Determination of the stiffness of the nuclear symmetry energy from isospin diffusion

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(Dated: January 6, 2014)

With an isospin- and momentum-dependent transport model, we find that the degree of isospin diffusion in heavy ion collisions at intermediate energies is affected by both the stiffness of the nuclear symmetry energy and the momentum dependence of the nucleon potential. Using a momentum dependence derived from the Gogny effective interaction, recent experimental data from NSCL/MSU on isospin diffusion are shown to be consistent with a nuclear symmetry energy given by $E_{\rm sym}(\rho) \approx 31.6(\rho/\rho_0)^{1.05}$ at subnormal densities. This leads to a significantly constrained value of about -550 MeV for the isospin-dependent part of the isobaric incompressibility of isospin asymmetric nuclear matter.

PACS numbers: 25.70.-z, 21.30.Fe., 21.65.+f, 24.10.Lx

Knowledge on the density dependence of nuclear symmetry energy is important for understanding not only the structure of radioactive nuclei [1, 2, 3, 4] but also many important issues in nuclear astrophysics [5, 6], such as nucleosynthesis during pre-supernova evolution of massive stars and the cooling of protoneutron stars. Although the nuclear symmetry energy at normal nuclear matter density is known to be around 30 MeV from the empirical liquid-drop mass formula [7, 8], its values at other densities are poorly known [9, 10]. Studies based on either microscopic many-body theories or phenomenological approaches have so far given widely divergent predictions on the density dependence of nuclear symmetry energy [11]. Empirically, the incompressibility of asymmetric nuclear matter is essentially undetermined [12], even though the incompressibility of symmetric nuclear matter at its saturation density $\rho_0 = 0.16$ fm^{-3} has been determined to be 231 ± 5 MeV from nuclear giant monopole resonances [13] and the equation of state at densities of $2\rho_0 < \rho < 5\rho_0$ has been constrained by measurements of collective flows in nucleus-nucleus collisions [14].

In light of the new opportunities provided by radioactive beams, a lot of interests and activities have recently been devoted to extract information on the density dependence of nuclear symmetry energy from reactions induced by such nuclei [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. In particular, isospin diffusion in heavy-ion collisions is found to depend sensitively on the density dependence of nuclear symmetry energy [27, 28, 29]. Within a momentum-independent transport model, in which the nuclear potential depends only on local nuclear density, the isospin diffusion data from recent experiments at the NSCL/MSU (National Superconducting Cyclotron Laboratory at Michigan State University) was found to favor a quadratic density dependence for the interaction part of nuclear symmetry energy [30]. This conclusion has stimulated much interest because of its implications to nuclear many-body theories and nuclear astrophysics. However, the nuclear potential acting on a nucleon is known to depend also on its momentum. For nuclear isoscalar potential, its momentum dependence is well-known and is important in extracting information about the equation of state of symmetric nuclear matter [14, 31, 32, 33, 34, 35, 36, 37, 38]. Very recently, the momentum-dependence of the isovector (symmetry) potential [11, 39, 40, 41] was also shown to be important for understanding a number of isospin related phenomena in heavy-ion reactions [42, 43, 44]. It is thus necessary to include momentum dependence in both the isoscalar and isovector potentials for studying the effect of nuclear symmetry energy on isospin diffusion. In this Letter, we shall show that the isospin diffusion data are consistent instead with a softer symmetry energy that is nearly linear in density within the momentum-dependent effective interaction used in present study.

Our study is based on an isospin-dependent transport model IBUU04 that uses experimental nucleon-nucleon cross sections in free space and includes the momentum dependence in both the isoscalar and isovector potentials [42]. Although the momentum dependence of the isoscalar potential is known empirically, that of the isovector potential is not as well determined [45]. In the IBUU04 model, it is based on the Gogny effective interactions. Specifically, the potential $U(\rho,\delta,\mathbf{p},\tau)$ for a nucleon with isospin τ (1/2 for neutrons and $-1/2$ for protons) and momentum $\mathbf{p}$ in asymmetric nuclear matter at total density ρ is given by [40]

$$\begin{aligned} U_{\rm MDI}(\rho,\delta,\mathbf{p},\tau) &= A_u\frac{\rho_{\tau'}}{\rho_0} + A_l\frac{\rho_\tau}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^\sigma(1-x\delta^2) \\ &- 8\tau x\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'} \\ &+ \frac{2C_{\tau,\tau}}{\rho_0}\int d^3\mathbf{p}'\frac{f_\tau(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2} \\ &+ \frac{2C_{\tau,\tau'}}{\rho_0}\int d^3\mathbf{p}'\frac{f_{\tau'}(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}, \quad (1) \end{aligned}$$

where ρ_τ and ρ'_τ denote proton or neutron density with $\tau \neq \tau'$; and $\delta \equiv (\rho_n - \rho_p)/\rho$ is the isospin asymmetry. The $f_\tau(\mathbf{r}, \mathbf{p})$ denotes the phase-space distribution function at coordinate $\mathbf{r}$ and momentum $\mathbf{p}$. The corresponding momentum-dependent interaction (MDI) leads to an incompressibility of $K_0 = 211$ MeV for the symmetric nuclear matter at saturation density. On the right hand side of Eq.(1), the first four terms with $\sigma = 3/4$ and $B = 106.35$ MeV describe the momentum-independent interaction. The terms with parameters $C_{\tau,\tau} = -11.7$ MeV and $C_{\tau,\tau'} = -103.4$ MeV describe the momentum-dependent interaction of a nucleon of isospin τ and momentum $\mathbf{p}$ with like and unlike nucleons in the background fields, respectively. With the parameter $\Lambda = 1.0 p_F^0$, where p_F^0 denotes nucleon Fermi momentum at ρ_0, the isoscalar potential $(U_n(\rho, p) + U_p(\rho, p))/2$ coincides with predictions from the variational many-body theory using inputs constrained by nucleon-nucleon scattering data [42], and the isovector potential $(U_n(\rho, p) - U_p(\rho, p))/2$ also agrees with the momentum dependence of the Lane potential extracted from low-energy nucleon-nucleus scattering experiments [42] and recent neutron-nucleus data at 96 MeV [46].

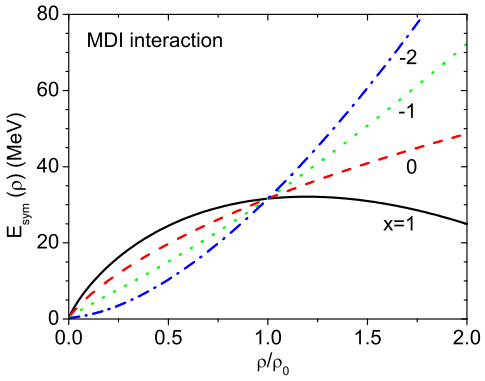

FIG. 1: (Color online) Density dependence of nuclear symmetry energy for different values of x parameter in Eq.(1).

The parameter x in Eq.(1) is introduced to mimic the various theoretical predictions for the density dependence of the nuclear symmetry energy $E_{\rm sym}(\rho)$, which is defined via the parabolic approximation to the nucleon specific energy in an isospin asymmetric nuclear matter [9, 10], i.e.,

$$E(\rho, \delta) = E(\rho, \delta = 0) + E_{\rm sym}(\rho)\delta^2 + \mathcal{O}(\delta^4). \quad (2)$$

With $x = 1$, for instance, the $E_{\rm sym}(\rho)$ is what predicted by a Hartree-Fock calculation using the Gogny effective interaction [40]. The parameters $A_l(x)$ and $A_u(x)$ are $A_l(x) = -120.57 + 2Bx/(\sigma + 1)$ and $A_u(x) = -95.98 - 2Bx/(\sigma + 1)$, respectively. Including also the well-known contribution from nucleon kinetic energies, i.e., $E_{\rm sym}^{\rm kin}(\rho) = (2^{2/3} - 1)\frac{3}{5}E_F^0(\rho/\rho_0)^{2/3} \approx 13.0(\rho/\rho_0)^{2/3}$,

TABLE I: The parameters F (MeV), G , $K_{\rm sym}$ (MeV), L (MeV), and $K_{\rm asy}$ (MeV) for different values of x.

x	F	G	$K_{\rm sym}$	L	$K_{\rm asy}$
1	107.232	1.246	−270.4	16.4	-368.8
0	129.981	1.059	−88.6	62.1	-461.2
−1	3.673	1.569	94.1	107.4	-550.3
−2	−38.395	1.416	276.3	153.0	-641.7

the density dependence of nuclear symmetry energy is shown in Fig. 1 for $x = 1$, 0, -1 and -2.

The interaction part of nuclear symmetry energy can be parameterized by $E_{\rm sym}^{\rm pot}(\rho) = F(x)\rho/\rho_0 + (18.6 - F(x))(\rho/\rho_0)^{G(x)}$, with $F(x)$ and $G(x)$ given in Table I for $x = 1$, 0, -1 and -2. Also shown are other characteristics of the symmetry energy, including its slope $L \equiv 3\rho_0 \left(dE_{\rm sym}/d\rho\right)_{\rho=\rho_0}$ and curvature $K_{\rm sym} \equiv 9\rho_0^2 \left(d^2E_{\rm sym}/d\rho^2\right)_{\rho=\rho_0}$ at ρ_0, as well as the isospin-dependent part $K_{\rm asy} \approx K_{\rm sym} - 6L$ [20] of the isobaric incompressibility of asymmetric nuclear matter $K(\delta) = K_0 + K_{\rm asy}\delta^2$ [47, 48].

For comparisons we have also constructed the following momentum-independent potential (SBKD) that has $K_0 = 200$ MeV [49] and exactly the same $E_{\rm sym}(\rho)$ as the MDI interaction:

$$\begin{aligned} U_{\rm SBKD}(\rho, \delta, \tau) \equiv & -356\ \rho/\rho_0 + 303\ (\rho/\rho_0)^{7/6} \\ & + 4\tau E_{\rm sym}^{\rm pot}(\rho) + (18.6 - F(x)) \\ & \times (G(x) - 1)(\rho/\rho_0)^{G(x)}\delta^2. \end{aligned} \quad (3)$$

Isospin diffusion in heavy ion collisions can in principle be studied by examining the average isospin asymmetry of the projectile-like residue in the final state. Since reactions at intermediate energies are complicated by pre-equilibrium particle emission and production of neutron-rich fragments at mid-rapidity, differences of isospin diffusions in mixed and symmetric systems are usually used to minimize these effects [30]. To study isospin diffusion in ^{124}Sn + ^{112}Sn reactions at $E = 50$ MeV/nucleon and an impact parameter of $b = 6$ fm, we thus also consider the reaction systems ^{124}Sn + ^{124}S and ^{112}Sn + ^{112}Sn and ^{124}Sn + ^{112}Sn at same energy and impact parameter as in Ref.[30]. The degree of isospin diffusion in the reaction ^{124}Sn + ^{112}Sn is then measured by [50]

$$R_i = \frac{2X_{^{124}\mathrm{Sn}+^{112}\mathrm{Sn}} - X_{^{124}\mathrm{Sn}+^{124}\mathrm{Sn}} - X_{^{112}\mathrm{Sn}+^{112}\mathrm{Sn}}}{X_{^{124}\mathrm{Sn}+^{124}\mathrm{Sn}} - X_{^{112}\mathrm{Sn}+^{112}\mathrm{Sn}}} \quad (4)$$

where X is the average isospin asymmetry $\langle\delta\rangle$ of the ^{124}Sn-like residue defined as the composition of nucleons with local densities higher than $\rho_0/20$ and velocities larger than 1/2 the beam velocity in the c.m. frame. A density cut of $\rho_0/8$ is found to give almost same results. In ideal case, the value of R_i ranges between 0.05 and 1 from complete mixing to full transparency.

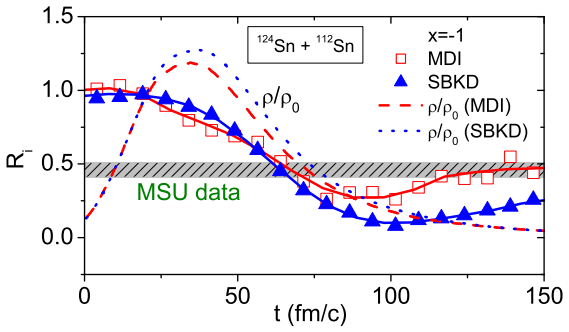

FIG. 2: (Color online) Time evolutions of R_i and average central density for MDI and SBKD interactions with $x = -1$.

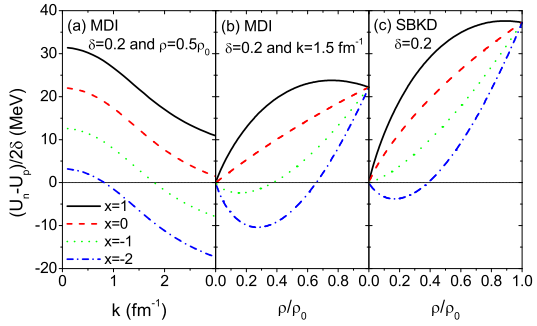

FIG. 3: (Color online) Symmetry potential as a function of momentum (a) or density (b) with the MDI interaction and SBKD interaction (c).

As an example of present more realistic calculations, we show in Fig. 2 the time evolutions of R_i and average central density calculated with $x = -1$ using both MDI and SBKD interactions. It is seen that the isospin diffusion process occurs mainly from about 30 fm/c to 80 fm/c corresponding to average central density from about $1.2\rho_0$ to $0.3\rho_0$. However, the value of R_i still changes slightly with time until after about 120 fm/c when projectile-like and target-like residues are well separated. This is partly due to the fact that the isovector potential remains appreciable at low density as shown in Fig. 3, where the symmetry potential $(U_n - U_p)/2\delta$ is shown as a function of momentum (panel (a)) or density (panel (b)) for the MDI interaction and as a function of density for the SBKD interaction (panel (c)). Also, evaluating isospin diffusion R_i based on three reaction systems, that have different time evolutions for the projectile residue as a result of different total energies and numbers of nucleons, further contributes to the change of R_i at low density. For the two interactions consider here, the main difference between the values for R_i appears in the expansion phase when densities in the participant region are well below ρ_0. The experimental data from MSU are seen to be reproduced nicely by the MDI interaction with $x = -1$, while the SBKD interaction with $x = -1$ leads to a significantly lower value for R_i as the strength of the momentum-independent potential is stronger (see Fig. 3), which has been shown to enhance the isospin diffusion [27, 29, 30].

To see how isospin diffusion depends on the density dependence of nuclear symmetry energy, we show in Fig. 4 the final saturated value for $1-R_i$, which measures the degree of isospin diffusion, as a function of $K_{\rm asy}$ for both MDI and SBKD interactions. It is obtained by averaging the value of $1 - R_i$ after 120 fm/c with error bars corresponding to its dispersion, whose magnitude is similar to the error band shown in Ref.[30] for the theoretical results from the BUU model. For the SBKD interaction without momentum dependence, the isospin diffusion decreases monotonically (i.e., increasing value for R_i) with increasing strength of $K_{\rm asy}$ as the corresponding isovector potential is mostly positive and decreases with increasing stiffness of $E_{\rm sym}(\rho)$ in the whole range of considered x parameter. The isospin diffusion is reduced when the momentum-dependent interaction MDI is used as the momentum dependence weakens the strength of symmetry potential except for $x = -2$. As seen in Fig. 3, the symmetry potential in the MDI interaction has the smallest strength for $x = -1$ as it is close to zero at $k \approx 1.5$ fm^{-1} and $\rho/\rho_0 \approx 0.5$, and increases again with further hardening of the symmetry energy, e.g., $x = -2$, when it becomes largely negative at all momenta and densities. The MDI interaction with $x = -1$ thus gives the smallest degree of isospin diffusion among the interactions considered in present study and reproduces the MSU data as already shown in Fig. 2.

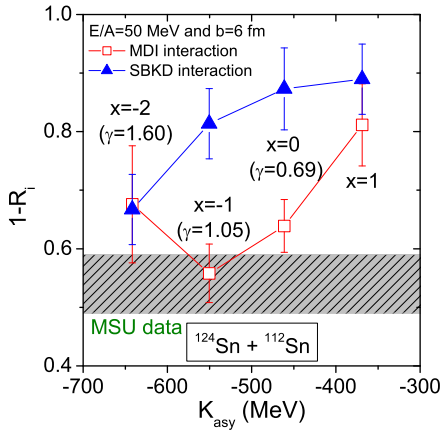

FIG. 4: (Color online) The degree of isospin diffusion as a function of $K_{\rm asy}$ with the MDI and SBKD interactions. γ is the parameter for fitting the corresponding symmetry energy with $E_{\rm sym}(\rho) = 31.6(\rho/\rho_0)^{\gamma}$.

The symmetry energy in the MDI interaction with $x = -1$ is $E_{\rm sym}(\rho) = 13.0(\rho/\rho_0)^{2/3} + 3.7\rho/\rho_0 + 14.9(\rho/\rho_0)^{1.57} \approx 31.6(\rho/\rho_0)^{1.05}$. It leads to a value of $K_{\rm asy} \approx -550$ MeV for the isospin dependent part of the isobaric incompressibility of asymmetric nuclear matter,

which should be compared to the published constraint of $-566 \pm 1350 < K_{\rm asy} < 139 \pm 1617$ MeV extracted earlier from studying giant monopole resonances [12].

It is worthwhile to mention that if the isoscalar part of the SBKD potential in Eq. (3) is replaced with the momentum-dependent MDYI potential of Gale *et al.* [33], which has a similar K_0 as those for the MDI and SBKD potentials, the resulting $R_i = 0.37 \pm 0.07$ is much closer to that obtained with the MDI ($R_i = 0.44 \pm 0.05$) than with the SBKD ($R_i = 0.19 \pm 0.06$) interaction for $x = -1$. The strongly repulsive momentum-dependent isoscalar potential thus reduces the effect of isovector potential on the reaction dynamics. Results from present study are therefore not much affected by the uncertainty in the momentum dependence of the isovector potential.

In summary, we have used an isospin- and momentum-dependent transport model to study isospin diffusion in heavy-ion collisions at intermediate energies. We find that the diffusion of isospins happens mostly during expansion stage when the density is below normal nuclear matter density. The momentum dependence in the nuclear potential plays an important role and affects the sensitivity of the degree of isospin diffusion to the density dependence of nuclear symmetry energy. Aside from the uncertainty due to residual effects of preequilibrium particle emission and cluster formation, the present study shows that within the context of the mean-field interactions considered here recent experimental data from NSCL/MSU on isospin diffusion is consistent with a nuclear symmetry energy that is nearly linear in density at subnormal densities. This leads to a significantly constrained value of about -550 MeV for the isospin-dependent part of the isobaric incompressibility of isospin asymmetric nuclear matter.

We would like to thank Enzo Greco for helpful discussions. This work was supported in part by the U.S. National Science Foundation under Grant Nos. PHY-0098805, PHY-0243571 and PHY0354572 as well as the Welch Foundation under Grant No. A-1358. LWC was also supported by the National Natural Science Foundation of China under Grant No. 10105008.

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[1] K. Oyamatsu *et al.*, Nucl. Phys. **A634**, 3 (1998).
[2] B.A. Brown, Phys. Rev. Lett. **85**, 5296 (2000).
[3] C.J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett **86**, 5647 (2001); Phys Rev. C **63**, 025501 (2001).
[4] R.J. Furnstahl, Nucl. Phys. **A706**, 85 (2002).
[5] H.A. Bethe, Rev. Mod. Phys. **62**, 801 (1990).
[6] J.M. Lattimer and M. Prakash, Astr. Phys. Jour. **550**, 426 (2001); Science **304**, 536 (2004).
[7] W.D. Myers and W.J. Swiatecki, Nucl. Phys. **A81**, 1 (1966).
[8] K. Pomorski and J. Dudek, Phys. Rev. C **67**, 044316 (2003).
[9] B.A. Li, C.M. Ko, and W. Bauer, topical review, Int. Jour. Mod. Phys. E **7**, 147 (1998).
[10] Isospin Physics in Heavy-Ion Collisions at Intermediate Energies, Eds. Bao-An Li and W. Udo Schröder (Nova Science Publishers, Inc, New York, 2001).
[11] I. Bombaci, in [10], p.35.
[12] S. Shlomo and D. H. Youngblood, Phys. Rev. C **47**, 529 (1993).
[13] D.H. Youngblood *et al.*, Phys. Rev. Lett. **82**, 691 (1999).
[14] P. Danielewicz *et al.*, Science **298**, 1592 (2002).
[15] B.A. Li, C.M. Ko, and Z.Z. Ren, Phys. Rev. Lett. **78**, 1644 (1997).
[16] B.A. Li and C.M. Ko, Nucl. Phys. **A618**, 498 (1997).
[17] V. Baran *et al.*, Nucl. Phys. **A632**, 287 (1998).
[18] H.S. Xu *et al.*, Phys. Rev. Lett. **85**, 716 (2000).
[19] W.P. Tan *et al.*, Phys. Rev. C **64**, 051901(R) (2001).
[20] V. Baran *et al.*, Nucl. Phys. **A703**, 603 (2002).
[21] M.B. Tsang *et al.*, Phys. Rev. Lett. **86**, 5023 (2001).
[22] B.A. Li *et al.*, Phys. Rev. C **64**, 054604 (2001).
[23] B.A. Li, Phys. Rev. Lett. **85**, 4221 (2000).
[24] Radioactive Nuclear Beams, a special volume of Nucl. Phys. **A693**, (2001), Ed. I. Tanihata.
[25] B.A. Li, Phys. Rev. Lett. **88**, 192701 (2002); Nucl. Phys. A**708**, 365 (2002).
[26] L.W. Chen, V. Greco, C.M. Ko, and B.A. Li, Phys. Rev. Lett. **90**, 162701 (2003); Phys. Rev. C **68**, 014605 (2003); L.W. Chen, C.M. Ko, and B.A. Li, *ibid* C **68**, 017601 (2003); Nucl. Phys. **A729**, 809 (2003).
[27] M. Farine *et al.*, Z. Phys. A**339**, 363 (1991).
[28] L. Shi and P. Danielewicz, Phys. Rev. C **68**, 064604 (2003).
[29] B.A. Li, Phys. Rev. C **69**, 034614 (2004).
[30] M.B. Tsang *et al.*, Phys. Rev. Lett. **92**, 062701 (2004).
[31] C. Gale, G. Bertsch, and S. Das Gupta, Phys. Rev. C **35**, 1666 (1987).
[32] G.M. Welke *et al.*, Phys. Rev., C **38**, 2101 (1988).
[33] C. Gale *et al.*, Phys. Rev., C **41**, 1545 (1990).
[34] Q. Pan and P. Danielewicz, Phys. Rev. Lett. **70**, 2062 (1993).
[35] J. Zhang *et al.*, Phys. Rev. C **50**, 1617 (1994).
[36] V. Greco *et al.*, Phys. Rev. C **59**, 810 (1999).
[37] P. Danielewicz, Nucl. Phys. **A673**, 375 (2000).
[38] D. Persram and C. Gale, Phys. Rev. C **65**, 064611 (2002).
[39] P.E. Hodgson, The Nucleon Optical Model, World Scientific, Singapore, 1994.
[40] C.B. Das, S. Das Gupta, C. Gale, and B.A. Li, Phys. Rev. C **67**, 034611 (2003).
[41] B.A. Li, Phys. Rev. C **69**, 064602 (2004).
[42] B.A. Li, C. B. Das, S. Das Gupta, and C. Gale, Phys. Rev. C **69**, 011603 (R) (2004); Nucl. Phys. **A735**, 563 (2004).
[43] J. Rizzo *et al.*, Nucl. Phys. **A732**, 202 (2004).
[44] L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. C **69**, 054606 (2004).
[45] V. Baran et al., Phys. Rep., submitted.
[46] J. Klug et al., Phys. Rev. C **67**, 0316001(R) (2003); **68**, 064605 (2003).
[47] M. Prakash and K. S. Bedell, Phys. Rev. C **32**, 1118 (1985).
[48] M. Lopez-Quelle *et al.*, Nucl. Phys. **A483**, 479 (1988).
[49] G.F. Bertsch *et al.*, Phys. Rev. C **29**, 673 (1984).
[50] F. Rami *et al.*, Phys. Rev. Lett. **84**, 1120 (2000).