_1} + \sqrt{\ell_2})^4 r^2}. \end{aligned} \quad (3.34)$$

Another simple example is provided by $\ell_1 = \ell_2 = \ell_3 = \ell$ and $\ell_4 = 0$, in which case we have

$$r_0 = \frac{1}{2} \ell, \quad 27g^2 \ell^2 = 1, \quad f = \frac{(2r - \ell)^2 (r + 4\ell)}{27\ell^2 r}. \quad (3.35)$$

For the case of all ℓ_i equal, the system reduces to the Einstein-Maxwell theory discussed in section 2. For these cases, the metric approaches AdS₂ × H² as r approaches r_0 .

It is straightforward to lift the general solution (3.33) to $D = 11$ by using the reduction ansatz obtained in [26]. The M-theory metric is given by

$$ds_{11}^2 = \Delta^{\frac{2}{3}} \left[r^2 ds_{\text{AdS}_2}^2 + f^{-1} dr^2 + (H_1 H_2 H_3 H_4)^{-1} f dx^2 + \frac{1}{g^2 \Delta} \sum_{i=1}^4 H_i \left(d\mu_i^2 + \mu_i^2 (d\phi_i + g(1 - H_i^{-1}) dx)^2 \right) \right], \quad (3.36)$$

where μ_i are spherical coordinates which satisfy $\sum_i \mu_i^2 = 1$. The warp factor Δ is given by

$$\Delta = H_1 H_2 H_3 H_4 \sum_{i=1}^4 H_i^{-1} \mu_i^2. \quad (3.37)$$

Since $\Delta > 0$ and $r \geq r_0 > 0$, the metric (3.36) has no power-law singularity.

4 Supersymmetry and first-order equations

4.1 Supersymmetry analysis for $D = 5$

We shall consider the case of $D = 5$ as an example. The supersymmetry transformations of the fermionic fields in the truncated $U(1)^3$ gauged five-dimensional supergravity are given by

$$\delta\lambda_\alpha = \left(\frac{1}{8} \frac{\partial X_i^{-1}}{\partial \phi_\alpha} \Gamma^{MN} F_{MN}^i - \frac{i}{4} \Gamma^M \partial_M \phi_\alpha + \frac{i}{2} g \sum_{i=1}^3 \frac{\partial X_i}{\partial \phi_\alpha} \right) \epsilon, \quad (4.1)$$

$$\delta\psi_M = \left(\nabla_M + \frac{i}{24} X_i^{-1} F_{NP}^i (\Gamma_M^{NP} - 4\delta_M^N \Gamma^P) + \frac{1}{6} g \sum_{i=1}^3 X_i \Gamma_M - \frac{i}{2} g \sum_{i=1}^3 A_M^i \right) \epsilon.$$

We take the metric ansatz (1.4) and write the three 1-form potentials as

$$A^i = -u_i dx, \quad (4.2)$$

where the u_i are functions only of r . From (4.1), we find that the first-order equations following from requiring supersymmetry are

$$\begin{aligned} \dot{\phi}_1^2 &= \frac{2}{3} g^2 (X_1 + X_2 - 2X_3)^2 - \frac{1}{6a^2} (\dot{u}_1 X_1^{-1} + \dot{u}_2 X_2^{-1} - 2\dot{u}_3 X_3^{-1})^2, \\ \dot{\phi}_2^2 &= 2g^2 (X_1 - X_2)^2 - \frac{1}{2a^2} (\dot{u}_1 X_1^{-1} - \dot{u}_2 X_2^{-1})^2, \\ \dot{a}^2 &= \frac{1}{9} a^2 g^2 (X_1 + X_2 + X_3)^2 + g^2 (u_1 + u_2 + u_3)^2 - \frac{1}{9} (\dot{u}_1 X_1^{-1} + \dot{u}_2 X_2^{-1} + \dot{u}_3 X_3^{-1})^2, \\ \dot{c}^2 &= \frac{1}{9} c^2 g^2 (X_1 + X_2 + X_3)^2 - \left[-1 + \frac{c}{6a} (\dot{u}_1 X_1^{-1} + \dot{u}_2 X_2^{-1} + \dot{u}_3 X_3^{-1}) \right]^2. \end{aligned} \quad (4.3)$$

In addition, the field equations for the three vector potentials imply

$$\dot{u}_1 = \frac{a X_1^2 q_1}{c^3}, \quad \dot{u}_2 = \frac{a X_2^2 q_2}{c^3}, \quad \dot{u}_3 = \frac{a X_3^2 q_3}{c^3}, \quad (4.4)$$

where the q_i are constants. It is straightforward to verify that our solution (3.25) satisfies these first-order equations and hence it is supersymmetric. The constants q_i are given by $q_i = 2\ell_i^2$. Note that, when all q_i are equal, we can set $X^i = 1$ and the system reduces to Einstein-Maxwell gauged supergravity. It is straightforward to verify that supersymmetry implies that $M = 2Q$, as stated in section 2.2.

It is worth mentioning that the first-order equations (4.3) and (4.4) do not necessarily imply the second-order equations of motion. To illustrate this, let us consider a simpler case with $u_1 = u_2 = 0$ and u_3 non-vanishing. In this case, we can set $\phi_2 = 0$, which reduces the equations to a single-charge and single-scalar system. Substituting the first-order equations into the equations of motion, we find that an additional algebraic equation has to be satisfied, namely

$$c^2 = \frac{q_3 e^{\frac{2}{\sqrt{6}} \phi_1}}{2(1 - e^{\frac{3}{\sqrt{6}} \phi_1})}. \quad (4.5)$$

It is easy to see that this constraint is consistent with the first-order equations. In the special case of $u_1 = u_2 = 0$ and $\phi_2 = 0$, the $\dot{\phi}_1$ and \dot{c} equations of (4.3) imply the following relation:

$$q_3 b^8 C'^2 - (q_3 b^2 - 6C)^2 b^2 C - 4g^2 C^3 (b^2 (b^3 - 1)^2 C'^2 - (b^3 + 2)^2 C^2) = 0, \quad (4.6)$$

where $C = c^2$, $b = e^{\phi_1/\sqrt{6}}$ and $C' \equiv dC/db$. The constraint (4.5) is a g -independent solution to (4.6). While (4.6) clearly admits a more general class of solutions with an integration constant, only the special solution (4.5), with a specific integration constant, is consistent with the second-order equations of motion. This provides an example in which the Killing spinor equations do not automatically give rise to a solution of the equations of motion. The fact that the constraint (4.5) is independent of g implies that the solution is supersymmetric for both $g = 0$ as well as non-vanishing g . (Of course, setting $g = 0$ requires appropriate Wick rotations of coordinates, after which the solution becomes an electric black hole in ungauged supergravity.)

Recall that in section 2 we obtained first-order equations with a superpotential construction for the AdS black holes and flux-branes of AdS Einstein-Maxwell theory.

We found that the superpotential approach could not be extended to cases where scalar fields are present in the gauged supergravity theories. This can be understood from the fact that the first-order system is then not of itself enough to determine the solution; additional algebraic equations are needed in these cases, which are not expected to arise from a superpotential description.

While $q_i = 2\ell_i^2$ are the charge parameters of the AdS black hole solutions, in the related AdS flux-branes it is no longer appropriate to view these as the natural charge parameters. To see this, let us consider the case in which (1.4) is the metric on $\text{AdS}_3 \times H^2$, for which $a = e^{\gamma\rho}$ and c is a constant. The metric can then be expressed as

$$ds_5^2 = \gamma^{-2} d\Omega_2^2 + c^2 ds_{\text{AdS}_3}^2, \quad (4.7)$$

where $d\Omega_2$ and $ds_{\text{AdS}_3}^2$ are the metrics of the unit H^2 and AdS_3 , respectively. Then we have

$$dA_{(1)}^i = -\dot{u}_i d\rho \wedge dx = -\frac{\dot{u}_i}{a\gamma^2} \Omega_{(2)} \equiv \tilde{q}_i \Omega_{(2)}, \quad (4.8)$$

where \tilde{q}_i is the charge as defined in [15, 16]. Thus, we find that

$$\tilde{q}_i = \frac{X_i^2 q_i}{c^3 \gamma^2}. \quad (4.9)$$

We expect that the complicated expression (3.28) becomes the same as the constraint found in [15, 16], once expressed in terms of the parameters \tilde{q}_i . As an illustration, let us consider the “two-charge” case, with ℓ_1 and ℓ_2 non-vanishing, but $\ell_3 = 0$. The $\text{AdS}_3 \times H^2$ solution can easily be found from the results in section 3.3, leading to

$$\tilde{q}_1 = \tilde{q}_2 = \frac{1}{2}(\ell_1 + \ell_2) = \frac{1}{2}g^{-1}. \quad (4.10)$$

This is precisely the same as the condition given in [15, 16]. It is rather surprising that the two-charge solutions with two ostensibly different charges in fact have equal charges in the more natural parameterisation.

4.2 Supersymmetry in $D = 7$

As a further example, we consider the supersymmetry of the $D = 7$ solutions. The supersymmetry transformations of the fermionic fields in the truncated $U(1) \times U(1)$

gauged seven-dimensional supergravity are given by

$$\begin{aligned}
\delta\lambda_1 &= \left(-\frac{1}{16}X_1^{-1}F_{MN}^1\Gamma^{MN}\sigma_{12} - \left(\frac{3\partial_M X_1}{4X_1} + \frac{2\partial_M X_2}{4X_2}\right)\Gamma^M + \frac{1}{4}g(X_1 - X_1^{-2}X_2^{-2}) \right)\epsilon, \\
\delta\lambda_2 &= \left(-\frac{1}{16}X_2^{-1}F_{MN}^2\Gamma^{MN}\sigma_{34} - \left(\frac{2\partial_M X_1}{4X_1} + \frac{3\partial_M X_2}{4X_2}\right)\Gamma^M + \frac{1}{4}g(X_2 - X_1^{-2}X_2^{-2}) \right)\epsilon, \\
\delta\psi_M &= \left(\nabla_M + \frac{1}{4}(X_1^{-1}F_{MN}^1\sigma_{12} + X_2^{-1}F_{MN}^2\sigma_{34})\Gamma^N + \frac{1}{4}gX_1^{-2}X_2^{-2}\Gamma_M \right. \\
&\quad \left. + \frac{1}{4}\Gamma_M\Gamma^N(X_1^{-1}\partial_N X_1 + X_2^{-1}\partial_N X_2) + \frac{1}{2}g(A_M^1\sigma_{12} + A_M^2\sigma_{34}) \right)\epsilon, \tag{4.11}
\end{aligned}$$

where σ_{12} and σ_{34} are generators in the Cartan subalgebra of $SO(5)$.

From these, and writing $A^i = -u_i dx$ as before, we find that the requirements of supersymmetry imply the following first-order equations:

$$\begin{aligned}
\left(\frac{3\dot{X}_1}{X_1} + \frac{2\dot{X}_2}{X_2}\right)^2 &= 4g^2(X_1 - X_1^{-2}X_2^{-2})^2 - \frac{\dot{u}_1^2}{a^2 X_1^2}, \\
\left(\frac{2\dot{X}_1}{X_1} + \frac{3\dot{X}_2}{X_2}\right)^2 &= 4g^2(X_2 - X_1^{-2}X_2^{-2})^2 - \frac{\dot{u}_2^2}{a^2 X_2^2}, \\
\left(\frac{2\dot{a}}{a} + \frac{\dot{X}_1}{X_1} + \frac{\dot{X}_2}{X_2}\right)^2 &= g^2 X_1^{-4} X_2^{-4} - \frac{1}{a^2} \left(\frac{\dot{u}_1}{X_1} + \frac{\dot{u}_2}{X_2}\right)^2 + \frac{4g^2(u_1 + u_2)^2}{a^2}, \\
\left(\frac{2\dot{c}}{c} + \frac{\dot{X}_1}{X_1} + \frac{\dot{X}_2}{X_2}\right)^2 &= g^2 X_1^{-4} X_2^{-4} - \frac{4}{c^2}. \tag{4.12}
\end{aligned}$$

Additionally, the field equations for the gauge fields imply

$$\dot{u}_1 = \frac{a X_1^2 q_1}{c^5}, \quad \dot{u}_2 = \frac{a X_2^2 q_2}{c^5}, \tag{4.13}$$

where q_i are constants. Our $D = 7$ flux-branes satisfy these first-order equations and hence are supersymmetric.

5 Analytical continuation to de Sitter spacetime

A large class of supersymmetric magnetic brane solutions supported by $U(1)$ gauge fields in AdS gauged supergravities have previously been investigated [7–16]. These solutions smoothly interpolate between $\text{AdS}_{D-2} \times \Omega^2$ (where $\Omega^2 = S^2$ or H^2) at the horizon and an AdS_D -type geometry in the asymptotic region, for $4 \leq D \leq 7$. The boundary geometry of the AdS_D -type metric is $\text{Minkowski}_{D-3} \times \Omega^2$.

De Sitter Einstein-Maxwell theory and, in a supergravity setting, Hull’s exotic * theories which have flux kinetic terms with the “wrong” sign [41], support the

de Sitter counterpart to these types of solutions. These are smooth cosmological solutions in which the proper time runs from an infinite past that is $dS_{D-2} \times S^2$ to an infinite future that is a dS_D -type spacetime with an $R^{D-3} \times S^2$ boundary [42]. These cosmological solutions can be obtained directly from the aforementioned magnetic brane solutions of standard gauged supergravity via analytical continuation.

An analogous analytical continuation can apply to our flux-branes. However, there is one important difference. As we shall see, the resulting $U(1)$ gauge fields still have standard kinetic terms, with the usual sign. The analytical continuation can be implemented by the replacements

$$g \rightarrow ig, \quad r \rightarrow it, \quad \ell_i \rightarrow i\ell_i, \quad ds_{\text{AdS}_d}^2 \rightarrow -ds_{S^d}^2. \quad (5.1)$$

Under these transformations, the functions H_i become

$$H \rightarrow H = 1 + \frac{\ell_i^{d-1}}{t^{d-1}}, \quad (5.2)$$

and hence the gauge fields $A_{(1)}^i = (1 - H_i^{-1}) dx$ remain real. It follows that the signs of the corresponding kinetic terms remain unaltered. The function f and the scalars X_i maintain the same form as in the flux-branes. Since the forms of these cosmological solutions are more or less the same as the corresponding flux-branes, we shall not repeat them here.

The transformation $g \rightarrow ig$ implies that the cosmological solution is asymptotic to a dS-type geometry at infinity, with an $S^d \times S^1$ boundary, rather than to an AdS-type geometry. Since the kinetic terms for the $U(1)$ gauge fields have the standard sign, it follows that our flux-brane solutions exist in standard de Sitter supergravity theories instead of those associated with the * theories. Cosmological solutions of such theories in $D = 5$ and $D = 4$ supported by only the scalar potential were studied in [43]. The solution are typically singular at certain point in the past. Our cosmological solutions however can be totally regular. While it is not clear how our scalar-coupled systems with $g \rightarrow ig$ could be obtained from string or M-theories, de Sitter Einstein-Maxwell gravity in $D = 5$ or $D = 4$ can arise from type IIB and M-theory, respectively. Here, we shall present non-trivial cosmological solutions to such theories obtained from first-order equations, even though supersymmetry is not expected.

Note that the cosmological solution can be obtained directly from the AdS black hole solutions, by sending g to ig , in which case, the coordinate r becomes automatically timelike without the Wick rotation. There is an “interior” region where the coordinate r is spatial. In this region the solution is singular. This is analogous to the Schwarzschild black hole, which behaves like a cosmological solution inside the horizon. The difference is that in our solution, the region with r being time-like is totally regular.

The Lagrangian for de Sitter Einstein-Maxwell gravity is given by

$$e^{-1} \mathcal{L} = R - \frac{1}{4} F_{(2)}^2 - (D-1)(D-2)g^2. \quad (5.3)$$

Note that the kinetic term for $F_{(2)}$ has the standard sign. Using the above analytical continuation, a cosmological solution can be obtained from a superpotential construction, giving⁷

$$ds_D^2 = -H^{-1} dt^2 + H dx^2 + t^2 ds_{S^{D-2}}^2, \quad H = g^2 t^2 - \left(1 - \frac{\ell^{D-3}}{t^{D-3}}\right)^2. \quad (5.4)$$

The solution is regular everywhere if the constraint

$$g^2 \ell^2 = (D-3)^2 (D-2)^{-\frac{2(D-2)}{D-3}} \quad (5.5)$$

is satisfied. This cosmological solution interpolates between $dS_2 \times S^{D-2}$ in the infinite past to a dS_D -type spacetime in the infinite future, *i.e.*

$$\begin{aligned} \tau \rightarrow -\infty : \quad ds_D^2 &= -d\tau^2 + e^{2\sqrt{D-2}g\tau} dx^2 + (D-2)^{\frac{2}{D-3}} \ell^2 d\Omega_{D-2}^2, \\ \tau \rightarrow \infty : \quad ds_D^2 &= -d\tau^2 + e^{2g\tau} (d\Omega_{D-2}^2 + dx^2). \end{aligned} \quad (5.6)$$

Similar results have been found in [44]. Here we only present the cosmological solution explicitly for the Einstein-Maxwell gravity with positive cosmological constant. The cosmological solutions for the more complicated system with scalar potentials can be easily obtained, as we have demonstrated above, from AdS black holes with analytical continuation. The behavior of these solutions are qualitatively the same as the one above.

⁷A general class of cosmological solutions in Einstein-Maxwell gravity with a cosmological constant were also constructed in [44].

Alternatively, we can consider the AdS black hole solution (2.7). We consider the case $\epsilon = -1$ so that $d\Omega_{D-2}^2$ is the metric of H^{D-2} , and perform the following analytical continuation:

$$r \rightarrow ir, \quad t \rightarrow x, \quad d\Omega_{D-2}^2 = -ds_{\text{dS}_{D-2}}^2. \quad (5.7)$$

The resulting metric is given by

$$ds_D^2 = H dx^2 + H^{-1} dr^2 + r^2 ds_{\text{dS}_{D-2}}^2, \quad (5.8)$$

where we have redefined

$$H = g^2 r^2 + 1 - \frac{M}{r^2} - \frac{Q^2}{r^4}. \quad (5.9)$$

This is a non-BPS solution of AdS Einstein-Maxwell theory which interpolates from $dS_{D-2} \times H^2$ at the near-horizon to an AdS_D -type geometry with the boundary of $dS_{D-2} \times S^1$. If x is periodic then $r \geq r_H$ and the geometry is completely smooth, where r_H is the horizon radius of the AdS black hole given by (2.7). Similar solutions have been discussed in [45–47].

6 Conclusions

We have discussed how electric black holes in gauged supergravity can be analytically continued to become magnetic flux-branes. We showed explicitly that these flux-branes can be supersymmetric and regular everywhere, interpolating between $AdS_{D-2} \times H^2$ at small distance to an AdS_D -type geometry, with an $AdS_{D-2} \times S^1$ boundary, in the asymptotic region. This differs from a previously-known interpolation from $AdS_{D-2} \times H^2$ to an AdS_D -type spacetime, where the asymptotic AdS_D -type geometry has an $M_{D-3} \times H^2$ boundary. Treating the AdS_{D-2} as the “internal” space, the new solution can be viewed as a supergravity dual describing the renormalization group flow of one-dimensional Euclidean conformal quantum mechanics. Alternatively, the solution can be viewed as a smooth embedding of AdS_{D-2} , suggesting a duality of certain conformal field theories in different dimensions and signature.

We have also used analytical continuation to obtain a smooth cosmological solution of de Sitter Einstein-Maxwell gravity, in which the kinetic term for the Maxwell field has the standard sign. Although there is no supersymmetry, the solution can arise

from a first-order system. It is regular at all times, smoothly running from $dS_2 \times S^2$ in the infinite past to a dS_D -type geometry, with a Euclidean-signatured $S^{D-2} \times S^1$ boundary, in the infinite future.

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A General equations of motion

Let us consider a theory with the Lagrangian

$$e^{-1} \mathcal{L}_D = R - \frac{1}{2} (\partial \vec{\phi})^2 - \frac{1}{4} \sum_i e^{\vec{a}_i \cdot \vec{\phi}} (F_{(2)}^i)^2 - V(\vec{\phi}). \quad (\text{A.1})$$

The equations of motion are given by

$$\begin{aligned} \square \vec{\phi} &= \frac{\partial V}{\partial \vec{\phi}} + \frac{1}{4} \sum_i \vec{a}_i (F_{(2)}^i)^2 e^{\vec{a}_i \cdot \vec{\phi}}, & d(e^{\vec{a}_i \cdot \vec{\phi}} * F_{(2)}^i) &= 0, \\ R_{\mu\nu} &= \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{D-2} V g_{\mu\nu} + \frac{1}{2} \sum_i e^{\vec{a}_i \cdot \vec{\phi}} \left((F^i)_{\mu\nu}^2 - \frac{1}{2(D-2)} (F^i)^2 g_{\mu\nu} \right). \end{aligned} \quad (\text{A.2})$$

We can construct a black hole solution with the following ansatz

$$\begin{aligned} ds_D^2 &= -a^2 dt^2 + d\rho^2 + c^2 d\Omega_{D-2}^2, \\ e^{\vec{a}_i \cdot \vec{\phi}} * F_{(2)}^i &= \lambda_i \Omega_{(D-2)}, \end{aligned} \quad (\text{A.3})$$

where $d\Omega_{D-2}^2$ is the metric of a unit sphere S^{D-2} , a unit hyperplane H^{D-2} or a torus T^{D-2} . Defining a vielbein basis $e^0 = a dt$, $e^1 = d\rho$ and $e^\alpha = c \tilde{e}^\alpha$, we find that the spin connection components are given by

$$\omega_{01} = -\dot{a} dt, \quad \omega_{1\alpha} = -\dot{c} \tilde{e}^\alpha, \quad \omega_{\alpha\beta} = \tilde{\omega}_{\alpha\beta}, \quad (\text{A.4})$$

and the Ricci tensor components are

$$\begin{aligned} R_{00} &= \frac{\ddot{a}}{a} + (D-2) \frac{\dot{a}\dot{c}}{ac}, & R_{11} &= -\frac{\ddot{a}}{a} - (D-2) \frac{\ddot{c}}{c}, \\ R_{\alpha\beta} &= \left(-\frac{\dot{a}\dot{c}}{ac} - \frac{\ddot{c}}{c} - (D-3) \frac{\dot{c}^2}{c^2} + \frac{(D-3)\epsilon}{c^2} \right) \delta_{\alpha\beta}, \end{aligned} \quad (\text{A.5})$$

where $\epsilon = -1, 1$ and 0 for H^{D-2} , S^{D-2} and T^{D-2} , respectively. In the above, a dot denotes a derivative with respect to ρ .

Thus, the equations of the motion for the black hole are given by

$$\begin{aligned}
\ddot{\vec{\phi}} + \left(\frac{\dot{a}}{a} + (D-2)\frac{\dot{c}}{c}\right)\dot{\vec{\phi}} &= \frac{\partial V}{\partial \vec{\phi}} - \frac{1}{2}c^{-2(D-2)} \sum_i \lambda_i^2 \vec{a}_i e^{-\vec{a}_i \cdot \vec{\phi}}, \\
\frac{\ddot{a}}{a} + (D-2)\frac{\dot{a}\dot{c}}{ac} &= -\frac{V}{D-2} + \frac{(D-3)}{2(D-2)} c^{-2(D-2)} \sum_i \lambda_i^2 e^{-\vec{a}_i \cdot \vec{\phi}}, \\
-\frac{\ddot{a}}{a} - (D-2)\frac{\ddot{c}}{c} &= \frac{V}{D-2} + \frac{1}{2}(\dot{\vec{\phi}})^2 - \frac{(D-3)}{2(D-2)} c^{-2(D-2)} \sum_i \lambda_i^2 e^{-\vec{a}_i \cdot \vec{\phi}}, \\
-\frac{\dot{a}\dot{c}}{ac} - \frac{\ddot{c}}{c} + \frac{(D-3)\epsilon}{c^2} - \frac{(D-3)\dot{c}^2}{c^2} &= \frac{V}{D-2} + \frac{1}{2(D-2)} c^{-2(D-2)} \sum_i \lambda_i^2 e^{-\vec{a}_i \cdot \vec{\phi}}.
\end{aligned} \tag{A.6}$$

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