

Suppressing the impact of a high tensor-to-scalar ratio on the temperature anisotropies

Carlo R. Contaldi^{a,b} Marco Peloso^c Lorenzo Sorbo^{d,e}

^aTheoretical Physics, Blackett Laboratory, Imperial College, London, SW7 2BZ, UK

^bCanadian Institute of Theoretical Physics, Toronto, M5S 3H8, On, Canada

^cSchool of Physics and Astronomy, University of Minnesota, Minneapolis, 55455, USA

^dDepartment of Physics, University of Massachusetts, Amherst, MA 01003, USA

^eInstitut de Physique Théorique, CEA, F-91191 Gif-sur-Yvette Cedex, France

E-mail: c.contaldi@imperial.ac.uk, peloso@physics.umn.edu,
sorbo@physics.umass.edu

Abstract. The BICEP2 collaboration has reported a strong B mode signal in the CMB polarization, which is well fit by a tensor-to-scalar ratio of $r \simeq 0.2$. This is greater than the upper limit $r < 0.11$ obtained from the temperature anisotropies under the assumption of a constant scalar spectral index n_s . This discrepancy can be reduced once the statistical error and the contamination from polarized dust are accounted for. If however a large value for r will be confirmed, it will need to be reconciled with the temperature anisotropies data. The most advocated explanation involves a variation of n_s with scales (denoted as running) that has a magnitude significantly greater than the generic slow roll predictions. We instead study the possibility that the large scale temperature anisotropies are not enhanced because of a suppression of the scalar power at large scales. Such a situation can be achieved for instance by a sudden change of the speed of the inflaton (by about 14%), and we show that it fits the temperature anisotropies and polarization data considerably better than a constant running (its χ^2 improves by ~ 7.5 over that of the constant running, at the cost of one more parameter). We also consider the possibility that the large scale temperature fluctuations are suppressed by an anti-correlation between tensor and scalar modes. Unfortunately, while such effect does affect the temperature fluctuations at large scales, it does not affect the temperature power spectrum and cannot, therefore, help in reconciling a large value of r with the limits from temperature fluctuations.

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1 Introduction

The BICEP2 experiment [1] has observed a B -mode polarization of the Cosmic Microwave Background (CMB) that can be well fit by a lensed- Λ CDM + tensor theoretical model, with tensor-to-scalar ratio $r = 0.2_{-0.05}^{+0.07}$, with $r = 0$ disfavored at 7σ . It is possible that the actual primordial component of the B -mode found by BICEP2 is smaller than $r = 0.2$. A number of tests were performed on the BICEP2 data to ensure that the the observed value is not due to any instrumental effects. Moreover, the lensing contribution to B -modes does not appear to be sufficiently large to explain the measured value. The signal, observed by BICEP2, peaks at $\ell \approx 100$, where the primordial signal is expected to dominate, whilst the lensing signal peaks at $\ell \approx 1000$.

Other potential contaminants are the Galactic synchrotron and polarized-dust emission. Whilst the former effect is negligible at the BICEP2 observing frequency, the polarized dust is a substantial contaminant. Although the area of sky probed by BICEP2 is very clean with respect to the total intensity emission by dust there is still much uncertainty in the level of polarised dust contamination due to the lack of observations. A number of models were considered by BICEP2 for the subtraction of the dust contamination which result in some shift in the maximum likelihood values in r . A further argument for the primordial nature of the signal is that cross-correlation between frequencies (specifically, the preliminary BICEP2 \times Keck cross-correlation shown in [1]) displays little change in the observed amplitude. This appears to indicate that frequency dependent foregrounds are not the dominant contributor to the observed B -modes. On the other hand, more recent works [2] (appeared after the first version of this manuscript) argue that the polarized dust contamination is likely stronger than what assumed in [1]. This could invalidate BICEP2 claim of a primordial origin of the observed signal, or at least reconcile it with the Planck limit [3] $r < 0.11$. In this work we assume that the BICEP2 noise estimate is correct. The data already collected by Planck should have the sensitivity and frequency range to definitely confirm or rule out the primordial nature of such a large signal.

If the primordial contribution to the B -modes is confirmed, it will represent the first detection of gravity waves from inflation [1]. This is of paramount importance, since - under the assumption that the observed gravity waves are those created by a period of quasi de-Sitter inflationary expansion (namely that $P_t \sim V/M_p^4$, where P_t is the tensor power spectrum, $V^{1/4}$ is the energy scale of inflation, and $M_p \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass) - it allows us, for the first time, to determine the energy scale of inflation. From

the parametrization $r \equiv P_t/P_s$, and from the measured value of the scalar power spectrum, $P_s \simeq 2.45 \times 10^{-9}$, one obtains the well known relation

$$V^{1/4} \simeq 2.25 \cdot 10^{16} \text{ GeV} \left(\frac{r}{0.2} \right)^{1/4} . \quad (1.1)$$

Therefore, if the B -mode signal observed by BICEP2 is due to inflationary vacuum modes, we have now learnt that inflation took place at the GUT scale.

As we already mentioned, taken at face value, the BICEP2 value is in strong tension with the 2σ limit $r < 0.11$ obtained by the Planck inflation analysis [3]. Such a limit however relies on the scaling of the temperature anisotropy data (supplemented by the WMAP large-scale polarization likelihood), and not on the direct measurement of the B -mode polarization. The $r < 0.11$ limit appears robust under the inclusion of several data sets (such as the ACT+SPT temperature data, BAO, and the Planck lensing [3]). However, it crucially relies in the assumption of a constant spectral tilt n_s .

Specifically, it is obtained from the Planck+ACT+SPT temperature data (with the Planck data supplemented by the WMAP large-scale polarization likelihood), under the assumption of constant spectral tilt $n_s = 0.960 \pm 0.007$ [3]. As discussed in [3], a more relaxed limit is obtained if n_s is allowed to vary with scale k . Specifically, it is customary to parametrize the scalar power spectrum as

$$P_s(k) \equiv P(k_0) \left(\frac{k}{k_0} \right)^{n_s - 1 + \frac{1}{2} \alpha_s \ln \frac{k}{k_0}} , \quad (1.2)$$

where $k_0 = 0.05 \text{ Mpc}^{-1}$, is the chosen pivot scale (this is the scale at which also r is defined) and the parameter α_s denotes the running of the scalar spectral tilt [4] with $\alpha_s = \frac{dn_s}{d \ln k}$.

If $\alpha_s \neq 0$, the $r < 0.11$ limit is relaxed to $r \lesssim 0.25$. From Figure 5 of [3] we infer that a value $\alpha_s \sim -0.02$ is required to reconcile the temperature data with $r = 0.2$. Such a large value of $|\alpha_s|$ is not a generic prediction of slow roll inflationary models. Indeed, in terms of the slow roll parameters

$$\epsilon \equiv \frac{M_p^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 , \quad \eta \equiv M_p^2 \frac{V_{,\phi\phi}}{V} , \quad \xi^2 \equiv M_p^4 \frac{V_{,\phi\phi\phi}}{V^2} , \quad (1.3)$$

where V denotes the potential of the inflaton ϕ and comma denotes a derivative, we have the well known slow roll relations

$$\begin{aligned} r &= 16 \epsilon , \quad n_s - 1 = 2\eta - 6\epsilon , \\ \alpha_s &= -2\xi^2 + \frac{r}{2} (n_s - 1) + \frac{3}{32} r^2 \simeq -2\xi^2 - 0.00025 , \end{aligned} \quad (1.4)$$

where $n_s = 0.96$, $r = 0.2$ has been used in the final numerical estimate. This is typically much smaller than the required value, since, as evident in (1.4), the running is generically of second order in slow roll.

In principle, models can be constructed in which the third derivative term ξ^2 is ‘‘anomalously large’’. However, besides being hard to motivate, it is difficult to maintain a large third derivative, while the first two derivatives are small, for a sufficiently long duration of inflation, [5, 6], so that the models in which a large running is achieved have potentials with

some bump-like feature or superimposed oscillations [5, 7, 8, 10, 11], or possess some peculiar aspects beyond standard scenarios [12–16].

In summary, it appears that $r = 0.2$ can be reconciled with the limits from the temperature anisotropies through a negative running, which is however of substantially larger magnitude than the generic slow roll prediction. It is possible that the value of r from the polarization will shift towards $r \sim 0.1$, in which case the tension with the temperature data can be relaxed (or disappear altogether). This can happen factoring in both the statistical uncertainty in the BICEP2 $r = 0.2_{-0.05}^{+0.07}$ result, and the decrease of r that appears in most of the model-dependent dust corrections [1].

Remarkably, r close to 0.15 appears as a prediction of the simplest models of inflation, such as chaotic inflation [17] and natural inflation [18]. Even if UV complete theories typically leads to a lower inflationary scale, it is possible to construct models that can evade the fundamental constraints which typically make high-scale inflation difficult to realize [19–22] and still display such simple potentials.

However, from a theoretical point of view, it is interesting to understand the implications that a large measured value for r from polarization would have for inflationary model building. In this work we discuss two additional possibilities (in addition to the already mentioned running of the spectral tilt) to suppress the large scale temperature signal in presence of a large $r \simeq 0.2$.

The first mechanism relies on the presence of a large scale suppression in the scalar power. A similar idea was already explored in [23], in order to address the suppressed power of the temperature anisotropies at the largest scales. The best fit to the first year WMAP data was obtained if the power drops to zero at scales $k \lesssim 5 \times 10^{-4} \text{ Mpc}^{-1}$ [24]. Such a strong suppression can for example occur if the universe is closed, with a curvature radius comparable to the horizon at the onset of inflation [25], or if the inflaton was in fast roll at the beginning of the last ~ 60 e -folds of inflation [23]. In this case, one also expects a suppression of the tensor signal at large scales, although this suppression is milder than that of the scalar power [26]. Here, for simplicity, we only consider a simple model for suppression in order to explore the viability of such a model in explaining the surprisingly discrepancy between TT and BB spectra. We do not assume that the power drastically drops to $\simeq 0$ at large scales, but that it decreases by a factor $(1 - \Delta)^2$ for k smaller than a given scale k_* (a similar analysis on Planck data only was performed in [27]). Our best fit is characterized by a $\sim 26\%$ drop in power that can be achieved for example by a change of slope of the inflaton potential, such that the inflaton goes slightly faster when the largest modes were generated. However, the system never leaves the slow-roll regime. Therefore, contrary to the situation studied in [23, 24] there is no discontinuity in the tensor power. Specifically, we study the model originally proposed by Starobinsky [28] for this change in slope (see also [29]). Alternatively, a change in power can result from a change of the sound speed of the inflaton perturbations [30] (see also [31, 32] for other models leading to a similar effect).

The second possibility that we discuss is a negative correlation between the scalar and tensor signal¹. This correlation will affect the sum of the scalar and tensor modes on the temperature anisotropy. Some degree of non-vanishing scalar-tensor correlation is expected in the presence of the breaking of Lorentz invariance, and for example it arises if the background expansion is anisotropic [36–39]. Unfortunately, we will see that such a correlation does not

¹In a previous version of this manuscript we reached a different conclusion. This section has been modified after the findings of [33, 34], see also [35].

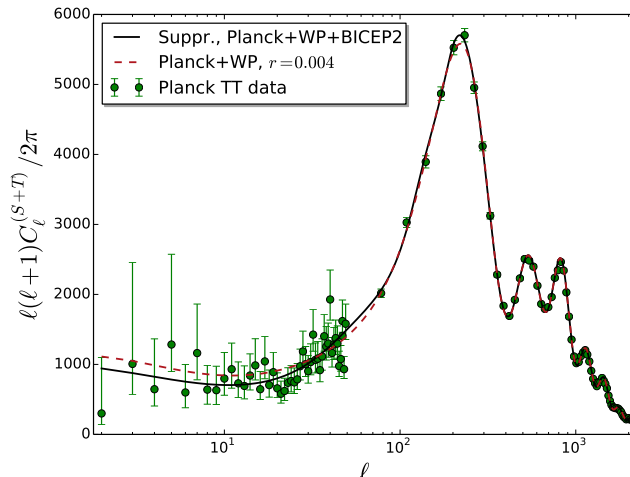


Figure 1. The total contribution to the TT spectrum from scalar and tensors. The black (solid) curve shows the best-fit suppression (or Starobinsky) model with $r = 0.18$ from the MCMC chains. The tensor contribution compensates for the loss of large-scale scalar power but also allows a better fit to the low points in the range $20 < \ell < 30$. The red (dashed) model is the best-fit model from the “base r ” Planck+WP MCMC runs with $r = 0.004$.

affect the temperature power spectrum, and will therefore not help reconcile the constraints on r from TT with those from BB spectra.

The plan of the paper is the following: In Section 2 we discuss the effects of a large scale suppression of the scalar power. In Section 3 we discuss the effects of an anti-correlation between scalar and tensor modes. In Section 4 we present our conclusions. Finally, in Appendix A we study a parametrization of the scalar power spectrum characterized by a step function suppression. This parametrization lacks the “ringing” effect that is typically encountered in concrete models (see Figure 2). The comparison between the two analyses shows that the ringing present in the Starobinsky model has a minor impact on the data.

2 Suppression of large scale scalar modes

A simple mechanism that produces a suppression at large scales is the Starobinsky model [28]

$$V(\phi) = \begin{cases} V_0 + C_+(\phi - \phi_*) & , \quad \phi \geq \phi_* \\ V_0 + C_-(\phi - \phi_*) & , \quad \phi \leq \phi_* \end{cases} \quad (2.1)$$

where we assume that all the parameters are positive, and $C_- < C_+$. As the inflaton ϕ rolls past ϕ_* during inflation, it goes from a region of greater (C_+) to a region of smaller (C_-) slope in the potential, and the scalar perturbations produced in this second stage, namely those with momentum $k > k_*$, where k_* is the momentum of the mode that leaves the horizon when $\phi = \phi_*$, have a greater power than those produced in the first stage, namely, those with

momentum $k < k_*$. Defining $\Delta \equiv \frac{C_+ - C_-}{C_+}$ and $\kappa \equiv k/k_*$, one obtains [28, 40]:

$$P_s(k) = A \left(\frac{k}{k_0} \right)^{n_s-1} \times \left\{ 1 + \frac{9\Delta^2}{2} \left(\frac{1}{\kappa} + \frac{1}{\kappa^3} \right)^2 + \frac{3\Delta}{2} \left(4 + 3\Delta - 3\Delta \frac{1}{\kappa^4} \right) \frac{1}{\kappa^2} \cos(2\kappa) + 3\Delta \left[1 - (1 + 3\Delta) \frac{1}{\kappa^2} - 3\Delta \frac{1}{\kappa^4} \right] \frac{1}{\kappa} \sin(2\kappa) \right\}, \quad (2.2)$$

with $A \equiv P(k_0)$, and k_0 is a pivot scale. The curly parenthesis in eq. (2.2) contains the precise suppression of power at large scales

$$\lim_{\kappa \rightarrow 0} P_s(k) = (1 - \Delta)^2 A \left(\frac{k}{k_0} \right)^{n_s-1}, \quad \lim_{\kappa \rightarrow \infty} P_s(k) = A \left(\frac{k}{k_0} \right)^{n_s-1}, \quad (2.3)$$

together with a “ringing” in the power that is typically obtained in models that give suppression of the power [23]. We show the power spectrum (2.2) for the best fit to the Planck and BICEP2 data in Figure 2. The best fit value $\Delta = 0.14$ (see below) results in a $\sim 26\%$ suppression of the power at large wrt short scales.

We assume that eq. (2.1) reproduces the potential close to the transition ($\phi \simeq \phi_*$), but we do not necessarily assume that the potential remains linear at all ϕ . For this reason, in eq. (2.2) we allow for an arbitrary spectral tilt n_s . In the data fitting, n_s has an impact on scales $k \gg k_*$, and there is no reason to assume that eq. (2.1) applies at ϕ arbitrarily far away from ϕ_* (in fact, we know that V needs to change from (2.1) so to have a minimum with $V_{\min} = 0$).

Moreover, we stress that the suppression of the power that is considered here is much smaller than the one studied in [23]. The goal of that work was to essentially suppress all the power at large scales, so to address the problem of the anomalously small observed quadrupole ($\ell = 2$). To achieve this, ref. [23] advocated a period of kination domination ($\dot{\phi}^2 \gg V$) before the onset of inflation. This also resulted in a very different evolution of H in this initial stage, and to a modification of the gravity wave power [26]. In the present situation we assume slow roll evolution before and after the transition, and a continuous potential at the transition. Therefore H is always slow roll evolving, and there is no transition in the tensor modes power. For this reason, we simply assume $P_T(k) = r A (k/k_0)^{n_t}$ and constrain the tensor spectral index to be given by the slow-roll consistency relation $n_t = -r/8$.

Using a modified version of the CAMB [41] package we calculate the total contribution to the TT spectrum from both scalar and tensor modes and the tensor contribution to the BB spectrum. The total contribution can then be fit to the combined Planck+WP+BICEP2 data set that include Planck TT data, WMAP9 polarisation data, and the new BICEP2 results. We do this by employing a modified version of CosmoMC [42] with extra parameters k_* and Δ included. For simplicity we carry out a MCMC exploration of the reduced parameter space spanned by τ the optical depth to reionization, n_s the scalar spectral index, $\ln(10^{10} A)$ the logarithm of the primordial curvature perturbation, r the tensor-to-scalar ratio, k_* , and Δ and keep all other cosmological and nuisance parameters fixed to the best-fit values of the “base r” Planck+WP run reported in [43]. The “base r” Λ CDM + tensor model will be the reference model for fit comparison. We adopt uniform prior ranges for the four parameters with $\tau = [0.01, 0.8]$, $n_s = [0.9, 1.1]$, $\ln(10^{10} A) = [2.7, 4]$, $r = [0, 0.8]$, $k_* = [0, 0.015]$, and $\Delta = [0, 1.0]$. We also compare to the Λ CDM + tensor + α_s model which has been highlighted as a way to reconcile the high tensor amplitude with the TT data. All runs include lensing effects when calculating model $C_{\ell S}$.

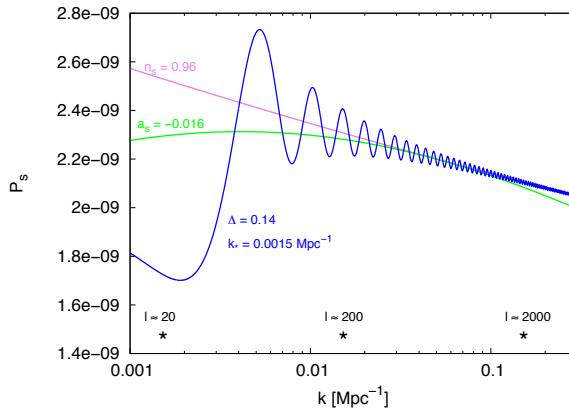


Figure 2. Primordial scalar power spectrum with the best fit parameters for (i) a constant spectral index, (ii) a constant running, and (iii) the inflation potential (2.1) with a $\Delta = 0.14$ change in slope. This corresponds to a $\sim 26\%$ suppression at large scales. The ℓ values indicated near the x -axis correspond to the multipole that is mostly sensitive to the mode with that momentum k .

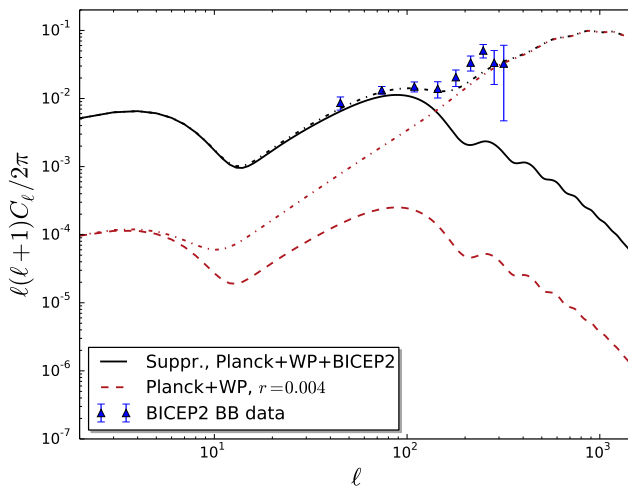


Figure 3. The BB spectra for the same two models shown in Figure 1. The Planck “base r ” best-fit model has a B -mode spectrum that is far below the BICEP2 detection, in the un-lensed limit $\ell < 100$. The suppression model is compatible with the detection having $r = 0.18$ whilst giving a better fitting TT spectrum. The best-fit parameters for the suppression model are $\Delta = 0.14$ and $k_* = 0.0015 \text{ Mpc}^{-1}$.

Figure 1 shows the total (scalar + tensor) contribution to the TT spectrum for the best-fit model obtain by Planck in their “base r ” run [43], with $r = 0.004$. This fit was obtained with the Planck+WP combination only. We compare this with the best-fit suppression model for the Planck+WP+BICEP2 data combination. The best-fit suppression parameters are $k_* = 0.0015 \text{ Mpc}^{-1}$, $\Delta = 0.14$, and the best-fit tensor-to-scalar ratio is $r = 0.18$. The

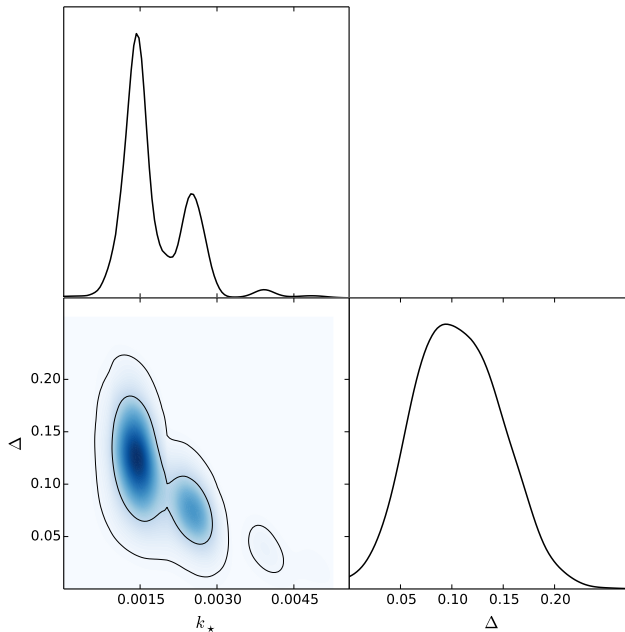


Figure 4. The marginalised, normalised, 1 and 2-D posteriors for k_* and Δ . The parameters are well constrained by the TT data. We recall that the best fit value $\Delta = 0.14$ corresponds to a $\sim 26\%$ suppression of the scalar power at large scales.

Planck+WP+BICEP2	ΔN_p	χ^2	$\Delta\chi^2$	r
Λ CDM + tensor	–	9853.104	–	0.18
Λ CDM + tensor + α_s	+1	9846.426	-6.68	0.18
Suppression	+2	9838.87	-14.23	0.18

Table 1. Relative changes in effective χ^2 values for the three models considered. The change in number of model parameters with respect to the Λ CDM + tensor model is shown as ΔN_p . The r column shows the best-fit value of the tensor-to-scalar ratio for each case. Only the case with suppressed scalar power can accommodate the observed amplitude of $r \sim 0.2$.

suppression model has the freedom to obtain a better fit than the conventional Λ CDM + tensor run by fitting the low power observed at $\ell < 30$. In particular the model allows a better fit to the low cluster of points in the range $20 < \ell < 30$. Including the BICEP2 results in the Λ CDM + tensor fit forces the tensor amplitude up to a best-fit value of $r = 0.18$. The effective $\chi^2 = -2 \ln L$ for the best-fit is significantly higher than the best-fit model in the suppression case. Table 1 shows the change in effective χ^2 relative to the Λ CDM + tensor model for fits including the Planck+WP+BICEP2 data combination. The change in χ^2 is indicative that the suppression model is *strongly* favoured over both standard and running case.² We attempt to quantify this by evaluating the Akaike Information Criterion AIC for the three models. The AIC is defined as

$$AIC = 2 N_p + \chi^2, \quad (2.4)$$

²Concerning the running, we find the best fit value $\alpha_s = -0.016$ and the marginalized constraints $\alpha_s = -0.028 \pm 0.010$.

Planck+WP+BICEP2	N_p	ΔAIC	Rel. Likelihood
Λ CDM + tensor	21	-10.617	2.45×10^{-5}
Λ CDM + tensor + α_s	22	-5.939	0.0026
Suppression	23	–	1.0

Table 2. Change in AIC for the three models. The relative likelihood can be interpreted, in this case, as the probability that the model minimizes the information required to describe the data relative to the *suppression* model that gives the best fit. The AIC attempts to properly penalise for any increase in number of parameters in the models.

where N_p is the number of parameters in the model. It attempts to properly take into account the penalty for using models with increasing number of parameters to describe a set of data. The relative likelihood between a model with minimum AIC value and a second model

$$L_{AIC} = \exp [(AIC_{\min} - AIC)/2] , \quad (2.5)$$

gives an estimate of the probability that the second model minimizes the information required to describe the data³. The results of this analysis are summarised in table 2 and show the suppression model is strongly favoured despite the extra parameters.

The results show that models with a suppression of scalar power at large scales are favoured with respect to ones that employ running of the scalar spectral index notwithstanding the additional parameter required in the suppression model. Both models do better than the Λ CDM + tensor model but the running case only marginally so given the additional parameter. This is not surprising since the case with running reduces power, for a negative α_s , on scales that are both lower *and* higher than the pivot scale of $k = 0.05 \text{ Mpc}^{-1}$, see Figure 2. The model with running is therefore not a natural one to appeal to if all that is required is to modify the scalar power on large scales.

Figure 3 shows the BB spectra for the same two models as in Figure 1. The spectra include the pure tensor contribution and the contribution from lensing. The BICEP2 data points are included for comparison. The conventional “base r” Planck best-fit model has a very low tensor spectrum which is incompatible with the BICEP2 detection whilst the suppression model has a best-fit value of $r = 0.18$ in agreement with BICEP2.

The suppression parameters are well constrained by the Planck data, as shown in Figure 4, with marginalised constraints; $k_* = (1.80_{-0.69}^{+0.86}) \times 10^{-3} \text{ Mpc}^{-1}$ and $\Delta = 0.106_{-0.013}^{+0.092}$. The null hypothesis that the suppression is either absent or occurs on scales larger than those observable seems to be ruled out strongly.

3 Scalar-tensor anticorrelations?

Since the temperature fluctuations in the CMB take contributions from both scalar and tensor perturbations whereas the B -modes are sourced only by the latter, one can wonder whether it is possible to reduce the temperature power spectrum without affecting the B -mode one by introducing an (anti)correlation between primordial tensors and scalars. In this case one would not need to break primordial scale invariance to effectively suppress the

³The *corrected AIC* also includes the effect of finite effective degrees of freedom N_d in the data but we cannot use it here as it is very difficult to define what N_d is for the PLANCK+WP data combination since it is a correlated data set whose likelihood is evaluated in both multipole and pixel space. However we know that $N_d \sim \mathcal{O}(10^3)$ and the correction goes as $\sim N_p^2/N_d$ and is therefore sub-dominant in this case.

temperature power spectrum *only at large scales* $\ell \lesssim 100$, since only at those scales the effect of tensors is not suppressed. On the other hand, such a correlation can be obtained only at the price of breaking Lorentz invariance. As a consequence, it would be interesting to explore possible connections to the large scale anomalies observed in WMAP and Planck. Here we will not study those implications but we focus on the main question of whether, and how, a primordial $\langle \zeta h_{ij} \rangle$ correlator can affect the temperature power spectrum. Nevertheless, we want to point out the following numerical coincidence. If the goal of the cross correlation is to reduce the impact of the tensor modes on the temperature anisotropies from $\sim 20\%$ to $\sim 10\%$, it should amount to $\sim 10\%$ of the total power. Interestingly, a $\lesssim \mathcal{O}(0.1)$ amount of statistical anisotropy, only at the largest scales, appears to be present in the WMAP [45] and in the Planck [46] temperature data.

As we will show below, a primordial scalar-tensor correlation will affect the observable $C_{lm\ell'm'} \equiv \langle a_{lm}^* a_{\ell'm'} \rangle$. However, as noted in [33, 34] (see also [35]), it will not affect the temperature power spectrum $C_l \equiv \frac{1}{2l+1} \sum_m \langle a_{lm}^* a_{lm} \rangle$, and will not, therefore, help suppressing the impact of a large value of r on the temperature anisotropies as they are measured by Planck.

In this section we will study the effect on $C_{lm\ell'm'}$ of a simple form of scalar-tensor correlator, assuming that the primordial correlation functions are scale invariant and that the scalars affect the temperature fluctuations only through the Sachs-Wolfe effect.

We define the coefficients

$$C_{lm\ell'm'} \equiv \int \frac{d\mathbf{k} d\mathbf{k}'}{(2\pi)^3} \int d\Omega_{\mathbf{p}} d\Omega_{\mathbf{p}'} \langle \frac{\delta T}{T}(\mathbf{k}, \mathbf{p})^* \frac{\delta T}{T}(\mathbf{k}', \mathbf{p}') \rangle Y_l^m(\mathbf{p}) Y_{\ell'}^{m'}(\mathbf{p}')^*, \quad (3.1)$$

where δT will receive contributions both from the scalar perturbations and from the tensors: $\delta T = \delta T^s + \delta T^t$. In the equation above \mathbf{p} is the unit vector directed along the line of sight.

The contribution to $C_{lm\ell'm'}$ from a primordial scalar-tensor correlation reads

$$C_{lm\ell'm'}^{\text{corr}} = \int \frac{d\mathbf{k} d\mathbf{k}'}{(2\pi)^3} \int d\Omega_{\mathbf{p}} d\Omega_{\mathbf{p}'} Y_l^m(\mathbf{p}) Y_{\ell'}^{m'}(\mathbf{p}')^* \times \left\langle \frac{\delta T^t}{T}(\mathbf{k}, \mathbf{p})^* \frac{\delta T^s}{T}(\mathbf{k}', \mathbf{p}') + \frac{\delta T^s}{T}(\mathbf{k}, \mathbf{p})^* \frac{\delta T^t}{T}(\mathbf{k}', \mathbf{p}') \right\rangle, \quad (3.2)$$

that is proportional to the correlator between ζ and h_{ij} , that we assume to take the form

$$\langle \zeta(\mathbf{k})^* h_{ij}(\mathbf{k}') \rangle = 2\pi^2 \frac{\delta(\mathbf{k} - \mathbf{k}')}{k^3} \Pi_{ij}{}^{ab}(\mathbf{k}') \Theta_{ab}, \quad (3.3)$$

where Θ_{ab} is some given constant background tensor. For simplicity, we assume that $\Theta_{ab} = \sigma \mathbf{v}_a \mathbf{v}_b$, for some constant vector \mathbf{v} (the generalization to an arbitrary tensor is straightforward) and with $\sigma = \pm 1$. It is then convenient to choose a coordinate system where $\mathbf{v} = (0, 0, v)$. The term $\Pi_{ij}{}^{ab}$ is the projector on the transverse-traceless modes, as required by the tensor nature of h_{ij} : $\Pi_{ij}{}^{ab} = \Pi_i^a \Pi_j^b - \frac{1}{2} \Pi_{ij} \Pi^{ab}$ with $\Pi_{ab} = \delta_{ab} - \hat{k}_a \hat{k}_b$ and with $\hat{\mathbf{k}} \equiv \mathbf{k}/k$.

An explicit calculation then shows that the only non vanishing coefficients are $C_{lm\ell m}^{\text{corr}}$ and $C_{lm\ell+2m}^{\text{corr}}$. The off-diagonal terms read

$$C_{lm\ell+2m}^{\text{corr}} = \frac{2\pi}{75} \frac{\sigma v^2}{l(l+1)} \frac{(7l^2 + 21l + 27)}{(l+2)(l+3)(2l+3)} \sqrt{\frac{((l+1)^2 - m^2)((l+2)^2 - m^2)}{(2l+1)(2l+5)}}, \quad (3.4)$$

whereas the diagonal terms are

$$C_{lm}^{\text{corr}} = -\frac{4\pi}{15} \frac{\sigma v^2}{l(l+1)(4l^2+4l-3)} (l^2+l-3m^2). \quad (3.5)$$

Since $\frac{1}{2l+1} \sum_{m=-l}^l m^2 = \frac{1}{3} l(l+1)$, the above result shows that the correlation has no effect on the coefficients $C_l \equiv \frac{1}{2l+1} \sum_m \langle a_{lm}^* a_{lm} \rangle$.

4 Conclusions

The BICEP2 experiment has detected a B -mode polarization signal in the CMB that can be explained by a lensed- Λ CDM + tensor theoretical model, with tensor/scalar ratio $r = 0.2_{-0.05}^{+0.07}$. While keeping in mind that the central value can be reduced with more statistics or with the subtraction of the dust-polarized signal, we discussed some possible mechanism that may explain how $r = 0.2$ can be reconciled with the upper limits on tensor modes from the temperature anisotropies measurements.

A possible way to reconcile this discrepancy is already offered in the Planck [3] and the BICEP2 [1] analyses, where a running of the spectral tilt is advocated. However, if the central value $r = 0.2$ will be confirmed, the required magnitude of the running is significantly greater than the generic slow-roll predictions. Motivated by this, we discussed two alternative possibilities for suppressing the large scales temperature anisotropies.

The first possibility involves a large scale suppression in the scalar power. This can for instance be achieved if the inflaton zero mode has a greater speed, or if the inflaton perturbations have a larger speed of sound, when the largest observable scales are produced. This resembles the study of [23], in which the power at very large drops to a negligible value due to a period of kinetic dominated regime at the beginning of the last $\sim 60 e$ -folds of inflation. Contrary to that case, here we study the possibility that a partial drop of scalar power occurs at scales that are larger than but comparable to the first acoustic peak. The best fit to the temperature data is obtained for a transition scale $k_* \simeq 1.5 \times 10^{-3} \text{ Mpc}^{-1}$, with a $\simeq 26\%$ decrease in power. This fit improves over the Planck ones with conventional models due to the suppression in power in the $10 \lesssim \ell \lesssim 30$ region. Moreover, the model easily fits the tensor-to-scalar ratio $r \sim 0.2$ observed by BICEP2. Quite interestingly, the χ^2 of the fit of this model to the Planck+WP+BICEP2 data improves by about ~ 7.5 with respect to the fit of the same data of a model with constant running. This is a considerable improvement, given that the suppression model has only one more parameter than the model with a constant running. The significance of the improvement is supported by the Akaike Information Criterion, see Table 2.

A second possibility is a negative correlation between tensor and scalar modes. Such a correlation is expected to occur, to some degree, in models with broken rotational symmetry. Such a situation actually appears to be present in the WMAP [45] and in the Planck [46] temperature data. Unfortunately, a primordial tensor-scalar correlation does not affect the temperature power spectrum, that is used by Planck to set limits on the tensor-to-scalar ratio, and cannot therefore help reconcile the bounds on r found from BICEP2 with those from Planck. It would be interesting to study whether, and under what conditions, such a conclusion might be avoided.

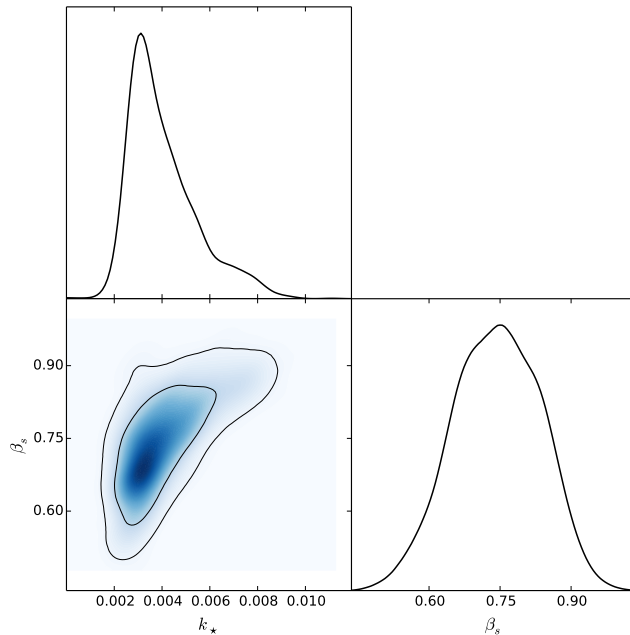


Figure 5. The marginalised, normalised, 1 and 2-D posteriors for k_* and β_s in eq. (A.1).

A Step-function suppression of the scalar power

In this Appendix we consider the possibility of a step-function suppression of the scalar power:

$$P_s(k) = A\beta_s k^{n_s-1}, \quad (\text{A.1})$$

with $\beta_s = 1$ for $k \geq k_*$ and $\beta_s < 1$ for $k < k_*$. This parametrization should be compared for instance with the power spectrum (2.2) obtained in the Starobinsky model (2.1), with the identification $\beta_s \equiv (1 - \Delta)^2$. As compared to that model, this parametrization does not have the “ringing” in the power visible in Figure 2. One could imagine alternative specific models that result in the suppression of power at large scales, as for instance a varying speed sound of the inflaton perturbations [30]. We expect that each different model will result in a different “ringing” structure. It is therefore instructive to compare the results presented in the main text with the results for a parametrization in which the ringing is absent.

We compare the parametrization (A.1) with the data, and obtain the results summarized in Figure 5. This result should be compared with the one shown in Figure 4 for the Starobinsky model. In this case we obtain marginalised constraints $k_* = (4.07_{-2.45}^{+0.70}) \times 10^{-3} \text{ Mpc}^{-1}$ and $\beta_s = 0.74_{-0.09}^{+0.11}$, with an improvement $\Delta\chi^2 = -14.23$ over the $\Lambda\text{CDM} + \text{tensor}$ fit. Upon the identification $\beta_s \equiv (1 - \Delta)^2$, the result for β_s translates into $\Delta = 0.14_{-0.06}^{+0.05}$. Both values of k_* and Δ are consistent with those obtained in the main text for the Starobinsky model (2.1). Moreover, the improvement in χ^2 is practically identical to that obtained in the main text for the more physically defined Starobinsky model. The small impact of the ringing effect on the data analysis is due to the fact that a range of values of k contributes to any given C_ℓ , and this softens the impact of the oscillation in the primordial power spectrum. This conclusion may not apply for models that present more marked oscillations.

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