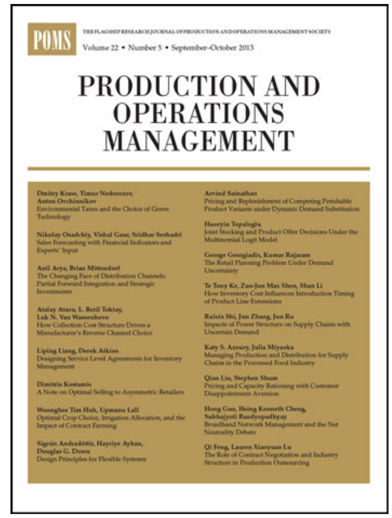
Accepted Manuscript



Title: Coordinating Lot Sizing Decisions Under Bilateral Information Asymmetry

Authors: Dimitris Zissis, George Ioannou, Apostolos Burnetas

DOI: https://doi.org/doi:10.1111/poms.13106

Reference: POMS 13106

To appear in: Production and Operations Management

Please cite this article as: Zissis Dimitris.,et.al., Coordinating Lot Sizing Decisions Under Bilateral Information Asymmetry. *Production and Operations Management* (2019), https://doi.org/doi:10.1111/poms.13106

Coordinating Lot Sizing Decisions Under Bilateral Information Asymmetry

Dimitris Zissis*
Norwich Business School, University of East Anglia, UK, D.zisis@uea.ac.uk
Department of Management Science & Technology, Athens University of Economics & Business, Greece, dzisis@aueb.gr

George Ioannou
Department of Management Science & Technology, Athens University of Economics & Business, Greece, ioannou@aueb.gr

Apostolos Burnetas
Department of Mathematics, University of Athens, Greece, aburnetas@math.uoa.gr

Abstract: We consider inventory management decisions when manufacturing and warehousing are controlled by independent entities. The latter possess private information that affects their choices and are allowed to communicate via a mediator who attempts to streamline their decisions without restricting their freedom. The mediator designs a mechanism based on quantity discounts to minimize the overall system costs, attempting to reach a win-win situation for both entities. Using the Revelation Principle we show that it is in the entities' self-interest to reveal their information and we prove that coordination is attainable even under bilateral information asymmetry. The acceptable cost allocation is not unique, providing adequate flexibility to the mediator during mechanism design; the flexibility may reflect the relative power of the entities and is quantified in our work by a series of computational experiments. Our approach is motivated by inventory management practices in a manufacturing group and, thus, it is directly applicable to real-life cases.

Keywords: voluntary participation; adverse selection; mechanism design; type-dependent reservation levels; communication; mediator

History: Received: September 2018; accepted: August 2019 by Prof. Panos Kouvelis after one revision.

doi: 10.1111/poms.13106

1 Introduction

One of the most important and active topics in the Operations Management literature is how system's entities interact to increase both their own profits as well as the overall system gains (Krajewski et al., 2016). Examples of such system entities can be business units within a large enterprise, firms or companies at the intra-enterprise level, and the multiple nodes of supply chains. However, all individual entities prefer to maximize solely their own profits, without taking into account the global optimum, contributing to an increase of the overall system costs and leading to non Pareto-optimal solutions (Cachon and Terwiesch, 2012). This is because competition drives decisions and every entity has different preferences, objectives, and information. Hence, decentralized settings are sought, since they are the only ones that can be implemented in systems with independent decision makers without restricting their freedom. Among such settings, the most preferred ones are those in which the payoffs of the participating entities are aligned with the system-wide objectives (Chen et al., 2001). Even though this is the preferable case, research approaches that simultaneously optimize both the entities' individual goals and the system efficiency under realistic assumptions are scarce (Cachon and Terwiesch, 2012). This is the central research question of our study, i.e., to examine if a system and its participating entities' objectives can be aligned under a decentralized decision making setting which can be applicable in real-life cases.

From the application viewpoint the paper is motivated by one of the largest cable producers in Europe and the issues related to production and inventory management across its business units. The group comprises four main subsidiaries and two affiliate companies with customers in Africa, Asia, Europe and North America; it operates six production plants while products and materials can be stored in seven different warehouses across Europe. The actual manufacturing cost of a product depends upon the production plant where it is made, while the holding cost is different at each warehouse. Across this group every subsidiary is an independent entity. This means that subsidiaries: i) possess private information that they are not willing to share without receiving incentives; and ii) decide on their strategies to maximize solely their profits. The subsidiaries have to deliver the orders/demand that the group receives by interacting with each other.

Decentralization of decisions is an inevitable part of managing large organizations (Lee and Whang, 1999); separate subsidiaries have different objectives and the importance of alignment for effective organizational performance is well recognized (Sabherwal et al., 2001). To coordinate an entire system and then share the total profits proportionally to the contributions of the subsidiaries is not an easy task, since subsidiaries have strong roots of independence and significant decision making power resides within their individual managerial teams. There are intra-organizational conflicts among those who make managerial decisions and those affected by them (Amit and Schoemaker, 1993). Of course, the Board of Directors (BoD) can exercise sufficient power to the subsidiaries and coordinate the group directions based on dictatorial collaboration (Drake and Schlachter, 2008), making tough decisions and coming into conflict with some of them. However, attempting to impose a course of action is not considered a wise policy; leaders should allow the path forward to reveal itself (Snowden and Boone, 2007). In the case of organizations/systems with separate corporate entities such as supply chains with multiple nodes, coordination is a utopian situation, as no single entity has the power to enforce a strategy that is optimal for the entire channel. Nevertheless, coordination is recognized in the literature as a desirable goal or best practice and significant research has been performed to show that channel coordination leads to the global optimal solution (Chopra and Meindl, 2016).

In this work we consider a decentralized decision making setting with two rational entities, interacting via a single product. We focus on designing a mechanism that allows the entities to coordinate their strategies. The proposed mechanism minimizes the channel costs and determines an acceptable cost allocation between the entities, based on their voluntary participation. To link the model to our industrial case the product can be power cables, manufactured in different production plants, each incurring a different cost and having limited capacity, and stored at different warehouses with different holding costs. The one entity is responsible for the cable manufacturing and can use any of the production plants based on the existing workload, while the second entity is responsible for the storage of the cables before the final delivery to the client. Therefore, our work is developed based on the assumption that both entities have discrete private information: the one about the actual production cost and the other about the real inventory holding cost. The entities are not willing to reveal their information a priory and use it as a negotiation tool to achieve a better deal (cost allocation) for themselves.

The entities are provided with the opportunity to communicate any private information they possess exclusively through a credible mediator (the BoD, in the industrial case of cable manufacturing). The role of a mediator is to facilitate entities to coordinate their decisions via communication. Information sharing is a critical factor for achieving coordination and reducing channel and individual costs. Communication takes place before the entities decide on their actions, without any restrictions (including misinformation and/or deception). This means that the entities are free to report whatever they want in order to optimize their individual objectives. Furthermore, the proposed communication is in effect for a limited period of time (one round), avoiding lengthy negotiation and inefficiency.

In this context, the contribution of our work is threefold: i) we develop a specific model of bilateral information asymmetry; ii) we introduce the notion of a mediator as a means of coordination, by designing a mechanism that facilitates communication between independent decision makers; iii) we prove that coordination in this setting is attainable, without the need to enforce centralized policies. Hence, the individual objectives can be aligned with the channel objectives, reducing the costs and eliminating inefficiencies.

The remainder of the paper is organized as follows: Section 2 provides the related literature and identifies research gaps. Section 3 describes the mathematical formulation of the problem and the game-theory perspective of the entities' interaction. In Section 4, we prove that coordination is always attainable when the independent decision makers communicate exclusively through a mediator, showing that the cost allocation between them is not unique. In Section 5, we conduct computational experiments concerning inventory holding cost, production cost and setup cost relationships, offering insights on the effect of the various parameters and providing information about the acceptable cost allocations. We conclude the paper with a general discussion and set future research avenues in Section 6.

2 Literature Review

During the last two decades, several papers have tackled problems associated with coordination among independent decision makers (Kouvelis et al., 2006). The latter could reduce the overall system costs and expect to achieve better individual profits, if they could coordinate their actions (Kanda et al., 2008). The importance of reducing overall costs instead of just tackling individual costs is also underlined both by private companies and academic researchers (Cachon and Terwiesch, 2012). Coordination is considered to be perfect when the total costs in a system with many decision makers is equal to the total costs if all the actions will be made by a single decision maker (Viswanathan and Wang, 2003). The challenge then, is to propose ways of coordination without restricting the entities' freedom, and align the individual and channel objectives.

A thorough literature review has revealed that almost all contributions make restrictive assumptions; for example, requiring contracts or assuming that all the entities have the same information. Li and Wang (2007) provided a comprehensive review of coordination based on the decision structure and the nature of demand. A number of papers have addressed decision making in the case of complete (or symmetric) information. It is well known in the literature that under complete information it is possible for the channel to be coordinated (Corbett et al., 2004). Coordination is often achieved in models where one entity provides appropriate incentives to the other in order to align the objectives of the latter with those of the channel. In such cases the problem can be modeled as a Stackelberg game, in which the leader secures all the gains from coordination for itself, by paying the follower just enough to force him/her to select the optimal decision for the channel (Corbett, 2001). The assumption that all entities have complete information is not always realized in practice, since independent entities tend to keep their cost structures or other internal information private.

In the framework of incomplete (or asymmetric) information, Cachon and Fisher (2000) examined the role of private information in a two-node model, addressing the way that information affects the entities' strategies and the effect of information sharing on the overall channel costs. Corbett and de Groote (2000) considered a two-node model where the retailer has private information on the holding cost, whereas the supplier assumes a continuous distribution on this parameter. Taking the perspective of minimizing the supplier's expected cost, they derived the quantity discount policy, which in general did not coordinate the channel. In the works of Zissis et al. (2015) and Kerkkamp et al. (2017) a discrete distribution of the holding cost was adopted. Furthermore, Cakanyildirim et al. (2012) considered discrete private information for the supplier's production cost. Ozer and Raz (2011) examined how the asymmetry of information affects the entire channel and addressed both the value of information and the competition in a model with one manufacturer and two competitive suppliers. Karabati and Sayin (2008) studied a single-supplier/multiple-buyer model proposing vertical information sharing that leads to better individual gains for all; however, they assumed that all the parties are truthful in information sharing because of their long-term relationship. Other academic researchers have addressed the case in which one entity possesses two-dimensional private information (Sucky, 2006; Pishchulov and Richter, 2016), assuming that both the retailer's ordering and holding costs are uncertain.

In two-node models where only one entity holds private information, the other one may be able to design and impose a mechanism as a screening device to induce the entity with information to reveal it. In such cases the problem can be modeled again as a Stackelberg game, but this time the entity without information should be the leader and design the mechanism. However, the leader-follower priority has a direct impact on the final cost/profit allocation. Generally, the leader pays an information rent to the follower in order to learn his/her private information. In reality who will act first depends on the relative power of the entities. In this case, it is not always possible to achieve perfect coordination, but both entities could reduce their own costs (Cakanyildirim et al., 2012; Zissis et al., 2015; Kerkkamp et al., 2017).

In the case of bilateral information asymmetry, it is not plausible to make any assumptions about priorities. The actual relative power of the entities may affect the cost/profit allocation after coordination is achieved, and, thus is disconnected from the coordination question. The Production and Operations Management literature on bilateral information asymmetry is sparse. However, multi-way information asymmetry is common in the Economics literature (Chatterjee and Samuelson (1987, 1988); Valley et al. (2002); Shneyerov and Wong (2010)). There are several works in Economics that study bilateral trade problems between a seller who owns an indivisible good and a potential buyer. Both entities have private valuations about the good and attempt to achieve as much profit as they can. In the above setting, Chatterjee and Samuelson (1987, 1988); Shneyerov and Wong (2010) addressed a bargaining game between the entities, while Valley et al. (2002) considered a double auction model, focusing on how communication facilitates the entities to achieve higher levels of efficiency. We adopt an Economics based formulation assuming bilateral information asymmetry, an assumption relevant in business environments with several decision makers.

The second stream of literature about coordination is related to contracts, that have long been considered an important tool to reach coordination. Contracts, in principle, bind the decision makers, but quite often are violated or non-fully respected in practice due to dynamic realities or changing conditions of the market. Li and Kouvelis (1999) studied risk sharing contracts in models with deterministic demand and price uncertainty. Corbett et al. (2004) examined three types of contracts in a two-node model and addressed the value of information. Ha and Tong (2008) studied two types of contracts and proved that the contract type affects the value of information sharing. Feng and Lu (2013) examined contracts in the setting of a Stackelberg game, where supplier is the leader and retailer the follower. We refer the reader to Cachon (2003) for a comprehensive review of contracts that achieve coordination in decentralized settings and Choi and Cheng (2011) for a recent contributed volume.

To achieve the alignment of individual and channel objectives in our study, we provide the entities with the opportunity to communicate any private information they possess. We employ a communication game, which can be viewed as a hybrid game between the two basic game categories, the non-cooperative and the cooperative games (Gibbons, 1992), as it combines properties from both (Myerson, 2007). This allows us to consider the overall channel costs (property on cooperative games) as the objective in a setting with independent decision makers that act to optimize their own costs (property on non-cooperative games). We assume that all the possibilities for communication are entirely controlled by a mediator, who: i) is credible; ii) does not coincide with any of the decision makers; and iii) serves a unique purpose of optimizing the channel. We use the Revelation Principle (framework proposed by Gibbard (1973); Myerson (1979, 1982)), as the technical approach that allows the derivation of statements about what rules are feasible in a communication game.

Obviously, opportunities for mutual benefits cannot be found unless the entities share honestly their information (Fiala, 2005). There are studies in which significant cost savings from information sharing have been reported (Moinzadeh, 2002; Karabati and Sayin, 2008). Furthermore, Inderfurth et al. (2013) conducted a laboratory experiment and showed that information sharing reduces the inefficiencies in decentralized settings. Hence, a crucial issue for coordination is to incorporate information sharing into the mechanism. Since each entity is the only one who knows its own information, the mediator should include incentives during the mechanism design phase in order to obtain the real information that the entities possess. These incentives are known as adverse selection (Myerson, 1982) and in our model are expressed via quantity discounts.

We select quantity discounts (transfer payment), since they are widely used in practice (for example, H. J. Heinz Company, according to Altintas et al. 2008), and require no additional information or physical flow between the parties beyond the initial transaction (Burnetas et al., 2007), in contrast to other policies (return policies, back-up agreements or quantity flexibility). Economies of scale are achieved through quantity discounts, yielding higher profits for several or even all the parties, while allowing each of them to make its own decisions (Choi et al., 2005). Weng (1995) examined in detail the use of quantity discounts in two-node models, while Munson and Rosenblatt (2001) explored the potential benefits of using quantity discounts on both ends of three-node models. We refer the reader to Choi et al. (2005) for a critical review of quantity discount models; an earlier but comprehensive review has been also performed by Benton and Park (1996). In the last years there are studies such as Kalkanci et al. (2011) and Davis et al. (2014), which report the results of behavioral experiments about how the managers use quantity discounts in practice.

3 Preliminaries

3.1 Model Description

We consider a model with two independent entities that interact via a single product. One entity can be thought of as a manufacturer denoted by S (referred to as he). The manufacturer is producing items in a lot-for-lot fashion and cannot accommodate finished goods inventory for long time periods due to limited storage capacity at his premises. Hence, completed lots of finished goods are directly forwarded to the other entity, who acts as a distributor (she) and is denoted by R. The distributor determines the order quantity (lot size), denoted by Q (a notation table is provided in the appendix). She places an order to the manufacturer, satisfying market demand D, under the objective to minimize her own cost. The market demand is assumed constant, exogenously defined and known to both entities (Corbett, 2001). As the demand is deterministic, shortages or backorders are not allowed, which is a standard assumption in the literature (Li and Wang, 2007). The entities must interact with each other; no alternatives for external interactions are allowed. They interact exclusively via order quantities, assuming that they are rational and risk neutral.

The manufacturer has a setup and a unit production cost, denoted by K_S and P_S, respectively; his cost is solely a function of the order quantity Q, and it can be expressed as:

$$TC_S(Q) = K_S D/Q + P_S D. \tag{3.1}$$

The distributor who decides on the order quantity, has an ordering and a unit holding cost. The ordering cost is denoted by K_R. The holding cost is assumed to be proportional to the production cost P_S (Bouchery et al., 2017), and is expressed as $P_S H_R$. In the remainder we will refer to H_R as the unit holding cost. Therefore, the distributor's cost function can be expressed as:

$$TC_R(Q) = K_R D/Q + H_R P_S Q/2. \tag{3.2}$$

Obviously, the distributor's cost is a function of her decision, Q. As the distributor is a rational decision maker, she selects the lot size that minimizes her own cost. $TC_R(Q)$ corresponds to an EOQ-type cost, thus the optimal lot size is $Q^R = \sqrt{2K_R D/H_R P_S}$ and her minimum cost is $TC_R(Q^R) = \sqrt{2K_R D H_R P_S}$. This leads to a manufacturer's cost $TC_S(Q^R) = K_S\sqrt{DH_R P_S/2K_R} + P_S D$. If the manufacturer could decide about the order quantity, he would favor large quantities because in this way he would reduce his own cost, since his cost (equation 3.1) is a decreasing function of the order quantity. The overall channel costs, denoted by $C_J(Q)$, are equal to the sum of manufacturer's and distributor's cost:

$$C_J(Q) = (K_R + K_S)D/Q + H_R P_S Q/2 + P_S D. \tag{3.3}$$

We observe that $C_J(Q)$ corresponds to an EOQ-type cost, with setup being the sum of K_R and K_S, and the optimal joint lot size is $Q^J = \sqrt{2(K_R + K_S)D/H_R P_S}$. In our work perfect coordination exists when the distributor decides on the optimal joint lot size Q^J and imposes it to the manufacturer. Obviously, $Q^J > Q^R$ and $C_J(Q^J) < C_J(Q^R)$. The difference $C_J(Q^R) - C_J(Q^J)$ denotes the maximum benefits that coordination may attain. A higher order quantity is also preferable to the manufacturer. However, this is achieved at the expense of increased distributor's cost, rendering the latter negative to such an option. Since the distributor decides on the order size, she should be provided with the appropriate incentives to raise the order level and achieve reduced overall costs as well. In our study we focus on employing quantity discounts for achieving that. In general, the quantity discounts (transfer payment) affect only the cost allocation between the entities and not the overall channel costs (Choi et al., 2005).

3.2 Bilateral Information Asymmetry

The case which we consider is a decentralized two-node system under information asymmetry on both sides. We model the bilateral information asymmetry by assuming that the production cost and the inventory holding cost can take discrete values, as different costs occur at different facilities. The assumption of discreteness is realistic in practical applications, where the cost or the prices could take one of a number of specific values (Lovejoy, 2006).

We consider two alternative choices; a high and a low value. The production cost P_S takes the low value P_d, with probability q and the high value P_u ($P_u > P_d$) with probability $1-q$, while the holding cost H_R takes the low value H_l, with probability p and the high value H_h ($H_h > H_l$) with probability $1-p$. The probability q can be considered as the probability that the cheap production facilities are out of capacity and the manufacturer must use a plant with a higher production cost per unit (P_u). Similarly, p can be considered as the probability that the cheap warehouse option is not available, leading the distributor to use an expensive one. In general q and p can be set based upon the ratio of the capacities of the two alternatives for both entities. The real costs depend on which production and warehousing facility will be used. The limited capacities prohibit the a priori assumption of "lower price selection" (Stevenson, 2015, Ch.5).

The entities are aware of the prior probability distributions of production and holding costs

$P(P_S = P_d) = q = 1 - P(P_S = P_u)$ and $P(H_R = H_l) = p = 1 - P(H_R = H_h)$. Additionally, the manufacturer knows the real value of P_S, while the distributor considers P_S as a discrete random variable with the prior distribution. Similarly, the distributor knows the real value of H_R, while the manufacturer considers it as a discrete random variable. According to the Bayesian formulation (Gibbons, 1992), the manufacturer can be of type-d or type-u and the distributor can be of type-l or type-h. Thus, four combinations of the entities' types arise: l/d, l/u, h/d, and h/u. Since the availabilities of the production and warehousing facilities are independent, the probabilities of cases l/d, l/u, h/d, and h/u are $pq, p(1-q), (1-p)q$, and $(1-p)(1-q)$, respectively.

The cost of each entity is a function of the order quantity (Q) and depends on his/her type, as each entity is aware of his/her own parameter value. Therefore, the costs of both entities are:

$$\text{Manufacturer's Cost} \begin{cases} TC_{S,d}(Q) = K_S D/Q + P_d D & \text{if he is type-d} \\ TC_{S,u}(Q) = K_S D/Q + P_u D & \text{if he is type-u,} \end{cases} \tag{3.4}$$

$$\text{Distributor's Cost} \begin{cases} TC_{R,l}(Q) = K_R D/Q + H_l P_S Q/2 & \text{if she is type-l} \\ TC_{R,h}(Q) = K_R D/Q + H_h P_S Q/2 & \text{if she is type-h.} \end{cases} \tag{3.5}$$

To conclude the discussion on the modelling of information asymmetry and how it affects operations, we make the following remarks. Our assumption on the manufacturer's production cost being private information does not mean that the distributor does not know how much she will pay per unit for the procurement of the actual product. This is determined by the wholesale price (real or transfer), which is known to all in advance and is not affected by the private information. The manufacturer private information is on his production cost, which affects the distributor's holding cost. In other words, the manufacturer sells the product at a fixed wholesale price, which is not under negotiation (for this reason, and since the demand is also known and constant, the actual procurement cost is not included in the cost functions).

The distributor's holding cost may generally be affected by the production mode (plant, process, etc.). Both elements (holding cost and production mode) are private information for the distributor and manufacturer, respectively. To operationalize this information asymmetry, we assume that the manufacturer's production cost is directly related to the production mode, and the distributor's unit holding cost is equal to a holding cost coefficient multiplied by the unit production cost (and not by the wholesale price as is usually assumed in the literature). Based on this setting, we assume that the manufacturer's private information is the actual unit production cost and the distributor's private information is the holding cost coefficient. Each of the above is kept private by the corresponding party and is used as leverage during the negotiations. It is only revealed after a final agreement is reached.

3.3 Communication

We allow the entities to communicate concerning any information they possess through a mediator. In this sense, revelation of information (truthful or not) is part of each entity's strategy. The mediator determines the order quantity and the cost allocation through quantity discounts as incentives to obtain the real information and coordinate the channel. The quantity discounts are given by the manufacturer to the distributor to induce the latter to amend her order decision.

Figure 1 summarizes the structure of our model. First, the mediator designs and announces a plan which describes the potential actions for all the possible combinations of the entities' types. Then, the mediator requests from both entities to participate in the plan by reporting confidentially their information to him. The entities are free to accept the mediator plan and report their

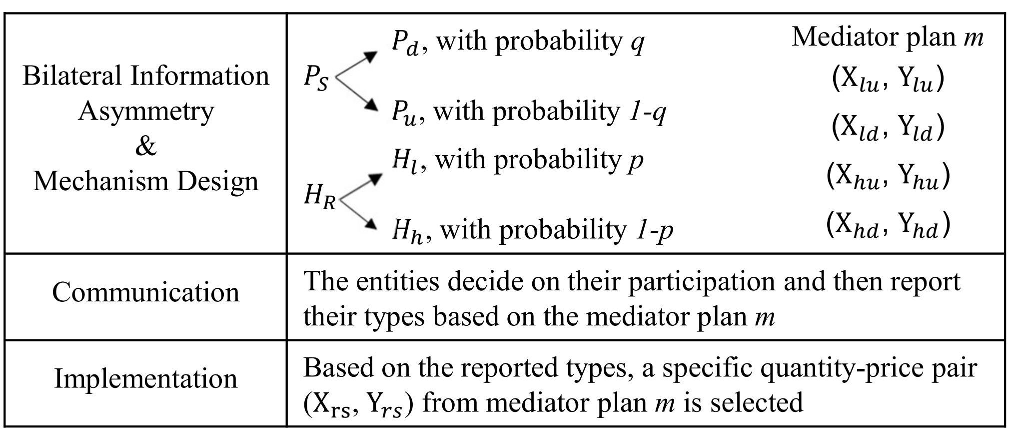


Figure 1: Structure of the model

The mediator cannot compel full participation and truthful behavior by the entities and anticipates that either of them may not participate or may lie to him in an attempt to manipulate the plan. Hence, the mediator should include appropriate incentives in the plan to promote participation and honesty. The incentives that ensure that entities participate are referred to as individual rationality or participation constraints (Myerson, 1979), while the incentives that ensure that the entities reveal their information are referred to as adverse selection (Myerson, 1982). Note that each entity is the only one who knows his/her own true type, and no one can prevent him/her from lying about it, since the entities may expect advantage from such a behavior. Hence, the reported types may not coincide with the real types. This means that misinformation and/or deception is also modeled and it can be considered as a possible choice by the entities.

After receiving the reports from both entities, the preannounced mediator plan specifies actions for them. The plan incorporates any rule that emanates from the entities' reports and enables the specification of actions. The mediator plan is a quantity discount pair (X, Y) that depends on the reported types. The entities could either participate in the mediator plan or refuse it (entities cannot alter the specific quantity-price pair or make a second report), avoiding lengthy negotiations. According to the Revelation Principle, it is sufficient to consider discounts such that the mediator sets one quantity-price pair (X, Y) for each possible combination of the entities' types, in order to distinguish them.

To employ the Revelation Principle, it is necessary to consider the reservation levels for both entities. We define the reservation levels as the costs that the entities will bear if they do not participate in the mediator plan (disagreement outcome). Since the process of ordering and product acquisition must be completed, the reservation level of each entity is equal to the highest cost it may have to pay if it does not accept the plan. Hence, the reservation levels are dependent on the information that the entities possess and are determined based on the model parameters (Cakanyildirim et al., 2012; Zissis et al., 2015). This means that the reservation levels are different for the low and the high value of the entities' information, which is a relevant assumption, as different information leads to different costs and business decisions.

The highest cost that the distributor is willing to bear happens when the production cost is high and the manufacturer does not give any discount to her. Since the distributor is aware of the real holding cost, her cost function under this case is: $K_R D/Q + H_l P_u Q/2$ when she is type-l, and $K_R D/Q + H_h P_u Q/2$ when she is type-h. To minimize her EOQ-type cost (rational decision maker), she orders quantity equal to $Q_l^R = \sqrt{2K_R D/H_l P_u}$, or $Q_h^R = \sqrt{2K_R D/H_h P_u}$, respectively. This results in the following costs, which are defined as the distributor's reservation levels, depending on her type: $C_{R,l}^+ = \sqrt{2K_R D H_l P_u}$, and $C_{R,h}^+ = \sqrt{2K_R D H_h P_u}$, respectively. The manufacturer's reservation level occurs when the distributor's order is equal to Q_h^R (minimum order quantity since the manufacturer's cost is a decreasing function of the order quantity). As the manufacturer is aware of the real production cost, his reservation level is: $C^+_{S,d} = K_S\sqrt{DH_hP_u/2K_R} + P_dD$ when he is type-d, and $C^+_{S,u} = K_S\sqrt{DH_hP_u/2K_R} + P_uD$ when he is type-u.

Given the reservation levels, the mediator designs a plan m as follows:

$$m = \{(X_{lu}, Y_{lu}), (X_{hu}, Y_{hu}), (X_{ld}, Y_{ld}), (X_{hd}, Y_{hd})\}, \tag{3.6}$$

which determines the quantity-price pair for each combination of entities' types, using the prior probability distributions of production and holding costs: $P(P_S = P_d) = q = 1 - P(P_S = P_u)$ and $P(H_R = H_l) = p = 1 - P(H_R = H_h)$, as he is not aware of their real values. The mediator objective is to minimize the expected value of the overall costs $E(C_J(Q))$ which is equal to:

$$E(C_J(Q)) = p(1-q)C_J(X_{lu}) + pqC_J(X_{ld}) + (1-p)(1-q)C_J(X_{hu}) + (1-p)qC_J(X_{hd}). \tag{3.7}$$

Recall that mediator plans are constrained by the adverse selection and the participation constraints.

In summary the entities act towards minimizing their own expected costs; they first decide whether they will participate in the mechanism and then report types that optimize their individual costs. Based on the reported types, a specific quantity and price (discount) pair of the mediator plan m is selected. Finally, orders are delivered under the selected quantity-price pair and every entity bears the corresponding cost based on the real cost elements.

3.4 Cable Case Description

To link the proposed model with the industrial case that motivated this work, we postulate two generic intermediate entities at the cable group. The first entity represents the production plants and is referred to in our model as the manufacturer, while the second entity represents the inventory management team and is referred to as the distributor. The manufacturer is responsible for the cable production and can use any of the group plants based on the capacity, the existing workload, and product requirements. The distributor is responsible for the storage of the cables before the final delivery to the client by selecting the warehousing facility that will be used based on the existing workload and the warehouse availability.

When the cable group receives an order (demand for a specific cable) from a client, the manufacturer and the distributor must agree about the lot size in which they will satisfy the demand and the final cost allocation between themselves. Both entities possess private information; the manufacturer about the production plant that will be used and the actual manufacturing cost, while the distributor about which warehouse will be used and the real holding cost. The entities use their information as a negotiation tool, seeking to achieve a better deal for themselves. Since the entities' objective is to achieve as high individual profits as possible, lengthy negotiations about the lot size and the final cost allocation are necessary in general; such negotiations may bring forward detrimental effects on the intra-organizational relationships, resulting in problematic realization of the production and delivery processes.

The BoD would like to streamline the entities' decisions and ensure the lowest level of costs for the group. It is quite important for the BoD to coordinate the decisions of its independent entities based on free participation and not on dictatorial collaboration, since the entities (business units) have significant power over resources and decisions. Hence, the BoD prefers to use policies without any enforcement. The BoD is aware of all the model parameters, without knowing which facilities will be used, since this information is assumed to be available only to the corresponding entity. Under our proposed model, the BoD is able to design a mediator plan in which it is defined what will be the order quantity and cost allocation (through the quantity discounts) based on all the possible combinations of the entities' information, seeking to optimize the overall channel costs.

The entities are informed about the plan and asked whether they will participate by reporting their information confidentially. If either of them declines, no discounts are implemented. The entities' objective is to optimize their individual costs. After consideration of the mediator plan, both entities announce confidentially their information to the BoD. Then, an order quantity and a discount are selected based on the reported information. Finally, the demand is satisfied and every entity pays the corresponding cost under the selected order quantity and the related discount based on the mediator plan. Note that the realized cost is based on the true cost elements and not on the reported ones, while the order quantity and the discount are based on the reported information.

4 Solution to the Mediator Problem

To reach coordination it is crucial for the entities to participate in the mediator plan and reveal their private information. The Revelation Principle asserts that any equilibrium of a communication game can be reached by an appropriate mechanism (Myerson, 1979). In our work, such a mechanism is a mediator plan, in which the mediator includes participation constraints and adverse selection incentives to induce entities to participate and reveal their information. The Revelation Principle guarantees that it is sufficient to consider only such mechanisms when devising the mediator plan. This restriction is significant, in the sense that this class is much smaller than the set of all feasible mechanisms and in general can be characterized by a finite number of inequalities, when there is a finite number of type combinations (Myerson, 1979).

Therefore, it is sufficient to consider mediator plans consisting of four quantity-price pairs (X, Y), one for each possible combination of entities' types. Both entities report that they are of a certain type, but they are free to report whatever type they desire or to decline the plan. Their criterion is to minimize their own expected cost, based on their prior distribution and conditional on their types (rational and risk neutral entities). Consequently, the entities' costs are functions of the mediator plan, the reported, and the real types. For example, the manufacturer's expected cost under mediator plan m when he reports type-d, given that he is type-u is:

$$\begin{aligned} C_S(m, d|u) &= p(TC_{S,u}(X_{ld}) + Y_{ld}) + (1-p)(TC_{S,u}(X_{hd}) + Y_{hd}) \\ &= p(K_S D/X_{ld} + Y_{ld}) + (1-p)(K_S D/X_{hd} + Y_{hd}) + P_u D, \end{aligned} \tag{4.1}$$

while the distributor's expected cost under mediator plan m when she reports type-h, given that she is type-l is:

$$\begin{aligned} C_R(m, h|l) &= q(TC_{R,l}(X_{hd}) - Y_{hd}) + (1-q)(TC_{R,l}(X_{hu}) - Y_{hu}) \\ &= q(K_R D/X_{hd} + H_l P_d X_{hd}/2 - Y_{hd}) + (1-q)(K_R D/X_{hu} + H_l P_u X_{hu}/2 - Y_{hu}). \end{aligned} \tag{4.2}$$

The other expected costs, i.e., $C_S(m, u|u)$, $C_S(m, d|d)$, $C_S(m, u|d)$, $C_R(m, l|l)$, $C_R(m, h|h)$, and $C_R(m, l|h)$ are similarly defined.

Since the entities could deny the mediator plan, their costs under any plan cannot exceed their reservation levels: $C^+_{R,l}, C^+_{R,h}, C^+_{S,d}$, and $C^+_{S,u}$ (participation constraints). Both entities prefer the solution under the mediator plan m when the following inequalities hold:

$$\begin{aligned} C_R(m, l|l) &\leq C^+_{R,l} \\ C_R(m, h|h) &\leq C^+_{R,h} \\ C_S(m, d|d) &\leq C^+_{S,d} \\ C_S(m, u|u) &\leq C^+_{S,u}. \end{aligned} \tag{4.3}$$

Moreover, to ensure that both entities report honestly their information, because it is in their self-interest to do so, the mediator should include adverse selection incentives in the mediator plan m, expressed as:

$$C_R(m, l|l) \leq C_R(m, h|l)$$

$$C_R(m,h|h) \le C_R(m,l|h)$$
$$C_S(m,d|d) \le C_S(m,u|d) \tag{4.4}$$
$$C_S(m,u|u) \le C_S(m,d|u).$$

According to the Revelation Principle, systems (4.3) and (4.4) of inequalities become constraints when the mediator designs a plan. The mediator objective is to design a plan that minimizes the expected value of the overall channel costs $E\left(C_J(Q)\right)$ (equation 3.7), i.e., he solves an optimization problem with the objective function $E(C_J(Q))$ under constraints (4.3) and (4.4). Hence, the mediator solves the following nonlinear optimization problem:

(M)

$$C_J^* = \min_{\{(X_{rs}\ge 0, Y_{rs}\ge 0), r=l,h,s=d,u\}} E(C_J(Q))$$

$$\begin{array}{llll}
\text{s.t.} & C_R(m,l|l) & \le & C^+_{R,l} \\
& C_R(m,h|h) & \le & C^+_{R,h} \\
& C_S(m,d|d) & \le & C^+_{S,d} \\
& C_S(m,u|u) & \le & C^+_{S,u} \\
& C_R(m,l|l) & \le & C_R(m,h|l) \\
& C_R(m,h|h) & \le & C_R(m,l|h) \\
& C_S(m,d|d) & \le & C_S(m,u|d) \\
& C_S(m,u|u) & \le & C_S(m,d|u).
\end{array}$$

In problem (M) the objective function coincides with the overall channel costs in a centralized model where a single decision maker controls both entities. Therefore, the minimum channel costs under a centralized setting is a lower bound on the optimal solution of the problem (M). If we prove that there exists a feasible solution $\{(X_{rs}, Y_{rs}), r = l, h$ and $s = d, u\}$ to (M) such that X_{rs} are equal to the coordination quantities, i.e., $Q^J_{r,s} = \sqrt{2(K_R + K_S)D/H_r P_s}$, then this solution is optimal for problem (M) and furthermore it allows the mediator to achieve channel coordination. The main result of this paper is that perfect coordination is indeed attainable, as it is stated in the following Theorem.

Theorem 1 *There exists an optimal solution of problem (M), in which*

$$X_{rs} = \sqrt{2(K_R + K_S)D/H_r P_s}, r = l, h \text{ and } s = d, u.$$

The proof of Theorem 1 follows directly from the intermediate properties in Lemma 1, and Proposition 1 below. The proofs are provided in the appendix. The main steps are outlined as follows: First, by setting $X_{rs} = Q^J_{r,s}$, the constraints of (M) become a system of linear inequalities in Y_{rs}. With appropriate changes of variables we transform this system into an equivalent one, with two variables only. In Proposition 1, first we establish a necessary and sufficient condition so that the last system is feasible and show that this condition is always true. In the remainder of the section we present the steps of the proof in detail.

Setting $X_{rs} = Q^J_{r,s}$, and after some simplifications the constraints of (M) are expressed as:

$$\begin{aligned}
qY_{ld} + (1-q)Y_{lu} \geq & G\sqrt{R_H}\left((1+F)\zeta - 2\sqrt{F}\right)\\
qY_{hd} + (1-q)Y_{hu} \geq & G\left((1+F)\zeta - 2\sqrt{F}\right)\\
pY_{ld} + (1-p)Y_{hd} \leq & G\frac{1-F}{\sqrt{F}}(1-\sqrt{R_P}\sqrt{F}\theta)\\
pY_{lu} + (1-p)Y_{hu} \leq & G\frac{1-F}{\sqrt{F}}(1-\sqrt{F}\theta)\\
(qY_{hd} + (1-q)Y_{hu}) - (qY_{ld} + (1-q)Y_{lu}) \leq & G\zeta(1-\sqrt{R_H})(F-\sqrt{R_H})\\
(qY_{hd} + (1-q)Y_{hu}) - (qY_{ld} + (1-q)Y_{lu}) \geq & G\zeta(1-\sqrt{R_H})(F-1/\sqrt{R_H})\\
(pY_{lu} + (1-p)Y_{hu}) - (pY_{ld} + (1-p)Y_{hd}) \geq & G\theta Z\\
(pY_{lu} + (1-p)Y_{hu}) - (pY_{ld} + (1-p)Y_{hd}) \leq & G\theta Z,
\end{aligned} \tag{4.5}$$

where: $G = \sqrt{DP_u H_h(K_R + K_S)}/\sqrt{2}$, $F = K_R/(K_R + K_S)$, $R_H = H_l/H_h$, $R_P = P_d/P_u$, $\zeta = q\sqrt{R_P} + 1 - q$, $\theta = p\sqrt{R_H} + 1 - p$, and $Z = (1-F)(\sqrt{R_P} - 1)$.

Under this reparametrization, it is true that F, R_H, R_P, ζ, and θ take values in the range $(0,1)$, while $Z < 0$. System (4.5) can be rewritten using the following variable transformations:

$$\begin{aligned}
y_l &= qY_{ld} + (1-q)Y_{lu}\\
y_h &= qY_{hd} + (1-q)Y_{hu}\\
y_d &= pY_{ld} + (1-p)Y_{hd}\\
y_u &= pY_{lu} + (1-p)Y_{hu}.
\end{aligned} \tag{4.6}$$

Variables y_l and y_h denote the expected discount to a distributor of type-l or type-h, respectively. Similarly y_d and y_u represent the expected discount paid to the distributor by the manufacturer of type-d or type-u, respectively. However, finding $y_l, y_h, y_d, y_u \geq 0$ that satisfy (4.6) does not necessarily mean that there exist feasible discounts in problem (4.5). Lemma 1 shows that this is true if and only if the new variables satisfy a linear relationship.

Lemma 1 *For every non negative numbers y_l, y_h, y_u, y_d there exist $Y_{rs} \geq 0, r = l, h$ and $s = d, u$ satisfying* (4.6) *if and only if*

$$py_l + (1-p)y_h = qy_d + (1-q)y_u. \tag{4.7}$$

Note that if equation (4.7) holds, then system (4.6) admits an infinite number of solutions Y_{rs} ($r = l, h$ and $s = d, u$) since the four linear equations are dependent, i.e., there are infinite choices of discounts for every set of values of y_l, y_h, y_u, y_d. This provides flexibility to the mediator when he designs the plan and can propose a range for each of the four discounts Y_{rs}.

Based on Lemma 1, coordination is attainable if and only if there exist non negative numbers y_l, y_h, y_u, y_d that satisfy the equivalent system of constraints (4.5) and equation (4.7). From the last two inequalities of (4.5), we have that $y_u = G\theta Z + y_d$. Substituting into equation (4.7) we obtain:

$$\begin{aligned} y_d &= py_l + (1-p)y_h - (1-q)G\theta Z \\ y_u &= py_l + (1-p)y_h + qG\theta Z. \end{aligned} \tag{4.8}$$

Therefore y_d and y_u are uniquely determined by y_l and y_h, which reduces the numbers of variables by two. Thus, it suffices to substitute y_d, y_u in system (4.5) from the system of equations (4.8), and seek $y_l, y_h \geq 0$, that satisfy system (4.5) and also result in $y_d, y_u \geq 0$. By doing this we obtain the following necessary and sufficient inequalities:

$$\begin{aligned} y_l &\geq max\{0, G\sqrt{R_H}\left((1+F)\zeta - 2\sqrt{F}\right)\} \\ y_h &\geq max\{0, G\left((1+F)\zeta - 2\sqrt{F}\right)\} \\ \zeta(1-\sqrt{R_H})(F - \frac{1}{\sqrt{R_H}})G &\leqslant y_h - y_l \leqslant \zeta(1-\sqrt{R_H})(F-\sqrt{R_H})G \end{aligned} \tag{4.9}$$

$$max\{0, -qG\theta Z\} \leqslant py_l + (1-p)y_h \leqslant min\{G\frac{1-F}{\sqrt{F}}\left(1-\sqrt{R_P F\theta}\right) + (1-q)G\theta Z, G\frac{1-F}{\sqrt{F}}\left(1-\sqrt{F}\theta\right) - qG\theta Z\}.$$

In the system of constraints (4.9) we have that: $-qG\theta Z > 0$ and the two terms inside the minimum are equal, which results in further simplification. In summary, to find a feasible solution to problem (M), it is necessary and sufficient to find non negative values of y_l and y_h such that:

$$\begin{aligned} y_l &\geqslant a^+ G\sqrt{R_H} \\ y_h &\geqslant a^+ G \\ d_1 G \leqslant y_h &- y_l \leqslant d_2 G \\ -qG\theta Z \leqslant p y_l &+ (1-p) y_h \leqslant eG, \end{aligned} \tag{4.10}$$

where: $a = (1+F)\zeta - 2\sqrt{F}$, $a^+ = max\{a, 0\}$, $e = \frac{1-F}{\sqrt{F}}(1-\sqrt{F}\theta) - q\theta Z = \frac{1-F}{\sqrt{F}}(1-\sqrt{F}\theta\zeta)$, $d_1 = \zeta(1-\sqrt{R_H})(F - 1/\sqrt{R_H})$, and $d_2 = \zeta(1-\sqrt{R_H})(F-\sqrt{R_H})$.

We have finally reduced the problem of finding a mediator plan to a system of linear inequalities in y_l and y_h. In Proposition 1 we show that this system is always feasible.

Proposition 1 *i) A necessary and sufficient condition for the system of constraints* (4.10) *to have a solution is that*

$$a^+ - p\min\{d_2, a^+(1-\sqrt{R_H})\} \leqslant e. \tag{4.11}$$

ii) Condition (4.11) *is always true.*

Based on Theorem 1, the mediator can always design an appropriate plan to coordinate entities' decisions and achieve the minimum overall channel costs. This means that there exists a feasible plan in which the individual objectives are aligned with the incentives of the entire channel, resulting to reduced individual costs. Therefore, there exist (nonnegative) discounts Y_{rs}, $r = l, h$ and $s = d, u$, which should be given by the manufacturer to the distributor to induce the latter to order the optimal joint lot size because it is in both entities self-interest. The difference between the overall channel costs under the mechanism and the uncoordinated case, i.e.,

$$C_J(Q^R) - C_J(Q^J) = \{2K_R + K_S - 2\sqrt{K_R(K_R+K_S)}\}\sqrt{DH_RP_S}/\sqrt{2K_R}, \tag{4.12}$$

is equal to the cost savings that can be achieved by channel coordination. These savings represent the coordination benefits that will be shared between the entities, since we assume that the mediator does not increase the cost.

A key finding which merits further attention is the infinite values of discounts that coordinate the channel (Lemma 1). This is related to the mediator flexibility during the mechanism design phase. The mediator can take into account secondary objectives. To avoid questions of relative power of the entities and how the coordination benefits are allocated between them, we focus on the extreme cases. Hence, in Section 5 we examine the cases when the manufacturer optimizes his benefits under the coordination (minimum values of discounts) and when the distributor optimizes her benefits under the coordination (maximum values of discounts). By studying the extreme cases, first we cover all the potential range of the acceptable cost allocations between the entities (as all the intermediate cases are possible according to different relative power of them), and then obtain insights on the minimum benefits that each entity can secure under the channel coordination.

5 Computational Experiments

The preceding analysis leads to several interesting questions regarding to the discounts that coordinate the channel. In this section, we conduct computational experiments which offer insights about the benefits and the sensitivity of the mediator plans that coordinate the entities' decisions, with respect to various model parameters. The computational experiments also assess the flexibility that the mediator has during the mechanism design phase.

Recall that the model involves the following nine independent parameters D, K_R, K_S, H_l, H_h, P_d, P_u, p, q, while the main finding is the channel coordination under bilateral information asymmetry via a mediator plan $m = \{(X_{rs}, Y_{rs}), r = l, h \text{ and } s = d, u\}$ (Theorem 1). According to Lemma 1, the corresponding discounts are not unique. The existence of multiple feasible solutions is a beneficial feature, since it provides the mediator with the adequate flexibility to take into account secondary objectives during mechanism design. Furthermore, multiple solutions allow the examination of different relative powers between the independent entities, which is important to a wide range of real word situations.

The discounts represent net payments from the manufacturer to the distributor; therefore, the manufacturer prefers as small discounts as possible and the distributor the opposite. We consider the difference between the minimum and the maximum values of the discounts as the mediator flexibility in designing a mechanism that ensures minimum overall channel costs. The experiments we perform provide us with insights about the mediator flexibility and how this affects cost allocation between the entities, indicating the feasible range of the acceptable payoffs under coordination. Based on the entities' relative power, each of them may be able to enforce a mediator plan that optimizes its individual costs (as a secondary objective) given the channel coordination (primary objective).

In the experiments we calculate the maximum and the minimum percentage of the overall channel costs that the distributor bears under coordination. The maximum and the minimum distributor's cost is presented, first as a function of the setup and ordering costs (Section 5.1), and then based on the information that the entities keep private (Section 5.2), assuming all other parameters constant in each case. Since in our model there are only two entities, the remaining cost (as a percentage) is paid by the manufacturer. Therefore, depending on the relative power of the entities, the actual plan that will be implemented enforces a cost allocation between the two extreme cases. The lower surface corresponds to the extreme case in which the distributor has the greatest possible relative power (i.e., implementation of the maximum values of discounts). Similarly, the higher surface indicates that the manufacturer has the greatest possible power (i.e., implementation of the minimum values of discounts). All the experiments have been performed under a large range of parameter values. Although we only present specific cases, the observations and insights we discuss are quite robust.

5.1 Impact of Setup and Ordering Costs

First we investigate how the setup and ordering costs affect the cost allocation between the entities and the mediator flexibility. In Figure 2 the x-axis corresponds to the ratio of the setup and ordering costs K_S/K_R, while the y-axis corresponds to the percentage of the overall channel costs that the distributor bears under coordination through the mediator. In the experiments we consider the ratio of K_S/K_R, and the ratio about the high and the low values of the information that both entities possess. Although one can argue that individual values of the costs are important when deriving actual costs, what really defines the direction of any managerial decision is the relative value of these costs. Additionally, we consider the case that there is no prior knowledge about the information that the entities possess; hence, we use a non informative prior about the low and the high value of both holding and production costs, i.e., $p = q = 1/2$.

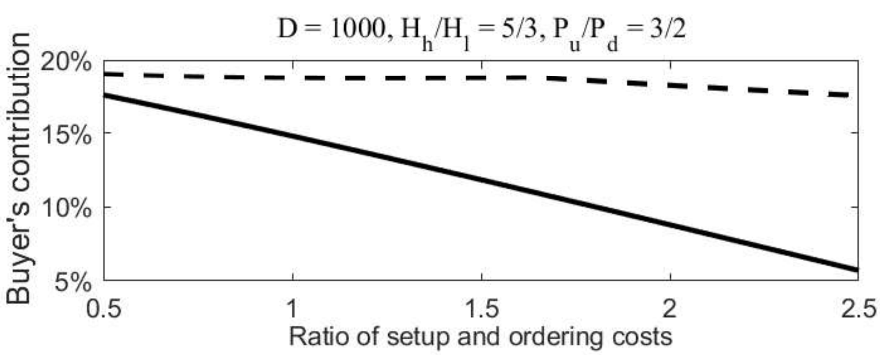


Figure 2: Range of distributor's cost percentage as a function of the ratio K_S/K_R.

In Figure 2 we observe that when the manufacturer can enforce his preferable mediator plan (depicted by the dashed line), he keeps his percentage contribution to the overall channel costs constant as the ratio K_S/K_R increases. In the case that the distributor can enforce her preferable mediator plan (depicted by the continuous line), she reduces her percentage contribution making the manufacturer to get hurt more significantly as the ratio of setup and ordering costs increases. We observe that the mediator flexibility increases for larger values of the ratio K_S/K_R, making the mediator more powerful. A particularly interesting observation that arises from Figure 2 is that as the ratio of setup and ordering costs decreases the mediator flexibility decreases as well. This happens since the actual decision maker about the order quantity is the distributor and, when K_R is significantly higher than K_S, her individual objective is almost aligned with the channel (and the mediator) objective.

5.2 Impact of Information Asymmetry

We investigate the impact of information asymmetry both on the cost allocation and the mediator flexibility. Regarding the information that the entities keep private (i.e., production or holding costs), we consider the ratio of the high and low values (P_u/P_d and H_h/H_l), since it represents a measure of the information asymmetry. Note that a ratio equal to 1 corresponds to the case of complete information on the specific cost, while larger values of the ratio indicate that the asymmetry is intense.

Figure 3 shows how the distributor's degree of information asymmetry (as depicted by the ratio of holding costs and the corresponding prior probability) affects the plan and the mediator flexibility. More specifically, in Figure 3:

- the x-axis corresponds to the distributor's information (i.e., H_h/H_l),
- the y-axis corresponds to the probability of the holding cost to take the low value,
- the z-axis corresponds to the percentage of the overall channel costs that the distributor bears under coordination through the mediator.

Similarly, Figure 4 shows how the cost allocation and mediator flexibility are affected by the manufacturer's private information and the corresponding prior probability. The x-axis corresponds to manufacturer's information (P_u/P_d), the y-axis corresponds the probability of the production cost to take the low value, and the z-axis corresponds to the percentage of the overall channel costs that the distributor bears under coordination. In both experiments, the two surfaces correspond

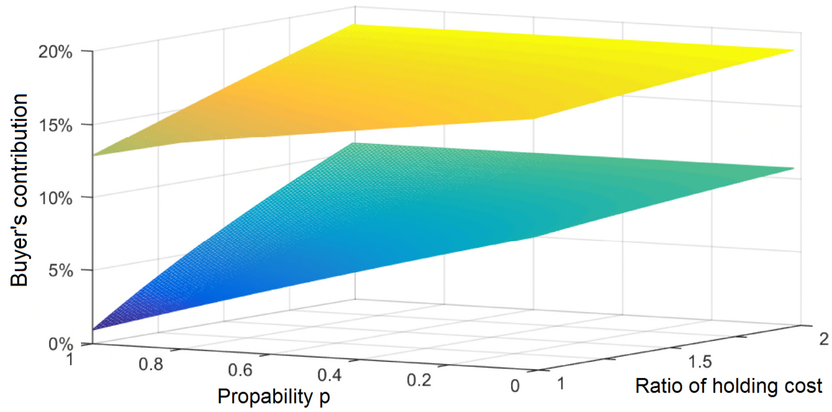


Figure 3: Range of distributor's cost percentage based on her private information.

to the minimum and maximum distributor's percentage contribution to the overall channel costs, under the mediator plan that coordinates the channel.

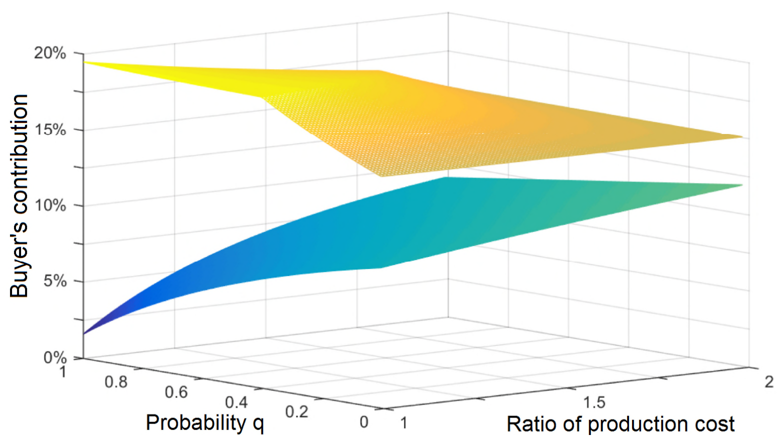


Figure 4: Range of distributor's cost percentage based on manufacturer's private information.

From Figures 3 - 4 as the asymmetry of either party decreases, i.e., the ratio of the holding and production costs take values close to 1, the mediator flexibility decreases. This is reasonable, because the elimination of information asymmetry moves one of the entities to have complete information on the other, restricting the mediator flexibility. For example, in Figure 3 the elimination of information asymmetry means that the manufacturer is able to reduce his contribution to the overall channel costs even when the distributor has the power to enforce her preferable mediator plan. The new insight we obtain from these figures is that as the information asymmetry increases, the significance of the entities' relative power is substantially increased as well.

In the last experiment we study the cost allocation and mediator flexibility as a function of both entities' private information by using a non informative prior about the low and the high value of the production and holding costs ($p = q = 1/2$). The mediator is only aware of the prior probability distributions of production and holding costs. Hence, the mediator is faced with the information asymmetry of H_h/H_l and P_u/P_d, when he designs the mechanism that optimizes the overall channel costs (objective of the mediator). In this experiment the range of both ratios is from 1 to 2 (Figures 5 and 6), which is a realistic range both for the production cost (Cakanyildirim et al., 2012) and the holding cost (Becerril-Arreola et al., 2013). Note that the case of complete information corresponds to both ratios being equal to 1.

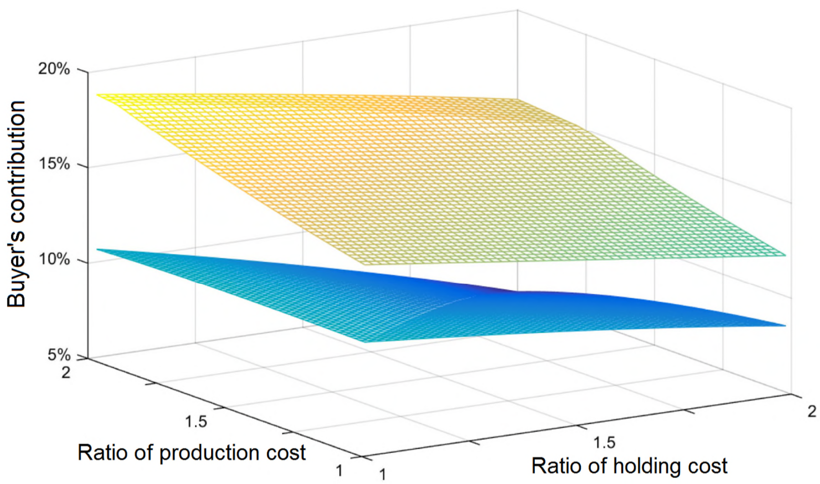


Figure 5: Range of distributor's cost percentage based on entities' information asymmetry.

In Figure 5 the lower surface corresponds to the extreme case in which the mediator optimizes the distributor's gains given the channel coordination, while the higher surface indicates the case in which the mediator provides the majority of the coordination benefits to the manufacturer. A fair solution could be for the mediator to share equally the additional benefits between entities, assuming that both of them have the same relative power (Figure 6).

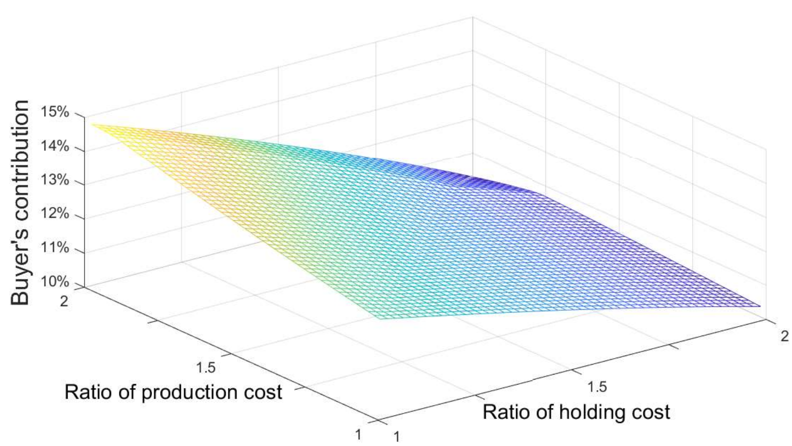


Figure 6: Distributor's cost percentage based on sharing the coordination benefits.

We observe from Figures 5 and 6 that as information asymmetry decreases, the mediator flexibility is reduced. Consistent with to the literature, in the case of complete information we know that the decision maker who has greater power is able to coordinate the channel and absorb all the benefits for itself (Corbett, 2001; Ha, 2001). This indicates that the relative power of the entities becomes crucial for the cost allocation under complete information.

5.3 Application to the Cable Case

In the industrial case of cable manufacturing, we proceed with the following numerical example. There is a customer order of 500 kilometers of power cables, which are manufactured either in Greece or in Romania, while they can be stored in any warehouse (see Figure 7 for the spatial of the locations of the warehousing and manufacturing facilities).

Figure 7: Production plants and warehouse locations.

The production in each country occurs a different cost; with the ratio of P_u/P_d being equal to 4/3. For the warehousing services we consider that there are two alternatives related to the cost with the ratio of H_h/H_l being equal to 3/2. We consider the case in which the capacities (either for the production or for the storage) of the two alternatives are equal; i.e. $p = q = 1/2$. The ratio of the setup and ordering costs (K_S/K_R) is considered to be equal to 3. The BoD is aware of all the model parameters, without knowing which facilities will be used for this specific customer order, since this information is available only to the entities. The BoD seeks to optimize the overall costs without enforcing any policy to the entities. In this example, we consider that both entities have the same relative power; hence, the coordination benefits will be shared equally between them. The optimal order quantities that minimize the overcall costs under complete information are: 115, 100, 94, 82 kilometers of cables based on the four possible combinations of the entities' types (l/d, l/u, h/d, and h/u).

According to our proposed model the BoD designs and announces to the entities the following quantity discounts: {(115, 778), (100, 665), (94, 676), (82, 482)} to coordinate the channel. A pair (X, Y) means that if the lot size is equal to X kilometers, the manufacturer will offer to distributor a discount (transfer payment) equal to Y monetary units. Then, the BoD requests from the entities to participate in the plan by revealing their private information, i.e., which facility will be used for this specific customer order. The entities are free to accept the plan and report whatever optimizes their own costs. We have proved that both entities will accept the plan and reveal their information, since this strategy is in their own self-interest. Based on the reports, a specific quantity-price pair from the proposed discounts is selected. Finally, the order of 500 kilometers of power cables is delivered and every entity bears the corresponding cost.

Consider that the plant that corresponds to the low production cost is full; thus, the production will be at the expensive plant, while the low warehousing facility is still available (l/u). The BoD ensures that using the quantity discounts: {(115, 778), (100, 665), (94, 676), (82, 482)}, the channel will be coordinated, reaching the minimum level of overall costs. This is attainable since the BoD optimizes the channel pie (implementation of the optimal lot size for the entire channel) and then shares the additional gains that arise from the coordination to both entities. The whole order of 500 kilometers of power cables will be manufactured in a lot for lot fashion of 100 kilometers each, leading to a win-win situation for both entities.

6 Concluding Remarks

Motivated by practical realities in a cable manufacturing group, we have considered inventory management decisions when manufacturing and warehousing are controlled by independent entities who possess private information that affects their choices. The idea arose from the fact that in business environments individual entities tend to keep cost structures or other internal information private in order to take advantage of a situation and achieve better individual gains. Even though in the Production and Operations Management literature there are several papers that examine cases in which either the information is common knowledge to all or only one entity possesses private information, research on bilateral information asymmetry is sparse. Our study provides insights into the effects of bilateral information asymmetry on channel coordination.

The proposed setting allows entities to communicate any information they possess via a mediator (third trusted party) who attempts to streamline their decisions and reach a solution beneficial for both, without restricting their freedom. Misinformation is also modeled in our work, since it is a possible choice by the entities if they anticipate to achieve more individual gains. The mediator designs a mechanism under the objective of minimizing the overall channel costs. Using the Revelation Principle we have proved that it is in both entities' self-interest to reveal their information and that channel coordination is attainable.

The intuitive explanation of achieving coordination is that the mediator is able to capture the entities' private information by including adverse selection incentives to the mechanism. The mediator completely eliminates the asymmetry of information, enabling coordination as it occurs in settings with complete information. Additionally, the mediator includes participation incentives in the mechanism for all the entities to accept the outcome, since they are free to deny it. We have proved that the entities accept the mechanism because their expected cost functions are minimized under this strategy.

The introduction of a mediator is crucial for two reasons. First, it allows to model the interaction under bilateral information asymmetry, since the mediator has to overcome that both entities have private information, during the mechanism design phase. Second, it enables coordination and reaching a Pareto-optimal solution by effectively leading the entities to consider the overall channel costs in a decentralized decision making setting. The entities can optimize the channel pie and then they share it by achieving a better payoff with regards to the uncoordinated case. The acceptable cost allocation given the optimal channel cost is not unique, a fact that provides flexibility to the mediator. This flexibility may reflect the relative power of the entities and we have provided computational experiments to quantify its effects on the entities' gains and the cost implications by examining the extreme cases (i.e., minimum and maximum values of discounts that achieve channel coordination). This allowed to obtain insights on the minimum benefits that the entities can secure under the coordination via the mediator.

A relevant question is who will act as a mediator and design a mechanism that optimizes the performance of the entire channel beyond the boundaries of an organization. In real supply chains this role can be played by: i) auditing firms, especially in the cases where are common between the supply chain nodes; ii) supervising authorities that consider the optimal for the entire chain; or iii) third - party companies that are paid based on the overall channel performance by providing exclusively services to the supply chain members.

Several directions seem promising for future research. The coordination result is affected by the assumption of type-dependent reservation levels. Since the entities are aware of their private information, this is a reasonable assumption. However, if the reservation levels are exogenously defined and/or type-independent, the possibility of coordination will depend on the specific values that will be used. We conjecture that in order for coordination to be achievable by an appropriate mechanism these values must be sufficiently high.

Another interesting extension would be to study models with more than two decision makers: either a single manufacturer with many distributors or a single distributor with many manufacturers. It is also worth considering models that include more than two decision makers from different tiers, e.g., supplier, wholesaler, etc. In both of these settings, determining the appropriate reservation levels is an interesting question by itself, since it depends on the degree of competition/cooperation among the entities, and this in turn may affect the existence of coordinating mechanisms. In other directions, bilateral information asymmetry on more than one dimensions may be considered (for example setup and production cost, ordering and holding cost, etc.). Finally, more general settings can be examined (multi-echelon inventory systems, multiple cost functions, etc.), considering policies other than quantity discounts.

Acknowledgments: The authors would like to thank the Department Editor, Prof. P. Kouvelis, the Senior Editor, and two anonymous referees for their comments and suggestions that helped to improve the context and the presentation of the material in the paper. Acknowledgments also to Dr. E. Zachariadis for his comments on an earlier version of this work. We are also indebted to the Cablel Hellenic Cables Group, the group CFO, and the group Strategic Business Planning Manager Dr. K. Pavlou for motivating our work and providing a sound industrial application.

Appendix

We have used the following notation throughout the paper:

Variables	**Definition**
D	Market Demand
Q	Order Quantity or Lot Size (Distributor's Decision)
(X, Y)	Quantity-Price Pair (Manufacturer's Decision)
K_S	Manufacturer's Setup Cost
K_R	Distributor's Ordering Cost
$P_S = \{P_d, P_u\}$	Production Cost (Manufacturer's Private Information)

$H_R = \{H_l, H_h\}$	Holding Cost (Distributor's Private Information)
q	Probability of Type-d Manufacturer
p	Probability of Type-l Distributor
$TC_S(Q), TC_R(Q)$	Manufacturer's, Distributor's Cost Function
$C_J(Q)$	Channel Cost Function
$Q^R = \{Q_l^R, Q_h^R\}$	Distributor's Optimal Lot Size without any Discounts
Q^J	Optimal Joint Lot Size
$C_R^+ = \{C_{R,l}^+, C_{R,h}^+\}$	Distributor's Reservation Level
$C_S^+ = \{C_{S,d}^+, C_{S,u}^+\}$	Manufacturer's Reservation Level
m	Mediator Plan
$C_R(m, \cdot\vert\cdot)$	Distributor's Expected Cost under Mediator Plan m
$C_S(m, \cdot\vert\cdot)$	Manufacturer's Expected Cost under Mediator Plan m

Proof of Lemma 1.

Suppose (4.6) has a non negative solution in Y_{rs}. Multiplying the first two equations by $p, 1-p$ and the last two by $q, 1-q$, respectively and adding, it follows that:

$$py_l + (1-p)y_h = pqY_{ld} + p(1-q)Y_{lu} + (1-p)qY_{hd} + (1-p)(1-q)Y_{hu} = qy_d + (1-q)y_u \equiv w.$$

Therefore, (4.7) is necessary for (4.6) to have a solution. To show that it is also sufficient, if y_l, y_h, y_u, y_d satisfy (4.7), then one solution of (4.6) is:

$$Y_{lu} = y_l y_u/w, Y_{hu} = y_h y_u/w, Y_{ld} = y_l y_d/w, Y_{hd} = y_h y_d/w. \quad \square$$

Proof of Proposition 1.

I) Note that (4.10) corresponds to six linear inequality constraints. The first four define an unbounded polyhedron K in the non negative quadrant of the (y_l, y_h) plane. We consider two cases for the form of this set and in each case examine when K has non empty intersection with the last two constrains. First, we observe $a^+G\sqrt{R_H} \leqslant a^+G$ and $d_1 < 0$, since $F < 1 < 1/\sqrt{R_H}$. Therefore, the point $(a^+G\sqrt{R_H}, a^+G)$ always satisfies $y_h - y_l = a^+G(1-\sqrt{R_H}) \geqslant 0 > d_1 G$ and may or may not satisfy $y_h - y_l \leqslant d_2 G$. We thus consider two cases:

Case A: $a^+(1-\sqrt{R_H}) \leqslant d_2$.

In this case, the point $(a^+G\sqrt{R_H}, a^+G) \in K$ (Figure 8). Furthermore condition (4.11) becomes $a^+ - pa^+(1-\sqrt{R_H}) \leqslant e$. Suppose this is not satisfied; then, for any $(y_l, y_h) \in K$

we have that $y_l \geqslant a^+G\sqrt{R_H}$, $y_h \geqslant a^+G$. Therefore $py_l + (1-p)y_h \geqslant G(a^+ - pa^+(1-\sqrt{R_H})) > Ge$; thus, the sixth constraint in (4.10) is violated, i.e., the system (4.10) is not feasible. On the other hand, if $a^+ - pa^+(1-\sqrt{R_H}) \leqslant e$, we can find a point $(y_l, y_h) \in K$ that satisfies the last constraint of (4.10). To do this, let $y_l = a^+G\sqrt{R_H}+\delta, y_h = a^+G+\delta$, with $\delta > 0$. Then, $y_h - y_l = a^+G(1-\sqrt{R_H}) \leqslant d_2G$ and $py_l + (1-p)y_h = a^+G\theta + \delta$. If we set $\delta = G(e - a^+\theta) > 0$, then the last inequality of (4.10) is satisfied with equality, then the fifth inequality is also satisfied.

Case B: $a^+(1-\sqrt{R_H}) > d_2$.

In this case, $(a^+G\sqrt{R_H}, a^+G) \notin K$, but $(a^+G - d_2G, a^+G) \in K$. For any $(y_l, y_h) \in K$ it is true that $y_l \geqslant a^+G - d_2G$ and $y_h \geqslant a^+G$. Then, by following an analogous reasoning as in Case A, we can find a solution that satisfies the last two constraints of (4.10) if and only if holds $a^+ - pd_2 \leqslant e$.

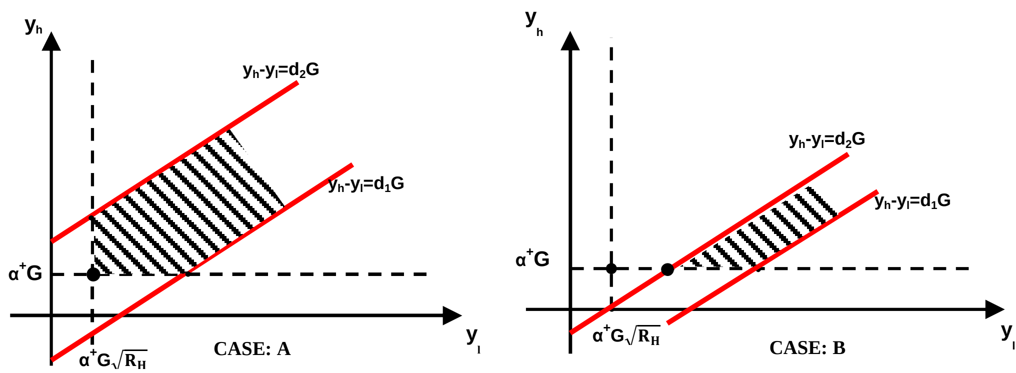


Figure 8: The two cases of condition (4.11).

II) To show that condition (4.11) is always true, we distinguish four separate cases, according to the value of $min\{d_2, a^+(1-\sqrt{R_H})\}$ and the sign of a.

Case 1: $a^+(1-\sqrt{R_H}) \leqslant d_2$.

In this case condition (4.11) can be written as $a^+\theta \leqslant e$.

Case 1a: $a \leq 0$. This means that $a^+ = 0$. Then (4.11) holds, since $e > 0$.

Case 1b: $a > 0$. This means that $a^+ = a$; thus, we must show that $a - pa(1-\sqrt{R_H}) \leqslant e$, i.e., $a\theta \leqslant e$, which after some algebra reduces to: $\theta(\zeta - \sqrt{F}) \leqslant (1-F)/(2\sqrt{F})$. If $\zeta - \sqrt{F} \leqslant 0$ then it is immediate. If $\zeta - \sqrt{F} > 0$ then $\theta(\zeta - \sqrt{F}) < \zeta - \sqrt{F} < 1 - \sqrt{F}$, since

F, θ and $\zeta \in (0,1)$. In addition, $(1-F)/(2\sqrt{F}) = (1+\sqrt{F})(1-\sqrt{F})/(\sqrt{F}+\sqrt{F}) > 1-\sqrt{F}$. Therefore, the inequality holds.

Case 2: $a^+(1-\sqrt{R_H}) > d_2$.

In this case condition (4.11) can be written as $a^+ - pd_2 \leqslant e$.

Case 2a: $a \leq 0$. We must show that $-pd_2 \leqslant e$. After substitutions and some algebra, the inequality becomes: $\zeta(1-\theta)(\sqrt{R_H}-F) \leq (1-F)(1/\sqrt{F}-\theta\zeta)$. For the left hand size, we have: $\zeta(1-\theta)(\sqrt{R_H}-F) \leq (\zeta-\zeta\theta)(1-F) \leq (1-\zeta\theta)(1-F)$, while for the right hand size, we have: $(1-F)(\frac{1}{\sqrt{F}}-\theta\zeta) \geqslant (1-F)(1-\theta\zeta)$. Thus, the inequality holds.

Case 2b: $a > 0$. We must show that $a - pd_2 \leqslant e$. After some algebra, the inequality becomes: $\zeta\left(1+\theta-(1-\theta)\sqrt{R_H}\right) \leq (1+F)/\sqrt{F}$. Since F, R_H, θ and $\zeta \in (0,1)$, it is easy to show that: $\zeta\left(1+\theta-(1-\theta)\sqrt{R_H}\right) \leq 2 \leq (1+F)/\sqrt{F}$. □

References

Altintas, N., Erhyn, F., and Tayur, S. (2008). Quantity discounts under demand uncertainty. *Management Science*, 54(4):777–792.

Amit, R. and Schoemaker, P. J. H. (1993). Strategic assets and organizational rent. *Strategic Management Journal*, 14(1):33–46.

Becerril-Arreola, R., Leng, M., and Parlar, M. (2013). Online retailers' promotional pricing, free-shipping threshold, and inventory decisions: A simulation-based analysis. *European Journal of Operational Research*, 230(2):272–283.

Benton, W. and Park, S. (1996). A classification of literature on determining the lot size under quantity discounts. *European Journal of Operational Research*, 92(2):219–238.

Bouchery, Y., Ghaffari, A., Jemai, Z., and Tan, T. (2017). Impact of coordination on costs and carbon emissions for a two-echelon serial economic order quantity problem. *European Journal of Operational Research*, 260(2):520–533.

Burnetas, A., Gilbert, S., and Smith, C. (2007). Quantity discounts in a single-period supply contracts with asymmetric demand information. *IIE Transactions*, 39(5):465–479.

Cachon, G. P. (2003). Supply chain coordination with contracts. In Graves, S. and de Kok, T., editors, *Handbooks in Operations Research and Management Science*. North Holland Press.

Cachon, G. P. and Fisher, M. (2000). Supply chain inventory management and the value of shared information. *Management Science*, 46(8):1032–1048.

Cachon, G. P. and Terwiesch, C. (2012). *Matching Supply with Demand: An Introduction to Operations Management, 3rd Edition*. Mc Graw Hill.

Cakanyildirim, M., Feng, Q., Gan, X., and Sethi, S. (2012). Contracting and coordination under asymmetric production cost information. *Production and Operations Management*, 21(2):345–360.

Chatterjee, K. and Samuelson, L. (1987). Bargaining with two-sided incomplete information: An infinite horizon model with alternating offers. *The Review of Economic Studies*, 54(2):175–192.

Chatterjee, K. and Samuelson, L. (1988). Bargaining under two-sided incomplete information: The unrestricted offers case. *Operations Research*, 36(4):605–618.

Chen, F., Federgruen, A., and Zheng, Y. (2001). Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management Science*, 47(5):693–708.

Choi, S. C., Lei, L., and Wang, Q. (2005). Quantity discounts for supply chain coordination. In K., C. A. and Eliashberg, J., editors, *Managing Business Interfaces. International Series in Quantitative Marketing*. Springer, Boston.

Choi, T. M. and Cheng, T. C. E. (2011). *Supply Chain Coordination under Uncertainty*. International Handbooks on Information Systems, Springer.

Chopra, S. and Meindl, P. (2016). *Supply Chain Management: Strategy, Planning, and Operation, 6th Edition*. Pearson.

Corbett, C. (2001). Stochastic inventory systems in a supply chain with asymmetric information: Cycle stocks, safety stocks, and consignment stock. *Operations Research*, 49(4):487–500.

Corbett, C. and de Groote, X. (2000). A supplier's optimal quantity discount policy under asymmetric information. *Management Science*, 46(3):444–450.

Corbett, C., Zhou, D., and Tang, C. (2004). Designing supply contracts: Contract type and information asymmetry. *Management Science*, 50(4):550–559.

Davis, A. M., Katok, E., and Santamara, N. (2014). Push, pull, or both? a behavioral study of how the allocation of inventory risk affects channel efficiency. *Management Science*, 60(11):2666–2683.

Drake, M. J. and Schlachter, J. T. (2008). A virtue-ethics analysis of supply chain collaboration. *Journal of Business Ethics*, 82(4):851–864.

Feng, Q. and Lu, L. X. (2013). Supply chain contracting under competition: Bilateral bargaining vs. stackelberg. *Production and Operations Management*, 22(3):661–675.

Fiala, P. (2005). Information sharing in supply chains. *Omega*, 33(5):419–423.

Gibbard, A. (1973). Manipulation of voting schemes: A general result. *Econometrica*, 41(4):587–601.

Gibbons, R. (1992). *A Primer in Game Theory*. Prentice Hall.

Ha, A. Y. (2001). Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation. *Naval Research Logistics*, 48(1):41–64.

Ha, A. Y. and Tong, S. (2008). Contracting and information sharing under supply chain competition. *Management Science*, 54(4):701–715.

Inderfurth, K., Sadrieh, A., and Voigt, G. (2013). The impact of information sharing on supply chain performance under asymmetric information. *Production and Operations Management*, 22(2):410–425.

Kalkanci, B., Chen, K., and Erhun, F. (2011). Contract complexity and performance under asymmetric demand information: An experimental evaluation. *Management Science*, 57(4):689–704.

Kanda, A., Deshmukh, S., et al. (2008). Supply chain coordination: perspectives, empirical studies and research directions. *International Journal of Production Economics*, 115(2):316–335.

Karabati, S. and Sayin, S. (2008). Single-supplier/multiple-buyer supply chain coordination: Incorporating buyers' expectations under vertical information sharing. *European Journal of Operational Research*, 187(3):746–764.

Kerkkamp, R., van den Heuvel, W., and Wagelmans, A. (2017). Two-echelon supply chain coordination under information asymmetry with multiple types. *Omega*, 76:137–159.

Kouvelis, P., Chambers, C., and Wang, H. (2006). Supply chain management research and production and operations management: Review, trends, and opportunities. *Production and Operations Management*, 15(3):449–469.

Krajewski, L., Ritzman, L., and Malhotra, M. (2016). *Operations Management Processes and Supply Chains, 11th Edition*. Pearson.

Lee, H. and Whang, S. (1999). Decentralized multi-echelon supply chains: Incentives and information. *Management Science*, 45(5):633–640.

Li, C. L. and Kouvelis, P. (1999). Flexible and risk-sharing supply contracts under price uncertainty. *Management Science*, 45(10):1378–1398.

Li, X. and Wang, Q. (2007). Coordination mechanisms of supply chain systems. *European Journal of Operational Research*, 179(1):1–16.

Lovejoy, W. S. (2006). Optimal mechanisms with finite agent types. *Management Science*, 52(5):788–803.

Moinzadeh, K. (2002). A multi-echelon inventory system with information exchange. *Management Science*, 48(3):414–426.

Munson, C. L. and Rosenblatt, M. J. (2001). Coordinating a three-level supply chain with quantity discounts. *IIE Transactions*, 33(5):371–384.

Myerson, R. (1979). Incentive-compatibility and the bargaining problem. *Econometrica*, 47(1):61–73.

Myerson, R. (1982). Optimal coordination mechanisms in generalized principal-agent problems. *Journal of Mathematical Economics*, 10(1):67–81.

Myerson, R. (2007). *Mechanism Design Theory, Scientific background on the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2007*. Royal Swedish Academy of Sciences, Stockholm.

Ozer, O. and Raz, G. (2011). Suplly chain sourcing under asymmetric information. *Production and Operations Management*, 20(1):92–115.

Pishchulov, G. and Richter, K. (2016). Optimal contract design in the joint economic lot size problem with multi-dimensional asymmetric information. *European Journal of Operational Research*, 253(3):711–733.

Sabherwal, R., Hirschheim, R., and Goles, T. (2001). The dynamics of alignment: Insights from a punctuated equilibrium model. *Organization Science*, 12(2):179–197.

Shneyerov, A. and Wong, A. C. L. (2010). Bilateral matching and bargaining with private information. *Games and Economic Behavior*, 68(2):748–762.

Snowden, D. and Boone, M. (2007). A leader's framework for decision making. *Harvard Business Review*, 85(11):69–76.

Stevenson, W. J. (2015). *Operations Management, 12th Edition.* McGraw-Hill, New York.

Sucky, E. (2006). A bargaining model with asymmetric information for a single supplier-single buyer problem. *European Journal of Operational Research*, 171(2):516–535.

Valley, K., Thompson, L., Gibbons, R., and Bazerman, M. H. (2002). How communication improves efficiency in bargaining games. *Games and Economic Behavior*, 38(1):127–155.

Viswanathan, S. and Wang, Q. (2003). Discount pricing decisions in distribution channels with price sensitive demand. *European Journal of Operational Research*, 149(3):571–587.

Weng, Z. K. (1995). Channel coordination and quantity discounts. *Management Science*, 41(9):1509–1522.

Zissis, D., Ioannou, G., and Burnetas, A. (2015). Supply chain coordination under discrete information asymmetries and quantity discounts. *Omega*, 53:21–29.