Critical densities for the Skyrme type effective interactions

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Abstract

We use the stability conditions of the Landau parameters for the symmetric nuclear matter and pure neutron matter to calculate the critical densities for the Skyrme type effective nucleon-nucleon interactions. We find that the critical density can be maximized by adjusting appropriately the values of the enhancement factor κ associated with isovector giant dipole resonance, the quantity L which is directly related to the slope of the symmetry energy and the Landau parameter G'_0. However, restricting κ, L and G'_0 to vary within acceptable limits reduces the maximum value for the critical density $\tilde{\rho}_{cr}$ by $\sim 25\%$. We also show that among the various quantities characterizing the symmetric nuclear matter, $\tilde{\rho}_{cr}$ depends strongly on the isoscalar effective mass m^*/m and surface energy coefficient E_s. For realistic values of m^*/m and E_s we get $\tilde{\rho}_{cr} = 2\rho_0$ to $3\rho_0$ ($\rho_0 = 0.16\text{fm}^{-3}$).

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There are many different parameterizations of the Skyrme type effective nucleon-nucleon interaction as recently reviewed in Ref. [1]. Values of the Skyrme parameters are usually obtained by fitting the results of the Hartree-Fock calculations for the binding energies, charge radii and spin-orbit splittings of a few closed shell nuclei to the observed ones using a least square procedure. The Skyrme parameters are also constrained to yield appropriate values for some properties of the infinite symmetric nuclear matter at the saturation density ρ_{nm}. Over time, several improvements were made by appropriately treating the center of mass correction to the binding energy, modifying the spin-orbit part of the interaction and including in the least square fit the equation of state (EOS) for the pure neutron matter obtained from a realistic interaction [2, 3]. Nevertheless, not all the Skyrme parameters are well determined in this way.

Recently, it has been shown in Ref. [4] that a more realistic parameterization of the Skyrme interaction can be obtained by subjecting it to stability requirements of the EOS defined by the inequality conditions for the Landau parameters for symmetric nuclear matter and pure neutron matter. In other words, few of the Skyrme parameters not well determined by the experimental data used in the least square procedure can be restricted by requiring that the inequality conditions are satisfied up to a maximum value for a nuclear matter density, also referred to as the critical density ρ_{cr}. A very recent systematic study carried out in Ref. [5] using several Skyrme interactions indicates that the density dependence of the symmetry energy coefficient J plays a critical role in determining the properties of neutron star. Out of 87 different parameterizations for the Skyrme interaction considered in Ref. [5] only 27 of them, having a positive slope for the symmetry energy coefficient at nuclear matter densities ρ up to $3\rho_0$ ($\rho_0 = 0.16\text{fm}^{-3}$), are found to be suitable for the neutron star model. Thus, it appears that the parameters of the Skyrme interactions can be better constrained by combining the findings of Refs. [4] and [5].

In the present work we study the dependence of ρ_{cr} on the nuclear matter saturation density ρ_{nm}, binding energy per nucleon B/A, isoscalar effective mass m^*/m, incompressibility coefficient K_{nm}, surface energy E_s, and symmetry energy coefficient J, as also considered in Ref. [4]. It must be emphasized that we determine ρ_{cr} in terms of the enhancement factor κ, the coefficient $L = 3\rho dJ/d\rho$ and the Landau parameter G_0' (at ρ_{nm}) instead of the combinations $t_i x_i$ ($i = 1, 2,$ and 3) of the Skyrme parameters t_i and x_i as used in Ref. [4]. Unlike the combinations $t_i x_i$, the quantities κ, L and G_0', which can be expressed in terms of the Skyrme parameters, are related to some physical processes. The enhancement factor κ accounts for the deviations from the Thomas-Reiche-Kuhn (TRK) sum rule in the case of the isovector giant dipole resonance. The value of κ at the ρ_{nm} is expected to be ~ 0.5 [2, 6]. The slope of the symmetry energy coefficient must be positive for ρ up to $3\rho_0$, so that resulting Skyrme interaction can be suitable for the study of the properties of neutron stars [5]. The Landau parameter G_0' must be positive at $\rho \leq \rho_{nm}$ in order to appropriately describe the position of the isovector $M1$ and Gamow-Teller states [6]. We find that by restricting the values of κ, L and G_0' within the acceptable limits, the maximum critical densities are lowered by about 25% compared to the ones obtained without such restrictions [4]. The constrains proposed in the present work not only maximizes the value of the critical density, but also ensures that the resulting Skyrme interaction can be used to study the bulk properties of finite nuclei as well as those of neutron stars.

Recently [1, 7] a generalized form for the Skyrme energy density functional has been obtained using the Hohenberg-Kohn-Sham approach. However, in the present work we restrict ourselves to the energy density functional associated with the commonly used Skyrme interaction [2, 8],

$$\begin{aligned} V_{12} &= t_0\left(1+x_0P_{12}^{\sigma}\right)\delta(\mathbf{r}_1-\mathbf{r}_2) \\ &+\frac{1}{2}t_1\left(1+x_1P_{12}^{\sigma}\right)\times\left[\overleftarrow{k}_{12}^{2}\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_2)\overrightarrow{k}_{12}^{2}\right] \\ &+t_2\left(1+x_2P_{12}^{\sigma}\right)\overleftarrow{k}_{12}\delta(\mathbf{r}_1-\mathbf{r}_2)\overrightarrow{k}_{12} \\ &+\frac{1}{6}t_3\left(1+x_3P_{12}^{\sigma}\right)\rho^{\alpha}\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)\delta(\mathbf{r}_1-\mathbf{r}_2) \\ &+iW_0\overleftarrow{k}_{12}\delta(\mathbf{r}_1-\mathbf{r}_2)(\overrightarrow{\sigma_1}+\overrightarrow{\sigma_2})\times\overrightarrow{k}_{12} \end{aligned} \tag{1}$$

where, P_{12}^{σ} is the spin exchange operator, $\overrightarrow{\sigma}_i$ is the Pauli spin operator, $\overrightarrow{k}_{12} = -i(\overrightarrow{\nabla}_1 - \overrightarrow{\nabla}_2)/2$ and $\overleftarrow{k}_{12} = -i(\overleftarrow{\nabla}_1 - \overleftarrow{\nabla}_2)/2$. Here, the right and left arrows indicate that the momentum operators act on the right and on the left, respectively. The Skyrme parameters t_i, x_i and α for a fixed value of W_0 can be expressed in terms of the quantities associated with the symmetric nuclear matter as follows [2, 4, 9].

$$\begin{aligned} t_0 &= \frac{8}{\rho_{nm}}\left[\frac{\left(-B/A+(2m/m^*-3)\left(\hbar^2/10m\right)k_f^2\right)\left(\frac{1}{27}K_{nm}-(1-6m^*/5m)\left(\hbar^2/9m^*\right)k_f^2\right)}{-B/A+\frac{1}{9}K_{nm}-(4m/3m^*-1)\left(\hbar^2/10m\right)k_f^2}\right. \\ &+ \left.\left(1-\frac{5m}{3m^*}\right)\frac{\hbar^2}{10m}k_f^2\right], \end{aligned} \tag{2}$$

$$t_1 = \frac{2}{3}\left[T_0 + T_s\right], \tag{3}$$

$$t_2 = t_1 + \frac{8}{3}\left[\left(\frac{1}{4}t_0 + \frac{1}{24}t_3\rho_{nm}^{\alpha}\right)\frac{2m^*}{\hbar^2}\frac{k_f}{\pi^2} + G_0'\right]\frac{\hbar^2}{m^*\rho_{nm}}, \tag{4}$$

$$t_3 = \frac{16}{\rho_{nm}^{\alpha+1}}\frac{\left(-B/A + (2m/m^* - 3)\left(\hbar^2/10m\right)k_f^2\right)^2}{-B/A + \frac{1}{9}K_{nm} - (4m/3m^* - 1)\left(\hbar^2/10m\right)k_f^2}, \tag{5}$$

$$x_0 = \frac{4}{t_0\rho_{nm}}\left[\frac{\hbar^2}{6m}k_f^2 - \frac{1}{24}t_3(x_3 + \frac{1}{2})\rho_{nm}^{\alpha+1} + \frac{1}{24}\left(t_2\left(4 + 5x_2\right) - 3t_1x_1\right)\rho_{nm}k_f^2 - J\right] - \frac{1}{2}, \tag{6}$$

$$x_1 = \frac{1}{t_1}\left[4\frac{\hbar^2\kappa}{m\rho_{nm}} - t_2(2 + x_2)\right] - 2, \tag{7}$$

$$x_2 = \frac{1}{4t_2}\left[8T_0 - 3t_1 - 5t_2\right], \tag{8}$$

$$x_3 = -\frac{8}{\alpha t_3\rho_{nm}^{\alpha+1}}\left[\frac{\hbar^2}{6m}k_f^2 - \frac{1}{12}\left((4 + 5x_2)t_2 - 3t_1x_1\right)\rho_{nm}k_f^2 - 3J + L\right] - \frac{1}{2}, \tag{9}$$

$$\alpha = \frac{B/A - \frac{1}{9}K_{nm} + (4m/3m^* - 1)\left(\hbar^2/10m\right)k_f^2}{-B/A + (2m/m^* - 3)\left(\hbar^2/10m\right)k_f^2}, \tag{10}$$

where,

$$T_0 = \frac{1}{8}\left(3t_1 + (5 + 4x_2)t_2\right) = \frac{\hbar^2}{m\rho_{nm}}\left(\frac{m}{m^*} - 1\right), \tag{11}$$

$$T_s = \frac{1}{8}\left[9t_1 - (5 + 4x_2)t_2\right], \tag{12}$$

and

$$k_f = \left(\frac{3\pi^2}{2}\rho_{nm}\right)^{1/3}. \tag{13}$$

In Eqs. (2)-(10), the various quantities characterizing the nuclear matter are the binding energy per nucleon B/A, isoscalar effective mass m^*/m, nuclear matter incompressibility coefficient K_{nm}, symmetry energy coefficient J, the coefficient L which is directly related to the slope of the symmetry energy coefficient ($L = 3\rho dJ/d\rho$), enhancement factor κ and Landau parameter G_0'. All these quantities are taken at the saturation density ρ_{nm}. It must be pointed out that we include consistently all terms in the energy density functional. In particular, the expression for the parameter G_0' used in the Eq. (4) is consistent with the energy density functional obtained using Eq. (1). For a more generalized Skyrme energy density functional, the expression for G_0' should be appropriately modified following Ref. [7]. Once, T_0 is known, T_s can be calculated for a given value of the surface energy E_s as [4],

$$E_s = 8\pi r_0^2 \int_0^{\rho_{nm}} d\rho \left[\frac{\hbar^2}{36m} - \frac{5}{36}T_0\rho + \frac{1}{8}T_s\rho - \frac{m^*}{\hbar^2}V_{\rm so}\rho^2\right]^{1/2} \left[B(\rho_{nm})/A - B(\rho)/A\right]^{1/2} \quad (14)$$

where, $B(\rho)/A$ is the binding energy per nucleon given by,

$$\frac{B(\rho)}{A} = -\left[\frac{3\hbar^2}{10m^*}k_f^2 + \frac{3}{8}t_0\rho + \frac{1}{16}t_3\rho^{\alpha+1}\right] \quad (15)$$

and,

$$r_0 = \left[\frac{3}{4\pi\rho_{nm}}\right]^{1/3}, \quad V_{\rm so} = \frac{9}{16}{W_0}^2. \quad (16)$$

The manner in which Eqs. (2) - (10) can be used to evaluate the Skyrme parameters t_i, x_i and α is as follows. First, the parameters t_0 and α can be obtained from Eqs. (2) and (10), respectively. Then, the parameter t_3 can be calculated using Eq. (5). Next, T_0 and T_s can be calculated using Eqs. (11) and (14), respectively. Once, the combinations T_0 and T_s of the Skyrme parameters are known, one can calculate the remaining parameters in the following sequence, t_1, t_2, x_2, x_1, x_3 and x_0.

The stability criteria requires that [10],

$$\mathcal{X}_l > -(2l+1), \quad (17)$$

where, $\mathcal{X}_l$ stands for the Landau parameters F_l, F_l', G_l and G_l' for a given multipolarity l. The Skyrme interaction only contains monopolar and dipolar contributions to the particle-hole interaction so that all Landau parameters are zero for $l > 1$. Thus, there are 12 different Landau parameters, i.e., F_l, F_l', G_l and G_l' ($l = 0, 1$) for the symmetric nuclear matter and $F_l^{(n)}$, $G_l^{(n)}$ ($l = 0, 1$) for the pure neutron matter. Each of these Landau parameters must satisfy the inequality condition given by Eq. (17). Using the expressions for the Landau parameters in terms of the Skyrme parameters, given in Refs. [4], one can obtain the values of the Landau parameters at any density for a given set of the Skyrme parameters. Thus, the critical density which is nothing but the maximum density up to which all the inequality conditions are met can be easily determined.

We have compiled the values of κ, L and G_0' for several parameterization of the Skyrme interaction presented in Refs. [2,3,11-16]. We find that the values of κ, L and G_0' vary over a wide ranges $0-2$, $40-160$ MeV and $-0.15-1.0$, respectively. This is due to the fact that the experimental data used in the least-square procedure to fit the values of the Skyrme parameters can not constrain well the values of these quantities. These quantities can be constrained by requiring a reasonable value for the critical density. In what follows, we shall refer to the set of standard values for six quantities $\rho_{nm} = 0.16\text{fm}^{-3}$, $B/A = 16$ MeV, $K_{nm} = 230$ MeV, $m^*/m = 0.7$, $E_s = 18$ MeV and $J = 32$MeV in short as the STD values for the nuclear matter input as used in Ref. [4].

We now study the dependence of ρ_{cr} on the ρ_{nm}, B/A, m^*/m, K_{nm}, E_s and J. For a given set of values for these quantities, we calculate the maximum value of ρ_{cr} by varying κ, L and G_0' at the ρ_{nm} within acceptable limits. We denote the maximum value of the critical density by $\tilde{\rho}_{cr}$. It must be emphasized here that the present work differs from the one performed in Ref. [4] by the fact that we constrain the values of κ, L and G_0', whereas, in Ref. [4], the value of $\tilde{\rho}_{cr}$ was calculated by varying the combinations $t_i x_i$ ($i = 1, 2$, and 3) of Skyrme parameters with no restrictions. We find that if the space of the parameters $t_i x_i$ is not restricted, it may lead to an unreasonable values of κ and L. As an example, for the STD values of nuclear matter input, $\tilde{\rho}_{cr}$ becomes $3.5\rho_0$ for $\kappa = 1.0$, $L = 36$ MeV and $G_0' = 0.20$ (at ρ_0). The value of G_0' seems reasonable, but the value of $\kappa = 1.0$ is a little too large [2, 6]. Further, we find that for $\rho > \rho_0$ the value of L decreases with increasing ρ and it becomes negative for $\rho > 1.6\rho_0$, which makes the interaction not favorable for the neutron star model.

In Fig. 1 we have displayed the results for $\tilde{\rho}_{cr}$ obtained by varying the various quantities associated with the nuclear matter around their standard values. To calculate $\tilde{\rho}_{cr}$ we allow for $0.25 \leq \kappa \leq 0.5$, $0 \leq L \leq 100$ MeV and $0 \leq G_0' \leq 0.5$ at the ρ_{nm}. We further demand that $L > 0$ at $3\rho_0$. Fig. 1 shows that $\tilde{\rho}_{cr}$ depends strongly on m^*/m and E_s. Whereas, $\tilde{\rho}_{cr}$ depends weakly on ρ_{nm}, B/A and K_{nm} and it is almost independent of J. These features for $\tilde{\rho}_{cr}$ are qualitatively similar to the ones presented in Ref. [4]. However, due to the restrictions imposed on the values of κ, L and G_0', the values of $\tilde{\rho}_{cr}$ becomes smaller than that obtained in Ref. [4] by up to 25%. For $m^*/m = 0.6$ (0.7) and keeping all the other nuclear matter quantities equal to their standard values, we get $\tilde{\rho}_{cr} = 4.5\rho_0$ $(2.8\rho_0)$ compared to $6\rho_0$ $(3.5\rho_0)$ obtained in Ref. [4].

It may be instructive to present the values of κ, L and G_0' required to obtained $\tilde{\rho}_{cr}$ for a given set of values for the nuclear matter input. We find that κ lies in the range of $0.45-0.5$ for variations in the nuclear matter input by up to $\pm 15\%$ relative to their standard values. These values of κ clearly reflects the fact that restricting κ to take values in the range of $0.25-0.5$ delimits the $\tilde{\rho}_{cr}$ to a lower value. In Figs. 2a and 2b we have plotted the values of L and G_0' (at ρ_{nm}), respectively, which are needed to yield the $\tilde{\rho}_{cr}$. We see that L varies from 20 to 60 MeV for different values of the nuclear matter input. For the STD values of nuclear matter input we find that $L = 47$ MeV at $\rho = \rho_0$. This value is quite large compared to the values of $L = 35, 27$ and 16 MeV associated with the the Skz0, Skz1 and Skz2 interactions [4], respectively, which were obtained for the same standard values of the nuclear matter input, but by varying the combinations t_1x_1 and t_2x_2 with no restrictions and keeping t_3x_3 fixed to some arbitrary values. We see from Fig. 2b that, except for J, the value of G_0' (at ρ_{nm}) depends strongly on the values of the various quantities associated with nuclear matter. The dependence of G_0' on the surface energy coefficient E_s is the most pronounced one. We note that E_s is mainly determined by the ground state properties of light nuclei. Thus, the center of mass correction to the binding energy and charge radii must be appropriately taken into account as they are very important for light nuclei and may affect the values obtained for E_s. We see from Fig. 2b that G_0' tends to vanish rapidly with increasing E_s.

As pointed out earlier (see from Fig. 1), the dependence of $\tilde{\rho}_{cr}$ on ρ_{nm}, J, and B/A is quite weak. Thus, it may be sufficient to calculate $\tilde{\rho}_{cr}$ as a function of m^*/m, E_s and K_{nm} only. In Fig. 3 we display our results for the variation of E_s versus m^*/m, obtained for fixed values of $\tilde{\rho}_{cr}$ with the remaining nuclear matter quantities kept equal to their standard values. It can be seen from Fig. 3 that for a fixed value of $\tilde{\rho}_{cr}$, E_s decreases with the increase in m^*/m. It is quite interesting to note that for $E_s = 18 \pm 1$ MeV (as most of the Skyrme interactions yield), $\tilde{\rho}_{cr} = 2\rho_0$ and $3\rho_0$ for $m^*/m = 0.72-0.85$ and $0.63-0.73$, respectively. For $\tilde{\rho}_{cr} = 4\rho_0$ one must have $m^*/m \sim 0.65$ for not too low value of E_s. The value of m^*/m is also constrained by the centroid energy of the isoscalar giant quadrupole resonance [17] which favors $m^*/m \geq 0.7$. Thus, for reasonable values of E_s and m^*/m, one may obtain a Skyrme interaction with $\tilde{\rho}_{cr} = 2\rho_0$ to $3\rho_0$.

In summary, we have used the stability conditions of the Landau parameters for the symmetric nuclear matter and pure neutron matter to calculate the critical densities for the Skyrme type effective nucleon-nucleon interactions. We find that the critical density can be maximized by appropriately adjusting the values of the enhancement factor κ, coefficient L and the Landau parameter G_0' as these quantities are not well determined by the Skyrme parameters, conventionally obtained by fitting the experimental data for the ground state properties of finite nuclei. We exploit the fact that i) The value of κ should be in the range of $0.25 - 0.5$, needed to describe the TRK sum rule for the isovector giant dipole resonance [2, 6], ii) $L > 0$ for $0 \leq \rho \leq 3\rho_0$; a condition necessary for a Skyrme interaction to be suitable for studying the properties of neutron star [5]. and iii) $G_0' > 0$ in order to reproduce the energies of the isovector $M1$ and Gamow-Teller states [6]. The maximum value of the critical density so obtained is lower by up to 25% compared to the ones obtained without any such restrictions [4]. We show that the critical density obtained for realistic values of the surface energy coefficient ($E_s = 18 \pm 1$ MeV) and isoscalar effective mass ($m^*/m = 0.7 \pm 0.1$) lie in the range of $2\rho_0$ to $3\rho_0$. Finally, we would like to remark that the pairing correlations were not included in the present work. It may be worthwhile to investigate the influence of the pairing correlations on the various quantities associated with the nuclear matter which may alter the value of the critical density.

Acknowledgments

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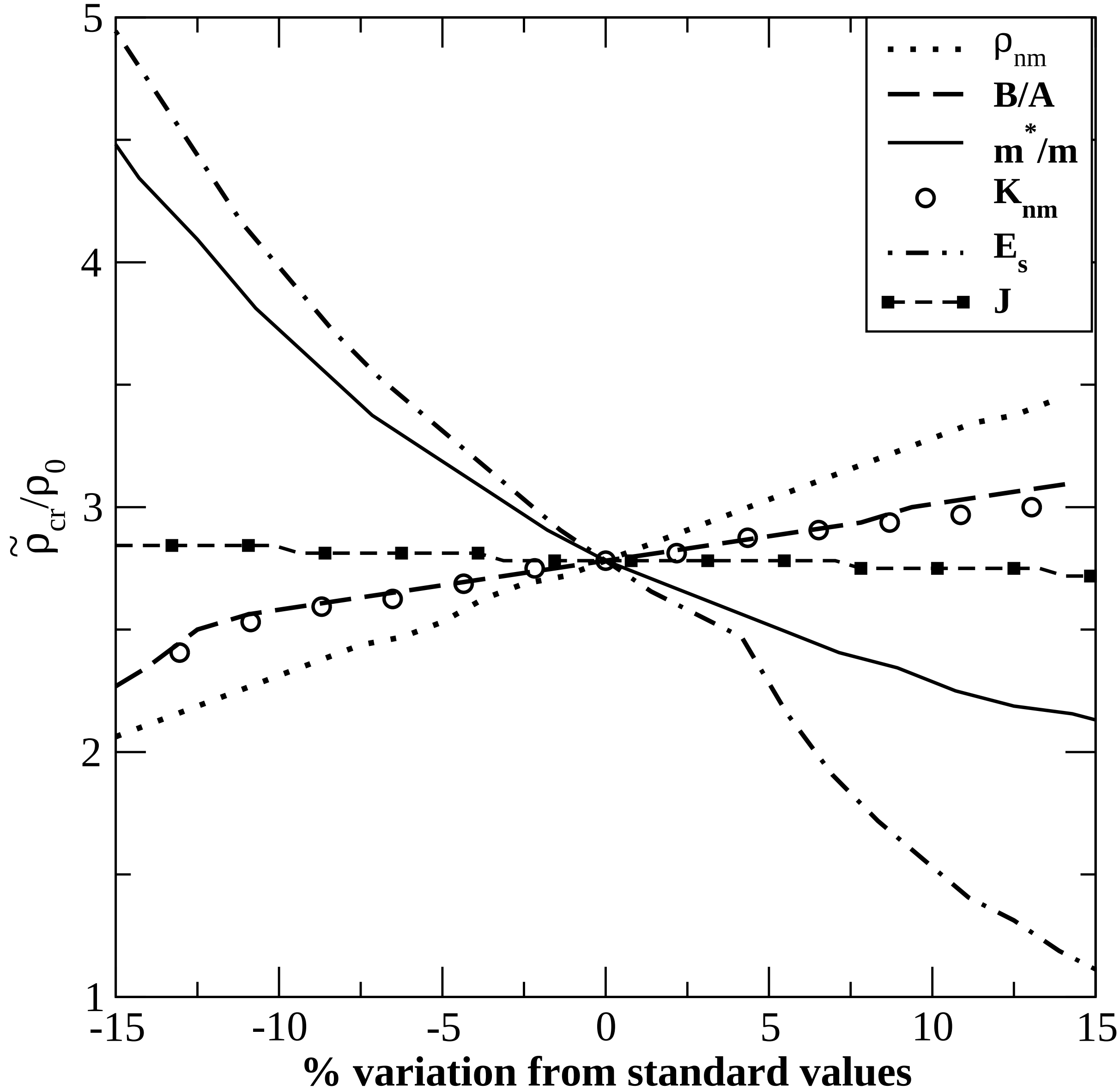
FIG. 1: The dependence of critical density $\tilde{\rho}_{cr}$ on the relative variation of ρ_{nm} (dotted), B/A (dashed), m^*/m (solid), K_{nm} (open circles), E_s (dashed-dot), and J (dashed-filled squares) around their standard values.

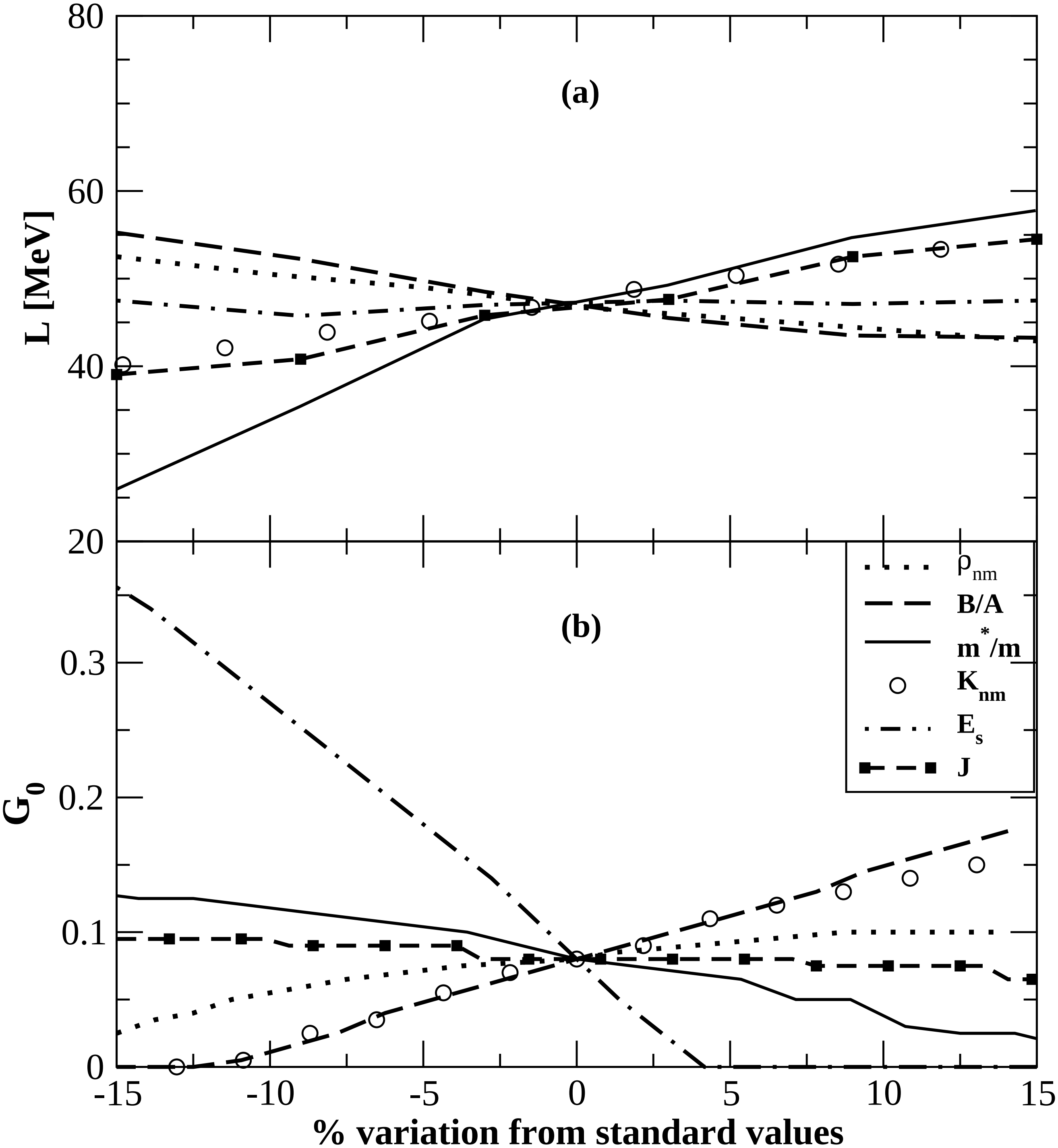
FIG. 2: Variation of (a) the coefficient L and (b) the Landau parameter G_0' as a function of the various quantities associated with nuclear matter at ρ_{nm}. The values of L and G_0' are determined by maximizing the critical density for a given set of values for the nuclear matter quantities.

FIG. 3: Variations of the surface energy coefficient E_s at ρ_{nm} as a function of the effective mass m^*/m for fixed values of the critical density $\tilde{\rho}_{cr} = 2\rho_0, 3\rho_0$ and $4\rho_0$ as labeled. All the other nuclear matter quantities are kept equals to their standard values.



ρnm
B/A
m*/m
Knm
Es
J
ρ̃cr/ρ0
% variation from standard values
5
4
3
2
1
-15
-10
-5
0
5
10
15

Fig.1



(a)
(b)
L [MeV]
G'0
80
60
40
20
0.3
0.2
0.1
0
-15
-10
-5
0
5
10
15
% variation from standard values
ρnm
B/A
m*/m
Knm
Es
J

Fig.2

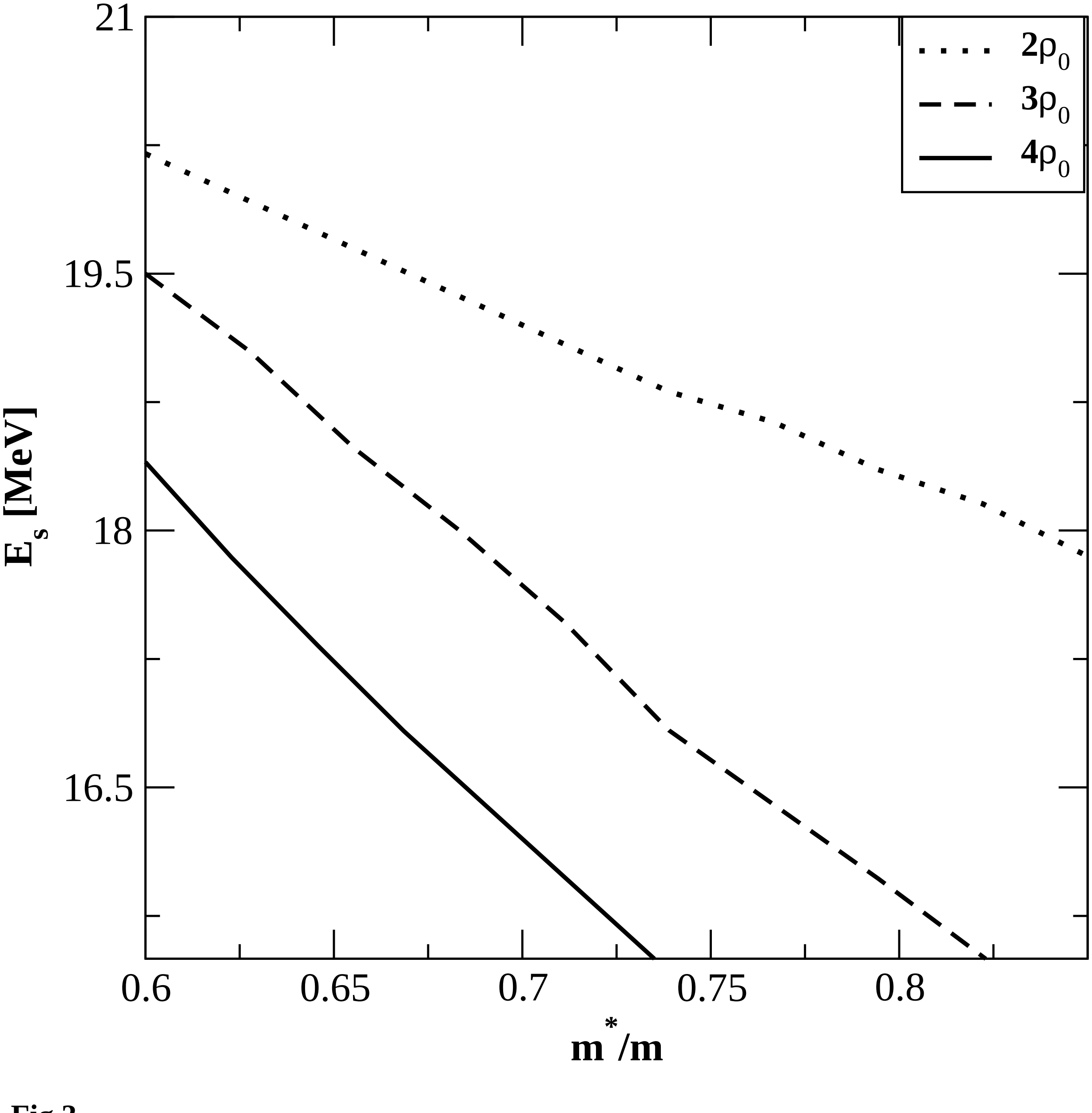


Fig.3