

# Bayesian Calibration of AquaCrop Model for Winter Wheat by Assimilating UAV Multi-Spectral Images

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## Abstract

Crop growth model plays a paramount role in smart farming management, which not only provides quantitative information on crop development but also evaluates various management strategies. A reliable model is desirable but challenging due to the presence of unknown and uncertain parameters; therefore, crop model calibration is significant to achieve its potentials. This work is focused on the calibration of AquaCrop model by leveraging advanced Bayesian inference algorithms and UAV multi-spectral images at field scales. In particular, aerial images with high spatial-temporal resolutions are first applied to obtain Canopy Cover (CC) value by using machine learning based classification. The CC is then assimilated into AquaCrop model and uncertain parameters could be inferred by Markov Chain Monte Carlo (MCMC). Both simulation and experimental validation are performed. The experimental aerial images of winter wheat at Yangling district from Oct/2017 to June/2018 are applied to validate the proposed method against the conventional optimisation based approach by Simulated Annealing (SA). 100 Monte Carlo simulations show that the root mean squared error (RMSE) of Bayesian approach yields a smaller parameter estimation error than optimisation approach. While the experimental results show that: (i) a good wheat/background classification result is obtained for the accurate calculation of CC; (ii) the predicted CC values by Bayesian approach are consistent with measurements by 4-fold cross validation, where the RMSE is 0.0271 smaller than optimisation approach (0.0514); (iii) in addition to parameter estimation, their distribution information is also obtained in the developed Bayesian approach, reflecting the prediction confidence. It is believed that the Bayesian model calibration, although is developed for AquaCrop model, can find a wide range of applications to various simulation models in agriculture and forestry.

*Keywords:* Unmanned Aerial Vehicle (UAV); Multispectral image; Machine learning; Model calibration; Bayesian inference

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## 1. Introduction

Agricultural crop states are paramount for smart farming management and food security. A timely and accurate estimation of canopy states has become an effective approach for crop monitoring, irrigation decision-making and yield management [1, 2]. In this regard, a reliable crop model is desirable for crop state estimation. However, due to the presence of unknown and uncertain parameters in spatial distribution of soil properties and crop parameters, the prediction performance of crop model degrades significantly if model parameters are chosen inappropriately [3].

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32 Consequently, model parameters need to be calibrated before its potentials being realised. To this end, the integration  
33 of crop models and remote sensing data are drawing ever-increasing research interest [4]. It is noted that the accuracy  
34 of remote sensing data plays an important role in enhancing the predictive power of the calibrated model. Therefore,  
35 remote sensing data of high spatial/temporal resolutions is desirable, and so UAV remote sensing is preferable in this  
36 regard.

37 Crop model, quantitatively simulating crop physiological development, is defined by mathematical formulations  
38 driven by carbon, water, and light [4, 5]. Various crop growth models are developed for various semantic applications  
39 in the literature [6] such as World Food Studies (WOFOST), Crop Estimation through Resource and Environment  
40 Synthesis-Wheat (CERES-Wheat), Decision Support System for Agro-technology Transfer-Cropping System Model  
41 (DSSAT), APSIM, STICS, CropSyst and AquaCrop model. In particular, AquaCrop model [7], a water-driven crop  
42 model, possesses a number of fine properties over others in terms of simplicity, robustness and accurateness. Therefore,  
43 this model is drawing increasing attention in precision agriculture applications such as crop monitoring, irrigation  
44 management and yield prediction [8, 9].

45 In terms of remote sensing data, different sensing platforms equipped with different sensors of various spa-  
46 tial/spectral resolutions are available in the literature such as satellite/manned-aircraft based [10] and Unmanned  
47 Aerial Vehicles (UAV) based ones [11]. Satellite/manned-aircraft remote sensing, although is suitable for large-scale  
48 applications, is usually constrained by its poor spatial resolution in farm-scale applications. UAV remote sensing, how-  
49 ever, is of low-cost, with a high resolution and good flexibility, and therefore has become an important complement  
50 to conventional remote sensing. It has been extensively applied to smart agriculture at field scales such as stress (e.g.  
51 disease, weed, drought) monitoring and crop parameter estimation [12, 13, 14].

52 In crop model calibration, the measurements are usually chosen as the easily-accessible dynamic states such as  
53 Leaf Area Index (LAI) in WOFOST [15], leaf nitrogen accumulation in WheatGrowth model [16], biomass and CC  
54 in AquaCrop [17]. As a key crop growth parameter, CC denotes the canopy percentage, which is defined as the  
55 fraction between plant foliage projection to horizontal surface and total ground area [18]. CC calculation, therefore,  
56 could be formulated as an image segmentation problem, where the pixels are classified into two classes including  
57 wheat and non-wheat. The proportion of wheat pixels in a given area can be treated as CC value. The commonly  
58 used approaches for CC calculation are threshold based and machine learning based approaches [19]. Threshold  
59 approach relies on a threshold of particular band or index [11]. This approach is relatively simple, however, is sensitive  
60 to environmental variations [20]. Machine learning approach instead relies on labelled data to segment the images  
61 without the requirement of a threshold. This approach usually results in better performance although at the expense  
62 of a relatively high computation/labelling workload [19]. Considering that computation cost is not a concern for offline  
63 crop model calibration, machine learning based approach is adopted in this work due to its better performance.

64 The emerging model calibration methods integrating crop models and remote sensing data have become an effective

65 approach for estimating crop parameters and simulating crop dynamics. **The dominant approach in the literature is**  
66 **optimization based model calibration [4].** In this approach, various optimisation algorithms are drawn to calibrate the  
67 model parameters by minimising the fitness (error) function, which is defined by the discrepancy between measurement  
68 data and predicted output by the model [4]. For example, particle swarm optimization (PSO) is adopted in [1] to  
69 calibrate AquaCrop model by using historical remote sensing data, based on which biomass and final yield are predicted  
70 before harvest. Moreover, other optimisation algorithms have also been employed such as simplex search algorithm,  
71 Least Squares Method (LSM), Genetic Algorithm (GA), Shuffled Complex Evolution (SCE-UA) [21, 22, 23]. The  
72 accuracy of SCE-UA is shown to be better than others, however, these algorithms may still easily get stuck in a local  
73 minima due to the complexity of the optimisation problem at hand. In addition, only a point estimate is returned in  
74 optimization approaches with no confidence information.

75 Different from optimization approaches, Bayesian approach can infer the posterior distribution of uncertain param-  
76 eters based on available information. The main idea of Bayesian calibration is to derive the posterior distribution of  
77 model parameters of interest by integrating the prior information and measurement information by Bayesian rule. The  
78 literature on Bayesian calibration for agricultural applications is sparse compared against optimization approaches.  
79 Still natural history model and forest model are calibrated in [24, 25] respectively, where the uncertain parameters  
80 are estimated by applying Markov Chain Monte Carlo (MCMC) algorithm. However, this approach has received little  
81 attention in the community of smart farming, which is the main aim of this study.

82 Winter wheat is one main crop in China (north China in particular), and therefore improving crop model simulation  
83 accuracy is significant for addressing the challenges in smart farming such as dynamic states prediction, irrigation  
84 management and yield prediction prior to harvest. Previous studies are mainly focused on optimisation approaches  
85 by using satellite or ground sensing data. In this approach, only point estimate of model parameters is available,  
86 where the confidence of the estimate is missing. However, very little literature information is available on model  
87 parameter estimation by Bayesian approach, particularly by assimilating UAV multispectral imagery at field scales.  
88 **Consequently, the aim of this study is to calibrate AquaCrop model by assimilating UAV multi-spectral aerial imagery**  
89 **using Bayesian calibration. The developed approach is compared against the conventional optimisation based approach**  
90 **(e.g. simulated annealing in particular), where both Monte Carlo (MC) simulation and experimental verification are**  
91 **conducted.** The main contributions of this work are summarized:

- 92 (1) State-of-the-art UAV multi-spectral image by RedEdge camera and DJI S1000 UAV are drawn to work out the  
93 key measurement variable (CC) of AquaCrop model by machine learning classification;
- 94 (2) Bayesian inference is drawn to integrate the AquaCrop model and remote sensing measurements so that the  
95 posterior distribution (instead of point estimate) of AquaCrop model parameters is obtained;
- 96 (3) Both Monte Carlo simulation and experimental validation are performed to verify the developed Bayesian cali-  
97 bration against conventional optimization based approach, where a promising result is obtained in term of model

parameter estimation and **CC prediction**.

## 2. Materials

In this section, materials related to the experimental work in this study are introduced, which mainly consist of the experiment site for winter wheat and UAV-camera system for multi-spectral image acquisition.

### 2.1. Experiment fields

The experiment was conducted in Caoxinzhuang experiment field (latitude:  $34^{\circ}306'N$ , longitude:  $108^{\circ}090'E$ , 499m a.s.l.), which belongs to Northwest A&F University located in Yangling city, Shannxi Province, China (see Fig 1 for the location). The soil property in this study is loessal soil with organic content of 8.0%–15.0%. The climate in the experimental region is characterized by semi-humid and semi-arid with a mean annual temperature, precipitation of  $12.9^{\circ}C$ , 635mm (especially from June to September), respectively.

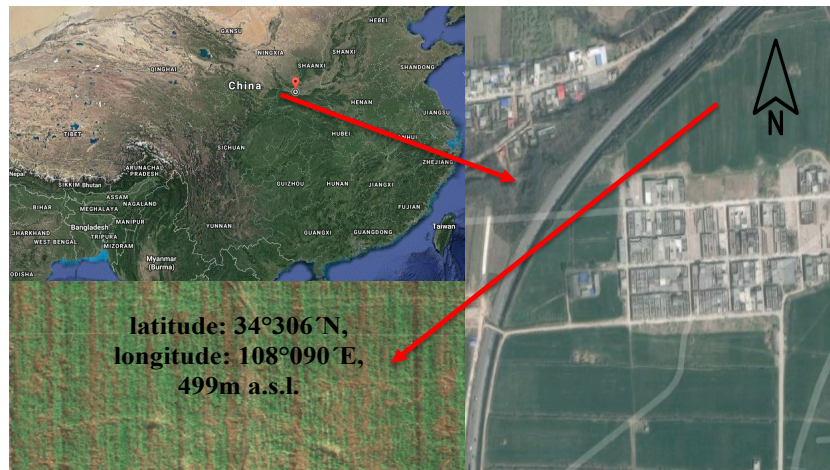


Figure 1: Geographic location of the experimental field for winter wheat.

In this study, one cultivars named Xiaoyan 22 developed by Northwest A&F university was selected and planted at a line spacing of 16cm with a rate of 30 g seeds/ $m^2$  from 5/October/2017 to early June, 2018. Local standard practise was implemented for field management, in addition, twice irrigation was carried out on 10/December/2017 and 13/March/2018 with no fertilizer. The meteorological data (one key input information of the AquaCrop model) can be downloaded from National Meteorological Information Center (<http://data.cma.cn>) and the basic soil data are also available on national Earth system Science Data Sharing Infrastructure (<http://www.geodata.cn>).

### 2.2. Multispectral aerial image

The area of the field is 5m by 10m and was investigated from 11/December/2017 to 23/May/2018, where eight UAV surveys were conducted to collect the aerial images. In this study, a five-band multi-spectral camera named RedEdge (MicaSense Company, Seattle, USA) is equipped on the commercial DJI Spreading Wings S1000 Octocopter

118 (DJI Company, Shenzhen, China) (see Fig 2). RedEdge camera outperforms conventional RGB camera in that:  
 119 (1) RedEdge camera possesses extra Rededge and NIR bands, providing extra spectral information for vegetation  
 120 classification; (2) calibration panel is adopted to calibrate the multispectral images, as a result, it is more robust  
 121 against environmental (illumination) variations. The specifications of the UAV is referred to [26] and the weight,  
 122 dimensions, image resolution of RedEdge camera are 135g,  $5.9\text{cm} \times 4.1\text{cm} \times 3.0\text{cm}$  and  $1280 \times 960$  pixels, respectively.

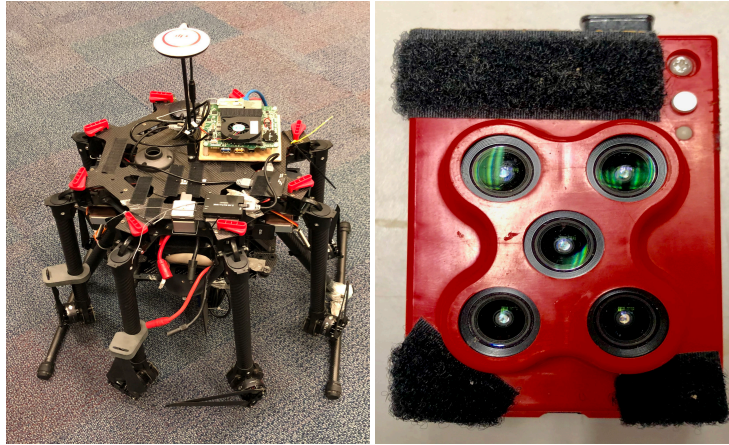


Figure 2: DJI S1000 with RedEdge Camera

123 In addition, RedEdge camera is fixed on a gimbal to attenuate the adverse effects of wind, so that high-quality  
 124 images can be captured during the survey. The spectral information of RedEdge camera is displayed in Table 1. Multi-  
 125 spectral images were obtained on winter wheat key developmental stages including tillering stage (11/December/2017  
 126 and 28/December/2017), green-up stage (23/March/2018), jointing stage (01/April/2018 and 17/April/2018), anthe-  
 127 sis stage (07/May/2018) and grain filling stage (15/May/2018 and 23/May/2018), respectively [27]. Each UAV aerial  
 128 image is with the necessary information for camera calibration and image stitching. An image of a reflectance cali-  
 129 bration panel was taken (at about 1m height) before and after each flight and used in the process of image calibration  
 130 to account for the side effects of environmental variations. In addition, commercial Pix4Dmapper software of version  
 131 4.2.27 is adopted to generate calibrated and georeferenced spectral reflectance data for CC calculation. The detailed  
 132 process is omitted and can be referred to Section 2.3 of [11]

Table 1: Spectral information of the RedEdge camera.

Band No.	Name	Center Wavelength	Bandwidth	Panel reflectance
1	Blue	475nm	20nm	0.57
2	Green	560nm	20nm	0.57
3	Red	668nm	10nm	0.56
4	NIR	840nm	40nm	0.51
5	RedEdge	717nm	10nm	0.55

### 133 3. Methodologies

134 In this section, the methodologies in this study are presented including CC calculation, wheat crop model and  
135 Bayesian calibration approach.

#### 136 3.1. CC calculation

137 The calculation of CC is first discussed. In this study, UAV remote sensing data (e.g. five-band multispectral  
138 image) is preferred due to its high spatial/spectral resolutions. The overall process is displayed in Fig 3, where each  
element is detailed in the following subsections.

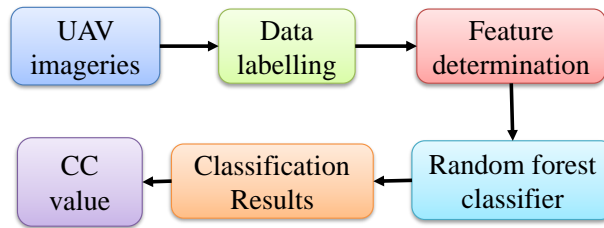


Figure 3: Overall framework for the canopy cover calculation.

139

#### 140 3.1.1. Data labelling and spectral analysis

141 In this study, CC calculation is formulated as a wheat/non-wheat two-class classification problem so that wheat  
142 pixel proportion can be calculated for the region of interest. One specific image acquired on 11/December/2017 is  
143 used as an illustration example. It is well known that supervised classification relies on labelled data for its training,  
144 which include wheat and non-wheat pixels in this study. In this work, wheat/non-wheat pixels are directly labelled  
for the five-band multispectral images in Matlab environment, where a sample image is displayed in Fig 4.

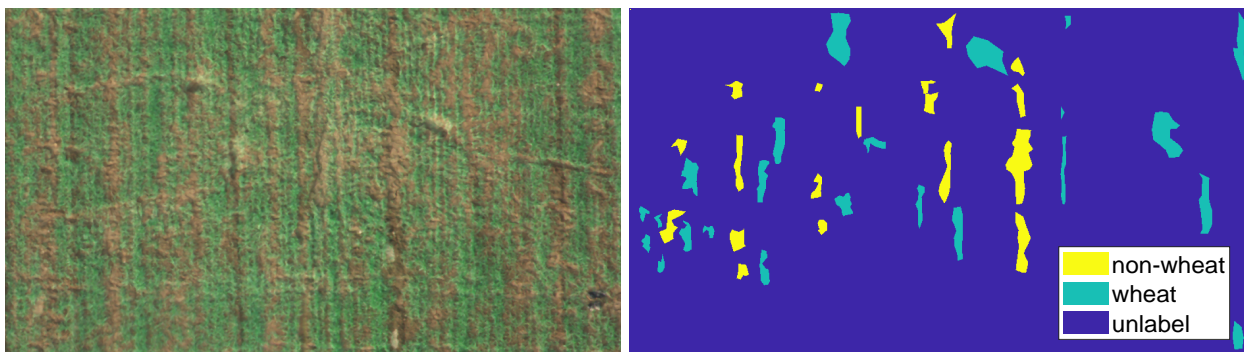


Figure 4: Original (left) and labelled (right) winter wheat on 11/Dec/2017

145

146 The spectral characteristics of the wheat/non-wheat pixels are also analysed, where the mean spectral reflectance  
147 values are shown in Fig 5. It can be seen that the green peak phenomenon is observed for wheat (green) crop where  
148 the value of Green band is higher than that of Blue and Red bands. In addition, wheat pixels also have a higher NIR

149 reflectance value than non-wheat pixels. The spectral differences provides important information for discriminating  
 150 wheat pixels from non-wheat pixels. Considering that there are only five spectral bands in the multispectral images,  
 151 all available bands are used as the features for the classification task in Section 3.1.2. If a large number of (redundant)  
 152 features are available, feature selection approaches in [28] can be drawn to reduce the computation load while preserving  
 the performance.

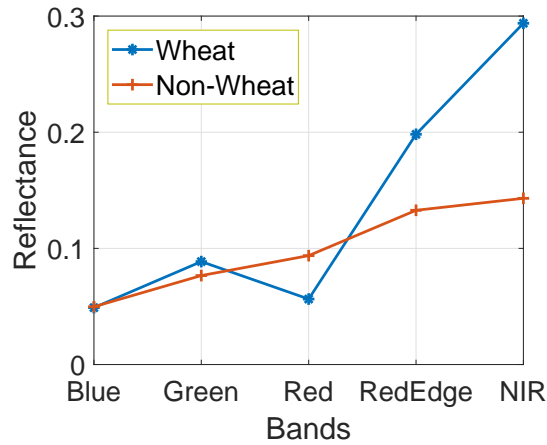


Figure 5: Average spectral reflectance value for two classes over five bands.

153

### 154 3.1.2. Image classification

155 Given labelled data in Section 3.1.1, a classifier is then required to perform the classification task so that new  
 156 aerial images can be automatically classified for CC calculation. A number of classifiers can achieve this task such  
 157 as Support Vector Machines (SVMs), neural network, nearest neighbour [10]. In this study, random forest classifier  
 158 is employed due to its high efficiency and accuracy, where the hyper-parameters are further automatically tuned by  
 159 Bayesian optimization [11]. The detailed algorithm is omitted, which is referred to [11, 29].

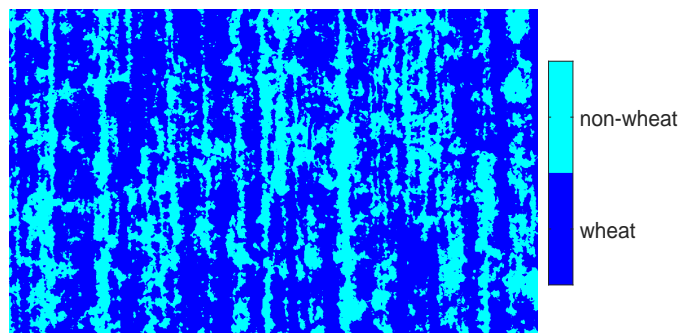


Figure 6: Classification map by the random forest classifier.

160 In this work, 70% and 30% of the labelled pixels are for training and testing respectively, where the classification  
 161 accuracy is calculated on testing dataset. The overall classification accuracy for the example image is 99%. The  
 162 trained model is then applied to the original image, where the classification map is shown in Fig 6. Then CC value

163 can be calculated by the formula in Eq 1.

$$CC = wp/(wp + nwp). \quad (1)$$

164 where  $wp$  and  $nwp$  denote the number of wheat and non-wheat pixels in the region of interest. Repeating the process,  
 165 the CC measurement values with classification accuracy over time are displayed in Table 2.

Table 2: Classification accuracy and canopy cover values over time

Acquisition Date	Overall Accuracy (%)	CC Measurement Value
11/12/2017	99	0.5896
28/12/2017	99.2	0.7182
23/03/2018	99.5	0.8983
01/04/2018	99.3	0.9319
17/04/2018	99.6	0.9225
07/05/2018	99.2	0.9124
15/05/2018	98.8	0.8726
23/05/2018	99.1	0.8155

### 166 3.2. Bayesian calibration for AquaCrop model

167 This section further discusses crop growth model and calibration method. The overall framework of the developed  
 168 Bayesian calibration for AquaCrop model is shown in Fig 7, which include AquaCrop function, Markov Chain Monte  
 169 Carlo (MCMC) method and result analysis. In this work, the CC measurement in Section 3.1 is chosen as observation  
 variable. Different elements of the proposed framework are detailed in the following sections.

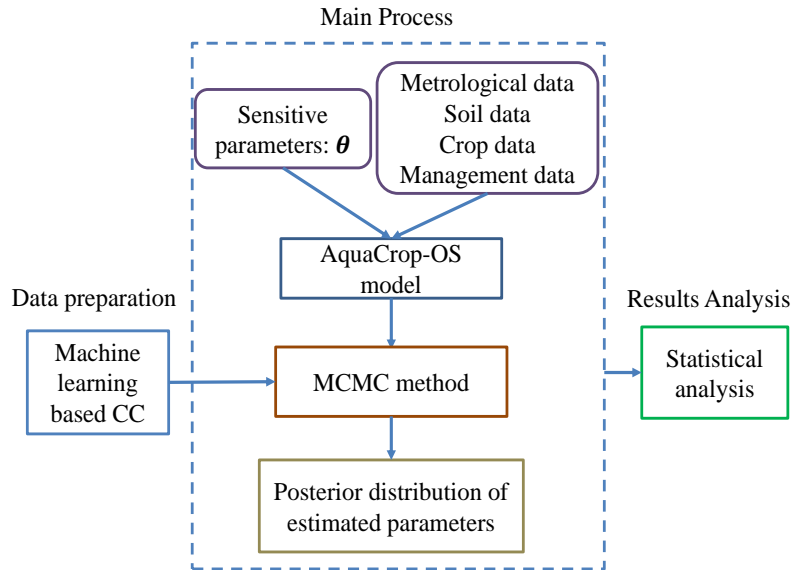


Figure 7: Framework of the proposed Bayesian calibration approach for AquaCrop model.



171 *3.2.1. AquaCrop-OS model*

172 AquaCrop crop growth model is developed in [7], which makes a good balance between model complexity and  
 173 model accuracy. It is a water balance based crop model simulating the interactions between weather, soil and crop  
 174 growth. In particular, the CC determines the water transpiration amount at expansion, ageing, conductance and  
 175 senescence stage, thus affecting the biomass production [30]. The crop’s daily aboveground biomass is generated by  
 176 normalised crop water productivity ( $WP^*$ ) from AquaCrop model. Biomass yield was determined by  $WP^*$  and the  
 177 ratio of crop transpiration ( $ET_i$ ) and reference evapotranspiration ( $ET_{0i}$ ) via Eq 2, and grain yield ( $Y$ ) is obtained by  
 178 multiplying the harvest index ( $HI$ ) by the biomass ( $B_i$ ) as in Eq 3.

$$B_i = WP^* \times \sum_{i=1}^N \frac{ET_i}{ET_{0i}}, \quad (2)$$

$$Y = B_i \times HI, \quad (3)$$

180 where  $WP^*$  is the normalised crop water productivity in  $g/m^2$ ;  $ET_i$  is daily crop transpiration in  $mm$ ;  $ET_{0i}$  is the  
 181 daily reference evapotranspiration in  $mm$ ;  $B_i$  is the cumulative biomass at  $i$ th day ( $ton/ha$ );  $HI$  is the harvest index;  
 182 and  $Y$  is grain yield ( $ton/ha$ ) at time  $i$ .

183 To facilitate the model application, an open-sourced version (named AquaCrop-OS model) was later developed in  
 184 Matlab environment[8]. This open-sourced model can be easily integrated with other approaches for various applica-  
 185 tions [9]. From a mathematics perspective, the dynamic system of AquaCrop model is a Markov process, where the  
 186 future status at  $t+1$  is only conditional on the current status at  $t$  rather than the past states [31]. Therefore, the  
 187 model could be simplified into Eq 4.

$$\begin{aligned} X_{t+1} &= F(X_t, \theta), \\ Y_t &= G(X_t) + \xi_t, \text{ with } \xi_t \sim N(0, \sigma) \end{aligned} \quad (4)$$

188 where  $F(\cdot)$  represents the crop model operator and  $X$  presents the canopy states (e.g. biomass, canopy cover, root  
 189 depth) on each simulated date.  $G(\cdot)$  denotes the measurement model with measurement noise  $\xi$  being with zero mean  
 190 and a proper covariance  $\sigma$ .

191 *3.2.2. Bayesian calibration method*

192 **Bayesian estimation theory:** the goal of Bayesian estimation is to update the probability distribution of the  
 193 sensitive parameters by integrating observation and prior [25]. Different from optimisation approach which derives  
 194 parameter estimation by minimizing the predefined objective function, Bayesian calibration derives the parameter  
 195 posterior distribution. [32]. In particular, the posterior distribution  $P(\theta|Y)$  is proportional to the prior parameter  
 196 distribution  $P(\theta)$  times the measurement likelihood function  $P(Y|\theta)$ , which is given by

$$P(\theta|Y) \propto P(\theta) \times P(Y|\theta),$$

197 where  $Y$  is the observational data and  $\theta$  represents the parameters to be estimated. To simplify the problem, the  
 198 likelihood function is defined as the error between observations and simulated model outputs (see Eq 5). More details  
 199 is given in Section 3.3.

$$P(Y|\theta) = P(E = Y - F(\theta)) \quad (5)$$

200 where  $F(\cdot)$  denotes the function of crop model conditional on parameter  $\theta$ ,  $E$  means the error.

201  
 202 **Markov Chain Monte Carlo (MCMC):** The MCMC process can effectively approximate the posterior distri-  
 203 bution function (PDF). The fundamental principle of Markov Chain is that the current sample value (at time  $t$ ) is  
 204 based on the past sample (at time  $t-1$ ), where determines whether the candidate is accepted or not with a probability.  
 205 The Monte Carlo (MC) sampling method is implemented to accurately evaluate the posterior PDF for the parameters  
 206  $\theta$ . The main purpose of MCMC is to generate a Markov Chain with a stable distribution of the target distribution.  
 207 This method can gather a series of samples at random walk generating a Markov Chain for the goal of parameter  
 208 distribution. Finally, one coverage chain with accepted parameters value will be achieved at an equilibrium status.  
 209 Several sampling methods have been proposed to accept or reject new states, the most popular one is Metropolis  
 210 Hastings (MH) sampling method [24, 25, 33].

211  
 212 **Delaying Rejection Adaptive Metropolis (DRAM):** To increase MH sampling performance, two variants  
 213 of MH algorithm named delaying rejection (DR) and adaptive metropolis (AM) were proposed. DR is capable of  
 214 modifying the standard MH algorithm to improve the estimation efficiency as this method employs considerable given  
 215 proposals and keep the reversibility in different stages [34]. In AM method, the covariance matrix of the Gaussian  
 216 proposal distribution is adjusted during the operation using the past chain. It can be demonstrated that the ergodicity  
 217 properties of the resulted samples still exist. AM is good at creating a Gaussian proposal distribution from the current  
 218 point in MCMC by computing the covariance matrix of the chain. The illustration of DRAM is given in Algorithm  
 219 1, where  $s_d$  is a parameter that only relies on the state space dimension  $d$  where equilibrium is defined and  $\varepsilon$  is a  
 220 constant bigger than zero.  $I_d$  denotes the  $d$  dimensional identify matrix.  $t_0$  denotes the initial non-adaptation time  
 221 and  $C_0$  is defined by our prior of the proposal covariance [34]. The combination of DR and AM can increase the  
 222 candidate acceptance probability and effectively improve the efficiency reaching to Markov Chain equilibrium. The  
 223 proof of DRAM realization is referred to [34, 35].

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**Algorithm 1:** DRAM Algorithm

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1 **Initialization:** Randomly select the initial parameters  $\theta^0$  for a chain length of  $M$  based on a symmetric transition kernel.

2 **Iteration:**  $i = 1$

3 **Sampling:**

for  $i = 0$  to  $M-1$

Construct Gaussian proposal

proposal mean = current  $\theta$

proposal covariance:

if  $i < i_0$ ,  $C_t = C_0$

else

$$C_t = s_d \text{Cov}(\theta^0, \theta^1, \dots, \theta^{i-1}) + s_d \varepsilon I_d$$

Randomly select the first stage proposal candidate parameter  $\theta^*$

Sample  $u \sim U[0,1]$

If  $u < \alpha = \min \left\{ 1, \frac{P(\theta^*)P(y|\theta^*)}{P(\theta^{i-1})P(y|\theta^{i-1})} \right\}$

$\theta^i = \theta^*$

otherwise

Construct the second stage proposal  $\theta^{**}$

Sample  $u \sim U[0,1]$

If  $u < \alpha = \min \left\{ 1, \frac{P(\theta^*)P(y|\theta^*)}{P(\theta^{i-1})P(y|\theta^{i-1})} \right\}$

$\theta^i = \theta^{**}$

otherwise

$\theta^i = \theta^{i-1}$

$i = i+1$

4 **Return to step 2**

---

### 224 3.3. Model calibration implementation

225 DRAM method is implemented to obtain the crop parameter distribution by using remotely sensed data and  
226 AquaCrop model in both theoretical and experimental way. The error distribution is assumed to Gaussian with zero  
227 mean and a proper variance, thereby the likelihood function in this study is formulated as

$$p(Y|\theta) = \prod_{i=1}^N \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(y_i - \hat{y}_i(x_k, \theta))^2}{2\sigma^2} \right\}, \quad (6)$$

228 where  $N$  is the total observation number and the  $y_i - \hat{y}_i(x_k, \theta)$  is the error between the measurement of dynamic  
229 states  $y_i$  and modelled states value by employing the crop model operator  $F(\cdot)$ . The variance ( $\sigma^2$ ) can be predefined  
230 or estimated along with model parameters [35].

## 231 4. Systematic validation

232 In this section, different model validation approaches including Monte Carlo simulation and real-world experiments  
233 are conducted to assess the performance of the developed Bayesian calibration against the conventional optimization  
234 based approach. **In particular, in MC simulation the parameters to be calibrated are used to assess the performance;**  
235 **while in real-world experiment, the measurable canopy cover is adopted to assess the performance.**

### 236 4.1. Monte Carlo simulation verification

237 Numerical Monte Carlo simulations are firstly conducted to evaluate the parameter estimation performance. Fol-  
238 lowing the exiting literature [17, 36, 37], variance-based Extended Fourier Amplitude Sensitivity Test (EFAST) is  
239 adopted to identify the sensitive parameters of AquaCrop model under different stresses. Then a ten-dimensional  
240 parameter vector, highly sensitive to CC and biomass, are selected

$$\theta = [sti, pse, wp, cgc, ccx, mat, eme, kcb, cdc, pop]^T. \quad (7)$$

241 The parameter definition and prior interval information are shown in Table 3. The default parameter values in  
242 AquaCrop-OS model are set to be truth.

243 To represent the noisy observation, the groundtruth CC data is added with a Gaussian measurement noise with  
244 zero mean and a variance of  $0.0005^2$ . The time period of simulation is consistent with the experiment period, which  
245 is from 05/October/2017 to 05/June/2018 and the data acquisition interval is 10 days. The simulation iteration is set  
246 as 5000, besides, 100 Monte Carlo experiments with random initial value and random noises is performed to test the  
247 robustness of both Bayesian and optimisation based calibration methods.

Table 3: Sensitive parameters with prior information for Monte Carlo simulation.

Parameters	Prior Information	Meaning
<i>sti</i>	(10,20)	Minimum growing degree days (degC/day) required for full biomass production
<i>pse</i>	(0.5,1)	Upper soil water depletion threshold for water stress effects on canopy senescence
<i>wp</i>	(30,40)	Water productivity normalized for $ET_0$ and $C0_2$ (g/m <sup>2</sup> )
<i>cgc</i>	(0.005,0.02)	Canopy growth coefficient
<i>ccx</i>	(0.82,0.98)	Maximum canopy cover fraction
<i>mat</i>	(1000,2500)	Growing degree days from sowing to maturity
<i>eme</i>	(60,100)	Growing degree days from sowing to emergence
<i>kcb</i>	(0.77,1.43)	Crop coefficient when canopy growth is complete but prior to senescence
<i>cdc</i>	(0,0.02)	Canopy decline coefficient
<i>pop</i>	(65000,85000)	Number of plants per hectare

#### 248 4.2. Experimental evaluation

249 In addition to MC simulation for parameter estimation, experimental validation is further considered. In this  
250 case, the time-series CC values learnt from multi-spectral image are used to estimate the uncertain parameters of  
251 AquaCrop-OS model. In order to test the capability of the developed algorithm, the prior information in Table 3 is  
252 reduced by increasing the uncertain parameter ranges as shown in Table 4. The iteration is also increased to 6000 to  
253 guarantee the convergence, this is because different from MC simulation fewer number of measurements are available  
254 in real-world experiments. The remaining settings of MCMC algorithm are the same as MC simulation.

Table 4: Sensitive parameters with prior information for experimental evaluation.

Parameters	Prior Information	Parameters	Prior information
<i>sti</i>	(3,20)	<i>mat</i>	(1500,3250)
<i>pse</i>	(0.35,1.85)	<i>eme</i>	(30,250)
<i>wp</i>	(5,40)	<i>kcb</i>	(0.5,2.8)
<i>cgc</i>	(0.004,0.02)	<i>cdc</i>	(0,0.06)
<i>ccx</i>	(0.82,0.99)	<i>pop</i>	(65000,95000)

## 255 5. Results

256 This section presents the comparative results. For MC simulation, parameter estimation performance is quantified  
257 in terms of mean estimation and root mean squared error (RMSE). While in experimental evaluation, RMSE is firstly  
258 calculated for CC estimation, and the estimated parameter posterior distributions are also shown.

259 5.1. Results of MC simulation

260 Monte Carlo analysis with random initial values and various noises is first performed for both Bayesian and  
 261 optimization approaches. For each MC simulation of the Bayesian approach, a Markov chain is constructed by using  
 262 MCMC, based on which the parameter estimation is calculated as the mean of the chain. Then mean parameter  
 263 estimation of the 100 MC simulations are calculated to assess the algorithm stability. On this basis, the estimation  
 264 error is defined by the following formula.

$$E_{opt} = \frac{|p_{opt} - p_t|}{p_t} * 100\%, \quad E_{bay} = \frac{|p_{bay} - p_t|}{p_t} * 100\%$$

265 where  $E_{opt}$  and  $E_{bay}$  denote the parameter estimation errors by optimisation and Bayesian methods, respectively.  $p_{opt}$   
 266 and  $p_{bay}$  represent the average calibrated parameters with  $p_t$  being the ground truth. The parameter estimations and  
 267 their error percentages are shown in Table 5.

Table 5: Mean of the estimated parameters and errors for 100 MC runs against ground truth via various methods

Parameters	Bayesian (error%)	Optimisation (error%)	Groundtruth
<i>sti</i>	12.7658(6.38)	14.8735(23.9)	12
<i>pse</i>	0.7066(2.41)	0.7172 (3.94)	0.69
<i>wp</i>	34.3915(2.05)	35.6457 (5.77)	33.7
<i>cgc</i>	0.0126 (0.82)	0.0125 (0.05)	0.0125
<i>ccx</i>	0.9625(0.26)	0.9539(0.63)	0.96
<i>mat</i>	1736 (2.14)	1845 (8.54)	1700
<i>eme</i>	82.0408 (2.55)	84.0569 (5.07)	80
<i>kcb</i>	1.0154 (3.29)	1.0649 (1.42)	1.05
<i>cdc</i>	0.0102 (1.90)	0.0100 (0.07)	0.01
<i>pop</i>	75239 (0.3185)	75933(1.240)	7500

268 It follows from Table 5 that in term of parameter estimation Bayesian approach outperforms optimization approach  
 269 for all parameters except *cgc*, *kcb*, and *cdc*. The performance is further quantified by using RMSE for the 100 MC runs,  
 270 where the results are displayed in Table 6. Similarly, it can be seen that Bayesian approach outperforms optimization  
 271 approach for all parameter estimation in term of RMSE except the parameter *eme*.

Table 6: RMSE of 100 Monte Carlo simulations via different methods

Parameters	Bayesian	Optimisation	Parameters	Bayesian	Optimisation
<i>sti</i>	0.9814	4.0684	<i>mat</i>	60.3164	411.1170
<i>pse</i>	0.0309	0.1361	<i>eme</i>	67.2370	65.1902
<i>wp</i>	0.9066	3.6541	<i>kcb</i>	0.0513	0.1942
<i>cgc</i>	0.0001	0.0004	<i>cdc</i>	0.0002	0.0031
<i>ccx</i>	0.0038	0.0317	<i>pop</i>	1745	6075

272 5.2. Results of experimental validation

273 In this section, experimental validation is conducted to further evaluate the performance. In particular, the key  
 274 state CC is adopted to validate the calibration accuracy. In order to avoid the problem of overfitting, k-fold cross  
 275 validation is adopted for the time-series data. The 8 experimental CC values are divided into  $k=4$  disjoint folds of  
 276 equal size, where  $k-1$  folds are for training and the remaining 1-fold is for testing [38]. Considering the particular  
 277 characteristics of the calibration problem in this study that observation data of the key stages should be preserved for  
 278 calibration, the dataset is divided into the particular  $k$  folds as shown in Fig 8. For example, when  $k = 1$  is chosen  
 279 for validation, the remaining ones are then for calibration so that the parameters can be estimated along with the  
 predicted CC values. This process is repeated for all four calibration/validation combinations.

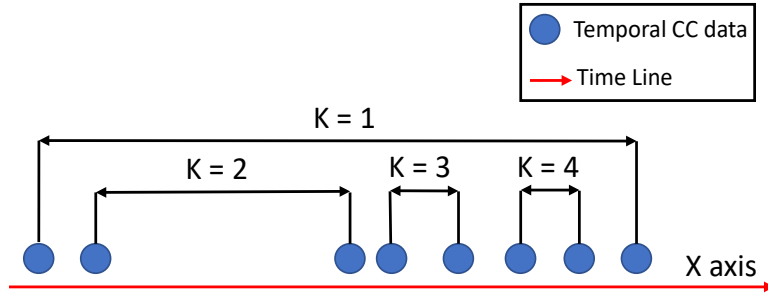


Figure 8: Conceptual explanation of K-fold cross validation datasets.

280

281 5.2.1. Markov Chain and Parameter Estimation

282 An example ( $k = 2, 3, 4$  folds for calibration and  $k = 1$  fold for validation) is illustrated in this part. In Bayesian  
 283 parameter estimation, the aim is to estimate the posterior probability distribution of parameters given observations  
 284 rather than a point estimate. By eliminating the burn-time (10% of the samples) in Markov Chain, it can be seen  
 285 from Fig 9 that all Markov chains converge to the corresponding equilibrium. Therefore, the posterior probability  
 286 density distribution of each parameters is reliable.

287 From the Markov Chain samples, the posterior distribution for each parameter can be represented by a histogram.  
 288 The normalized probability density of each estimated parameter with original prior information (red line) is displayed

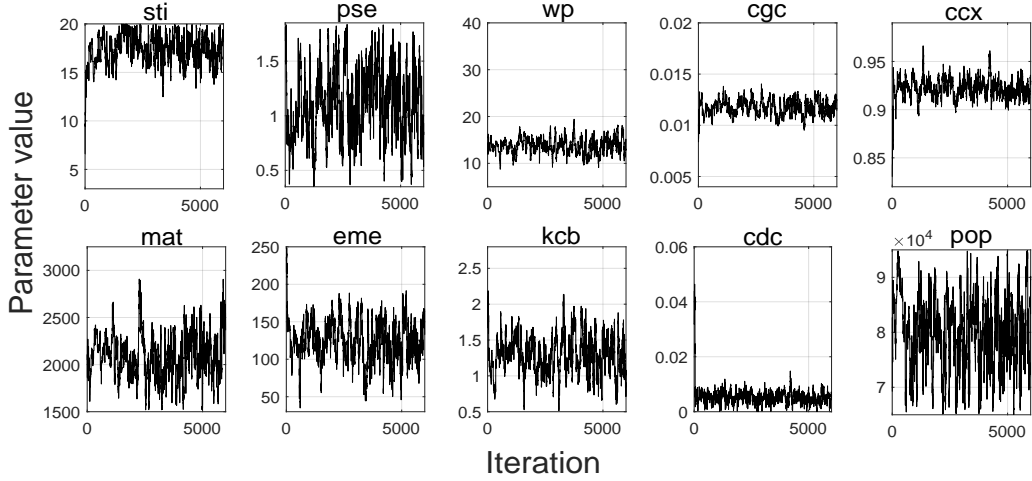


Figure 9: Markov Chain of each parameter using  $k = 2, 3, 4$  data for calibration

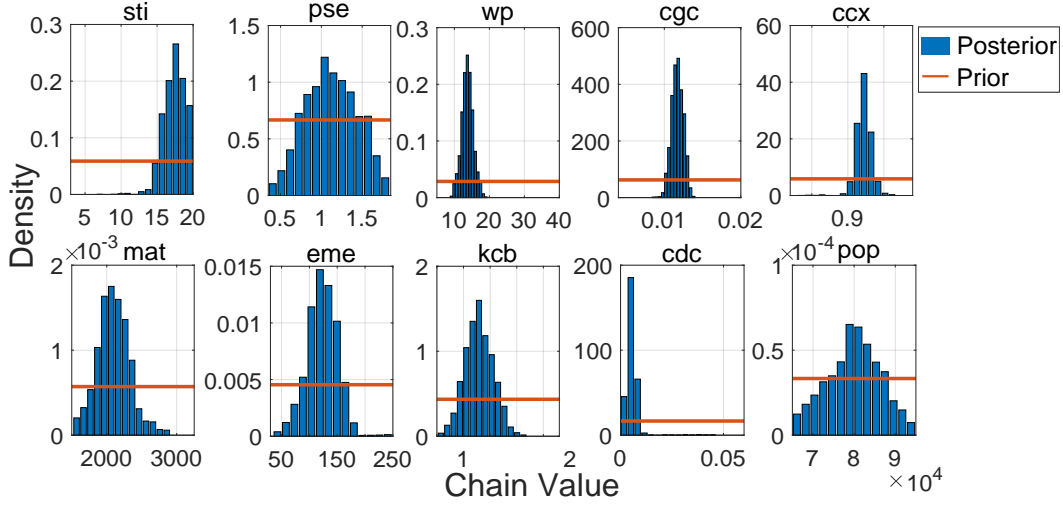


Figure 10: Normalized probability distribution of each parameter using  $k = 2, 3, 4$  folds data for calibration.

289 in Fig 10. It can be seen that the uniform prior distribution has been transformed into posterior distribution by  
 290 integrating the measurements into the AquaCrop crop model. From the distributions, parameter estimate (e.g. mean,  
 291 mode) can be derived and more importantly the confidence of parameter estimation can also be quantified. The  
 292 confidence rule is that the less spread the distribution is, the more reliable the parameter estimation is. However, the  
 293 optimization based approach can only provide a point estimate without confidence information (see, Fig 11).

294 It can also be seen from Fig 10 that  $pse$  and  $pop$  are with a large variance. There exist several possible reasons.  
 295 First, it may be due to the lack of calibration data in the sensitive growth stages. Secondly, the number of observations  
 296 may be not enough for the estimate of 10 dimensional parameter vector. The estimated parameters for both Bayesian  
 297 (e.g. mean value) and optimization (e.g. point estimate) methods are calculated and shown in Table 7, which are  
 298 used for CC prediction in Section 5.2.2.



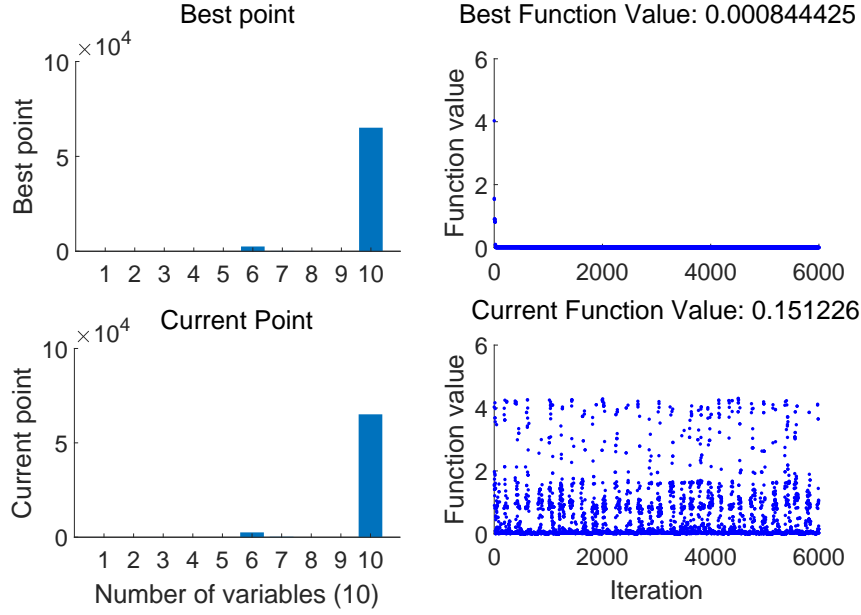


Figure 11: Simulated Annealing based parameter point estimate.

Table 7: Estimated parameters by both Bayesian and optimization methods

Parameters	Bayesian	Optimisation	Parameters	Bayesian	Optimisation
<i>sti</i>	17.3898	19.7103	<i>mat</i>	2096	2518
<i>pse</i>	1.1206	0.8055	<i>eme</i>	123.3688	171.1030
<i>wp</i>	13.7527	11.8404	<i>kcb</i>	1.2902	2.7902
<i>cgc</i>	0.0117	0.0135	<i>cde</i>	0.0055	0.0059
<i>ccx</i>	0.9221	0.9267	<i>pop</i>	79900	65117

### 299 5.2.2. CC estimation

300 The CC estimation over the whole growth season by using both Bayesian and SA optimization approaches is  
 301 conducted, where the results under different datasets for calibration are displayed in Fig 12. In particular, the  
 302 coloured lines denote the estimated CC curve for each day.

303 It can be seen that both approaches can obtain a relatively smooth CC estimate. However, in comparison to  
 304 SA optimization approach, Bayesian approach obtains a more reliable results when different calibration datasets are  
 305 adopted. However, when  $k = 1, 2, 4$  folds data are chosen for calibration, optimization based approach leads to a poor  
 306 CC estimate, which substantially deviates from groundtruth data. The main reason is that optimization approach  
 307 aims at minimizing the error between measurement data and model output data, which will result in poor performance  
 308 (e.g. local minima due to the complex optimization problem, poor generalization due to the problem of overfitting)  
 309 when inappropriate observations are chosen. While if no sufficient dataset is available for Bayesian approach, one can  
 310 easily observe this by inspecting the parameter estimation confidence (e.g. the spread of the parameter distribution).

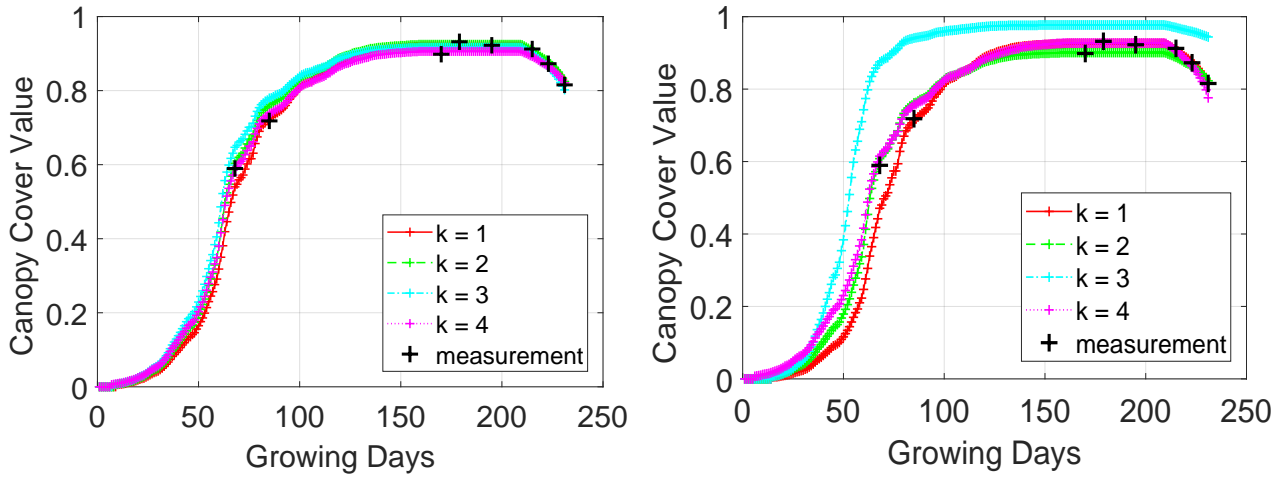


Figure 12: Canopy cover estimation of Bayesian method(left) and optimisation method(right) using different calibration  $k$  dataset

311 Compared to field observations, it can also be seen that Bayesian calibration, building a predictive model by fusing field  
 312 observations and crop growth model, can also provide CC prediction for days when field observation are unavailable.

313 *5.2.3. Regression analysis*

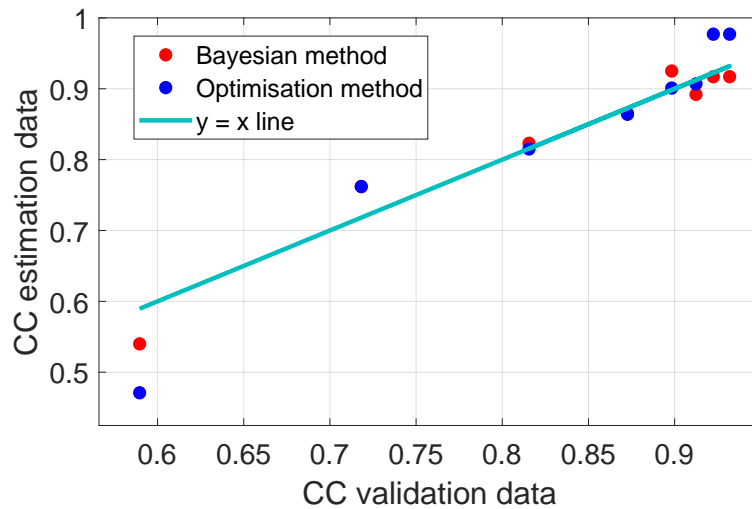


Figure 13: Comparison of estimated CC and validation CC in 2017-2018 year.

314 The CC estimation against ground truth CC data for different approaches under different datasets for calibration  
 315 is also displayed in Fig 13. X-axis and y-axis represent the ground truth validation data and the estimated CC values;  
 316 the red and blue points represent the estimated CC by using MCMC Bayesian and SA optimization approaches. It  
 317 can be visually seen that the results of Bayesian approach are closer to  $y = x$  line than SA optimization approach.

318 The RMSE values are also summarised for two approaches in Table 8. It can also be seen that MCMC Bayesian  
 319 approach results in a smaller RMSE value (in comparison with validation CC) than SA optimization approach.

Table 8: Summary of optimisation and Bayesian based calibration: regression results

Method	Sensor	Dynamic States	RMSE
Bayesian	RedEdge Camera	Canopy Cover	0.0271
Optimisation	RedEdge Camera	Canopy Cover	0.0514

## 6. Conclusion and future work

This study introduces a Bayesian framework to assimilate UAV remote sensing images into AquaCrop model so that a more reliable crop model is obtained for crop monitoring. High spatial/spectral multispectral images are first used to calculate the canopy cover by using supervised classification algorithms. Then the remote sensing information is accommodated by Markov Chain Monte Carlo so that the posterior parameter distributions are obtained. Then a systematic validation is conducted, [which include Monte Carlo simulations to assess parameter estimation performance and experimental 4-fold cross validation to evaluate canopy cover prediction performance](#). The Bayesian approach is also compared against the widely used optimization based approach. [Comparative results show that](#) both approaches are capable of estimating sensitive parameters and predicting canopy cover with a high accuracy. However, only point estimate is obtained by optimization approach, while Bayesian approach can return parameter posterior distribution [reflecting estimation confidence](#). Bayesian approach also obtains a smaller root mean square error for parameter estimation and canopy cover prediction than optimization based approach. In addition, Bayesian approach is less sensitive to the selection of data points for calibration. Although the results are very promising, there is also room for further improvement, which are summarized as below.

- (1) This work is mainly focused on algorithm development and its initial validation by using a small field, algorithm validation by large fields will be more convincing;
- (2) More advanced Bayesian inference algorithms can be developed to further improve the performance (e.g. reducing the computation load).

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