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# Search for C P violation in the $D^+ \rightarrow \pi^+ \pi^0$ decay at Belle

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Search for  $CP$  violation in the  $D^+ \rightarrow \pi^+ \pi^0$  decay at Belle

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We search for  $CP$  violation in the charged charm meson decay  $D^+ \rightarrow \pi^+\pi^0$ , based on a data sample corresponding to an integrated luminosity of  $921 \text{ fb}^{-1}$  collected by the Belle experiment at the KEKB  $e^+e^-$  asymmetric-energy collider. The measured  $CP$ -violating asymmetry is  $[+2.31 \pm 1.24(\text{stat}) \pm 0.23(\text{syst})]\%$ , which is consistent with the standard model prediction and has a significantly improved precision compared to previous results.

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Within the standard model (SM), the violation of charge-parity ( $CP$ ) symmetry in the charm system is expected to be small [ $\mathcal{O}(10^{-3})$ ] owing to suppression from the Glashow-Iliopoulos-Maiani mechanism [1]. These order-of-magnitude estimates [2] suffer from large uncertainties [3] due to nonperturbative long-distance effects resulting from a finite charm-quark mass. The problem came to the fore in 2012, when the world average of the difference in  $CP$ -violating asymmetries between  $D^0 \rightarrow K^+K^-$  and  $D^0 \rightarrow \pi^+\pi^-$  decays was measured to be  $\Delta A_{CP} = (-0.656 \pm 0.154)\%$  [4]; here, each asymmetry is

$$A_{CP}(D \rightarrow f) = \frac{\Gamma(D \rightarrow f) - \Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(\bar{D} \rightarrow \bar{f})}, \quad (1)$$

where  $\Gamma(D \rightarrow f)$  and  $\Gamma(\bar{D} \rightarrow \bar{f})$  are the decay rates for a given process and its  $CP$  conjugate, respectively. This led to much discussion as to whether the result was consistent with the SM or a signature of new physics (NP). Though the current  $\Delta A_{CP}$  value is consistent with zero [5], it is important to study those decay channels expected by the SM to exhibit negligible  $CP$  violation.

Singly Cabibbo-suppressed decays like  $D^+ \rightarrow \pi^+\pi^0$  [6] are excellent candidates to probe  $CP$  violation in the charm sector [7]. Such decays require additional strong and weak phases besides those in the tree diagram to have a sizable  $CP$  asymmetry. The phases can appear in either a strong or an electroweak loop (e.g., box diagram). As the former produces only isospin singlets, it cannot contribute to the  $I = 2$  final state of  $\pi^+\pi^0$ . On the other hand, electroweak loop diagrams have too small an amplitude of  $\mathcal{O}(10^{-6})$  for the interference to manifest  $CP$  violation. Any  $CP$  asymmetry found in these channels would therefore point to NP [7,8]. In particular, the authors of Ref. [7] suggested looking for  $CP$  violation in  $D^+ \rightarrow \pi^+\pi^0$  as well as

verifying a sum rule that relates individual asymmetries of the three isospin-related  $D \rightarrow \pi\pi$  decays as potential NP probes. The sum rule, which reduces the theoretical uncertainty due to strong interaction effects, can be characterized by the ratio

$$R = \frac{|\mathcal{A}_1|^2 - |\bar{\mathcal{A}}_1|^2 + |\mathcal{A}_2|^2 - |\bar{\mathcal{A}}_2|^2 - \frac{2}{3}(|\mathcal{A}_3|^2 - |\bar{\mathcal{A}}_3|^2)}{|\mathcal{A}_1|^2 + |\bar{\mathcal{A}}_1|^2 + |\mathcal{A}_2|^2 + |\bar{\mathcal{A}}_2|^2 + \frac{2}{3}(|\mathcal{A}_3|^2 + |\bar{\mathcal{A}}_3|^2)}, \quad (2)$$

where  $\mathcal{A}_1$ ,  $\mathcal{A}_2$ , and  $\mathcal{A}_3$  are the amplitudes of  $D^0 \rightarrow \pi^+\pi^-$ ,  $D^0 \rightarrow \pi^0\pi^0$ , and  $D^+ \rightarrow \pi^+\pi^0$ , respectively;  $\bar{\mathcal{A}}_1$ ,  $\bar{\mathcal{A}}_2$ , and  $\bar{\mathcal{A}}_3$  are those of their  $CP$  conjugates. The amplitudes are normalized so that

$$|\mathcal{A}_k|^2 \propto \frac{\mathcal{B}_k}{\tau_{0(+)} p_k}, \quad (3)$$

where  $\mathcal{B}_k$  is the branching fraction of the decay  $D \rightarrow \pi_i\pi_j$ ,  $\tau_{0(+)}$  is the appropriate  $D^0(D^+)$  lifetime, and

$$p_k = \frac{\{[m_D^2 - (m_i + m_j)^2][m_D^2 + (m_i - m_j)^2]\}^{\frac{1}{2}}}{2m_D}, \quad (4)$$

is the breakup momentum in the  $D$  rest frame. The indices  $i$  and  $j$  correspond to the pion daughters. As the masses of the charged and neutral species of the  $D$  or  $\pi$  mesons are close to each other, we consider all  $p_k$  values to be equal. We use Eqs. (3)–(4) and the relation

$$|\mathcal{A}_k|^2 - |\bar{\mathcal{A}}_k|^2 = A_{CP}(|\mathcal{A}_k|^2 + |\bar{\mathcal{A}}_k|^2) \quad (5)$$

to rewrite Eq. (2) as

$$R = \frac{A_{CP}(D^0 \rightarrow \pi^+\pi^-)}{1 + \frac{\tau_{D^0}}{\mathcal{B}_1} \left( \frac{\mathcal{B}_2}{\tau_{D^0}} + \frac{2}{3} \frac{\mathcal{B}_3}{\tau_{D^+}} \right)} + \frac{A_{CP}(D^0 \rightarrow \pi^0\pi^0)}{1 + \frac{\tau_{D^0}}{\mathcal{B}_2} \left( \frac{\mathcal{B}_1}{\tau_{D^0}} + \frac{2}{3} \frac{\mathcal{B}_3}{\tau_{D^+}} \right)} - \frac{A_{CP}(D^+ \rightarrow \pi^+\pi^0)}{1 + \frac{3}{2} \frac{\tau_{D^+}}{\mathcal{B}_3} \left( \frac{\mathcal{B}_2}{\tau_{D^0}} + \frac{\mathcal{B}_1}{\tau_{D^0}} \right)}. \quad (6)$$

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If the value of  $R$  is consistent with zero while the  $CP$  asymmetry in  $D^+ \rightarrow \pi^+\pi^0$  is nonzero [7], it would be an NP signature.

A test of the above sum rule requires the measurement of the time-integrated  $CP$  asymmetries  $A_{CP}(D^0 \rightarrow \pi^+\pi^-)$ ,  $A_{CP}(D^0 \rightarrow \pi^0\pi^0)$ , and  $A_{CP}(D^+ \rightarrow \pi^+\pi^0)$ . The current world average of  $A_{CP}(D^0 \rightarrow \pi^+\pi^-)$  is  $(+0.13 \pm 0.14)\%$  [9]. Three years ago, Belle measured  $A_{CP}(D^0 \rightarrow \pi^0\pi^0)$  as  $[-0.03 \pm 0.64(\text{stat}) \pm 0.10(\text{syst})]\%$  [10]. However the charged-mode asymmetry measured by CLEO has an uncertainty of 2.9% [11] and therefore limits the precision with which the above sum rule can be tested.

We present herein an improved measurement of  $CP$  asymmetry for the channel  $D^+ \rightarrow \pi^+\pi^0$  using the full  $e^+e^-$  collision data sample recorded by the Belle experiment [12] at the KEKB asymmetric-energy collider [13]. The data sample was recorded at three different center-of-mass (CM) energies: at the  $\Upsilon(4S)$  and  $\Upsilon(5S)$  resonances and 60 MeV below the  $\Upsilon(4S)$  peak, with corresponding integrated luminosities of 711, 121, and 89  $\text{fb}^{-1}$ , respectively. The detector components relevant for the study are a tracking system comprising a silicon vertex detector and a 50-layer central drift chamber (CDC), a particle identification device that consists of a barrel-like arrangement of time-of-flight scintillation counters (TOF) and an array of aerogel threshold Cherenkov counters (ACC), and a CsI(Tl) crystal electromagnetic calorimeter (ECL). All these components are located inside a superconducting solenoid that provides a 1.5 T magnetic field.

For the measurement, we consider an exclusive sample of  $D^\pm$  mesons tagged by  $D^{*\pm} \rightarrow D^\pm\pi^0$  decays, and another that is not tagged by the  $D^{*\pm}$  decays. The former sample has a better signal-to-noise ratio while the latter has more events. For optimal sensitivity, we combine their asymmetry measurements.

From a simultaneous fit to the invariant-mass ( $M_D$ ) distributions of the  $\pi^\pm\pi^0$  samples, we determine the raw asymmetry

$$A_{\text{raw}}^{\pi\pi} = \frac{N(D^+ \rightarrow \pi^+\pi^0) - N(D^- \rightarrow \pi^-\pi^0)}{N(D^+ \rightarrow \pi^+\pi^0) + N(D^- \rightarrow \pi^-\pi^0)}, \quad (7)$$

where  $N(D^+ \rightarrow \pi^+\pi^0)$  and  $N(D^- \rightarrow \pi^-\pi^0)$  are the yields for the signal and its  $CP$ -conjugate process, respectively.  $A_{\text{raw}}^{\pi\pi}$  has three contributing terms:

$$A_{\text{raw}}^{\pi\pi} = A_{CP}^{\pi\pi} + A_{FB} + A_\epsilon^{\pi^\pm}. \quad (8)$$

The first term,  $A_{CP}^{\pi\pi}$ , is the true asymmetry. The forward-backward asymmetry,  $A_{FB}$ , arises due to interference between the amplitudes mediated by a virtual photon, a  $Z^0$  boson, and higher-order effects [14–16] in  $e^+e^- \rightarrow c\bar{c}$ . It is an odd function of the cosine of the  $D^{*\pm}$  polar angle,  $\theta^*$ , in the CM frame. The pion-detection efficiency

asymmetry,  $A_\epsilon^{\pi^\pm}$ , is a function of the  $\pi^\pm$  momentum and polar angle.

We make use of the high-statistics normalization channel  $D^+ \rightarrow K_S^0\pi^+$  to correct the measured asymmetry for  $A_{FB}$  and  $A_\epsilon^{\pi^\pm}$ . As both signal and normalization decays arise from the same underlying process,  $A_{FB}$  should be identical for them. This assumption has been verified by checking the consistency of the  $\cos\theta^*$  distribution between the two decays. Further, we expect  $A_\epsilon^{\pi^\pm}$  to be the same if the two channels have similar pion momentum and polar-angle distributions. The angle distributions for the two channels are found to be identical. Though there is a small difference between the momentum distributions, it has been verified to have a negligible impact on the measurement. The raw asymmetry for the normalization channel is thus

$$A_{\text{raw}}^{K\pi} = A_{CP}^{K\pi} + A_{FB} + A_\epsilon^{\pi^\pm}, \quad (9)$$

where  $A_{CP}^{K\pi}$  is the  $CP$  asymmetry of  $D^+ \rightarrow K_S^0\pi^+$ ; this has been measured to be  $[-0.363 \pm 0.094(\text{stat}) \pm 0.067(\text{syst})]\%$  [17], including the  $CP$  asymmetry induced by  $K^0 - \bar{K}^0$  mixing and the difference in interactions of  $K^0$  and  $\bar{K}^0$  mesons with the detector material. The difference in the raw asymmetries is

$$\Delta A_{\text{raw}} \equiv A_{\text{raw}}^{\pi\pi} - A_{\text{raw}}^{K\pi} = A_{CP}^{\pi\pi} - A_{CP}^{K\pi}, \quad (10)$$

which leads to

$$A_{CP}^{\pi\pi} = A_{CP}^{K\pi} + \Delta A_{\text{raw}}. \quad (11)$$

Monte Carlo (MC) simulated events are used to devise and optimize the selection criteria; the size of the MC sample corresponds to an integrated luminosity six times that of the data. We perform the optimization by maximizing the signal significance,  $N_{\text{sig}}/\sqrt{N_{\text{sig}} + N_{\text{bkg}}}$ , where  $N_{\text{sig}}$  ( $N_{\text{bkg}}$ ) is the number of signal (background) events expected within a  $\pm 3\sigma$  window ( $\sigma = 15.3 \text{ MeV}/c^2$ ) around the nominal  $D$  mass [9]. The branching fraction of the signal channel used in the  $N_{\text{sig}}$  calculation is the current world average,  $1.24 \times 10^{-3}$  [9]. The background level is corrected for a possible data-MC difference by comparing yields in the  $M_D$  sidebands of 1.70–1.76 and 1.92–2.00  $\text{GeV}/c^2$ .

Charged-track candidates must originate from near the  $e^+e^-$  interaction point (IP), with an impact parameter along the  $z$  axis and in the transverse plane of less than 3.0 and 1.0 cm, respectively. (The  $z$  axis is the direction opposite the  $e^+$  beam.) They must have a momentum greater than 840  $\text{MeV}/c$ . They are treated as pions if the likelihood ratio,  $\mathcal{L}_\pi/(\mathcal{L}_\pi + \mathcal{L}_K)$ , is greater than 0.6, where  $\mathcal{L}_\pi$  and  $\mathcal{L}_K$  are the pion and kaon likelihoods, respectively. These are calculated with information from the CDC, TOF and ACC. This requirement, when applied to charged particles with a

momentum distribution similar to that of the signal decay, yields a pion identification efficiency of approximately 88% and a kaon-to-pion misidentification probability of about 7%.

The high-momentum (“hard”)  $\pi^0$  candidates that would originate from two-body  $D$  decay are reconstructed from pairs of photons by requiring the diphoton invariant mass to be within  $\pm 16$  MeV/ $c^2$  of the nominal  $\pi^0$  mass [9]. The hard  $\pi^0$  daughter photons in the barrel, forward- and backward-endcap regions of the ECL are required to have an energy greater than 50, 100 and 150 MeV, respectively. (The barrel, forward- and backward-endcap regions span the polar angle ranges 32.2–128.0°, 12.4–31.4° and 130.7–155.1°, respectively.) The thresholds for the endcap photons are higher due to the higher beam background. The hard  $\pi^0$  must have a momentum greater than 1.06 GeV/ $c$ .

Charged  $D$  meson candidates are formed by combining a charged-pion with a hard- $\pi^0$  candidate, and requiring the resultant  $M_D$  distribution to lie within  $\pm 200$  MeV/ $c^2$  of the nominal  $D$  mass [9]. For  $D^{*+}$  reconstruction in the tagged sample, low-momentum (“soft”)  $\pi^0$  candidates are reconstructed from a pair of photon candidates whose energy criteria are optimized for each ECL region; the corresponding values are listed in Table I. The soft- $\pi^0$  invariant mass is required to be within an optimized window, 125–143 MeV/ $c^2$ . It is verified during optimization that the  $\pi^0$  mass distributions in simulations are in agreement with control data consisting of a high-statistics sample of  $D^+ \rightarrow K^- \pi^+ \pi^+$  decays, with the  $D^+$  arising from  $D^{*+} \rightarrow D^+ \pi^0$ .

For the tagged sample,  $D^*$  candidates are formed by combining  $D$  mesons with soft  $\pi^0$  candidates such that the mass difference between the  $D^*$  and  $D$  candidates,  $\Delta M$ , lies within an optimized window of 139 – 142 MeV/ $c^2$ . This corresponds approximately to a  $\pm 1.5\sigma$  signal region, where  $\sigma$  is the  $\Delta M$  resolution. For the fit to extract  $A_{CP}$  (described below), two intervals of  $D^*$  CM momentum with different signal-to-background ratios are chosen:  $p_{D^*}^* > 2.95$  GeV/ $c$  and  $2.50$  GeV/ $c < p_{D^*}^* < 2.95$  GeV/ $c$ . The first corresponds to an optimized  $p_{D^*}^*$  criterion with maximal signal significance. The second interval is added to increase the statistical sensitivity of the measurement, while ensuring that the lower bound excludes  $D^*$  mesons from a  $B$ -meson decay, as the latter might introduce a nontrivial  $CP$  asymmetry.

After the above selection criteria are applied, we find that about 3% of events have multiple  $D^*$  candidates. We

TABLE I. Optimized requirements on the soft- $\pi^0$  photon energies (ECL region) in the tagged sample.

Case	$E_{\gamma 1}$ criterion	$E_{\gamma 2}$ criterion
1	>46 MeV (barrel)	>46 MeV (barrel)
2	>36 MeV (barrel)	>68 MeV (forward endcap)
3	>30 MeV (barrel)	>44 MeV (backward endcap)

perform a best-candidate selection (BCS) to remove spurious  $D^*$  candidates formed from fake soft- $\pi^0$  mesons. This is done by retaining, for each event, the candidate whose  $\Delta M$  value lies closest to the mean of the  $\Delta M$  distribution, 140.69 MeV/ $c^2$ . For events with multiple  $D^*$  candidates, with at least one of them being the true candidate, the BCS successfully identifies the correct one around 65% of the time. As the spurious  $D^*$  candidates also correspond to true  $D$  candidates, this component peaks in the  $M_D$  distribution. By performing the BCS, we ensure that only one  $D$  candidate is selected per event, and so avoid overestimating the signal component in the  $M_D$  fits.

If there are no suitable  $D^*$  candidates found in an event, the charged  $D$  candidates, if any, are considered for the untagged sample. Here, we require that the  $D$  CM momentum be above an optimized threshold of 2.65 GeV/ $c$ . In case there are multiple  $D$  candidates in the event, the one with the daughter  $\pi^0$  candidate having a reconstructed mass closest to the nominal  $\pi^0$  mass [9] is chosen. If there are still multiple surviving candidates, the one whose charged-pion daughter has the smallest transverse impact parameter is retained. About 2% of events in the untagged sample have multiple  $D$  candidates; for such events, with at least one of them being the true candidate, the BCS successfully identifies the correct one around 66% of the time.

For the normalization channel, we reconstruct  $K_S^0$  candidates from pairs of oppositely charged tracks that have an invariant mass within 30 MeV/ $c^2$  ( $\pm 5\sigma$ ) of the nominal  $K_S^0$  mass. The transverse impact parameter of the track candidates is required to be larger than 0.02 cm for high-momentum ( $>1.5$  GeV/ $c$ ) and 0.03 cm for low-momentum ( $<1.5$  GeV/ $c$ )  $K_S^0$  candidates. The  $\pi^+ \pi^-$  vertex must be displaced from the IP by a minimum (maximum) transverse (longitudinal) distance of 0.22 cm (2.40 cm) for high-momentum candidates and 0.08 cm (1.80 cm) for the remaining candidates. The direction of the  $K_S^0$  momentum must be within 0.03 rad (0.10 rad) of the direction between the IP and the vertex for high-momentum (remaining) candidates. The surviving  $K_S^0$  candidates are kinematically constrained to their nominal masses [9]. Candidate events for the  $D^+ \rightarrow K_S^0 \pi^+$  channel are selected with essentially the same requirements as for signal, except that we require the  $D$  candidate mass to lie within  $\pm 80$  MeV/ $c^2$  of the nominal  $D$  mass; the tighter criterion is due to the better mass resolution with an all-charged final state. Similar to the signal channel described earlier, nonoverlapping tagged and untagged samples are formed.

A fitting range of 1.68–2.06 GeV/ $c^2$  in  $M_D$  is chosen for the signal  $D \rightarrow \pi\pi$  channel. For the tagged sample, a simultaneous unbinned maximum-likelihood fit of the two  $p_{D^*}^*$  intervals and oppositely charged  $D$  meson candidates is performed. Similarly, for the untagged sample, a simultaneous binned maximum-likelihood fit of oppositely charged  $D$  meson candidates is done. We use a combination

of a Crystal Ball (CB) [18] and a Gaussian function to model the signal peak for both tagged and untagged fits. The background in the tagged fit is parametrized by the sum of a reversed CB and a linear polynomial, while that for the untagged fit uses a quadratic rather than a linear polynomial. All signal shape parameters for the tagged fit are fixed to MC values except for an overall mean and a width scaling factor, which are floated. We introduce the scaling factor to account for the possible difference between data and simulations. For the untagged fit, all shape parameters are fixed to MC values, aside from the overall mean, which is floated, and the width scaling factor, which is fixed from the tagged-data fit. For the background, the cutoff and tail parameters of the reversed CB are fixed from MC events, and all other shape parameters are floated. For the tagged fit, the two  $p_{D^*}^*$  intervals are required to have a common signal asymmetry but have separate background asymmetries. For the tagged sample, the total signal yield obtained from the fit is  $6632 \pm 256$  with  $A_{\text{raw}}^{\pi\pi} = (+0.52 \pm 1.92)\%$ ; the corresponding results for the untagged sample are  $100934 \pm 1952$  and

$(+3.77 \pm 1.60)\%$ . The quoted uncertainties are statistical. Figures 1 and 2 show the projections of the simultaneous fit performed on the tagged and untagged data samples, respectively.

For the  $D^+ \rightarrow K_S^0 \pi^+$  normalization channel, a fitting range of  $1.80\text{--}1.94 \text{ GeV}/c^2$  is chosen and the simultaneous fits for the tagged sample, with two  $p_{D^*}^*$  intervals, and the untagged sample are performed as for the  $D \rightarrow \pi\pi$  signal channel. The narrower fitting range can be afforded because of the better  $D$ -mass resolution. The signal peak is modeled with the sum of a Gaussian and an asymmetric Gaussian function, with all shape parameters floated. The background shape is parametrized with a linear polynomial, whose slope is floated. The total signal yield obtained from the tagged fit is  $68434 \pm 308$  with  $A_{\text{raw}}^{K\pi} = (-0.29 \pm 0.44)\%$ ; the corresponding results for the untagged sample are  $982029 \pm 1797$  and  $(-0.25 \pm 0.17)\%$ . The quoted uncertainties are again statistical. Figure 3 shows the projections of the simultaneous fit performed on the tagged and untagged data samples.

From the results of the fit to the signal and normalization channels, we calculate  $\Delta A_{\text{raw}}$  (tagged) =  $(+0.81 \pm 1.97 \pm 0.19)\%$  and  $\Delta A_{\text{raw}}$  (untagged) =  $(+4.02 \pm 1.61 \pm 0.32)\%$ . The first uncertainty quoted in each measurement is

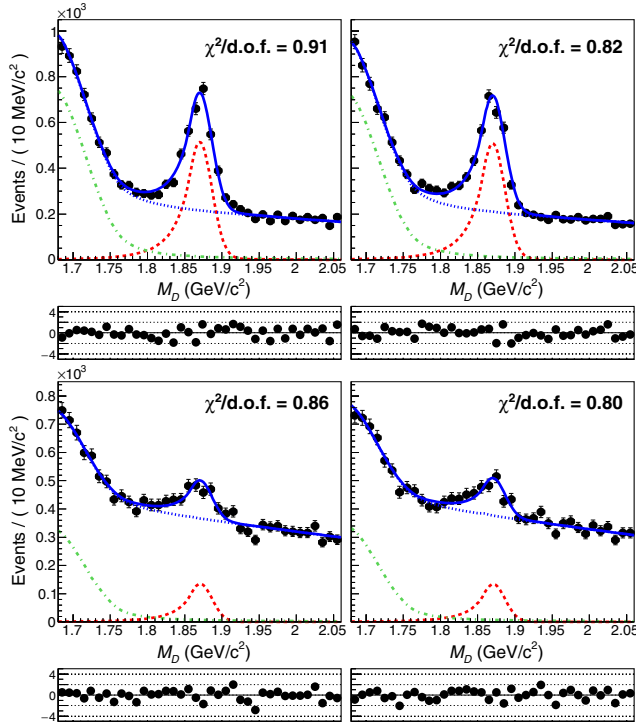


FIG. 1. Invariant mass distributions for the  $\pi^{\pm}\pi^0$  system for the tagged  $D \rightarrow \pi\pi$  sample in the intervals  $p_{D^*}^* > 2.95 \text{ GeV}/c$  (top) and  $2.50 \text{ GeV}/c < p_{D^*}^* < 2.95 \text{ GeV}/c$  (bottom). Left (right) panels correspond to  $D^+$  ( $D^-$ ) samples. Points with error bars are the data. The solid blue curves are the results of the fit. The red dashed, blue dotted and green dash-dotted curves show the signal, total- and peaking-background contributions, respectively. The normalized residuals are shown below each distribution, and the post-fit  $\chi^2$  per degree of freedom ( $\chi^2/\text{d.o.f.}$ ) is given in each panel.

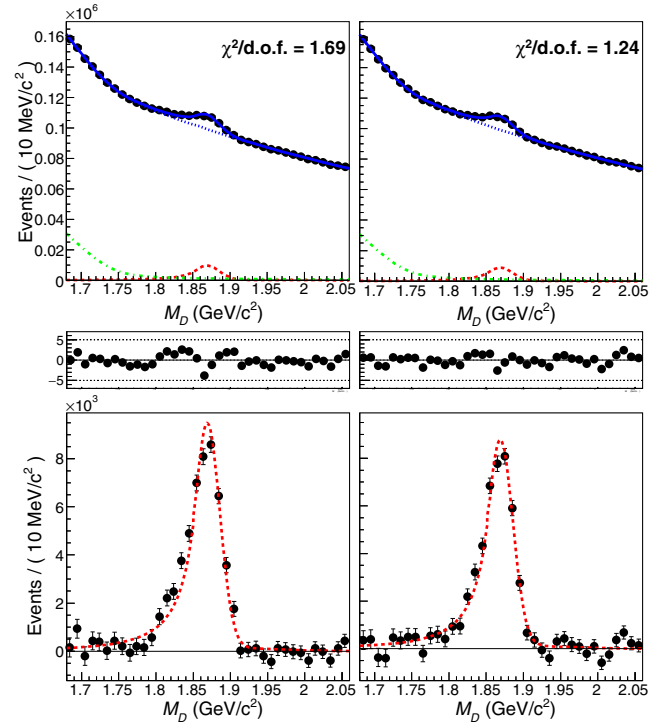


FIG. 2. Invariant mass distributions for the  $\pi^{\pm}\pi^0$  system for the untagged  $D \rightarrow \pi\pi$  sample. The top two panels are the full distributions with signal and background components, while the bottom two show the corresponding background-subtracted distributions. Left (right) panels correspond to  $D^+$  ( $D^-$ ) samples. Points with error bars, colored curves, and residual plots are described in the caption of Fig. 1.

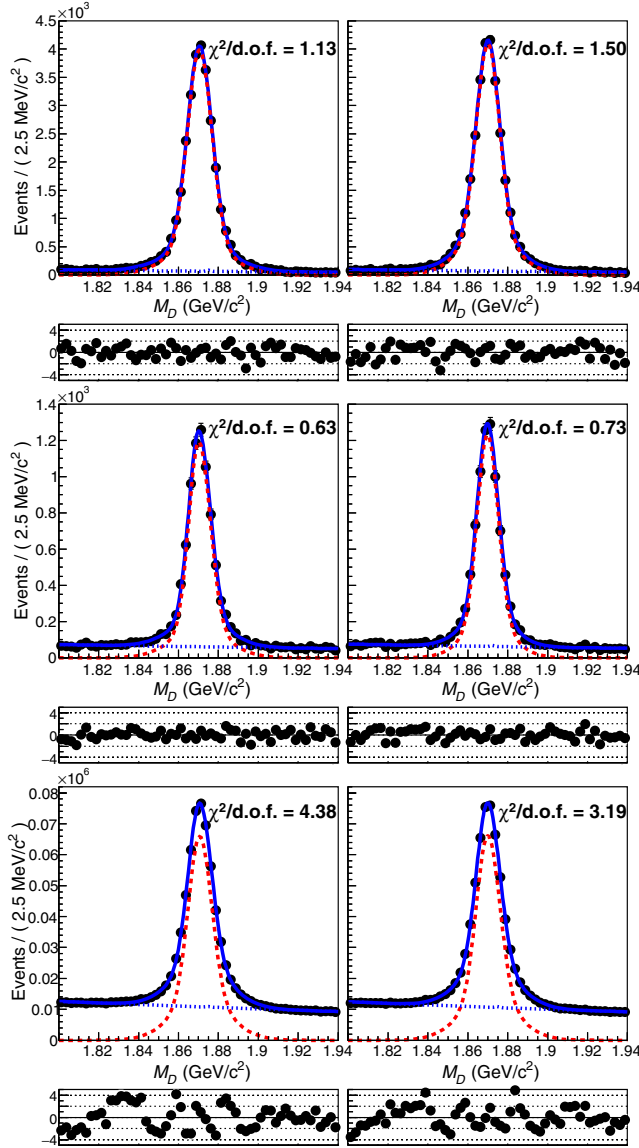


FIG. 3. Invariant mass distributions for the  $K_S^0\pi^\pm$  system for the normalization channel,  $D \rightarrow K_S^0\pi^\pm$ , in the intervals  $p_{D^*}^* > 2.95$  GeV/c (top) and  $2.50$  GeV/c  $< p_{D^*}^* < 2.95$  GeV/c (middle) for the tagged sample, and for the untagged sample (bottom). Left (right) panels correspond to  $D^+$  ( $D^-$ ) samples. Points with error bars, colored curves, and residual plots are described in the caption of Fig. 1.

statistical and the second is systematic (see below). A combination of the two [19] gives

$$\Delta A_{\text{raw}} = (+2.67 \pm 1.24 \pm 0.20)\%, \quad (12)$$

which, in conjunction with the world average of  $A_{CP}(D^+ \rightarrow K_S^0\pi^+)$  [9], results in

$$A_{CP}(D^+ \rightarrow \pi^+\pi^0) = (+2.31 \pm 1.24 \pm 0.23)\%. \quad (13)$$

The major sources of systematic uncertainty for the  $A_{CP}$  measurement are (i) uncertainty in the signal and

background shapes for the  $D \rightarrow \pi\pi$  fits, (ii) uncertainty in modeling the peaking-background shape, and (iii) uncertainty in the  $A_{CP}$  measurement for the normalization channel. Source (i) arises from fixing some of the shape parameters to MC values. Its contribution to the systematic uncertainties is estimated by constructing an ensemble of fits, randomizing the fixed parameters with Gaussian distributions whose mean and width are set to MC values and then extracting the RMS of the  $A_{\text{raw}}$  distribution obtained from the fits. The peaking background of source (ii) is due to misreconstructed  $D$  or  $D_s$  meson decays and exhibits a broad peaking structure shifted to the left of the signal peak (Figs. 1 and 2). As it is only partially present in the fitting range, the reversed-CB shape is subject to uncertainty. We vary the lower  $M_D$  threshold between 1.68 to 1.72 GeV/ $c^2$  in steps of 10 MeV/ $c^2$  and then refit to assess the impact on the signal's  $A_{CP}$  determination. For source (iii), we rely on the world average of  $A_{CP}(D^+ \rightarrow K_S^0\pi^+)$  [9]. The various sources of systematic uncertainties and their values are listed in Table II. The total uncertainty is  $\pm 0.23\%$ .

In summary, we have measured the  $CP$ -violating asymmetry  $A_{CP}$  for the  $D^+ \rightarrow \pi^+\pi^0$  decay using 921 fb $^{-1}$  of data, with the combined result from two disjoint samples: one tagged by the decay  $D^{*+} \rightarrow D^+\pi^0$  and the other untagged. After correcting for the forward-backward asymmetry and detector-induced efficiency asymmetry, based on the normalization channel  $D^+ \rightarrow K_S^0\pi^+$ , we obtain  $A_{CP}(D^+ \rightarrow \pi^+\pi^0) = [+2.31 \pm 1.24(\text{stat}) \pm 0.23(\text{syst})]\%$ . The result is consistent with the SM expectation of null asymmetry and improves the precision by more than a factor of two over the previous measurement [11]. Inserting this result into Eq. (6) along with the current world averages of  $A_{CP}$  and  $\mathcal{B}$  for  $D^0 \rightarrow \pi^+\pi^-$  [9] and  $D^0 \rightarrow \pi^0\pi^0$  [10] decays, as well as  $\tau_{0(+)}$  [9], we obtain  $R = (-2.2 \pm 2.7) \times 10^{-3}$ . The isospin sum rule holds to a precision of three per mille, putting constraints on the NP parameter space [7]. As the statistical error of  $A_{CP}(D^0 \rightarrow \pi^0\pi^0)$ , as well as of our result, dominate the total uncertainty on  $R$ , we expect a substantial improvement in testing the sum rule from the upcoming Belle II experiment [20].

TABLE II. Summary of systematic uncertainties (%) on  $A_{CP}$ .

Source	$D \rightarrow \pi\pi$ tagged	$D \rightarrow \pi\pi$ untagged
Signal shape	$\pm 0.02$	$\pm 0.23$
Peaking background shape	$\pm 0.19$	$\pm 0.22$
$\Delta A_{\text{raw}}$ measurement	$\pm 0.19$	$\pm 0.32$
$A_{CP}(D \rightarrow K_S^0\pi)$ measurement		$\pm 0.12$
Total (combined $A_{CP}$ measurement)		$\pm 0.23$



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