Представляє практичний інтерес створення моделей магнітного поля квадрупольного електромагніту, які дозволяють коригувати середньо-інтегральні коефіцієнти магнітної індукції за допомогою зміни геометричних параметрів конструкції магніту. Мета роботи – розробка методу оптимізації конструкції квадрупольного електромагніту з надпровідною обмоткою за критерієм мінімуму величин неквадрупольних середньо-інтегральних коефіцієнтів магнітної індукції в апертурі. Практичне застосування методу дозволяє оптимізувати конструкцію квадрупольного електромагніту для мінімізації середньо-інтегральних по довжині коефіцієнтів магнітної індукції на основі розрахунку геометричних параметрів ярма і обмотки.

Отримані аналітичні вирази для розрахунку мінімізованих середньо-інтегральних по довжині коефіцієнтів магнітної індукції, що створюються всередині апертури квадрупольного електромагніту засновані на їх пропорційності вкладам від струмової обмотки в залежності від її положення відносно ярма. Встановлені експериментальним шляхом взаємозв'язки між середньо-інтегральними по довжині коефіцієнтами магнітної індукції і параметрами конструкції покладені в основу процедури практичного застосування способу поліпшення однорідності градієнта магнітної індукції квадрупольного електромагніту. Отримані вирази дозволяють розраховувати необхідну корекцію геометричних параметрів вже існуючої конструкції для оптимізації магнітного поля всередині апертури квадрупольних електромагнітів по заданим середньо-інтегральним коефіцієнтам.

Приведені результати оптимізації конструкції магнітоактивної частини триплету квадрупольних електромагнітів прискорювального комплексу проекту NICA. Оптимізація проводилась на основі запропонованого способу, з мінімізацією до рівня 10^{-5} неквадрупольних середньо-інтегральних коефіцієнтів поперечних складових магнітної індукції, створюваної в апертурі електромагніту з надпровідною обмоткою

Ключові слова: пучок частинок, квадрупольний електромагніт, коефіцієнт магнітної індукції, надпровідна обмотка

UDC 621.317.44
DOI: 10.15587/1729-4061.2018.142163

DEVELOPMENT OF THE TECHNIQUE FOR IMPROVING THE STRUCTURE OF A MAGNETIC FIELD IN THE APERTURE OF A QUADRUPOLE ELECTROMAGNET WITH A SUPERCONDUCTING WINDING

A. Getman
PhD, Senior Researcher
Department of Theoretical Electrical Engineering
National Technical University "Kharkiv Polytechnic Institute"
Kyrpychova str., 2, Kharkiv, Ukraine, 61002
E-mail: getmanav70@gmail.com

1. Introduction

To solve the problem on the formation of transverse size and the transportation of a beam of particles in accelerators, electromagnets are typically used. The force impact induced on the particle beam depends on the structure of a magnetic field generated by the electromagnet. In order to change the cross-section shape of the beam of particles, the quadrupole electromagnets with a uniform transverse gradient of magnetic flux density are applied. In this case, the force influence of the transverse magnetic field on the beam of particles is similar to the focusing and defocusing effect of the optical lens. Deviations of the gradient of a magnetic field from the constant value, inherently arising in the aperture of an electromagnet, are analogous to the optical lens defects in terms of deterioration in the quality of focusing and forming the transverse size of a beam of particles.

Therefore, the transverse gradient of magnetic flux density, created in the aperture of a quadrupole electromagnet, is imposed with strict requirements for its homogeneity. The magnitude of maximum relative deviation of the gradient of magnetic flux density is imposed with a limitation not to exceed 10^{-4}.

On the other hand, the application in the structure of the quadrupole electromagnet of superconducting windings significantly changes the requirements for the accuracy of its fabrication. Even for the well-established and widely-used classical scheme with an iron yoke, the standard requirement, rendering a hyperbolic shape to the surface of its poles, is not insufficient. This is the consequence of the dramatically increased contribution to the gradient of magnetic flux density from the current winding at its superconducting implementation.

The practical need to meet the requirements for a magnetic field makes it a relevant task to develop the appropriate methods for the optimization of structure of the quadrupole electromagnets.

2. Literature review and problem statement

Solving the problem on prolonging the "lifetime" and ensuring the required shape of a beam of particles in the orbit of circular accelerators leads to the toughening of requirements to the quality of magnetic flux density generated by electromagnets [1].

To assess the quality of the generated magnetic field, its model is used, built for two transverse components of magnetic flux density in the form of a series [2]. For the quadrupole electromagnet, all non-quadrupole coefficients of the series have a sense of the relative error in the formation of a magnetic field and thus require minimization. Practical application of a magnetic flux density series' coefficients is predetermined by the need to represent a force impact of the field on a beam of particles in the form that is convenient to calculate its trajectory in the orbit of an accelerator [3]. It is therefore necessary to find a technique to practically determine the values for coefficients of magnetic flux density.

For a particular case of the 2D magnetic field distribution, the application of the magnetic flux density description in the form of a series is similar to its representation based on the cylindrical (polar) harmonics of scalar potential [4]. In this case, the introduced coefficients of the series of two projections of magnetic flux density are the constants that are proportional to the amplitude coefficients of cylindrical harmonics.

However, because the magnetic field in an electromagnet aperture has a 3D distribution and it is not plane-parallel, coefficients of the series of magnetic flux density turn out to be a function of the longitudinal coordinate [5].

The practical determination of functional dependence on the longitudinal coordinate for the magnetic flux density coefficients is not possible purely analytically. Such a dependence is found either by calculating the field using numerical methods [6] or directly measuring the magnetic flux density in the aperture of a working sample [7].

A special feature of the search for an optimal structure based only on the results of calculation of magnetic flux density using numerical methods is the need to obtain results for a very large number of the structure's variants with many discretely variable parameters for the yoke and the winding [8]. On the other hand, conducting an experimental testing of the longitudinal distribution of the magnetic flux density coefficients for the created set of the prototypes of an electromagnet is the most expensive variant for optimizing the structure based on the predefined magnetic characteristics. Therefore, the experimental determination of the magnetic flux density coefficients in the process of designing and constructing the electromagnets is performed in order to exercise ultimate control over their magnetic characteristics [9].

One of the variants for solving the task on ensuring the specified magnetic characteristics of an electromagnet is the application of methods based on the relationships between magnetic flux density coefficients, medium-integral in length, and structural parameters of the electromagnet. Although such an approach to resolving a task is described for a magnetic flux density component, generated by the winding only, the methods for the optimization of the structure of the entire magnetoactive part of the electromagnet need to be developed [10].

3. The aim and objectives of the study

The aim of this study is to develop a method for optimizing the structure of a quadrupole electromagnet with a superconducting winding based on the criterion for a minimum of magnitudes of non-quadrupole medium-integral coefficients of magnetic flux density in aperture. This would make it possible to solve the problem on the structure optimization of a quadrupole electromagnet by minimizing magnetic flux density coefficients, medium-integral in length, on the basis of the calculation of geometrical parameters of the yoke and winding.

To accomplish the aim, the following tasks have been set:

– to adapt a 3D model of the magnetic field of a quadrupole electromagnet in order to search for the parameters for its structure that would ensure the minimization of magnetic flux density coefficients;

– to derive empirical expressions for calculating the non-quadrupole medium-integral coefficients of magnetic flux density based on the geometrical parameters of the yoke and the winding of a quadrupole electromagnet;

– to propose a procedure for searching for the parameters in the design of the yoke and winding that would ensure finding the preset magnetic characteristics of a quadrupole electromagnet.

4. Materials and methods of research

This paper investigates the spatial structure of the magnetic field generated by the quadrupole electromagnet (Fig. 1) with a superconducting current winding (Fig. 2).

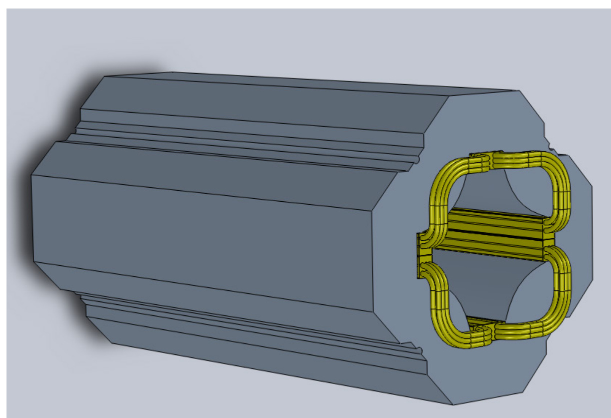

Fig. 1. Quadrupole electromagnet with a superconducting winding

Similar to the plane-parallel magnetic field, I consider, in the form of a series, only two transverse projections of magnetic flux density B_ρ and B_φ in the cross-sectional planes of electromagnet, according to [2]:

$$B_\phi(\rho,\phi,z)+iB_\rho(\rho,\phi,z)= \\ =B_0(z)\sum_{n=1}^{\infty}\left[b_n(z)+ia_n(z)\right]\frac{(\rho)^{n-1}}{R_{ref}^{n-1}}e^{in\phi}, \quad (1)$$

where ρ, φ, z are the cylindrical coordinates of the observation point of the magnetic field; $B_0(z)$ is the main value of magnetic flux density; $b_n(z)$, $a_n(z)$ are the normal and the skew coefficients of magnetic flux density of power n.

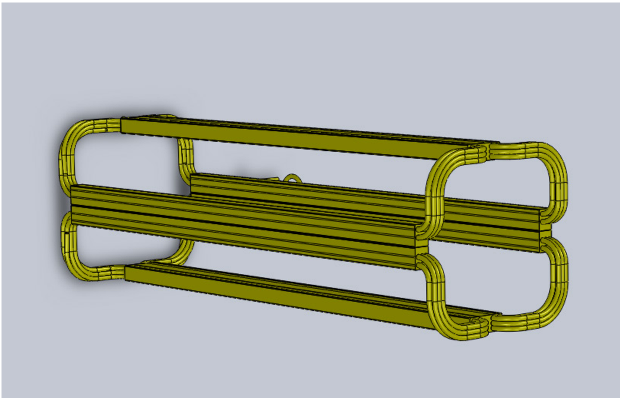

Fig. 2. Superconducting winding of a quadrupole electromagnet

The reference radii R_{ref} is understood as the radius of the circle, along which, based on the magnetic flux density data, the coefficients $b_n(z)$, $a_n(z)$ were determined.

The magnetic flux density coefficients' values, medium-integral in length, from (1) are introduced based on [2]:

$$b_n^* = \frac{\int_{-\infty}^{\infty} b_n(z)B_0(z)\mathrm{d}z}{\int_{-\infty}^{\infty} B_0(z)\mathrm{d}z} = \frac{\int_{-\infty}^{\infty} b_n(z)B_0(z)\mathrm{d}z}{B_c L_{eff}}, \quad (2)$$

$$a_n^* = \frac{\int_{-\infty}^{\infty} a_n(z)B_0(z)\mathrm{d}z}{\int_{-\infty}^{\infty} B_0(z)\mathrm{d}z} = \frac{\int_{-\infty}^{\infty} a_n(z)B_0(z)\mathrm{d}z}{B_c L_{eff}}, \quad (3)$$

where B_c is the main value for magnetic flux density in the central cross-section of the electromagnet; L_{eff} is the effective length of the electromagnet, introduced in the form:

$$L_{eff} = \frac{\int_{-\infty}^{\infty} B_0(z)\mathrm{d}z}{B_c}. \quad (4)$$

For definiteness, let us consider the normal quadrupole electromagnet and, accordingly, only normal $b_n(z)$, from the series (1), coefficients of the magnetic field generated in the aperture, in the coordinate system associated with the center of the electromagnet whose longitudinal axis coincides with the applicates axis.

5. A technique for the optimization of design of the magnetoactive part of a quadrupole electromagnet

Consider one of the possible techniques to minimize the contribution of senior (non-quadrupole) magnetic flux density coefficients, medium-integral in length, generated within the aperture of the quadrupole electromagnet that has a classical circuit of the magnetoactive part with an iron yoke and superconducting current winding. Represent a given technique in the form of a procedure that consists of two stages.

The first stage is based on the results of performing the initial 3D calculation of magnetic flux density inside the aperture of the simplified model of a quadrupole electromagnet. To perform the calculation, one assigns the design of the central cross-section, the structure and lengths of the yoke and the current winding without the end elements and interturn transitions.

The design of the central cross-section of an electromagnet is chosen based on the initial requirements to the magnitude of the generated gradient of magnetic flux density using 2D analytical models [12], considering the practice of development of the earlier constructed samples and prototypes.

Let us use as the assigned data the gradient and radius of aperture R_a, considering it being equal to the distance from the longitudinal axis of the electromagnet to the pole (Fig. 3), with an internal cross-section in the shape of a hyperbola [3].

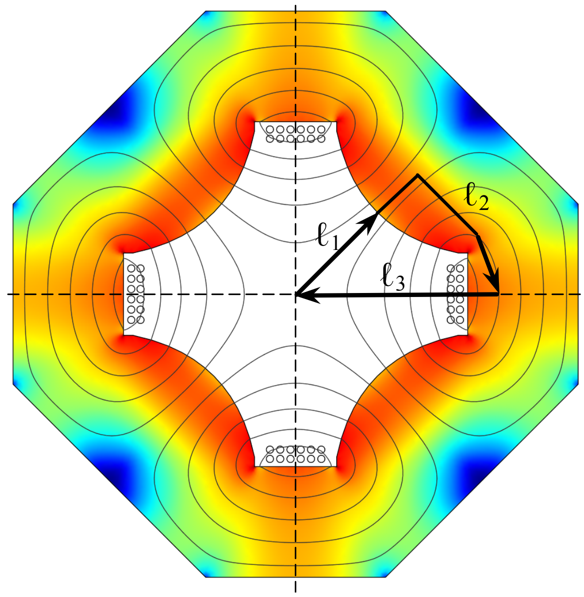

Fig. 3. Sections of the closed contour for the preliminary calculation of ampere-turns in a quadrupole electromagnet

The number of ampere-turns, required to generate a field with the assigned gradient, can be determined based on the law of full current [11]:

$$\oint_L \vec{H}\mathrm{d}\vec{l} = \int_S \vec{j}\mathrm{d}\vec{s} = NI, \quad (5)$$

where NI is the desired number of ampere-turns in the quadrupole winding.

When choosing a path for the integration according to Fig. 3, the closed integral from (5) can be represented as a sum of three integrals of the related sections [12]. Let us assume that the integral of the second section ℓ_2 is negligible due to the low magnetic resistance of the iron yoke. The integral at the third section ℓ_3 is zero due to the perpendicularity of the integration line and the direction of the magnetic field. The value of the closed integral from (5) will then be defined only by the integral at the first section ℓ_1 from the axial axis of the electromagnet to the center of the yoke's pole. Considering the gradient of magnetic flux density G=const inside the aperture a constant magnitude, the radial projection of the field intensity will be expressed in the form:

$$H_\rho(\rho) = \frac{G}{\mu_0}\rho. \quad (6)$$

Finally, the following expression will be obtained for the winding's ampere-turns:

$$NI = \oint_L \vec{H} d\vec{l} \approx \int_0^{R_a} H_\rho(\rho) d\rho = \frac{G R_a^2}{2\mu_0}. \quad (7)$$

Knowing the number of turns and the required effective length of the quadrupole electromagnet, the length of the longitudinal sections of the winding's turns L_{win} should be chosen based on the expression derived in [10]:

$$L_{win} = \frac{\left(\sqrt{(L_{eff})^2 + (2R_a)^2}\right)^3}{(L_{eff})^2 + 6(R_a)^2}. \quad (8)$$

Considering that there should be a technological gap s between the yoke and the intended place for the end elements of the winding, the yoke turns out to be shorter than the winding (Fig. 4). To account for a given design feature when calculating the length of the yoke L_{yok}, one can apply the approximate ratio of one to two for the contributions to the gradient of magnetic flux density from the current winding and yoke. The following expression for the effective length L_{eff} of the electromagnet then holds in the first approximation:

$$L_{eff} \approx \frac{2}{3} L_{yok} + \frac{1}{3} L_{win} = \frac{2}{3}(L_{win} - s) + \frac{1}{3} L_{win}. \quad (9)$$

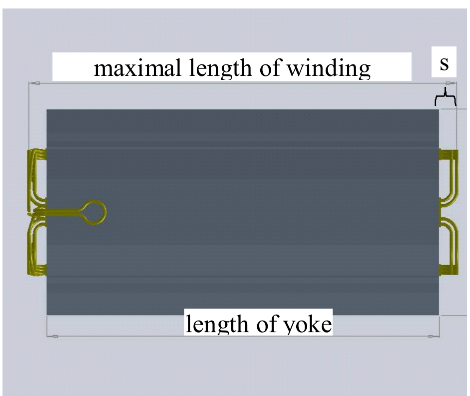

Fig. 4. Technological gap s between the yoke and the end of winding

The parameters for a quadrupole electromagnet, obtained in this fashion, will suffice to build its simplified 3D model (symmetrical relative to the central cross-section with missing interturn transitions and end elements of the winding) and to perform numerical calculation of magnetic flux density in the aperture. The magnetic flux density, derived as a result of numerical calculation, in the central cross-section on a circle of radius R_{ref} is subject to mathematical processing using a Fourier analysis to obtain the values for coefficients $b_n(z=0)$. As described in [2], for example, using the radial projections of magnetic flux density on a circle of radius R_{ref} at N points for computing a coefficient $b_n(z=0)$ based on expression:

$$b_n(z=0) \approx \frac{\sum_{k=1}^{N} B_\rho(R_{ref}, \phi_k, 0) \sin n\phi_k}{\sum_{k=1}^{N} B_\rho(R_{ref}, \phi_k, 0) \sin 2\phi_k}, \quad (10)$$

where R_{ref}, φ_k, $z=0$ are the cylindrical coordinates of the k-th point on the circle in the central cross-section.

To minimize the derived values for coefficients $b_n(z=0)$ with $n=6$ and with $n=10$, one should employ the dependence of their values on the position (coordinates) of longitudinal elements w of the current winding's turns. The new position of all longitudinal elements in the winding (offset relative to the starting one by $\Delta\rho$ and $\Delta\varphi$ quadrupole-symmetrical for all four pairs of the longitudinal elements of turns) can be calculated from equalities:

$$b_6(z=0) = \sum_{i=1}^{w} \left(\frac{R_{ref}}{\rho_i}\right)^4 \frac{\cos 6(\phi_i)}{\cos 2(\phi_i)} - \sum_{i=1}^{w} \left(\frac{R_{ref}}{\rho_i + \Delta\rho}\right)^4 \frac{\cos 6(\phi_i + \Delta\phi)}{\cos 2(\phi_i + \Delta\phi)},$$

$$b_{10}(z=0) = \sum_{i=1}^{w} \left(\frac{R_{ref}}{\rho_i}\right)^8 \frac{\cos 10(\phi_i)}{\cos 2(\phi_i)} - \sum_{i=1}^{w} \left(\frac{R_{ref}}{\rho_i + \Delta\rho}\right)^8 \frac{\cos 10(\phi_i + \Delta\phi)}{\cos 2(\phi_i + \Delta\phi)}. \quad (11)$$

The solutions, found, for example, graphically (11), to $\Delta\rho$ and $\Delta\varphi$, define the new coordinates for the position of longitudinal sections of the current winding's turns. The new coordinates for the longitudinal sections of turns are again introduced to the simplified model of a quadrupole electromagnet, and a new numerical calculation of magnetic flux density is performed followed by the computation of coefficients in the central cross-section. Thus, the cycle of the described procedure is repeated several times (typically less than five) in order to reduce coefficients $b_n(z=0)$ of magnetic flux density that have $n=6$ and $n=10$. The need to repeat the procedure is explained by the nonlinear dependence of the iron yoke's contribution to the magnetic flux density coefficients on the position of the longitudinal sections of the winding.

In this case, one could theoretically obtain the arbitrarily small values for the decreased magnetic flux density coefficients. However, it should be borne in mind that the values for $\Delta\varphi$ and $\Delta\rho$, calculated based on (11), cannot be arbitrarily small in reality. Anyway, they may not be less than ε, a technological error in the fabrication and laying of the current winding's turns. Thus, one could assign, over a cycle of application of the described procedure, a natural condition to terminate the calculation (leaving the cycle).

Thus, the first stage of the method ends with the calculated parameters for the simplified model of a quadrupole electromagnet with the minimized coefficients of magnetic flux density $b_6(0)$ и $b_{10}(0)$) in the central cross-section.

The second stage of the method implies accounting for the effect of end elements of the current winding on the magnetic characteristics of a quadrupole electromagnet.

The simplified model, constructed at the first stage, is complemented with the interturn transitions and end elements of the winding whose design parameters are selected based on the practice of constructing such windings, taking into account the feasibility of manufacturing technology.

To calculate values for the medium-integral coefficients of magnetic flux density, one carries out first numerical calculation of magnetic field in the aperture both inside an electromagnet and outside it at a distance up to $L_{eff}/4$ from the ends of the winding. Next, the magnetic flux density, resulting from numerical calculation, should be processed mathematically using a Fourier analysis similarly to (10) in order to derive functional dependences (Fig. 5) of values for coefficients $b_n(z)$ on the applicate. To this end, one should apply the values, derived via numerical calculation, for radial projection of magnetic flux density on the circles of radii R_{ref} in all cross-sections, to be selected, starting with the central, with a longitudinal step of the order of 1 mm.

Based on the derived functional dependences (Fig. 6) for values $b_n(z)$, in accordance with (2), one calculates the medium-integral coefficients of magnetic flux density, and obtains the first calculated value for L_{eff} using (4).

The values for the medium-integral coefficients of magnetic flux density, obtained as a result of the first calculation, typically differ greatly from values in the central cross-section. This is due primarily to the edge effects in the representation of a non- plane-parallel magnetic field based on series (1). The given typical dependences clearly show significant changes in the magnetic flux density coefficients $b_n(z)$ at the edges of the electromagnet (half the length of the yoke in Fig. 5, 6 corresponds to z=415 mm).

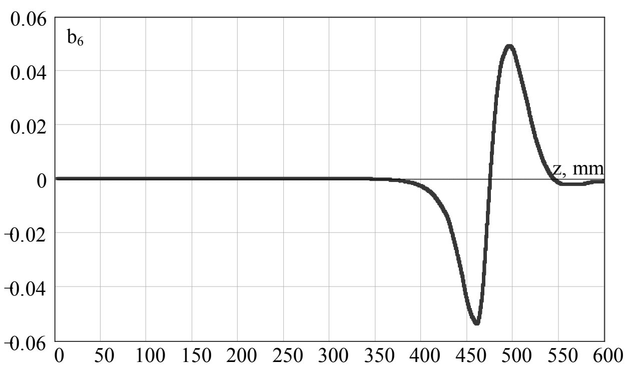

Fig. 5. A typical dependence of coefficient b_6 on distance of the transversal section to the center of the quadrupole electromagnet at L_{eff}=1 m

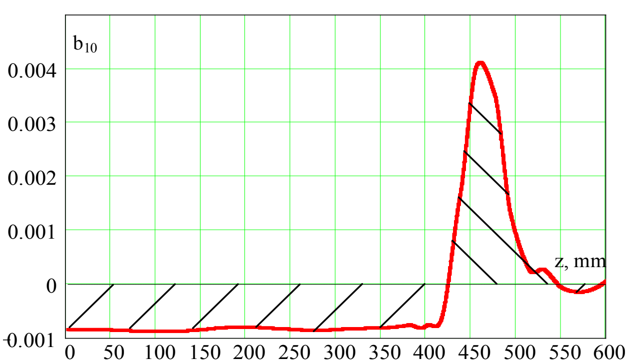

Fig. 6. Dependence (red line) of coefficient b_{10} on distance of the cross-section to the center of a quadrupole electromagnet at L_{eff}=1 m

Thus, the longitudinal dependence $b_6(z)$ of a magnetic flux density coefficient is characterized by the presence of roughly the same minimum and maximum. It is obvious that changing the geometry of an electromagnet's design could be used to achieve a longitudinal dependence $b_6(z)$ such as to minimize the medium-integral coefficient b_6^*, the longitudinal integral according to (2). To this end, let us use the empirically established dependence of the medium-integral coefficient b_6^* on the technological gap s between the yoke and the winding's end elements:

$$b_6^* \approx ks + p. \qquad (12)$$

Empirical coefficients k and p from (12) could be found using the calculation of magnetic characteristics for two identical variants of design of the electromagnet with slightly different (about 5 mm) technological gaps. The technological gap, found at a zero right part in (12), would then ensure the minimum value for medium-integral coefficient b_6^*.

The practice of calculation showed the independence of remaining medium-integral coefficients when changing a technological gap. However, a change in the distance between the yoke and the winding's end elements results in a change in the effective length of electromagnet L_{eff}. To correct it, the minimization of medium-integral coefficient b_6^* should be followed by the same change in the longitudinal sizes of the yoke and winding, leaving the found technological gap unchanged.

The longitudinal dependence of coefficient $b_{10}(z)$ has a feature in the form of a single maximum at the end of the electromagnet (Fig. 6). That is why one of the variants to minimize the medium-integral coefficient b_{10}^* is to change the values for coefficient $b_{10}(z)$ in the central part of an electromagnet so that the differently shaded areas (Fig. 6) are equal. That could be achieved by changing a position of the longitudinal sections of the winding's turns by finding $\Delta\rho$ and $\Delta\varphi$ based on expressions:

$$0 = \sum_w \frac{L_w}{\rho_{0w}}\left(\frac{R_{ref}}{\rho_{0w}}\right)^5 \cos 6(\phi_{0w}) - -\sum_w \frac{L_w}{\rho_{0w}+\Delta\rho}\left(\frac{R_{ref}}{\rho_{0w}+\Delta\rho}\right)^5 \cos 6(\phi_{0w}+\Delta\phi),$$

$$b_{10}^* = \frac{\sum_w \frac{L_w}{\rho_{0w}}\left(\frac{R_{ref}}{\rho_{0w}}\right)^9 \cos 10(\phi_{0w}) - -\sum_w \frac{L_w}{\rho_{0w}+\Delta\rho}\left(\frac{R_{ref}}{\rho_{0w}+\Delta\rho}\right)^9 \cos 10(\phi_{0w}+\Delta\phi)}{\sum_w \frac{L_w R_{ref}}{(\rho_{0w})^2}\cos 2(\phi_{0w})}, \qquad (13)$$

where L_w is the length of the longitudinal sections of the w-th turn of the winding; ρ_{0w}, φ_{0w} are the cylindrical coordinates of starting position of the longitudinal sections of the w-th turn of the winding.

The first of the equations in (13) is a requirement for the invariability of b_6^* when searching for a new position of the longitudinal elements, which minimizes the initial value of b_{10}^* in the second equality.

6. Results of studying the structure of the magnetic field of a quadrupole electromagnet

The method described above can be represented in the simplified form in the form of a procedure for searching for the design parameters of a quadrupole electromagnet, which would ensure the preset minimized values for the non-quadrupole magnetic flux density coefficients in the aperture. The algorithmic scheme for such a procedure is shown in Fig. 7.

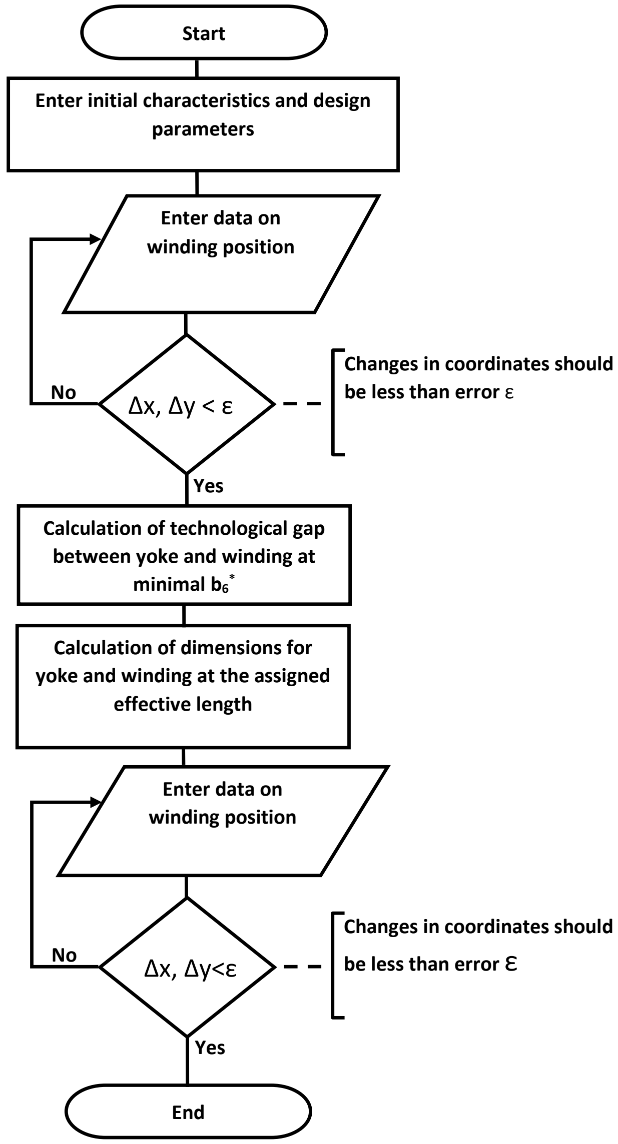

Fig. 7. Scheme of the algorithm to search for design parameters and to calculate the magnetic characteristics of a quadrupole electromagnet

The first stage of the proposed technique ends by leaving the upper cycle at a decrease in the magnitude of correction of the turns' position in the central cross-section to a value smaller than the error of laying. Changes in the dimensions of the yoke and winding at the magnetic flux density coefficients, minimized in the center, do not lead to a change in their medium-integral values.

Verification of the proposed method was performed when searching for three design variants of the quadrupole electromagnet with effective lengths of 0.85 m, 1.0 m, 1.65 m, respectively, at a rated current value of 8.25 kA in a superconducting winding. The result of application of the described approach is the derived geometrical parameters of the quadrupole triplet, given in Table 1. Parameters for the electromagnets were searched for based on the criterion for a minimum of values (less than 10^{-4}) of the non-quadrupole coefficients of magnetic flux density, medium-integral in length, on a reference radius of 60 mm inside the aperture.

Table 1

Values for characteristics of the triplet quadrupole electromagnets at a supply current of 8.25 kA

Yoke length	mm	779	929	1579
Maximal length of winding	mm	880.0	1,030.0	1,680.0
Effective length, L_{eff}	mm	850.2	1,000.3	1,650.0
b_6^*		$-0.27\cdot10^{-4}$	$-0.27\cdot10^{-4}$	$-0.17\cdot10^{-4}$
b_{10}^*		$-0.25\cdot10^{-4}$	$-0.32\cdot10^{-4}$	$-0.54\cdot10^{-4}$
b_{14}^*		$-0.7\cdot10^{-4}$	$-0.7\cdot10^{-4}$	$-0.7\cdot10^{-4}$

7. Discussion of results of the optimization of the structure of magnetic field of a quadrupole electromagnet

The proposed technique, in contrast to the known ones based on a 2D model of magnetic flux density in the central cross-section, makes it possible to directly adjust values for the magnetic flux density coefficients, medium-integral in length, which are required to calculate the dynamics of a beam of particles in the accelerator. In this case, the necessary changes for the non-quadrupole medium-integral coefficients could be introduced in the form of a slight correction to the superconducting winding design when building an electromagnet during testing.

It follows from a comparison of values for the medium-integral coefficient b_6^*, given in Table 1, that it is almost constant at insignificant changes in the overall length of an electromagnet. However, at a significant (twofold) change in the length of yoke b_6^*, the changes become larger than a calculation error. Moreover, it turned out that these changes also depend on the magnitude of current in the winding. The changes are minimal at a maximum (11 kA) current in the winding and increase at its decrease to 3×10^{-4}. This suggests that the reason is the effect of magnetization of the central part of the iron yoke by the winding's end elements.

In contrast to b_6^*, it was established for the medium-integral coefficients of magnetic flux density b_n^* at $n>6$ that a change in the value of current supplied to winding does not lead to a marked deterioration in these quality criteria of a magnetic field inside the aperture of quadrupole magnets of the triplet.

The described technique was applied as a theoretical framework when optimizing design of the triplet of quadrupole magnets for the final focus of the collider NICA.

8. Conclusions

1. A technique for reducing the non-quadrupole medium-integral coefficients of magnetic flux density in the aperture of a quadrupole electromagnet has been proposed. Underlying the technique are the analytical expressions for the calculation of medium-integral coefficients based on the position of longitudinal sections of turns in a superconducting winding. That makes it possible, by performing a small correction (the order of 1 mm) of the winding's size, to minimize an error in the generation of the specified spatial structure of magnetic field.

2. An analysis of results of numerical calculations of the magnetic field for various variants of dimensions of

the yoke and the superconducting winding of a quadrupole electromagnet has established the dependence of the medium-integral coefficients of magnetic flux density in the aperture on dimensions of the structure. The derived analytical expressions simplify the search for geometrical parameters of electromagnet design with improved magnetic characteristics.

3. Parameters for the triplet of quadrupole magnets with the aperture of radius R_a=90 mm at a current of 8.25 kA were calculated. Using the proposed technique has enabled the optimization of design of quadrupole magnets. For the non-quadrupole medium-integral coefficients of magnetic flux density, the values not exceeding $0.3\cdot10^{-4}$ (at n=6) and not larger than $0.6\cdot10^{-4}$ (at n=10) were derived.

Referenses

1. Russenschuck S., Tortschanoff T. Mathematical optimization of superconducting accelerator magnets // IEEE Transactions on Magnetics. 1994. Vol. 30, Issue 5. P. 3419–3422. doi: https://doi.org/10.1109/20.312673
2. Russenschuck S. Differential Geometry Applied to Coil-End Design // Field Computation for Accelerator Magnets: Analytical and Numerical Methods for Electromagnetic Design and Optimization. Wiley, 2010. P. 609–636. doi: https://doi.org/10.1002/9783527635467.ch19
3. Design Challenges for a Wide-Aperture Insertion Quadrupole Magnet / Russenschuck S., Auchmann B., Perez J. C., Ramos D., Fessia P., Karppinen M. et. al. // IEEE Transactions on Applied Superconductivity. 2011. Vol. 21, Issue 3. P. 1674–1678. doi: https://doi.org/10.1109/tasc.2011.2105453
4. Fischer E., Khodzhibagiyan H. G., Kovalenko A. D. Full Size Model Magnets for the FAIR SIS100 Synchrotron // IEEE Transactions on Applied Superconductivity. 2008. Vol. 18, Issue 2. P. 260–263. doi: https://doi.org/10.1109/tasc.2008.922261
5. Full size prototype magnets for heavy ion superconducting synchrotron SIS100 at GSI: status of manufacturing and test at JINR / Kovalenko A., Agapov N., Alfeev A., Bychkov A., Gromov A., Khodzhibagiyan H. et. al. // EPAC'08. Genoa, 2008. P. 2443–2445.
6. Numerical Analysis of the Operation Parameters of Fast Cycling Superconducting Magnets / Fischer E., Schnizer P., Kurnyshov R., Schnizer B., Shcherbakov P. // IEEE Transactions on Applied Superconductivity. 2009. Vol. 19, Issue 3. P. 1266–1269. doi: https://doi.org/10.1109/tasc.2009.2018746
7. Magnetic Field Measurement for Synchrotron Dipole Magnets of Heavy-Ion Therapy Facility in Lanzhou / Yang W., Zhang X., Han S., Yang J., Pei C., Yang L. et. al. // IEEE Transactions on Applied Superconductivity. 2014. Vol. 24, Issue 3. P. 1–4. doi: https://doi.org/10.1109/tasc.2013.2289953
8. Magnetic Field Design of the Dipole for Super-FRS at FAIR / Yao Q. G., Ma L. Z., Zhang X. Q., He Y., Wu W., Moritz G. et. al. // IEEE Transactions on Applied Superconductivity. 2010. Vol. 20, Issue 3. P. 172–175. doi: https://doi.org/10.1109/tasc.2009.2038890
9. Magnetic Field Analysis for Superferric Accelerator Magnets Using Elliptic Multipoles and Its Advantages / Schnizer P., Schnizer B., Akishin P., Fischer E. // IEEE Transactions on Applied Superconductivity. 2008. Vol. 18, Issue 2. P. 1605–1608. doi: https://doi.org/10.1109/tasc.2008.920636
10. Getman A. Cylindrical harmonic analysis of the magnetic field in the aperture of the superconducting winding of an electromagnet // Eastern-European Journal of Enterprise Technologies. 2018. Vol. 1, Issue 5. P. 4–9. doi: https://doi.org/10.15587/1729-4061.2018.123607
11. Vanderlinde J. Classical electromagnetic theory. Springer, 2004. 420 p. doi: https://doi.org/10.1007/1-4020-2700-1
12. Wolff S. Superconducting accelerator magnet design // AIP Conference Proceedings. 1992. Vol. 249, Issue 2. doi: https://doi.org/10.1063/1.41989