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Integrating Knowledge for Instruction: A Tale of Two Teachers

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Abstract: Teacher knowledge is a critical factor that influences pedagogical decisions. If we want teachers to make appropriate choices in the classroom we must know and understand the types of knowledge used during this decision-making process. To this end, we sought to understand how, and the extent to which, two 5th grade teachers drew upon and integrated their knowledge of mathematics, learners, and pedagogy while teaching. Stimulated recall interviews were analyzed to uncover the types of knowledge and interactions that occurred. Both teachers primarily used their knowledge of learners and pedagogy, with the knowledge of mathematics playing a supportive role. In addition, the teachers integrated their knowledge in one of two ways: a) one knowledge type was used to justify or explain a statement about a second knowledge type and b) a discussion of one knowledge type lead to an implication or reflection about a second knowledge type. These interactions allowed the teachers to use and build their connected knowledge. Understanding how teachers integrate and use their knowledge has implications for the structure of teacher professional development.

Keywords: Teacher Knowledge; Decisions-Making; Knowledge Integration; Knowledge of Learners; Knowledge of Mathematics; Knowledge of Pedagogy.

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Introduction

Teaching is a complex endeavor where numerous pedagogical decisions are made each day. When preparing for a lesson, teachers decide which mathematical concepts will be taught, the tasks and activities that promote the learning of these concepts, and how students will participate in these learning opportunities. During the lesson, teachers make multiple in-the-moment decisions as they adjust to student thinking and other classroom situations. After the lesson, teachers reflect on the lesson and assessments to determine what students have learned and the direction for the next day's lesson. This is not an exhaustive list of teacher decisions, but is illustrative of how teachers draw upon their knowledge of mathematics, learners, and pedagogy for the purpose of instruction. The knowledge teachers possess guides them in each step of the decision-making process. This paper describes how two teachers integrated their knowledge of mathematics, pedagogy, and learners while making and reflecting on decisions made in their classrooms.

Teacher knowledge is a critical component of teacher decision-making. However, successful decision-making requires more than simply understanding relevant content. Ball (2000) stated that, "although some teachers have important understandings of the content, they often do not know it in ways that help them hear students, select good tasks, or help all their students learn" (p. 243). This observation identifies an important dilemma for mathematics educators, how do we help teachers integrate and use knowledge so that it can be successfully applied in instructional settings? Hill et al. (2008) noted that "there *is* a powerful relationship between what a teachers knows, how she knows it, and what she can do in the context of instruction" (p. 496). In order to help teachers make effective pedagogical decisions in the classroom, we must gain further insight into how teachers' draw upon their knowledge in making these decisions. We must understand the mechanisms that allow teachers to go beyond simply possessing knowledge, to using it for educational purposes. As White et al (2013) stated, "there is a need to clarify the difference between teachers' theoretical knowledge and knowledge that arises from the teaching experience" (p. 394).

Hiebert, Gallimore, and Stigler (2002) stated that teachers do not separate knowledge as researchers do, but weave their knowledge together around problems of practice. Fennema and Franke (1992) commented that, "[teacher] knowledge is not monolithic. It is a large, integrated, functioning system with each part difficult to isolate … some have studied knowledge as integrated, but most have not" (p. 148). With this in mind, mathematics teacher educators need insights into how teachers' integrate their knowledge while making pedagogical decisions. Although researchers have studied knowledge integration (e.g., Ball & Bass, 2000; Even, 1999; Steele & Hillen, 2012; Wilson, 1994), the importance of integration to the effective application of teacher knowledge to the problems of practice necessitates that it be given more attention. For the purposes of this paper, *knowledge integration* is defined as developing or drawing upon connections between different domains of knowledge (e.g., mathematics, pedagogy, learners) for the purpose of making instructional decisions.

In this paper, we examine the connections two 5th grade teachers made between their knowledge of mathematics, learners, and pedagogy. Using a case study, we sought to answer the following research questions:

1. To what extent did two teachers draw upon their knowledge of mathematics, learners, and pedagogy when reflecting on, and making instructional decisions?
2. How did two teachers integrate their knowledge of mathematics, learners, and pedagogy when reflecting on, and making instructional decisions?

Theoretical Perspective

One line of teacher knowledge research focuses on classifying the different types of teacher knowledge used in the teaching process (e.g. Ball, Thames, & Phelps, 2008; Rowland et al, 2009; Shulman, 1987). Shulman (1987) proposed the existence of multiple forms of teacher knowledge. They include, but are not limited to, knowledge of content, general pedagogy, curriculum, learners and their characteristics, educational contexts, and education goals. In addition, Shulman proposed a unique form of knowledge, pedagogical content knowledge (PCK), which is of particular interest to the field of teacher education. Shulman stated that PCK "represents the *blending* of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman 1987, p. 8, emphasis added). The word "blending" is important to our work as it connotes the integrated types of knowledge that teachers use.

Based on their observations of mathematics teaching, Ball, Thames, and Phelps (2008) proposed a model of Mathematical Knowledge for Teaching consisting of several categories of knowledge that a mathematics teacher must possess. Three categories related to Shulman's (1987) notion of PCK are Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and the Knowledge of Content and

Curriculum (KCC). Again, as the names imply, these categories suggest a blending of different forms of knowledge for teaching. Whether this blending occurs naturally, or through teacher development, the process of integrating teacher knowledge, and methods of measuring this process, deserves more attention. We must gain an understanding of how teachers connect the various types of knowledge (mathematics, students, teaching, and curriculum) to form and use PCK, KCS, KCT, and KCC.

While the work of Shulman (1987) and Ball, Thames, and Phelps (2008) are important in identifying the unique and essential types of knowledge a mathematics teacher must possess, we also need to understand how this knowledge is used in the act of teaching. Bishop (1976) noted that, "decision making … is an activity which seems to me to be at the heart of the teaching process. If I can discover how teachers go about making their decisions then I shall understand better how teachers are able to teach" (p. 142). Decision-making is a critical process in the act of teaching that requires more than simply possessing knowledge, but understanding how to use that knowledge. This shift to viewing teacher knowledge as a process, rather than an object, is evident in the research on teacher noticing. Sherin, Jacobs, and Phillipp (2011) noted that, "the word noticing names a process rather than a static category of knowledge" (p. 5). With this in mind, our goal was to describe the *process* of knowledge integration for two teachers during instruction.

Bishop and Whitefield (1972) proposed that decisions are made using a framework or schema. The main operation of a schema is to store knowledge through a network of connected pieces of knowledge called "elements" (Marshall, 1995, p. 43). The more connections that exist within a schema, the stronger and more useful the schema will be. With respect to problem solving, Marshall (1995) proposed that mathematics students have different schemata for various purposes that are connected to each other. If two schemata are connected, then activating one schema during problem solving can activate the other. We hypothesize that teachers' schemata operate in a similar manner. If a teacher has a variety of teaching-related schemata with well-formed connections, then each schema becomes more useful during the act of teaching, resulting in more informed decisions (Arnon et al, 2014). Decision-making is one means by which teachers develop and reinforce the connections they use in the act of teaching (Barrett & Green, 2009).

Schema theory is a promising means to examine how teachers use and integrate their knowledge during the act of teaching. In order to develop an analytic framework to investigate teacher knowledge we combined the idea of schema theory with the common subsets of teacher knowledge identified by mathematics education researchers. Shulman (1987) and Hill, Ball, & Shilling (2008) describe teacher knowledge (PCK and KCT, KCS, and KCC) as integrated in nature containing elements of the knowledge of mathematics, learners, and pedagogy. We believe that as teachers use, integrate, and reflect on their knowledge of mathematics, learners, and pedagogy they develop and refine their PCK, KCC, KCS, and KCT. In essence, we believe the process of schema refinement and integration plays an important role in the development of teacher knowledge that is unique to mathematics.

In order to develop and promote the teacher knowledge that is unique to mathematics we need to understand the processes of blending knowledge and building schemata. To this end, we begin by identifying three broad types of teacher knowledge and then focus on how these distinct knowledge types are being integrated and connected.

The three types of knowledge that are the focus of our analytic framework are the knowledge of mathematics (M), the knowledge of learners (L), and the knowledge of pedagogy (P). We define the three types of knowledge as follows:

Knowledge of Mathematics (M)	A teacher statement where the **primary focus** is about mathematics content. The teacher discusses a mathematical topic connected to the mathematical goal of the lesson, including connections and relationships between ideas. Included in this category are the ways and means of justifying and providing proof for these ideas, the teacher's personal views or ways of thinking about the topic, and the mathematical topics needed for instruction.
Knowledge of Learners (L)	A teacher statement where the **primary focus** is about mathematical learners. The teacher shares student thinking—what she observed students thinking as well as how she expects students to think. In addition, this category includes conversations about student characteristics, habits, understanding, or misunderstandings that may influence the thinking of students. The knowledge of learners could be, but does not have to be, specific to mathematics learning.
Knowledge of Pedagogy (P)	A teacher statement where the **primary focus** is about pedagogy. The teacher mentions tasks, curriculum, and questions that were used to further the goals of the lesson. Included in this category are comments centered on the implementation of the lesson or decisions regarding the flow of the lesson. The pedagogy could be, but does not have to be, specific to mathematics teaching.

Table 1. Definitions for knowledge of mathematics, learners, and pedagogy.

Our definitions of M, L, and P are broader than, but related to, many of the categorizations of knowledge described in the preceding paragraphs. Given our broad definitions, the focus of our analytic framework lies in the interactions among these knowledge sets and the processes teachers use to integrate this knowledge. Although knowledge integration is not synonymous with PCK or MKT, it works synergistically with these models to providing insights into how these knowledge types are developed.

From our viewpoint, teachers make decisions in the classroom, in part, by utilizing connections. Connections between different knowledge types are activated as teachers interact with the context and content of the classroom. The more connections teachers have in their schemata, the more information they have available to make decisions. As Bishop and Whitfield (1972) noted, a teacher's decision-making process is the catalyst for the activation of various schemata.

Our work is related to Ma's (1999) concept of knowledge packages. In Ma's (1999) work she described a teacher's knowledge of a topic by mapping out the connections between the various mathematical components. Ma (1999) stated, "The purpose of a teacher in organizing knowledge in such a package is to promote a solid learning of a certain topic" (p. 19). The knowledge a teacher utilizes in making decisions draws upon the knowledge packages described by Ma (1999), but also includes other types of knowledge, such as the knowledge of learners and the knowledge of pedagogy. Our model expands on the work of Ma (1999) to describe connections teachers make between their knowledge of mathematics, learners, and pedagogy, a three-dimensional knowledge package.

We pose the following example to illustrate how a teacher's schemata are activated and used. Consider a teacher who is teaching the mathematical concept of slope. The teacher's knowledge of mathematics will come into play as she considers the learning objectives for her lesson. She will have to identify and understand the mathematical "big ideas" of slope in order to define these objectives. The teacher will then need to examine her knowledge of learners in order to know what prior knowledge students have about slope and what potential misconceptions might arise during the lesson. The teacher will also have to examine her knowledge of pedagogy to choose teaching methods and tasks to achieve the objectives of the lesson. The teacher's knowledge of P is linked to her knowledge of L in order to create a lesson that is tailored for the particular students in her current class. The teacher's knowledge of P is connected to her knowledge of M to determine which teaching acts help promote an understanding of a particular mathematical topic. The knowledge of L and knowledge of M are connected when the teacher asks herself what makes the topic of slope difficult for students to learn. This is just one example of how a teacher might draw upon her integrated knowledge while teaching a lesson on slope.

Participants and Setting

There were two participants in this study, Amanda and Emily. Both participants taught fifth-grade at a school located in a mid-sized town in the Midwest. At the time of the study, Amanda was in her second year of teaching while Emily was in her seventh. Amanda's lessons often started with a warm-up to review basic skills. Following the warm-up, instruction continued with Amanda asking questions that pushed for student reasoning. She often prompted her students to slow down and think, providing considerable wait time for the students to respond. She also established social norms for student discourse. Students often said, "I disagree with Kyle because _____." or "I agree with Ben because _____." The students appeared comfortable disagreeing with each other respectfully and most, if not all, appeared engaged in the lessons. Amanda often had her students work individually, in small groups, and as a whole class during the progression of the lesson.

Emily frequently began her lessons by collecting homework and having students discuss topics they did not understand. After discussing homework, the class often gathered on the carpet in the front of the room to discuss the mathematical activity of the day. During instruction, Emily varied how her students worked (e.g., whole class, small group, pairs). Emily asked questions and encouraged her students to explain their thinking to her and their peers. Emily made a concerted effort to investigate student thinking, even when it contained faulty logic, and to make connections to how other students were thinking about the problem.

Data Collection and Analysis

A case study (Yin, 2003) was conducted to investigate how Amanda and Emily drew upon their knowledge of mathematics, learners, and pedagogy. Each teacher was observed and videotaped as they taught a series of three consecutive lessons on the same subject. The teachers collaborated in the development of these three lessons. Following each observation, a stimulated-recall interview was conducted where the researcher and the teacher watched the videotaped lesson. As they watched the video during the interview, the teachers were instructed to reflect on what occurred during the lesson and to comment on their thought process as they made instructional decisions. These interviews were audio recorded and transcribed. It must be noted that although the teachers were reflecting on their decisions while they watched the videos, this does not necessarily mean their reflection was an accurate portrayal of the knowledge used while actually making these decisions in the moment.

Following data collection and transcription, the data were retrospectively analyzed using a data reduction approach (Miles, Huberman, & Saldana, 2014). The analysis occurred in three separate phases.

Phase 1. Each transcript was initially read to identify individual episodes, defined to be sections of the transcript where the teacher was discussing a single thought.

Phase 2. Following the identification of episodes, the constant comparative method (Glaser & Strauss, 1967; Strauss & Corbin, 1990) was used to identify individual statements about mathematics (M), learners (L), and pedagogy (P). After initial definitions were developed, a second researcher coded portions of the transcripts to test the viability of the definitions. Discrepancies in coding were discussed, and definitions adjusted, until all disagreements were resolved. All transcripts were recoded using the updated and finalized definitions, see Table 1.

Phase 3. To gauge the extent of knowledge interaction, episodes were divided based upon the particular combination of knowledge used (e.g., M&L, M& P, L&P). To investigate the nature of knowledge interactions, episodes containing more than one form of knowledge were further analyzed. Free coding was employed to identify themes in the ways the teachers were integrating or using multiple knowledge types. These categories were tested and discrepancies in coding were discussed, and categories adjusted, until all disagreements were resolved. After all episodes were coded, categories were further analyzed to uncover patterns of knowledge integration.

Findings

The goal of this study was to understand how two fifth grade teachers integrated their knowledge of mathematics, learners, and pedagogy while making decisions in their classrooms. In order to identify this integration, our first step was to gauge the extent to which Amanda and Emily had the opportunity to integrate their knowledge. To accomplish this we first took the identified episodes—narratives discussing a single issue or thought—and tallied the number of episodes that contained the various combinations of knowledge (e.g. ML, LP). Thus, if an episode contained statements that were coded as knowledge of mathematics and statements coded as knowledge of learners, it would have been identified as an ML episode. For example, in the following episode Amanda described an instance where her students were multiplying a number by 98.

Transcript	Code
Amanda: **The next day I came in and the kid goes times 100 minus two groups. You know, I was like, "Man, why didn't I see that?**	L
Why didn't I see that it was really close to the landmark of 100? My mind is so one tracked, I learned a new way to do it through them.	M

Table 2. An example of an episode containing knowledge of mathematics and learners.

Instead of using the traditional algorithm for multiplication, Amanda's student multiplied the number by 100 and subtracted twice the original number from the result. This was not what Amanda had expected; she had completed the problem in her mind using the standard algorithm. Amanda commented that her traditional background created roadblocks to how she thought and that she learned a considerable amount from listening to her students. In this example, her examination of student thinking (L) led to new personal insights about mathematics (M).

It must be noted that this analysis only confirms that multiple knowledge forms were used during an episode, not that a direct connection was made between the different knowledge types. Hence, it is possible that a teacher discussed two different forms of knowledge during the episode, but never integrated them. This analysis provides an estimation or upper bound of the teachers' knowledge integration. Table 3 displays the number of episodes, and the percentage of those episodes, that contained the different knowledge combinations.

	M	L	P	ML	MP	LP	MLP
Amanda	3 2.5%	16 13.2%	10 8.3%	17 14.0%	3 2.5%	39 32.3%	33 27.3%
Emily	2 1.8%	14 12.5%	9 8.0%	15 13.4%	9 8.0%	46 41.1%	13 11.6%
Total	**5** **2.2%**	**30** **12.9%**	**19** **8.2%**	**32** **13.7%**	**12** **5.2%**	**85** **36.5%**	**46** **19.7%**

Table 3. Amanda and Emily's integrated use of knowledge.

From the data, we see that most episodes involved more than one type of knowledge, with the mixture of knowledge of learners and knowledge of pedagogy being the most prevalent (36.5%). The combination of knowledge of mathematics and knowledge of pedagogy was the least common knowledge combination at 5.2%. There were also differences in knowledge use between the teachers. Amanda used all three knowledge types more often than Emily (27.3% to 11.6%) and Emily used the combination of only mathematics and pedagogy more often than Amanda (8.0% to 2.5%).

Table 4 displays the number of episodes that drew upon a single knowledge type versus those that drew upon multiple knowledge types.

	Single Knowledge Type	Multiple Knowledge Types
Amanda	29 24.0%	92 76.0%
Emily	25 22.3%	83 74.1%
Total	**50** **23.2%**	**174** **75.1%**

Table 4. Number of episodes using single and multiple knowledge types

Amanda and Emily used multiple knowledge forms approximately 75 percent of the time. These results indicate the prevalence of the use of multiple knowledge forms during the processes of decision-making and reflection. Although these data suggests that Amanda and Emily used their knowledge in an integrated manner, we need further information to determine the ways in which this knowledge was used. What was the role of the different knowledge forms in the decision-making process? Why and how were the teachers integrating their knowledge? Identifying the role each knowledge type played in the teaching process is the focus of the next section.

The Nature of Knowledge Integration

It was not surprising that the teachers in this study used their knowledge of pedagogy, mathematics, and learners in concert. The challenge was to characterize *how* this knowledge was used. During the last stage of our analysis we looked for themes in the ways the teachers were integrating or using multiple knowledge types. Our analysis identified two distinct types of interactions: (a) one knowledge type was used to *justify or explain* a claim about a second knowledge type, and (b) a discussion focused on one knowledge type led to an *implication or reflection* about a second knowledge type. We hypothesize that these roles could be used within a single knowledge type (i.e., a teacher uses knowledge of learners to justify a statement about learners). However, this was not the purpose of the present study. Examples of these two roles of knowledge integration are provided in the paragraphs that follow.

In the following episode, Amanda used her knowledge of pedagogy to justify a comment about one of her students.

Transcript	Code
***Amanda:* I was surprised that Kate raised her hand and was giving answers. She usually does not participate. It was like a reversal.**	L
***Researcher:* Why do you think she was participating?**	
***Amanda:* I think it is because we are using a context, we are using blocks, we are showing it, we are showing it in two different formats so we are including two totally different types of thinkers.**	P

Table 5. An example of integrating knowledge for the purpose of justification.

In this episode, Amanda begins by making a claim about one of her learners, "I was surprised that Kate raised her hand and was giving answers. She usually does not participate." At this point the focus of conversation was about one of her learners. After being prompted, she provided an explanation, or justification, for why this might be the case, "I think it is because we are using a context … we are showing it in two different formats so we are including two totally different types of thinkers." In this episode the initial focus was on her knowledge of learners (L) and the statement involving pedagogy was used to justify her statement about a learner.

In contrast, there were episodes where a discussion involving one aspect of teacher knowledge led to an implication or reflection involving a second. In the following episode, see Table 6, Amanda made a comment about her knowledge of learners that led to a reflection about her knowledge of mathematics.

Transcript	Code
***Amanda:* They have such difficulties coming up with a rule. They want it to be clear; I think I want it to be. They want to be able to say, "I see what it is doing, there has to be something I can do to tell you really quick what the answer is."**	L
Which is funny, Emily and I talked about the straw problem for a second—I was so frustrated. How does this stupid pattern work? It was frustrating me that I couldn't come up with the formula. I was trying to make these connections, we kept saying, "Do you include the first straw to make a rule or do you make that a separate part of your formula?"	M

Table 6. An example of integrating knowledge to make an implication.

Amanda initially described her students' desire to find a rule, which was coded as knowledge of learners (L). "They have such difficulties coming up with a rule. They want it to be clear." The students wanted a rule that could quickly provide them with the answer. After this initial statement about learners, the conversation shifted to a second knowledge type, the knowledge of mathematics (M). At this point, Amanda described her personal struggle in developing a rule for the problem. "How does this stupid pattern work? It was frustrating me that I couldn't come up with a formula. I was trying to make these connections." This use of a secondary knowledge type was not for the purpose of justification; rather it was a reflection, or a consequence, brought about by the original statement concerning learners.

A variety of potential reasons exist for Amanda's use of mathematics in the episode depicted in Table 6. It may be that Amanda was investigating the mathematics of the problem in order to understand why it was difficult for her students to obtain a rule. If she would have been successful in her mathematical investigation, it is possible that she could have generated a justification for why students struggle with this particular rule. Alternatively, the discussion of her students' difficulties may have simply triggered a connection or implication that was of interest to her, causing a momentary shift in her focus. In either case, the investigation or discussion of the secondary knowledge type was not connected directly back to, and was not used to explain, the original claim about learners.

Two distinct types of interactions (*integrating to justify* and *integrating to imply or reflect*) emerged from the data. Given these categories of knowledge interaction, the data were organized to display the occurrence of these interactions among the knowledge of mathematics, learners, and pedagogy. We will begin by providing the data involving integrating to justify.

Integrating to Justify. When a teacher integrates to justify they are using one form of knowledge to justify or explain a statement involving a second. In Table 7, we provide the teachers' use of integrating to justify. The first column provides the knowledge type of the teachers' initial statements and the second column provides the knowledge type used to justify these initial statements. For instance, the first row provides all interactions where a teacher made a mathematical statement and then justified it using her knowledge of learners—each teacher had one interaction of this type. The unit of analysis for this data is an interaction between two knowledge types. Many episodes contained multiple interactions, and in some cases these interactions occurred in chains. For instance, a chain in which statement A justified B, and then statement B justified C was broken up into two different interactions. In addition, if two statements A and B both justified C, this was also divided into two interactions.

Integrating to Justify				
Statement	**Justification**	Amanda	Emily	Total
M	**L**	1	1	2
M	**P**	0	0	0
L	**M**	38	10	48
L	**P**	23	13	36
P	**M**	8	8	16
P	**L**	19	15	34

Table 7. Frequency of Integrating to Justify for Amanda and Emily.

Several patterns emerged in the role that each data type played in the *integrating to justify* interactions. For instance, as these two teachers reflected on their instruction they used mathematics to justify a claim about learners or pedagogy, but rarely made initial statements about mathematics. Table 8 provides the role of each knowledge type while integrating to justify. The first row provides the role that the knowledge of mathematics played while integrating to justify. The knowledge of mathematics occurred 66 times with mathematics being used to justify another statement in 64 of them.

Role of Knowledge Type in Integrating to Justify		
Knowledge Type	Statement	Justification
M	2	64
L	84	36
P	50	36

Table 8. Role of Knowledge Type in Integrating to Justify

As the teachers *integrated to justify* the object of these justifications were primarily statements about learners (84) and pedagogy (50). Hence, the teachers initial focus in these instances tended to be about pedagogy and learners. However, when the teachers justified these statements about learners and pedagogy, mathematics was the most common knowledge type used (64), but the knowledge of learners and pedagogy were also used to justify statements. It must be noted that this data only describes integrating to justify with two different knowledge types, which is the focus of this study; it does not include instances where the initial statement and the justifying statement are of the same knowledge type. Hence, these data may not be reflective of all justifications made by these teachers during their reflections.

Integrating to Imply. As stated earlier, *integrating to imply* is when a discussion focused on a single knowledge type leads to an implication or reflection involving a second knowledge type. Hence, in table 9 below, the first column provides the knowledge type of the original teacher statement and the second column provides the knowledge type of the implication or reflection made from that original statement. For instance, the first row provides all the interactions in which a teacher made a mathematical statement that led to an implication about learners—Emily had six interactions of this type, Amanda had none. Again, the unit of analysis for this data is an interaction between two different knowledge forms.

Integrating to Imply & Reflect				
Statement	**Implication/ Reflection**	Amanda	Emily	Total
M	**L**	0	6	6
M	**P**	1	2	3
L	**M**	5	5	10
L	**P**	22	15	37
P	**M**	0	1	1
P	**L**	11	12	23

Table 9. Frequency of Integrating to Imply for Amanda and Emily.

Amanda and Emily’s use of integrating to imply was similar. The only distinction that can be observed is that Emily more readily started off discussing mathematics (8) as compared to Amanda (1). For both teachers, the most common *integrating to imply* interaction was a statement about learners leading to an implication about pedagogy. The second most common interaction for both teachers was a statement about pedagogy leading to an implication about learners. Table 10 provides the role for each knowledge type while integrating to imply.

Role of Knowledge Type in Integrating to Imply		
Knowledge Type	Statement	Implication
M	9	11
L	47	29
P	24	40

Table 10. Role of Knowledge Type in Integrating to Imply

The teachers' integrating to imply statements were primarily about learners (47) and pedagogy (24), which is a pattern also observed for integrating to justify. However, in stark contrast to integrating to justify, the implications of these original statements tended to revolve around the teachers' knowledge of learners and pedagogy. Again, it must be noted that this data only describes integrating to imply with two different knowledge types, it does not include instances where the initial statement and the implication are of the same knowledge type.

Discussion

In the previous section we identified and described two distinct ways in which Amanda and Emily integrated their knowledge, *integrating to justify* and *integrating to imply and reflect*. We found that how the different knowledge forms were used in these two processes noteworthy. For example, Amanda and Emily primarily used their knowledge of mathematics in a supportive role to justify statements about learners or pedagogy, although Emily did use it several times while *integrating to imply* (both as a statement and an implication). 97% of the *integrating to justify* interactions involving mathematics used mathematics to justify a statement about learners or pedagogy. Overall, mathematics was used in 48.5% of the *integrating to justify* interactions. If it is true that the knowledge of mathematics is often used in a supportive role to justify statements about pedagogy and learners, how do we incorporate this fact into the mathematical preparation of teachers? Would it be beneficial to teachers if they learned mathematics in ways that resemble how they will use it in the classroom? This finding may help to explain why subject matter knowledge alone does not ensure effective teaching performance (Kahan, Cooper, and Bethea, 2003) and suggests that *how* mathematical knowledge is used in conjunction with other forms of knowledge is critical for instruction (Hill et al., 2008).

Knowledge of learners was the prominent knowledge type used while *integrating to justify* and *integrating to imply*--94.8% of these integrated episodes contained the knowledge of learners. There were few episodes that contained the combination of only mathematics and pedagogy. This finding suggest that the knowledge of learners may have played a role in helping Amanda and Emily integrate their knowledge of mathematics and pedagogy and reinforces the need for teachers to understand and utilize student thinking in teaching. Projects such as Cognitively Guided Instruction (Carpenter et al., 1999) and Integrating Mathematics and Pedagogy emphasize understanding student thinking in order to teach effectively. Both projects used student thinking as a catalyst to connect teachers' knowledge of mathematics and pedagogy.

If it is true that the knowledge of learners plays a critical role in a teacher's ability to integrate mathematics and pedagogy, how do we find more opportunities to infuse student thinking into our mathematics methods courses? Borko, Livingston, McCaleb, and Mauro (1988) found that a lack of understanding of student thinking limited the development of effective teaching methods in teachers. Furthermore, some document that pre-service teachers are unaware of the informal knowledge that students bring to the classroom (Borko & Putnam, 1996; Grouws & Schultz, 1996). We hypothesize that using the knowledge of learners can serve as a catalyst for integrating knowledge.

We note that the data for this study comes from teachers' reflections of their teaching and not other aspects of their job, such as planning. We recognize that the role of mathematics and learners may be different for other areas of teacher practice. We also recognize that these data reflect the knowledge used by only two teachers, but felt their use of mathematics and learners was particularly interesting.

One of our goals for this paper was to describe the *process* of knowledge integration. From this viewpoint, teacher knowledge is not a static object, but one of dynamic schema engagement and development. In relation to this active, constantly changing schema, *integrating to justify* and *integrating to imply* each play an important role. Many of the *integrating to justify* interactions occurred during the teachers' decision-making process; supporting the prominent role that decision-making plays in the use of integrated knowledge. However, we hypothesize that the teachers were not necessarily building connections within their schema at this point, but instead using their preexisting integrated knowledge to inform their decisions. Hence, we hypothesize that the process of *integrating to justify*, in addition to being connected to decision-making, plays a role in reinforcing existing integrated knowledge and schema.

If *integrating to justify* and decision-making are means by which teachers use their integrated knowledge, how is integrated knowledge developed? It is our contention that when teachers are *integrating to imply and reflect* they are generating new connections among their different knowledge sets. In essence, *integrating to imply and reflect* can be seen as a process that allows teachers to build and adapt the connected knowledge structures that are then used in the decision-making process. Barrett and Green (2009) stated that, "through this process of reflection, teachers transform their knowledge into active, classroom practice that continually evolves as they encounter new situations and reconsider past experiences in light of more recent experiences" (2009, p.

19). Hence, we hypothesize that the process of *integrating to imply and reflect* plays an important role in the generation of integrated knowledge.

Conclusion

In this manuscript we characterized how two fifth-grade teachers drew upon their knowledge of mathematics, learners, and pedagogy to provide instruction. This study built upon the assumption that knowledge integration is of fundamental importance to the practice of teaching. Hiebert, Gallimore, and Stigler (2002) stated, “Another characteristic of knowledge that is linked with practice is that it is integrated and organized around the problems of practice” (p.6). In essence, the knowledge that teachers’ use in the classroom is messy, integrated, and intimately tied to the context in which it is being used. The data from this study provides a snapshot of this complexity; approximately 75 percent of the teacher episodes contained multiple forms of knowledge. If this is indeed true of the larger teacher population, then the study of isolated forms of teacher knowledge, without consideration for how they will be integrated, may result in findings that are difficult to translate to the practice of teaching.

Hiebert, Gallimore, and Stigler (2002) continue, “In practitioner knowledge, all of these types of knowledge are intertwined, organized not according to type, but according to the problem the knowledge is intended to address” (p. 6). We experienced this difficulty first hand as our identification of specific knowledge types was challenging, primarily due to the integrated nature of our teachers’ comments. However, Hiebert, Gallimore, and Stigler (2002) provided an interesting comment about practitioner knowledge being organized, “according to the problem the knowledge is intended to address” (p. 6). We expand on this notion to suggest that the classification of practitioner knowledge, which is often integrated in nature, be organized around the purpose for this knowledge use and integration. Our notions of *integrating to justify* and *integrating to imply* may be two particular purposes that would fit such an organization scheme.

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