View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Texas A&M Repository

ACT-8-14, MIFPA-14-22

Aligned Natural Inflation and Moduli Stabilization from Anomalous $U(1)$ Gauge Symmetries

Tianjun Li ,[1,2] Zhijin Li,[3] and Dimitri V. Nanopoulos[3,4,5]

[1] *State Key Laboratory of Theoretical Physics and Kavli Institute for Theoretical Physics China (KITPC), Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China*

[2] *School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu 610054, P. R. China*

[3] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, TX 77843, USA*

[4] *Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA*

[5] *Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece*

arXiv:1407.1819v1 [hep-th] 7 Jul 2014

Abstract

To obtain natural inflation with large tensor-to-scalar ratio in string framework, we need a special moduli stabilization mechanism which can separate the masses of real and imaginary components of Kähler moduli at different scales, and achieve a trans-Planckian axion decay constant from sub-Planckian axion decay constants. In this work, we stabilize the matter fields by F-terms and the real components of Kähler moduli by D-terms of two anomalous $U(1)_X \times U(1)_A$ symmetries strongly at high scales, while the corresponding axions remain light due to their independence on the Fayet-Iliopoulos (FI) term in moduli stabilization. The racetrack-type axion superpotential is obtained from gaugino condensations of the hidden gauge symmetries $SU(n)\times SU(m)$ with massive matter fields in the bi-fundamental respresentations. The axion alignment via Kim-Nilles-Pelroso (KNP) mechanism corresponds to an approximate S_2 exchange symmetry of two Kähler moduli in our model, and a slightly S_2 symmetry breaking leads to the natural inflation with super-Planckian decay constant.

PACS numbers: 04.65.+e, 04.50.Kd, 12.60.Jv, 98.80.Cq

I. INTRODUCTION

Natural inflation was proposed to explain the unnatural flatness of inflationary potential which is introduced *ad hoc* at tree level and remains flat under radiative corrections [1]. The flatness of inflationary potential is protected by continuous shift symmetry of an axionic field ϕ. But this symmetry is spontaneously broken to a discrete shift symmetry at inflation scale, and the following inflationary potential is generated

$$V(\phi) = \Lambda^4(1 \pm \cos(\frac{\phi}{f})), \tag{1}$$

which is invariant under the discrete shift symmetry $\phi \to \phi + 2\pi f$ with f as an axion decay constant. Recent observation of the B-mode polarization by the BICEP2 Collaboration suggests a large tensor-to-scalar ratio $r = 0.16^{+0.06}_{-0.05}$ excluding the dust effects [2], which can be obtained from natural inflation with trans-Planckian axion decay constant $f \sim O(10)\,\mathrm{M_{Pl}}$, where $\mathrm{M_{Pl}} = 2.4 \times 10^{18}$GeV is the reduced Planck mass [3]. While the value of r could be much smaller [4, 5] if the dust polarization effect plays more important role than estimated in Ref. [2]. Besides, natural inflation with large r also agrees with the Planck observations [6], in which a lower bound of trans-Planckian axion decay constant is needed $f \geqslant 5\mathrm{M_{Pl}}$.

It is an attractive destination to realize natural inflation in string theory or its efffective no-scale supergravity (SUGRA) theory. Axions arise from anti-symmetric tensor fields in string theory through compactification of the extra space dimensions, and may play important roles in cosmology and particle physics [7]. A more fundamental reason for stringy inflation is that, according to the Lyth bound obeyed by general single field slow-rolling process [8], large tensor-to-scalar ratio requires trans-Planckian excursion during inflation. Therefore, corrections from the Planck-suppressed operators are non-ignorable, and a reliable inflation theory has to be constructed based on the ultra violet (UV) complete theory such as string theory.

Axion (as imaginary component of Kähler modulus T) inflation in string theory needs a moduli stabilization mechanism, which can separate the masses of real and imaginary components of Kähler modulus T at different scales. Specifically, the real component of modulus should be frozen during inflation, so it obtains a large mass from modulus stabilization: $M_{\mathrm{Re(T)}} > H$, where H is the Hubble constant during inflation, at the order 10^{14} GeV from the BICEP2 results, while the "effective mass" of axion $M_{\mathrm{Im(T)}} = \Lambda^2/f$ is of order 10^{13} GeV. In the well-known KKLT mechanism [9], once the real component of modulus is

stabilized, the imaginary component obtains a large mass comparable to the real component which destroys axion inflation [10].

Moduli stabilization, which is consistent with axion inflation, has been proposed recently in Refs. [11, 12]. In these works, an anomalous $U(1)_X$ gauge symmetry has been introduced to split the masses of real and imaginary components of Kähler moduli. The anomalous $U(1)_X$ gauge symmetry introduces the moduli-dependent Fayet-Iliopoulos (FI) term as a result of the non-trivial moduli transformation under $U(1)_X$. The D-term scalar potential of $U(1)_X$ depends only on the real component of moduli, and actually is close to string scale by taking suitable gauge charges. Consequently, the D-term flatness provides a strong stabilization on real component of moduli once the $U(1)_X$ charged matter fields are stabilized by F-terms. The stabilization of matter fields can be directly done by tree-lever superpoential. While axion potential for natural inflation is obtained from non-perturbative effects, so it is generally much weaker than the perturbative terms [12]. In the model, besides moduli stabilization consistent with axion inflation, $U(1)_X$ symmetry also provides an elegant solution to the problem of super-Planckian axion decay constant. A general review on the anomalous $U(1)$ gauge symmetry in SUGRA and its applications on cosmology is provided in Ref. [13].

However, the super-Planckian axion decay constant in natural inflation required by the BICEP2 results is problematic in string theory. String theory predicts the axion decay constants cannot surpass the string scale [7, 14, 15], while as a controllable theory, the scale of weakly coupled string theory should be lower than the Planck scale. The Kim-Nilles-Pelroso (KNP) mechanism was proposed to obtain the effective super-Planckian axion decay constant [16]. In this proposal, two axions with sub-Planckian decay constants are aligned to have a flat direction along which the effective decay constant can be large enough for natural inflation. Another solution to this problem was provided in Ref. [12] based on an anomalous $U(1)$ gauge symmetry. In this work the axion decay constant is directly determined by the flatness of anomalous $U(1)$ D-term, and can be super-Planckian by taking a reasonably large condensation gauge group.

Recently, a lot of works on the KNP mechanism have been done after the BICEP2 results [17–21]. Specifically, in Ref. [20] a stringy geometrical realization of aligned axions was proposed based on the assumption that the moduli are well stabilized. While a complete realization of the KNP mechanism based on string framework, which is consistent with moduli stabilization, is still absent.

In this paper, we will realize both the KNP mechanism and moduli stabilization in string inspired no-scale supergravity with anomalous gauge symmetries $U(1)_X \times U(1)_A$. Similar to our previous study [11, 12], we stabilize the matter fields by F-term potential and the moduli by the D-terms of anomalous $U(1)$ symmetries. Two axions are imaginary components of two Kähelr moduli, which transform non-trivially under two anomalous gauge symmetries $U(1)_X \times U(1)_A$. The axion superpotential is of racetrack-type, and is from gaugino condensations of two gauge groups $SU(n) \times SU(m)$ with massive quark representations. The alignment of axions as in the KNP proposal, actually corresponds to an approximate S_2 exchange symmetry between two Kähler moduli. This S_2 symmetry is explicitly broken slightly so that one linear combination of the axions gives us the natural inflation with super-Planckian effective decay constant.

This paper is organized as follows. In Section 2 we present string inspired no-scale SUGRA with anomalous gauge symmetries $U(1)_X \times U(1)_A$. In Section 3 we provide stabilizations of matter fields and Kähler moduli based on F-term and D-term potentials, respectively. In Section 4 we show that the aligned natural inflation can be realized by superpotential obtained from gaugino condensations of hidden gauge groups. Conclusions are given in Section 5.

II. ALIGNED AXIONS IN NO-SCALE SUGRA

We start from the no-scale type SUGRA with Kähler potential

$$K = -\ln(T_1 + \bar{T}_1) - \ln(T_2 + \bar{T}_2) + \phi_i \bar{\phi}_i + \chi_i \bar{\chi}_i + \psi_i \bar{\psi}_i + X_i \bar{X}_i + Y_i \bar{Y}_i \ , \tag{2}$$

where $i = 1, 2$, and T_1 and T_2 are Kähler moduli. No-scale SUGRA [22] was realized naturally in the compactifications of weakly coupled heterotic string theory [23] or M-theory on S^1/Z_2 [24]. For the third Kähler modulus T_3, we assume that it is neutral under anomalous gauge symmetries $U(1)_X \times U(1)_A$ and then is ignored in this work. Two axions θ_i are the imaginary parts of the Kähler moduli, *i.e.*, $\theta_i \equiv \mathrm{Im}(T_i)$, $i = 1, 2$.

As we know, n stacks of D7-branes, which wrap a 4-cycles of the Calabi-Yau space, gives $U(n)$ gauge group. As in the KKLT scenario, the condensation gauge group is $SU(n)$, and typically there is another anomalous $U(1)$ gauge symmetry. For two Kähler moduli, there can be two copies of such gauge sectors, saying $SU(n) \times SU(m) \times U(1)_X \times U(1)_A$. In particular,

two anomalous $U(1)$ gauge symmetries can play special role in moduli stabilization and inflation.

We assume that the vector-like massive quarks are in the fundamental representations of Yang-Mills gauge groups $SU(n)\times SU(m)$. After integrating out the heavy chiral superfields, the gaugino condensations of gauge groups $SU(n)\times SU(m)$ generate the following effective superpotential

$$W = A\phi_1^{\frac{1}{m}} e^{-(aT_1+bT_2)} + B\phi_2^{\frac{1}{n}} e^{-(aT_1+bT_2+cT_2)}, \tag{3}$$

where ϕ_1 and ϕ_2 are matter fields charged on both $SU(n)$ and $SU(m)$ [25–27]. Here, we have assumed that the gauge kinetic functions of $SU(n)$ and $SU(m)$, which relate to the gauge anomaly cancellations of $SU(n)^2\times U(1)_a$ and $SU(m)^2\times U(1)_a$, are

$$f_{SU(n)} \propto aT_1 + bT_2 + cT_2 \ , \tag{4}$$

$$f_{SU(m)} \propto aT_1 + bT_2 \ . \tag{5}$$

The superpotential in Eq. (3) and moduli dependent D-term are employed to stabilize the matter fields and real parts of the moduli, respectively. Once the matter fields ϕ_i obtain vacuum expectation values (VEVs), the real part of the Kähler moduli are fixed by the D-term flatnesses. The effects of non-perturbative terms on moduli stabilization and inflation based on anomalous $U(1)$ have been studied in Refs. [28–33]. In these works, normally the D-terms are non-cancellable so that they can uplift the AdS vacua to dS vacua from moduli stabilization. The non-cancellability of anomalous $U(1)$ D-term arises from the massless fundamental representation of condensation gauge group. While here the cancellable anomalous $U(1)$ D-terms are preferred, the condensation gauge groups are equipped with massive fundamental representations. Recently, under the stimulation from the BICEP2 results, the potential roles of gaugino condensation on inflation have been studied in [34, 35].

In our strategy, fields ϕ_i are stabilized by their superpotential terms, which break anomalous $U(1)_X$ (also the continous shift symmetry of Kähler modulus) spontaneously. This procedure was first proposed in Ref. [12] for string-inspired no-scale SUGRA, and later it was applied to the case of the minimal SUGRA [36].

	T_1	T_2	X_i	Y_i	ϕ_1	ϕ_2	χ_1	χ_2	ψ_1	ψ_2
$U(1)_X$	δ_X^1	δ_X^2	0	0	q	q	$-q$	$-q$	q	q
$U(1)_A$	δ_A^1	δ_A^2	0	0	$\tilde{q}$	$2\tilde{q}$	$-\tilde{q}$	$-2\tilde{q}$	$\tilde{q}$	$2\tilde{q}$

TABLE I: $U(1)_X \times U(1)_A$ charges of the Kähler moduli and matter fields.

A. $U(1)_X \times U(1)_A$ Gauge Invariance

The $U(1)_a$ charges of the Kähler moduli and matter fields are provided in Table I, so the overall superpotential, which is invariant under gauge transformations of $U(1)_a$ with $a \in \{A, X\}$, is given by

$$\begin{aligned} W = w_\star + A\phi_1^{\frac{1}{m}} e^{-(aT_1+bT_2)} + B\phi_2^{\frac{1}{n}} e^{-(aT_1+bT_2+cT_2)} \\ +X_i(\phi_i\chi_i - \lambda_1) + Y_i(\psi_i\chi_i - \lambda_2) + c_0\psi_i\chi_i, \end{aligned} \tag{6}$$

in which the constant term $w_\star$ is arising after integrating out all complex-structure moduli. Similar to Ref. [12], we may realize the above superpotential or its equivalent.

The matter fields $z_n \in \{\phi_i, \chi_i, \psi_i\}$ transform linearly under $U(1)_a$, while Kähler moduli shift under the $U(1)_a$ gauge transformations

$$\begin{aligned} T_i &\to T_i + i\delta_a^i \epsilon, \\ z_n &\to z_n e^{i\epsilon q_{z_n}^a}. \end{aligned} \tag{7}$$

The $U(1)_a$ gauge invariance of matter couplings in (6) is clearly based on the charges provided in Table I. While for the non-perturbative terms, gauge invariance requires the following condition

$$\begin{pmatrix} a & b \\ a & b+c \end{pmatrix} \begin{pmatrix} \delta_\alpha^1 \\ \delta_\alpha^2 \end{pmatrix} = \begin{pmatrix} \frac{q_\alpha^1}{m} \\ \frac{q_\alpha^2}{n} \end{pmatrix},$$

where $\alpha \in \{A, X\}$ and q_α^i means the $U(1)_\alpha$ charge of ϕ_i. Without c the equation is degenerate and the moduli charges cannot be uniquely determined based on charges of the matter fields.

In the superpotential (6), the parameter c is very small. There is an exact S_2 exchange symmetry

$$aT_1 \leftrightarrow bT_2, \tag{8}$$

in Kähler potential K and also in superpotential W without term cT_2. The exact S_2 symmetry corresponds to exact aligned axions, then the potential is independent with $bT_1 - aT_2$,

which becomes an exact flat direction in the scalar potential. The S_2 symmetry is broken explicitly by cT_2 while reserves approximately as long as $c \ll b$. This approximate symmetry is crucial that it provides sufficient flat potential for inflation.

In this work, we will take $m = n+1$ for simplicity, besides, the approximate S_2 symmetry is useful to simplify calculations.

Given the charges of ϕ_i in Table I, the $U(1)_a$ charges of moduli T_i are uniquely fixed

$$\begin{aligned} \delta_X^1 &= \frac{q}{a(n+1)}(1-\frac{b}{cn}), & \delta_X^2 &= \frac{q}{cn(n+1)}, \\ \delta_Y^1 &= \frac{\tilde{q}}{a(n+1)}(1-\frac{b(n+2)}{cn}), & \delta_Y^2 &= \frac{\tilde{q}(n+2)}{cn(n+1)}. \end{aligned} \tag{9}$$

In this work the degree of gauge group n is taken as $O(10)$, while the ratio b/c is close to $O(10^2)$, so we have $1-\frac{b}{cn} < 0$, *i.e.*, the $U(1)_a$ charges of modulus T_1 are negative in unit q or $\tilde{q}$. Negative charges of T_1 are directly determined by the smallness of ratio c/b, i.e., the approximate S_2 symmetry between two Kähler moduli. This property is greatly appreciated in quantum anomaly cancellation, as will be shown later.

B. Quantum Anomaly Cancellation of $U(1)_X \times U(1)_A$

For a consistent gauge theory, all quantum anomalies associated with $U(1)_a$ should be cancelled. For gauge sectors $U(1)_X \times U(1)_A$, the gauge anomalies contain the cubic $U(1)_a^3$ anomaly, the gravitational $U(1)_a$ anomaly and the mixed $U(1)_a^2 \times U(1)_b$ anomaly. The overall fermionic contributions on these gauge anomalies are non-zero, to keep theory free of gauge anomaly, the Green-Schwarz contributions on gauge anomaly [37] are necessary.

For gravitational $U(1)_a$ gauge anomaly, the fermionic contributions are

$$\begin{aligned} \mathrm{Tr}\ q_X &= \sum_z q_z = 2q, \\ \mathrm{Tr}\ q_A &= \sum_z \tilde{q}_z = 3\tilde{q}. \end{aligned} \tag{10}$$

The gravitational anomalies are cancelled by higher derivative terms R^2. The mixed anomalies such as $U(1)_a^2 \times U(1)_b$ and $SU(n) \times U(1)_a$ can be cancelled as well.

For cubic $U(1)_a^3$ anomalies, the fermionic contributions are

$$\text{Tr } q_X^3 = \sum_z q_z^3 = 2q^3, \tag{11}$$

$$\text{Tr } q_A^3 = \sum_z \tilde{q}_z^3 = 9\tilde{q}^3. \tag{12}$$

Gauge kinetic functions of $U(1)_a$ are

$$f_{U(1)_a} = k_a^1 T_1 + k_a^2 T_2, \tag{13}$$

in which k_a^1 are positive parameters. Consequently, the gauge kinetic terms are

$$\int d^2\theta \; k_a^i T_i W_a^2 \; , \tag{14}$$

where W_a is the $U(1)_a$ gauge field strength. There are two parts in the gauge kinetic term: $Re(f)F^2$ and $Im(f)F\tilde{F}$. The first part is $U(1)_a$ invariant, while the second part shifts under $U(1)_a$ due to non-trivial $U(1)_a$ gauge transformations of Kähler moduli T_i, actually the $U(1)_a$ variation of gauge kinetic term cancels the cubic gauge anomaly from fermionic contributions. Vanishing of cubic $U(1)_a^3$ anomaly requires

$$k_a^i \delta_a^i = -\frac{1}{48\pi^2} \text{Tr } q_a^3, \tag{15}$$

where an extra coefficient 1/3 is introduced as a symmetry factor of $U(1)_a^3$ anomaly graphs. Specifically, we have

$$k_X^i \delta_X^i = -\frac{1}{24\pi^2} q^3, \tag{16}$$

$$k_A^i \delta_A^i = -\frac{3}{16\pi^2} \tilde{q}^3. \tag{17}$$

Here all the coefficients k_a^i are positive. The above conditions cannot be fulfilled unless for each $U(1)_a$, at least one of the Kähler moduli has negative charge (with unit of q or $\tilde{q}$). Fortunately, as shown in (9), we do have one Kähler modulus T_1 whose $U(1)_a$ charges are negative resulting from the approximate S_2 symmetry. Then for any values of $q/\tilde{q}$, it is easy to adjust the parameters k_a^i so that the cubic $U(1)_a^3$ anomalies vanish.

III. THE KÄHLER MODULI AND MATTER FIELD STABILIZATION

The matter fields are stabilized by F-term potential. Once the matter fields obtain VEVs, the real components of the moduli are fixed by vanishing of $U(1)_a$ D-terms. Normally field

stabilization happens at scale much higher than inflation scale. The couplings between matter fields and inflation potential can only slightly modify the VEVs of matter fields, and more detailed analysis is presented in Ref. [11, 12]. Here, we just ignore the scalar potential relating to inflation at this stage.

The F-term scalar potential is determined by the Kähler potential K and superpotential W

$$V_F = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3W\bar{W}), \tag{18}$$

in which $K^{i\bar{j}}$ is the inverse of the Kähler metric $K_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$ and $D_i W = W_i + K_i W$. The D-term scalar potential is given by

$$V_D = \frac{1}{2} D_a D^a, \tag{19}$$

in which the gauge indices a are raised by the form $[(Ref)^{-1}]^{ab}$, and f is the gauge kinetic function. The D_a components are

$$D_a = iK_i X_a^i + i\frac{W_i}{W} X_a^i, \tag{20}$$

where X_a^i are the components of Killing vectors $X_a = X_a^i(\phi)\partial/\partial\phi^i$ which generate the isometries of the Kähler manifold that are gauged to form $U(1)_a$. If the superpotential W is gauge invariant instead of gauge covariant, the D_a components reduce to

$$D_a = iK_i X_a^i. \tag{21}$$

For the $U(1)_a$ charged matter fields z_n, they transform linearly under $U(1)_a$, and the Killing vectors linearly depend on the matter field z_n

$$X_a^{z_n} = iq_{z_n}^a z_n. \tag{22}$$

For the Kähler moduli T_i, they shift under $U(1)_a$ gauge transformations, and the Killing vectors are

$$X_a^{T_i} = i\delta_a^i, \tag{23}$$

which are purely imaginary constants.

A. Matter Field Stabilization

Considering the renormalizable matter couplings in (6), it is clear that the neutral matter fields X_i and Y_i have global minimum at the origin, while the charged matter fields ϕ_i, χ_i

and ψ_i obtain non-vanishing VEVs. During inflation, these matter fields will evolve to the global minimum very fast in consequence of the F-term exponential factor $e^{z_n\bar{z}_n}$ and the large masses obtained from the matter couplings in (6).

Part of the F-term potential is

$$\begin{aligned} V_{\rm m} = e^K(|\phi_i\chi_i - \lambda_1|^2 + |\psi_i\chi_i - \lambda_2|^2 + c_0^2(|\psi_i|^2 + |\chi_i|^2) \\ + 2c_0(\psi_i\chi_i\bar{W} + \bar{\psi}_i\bar{\chi}_i W) + \cdots), \end{aligned} \tag{24}$$

where we have ignored the terms proportional to X_i, Y_i or containing ϕ_i, χ_i, ψ_i while several orders smaller. The small term $2c_0(\psi_i\chi_i\bar{W} + \bar{\psi}_i\bar{\chi}_i W)$, although ignorable for field stabilization, has considerable contribution to inflation potential.

As shown in [12], for $\lambda_2 \gg c_0^2$, above potential admits a global minimum at

$$\begin{aligned} |\psi_i| = |\chi_i| \simeq \sqrt{\lambda_2}, \\ |\phi_i| \equiv r_i \simeq \frac{\lambda_1}{\sqrt{\lambda_2}}. \end{aligned} \tag{25}$$

The $U(1)_a$ gauge symmetries are broken spontaneously by non-zero VEVs. Through matter field stabilization, potential $V_{\rm m}$ obtains VEVs as well and uplifts the vacuum energy

$$\langle V_{\rm m}\rangle = e^{\langle K\rangle}c_0^2(\langle\psi_i\rangle^2 + \langle\chi_i\rangle^2) \simeq 4c_0^2\lambda_2 e^{\langle K\rangle}. \tag{26}$$

Normally the non-perturbative superpotential associated with no-scale Kähler potential leads to the AdS vacua. To obtain the Minkowski or dS vacua, an uplifting mechanism is needed. Here, the positive vacuum energy obtained from matter field stabilization provides a natural solution to this problem.

Up to now we have ignored the effects of the lower order terms on matter field stabilization. In [12] these effects have been studied, it was shown that the small couplings can only lead to tiny corrections on these VEVs, and reduce the vacuum energy slightly, which are totally ignorable in a general estimation.

B. Moduli Stabilization from $U(1)_X \times U(1)_A$ D-terms

According to the $U(1)_a$ charges provided in Table I, the D-term potentials associated with $U(1)_a$ are

$$V_{Da} = \frac{1}{2f_{U(1)_a}}(-\frac{\delta_a^1}{T_1+\bar{T}_1} - \frac{\delta_a^2}{T_2+\bar{T}_2} + q_{z_n}^a z_n\bar{z}_n)^2, \tag{27}$$

where $z_n \in \{\phi_i, \chi_i, \psi_i\}$ and $q^a_{z_n}$ is $U(1)_a$ charge of field z_n. Gauge kinetic function $f_{U(1)_a}$ is provided in (13). The matter fields z_n obtain VEVs through F-term stabilization, then the $U(1)_a$ D-terms become

$$\begin{aligned} V_{DX} &= \frac{1}{2f_{U(1)_X}}(-\frac{\delta^1_X}{T_1+\bar{T}_1} - \frac{\delta^2_X}{T_2+\bar{T}_2} + qr_1^2 + qr_2^2)^2, \\ V_{DA} &= \frac{1}{2f_{U(1)_A}}(-\frac{\delta^1_A}{T_1+\bar{T}_1} - \frac{\delta^2_A}{T_2+\bar{T}_2} + \tilde{q}r_1^2 + 2\tilde{q}r_2^2)^2, \end{aligned} \tag{28}$$

which are vanished at vacuum. Together with the charges of moduli in (9), the real components of Kähler moduli $T_i \equiv T_{Ri} + i\theta_i$ can be uniquely determined at vacuum

$$\begin{aligned} \frac{1}{2a\langle T_{R1}\rangle} &= (n+1)r_1^2 + nr_2^2, \\ \frac{1}{2b\langle T_{R2}\rangle} &= (n+1)r_1^2 + nr_2^2 + \frac{cn}{b}r_2^2. \end{aligned} \tag{29}$$

Even though the $U(1)_a$ gauge symmetries are broken after field stabilization, the approximate discrete symmetry S_2 is sustained. The symmetry breaking factor is very small, $c \ll b$, therefore the VEVs of T_i satisfy $a\langle T_{R1}\rangle \simeq b\langle T_{R2}\rangle \equiv r/2$ which is guaranteed by the approximate S_2 symmetry.

The imaginary components θ_i remain free in the perturbative potential, so actually they only appear in the potential through non-perturbative effects.

IV. INFLATION POTENTIAL

Field stabilization is happened at scale much higher than the inflation scale. After stabilization, we get the following effective superpotential

$$W = w_0 + A\phi_1^{\frac{1}{m}} e^{-(aT_1+bT_2)} + B\phi_2^{\frac{1}{n}} e^{-(aT_1+bT_2+cT_2)}, \tag{30}$$

where $w_0 = w_\star + 2c_0\lambda_2$, and there is a positive cosmology constant term $4c_0^2\lambda_2 e^{\langle K\rangle}$, which is necessary to uplift the AdS vacua to Minkowski or dS vacua.

Denoting $W_{np} = A\phi_1^{\frac{1}{m}} e^{-(aT_1+bT_2)} + B\phi_2^{\frac{1}{n}} e^{-(aT_1+bT_2+cT_2)} \equiv w_1 + w_2$, we have $W_{T_1} = -aW_{np}$, $W_{T_2} = -bw_1 - (b+c)w_2 \simeq -bW_{np}$. The F-term scalar potential includes

$$\begin{aligned} D_1 W D_{\bar{1}} \bar{W} K^{1\bar{1}} =& a^2(T_1+\bar{T}_1)^2 W_{np}\bar{W}_{np} \\ &+ a(T_1+\bar{T}_1)(w_0(W_{np}+\bar{W}_{np}) + 2W_{np}\bar{W}_{np}) + W\bar{W}, \end{aligned} \tag{31}$$

and

$$
\begin{aligned}
D_2 W D_{\bar{2}} \bar{W} K^{2\bar{2}} =& b^2 (T_2 + \bar{T}_2)^2 W_{np} \bar{W}_{np} \\
&+ b(T_2 + \bar{T}_2)(w_0 (W_{np} + \bar{W}_{np}) + 2 W_{np} \bar{W}_{np}) + W \bar{W}.
\end{aligned}
\tag{32}
$$

Therefore, parts of the moduli contributions on F-term potential are

$$
V_T \propto 2r^2 W_{np} \bar{W}_{np} + 2r(w_0 (W_{np} + \bar{W}_{np}) + 2 W_{np} \bar{W}_{np}) + 2 W \bar{W}, \tag{33}
$$

in which we have used the approximate S_2 symmetry $\langle a T_{R1} \rangle \simeq \langle b T_{R2} \rangle \equiv r/2$.

For the matter fields ϕ_i, we have

$$
\begin{aligned}
W_{\phi_1} &= \frac{w_1}{m \phi_1}, \\
W_{\phi_2} &= \frac{w_2}{n \phi_2},
\end{aligned}
\tag{34}
$$

where the terms proportional to X_i are ignored. The F-term potential contains

$$
\begin{aligned}
V_{\phi_1} &= W_{\phi_1} \bar{W}_{\bar{\phi}_1} + \phi_1 W_{\phi_1} \bar{W} + \bar{\phi}_1 W \bar{W}_{\bar{\phi}_1} + \phi_1 \bar{\phi}_1 W \bar{W} \\
&= \frac{1}{m^2 r_1^2} w_1 \bar{w}_1 + \frac{1}{m} (W \bar{w}_1 + w_1 \bar{W}) + \phi_1 \bar{\phi}_1 W \bar{W},
\end{aligned}
\tag{35}
$$

and

$$
\begin{aligned}
V_{\phi_2} =& W_{\phi_2} \bar{W}_{\bar{\phi}_2} + \phi_2 W_{\phi_2} \bar{W} + \bar{\phi}_2 W \bar{W}_{\bar{\phi}_2} + \phi_2 \bar{\phi}_2 W \bar{W} \\
=& \frac{1}{n^2 r_2^2} w_2 \bar{w}_2 + \frac{1}{n} (W \bar{w}_2 + w_2 \bar{W}) + \phi_2 \bar{\phi}_2 W \bar{W}.
\end{aligned}
\tag{36}
$$

Terms like $\phi_i \bar{\phi}_i W \bar{W}$ will be dropped in the following discussions as they are several orders smaller than others.

F-term potential is separated into two parts: these independent of axions V_1 and these depending on axions V_2. For the axion-independent part, it is

$$
\begin{aligned}
V_1 =& e^K \{ \frac{1}{m^2 r_1^2} w_1 \bar{w}_1 + \frac{2}{m} w_1 \bar{w}_1 + \frac{1}{n^2 r_2^2} w_2 \bar{w}_2 + \frac{2}{n} w_2 \bar{w}_2 \\
&+ 2r^2 (w_1 \bar{w}_1 + w_2 \bar{w}_2) + 4r (w_1 \bar{w}_1 + w_2 \bar{w}_2) - (w_0^2 + w_1 \bar{w}_1 + w_2 \bar{w}_2) + 4 c_0^2 \lambda_2 \} \\
=& e^K \{ (\frac{1}{m^2 r_1^2} + \frac{2}{m} + 2r^2 + 4r - 1) w_1 \bar{w}_1 \\
&+ (\frac{1}{n^2 r_2^2} + \frac{2}{n} + 2r^2 + 4r - 1) w_2 \bar{w}_2 - w_0^2 + 4 c_0^2 \lambda_2 + 8 c_0 \lambda_2 w_0 \},
\end{aligned}
\tag{37}
$$

while the axion-dependent part is

$$
\begin{aligned}
V_2 =& e^K \{ (\frac{4 c_0 \lambda_2}{w_0} + \frac{1}{m} + 2r - 1) w_0 (w_1 + \bar{w}_1) + (\frac{4 c_0 \lambda_2}{w_0} + \frac{1}{n} + 2r - 1) w_0 (w_2 + \bar{w}_2) \\
&+ (\frac{1}{m} + \frac{1}{n} + 2r^2 + 4r - 1)(w_1 \bar{w}_2 + w_2 \bar{w}_1) \}.
\end{aligned}
\tag{38}
$$

Terms proportional to $4c_0\lambda_2$ in V_1 and V_2 are obtained from the matter field stabilization.

To fit with observations, the parameters are taken as $m \sim O(10)$, $c_0 \sim O(10^{-2})$, $\lambda_2 \sim 10^{-2}$, $w_0 \sim 10^{-3}$, and $r \sim O(10)$. Hence, we have $1/m \sim 4c_0\lambda_2/w_0 \ll r$. Besides, according to Eq. (29), $r = 2aT_{R1} \simeq 1/2mr_1^2$, the factor $1/m^2r_1^2 \simeq 2r/m \ll r^2$ is insignificant. The lower order terms will be dropped in the preliminary estimation.

Employing the formula of w_1 and w_2 in the above equations and ignoring the lower order terms, we rewrite scalar potentials as follows

$$\begin{aligned} V_1 = \frac{1}{4T_{R1}T_{R2}}\{&(2r^2+4r-1)A^2r_1^{\frac{2}{m}}e^{-2r} \\ &+(2r^2+4r-1)B^2r_2^{\frac{2}{n}}e^{-2r} - w_0^2 + 4c_0^2\lambda_2\}, \end{aligned} \tag{39}$$

and

$$\begin{aligned} V_2 = \frac{1}{2T_{R1}T_{R2}}\{&(2r-1)w_0Ar_1^{\frac{1}{m}}e^{-r}\cos(a\theta_1+b\theta_2) \\ &+(2r-1)w_0Br_2^{\frac{1}{n}}e^{-r}\cos(a\theta_1+b\theta_2+c\theta_2) \\ &+(2r^2+4r-1)ABr_1^{\frac{1}{m}}r_2^{\frac{1}{n}}e^{-2r}\cos(c\theta_2)\}, \end{aligned} \tag{40}$$

where we have used $e^{\langle z_n\bar{z}_n\rangle} \simeq 1$.

A and B are parameters that depend on the details of non-perturbative effects. Here we may simply assume they are close to each other, and $Ar_1^{\frac{1}{m}}e^{-r} \simeq Br_2^{\frac{1}{n}}e^{-r} \sim 10^{-4}$.

At the global minimum, the scalar potential $V_1 + V_2$ decreases to

$$\langle V_1+V_2\rangle = \frac{1}{4\langle T_{R1}T_{R2}\rangle}(4c_0^2\lambda_2 - w_0^2 - 4(2r-1)w_0Ar_1^{\frac{1}{m}}e^{-r}). \tag{41}$$

Without uplifting term from matter field stabilization, the non-perturbative superpotential with no-scale-type Kähler potential admits an AdS vacuum, as expected. The constant term $4c_0^2\lambda_2$ elevates the AdS vacuum to Minkowski vacuum under a constraint

$$4c_0^2\lambda_2 = w_0^2 + 4(2r-1)w_0Ar_1^{\frac{1}{m}}e^{-r}, \tag{42}$$

and the scalar potential can be simplified as

$$\begin{aligned} V = 2\Lambda_1^4 + \Lambda_2^4 + \Lambda_1^4\cos(a\theta_1+b\theta_2) \\ + \Lambda_1^4\cos(a\theta_1+b\theta_2+c\theta_2) + \Lambda_2^4\cos(c\theta_2). \end{aligned} \tag{43}$$

Giving $w_0 = 2\times10^{-3}$, $T_{Ri} = 20$, $r = 10$, $Ar_1^{\frac{1}{m}}e^{-r} \simeq 2\times10^{-4}$, we have $\Lambda_{1,2}^4 \approx 10^{-8}$ which agree with the BEICEP2 observations.

Because the kinetic terms of Kähler moduli are non-canonical, the field transformations are needed to determine the physical axion decay constants. The kinetic terms are

$$L_K = \frac{1}{(T_i + \bar{T}_i)^2}\partial_\mu T_i \partial^\mu T_i = \frac{1}{4T_{Ri}^2}(\partial_\mu T_{Ri}\partial^\mu T_{Ri} + \partial_\mu \theta_i \partial^\mu \theta_i). \tag{44}$$

Taking field re-scale $\theta_i \to \sqrt{2}T_{Ri}\theta_i$, and using $aT_{R1} = bT_{R2} = r/2$, we get

$$\begin{aligned} V = {} & 2\Lambda_1^4 + \Lambda_2^4 + \Lambda_1^4 \cos(\frac{r}{\sqrt{2}}(\theta_1 + \theta_2)) \\ & + \Lambda_1^4 \cos(\frac{r}{\sqrt{2}}(\theta_1 + \theta_2 + \frac{c}{b}\theta_2)) + \Lambda_2^4 \cos(\frac{c}{\sqrt{2}b} r\theta_2), \end{aligned} \tag{45}$$

where the axions θ_i now have canonical kinetic terms. Redefining the axions $\varphi_{1,2} = (\theta_1 \pm \theta_2)/\sqrt{2}$, we get

$$\begin{aligned} V = {} & 2\Lambda_1^4 + \Lambda_2^4 + \Lambda_1^4 \cos(r\varphi_1) \\ & + \Lambda_1^4 \cos((1 + \frac{c}{2b})r\varphi_1 - \frac{c}{2b} r\varphi_2) + \Lambda_2^4 \cos(\frac{c}{2b} r(\varphi_1 - \varphi_2)). \end{aligned} \tag{46}$$

The effective mass of axion φ_1 is

$$m_{\varphi_1} = r\Lambda_1^2 \approx 10^{-3} \gg H, \tag{47}$$

where H is the Hubble constant during inflation, and its value is about 10^{-4} in Planck units based on the BICEP2 results. Therefore, axion φ_1 is frozen out during inflation, and another axion φ_2 drives the observed inflationary process if its decay constant $f = 2b/cr$ is of order $O(10)$. Although $r \sim O(10)$, we have $c \ll b$, so we can easily get a large effective decay constant $f \sim 10$ by adopting a small S_2 symmetry breaking factor $c/b \sim 10^{-2}$.

V. CONCLUSIONS

We have proposed a concrete model to realize aligned axion inflation [16] for natural inflation with moduli stabilization based on two anomalous $U(1)$ gauge symmetries. String theory provides abundant axion landscapes, and the natural inflation driven by aligned axions are expected to be true of certain choices in the axion landscapes [17–21]. For the axions as imaginary components of Kähler moduli, generally they appear in the potential through non-perturbative effects. Inflation driven by these axions needs subtle moduli stabilization since it requires to fix real components of moduli while keep axions sufficient light.

Similar to Refs. [11, 12], we employed the anomalous $U(1)$ gauge symmetries for moduli stabilization. Kähler moduli transform non-trivially under gauge symmetries $U(1)_X \times U(1)_A$, and lead to moduli-dependent FI terms in D-term potential associated with $U(1)_X \times U(1)_A$. The condensation hidden sectors $SU(n) \times SU(m)$ are assumed to have massive fundamental representations, from which the gaugino condensations introduce race-track type superpotential and cancellable D-terms. Since the D-terms depend on real components of Kähler moduli only, their cancellations at vacuum state lead to strong stabilizations on the real components of Kähler moduli. The axions, which are imaginary components of Káhler moduli, remain light.

We introduced renormalizable matter couplings for matter field stabilization. Prior to D-term moduli stabilization, the matter fields have to be stabilized and obtain non-zero VEVs. This is done by Higgs-like matter couplings. Gauge symmetries $U(1)_X \times U(1)_A$ are spontaneously broken by VEVs of charged matter fields, besides, the continuous shift symmetries of Kähler moduli are spontaneously broken into discrete shift symmetries. Field stabilization in this model also provides a natural mechanism for uplifting the AdS vaccum to Minkowski or dS vaccum, *i.e.*, it introduces large positive vaccum energy, which is suitable for elevating the AdS vaccum arising from non-perturbative superpotential.

We showed that the alignment of axions in the KNP mechanism corresponds to an approximate S_2 symmetry between two Kähler moduli. The S_2 symmetry is approximate as it is explicitly broken by a small factor c. Different from the $U(1)$ sectors, the discrete S_2 symmetry is sustained after spontaneously gauge symmetry breaking and field stabilization. After field stabilization, the potential is determined by two axions through non-perturbative effects. The decay constants of the two axions, which are determined by the moduli stabilization and canonical field transformation, are close to $f_i = 1/r$, just about the string scale. In consequence of S_2 symmetry, the potential forms a steep direction along $\theta_1 + \theta_2$, while its orthogonal direction $\theta_1 - \theta_2$ is flat, and is suitable for inflation by taking small S_2 symmetry broken factor c.

Acknowledgments

The work of DVN was supported in part by the DOE grant DE-FG03-95-ER-40917. The work of TL is supported in part by by the Natural Science Foundation of China under grant

numbers 10821504, 11075194, 11135003, and 11275246, and by the National Basic Research Program of China (973 Program) under grant number 2010CB833000.

---

[1] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990). F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. D **47**, 426 (1993) [hep-ph/9207245].

[2] P. A. R. Ade *et al.* [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO].

[3] K. Freese and W. H. Kinney, arXiv:1403.5277 [astro-ph.CO].

[4] M. J. Mortonson and U. Seljak, arXiv:1405.5857 [astro-ph.CO].

[5] R. Flauger, J. C. Hill and D. N. Spergel, arXiv:1405.7351 [astro-ph.CO].

[6] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].

[7] P. Svrcek and E. Witten, JHEP **0606**, 051 (2006) [hep-th/0605206].

[8] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997) [hep-ph/9606387].

[9] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003) [hep-th/0301240].

[10] R. Kallosh, Lect. Notes Phys. **738**, 119 (2008) [hep-th/0702059 [HEP-TH]].

[11] T. Li, Z. Li and D. V. Nanopoulos, arXiv:1405.0197 [hep-th].

[12] T. Li, Z. Li and D. V. Nanopoulos, arXiv:1405.1804 [hep-th].

[13] P. Binetruy, G. Dvali, R. Kallosh and A. Van Proeyen, Class. Quant. Grav. **21**, 3137 (2004) [hep-th/0402046].

[14] K. Choi and J. E. Kim, Phys. Lett. B **154**, 393 (1985) [Erratum-ibid. **156B**, 452 (1985)].

[15] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, JCAP **0306**, 001 (2003) [hep-th/0303252].

[16] J. E. Kim, H. P. Nilles and M. Peloso, JCAP **0501**, 005 (2005) [hep-ph/0409138].

[17] K. Choi, H. Kim and S. Yun, arXiv:1404.6209 [hep-th];

[18] T. Higaki and F. Takahashi, arXiv:1404.6923 [hep-th];

[19] R. Kappl, S. Krippendorf and H. P. Nilles, arXiv:1404.7127 [hep-th];

[20] C. Long, L. McAllister and P. McGuirk, arXiv:1404.7852 [hep-th].

[21] X. Gao, T. Li and P. Shukla, arXiv:1406.0341 [hep-th].

[22] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B **133**, 61 (1983); J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **134**, 429 (1984);

J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B **241**, 406 (1984); Nucl. Phys. B **247**, 373 (1984); A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. **145**, 1 (1987).

[23] E. Witten, Phys. Lett. B **155**, 151 (1985).

[24] T. Li, J. L. Lopez and D. V. Nanopoulos, Phys. Rev. D **56**, 2602 (1997).

[25] T. R. Taylor, G. Veneziano and S. Yankielowicz, Nucl. Phys. B **218**, 493 (1983).

[26] D. Lust and T. R. Taylor, Phys. Lett. B **253**, 335 (1991).

[27] B. de Carlos, J. A. Casas and C. Munoz, Phys. Lett. B **263**, 248 (1991).

[28] E. Dudas and S. K. Vempati, Nucl. Phys. B **727**, 139 (2005) [hep-th/0506172].

[29] G. Villadoro and F. Zwirner, Phys. Rev. Lett. **95**, 231602 (2005) [hep-th/0508167].

[30] A. Achucarro, B. de Carlos, J. A. Casas and L. Doplicher, JHEP **0606**, 014 (2006) [hep-th/0601190].

[31] Z. Lalak, G. G. Ross and S. Sarkar, Nucl. Phys. B **766**, 1 (2007) [hep-th/0503178].

[32] Z. Lalak, O. J. Eyton-Williams and R. Matyszkiewicz, JHEP **0705**, 085 (2007) [hep-th/0702026 [HEP-TH]].

[33] P. Brax, A. -C. Davis, S. C. Davis, R. Jeannerot and M. Postma, JCAP **0801**, 008 (2008) [arXiv:0710.4876 [hep-th]].

[34] M. Dine, P. Draper and A. Monteux, arXiv:1405.0068 [hep-th].

[35] K. Yonekura, arXiv:1405.0734 [hep-th].

[36] A. Mazumdar, T. Noumi and M. Yamaguchi, arXiv:1405.3959 [hep-th].

[37] M. B. Green and J. H. Schwarz, Phys. Lett. B **149**, 117 (1984).