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"What Was *Really* Accomplished Today?"

Mathematics Content Specialists Observe a Class for Prospective K–8 Teachers

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Abstract: *One of the important activities mathematics teacher educators engage in is the development of teachers at both the in-service and pre-service levels. Also of importance is the professional development of these professional developers. In the summer of 2004, a summer institute was held that allowed mathematics teacher educators watch the teaching of a mathematics content course for prospective K–8 teachers. This paper examines the manner in which a specific group of mathematics content specialists experienced this professional development.*

Key words: pre-service teacher education; mathematics content for teachers; middle school mathematics; teacher professional development

Introduction

One of the important activities mathematics teacher educators engage in is the development of teachers at both the inservice and preservice levels. Teacher educators in the field of mathematics education work in a variety of settings, including university departments of

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mathematics, schools or colleges of education, local school districts or state departments of education, or private organizations. These teacher educators may engage mathematics teachers in mathematical content, pedagogical strategies and techniques, or some combination of content and pedagogy. They design experiences to help teachers improve their understanding of mathematics and develop their pedagogical practices. Learning to teach is an ongoing process and teachers at both the preservice and inservice stages are expected to engage in professional development activities to further refine and improve their practice.

Teacher educators however, are also first and foremost teachers. So if learning to teach is a continual process, it makes sense to speak of the ongoing development not only for classroom teachers, but for teacher educators, as well. This article discusses a professional development opportunity designed to allow mathematics teacher educators to examine, explore and discuss the development of preservice teachers in the context of mathematical knowledge for teaching (MKT) (Hill, Rowan, & Ball, 2005). The idea of formal professional development for teacher educators is a relatively recent phenomenon in the field of mathematics education and little is known about how teacher educators might engage in improving their own practice or how one might design a professional development experience for participants from these varied backgrounds and work settings. Thus, as we examine the manner in which these teacher educators experienced the professional development, it is critical to ask who these teacher educators are, what they think and believe about the practice of teaching and learning, and how they might examine and improve their practice.

Institute Description

During the summer of 2004, The Center for Proficiency in Teaching Mathematics (CPTM), a NSF-funded research effort at the University of Georgia and the University of Michigan, held an eight day summer institute entitled “Developing Teachers’ Mathematical Knowledge for Teaching”. The internal language of CPTM planners often described the purpose of the institute as *the professional development of professional developers.* The institute had 65 participants, along with numerous special guests, doctoral students as participant-researchers or staff, outside observers, and part-time visitors. Participants included mathematicians; university-based mathematics educators from departments of mathematics or schools or colleges of education; school-, district-, and state-level professional developers; and representatives of independent professional development organizations.

The institute centered on a mathematics content course for prospective K–8 teachers. The specific content of the course was the conceptual understanding of fractions. This setting was used as a context to study the learning of mathematics teacher educators. Some of the overarching questions to be answered by CPTM included "What do teacher developers know and believe? What do they need to learn? What is challenging about their work that many do not learn simply from experience? What content knowledge do they need? What do they need to know about their own learners and about how to relate to them effectively?"(Sztajn, Ball, & McMahon, 2006).

The mathematics content course met in a large banquet room, which allowed participants to observe the lessons. Participants were not allowed to interact with members of the class or to interrupt the proceedings of the class. Before class each day, the entire group of participants met for forty-five minutes with the instructor or with one of the institute planners to examine the lesson plan for the day. In these *Preparation* sessions, participants were encouraged to question the instructor as to her goals and plans and to make specific suggestions for how each class might be conducted. Participants also met with the instructor or with institute planners for forty-five minutes immediately after each class to discuss their observations. For these *Lab Analysis and Discussion* sessions, participants were divided into three, approximately equal, focus groups. For three days of the institute, each focus group was given a specific assignment during the lab class, either to focus on the mathematics of the class, the teaching of the class, or student learning in the class, so that over the course of the three days, each focus group attended to each of the three foci.

Data Sources and Research Questions

Each of the lab classes was videotaped from multiple perspectives, as were the Preparation sessions and the Lab Analysis and Discussion sessions with participants. Participants were each given a notebook in which to record their observation notes and comments on the lab classes. At various times during the institute, they were also given prompts for reflection or for their reaction to specific events relative to the planning or implementation of the lab class. These notebooks were collected, scanned, and returned to the participants. Field notes on individual participants as well as whole group field notes during the Preparation and the Lab Analysis and Discussion sessions were taken.

> The overall goal of studying the Summer Institute is to better understand the interactions between the participants and the learning opportunities they experience in this professional development initiative. To this regard, we will carefully examine who the institute participants are, as well as the planning and development of experiences in which they will engage this summer. (Ball, Sztajn, & McMahon, CPTM internal document, June 2004)

During the institute, participants were asked to sign a consent form in which the research goal was again stated as "to better understand the interactions between the participants and the learning opportunities they experience in this professional development initiative." Specific research questions of the institute related to the goals and evolving design of the institute's curriculum, the ways in which participants with differing characteristics interacted with the curriculum and how those interactions might change during the institute, and how participants viewed the mathematical knowledge and work of elementary preservice teachers and how those views might change during the institute. Interest in participants' backgrounds is expressed in the goal statement above and is implicit in each research question stated above.

We chose to focus on the question "How do participants with differing characteristics view the mathematical knowledge and work of preservice elementary teachers?" In this paper, we describe the identification of a particular subgroup from within the 65 participants and analyze selected data to address the question. We present evidence of how seven *mathematics content specialists* looked at and responded to a laboratory class for prospective K–8 mathematics teachers during a one-week professional development institute for teacher educators.

Participant Selection

We decided to focus on a group of mathematics educators that we called *mathematics content specialists.* This subgroup of participants were selected based on the following criteria:

1. Work in a department of mathematics
2. Do not teach mathematics pedagogy courses for K–8 preservice teachers
3. Teach mathematics courses, though not necessarily for K–8 preservice teachers.

We found 21 such participants in the institute. These 21 mathematics content specialists were equally divided among three subgroups. We selected one of the subgroups for our study. Our choice of one particular subgroup was based on our observations that several of the participants in this group were particularly vocal in group discussions during the institute. They seemed open to sharing ideas and opinions on each of the three foci of the institute: student learning, teaching, and mathematics. Descriptive information on these seven people, taken from their applications to attend the institute, is shown in Table 1.

Table 1:

Seven content specialists from Focus Group 1 and selected characteristics.

Pseudonym	# of math courses taught to K–8 PSTs	# of math education courses taught to K–8 PSTs	# of other courses taught to K–8 PSTs	# of other math courses taught	# of other math education courses taught	# of other courses taught	# of math courses you plan to teach to K–8 PSTs	# of math education courses you plan to teach to K–8 PSTs	# of other courses you plan to teach to K–8 PSTs	K–8 teaching experience (in years)
Donna	10	0	0	5	3	0	4	0	0	0
Sharona	4	0	0	8	0	0	4	0	0	0
Lona	3	0	0	8	0	1	2	0	0	0
John	3	0	0	11	0	0	4	1	0	0
Emily	1	0	0	6	0	0	2	0	0	0
Darryl	0	0	0	9	0	0	1	0	0	0
James	0	0	0	5	0	0	1	0	0	0

Each of these participants reported that he or she encountered K–8 preservice teachers only in mathematics content courses, if at all. None had K–8 teaching experience or taught mathematics education courses for K–8 preservice teachers, and only one planned to teach an education course for K–8 preservice teachers in the future. Hereafter, when we use the term *participants*, we refer only to the seven mathematical content specialists chosen for the analysis in this paper.

Analysis and Discussion

In order to examine how the participants viewed the mathematical knowledge and work of the preservice teachers we used researcher field notes, written transcriptions of discussions that were held after Lab Analysis and Discussion sessions as well as participant's individual notebook entries. For most of the participants these notebook entries do not contain a reflective or critical component. They are, rather, lists of events and direct quotations of statements made by students in the class, much like the notes one might take during a lecture without comment or reflection. They consist of a simple record of what happened and what was said. There are exceptions in that some of the seven participants wrote comments beside their observations or wrote summary reflections after the lab class.

The Cookie Jar Problem

In order to analyze participants' views of the mathematical knowledge and work of the preservice teachers, we examined a particular event within the lab class and the participants' reactions to that event. Even though this event occurred on a single day of the course, discussion of the event continued throughout the three days in which our participants attended to teaching, learning, and mathematics. Students in the lab class were presented with the following problem on the second day of the class:

> There was a jar of cookies on the table. Kim was hungry because she hadn't had breakfast, so she ate half the cookies. Then Stan came along and noticed the cookies. He thought they looked good, so he ate a third of what was left in the jar. Nita came by and decided to take a fourth of the remaining cookies with her to her next class. Then Karen came dashing up and took a cookie to munch on. When Patty looked at the cookie jar, she saw that there were two cookies left. "How many cookies were in the jar to begin with?" she asked Kim (lesson plans).

Students worked in pairs and individuals presented solutions for class discussion. The event on which we wish to focus is a presentation by one of the students, Tessa; however, in order to place her presentation in context, we first briefly describe the preceding presentations.

Stan presented a solution to the cookie jar problem in the form of a sketch, as shown in Figure 1. Stan solved the problem by working backward through the given information. That is, he first drew the two squares in the upper left box to represent the two cookies Patty saw in the end. He then drew the lower left square to represent the one cookie taken by Karen. The middle square on the bottom row represents the cookie taken by Nita, or one-fourth of the cookies remaining. He then drew the two squares in the middle to represent one-third of the remaining cookies taken by Stan. His final step was to draw the six squares on the right to represent the one-half of the cookies first taken from the initial state of the cookie jar. His answer to the question then became a simple matter of counting the squares he had drawn to represent the cookies to arrive at an answer of 12.

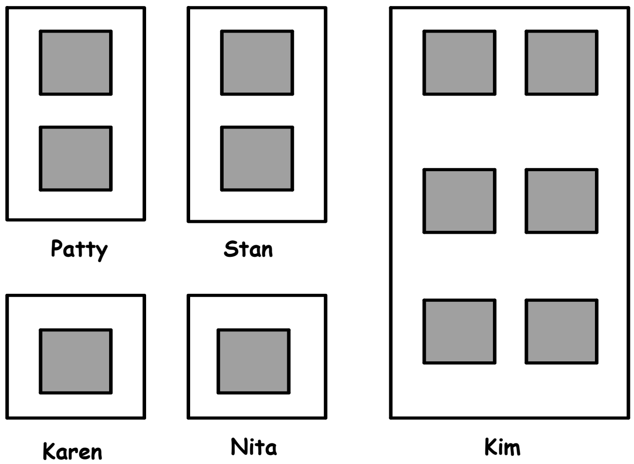

Figure 1: Stan's solution to the cookie jar problem by working backward.

Nita, as shown in Figure 2, presented an algebraic solution. Since our intent in this paper is merely to use these earlier solutions to illustrate the context for Tessa's later work, we do not discuss Nita's algebraic solution in detail. However, we should comment that Nita's incorrect answer was the result of a mistake of taking a fraction of the cookies *remaining* versus taking a fraction of the cookies already taken. Students seemed to be enamored of Nita's approach and discussion of this algebraic solution consumed a considerable portion of class time. The cookie jar problem was revisited at the beginning of the next class. Students again worked in pairs to produce alternate solutions. Sharon worked with Nita to produce a solution that Shelly presented to the class, as shown in Figure 3.

$$\left(\frac{1}{2}x-\frac{1}{3}\left(\frac{1}{2}x\right)-\frac{1}{4}\left(\frac{1}{3}\right)\left(\frac{1}{2}x\right)-\frac{1}{4}\left(\frac{1}{3}\right)\left(\frac{1}{2}x\right)\right)-1=2$$

$$\left(\frac{1}{2}x-\frac{1}{3}\left(\frac{1}{2}x\right)-\frac{1}{4}\left(\frac{2}{3}\right)\left(\frac{1}{2}x\right)\right)-1=2$$

...

$$x-3=\frac{1}{2}x+\frac{1}{6}x+\frac{1}{24}x$$

$$x-3=\frac{12}{24}x+\frac{4}{24}x+\frac{1}{24}x$$

$$x=\frac{17}{24}x+\frac{72}{24}$$

$$x\left(\frac{24}{17}\right)=\frac{72}{24}$$

$$x\approx 2.12$$

Figure 2: Nita's algebraic solution.

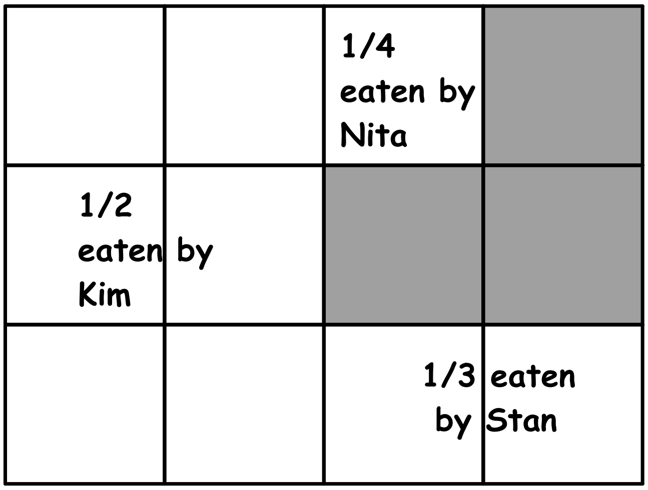

Figure 3: Shelly's solution to the cookie jar problem

Unlike Stan's solution, Shelly worked forward through the problem statement by first drawing the large outer rectangle to represent the cookie jar in its original state. She drew a vertical segment to divide the rectangle in half, and labeled the left half as "1/2 eaten by Kim." She divided the right half into thirds horizontally and labeled the bottom third as "1/3 eaten by Stan." She then divided the upper square on the right side into fourths and labeled the unshaded square as "1/4 eaten by Nita." She reasoned that, since there were three small squares remaining, these must represent the one cookie eaten by Karen and the two cookies remaining in the jar. She concluded that each small square must represent a single cookie, and was thus able to divide the larger rectangle into 12 small squares to arrive at an answer of 12 cookies originally in the jar. Shelly's solution might be characterized as an area model for the solution to the cookie jar problem, and it is in this context that Tessa offered her solution.

In Figure 4, we see Tessa's solution to the cookie jar problem. As she produced this sketch, she mapped the words of the problem onto her emerging representation. First, she drew a circle to represent all the cookies in the cookie jar at the beginning of the problem. She then drew

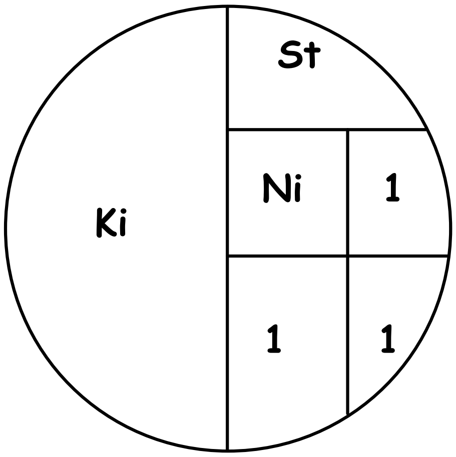

Figure 4: Tessa's solution

a vertical segment and labeled the left part as "Ki" to represent the one-half of the cookies taken by Kim. She then drew the two horizontal segments in order to "divide this into three parts" and labeled the top portion as "St," to represent the one-third of the remaining cookies eaten by Stan. Tessa then divided the remaining, unlabeled, portion of the circle into four parts by drawing a vertical segment. She experienced some confusion about how to label these four new parts, and initially placed the numeral 1 in the lower right portion before erasing it to rethink her solution. Her confusion seemed to remain until the instructor of the course said, "Who took one-fourth of the remaining cookies?" With this question, Tessa labeled one of the remaining parts as "Ni," for the one-fourth of the remaining cookies eaten by Nita. She then quickly placed a "1" in each of the remaining parts to represent the one cookie taken by Karen and the two cookies remaining. She then had no trouble arriving at an answer of 12 cookies for the number of cookies originally in the cookie jar.

One student in the lab class commented that when Tessa drew the horizontal segments in the right portion of her circle, she did not really create equal parts. Tessa's response was, "I don't do very many math problems. Although I've seen the pie diagrams, it just didn't occur to me. When I drew this [the thirds], I knew it didn't look like they were equal portions, but you can certainly take a third of a half. I decided not to worry about them not being proportional. But it did bug me and I was hoping I wouldn't have to deal with any algebra …." The instructor interrupted to ask, "Are you saying your drawing is or is not representing equal parts, or are you just not worrying about it, or …." Tessa replied, "It works for *me.*" There was a short discussion in which some students saw Tessa and Shelly's representations as equivalent and others expressed a personal, but not mathematical, preference for Shelly's rectangular representation. The discussion concluded when one student commented that this way made sense to Tessa and that was all that mattered. We will use participants' comments on Tessa's partition of the semicircular region into three parts, as well as the cookie jar problem in general, as a context to examine, the participants' views of the mathematical knowledge and work of the preservice teachers.

The Cookie Jar Problem and Mathematical Knowledge

In general, the participants focused on correctness of mathematics and clarity of explanations. Each of the participants noted that one or another of the students in the lab class *got it* or *didn't get it*, or *understood* or *didn't understand*. We can only infer that the participants made these dichotomous judgments against some absolute standard of mathematical correctness. Similarly, participants described student presentations of solutions as *clear* or *elegant,* as opposed to *confused*, *fuzzy,* or *muddled*. Again, these characterizations of explanation seem relative to some predetermined and absolute standard. We also wish to state here what others at the institute suggested to the mathematics content specialists, namely, that they seemed to concentrate on what the students in the lab class did not know, rather than what the students did know. Interpreted another way, the participants considered mathematics from their own understanding rather than attempting to understand the subtleties of, or create a model of, each student's understanding of the problem. From a beliefs standpoint, this type of constructivist stance would indicate a viewpoint categorized by Kuhs and Ball (1986) as content-focused with an emphasis on conceptual understanding. The content specialists' responses however, with their allusions to absolutism are an indicator of content-focused with an emphasis on performance model of mathematics teaching. Further evidence of this classification is presented in the following expositions.

Each of the seven participants noted Tessa's incorrect division of the semicircle into thirds and that no one in the lab class had strongly objected to her division. Four of the participants seemed to have dropped the issue after the class discussion that, "It works for Tessa, and that's all that matters," and one of these four, Sharona, noted that Tessa's drawing was a *tool* for solving the problem, implying that a blunt tool, so to speak, was acceptable if it accomplished the job at hand. The other three participants, however, were more adamant in their objection to allowing Tessa's mistake to pass uncorrected. James, for example, questioned in his summarizing notes for the day, “Does Tessa’s misleading area drawing ever get corrected? In this course do we cultivate skepticism and constructive self-criticism? What about analysis? Or is it all just validation?” We take James’ comments, along with those of other participants, as further evidence that the participants expected the use, presumably by the instructor, of some external standard of correctness and explanation.

During a focus group discussion of this episode, one of the CPTM staff suggested to James, Darryl, and John that perhaps Tessa did not imply an area model in her division of the semicircle. Rather, in her explanation Tessa stated that she needed to divide the semicircle into “three parts” and proceeded to do so, consistent with a *discrete* model of the cookie jar problem. Thought of as a discrete situation, there is no real need for the cookies, and therefore the parts of the diagram, to be of equal size. Some of the participants strongly stated that this interpretation of Tessa’s solution was far beyond what she was actually thinking and that it was “not appropriate to make excuses and justifications for what Tessa did. She was a student in the class.” (James, LAD discussion). We interpret James’ comment that Tessa “was a student in the class” to mean that she should be judged, based on the physical and oral evidence she produced, against some preexisting standard rather than on any model of her understanding as created by an observer.

Another example of the content specialists’ beliefs came about during a discussion on the role of definition. To solve the cookie jar problem, one must resolve, either explicitly or implicitly, the issue of what came to be known at the institute as *the shifting whole.* That is, the language of the problem requires that the *unit* for each fraction of cookies be redefined at successive steps in the problem. When Stan takes one third of the cookies left in the jar, and later when Nita takes one fourth of the remaining cookies, the solver must realize that as operators, these fractions do not operate on the same quantities. One might also need to consider that, with certain solution strategies such as the algebraic one shown in Figure 2, one third of the cookies remaining are equivalent to two thirds of the cookies taken, for example. In either of these situations, the unit associated with each fraction changes, or shifts.

Concern over the notion of the shifting whole initiated a discussion concerning the definition of fraction. One of the stated goals of the lab class was to explore the concept of the *unit* in understanding fractions and there seemed to be some concern among participants that this concept was not addressed explicitly, but rather left for each student to develop individually through exploration of the cookie jar, and other problems. The data suggest that, for the institute participants as a whole, there was an issue with not providing the students in the lab class with a definition of fraction. Though this omission is a pedagogical decision, the concern over the need for a definition implies a view of mathematics as growing from given definitions and postulates rather than from experience. The participants noted this issue of the *shifting whole* and the call for a *clear* definition of fraction. We discuss this issue further in a later section.

The Cookie Jar Problem and Student Work

Participants' focus on mathematics carried over into their observations of student work. Participants noted students' fluid or hesitant use of mathematical language, correct or incorrect representations, and clarity or fuzziness of explanation. Each of the participants commented on Tessa's non-proportional division of the semicircle into thirds and five of them noted that there seemed to be little concern among the students over her mathematically incorrect representation.

In general, participants seemed surprised by students' lack of acceptance of a pictorial or geometric solution and explanation. Five of the seven mathematics content specialists noted students' focus on the presented algebraic solution to the cookie jar problem. In particular, James, John, Darryl, and Sharona each commented, in some way, that Stan's initial pictorial solution was elegant and convincing, yet, as James stated,

> I was struck by the value that they put on the mathematics. The first solution was clear, and so there was almost no need for discussion, but they seem to feel that the algebraic argument, even though they demonstrated very clearly that it was tricky and error prone [was necessary to 'prove' the solution was correct.] The thing that struck me was that they put down their initial solution and even excused it as being the kind of lame production [one might expect from] people who have their mathematical background.

One of the students actually made the comment after presenting a graphical representation of the solution that, "Of course you would need to prove it with algebra." There is no indication in the data that the participants discussed how students might have come to hold either a view of algebra as some sort of ultimate arbiter of truth or of graphical or geometric solutions as somehow inadequate. Understanding however, that one facet of a classroom teacher's pedagogical content knowledge is his understanding of what his students know and how they learn, it is important to pay attention to how the mathematical content specialists viewed the students in the lab class. Specifically, they were surprised by how quickly the pictorial solution was dismissed by the students due to its apparent sophistication.

Discussions of classroom discourse, language and clarity, mentioned above relative to mathematics, also appeared in comments about students. Donna commented about the lab class's "impressive student interaction, especially for the first day" and Lana noted how, on the first day "important patterns [were being] established for the course". The same participants also commented in a whole group discussion about the focus of the lab class on student mathematical explanations. Donna commented on the "fuzziness" of the student explanations and what was expected in their explanations. She also took detailed notes on individual students during one of the lab classes and noted how each student explained or represented a solution. She commented in her notebook:

> I know that when I started teaching fractions some of the connections between division and other models of fractions were not obvious to me—need to admit that, so of course it's hard for the students too; it's just hard."

She seemed to understand the difficulty these students had with the mathematics content because she had her own problems with teaching it. Whether or not this was a new revelation for her as a result of this experience was not clear from the data. Two content specialists also commented on the importance of the students using correct mathematical language. Lana commented in her notebook that she was impressed by the students' mathematical language. James commented on correct language as a goal of the course, noting that getting such precision "is really, really hard and a short course like this can't achieve this."

Conclusions, Discussion, and Implications

Professional development for teacher educators beyond their initial preparation is a recent consideration in mathematics education. Those who plan and implement such professional development situations must take into account the intended consumers of their products. In this paper, we examined a particular professional development institute and identified a set of its attendees whom we designated as *mathematics content specialists.* We attempted to analyze the ways in which these participants viewed the mathematical knowledge and work of preservice mathematics teachers.

Content Specialists and Mathematical Knowledge

When the content specialists were asked to focus on the mathematical knowledge demonstrated by the students in the lesson, they focused on the correctness of the mathematics and the clarity of explanations. The content specialists were concerned that there was no explicit discussion in the class about the shifting whole in the cookie jar problem nor was a definition for fraction ever given to the students. Other participants at the institute suggested that the content specialists seemed to focus more on the mathematics that the students did not know rather than on the mathematics that they did know.

The content specialists seemed to consider the mathematics based on their own understandings instead of attempting to understand the mathematics of the students. They seemed to make dichotomous judgments based on whether a student *got it* or *didn't get it* or *understood* or *didn't understand.* More importantly, they seemed to base these judgments on some absolute standard of mathematical correctness. From one participant in particular, it seemed that as students in the class, they were meant to be judged based on this preexisting standard of mathematics that seemed to be the mathematics as understood by the content specialist.

Content Specialists and Mathematical Work

When the content specialists were asked to focus on the mathematical work of the students, their journals noted students' use of mathematical language, representations, and explanations. Each commented on Tessa's non-proportional division of the semi-circle into thirds, and five of them commented on the lack of concern among the other students about the incorrect representation. Five of the participants also commented about the elegance and convincing pictorial solution given by Stan but that the students seemed to focus on the importance of using algebra to "prove" the solution was correct even though the algebra of the problem was found to be error prone. The participants seemed to think that the students in the class did not value the pictorial representation as much as the algebraic representation because the picture did not seem to be mathematical enough.

The content specialists seemed baffled by the students' inability to accept Stan's pictorial solution as a correct mathematical approach. Instead the students made an excuse for the pictorial representation being one that someone with a lesser mathematical background would produce instead of using algebra. There was no data showing that the participants discussed or thought about how these students would come to this conclusion; however, it seems very similar to the content specialists wanting to judge the students based on pre-existing standards. So, perhaps, the students were also aware of some pre-existing standard of which they were to be judged.

Implications for Future Professional Development of Professional Developers

With these conclusions in mind, we suggest that those who plan and implement future professional development that include mathematics content specialists as participants consider some explicit orientation and discussion of the philosophies that guided the design of the institute. We do not suggest such orientation necessarily include debate over the correctness or desirability of any particular philosophy, but merely should indicate that a particular philosophy is explicitly in play for the work of the institute and one aspect of participation is to observe and discuss where and how that philosophy operates in practice.

Second, we believe that this institute for professional development did indeed, for the participants in this study, "create a high level of cognitive dissonance to upset the balance between teachers' beliefs and practices and new information or experiences about students, the content, or learning" (Loucks-Horsley et al., 2003, p. 45). However, based on the results of this study, we suggest that planners of future professional development for teacher educators should better anticipate and take advantage of perturbations that might arise relative to the beliefs that teacher educators bring with them. Hopefully, other data from this institute will elaborate on how these and other participants viewed and interacted with the intended professional development curriculum.

Finally, we see evidence that the participants in this study were keen observers of mathematics, mathematics learning, and mathematics teaching, if from a particular viewpoint. Though there is some evidence that participants considered their own practice as they participated in the institute, what is not evident is whether or not any participant considered changes, or actually changed, either his or her teaching practices or views of student learning. This is not to conclude that participants were not reflective, but merely to note that asking participants to observe a mathematics course and to record observations in a notebook or to discuss observations with other participants may not be sufficient to open participants' own practices to the careful examination that might lead to changes in practice. We suggest that future professional development experiences that target teacher educators should include some structure to open participants' practices to discussion and to examine those practices in a more public way. We also suggest that future professional development include some device to evaluate the effectiveness of the program, assuming that teacher educator change is one goal of professional development. This evaluation might include post-program surveys, interviews, or data from participants' own teaching, such as course syllabi, selected mathematical tasks and assessment, or classroom observations.

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