


Summer 2010

# A Study of the Relationships between Epistemological Beliefs and Self-Regulated Learning among Advanced Placement Calculus Students in the Context of Mathematical Problem Solving

James Clinton Stockton  
*Kennesaw State University*

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A STUDY OF THE RELATIONSHIPS BETWEEN EPISTEMOLOGICAL BELIEFS  
AND SELF-REGULATED LEARNING AMONG ADVANCED PLACEMENT \*  
CALCULUS STUDENTS IN THE CONTEXT OF  
MATHEMATICAL PROBLEM SOLVING

by

James Clinton Stockton

A Dissertation

Presented in Partial Fulfillment of Requirements for the

Degree of

Doctor of Education

In

Leadership for Learning

Teacher Leadership

In the

Bagwell College of Education

Kennesaw State University

Kennesaw, GA

2010

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## Dissertation Signature Page

**The dissertation of**

**James C. Stockton**

CANDIDATE NAME/KSU ID

Adolescent Education - Mathematics

CONCENTRATION

**Titled:**

**A Study of the Relationships Between Epistemological Beliefs and Self-Regulated Learning Among Advanced Placement Calculus Students in the Context of Mathematical Problem Solving**

**submitted to the Bagwell College of Education in partial fulfillment of the requirements for the degree of:**

**Doctor of Education**

**has been read and approved by the Committee:**

*Paul L. Gandy*  
\_\_\_\_\_  
DISSERTATION CHAIR SIGNATURE

*7/7/10*  
\_\_\_\_\_  
DATE

*Gleece H. Terry*  
\_\_\_\_\_  
COMMITTEE MEMBER SIGNATURE

*7/7/10*  
\_\_\_\_\_  
DATE

*Mundy Blanche*  
\_\_\_\_\_  
COMMITTEE MEMBER SIGNATURE

*7/7/10*  
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COMMITTEE MEMBER SIGNATURE

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## DEDICATION

*I dedicate this dissertation to my wonderful wife, Stacey, and children, Joshua, Julia, and Olivia, who have supported me throughout my graduate education.*

## ACKNOWLEDGEMENTS

An endeavor of this magnitude requires significant advisement and support. I would like to thank my dissertation chair, Dr. Mary Garner, for providing guidance, resources, and encouragement. She demanded a high standard of my work, which encouraged me to persevere and produce a quality document. I would also like to thank my dissertation committee members, Dr. Wendy Sanchez and Dr. Alice Terry, for providing specialized feedback and encouragement. Dr. Sanchez's insights into mathematical beliefs and qualitative methods were invaluable to this study. Dr. Terry's expertise in self-regulated learning and qualitative methods informed every aspect of this study. Special thanks also goes to Dr. Nita Paris and Dr. Harriet Bessette, whose vision of this program has finally been realized. Without their dedication and perseverance, the doctor of education degree program would not have been possible at Kennesaw State University. I would also like to thank my father, Jim Stockton, for always believing in me and encouraging me throughout this experience. He was always interested to hear of my latest accomplishments and provided support for my endeavors in many ways. Finally, my wife, Stacey Stockton, deserves credit for having the most significant influence on my dissertation journey. She freely gave her time, resources, and support to ensure my success. The dissertation presented here was made all the better by my wife's love and encouragement.

## ABSTRACT

# A STUDY OF THE RELATIONSHIPS BETWEEN EPISTEMOLOGICAL BELIEFS AND SELF-REGULATED LEARNING AMONG ADVANCED PLACEMENT CALCULUS STUDENTS IN THE CONTEXT OF MATHEMATICAL PROBLEM SOLVING

by

James Clinton Stockton

Secondary mathematics educators advocating constructivist-oriented instruction face the dilemma of developing students' problem-solving skills. Students' epistemological beliefs and self-regulated learning (SRL) processing capacity influence mathematical problem-solving prowess. This multiple-case study explored the relationships between epistemological beliefs and SRL processing while advanced mathematics students engaged in problem-solving tasks and investigated students' SRL strategy use, heuristic strategy use, and problem-solving performance. Data sources included think-aloud and interview transcriptions, student work, and classroom observation protocols. Validity and reliability were enhanced via member-checking interviews, triangulation, peer review, and completion of a case study database. Five major findings emerged from the data: (1) participants' *unique/arbitrary* beliefs regarding problem solutions, *procedural/conceptual* beliefs in problem solving, and *empirical/rational* beliefs in problem solving were related to various facets of SRL processing; (2) differences in SRL strategy use were noted dependent upon cognitive load of problem-solving tasks; (3) heuristic strategy use was

related to participants' mathematical problem-solving beliefs; (4) problem-solving performance was related to participants' mathematical problem-solving beliefs; (5) discrepancies were noted between espoused beliefs and manifested beliefs among participants with non-availing beliefs. Recommendations for practicing mathematics educators include the assessment and development of students' mathematical epistemological beliefs and SRL processing capacity, differentiation of cognitive load for tasks based on assessments of students' cognitive capacity, and professional development training for teachers. Further research is needed which involves students of various achievement levels and extends methodologies to grounded theory or structural equation modeling. Additionally, a request is made for more research from classroom teachers.

*Keywords:* epistemological beliefs; gifted students; mathematics education; mathematical problem solving; self-regulated learning



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## CHAPTER I

### INTRODUCTION

This study investigated the relationships between epistemological beliefs and self-regulated learning (SRL) processing while students engaged in mathematical problem-solving tasks. Data collected from six students selected from an Advanced Placement (AP)\* Calculus BC course provided detailed narratives and cross-case analysis explaining the phenomenon. Hopefully the results of this study will inform pedagogical practice for the purpose of fostering improved problem solving skills in secondary mathematics students.

This chapter initially addresses the background and context of the study. Then a statement of the problem and purpose of the study leads to the research questions that drove the inquiry. The chapter also includes a brief overview of the research design, assumptions inherent in the study, the rationale for conducting the study, and significance of the study. This chapter concludes with definitions of key terms and limitations and delimitations inherent in the study.

#### Background and Context

For the purposes of this study, problem solving was viewed as an activity that facilitates student learning of mathematics. The implication of this stance is that a study relating self-regulation, epistemology, and problem solving may inform pedagogical practice for the purpose of fostering increased student learning of mathematics. Early

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mathematical problem-solving theorists and researchers suggested that issues of control and mathematical beliefs are connected to the successful completion of mathematical problems (e.g., Polya, 1957; Schoenfeld, 1982, 1983, 1985, 1988, 1989). More recently, theorists and researchers have infused contemporary self-regulated learning (SRL) theory and epistemological beliefs into the study of mathematical problem solving (Hofer, 1999; Muis, 2004, 2008). This study extended the exploration of these constructs using a multiple-case study design in an advanced high school mathematics course.

SRL was viewed as students' autonomous control of learning experiences via the following phases: definition of the task, forethought, performance control, and self-reflection (Winne & Hadwin, 1998; Zimmerman, 2000). Based on a review of literature, mathematical problem-solving-based epistemological beliefs were assumed to exist on a continuum and were identified as follows: rational/empirical approaches, nature of problem solutions, duration of problem-solving, procedural/conceptual approaches, importance/usefulness of mathematics, and effort/inherent mathematical ability (Kloosterman & Stage, 1992; Muis, 2004; Royce & Mos, 1980; Schoenfeld, 1983, 1985, 1992). To appropriately define the background and context of issues germane to the study, this section will present a summary of student learning issues relative to SRL, epistemology, and problem solving.

### *Philosophical Viewpoint of Student Learning*

Much debate has revolved around how students *do* and *should* learn mathematics. The camps range from back-to-basics rote learning to radical constructivism (Steffe & Kieren, 1994). The researcher's philosophical perspective closely resembles von Glasersfeld's trivial constructivism (as cited in Steffe & Kieren, 1994), which implies



that students can and do construct their own mathematical meanings, and carries the assumption that researchers and practitioners can identify, study, and enhance students' mathematical ways of knowing. Based on its alignment with this philosophical perspective, the definition of student learning defined by the National Council of Teachers of Mathematics (NCTM) was used throughout this paper: "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p. 20).

This study utilized the definition of mathematical problem solving provided by Schoenfeld (1985), who stated, "By definition, problem situations are those in which the individual does not have ready access to a (more or less) prepackaged means of solution" (p. 54). Due to the novel student experiences inherent in a constructivist perspective, Schoenfeld's definition implied that mathematical problem solving is at the heart of students' learning of mathematics. Student learning will now be situated within the interrelated constructs of problem solving, epistemology, and SRL.

#### *Mathematical Problem Solving and SRL*

"For mathematics education and for the world of problem solving [Polya's works] marked a line of demarcation between two eras, problem solving before and after Polya" (Schoenfeld, 1987, p. 283). Polya (1957) developed a problem solving system, *understand, plan, execute, and check*, which remarkably foreshadowed every phase of the SRL model described above. The theoretical basis of Polya's problem-solving system was an extensive list of heuristics, or problem-solving strategies. The main focuses of his problem-solving works were the application and adaptation of these heuristic strategies, teacher-student dialogues, and internal dialogues of students solving problems.

Drawing upon the works of Polya, Schoenfeld studied the problem solving practices of undergraduate and high school students. Schoenfeld's (1985) mathematical problem solving framework included the aspect of *control*, which he further subdivided into *reading, analyzing, exploring, planning, implementing, and verifying*. Schoenfeld's (1982, 1985) findings suggested that many students do not enter college with the appropriate control skills for problem solving, but usage and adaptation of heuristic skills can be taught to students. Specifically, many students who engaged in *wild goose chases* involving empirical meanderings without curtail were unable to solve his problems (Schoenfeld, 1982, 1985). Schoenfeld's (1988, 1989) suggested that the extensive focus on structure and standardized testing that pervades many high school classrooms is the cause of such ill preparation. It should be noted that these studies were conducted at a New York high school with an excellent track record for high student achievement on the standardized Regents exam. Thus, Schoenfeld's findings suggested a significant disconnect between the learning implied by standardized exam results and the learning suggested by a constructivist perspective. In fact, Schoenfeld (1988) stated, "It is pretty clear what mathematical thinking is not: the rote memorization of facts and procedures as often practiced in our classrooms, and as reified by current texts and examinations" (p. 164).

In 2000, the NCTM published *Principles and Standards for School Mathematics*, a culmination of over a decade of reform efforts. (It should be noted that Schoenfeld was a contributing author to the grades 9-12 chapters of the volume.) The suggestions in *Principles and Standards* reflected a constructivist, conceptually-oriented view of teaching and learning mathematics. For the *problem solving* standard, NCTM (2000)

suggested that “instructional programs from prekindergarten through grade 12 should enable all students to apply and adapt a variety of appropriate strategies to solve problems” and “monitor and reflect on the process of mathematical problem solving” (p. 334). Adapting, monitoring, and reflecting are key cognitive actions inherent in the model of SRL used in this study. From a constructivist viewpoint, then, student learning that is fostered through mathematical problem solving may be partially dependent upon the attainment of self regulatory skills.

With respect to general educational psychology, the origins of SRL can be traced to Vygotsky’s (1978) notions of self-talk and Bandura’s (1986, 1997) social cognitive theory. Briefly, “self regulation refers to self-generated thoughts, feelings, and actions that are planned and cyclically adapted to the attainment of personal goals” (Zimmerman, 2000, p. 14). For the purposes of this study, SRL was viewed as a cyclic, recursive process that involves the following phases: definition of the task, forethought, performance control, and self-reflection (Winne & Hadwin, 1998; Zimmerman, 2000). *Definition of the task* and *forethought* are pre-action phases and include such activities as defining the problem-solving space, setting goals, and developing plans. *Performance control* is the action phase and includes application and monitoring of appropriate strategies, while applying self-control throughout the learning experience. *Self-reflection* is the post-action phase and involves evaluation of performance and determination of causal attributions for both successful and unsuccessful aspects of the completed task. Typically, judgments are made assessing the effectiveness of the learning strategies used, which provides internal feedback affecting further action for the current learning task (if needed) and all other future learning tasks. Students’ actions may indicate structured

adherence to the model, recursive, cyclic patterns of behavior within the phases of the model, or very little adherence to the model.

De Corte, Verschaffel, and Op 'T Eynde (2000) suggested a model for “self-regulated mathematical learning and problem solving” to encompass more contemporary views of SRL (p. 693). A major component of the model was students’ mathematical dispositions, which included the application of heuristic strategies and self-regulatory skills. For the purposes of this study, heuristic strategies were viewed as domain-specific strategies that may be enacted during the performance control phase of SRL. Students’ mathematical beliefs are another major component of the dispositional perspective, which segues into the next section.

#### *Mathematical Problem Solving and Epistemology*

In addition to issues of control, students’ epistemological beliefs are important factors in the successful navigation of a problem space (Kloosterman & Stage, 1992; Muis, 2004, 2008; Schoenfeld, 1983, 1985, 1992). Polya’s (1957) contributions to problem solving dealt very little with issues of personal epistemology, instead focusing on teacher-student dialogue and heuristic strategy application and adaptation. This is consistent with historical reviews of mathematics education, which cite the early to mid-twentieth century as an era of the philosophy of knowledge transfer (D’Ambrosio, 2003).

Schoenfeld (1983, 1985, 1992) is generally credited with formally introducing personal epistemology to the mathematical problem-solving dialogue. He suggested that students’ “mathematical worldviews” are key components to the successful completion of problem-based tasks (Schoenfeld, 1985, p. 186). Schoenfeld dichotomously defined students’ mathematical problem solving beliefs as rational (logical and analytical

approach) or empirical (observational and perceptual approach). In general, Schoenfeld's studies suggested that rational problem solvers exert more control and are more successful than their empirical counterparts. Typical empirical students' beliefs included the assumptions that formal mathematics is not needed during problem solving, mathematics problems are solved quickly or not at all, mathematical discovery is only possible for geniuses, a unique solution exists for all mathematics problems, and an algorithmic, procedural method is available for all mathematics problems (Schoenfeld, 1985, 1992). These non-availing mathematical beliefs would surface later as both general and domain-specific epistemological beliefs literature expanded to a multi-dimensional perspective (Hofer, 2000; Hofer & Pintrich, 1997; Muis, 2004; Muis, Bendixen & Haerle, 2006; Schommer, 1990).

Schommer (1990) was the first to suggest that epistemological beliefs may exist as a system of independent dimensions. Much theoretical and empirical work led to the development of a contemporary system of epistemological beliefs: certainty of knowledge, simplicity of knowledge, sources of knowledge, and justification for knowing (Hofer, 2000; Hofer & Pintrich, 1997). Additionally, domain-specificity has been suggested as a major factor in the study of students' epistemological beliefs (Hofer & Pintrich, 1997; Muis, 2004; Muis, Bendixen & Haerle, 2006). The main justification for examining beliefs from a domain-specific perspective is based on the assumption that students' beliefs vary with respect to the domain of study. In the field of mathematics education, there is relatively consistent agreement that students' adherence to non-availing mathematical beliefs are generally detrimental to learning and performance (Muis, 2004; NCTM, 2000; Schoenfeld, 1988, 1989). Muis (2004) suggested that a

system of mathematical beliefs may include the “nature of mathematics knowledge, justifications of mathematics knowledge, sources of mathematics knowledge, and acquisition of mathematics knowledge” (p. 326). Within this synthesis, Muis (2004) also suggested that students’ epistemological beliefs impact cognition and motivation, which foreshadowed her future work relating personal epistemology and SRL.

### *SRL and Epistemology*

Recently, SRL processing and epistemological beliefs have begun to appear as interrelated constructs in the literature (Muis, 2007, 2008; Muis & Franco, 2009; Schommer-Aikins, 2004). Schommer-Aikins (2004) hypothesized that a reciprocal relationship may exist between epistemological beliefs and SRL. Students’ initial beliefs may affect the degree of self-regulatory actions taken in the classroom and subsequently, the potential feedback loop inherent to SRL processing may affect, or possibly alter, the initial beliefs. Muis (2007) presented a model introducing epistemological beliefs into Winne and Hadwin’s (1998) SRL model. She suggested that students’ epistemological beliefs are enacted during the task definition and goal-setting phases. These beliefs may subsequently affect other aspects of self-regulatory processing (i.e., self-monitoring, self-reflection). Consistent with Schommer-Aikins’ theoretical assertion, Muis also suggested that a reciprocal relationship may exist between students’ epistemological beliefs and SRL. Muis and colleagues have begun the work of studying the relationships that exist between SRL and epistemology (Muis, 2008; Muis & Franco, 2009). Their mixed-methods findings were promising but suggested that further research is needed utilizing think-aloud and interview methods, specifically with respect to the definition of the task phase of SRL.

### *Problem Solving, SRL, and Epistemology*

To my knowledge the only study investigating the relationships among mathematical problem solving, SRL, and epistemology is Muis' (2008) study of Canadian college students. Muis conducted a mixed-methods study involving questionnaires, think-aloud protocols, and interview protocols. Her findings confirmed Schoenfeld's assertion that rational problem solvers are more successful and engage in higher levels of SRL processing than empirical problem solvers. Her findings also suggested that epistemological beliefs are enacted during the definition of the task phase of SRL and may influence learning standards and strategies. Limitations to the study were the domain-generalty of the epistemological questionnaire used and the lack of attention to more contemporary views of multi-dimensional aspects of epistemological beliefs. The current study attempted to extend the scholarly dialogue concerning connections between mathematical problem solving, epistemology, and SRL. The current study extended Muis' (2008) work by infusing more contemporary, domain-specific epistemological beliefs into the theoretical framework. Additionally, the current study investigated high school students, an often underrepresented group in mathematics education research, in their authentic learning environment.

#### Problem Statement

Research has demonstrated that a relationship exists between SRL and epistemological beliefs in multiple contexts (Bråten & Strømsø, 2005; Hofer, 1999; Muis, 2008) . Research has also demonstrated that successful problem-solvers typically exert control over the problem space and have availing epistemological beliefs (Muis, 2008; Perels, Gürtler, & Schmitz, 2005; Schoenfeld, 1983, 1985, 1988, 1989). Unfortunately,

students continue to enter university mathematics courses without adequate problem-solving skills. Despite this fact, a significant gap exists in the relevant literature for secondary mathematics education. Therefore, this study utilized domain-specific beliefs, contemporary views of SRL and epistemology, and authentic problem-solving tasks in an effort to provide further insights into relationships between secondary mathematics students' beliefs and self-regulatory problem-solving practices.

#### Statement of Purpose and Research Questions

The purpose of this study was to explore the SRL practices of six advanced mathematics students in relation to mathematical epistemological beliefs while engaged in problem-solving tasks. From a constructivist standpoint, it was assumed that engagement in mathematical problem solving could potentially lead to learning. So, furtherance of our understanding of the factors involved in both successful and unsuccessful students' problem-solving engagement should lead to pedagogical initiatives aimed at improving student learning. This study explored these issues by answering the following research questions:

1. How are students' epistemological beliefs related to self-regulatory processing practices during engagement in mathematical problem-solving tasks?
2. What self-regulation strategies do students employ while preparing for the AP Calculus exam and engaging in problem-solving episodes?
3. What epistemological beliefs influence students' choice and use of heuristic strategies to solve mathematical problems?
4. How are self-regulated learning strategies and epistemological beliefs related to student performance on problem-solving tasks?



## Research Design Overview

Prior to data collection, IRB approval was obtained from the local school district and university to study six students' experiences with respect to the proposed problem. A multiple-case study design was selected to provide a deep, fine-grained exploration of students' SRL processing practices relative to their mathematical epistemological beliefs (Creswell, 2007; Yin, 2008). Six students were selected to serve as case participants. The term *case* referred to a student who participated in the learning activities in this study and whose data were collected by the researcher. Due in large part to his mathematical background, the researcher applied the postpositivist qualitative paradigm to the study. Thus, research practices included accepting multiple realities to explain phenomena, applying rigorous data collection and analysis procedures, and reporting findings using a scientific structure (Creswell, 2007).

Multiple types of data were collected from the six participants selected for this study. Data collection occurred in four different phases, with the first involving the administration of three quantitative surveys measuring students' self-reported epistemological beliefs and SRL aptitude. Second, students prepared for the AP Calculus exam during class by working select problems based on knowledge taught the prior semester. Field observation transcriptions and AP exam practice journals served as data sources. The third phase of data collection involved the six participants engaging in two *think-aloud* problem-solving sessions (Ericsson & Simon, 1993). Immediately following each session, participants completed retrospective interviews to discuss various aspects of their navigation through the problem space. Finally, soon after completion of data

collection, participants engaged in member-checking interviews to review narrative draft reports and discuss initial findings.

Data analysis occurred concurrently with data collection to ensure that accurate recollections of events were documented, allow opportunities for addressing discrepancies or misconceptions, and inform the researcher as to whether alterations in the research design were needed. Descriptive statistics from the quantitative surveys aided in participant selection and provided initial, albeit self-reported, evidence of students' self-regulatory prowess and epistemological beliefs. For the qualitative data, an extensive codebook was developed from a thorough review of the literature and was used to code each piece of data. Various matrices provided multiple perspectives for both individual and cross-case analyses and led to rich, thick narrative descriptions. Reliability was ensured by creating a case study database, establishing a chain of evidence, and conducting a peer review of the coding scheme. The validity of the findings was supported by triangulating data sources, conducting member-checking interviews, and developing rich, thick descriptions (Creswell, 2007; Miles & Huberman, 1994; Yin, 2008).

#### Limitations and Delimitations

Limitations of this study existed that were beyond the researcher's control. First, the results of this small-scale case study may not be generalized to all secondary mathematics students. Rather, the results provided a deeper, fine-grained description of advanced secondary mathematics students' experiences that is much needed in the literature. Another limitation was my role as both teacher and researcher. Participants may have reacted differently in certain situations than if an outside researcher had

conducted the study. Although this dual role provided particularly poignant insights into students' actions, the researcher was careful not to introduce bias into the analysis of the findings, but reported actual students' actions and intentions. Finally, research methods may have affected the results of this study. Specifically, issues of group dynamics may have affected results during the in-class portion of the study and think-aloud protocols, although a widely used method for assessing cognitive activities, may not have produced a complete report of each participant's thinking (Ericsson & Simon, 1993).

The desire to gain a deep, fine-grained description of students' actions led to certain necessary delimitations for the study. First, the study included six students to enable deep analysis of all data and provide rich descriptions. Second, only high school students were selected as this group seems most neglected in the literature. Third, all six students were selected from one intact class to introduce rich contextual descriptions and social interactions that are important to both epistemological and SRL constructs.

#### Assumptions

Four major assumptions were made with respect to this study based on the review of literature and the researcher's nine-year experience as a high school mathematics teacher. The first assumption was that most AP Calculus students have an innate desire to learn mathematics. Those who are not learning-driven are typically grade-driven, so all students have some motivation to perform at a high level. Second, mathematical learning can be achieved, and possibly enhanced, by the act of problem solving. This assumption was derived from suggestions made by NCTM's (2000) *Principles and Standards*. Third, heuristic strategies, when applied properly, enhance problem-solving prowess. Polya (1957) introduced heuristics as a viable pedagogical tool and then Schoenfeld's (1985)

findings suggested that heuristics can be taught to students and have the potential to positively affect problem-solving performance. Fourth, SRL processing enhances students' overall educational experiences, particularly in the mathematics classroom. This assumption, which is crucial to the rationale for conducting the study, has been in place among social cognitive theorists since the conception of SRL (Bandura, 1986; Zimmerman, 1989).

### Rationale and Significance

The rationale for conducting this study stemmed from my desire to see students learn mathematical concepts by engaging in mathematical problem solving tasks. As discussed above, students' mathematical epistemological beliefs and self-regulatory strategy use may be related to their problem-solving performance (Muis, 2008; Schoenfeld, 1985). Additionally, the active construction of mathematical knowledge is dependent upon students' problem-solving prowess (NCTM, 2000). Then, from a practitioner's point of view, justification for the study is established based on potential student learning and achievement benefits.

This study has the potential to significantly contribute to theory and practice. Work has recently been undertaken to establishing relational ties between epistemological beliefs and SRL (Muis, 2007, 2008; Muis & Franco, 2009). This study sought to extend these works. Specifically, this case study was designed to provide evidence of the relationships between epistemological beliefs and SRL at a finer grain size by closely examining students' authentic experiences. For the practicing educator, the study should provide formative groundwork for the development of pedagogical interventions for developing students' availing mathematical beliefs and self-regulatory

practices. My desire is that the study will contribute to general educational psychology theory, mathematics education theory, and mathematics teachers' arsenals of best practices.

### Definitions of Terms

The terms below represent the major constructs that are important to this study. Operational definitions are provided to clarify how each term was used throughout the study. A more detailed and extensive description and analysis of each construct and its component parts may be found in *Chapter II: Review of Relevant Literature*.

#### *Epistemological Beliefs*

Hofer and Pintrich (1997) described “personal epistemological development and epistemological beliefs” as “how individuals come to know, the theories and beliefs they hold about knowing, and the manner in which such epistemological premises are a part of and an influence on the cognitive processes of thinking and reasoning” (p. 88). This study examined both students' general and mathematics-specific epistemological beliefs from a contemporary, multi-dimensional viewpoint. The general epistemological beliefs dimensions included: certainty of knowledge, simplicity of knowledge, sources of knowledge, and justification for knowing (Hofer, 2000; Hofer & Pintrich, 1997). The mathematical problem-solving epistemological beliefs dimensions included: rational/empirical problem-solving, unique/arbitrary problem solutions, duration of problem-solving, procedural/conceptual approach, importance/usefulness of mathematics, and effort/inherent mathematical ability (Kloosterman & Stage, 1992; Muis, 2004; Royce & Mos, 1980; Schoenfeld, 1983, 1985). From a multi-dimensional epistemological

beliefs perspective, students may hold various, even contradictory, beliefs depending on both contextual and domain-related issues.

### *Heuristic Strategies*

Generally considered the father of the modern study of heuristics, Polya (1957) defined heuristic strategies as “the process of solving problems, especially the *mental operations typically useful* in the process” (p. 129–130). Polya provided an extensive list of heuristic strategies and suggested that both teachers and students of mathematics could benefit from serious consideration of the use of heuristics when solving problems.

### *Mathematical Problem*

For the proposed study, the definition of a mathematical problem is a scenario or situation proposed such that a prescribed solution path is not readily available to the solver (Schoenfeld, 1985). This definition differentiates between a mathematical problem and a mathematical exercise, which simply involves applying prescribed procedures. This distinction is important in that problems requiring application of pre-scripted knowledge (exercises) imply a rote-memorization approach to learning, which is in direct contrast to the constructivist philosophy of learning that is the basis of the study.

### *Self-Regulated Learning (SRL)*

Zimmerman (2002) defined SRL in the following manner:

Self-regulation is not a mental ability or an academic performance skill; rather it is the self-directive process by which learners transform their mental abilities into academic skills. Learning is viewed as an activity that students do for themselves in a *proactive* way rather than as a covert event that happens to them in reaction to teaching. (p. 65)

For this study, the SRL process referenced by Zimmerman will include the following phases: definition of the task, forethought, performance control, and self-reflection (Winne & Hadwin, 1998; Zimmerman, 2000). Self-regulating students tend to navigate a problem space by recursively and cyclically applying the SRL phases as needed.

## CHAPTER II

### REVIEW OF RELEVANT LITERATURE

This study examined how advanced mathematics students' epistemological beliefs are related to self-regulated learning (SRL) processing while engaging in problem-solving tasks. To complete a case study investigating such complex phenomena, a thorough review and critique of relevant literature was required (Yin, 2008). Thus, this chapter develops a literature-based analysis and synthesis of issues relevant to epistemological beliefs, SRL, and mathematical problem-solving.

Initially, the focus was on the broad spectrum of SRL literature and found highly-developed theoretical constructs (e.g., Butler & Winne, 1995; Garcia & Pintrich, 1994; Pintrich, 2000; Winne & Hadwin, 1998, 2008; Zimmerman, 1989, 2000) and empirical studies relating SRL to various facets of education (e.g., Greene & Azevedo, 2009; Hadwin, Boutara, Knoetzke, & Thompson, 2004; Muis, 2008; Usher, 2009). As my review began to narrow, I discovered that current researchers were requesting studies investigating closely the specific mechanisms of students' application of SRL processes and strategies (Hadwin, Boutara, Knoetzke, & Thompson, 2004; Winne & Jamieson-Noel, 2003; Winne & Perry, 2000; Zimmerman, 2008). Thus, this study evolved into a qualitative case study to gain a rich description of individual students' use of SRL processes, as opposed to a broad paint stroke of general SRL usage by a large sample of



students (Creswell, 2007; Yin, 2008). This study is expected to augment current models of SRL and uncover new perspectives for modeling SRL.

Further narrowing of the literature review revealed a topic of SRL study related directly to mathematics education—problem solving (De Corte, Verschaffel & Op ’T Eynde, 2000; Muis, 2008; Schoenfeld, 1992). Then, the research topic was narrowed to the study of the relationships between epistemological beliefs and SRL processing of advanced high school mathematics students engaged in problem-solving episodes. Additional probing revealed that critical thinking and problem solving are influenced by students’ epistemological beliefs (Hofer, 1999, 2000; Hofer & Pintrich, 1997; Kloosterman & Stage, 1992; Muis, 2004, 2007, 2008; Schoenfeld, 1983, 1985, 1989). Particularly influential to the overall design and content of the proposed study was a mixed methods study by Muis (2008), which examined the complex weave of mathematics students’ epistemic profiles, SRL processing, and problem solving capacity. This study attempted to extend Muis’ work by examining SRL, epistemology, and problem solving at a finer grain size and by examining students’ epistemological beliefs from a more contemporary, domain-specific perspective.

The ensuing review of relevant literature is topically segmented and incrementally builds a theoretical framework for the study. The main topics relevant to this study include SRL, epistemology, and problem solving. Each topic will be analyzed and synthesized with regard to the field of mathematics education. The presentation of relevant literature is divided into the following sections: (1) theoretical analysis of SRL, (2) theoretical analysis of epistemology, and (3) theoretical analysis of problem solving.

The review culminates in a description of the theoretical framework developed from the literature reviewed.

### Topical Review of Relevant Literature

#### *Theoretical Analysis of SRL*

The purpose of this section of the literature review is to provide an analysis of general theory of SRL relevant to the study. As a branch of educational psychology, SRL has been widely researched and theoretically developed. SRL originated from Vygotsky's (1978) notions of *inner speech* and Bandura's (1986) social cognitive theory. Ormrod (2008) described the connection between Vygotsky's theories on inner speech and SRL as follows:

In Vygotsky's view, such self-talk (also known as *private speech*) plays an important role in cognitive development. By talking to themselves, children learn to guide and direct their own behaviors through difficult tasks and complex maneuvers in much the same way that adults have previously guided them. Self-talk eventually evolves into *inner speech* [italics added], in which children "talk" to themselves mentally rather than aloud . . . We are essentially talking about *self-regulation* here. (p. 331)

Students who practice inner speech are essentially applying the self-monitoring process of SRL, which involves evaluating the effectiveness of learning goals and cycling back through SRL processes if needed. Several cognitive and affective processes are common to most SRL models (Butler & Winne, 1995; Garcia & Pintrich, 1994; Pintrich, 2000; Winne & Hadwin, 1998; Zimmerman, 2000). The ensuing theoretical analysis of SRL contains a description and analysis of the processes involved in the model used in this

study, qualitative studies of SRL processing, and relationships between SRL, external feedback, and motivation.

### *Self-Regulated Learning (SRL) Processing Model*

SRL is a highly-developed theory within educational psychology that describes students' control of learning. Students may control, or regulate, virtually any aspect of their learning, including, but not limited to: cognition, metacognition, motivation and affect, emotion, and behavior (Boekaerts & Niemivirta, 2000; Pintrich, 2000). This study focused on SRL processing of cognitive and metacognitive navigation through problem-solving tasks.

Multiple models to describe SRL processing have been developed. For this study, Zimmerman's (2000) cyclic model served as the main framework for describing SRL processing. However, particularly salient constructs from other models were integrated into Zimmerman's model (Butler & Winne, 1995; Garcia & Pintrich, 1994; Pintrich, 2000; Winne & Hadwin, 1998). Steeped in the tenets of social cognitive theory, Zimmerman's (2000) cyclic model involves the following three phases: (1) forethought, (2) performance control, and (3) self-reflection. For the model used in this study, Winne and Hadwin's (1998) *definition of the task* phase precedes these three phases. These four phases of SRL will serve to organizationally subdivide this section of the literature review.

*Definition of the task.* Winne and Hadwin (1998) posited that self-regulating students develop a definition of the task prior to a specific goal-setting and planning phase. The definition of the task is "a perception about features of the task" and involves students' development of inferences and preliminary goals relative to the task (p. 283).

Portions of the task definition will remain intact throughout the task, while others will be eliminated as additional information is gleaned from future SRL processes.

*Forethought.* The forethought phase involves developing goals and planning activities for the purpose of completing the learning task. Particularly well-attuned self-regulators will set distal learning goals and then evaluate their progress via proximal process goals (Zimmerman, 1989, 2000). Process goals serve as standards for measuring task progression and provide the learner with parameters to assess levels of success. More specifically, goals may be classified as *mastery-* (focused on learning and understanding) or *performance-* (focused on doing better than others) oriented (Pintrich, 2000). Further delineation of performance goals yields an *approach* focus (motivated to demonstrate superiority over peers) and an *avoidance* focus (motivated to avoid failure). Pintrich (2000) pointed out that mastery- and performance-approach goal orientations tend to produce increased SRL processing in students, whereas students with a performance-avoidance goal orientation tend to demonstrate inferior cognitive processing skills. Although mastery goals may also be subdivided into the categories *approach* (focused on comprehension and learning) and *avoidance* (focused on avoidance of misunderstanding), researchers typically do not address this distinction. Pintrich (2000) speculated that students with a mastery-avoidance approach may use less adaptive monitoring processes due to a focus on not making mistakes rather than on deep learning. Furthermore, SRL processing is not linear. Thus, students may set goals prior to a learning task or during latter stages of the task based on future SRL processing (Boekaerts & Niemivirta, 2000; Pintrich, 2000).

The planning stage involves the selection of strategies best suited to the learning task based on goal-driven standards for learning. Additionally, self-regulatory strategies vary with respect to both individuals and learning contexts. Since adjustments are necessary for most learning tasks, self-regulating students must develop a plan to monitor their progress as they navigate through the learning task (Boekaerts & Niemivirta, 2000; Zimmerman, 1989, 2000).

Schunk (1996) reported the results of two empirical studies analyzing the effects of goal-setting and self-evaluation on fourth-grade students' self-efficacy and achievement. Schunk adopted Dweck and Leggett's differentiations of goal profiles (as cited in Schunk, 1996, p. 361) for the study: "A *learning goal* refers to what knowledge and skills students are to acquire; a *performance goal* denotes what task students are to complete." These goal types are respectively synonymous with the *mastery* and *performance* goal orientations discussed above. In both studies, students were learning fraction skills and were divided into four groups. The groups consisted of students who were taught to set *learning goals*, students who were taught to set *learning goals and practice self-evaluation*, students who were taught to set *performance goals*, and students who were taught to set *performance goals and practice self-evaluation*. The reason for developing such groups was to better examine the effects of the differing types of goals and the presence or absence of self-evaluation practices. Schunk used a pretest-posttest model with assessments measuring students' goal orientation, self-efficacy, skill, and motivation. The findings from both studies suggested that setting learning goals enhances students' task (or mastery) goal orientation, skill, self-efficacy and motivation. The achievement results were mixed in that only the second study showed a significant

increase in achievement for students who set learning goals and participated in self-evaluation, in comparison to students who set performance goals and participated in self-evaluation.

Ablard and Lipschultz (1998) investigated high-achieving students' SRL processing with respect to goal orientation. One rationale for conducting the study was the suggestion from prior research that performance levels of some gifted students indicate underachievement (Risemberg & Zimmerman, 1992). Ablard and Lipschultz suggested that a lack of SRL processing capacity may be a factor in gifted students' underachievement. To investigate their claim, the authors conducted a study of 222 seventh-grade students who scored in the top 3% on grade-level assessments. Quantitative data were generated from a variety of SRL and goal orientation questionnaires and protocols.

The results of the study indicated that, although all students were high-performers, significant variation existed with respect to goal orientation and SRL strategy use. Students with high mastery goals demonstrated significantly higher use of SRL strategies than their low mastery-goal setting peers. In fact, of all the variables analyzed, mastery goal orientation accounted for the most variation in SRL strategy use. Ablard and Lipschultz (1998) suggested that some high-achieving students succeed without the use of SRL strategies and thus, relationships between SRL and achievement are complex. The authors tempered their findings by pointing out that data was obtained via self-report instrumentation. Thus, students may have indicated learning strategies that they were aware of, but did not actually use. The authors suggested that future research should investigate SRL strategies students actually apply compared to strategies they report.

*Performance control.* During performance control, the learner utilizes self-control and self-observation to enact the plan developed in the forethought phase. Zimmerman (2000) identified the following self-control processes: “self-instruction, imagery, attention focusing, and task strategies” (p. 18). Self-instruction is the act of describing the process for completing a learning task while engaged in the task. Self-instruction may be a physical (e.g., self-talk, writing mathematical formulas before applying them) or mental (e.g., mental rehearsal of algebraic simplification steps, recalling typical pitfalls encountered during previous work) phenomenon.

Students who utilize imagery self-control develop mental images to aid in the completion of a task. An example would be a student who separates a composite function into its component parts and uses mental images of the graphs of each function to solve a particularly difficult limit in calculus. Attention focusing involves ignoring external (e.g., a noisy classroom during group work) or internal (e.g., the nervous excitement of playing in tonight’s football opener) stimuli and consciously focusing on the learning task. Task strategies involve reducing a learning task to its component parts and developing a personalized systematic representation that makes sense to the learner.

Self-observation, another component of the performance control phase, involves the student monitoring the learning task and assessing focused aspects of performance. Proximal goal-setting during the forethought phase facilitates purposefully selective self-observation. Zimmerman (2000) suggested the following features of effective self-observation: (a) self-feedback should be provided concurrently with the task, (b) feedback should inform the level of performance, (c) self-observations should be accurate portrayals, and (d) self-observations should focus on performance accomplishments

instead of deficits. Butler and Winne (1995) suggested that self-monitoring (or self-observation) naturally elicits internal feedback, which provides a bridge between goal-based expectations and actual performance. Operationally, Butler and Winne define internal (or self-) feedback as “conditional knowledge that bridges past performance to the next phase of engaging with a task” (p. 260). So, internal feedback is a product of self-observation and when compared to a goal-based standard, yields action on the part of the learner. This action is the subject of Zimmerman’s (2000) final phase in the SRL cycle, as described below. External feedback, or feedback provided by others, may also affect SRL processing and is discussed in detail in a later section of this chapter.

In a recent empirical study, Greene and Azevedo (2009) used think-aloud protocols to study middle and high school students’ use of macro-level SRL processing of complex systems of information. Throughout the study, the authors used a model of SRL developed in part by Azevedo, which expands Winne and Hadwin’s (1998) and Pintrich’s (2000) theories of SRL into 35 specific micro-level processes that fall under five main processes similar to the model used in this study. The study involved 219 middle and high school students investigating the circulatory system via a hypermedia learning environment (HLE) while thinking aloud and taking a pre- and post-test in the form of a mental model essay. The mental model pre-and posttests were scored based on a 0 to 12 point scale. The think-aloud transcriptions, which totaled 8760 minutes in duration, were encoded using the 35-component model of SRL, then categorized based upon the five macro-levels of the same model. Inter-rater agreement was high for both the essay scoring and the coding.



Using cumulative logit ordinal logistical regression modeling, Greene and Azevedo (2009) found significant differences in learning performance between middle and high school students, with high school students tending to have a more sophisticated mental model. Additionally, prior domain knowledge was significantly associated with the production of more sophisticated mental models. Finally, self-monitoring was the only SRL process that was significantly associated with the production of a more sophisticated mental model. These results suggest that monitoring is a key component of all aspects of SRL and is important in promoting student learning. The authors identified limitations of their study: lack of clarity as to the influence of think-aloud protocol on metacognitive activity in students, limited scope of the topic of study to one domain, and lack of instructional aids embedded into the HLE. The authors also suggested that future research should investigate SRL at multiple grain sizes and provide objective data regarding student SRL processing.

*Self-reflection.* Finally, during the self-reflection phase, students make self-evaluations and consider causal attributions in terms of their performance during the learning task (Zimmerman, 2000). Self-evaluations may use mastery criteria, which imply that the learner sets incremental performance markers ultimately leading to becoming expert in the task. Self-regulated learners who set process goals during the forethought phase naturally become mastery-focused self-evaluators. Alternatively, students may base their self-evaluation of current functioning on prior performance of similar tasks. Finally, students may utilize normative criteria during self-evaluation, which involves comparing their performance to that of others. Self-regulated learners

who set outcome goals tend to apply normative criteria to their evaluations and may find themselves focusing more on negative aspects of their performance.

Self-regulating students identify causal attributions based upon their self-evaluations, perceived self-efficacy, and/or the learning environment (Bandura, 1997; Zimmerman, 2000). Attributions that focus on deficits in the learner's ability tend to discourage future self-regulation; whereas attributions that focus on insufficient strategy use tend to promote motivation to alter future behavior to achieve self-set learning standards. As the cyclic SRL processing draws to an end, the student either makes an adaptive or defensive inference based upon progression through the learning task. Adaptive inferences lead students to improved understanding of the interplay between SRL phases with respect to the current task and improved application of SRL processing for future learning tasks (Winne & Hadwin, 1998; Zimmerman, 2000). Other students may choose a defensive inference, which protects the learner from adverse evaluations, but simultaneously stifles SRL processing. Thus, the cyclic nature of Zimmerman's (2000) model is longitudinal in that students develop SRL skills over time based on their commitment to attaining personal learning and achievement standards.

#### *Qualitative Studies of SRL Processing*

SRL processing may be described along a developmental continuum. Hadwin, Boutara, Knoetzke, and Thompson (2004) studied Canadian college students' SRL processing from a developmental perspective. The authors utilized self-report questionnaires, self-evaluations, and trace data obtained from a hypermedia program to develop case studies describing students' actual SRL processing over a series of events. Participants were 8 students chosen as cases out of 50 undergraduates enrolled in an

instructional psychology course. The participants were chosen based upon performance on three exams. Each participant was categorized with respect to performance on the three exams as either a low, average, high, or improved performer. The unit of analysis became cross-case comparisons of each group, as opposed to individual case studies. Trace data were collected from hypermedia software called CoNoteS2, which was experimental software specifically designed to gather fine-grained data of student control of learning during a specified task. The remaining data were collected through three exams, weekly self-reflections, and a final self-reflection and analysis.

During the four-week study, students studied content in each of three chapters presented by CoNoteS2 for one hour, completed a self-reflection, and took an exam over the content. Each learning event occurred once during the first three weeks and students were required to complete a one-page final self-reflection and analysis to be submitted for grading. Findings indicated significant variability in SRL processing within each performance group. For example, within the high-performing pair, one student displayed a learning-oriented approach while the other student tended more toward a performance-oriented approach. Although the first student demonstrated much higher-order cognitive processing, both students performed well on the exams.

Both the high and improved performance groups demonstrated deeper approaches to studying. However, the high performers demonstrated these skills throughout the three units; whereas, the improved performers developed their skills over time. Finally, the authors developed a continuum of SRL ratings: low, emerging, and high. Participants were given an overall rating based on their predominant rating classification in terms of skill, will, and self-regulation. Although one high achiever had the highest rating and one

low achiever had the lowest rating, results were inconsistent in between. Hadwin et al. suggested that this inconsistency indicates that students' performance may not be a good indicator of SRL processing prowess.

Despite the loss of generality inherent in qualitative designs, Hadwin et al. (2004) suggested that students in early developmental stages of SRL (low overall rating) appear to have difficulty accurately assessing their learning processes and products.

Additionally, findings support the growing literature base that SRL study designs need to incorporate multiple sources of data. The authors pointed out that by collecting and analyzing data over a series of events, patterns of SRL processing and development emerged over time. Such patterns may not have been uncovered by a study using performance data that emerges from a single event. The authors provided calls for future research as follows: similar qualitative studies involving longer duration, more learning events, and/or more participants, studies investigating the relationship between goal orientations and SRL development, and studies involving more grade-bearing learning tasks while controlling for confounding instructional variables.

Issues of self-efficacy often arise when considering students' capacity to self-regulate. Usher (2009) studied middle school students' sources of self-efficacy and relations to issues of race, gender, and SRL processing. Eight students participated in the study and were representative of every combination of four subgroups: African American females, African American males, White females, and White males, and two self-efficacy profiles, high and low. Teachers and parents were also included as participants to gain additional perspectives on students' functioning. Data collection instruments consisted of semi-structured interview protocols for students, parents, and teachers. Internal validity

was addressed via triangulation from multiple sources, member checking, maintaining an audit trail, and peer review (intercoder reliability). External validity was addressed via rich descriptions of participants' experiences.

The findings of Usher's (2009) study indicated that middle school mathematics students' self-efficacy profiles are derived from performance interpretations, peer and adult influences, physiological influences, and self-regulatory activity. Additionally, Usher's findings suggested that self-efficacy and SRL are reciprocally linked, which is in line with Bandura's (1997) assertions. In other words, students' degree of self-efficacy tends to predict their self-regulatory prowess. Inversely, the level of SRL processing practiced by a student tends to appropriately affect their self efficacy beliefs. Due to the in-depth nature of qualitative data collection and analysis, Usher was able to identify one student who did not participate in self-regulatory study habits, yet had high self-efficacy and performed well in mathematics. Additionally, this student's family members all had poor mathematics skills, yet he used this as motivation to excel and thus, improved his mathematical self-efficacy. This finding contradicted Usher's assertions and the theoretical framework that she developed. Usher used these surprising findings to justify her choice of qualitative methods for the study. Of additional importance to the current study is the fact that the aforementioned student was on an advanced mathematics track, thus implying that advanced mathematics students may not adhere strictly to theoretical SRL propositions.

#### *External Feedback and SRL*

Feedback can be divided into two major categories: internal and external. Internal feedback is controlled by the learner and occurs naturally as the learner participates in the

self-monitoring process of SRL. External feedback is provided to the learner by teachers, peers, and other sources in the event that the established learning goals are not being successfully executed (Butler & Winne, 1995). Butler and Winne (1995) developed a research-based model for SRL that incorporates external feedback. For the purposes of this study, the model provides clear connectivity between both internal and external feedback and the processes of SRL. Butler and Winne suggested that external feedback initiates internal, cognitive processing conducive to self regulation.

External feedback is supplied as a result of the provider's evaluation of learner performance. "If external feedback is provided, that additional information may confirm, add to, or conflict with the learner's interpretations of the task and the path of learning" (Butler & Winne, 1995, p. 248). Students are then able to evaluate both internal and external feedback to determine necessary adjustments for the next learning task. So, external feedback should be provided while the student is engaged in the learning task so that the learner can process the feedback during the self-monitoring and self-evaluation phases.

External feedback can be further subdivided into cognitive and outcome categories. Cognitive feedback has been suggested as preferable to outcome feedback (Butler & Winne, 1995). Outcome feedback simply involves informing the learner if their performance is accurate; cognitive feedback refers to information provided to the learner that suggests learning cues that "may help students identify cues and monitor task engagement" (p. 253). Therefore, teacher and peer feedback should be constructed to encourage critical thinking and metacognitive activity. Finally, Butler and Winne found gaps in the literature and included a call for more research "that integrates instruction,

self-regulation, feedback, and knowledge construction” (p. 275). The authors further requested that research should be “fine-grained analyses” of “single students . . . that lead to successfully updated, improved performance” (p. 276).

In a synthesis of feedback and SRL literature, Nicol and Macfarlane-Dick (2006) expanded Butler and Winne’s (1995) model. Most salient to the Nicol and Macfarlane-Dick model is the development of “seven principles of good feedback,” as follows:

Good feedback practice:

1. helps clarify what good performance is (goals, criteria, expected standards);
2. facilitates the development of self-assessment (reflection) in learning;
3. delivers high quality information to students about their learning;
4. encourages teacher and peer dialogue around learning;
5. encourages positive motivational beliefs and self-esteem;
6. provides opportunities to close the gap between current and desired performance;
7. provides information to teachers that can be used to help shape teaching. (p. 205)

The seven principles provide educators and researchers a standard by which to guide feedback practices. Reflecting on their assertions, Nicol and Macfarlane-Dick noted that research on the quality of external feedback is lacking, which may cause problems with the implementation of principle number three. The authors suggested that “good quality external feedback is information that helps students troubleshoot their own performance and self-correct: that is, it helps students take action to reduce the discrepancy between their intentions and the resulting effects” (p. 208). This definition is learner-focused and

steeped in SRL theory, providing practitioners with further guidance for feedback practices, despite the lack of a research-based description. Nicol and Macfarlane-Dick identified gaps in the research base that yielded calls for further research. Recognizing that their synthesis and resulting analysis was not exhaustive, a request for research to “refine these principles, identify gaps and to gather further evidence about the potential of formative assessment and feedback to support self-regulation” was suggested (p. 215).

In a case study analysis, Cleary and Zimmerman (2004) assessed the implementation of the Self-Regulated Empowerment Program (SREP), which is a program designed to help educators foster positive, self-motivating learning experiences for students. The SREP provides students a self-regulated learning coach (SRC) to identify academic deficiencies, provide instruction fostering SRL cyclic processing, and generate continuous and immediate feedback. The ultimate goal of the SRC was to empower students to be responsible for their own learning.

A single case study was presented that detailed a female student struggling in a science course who had been introduced previously to other interventions, resulting in very little success. The SREP provided her with guidance that initiated a cyclic SRL processing system to aid in learning tasks (Zimmerman, 1989, 2000). The results of the case study analysis showed that the SREP fostered autonomy and self-directed learning practices. The student also showed increased achievement due in part to the program. The researchers did quantify their results by stating that the SREP is not an all-inclusive program and would be more successful if introduced in conjunction with other interventions. Cleary and Zimmerman’s (2004) comments on teacher feedback provided insight into the interrelationships inherent in SRL and external feedback, as follows:



The type of feedback that students receive from teachers also will influence their ability to reflect on performance outcomes. For example, teachers who do not provide students with strategic feedback or with a clear explanation of their specific errors will make it more difficult for students to understand why they are performing poorly and what they need to do to improve. (p. 548)

So, according to Cleary and Zimmerman, educators can impact students' self-regulatory actions and feedback provides a means for fostering such behavior.

In a study involving an online university course in the Netherlands, van den Boom, Paas, and van Merriënboer (2007) investigated the effects of student reflections with external feedback on SRL and performance. The SRL framework used in the study consisted of a decomposition of SRL processing using theory similar to that found in Zimmerman's (2000) cyclic model. The decomposition provided micro-level processes for each of the constructs from the model used in the study. Participants were 49 students studying psychology via a distance teaching university, who were assigned to the following random groups: control, reflection with peer feedback, and reflection with tutor feedback. Reflective protocols were developed using the decomposition of SRL processing described above and consisted of prompts intended to elicit student reflection of their learning process. Peer feedback was to be generated via electronic newsgroups concerning students' discussion posts generated from the reflective protocols. Tutor feedback was provided via direct email from the students to their assigned tutor. Students' SRL functioning, achievement, and appraisal of the learning experience were measured by the Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich,

Garcia, Smith, & McKeachie, 1991), the final exam for the course, and an evaluation questionnaire.

Upon the completion of the study, van den Boom et al. (2007) chose to rename the peer-feedback group as the *reflection-without-feedback* group since very little peer feedback dialogue was generated by the group. This led to the observation that more research is needed to determine how to better elicit feedback dialogue amongst peer groups. Findings indicated a significant difference between the reflection-without-feedback and tutor-feedback groups' frequencies of reflective activities. In fact, the peer-feedback group demonstrated very little attention to developing reflective dialogue. In terms of SRL functioning, the analysis conducted on the MSLQ data revealed significant differences in only two of the six scales: Value and Test Anxiety. The Value scale of the MSLQ contains three subscales: Intrinsic Goal Orientation, Extrinsic Goal Orientation, and Task Value. In both cases, the reflective groups scored higher than the control group. Only with respect to the test anxiety scale did the tutor-feedback group score significantly higher than the peer-feedback group. Although in partial agreement with their hypothesis that feedback would foster SRL processing, the authors suggested that inflexibility of the course and/or possible incongruities between the aspects of SRL being assessed and the MSLQ may explain why the findings indicated significant differences for only two of the scales of the MSLQ. In terms of achievement, the tutor-feedback group scored significantly higher on the exam than the peer-feedback group, but no significant difference was found between the control group and the combined reflective groups. The evaluation questionnaire revealed that both reflective groups valued the learning experience, but no significant differences were found amongst their responses.

Finally, van den Boom et al. (2007) suggested that researchers investigate peer-feedback in conjunction with collaborative learning and that similar studies be developed using mixed methods designs. The authors also suggested that educators implement reflective dialogue into online courses and utilize scaffolding to gradually shift the feedback responsibilities to students. The authors provided three limitations for their study, as follows: unaccountable factors in the design of the study, questionable credibility of the MSLQ (or any self-report questionnaire) as the main source of data, and conclusions being limited to higher education online learning environments.

#### *Motivation and SRL*

In addition to cognitive aspects of SRL, Garcia and Pintrich (1994) suggested that motivation is an integral part of any SRL model. Garcia and Pintrich also suggested that motivation depends on students' goals, orientation for learning, and beliefs about task difficulty. Further, they claimed that motivation is related to students' knowledge of self, which includes self-schemas (i.e., students' beliefs about themselves). They also introduced motivational strategies within the framework of SRL, which included self-handicapping, defensive pessimism, self-affirmation, and attributional style.

According to Garcia and Pintrich (1994), self-handicapping is an anticipatory coping mechanism for students with fragile self-schemas, which involves students purposely failing to exert effort to maintain their self-worth. An example would be a student who exerts low effort on a learning task knowing that obtaining a high grade will be positive, but obtaining a low grade can be excused for lack of time spent on the task. Defensive pessimism is also anticipatory but involves the exertion of high effort to compensate for negative self-schemas. Students who practice defensive pessimism

typically demonstrate high motivation and self-regulatory skills but set extremely low expectations to avoid anxiety and compensate for poor self-efficacy. The authors noted that the presence of SRL processing and low self-efficacy goes against research and literature suggesting that students need high self-efficacy to engage in SRL (Bandura, 1997; Usher, 2009; Zimmerman, 1989, 2000). Self-affirmation is a reactive strategy that involves the learner activating positive self-conceptions to counteract a negative evaluation. For example, a student may receive a low grade on a mathematics quiz but remind himself that he is in a prestigious program of study. Finally, attributional style is a reactive strategy that involves the student with a well-defined self-schema responding to events based upon multiple habitual experiences. Each of these strategies may be employed by self-regulating students when faced with a particular academic task that demands motivational regulation.

Wolters (1998) conducted a mixed-methods study examining the degree to which college students regulate motivation while participating in tasks deemed irrelevant, boring, or difficult. Overall, it was found that students did use a variety of techniques to regulate motivation. In fact, Wolters identified 14 different descriptors for coping strategies utilized by students to regulate motivation. Of the 14 codes developed by Wolters, the most often used coping strategies fell under the code of *cognition* and most of these instances occurred when the task condition was *highly difficult*. This finding indicates that students tend to exhibit self-regulatory practices when the task is more difficult.

Wolters (1998) also examined student self-regulation of motivation with respect to goal profiles. Recall that the focus of learning goals is on knowledge and skill

attainment, whereas the focus of performance goals is on task completion. Findings indicated that intrinsic motivation strategies were positively related to learning goal orientation and that extrinsic motivation strategies were positively related to performance goal orientations. Intrinsic motivation strategies were also positively related to cognitive strategy use, whereas extrinsic motivation strategies were not related to cognitive strategy use. Overall, Wolters suggested that the students demonstrated a systematic means of regulating motivation and that their regulation was similar to cognitive models of SRL.

Wolters (1999) conducted another study on the regulation of motivation in high school students. This quantitative study utilized a questionnaire developed by the author based upon the MSLQ (Pintrich et al., 1991). Factor analysis revealed the following statistically valid factors developed by Wolters (1999) from the SRL and motivation literature:

- *Self-consequating* questions were related to students' "self-provided extrinsic rewards for reinforcing their desire to finish academic tasks."
- *Environmental control* questions were related to students' "avoiding or reducing distractions as a means of ensuring their completion of academic tasks."
- *Interest enhancement* questions were related to students' "tendency to make the task into a game, or more generally to make it more immediately relevant, enjoyable, or fun to complete."
- *Performance self-talk* questions were related to students' use of "subvocal statements or thoughts designed to increase their desire to complete the task by intensifying their focus on performance goals."

- *Mastery self-talk* questions were related to students' "tendency to focus or make salient their desire to learn or master task materials in order to increase their level of motivation." (p. 287).

The strong relationships among factors imply that overall students who apply one motivational strategy tend to apply others. Results indicated that students differentiate the frequency of motivational regulation strategy use, as indicated by the following highest-to-lowest ordinal list: performance self-talk, environmental control, self-consequating, mastery self-talk, and interest enhancement. All five of the strategies exhibited moderate to strong positive correlations with cognitive and metacognitive SRL practices and effort, but only performance self-talk was significantly related to GPA. Also, mastery self-talk was significantly related to more elaborate forms of self-regulation; whereas performance self-talk was significantly related to lower-level cognitive aspects of SRL. This seems to go against theoretical suppositions that higher-order cognition is related to higher student achievement. These findings must be tempered with the fact that self-report was the only instrument used in the study.

Recent SRL literature has brought to the forefront the theoretical underpinnings of motivation with respect to SRL (Schunk & Zimmerman, 2008). Zimmerman and Schunk (2008) provided a clear description of the need for motivational-based studies of SRL:

Although SRL interventions produced successful outcomes in classroom settings, they often failed to sustain students' use of these processes in less-structured environments. This limitation has led researchers to focus on students' sources of motivation to self-regulate . . . (p. 2)

A more specific rationale for investigating motivation within a study of SRL is that students who are able to effectively self-monitor learning tasks but lack the motivation to alter practices do not receive full benefits of SRL.

Research has suggested that motivation is a key component in eliciting student transitions through phases within the SRL processing sequence (Wolters, 1998, 1999). Winne and Hadwin (2008) identified such changes and their relation to SRL, stating, “We refer to making such changes as regulating a *motivational state* [italics added]. We posit that regulating a motivational state follows a similar process to regulating other aspects of learning” (p. 306). Findings that suggest a link between SRL and motivation have significant implications for practicing educators. If what Winne and Hadwin posited above is true (i.e., if motivational state can be self-regulated) then practitioners could potentially develop methodologies for fostering autonomous, self-motivating students.

Specifically, Winne and Hadwin (2008) requested studies tying motivational state to SRL:

Research needs to examine more thoroughly (a) the types of goals and standards students adopt with respect to motivational state, (b) strategies they actually employ to regulate motivational state, and (c) the degree to which they are metacognitively aware of the goals, standards and strategies used to monitor and change motivational states during learning activities. (p. 308).

In addition to pointing out key gaps in the current body of literature, the authors also stated that such studies must examine thoroughly the actions students actually engage in with respect to SRL processing. Many of the current studies in the field merely report the

intentionality of students to participate in SRL but never follow up on the goals and plans established by participating students.

### *Theoretical Analysis of Epistemology*

“*Epistemology* [italics added] is an area of philosophy concerned with the nature and justification of knowledge” (Hofer & Pintrich, 1997, p. 88). Educational psychologists are interested in the effects that students’ epistemologies have on cognition, affect, and ultimately student achievement and learning. Educational epistemological research and theory can be divided into two main categories: the development of epistemological beliefs over time and the exploration and theory of a multidimensional interpretation of epistemological beliefs (Hofer & Pintrich, 1997). Perry (1970) is generally given credit for beginning the developmental epistemology movement (and the study of personal epistemology, in general) and Schommer (1990) is generally credited with initiating the study of epistemology through the lens of independent, multi-dimensional beliefs. These seminal works would eventually filter down to two specific issues relevant to this study: the domain specificity of epistemological beliefs (Hofer & Pintrich, 1997; Muis, Bendixen & Haerle, 2006) and the ramifications of students’ personal epistemological beliefs on mathematical problem solving (Kloosterman & Stage, 1992; Muis, 2004, 2008; Schoenfeld, 1983, 1985, 1988, 1989, 1992). In her contemporary synthesis of mathematics-based personal epistemology research, Muis (2004) divided students’ mathematical beliefs into two categories with respect to learning: availing and non-availing. “An availing belief is one that is positively related to quality learning and achievement, and a nonavailing belief is one that does not affect learning or achievement in a positive way” (p. 324). These categories will be used



throughout the ensuing discussions of epistemology and mathematical problem solving. This section will present a literature-based, theoretical analysis of personal epistemology. The section will be subdivided as follows for organizational purposes: (1) multi-dimensional beliefs view of epistemology, (2) issues of domain with respect to epistemology, and (3) mathematical problem solving and epistemology.

*Multi-Dimensional Beliefs View of Epistemology*

Schommer (1990) suggested a framework for investigating epistemology based on multiple, independent dimensions of epistemological beliefs. Her study involving university and junior college students utilized a questionnaire to investigate her hypothesis that “epistemological beliefs are a system of more or less independent beliefs” (p. 499). Her findings suggested that epistemological beliefs can be divided into four independent dimensions: (1) innate ability (“the ability to learn is innate rather than acquired”), (2) simple knowledge (“knowledge is simple rather than complex”), (3) quick learning (“learning is quick or not at all”), and (4) certain knowledge (“knowledge is certain rather than tentative”) (p. 499). Additionally, Schommer found that quick learning predicted oversimplified conclusions, underperformance on mastery assessments, and overestimation of understanding. Certain knowledge predicted distortion of the reality that knowledge is relative and contextual to compensate for the rigid aspect of this belief.

Since Schommer’s (1990) findings, researchers have sought to integrate a multi-dimensional model of epistemological beliefs with other cognitive and affective models of learning (Hofer, 2004a; Hofer & Pintrich, 1997; Muis, 2007; Schommer-Aikins, 2004). In fact, Schommer-Aikins (2004) stated, “The need for an *embedded systemic model* of epistemological beliefs, that is, a model that includes many other aspects of

cognition and affect, comes from the assumption that epistemological beliefs do not function in a vacuum” (p. 23). Of particular interest to this study are epistemological beliefs models that incorporate metacognitive, affective, and self-regulatory aspects.

Hofer and Pintrich (1997) acknowledged the theoretical strides of Schommer’s (1990) work, but questioned the *innate ability* belief dimension. They argued that innate, or “fixed, ability beliefs concern the nature of intelligence as a personal, psychological trait of an individual” and should, therefore, be considered a separate construct from epistemological beliefs (p. 109). They also acknowledged Schommer’s contribution of devising a questionnaire for measuring personal epistemology but pointed out the construct validity issues that have plagued the questionnaire’s use in her studies. In fact, Hofer and Pintrich questioned whether or not epistemological beliefs can be measured via questionnaire.

Upon examining models ranging from Perry’s (1970) developmental model to Schommer’s (1990) multi-dimensional model, Hofer and Pintrich (1997) suggested the following general framework for epistemological beliefs: (1) nature of knowledge, which includes certainty of knowledge and simplicity of knowledge, and (2) process of knowing, which includes sources of knowledge and justification for knowing. Hofer and Pintrich provided comprehensive definitions for the four subdivisions of the general framework:

- *Certainty of knowledge*. The degree to which one sees knowledge as fixed or fluid.

- *Simplicity of knowledge.* As conceptualized by Schommer, knowledge is viewed on a continuum as an accumulation of facts or as highly interrelated concepts.
- *Source of knowledge.* At the lower levels of most of the models, knowledge originates outside the self and resides in external authority, from whom it may be transmitted. The evolving conception of knower, with the ability to construct knowledge in interaction with others, is a developmental turning point of most models reviewed.
- *Justification for knowing.* This dimension includes how individuals evaluate knowledge claims, including the use of evidence, the use they make of authority and expertise, and their evaluation of experts. (pp. 119–120)

These general epistemological beliefs dimensions provide a framework for analyzing student beliefs during any learning episode and may be applied to domain-specific inquiries.

Additionally, Hofer and Pintrich (1997) suggested that epistemological beliefs may be related to cognition and motivation. In particular, they suggested that epistemological beliefs may be tied to the goal-setting phase of self-regulated learning models (e.g., Winne & Hadwin, 1998; Zimmerman, 2000). Their suggestions prompted epistemology researchers and theorists to develop and investigate integrated models of epistemological beliefs. These innovations include, but are not limited to, models relating epistemological beliefs to metacognition, metacognitive monitoring, and self-regulated learning (Hofer, 2004a; Muis, 2007; Schommer-Aikins, 2004).

*Issues of Domain with Respect to Epistemology*

One core issue facing the research of epistemology is determining whether epistemological beliefs are domain-general, domain-specific, or a combination of the two. In other words, are students' beliefs of knowledge and knowing similar regardless of context (domain-general), or are there differences in students' epistemological beliefs dependent upon context (domain-specific)? With respect to problem solving, Schraw (2001) suggested that the importance of answering this question is "that individuals may develop different epistemological commitments across domains depending upon the extent to which they possess expert knowledge" (p. 453).

In a synthesis of epistemological studies related to academic domain issues, Muis, Bendixen, and Haerle (2006) found support for both domain-general and domain-specific views of epistemological beliefs. Based upon the extensive review, Muis et al. developed the theory of integrated domains in epistemology (TIDE) framework for academic studies and interventions relating to student epistemology. The TIDE model "provide[s] a theoretical framework from which to discuss broader relations among epistemic beliefs and various facets of cognition, motivation, and achievement" (p. 30). Throughout the model, general and domain-specific epistemological beliefs work in tandem and are reciprocally related. Additionally, development of epistemological beliefs is assumed to occur through the four dimensions suggested by Hofer and Pintrich (1997). The authors suggested that domain-specific beliefs socially develop and evolve through school experiences, but general ways of knowing developed before entering school remain and continue to influence subsequent learning experiences. As students progress through school, domain-specific beliefs tend to become more influential. Muis et al. suggested

that “as individuals become more specialized in a particular domain, which typically begins in upper-level high school, their academic epistemic beliefs are more representative of their focal domain” (p. 35). Muis et al. concluded that researchers should consider domain when developing contextually dependent studies involving personal epistemology, including the choice and use of questionnaires.

Royce and colleagues’ findings are particularly salient for viewing students’ beliefs about mathematics knowledge as variant with respect to domains (Diamond & Royce, 1980; Royce & Mos, 1980; Wardell & Royce, 1978). Royce and colleagues examined three epistemological constructs: (1) rationalism (logical and analytical cognition), (2) empiricism (observational and perceptual cognition), and (3) metamorphism (insightful and symbolic cognition). Royce and Mos (1980) developed the Psycho-Epistemological Profile (PEP) to determine an individual’s epistemological persuasion. The PEP provides a score for each of the three epistemological constructs described above. The highest of the three scores indicates the individuals’ epistemic persuasion, or profile. Additionally, Royce and Mos (1980) found that mathematics professors tended to demonstrate rational epistemic qualities, which distinguishes mathematics from other disciplines such as science whose professors depended mainly upon empiricism. It should be noted that not all participants of domain-specific activities model the more generally dominant patterns of epistemic beliefs for that domain (Muis, et al., 2006; Muis, 2008). Students tend to exhibit this disconnect between actual beliefs and domain-dominant beliefs, which provides a window of opportunity to observe relationships between students’ actual epistemic profile, SRL prowess, and performance during mathematical problem-solving tasks.

Hofer (2000) investigated the dimensionality of epistemological beliefs with respect to academic discipline, or domain, in a study involving first-year college students in psychology and science. A portion of the study involved the testing of the Discipline-Focused Epistemological Beliefs Questionnaire (DFEBQ) designed by the author. Her intention was that the DFEBQ would measure students' levels of epistemological beliefs as described by Hofer and Pintrich (1997): certainty of knowledge, simplicity of knowledge, source of knowledge, and justification for knowing. After conducting a factor analysis, the following dimensions emerged from the questionnaire: certain/simple knowledge, justification for knowing: personal, source of knowledge: authority, and attainability of truth. Both certainty and simplicity of knowledge loaded onto the same dimension and thus, the two dimensions were collapsed to one. Thus, the certain/simple knowledge dimension may be assigned the combined definition of certainty and simplicity of knowledge provided by Hofer and Pintrich (1997) above. The justification for knowing and sources of knowledge items only loaded with respect to non-availing beliefs. Specifically, the justification dimension only factored with regard to personal opinion and experience, as opposed to the more availing belief that knowledge should be justified based on the evaluation of evidence and expertise. Similarly, the sources of knowledge dimension only factored with respect to external authority, as opposed to the more availing belief that knowledge may be attained via the interactive construction of knowledge. Finally, attainability of truth was an unexpected dimension whose items were expected to load onto certainty of knowledge. The results of the study revealed that significant differences existed between the science and psychology students for each belief scale. For example, students held significantly higher beliefs that science

knowledge was more certain than psychological knowledge. Hofer (2000) suggested that, while students hold domain-general epistemological beliefs, first-year college students are able to discriminate among beliefs based on domain. Finally, Hofer suggested factor loadings indicated that additional work is needed to improve epistemological self-report questionnaires, implying that interviews and other qualitative methods may be more suitable for studying epistemology.

Hofer continued to investigate the dimensionality and domain-specificity of epistemological beliefs using think-aloud protocols (Hofer, 2004a) and case study methods (Hofer, 2004b). Applying a framework encompassing metacognition and epistemology, Hofer (2004a) conducted a series of studies with high school and college students, which involved thinking aloud while conducting computer searches for a science unit. The framework situated certainty and simplicity of knowledge within a *metacognitive knowledge* construct, source of knowledge and justification of knowing within a *metacognitive judgments and monitoring* construct, and identified self-regulation as the regulation of cognition during knowledge construction. Her prior work revealed problems assessing students' epistemological beliefs via self-report questionnaires, particularly the justification of knowing and source of knowledge dimensions (Hofer, 2000). Think-aloud protocol analysis yielded evidence of all four beliefs dimensions and provided evidence that beliefs operate interactively and tie to student motivation. Despite methodological concerns, Hofer (2004a) suggested "thinking aloud, however imperfect, may be the best means of learning about the actual, situated nature of epistemic thinking" (p. 51).

Hofer (2004b) conducted a case study of 25 first-year university students enrolled in two different chemistry courses. Thirteen of the students were enrolled in an advanced organic chemistry course focusing on student construction of knowledge via group activities. The remaining twelve students were in a general chemistry course focusing on mathematical procedures and measurement. In the latter course, the professor placed heavy emphasis on the text and review sessions as means for preparing students for eventually curved open-ended exams. Hofer and Pintrich's (1997) four dimensions of epistemological beliefs were analyzed within the classroom context using semi-structured interviews, classroom observations, and pedagogical artifacts. With respect to *simplicity of knowledge*, a common theme amongst many students was the preference for multiple-choice exams, which typically assessed more simplistic, concrete topics instead of the open-ended questions that were prevalent on both chemistry courses' exams. Although not to be generalized, findings indicated that students in the more constructivist-oriented course tended to adapt more readily to the critical thinking and synthesis required of the exams. The students in the more procedurally-based general chemistry course tended to struggle throughout the course.

In terms of the continuum of *certainty of knowledge*, the results of the study suggested most students had moved past the extreme dualistic nature of knowledge but were not sophisticated enough to critically evaluate knowledge to develop improved interpretations. Such students tended to accept all knowledge as opinion and assigned equal validity to any claim. Students in the study held fairly unsophisticated views of *source of knowledge* and *justification for knowing*. Textbooks were generally cited as authoritative sources of knowledge despite college professors' credentials and



preparation. Additionally, despite the opportunity in the organic chemistry course to attend study groups and construct their own knowledge, few of the students in the study participated. Finally, few students comprehended the mechanics of scientific inquiry, which is the main source for justifying knowledge in chemistry.

### *Epistemological Beliefs and SRL*

Building on prior work suggesting connections between epistemological beliefs and SRL (e.g., Hofer, 2004a; Hofer & Pintrich, 1997; Schommer-Aikins, 2004), Muis (2007) proposed a model integrating epistemological beliefs into Winne and Hadwin's (1998) model of SRL. Specifically, she theorized that epistemological beliefs are enacted at the *definition of task* phase and "may influence the standards set for a task," which directly impacts the goals that students set (Muis, 2007, p. 180). In turn, standards affect students' enactment and evaluation of strategies used to complete the task. Thus, Muis' (2007) proposition implied that epistemological beliefs may be related to all phases of SRL, as defined by Winne and Hadwin (1998). Additionally, Muis suggested that a reciprocal relationship may exist between epistemological beliefs and SRL. Internal feedback generated at the monitoring phase of SRL is compared to standards set and ultimately informs cognitive and affective schemas, which may include epistemological beliefs. Finally, Muis suggested that studies examining the relationships between epistemology and SRL combine quantitative and qualitative techniques, including think-aloud protocols and interviews.

In a study directly applying Muis' (2007) model of SRL and epistemological beliefs, Muis and Franco (2009) studied 201 undergraduate educational psychology students. Hofer and Pintrich's (1997) four-dimension framework for epistemological

beliefs was used to analyze students' beliefs. Data were collected with regard to students' SRL processing, goal-orientation, and epistemological beliefs. Qualitative data consisted of students' reflections on three course assessments. As part of the coursework, students were required to reflect on what they had learned and how they could apply the knowledge to teaching or learning situations. Quantitative data included the MSLQ (Pintrich et al., 1991), DFEBQ (Hofer, 2000), and a goal-orientation questionnaire. Qualitative data were coded based on a coding scheme developed from Hofer and Pintrich's (1997) epistemological beliefs. Structural equation modeling was applied to the quantitative data to obtain a system of weighted, causal paths between the myriad of variables. The resultant model demonstrated a moderate to good fit and suggested that epistemological beliefs influence goal-orientation, then goal-orientation influences SRL strategy use, and finally, SRL strategy use influences student achievement.

A surprising result was that the *source of knowledge* belief was not a factor in the model. Muis and Franco (2009) suggested that the applied nature of educational psychology may explain the lack of a significant causal path. Of particular interest are the findings relating epistemological beliefs to goal orientation (see Table 1). Muis and Franco's findings suggested that a relationship exists between goal-orientation and three of Hofer's (2000) epistemological beliefs categories from the DFEBQ. In addition to the lack of source of knowledge dimension, the relationship between *attainability of truth* and performance- and mastery-approach goal orientations was also surprising. This relationship contradicted Muis' hypothesis that a constructivist orientation to learning would be more conducive to students' adoption of mastery-approach goals since the belief that truth is attainable is generally considered to be non-availing and less

constructivist-based (Hofer & Pintrich, 1997). Muis and Franco suggested that the type and difficulty-level of tasks may affect the depth of SRL processing, including goal-setting and orientation. Additionally, these findings suggested that domain-specificity may play a role in determining the relationship between epistemological beliefs and SRL. Finally, Muis and Franco called for research examining task-definition and domain-specificity using think-aloud protocols and interviews.

Table 1

*Interrelated Qualities of Goal-Setting and Epistemological Beliefs*

Belief Dimension	Student profile	Goal preference
Certainty/simplicity	Certain/simple	P-AP, P-AV
Certainty/simplicity	Tentative/complex	M-AP
Justification	Expert justification	P-AP, M-AV
Attainability of truth	Truth is attainable	P-AP, M-AP

*Note.* Belief dimensions represent Hofer's (2000) belief scales from the DFEBQ. Certainty of knowledge and simplicity of knowledge loaded onto a single factor and are thus, shown as a single dimension. P-AP = performance-approach; P-AV = performance-avoidance; M-AP = mastery-approach; M-AV = mastery-avoidance

Neber and Schommer-Aikins's (2002) study focused on SRL and epistemological beliefs for gifted students. The study involved both elementary and secondary students and investigated, among other things, the causal relationships between epistemological

beliefs and SRL. The quantitative study utilized the MSLQ (Pintrich et al., 1991) and Schommer's epistemological beliefs measure for high school students as the main instruments for data collection and subsequent MANOVA analyses. Three surprising results were found suggesting that high school students' epistemological beliefs are no more advanced than elementary students, high school students demonstrate poor SRL processing in physics, and goal-orientation is a very weak predictor for SRL processing. Additionally, *success does not require work* was the only epistemological belief significantly related to SRL and subsequently included in the causal path model.

Neber and Schommer-Aikins (2002) suggested that the constraining environment of high-school physics, in contrast to the more constructive elementary science courses, was the main cause of the differences in SRL practices. The authors also suggested that SRL in science could be promoted by increasing students' opportunities to actively engage in investigative studies. Finally, the weak effects of epistemological beliefs on SRL strategy use may be attributed to the use of a domain-general questionnaire in a domain-specific study.

Bråten and Strømsø (2005) investigated relationships between epistemological beliefs and SRL amongst Norwegian college students, 178 majoring in management and 108 in education. The quantitative study utilized self-report questionnaires. In contrast to Neber and Schommer-Aikins' (2002) findings, the results suggested that epistemological beliefs were significant predictors of SRL strategy use and "should be included in models of self-regulated learning" (Bråten and Strømsø, 2005, p. 559). These results were obtained despite using a domain-general questionnaire. However, the authors were unable to establish domain related differences amongst the students, which may be attributable

to the domain-generalty of the instrument. Additionally, naïve epistemological beliefs negatively related to self-efficacy and mastery goal orientation. The authors presented multiple calls for research, including a call for more research utilizing classroom observations, think-aloud protocols, and interviews to “provide dynamic, in-depth views of epistemological beliefs and their relations to other constructs” (p. 561).

Research suggests that epistemological beliefs are related to academic achievement. In a quantitative study involving approximately 1600 students from Spain, Cano (2005) reported that quick and simple knowledge directly, negatively impacted student achievement. Additionally, surface approaches to learning negatively impacted student achievement; whereas deeper approaches to learning positively affected student achievement. Cano also found that high school students tend to show a significant decrease in deep-level approaches to learning. One possible rationale for this disturbing result is that secondary students may have become institutionalized and learned “to navigate the choppy waters of the curriculum” (Cano, 2005, p. 215). Schoenfeld (1988, 1989) echoed this opinion in his work on problem solving, discussed in detail below.

#### *Theoretical Analysis of Problem Solving*

According to the National Council of Teachers of Mathematics (NCTM), “solving problems that have been strategically chosen and carefully sequenced is a fundamental vehicle for *learning* mathematical content [italics added]” (NCTM, 2000, p. 335). The constructivist viewpoint taken by NCTM assumes that problem solving can be a form of learning, given that students are engaged in the development of their own knowledge. Unfortunately, Schoenfeld’s (1988, 1989) studies of high school geometry students indicated that students tend to rely heavily on rote memory and empirical musings, fail to

assess the validity of mathematical reasoning while problem solving, and lack the control to curtail *wild goose chases*. More contemporary studies have suggested that such behaviors are still prevalent (Lerch, 2004; Muis, 2008), but provide hope that pedagogical interventions are feasible and conducive to increasing students' problem solving prowess and self-regulatory practices (de Corte, Verschaffel, & Op 'T Eynde, 2000; Perels, Gürtler, & Schmitz, 2005).

For this study, Schoenfeld's (1985) problem-solving framework was appropriate for analyzing students' problem-solving endeavors. Schoenfeld's interest in problem solving can be credited to George Polya, the father of the modern problem-solving movement. Although problem solving has always been inherent in mathematics, George Polya is generally given credit for ushering in the modern era of the *study* of mathematical problem solving (Schoenfeld, 1987). Even with publication dates for most of his problem-solving works surpassing four decades, Polya's general methods for teaching and learning mathematical problem solving, including an extensive list of heuristic (problem-solving) strategies, are still in use in classrooms today. Schoenfeld (1985), extending the work of Polya, presented a problem-solving framework that brought student control and beliefs issues to the forefront, suggesting that self-regulatory processing and epistemology may be an important aspect of problem solving. More recently, Muis' (2008) findings have extended Schoenfeld's work to encompass more contemporary SRL theory and have suggested that student beliefs, self-regulatory processing, and problem-solving capacity are interrelated constructs.

The design of this study necessitated the need for an analysis of task definition and group dynamics. Mathematical problem solving tasks may be defined according to

cognitive demand (Stein, Smith, Henningsen, & Silver, 2000) and structure (Lodewyk, 2007; Lodewyk, Winne, & Jamieson-Noel, 2009). A framework for evaluating small group problem-solving episodes presented by Artzt and Armour-Thomas (1992) proved useful for evaluating self-regulatory aspects of students engaged in group work. This section of the literature review is divided as follows: (1) issues of task definition in mathematical problem solving, (2) Schoenfeld's (1985) mathematical problem-solving model, (3) issues of group dynamics in mathematical problem solving, and (4) Muis's (2008) study of mathematical problem solving, epistemology, and SRL.

#### *Issues of Task Definition in Mathematical Problem Solving*

When developing learning tasks, teachers and researchers should consider the cognitive demand required of students for completing the task. Stein, Smith, Henningsen, and Silver (2000) suggested the following categorical scheme for assessing a task's cognitive demand: lower-level demands—*memorization* tasks and *procedures without connections* tasks; higher-level demands—*procedures with connections* tasks and *doing mathematics* tasks. *Procedures without connections* tasks require students to apply algorithmic procedures, which are devoid of deeper connections to concepts underlying the task. *Procedures with connections* tasks make connections to underlying concepts and require significant cognition as students adapt procedures to task conditions. A constructivist, SRL-infused, problem-solving approach to mathematics teaching and learning implies the need to develop tasks with a high cognitive demand (Lodewyk, 2007). The *doing mathematics* tasks most closely resemble the types of problem-solving activities appropriate to the constructivist view of mathematical learning. In fact, one

rationale suggested by Stein et al. for classifying a task as *doing mathematics* is “there is no predictable pathway suggested by the task and it requires complex thinking” (p. 21).

In practice, tasks used by mathematics teachers will necessarily shift from one end of the cognitive continuum to the other (Stein et al., 2000). Teachers need to keep track of the frequency of use for each level and keep learning goals in mind as tasks are developed. Finally, teachers should also consider the most appropriate task level to use based on student abilities and needs (Stein et al., 2000).

Lodewyk and colleagues investigated task structure with respect to epistemology and SRL (Lodewyk, 2007; Lodewyk, Winne, & Jamieson-Noel, 2009). In terms of structure, tasks are generally defined as either *well-structured tasks* (WST) or *ill-structured tasks* (IST). Lodewyk (2007) suggested that a WST “is usually more clearly formulated and presented, and requires straightforward operations, providing the necessary information, algorithms, and precise criteria to indicate how the completed learning task will be assessed to aid the learner in searching for a suitable answer;” whereas an IST “makes less obvious the operations to be used, is more ambiguous (does not have a clearly identifiable answer), and does not provide the necessary information (clear instructions), algorithms (cues and resources), or evaluative criteria for students to determine if they are solving the problem correctly” (p. 311).

Both studies discussed here drew from the same sample of tenth grade science students in western Canada (Lodewyk, 2007; Lodewyk et al., 2009). Each student completed a WST and an IST in partial completion of a science unit on cancer. To complete the WST, students typed an essay on cancer and were provided with specific subgoals, organized resources, and grading criteria. In contrast, the IST required students



to develop a stance as to whether government should focus on the treatment or prevention of cancer. The task was more ambiguous, cognitively-demanding, and reflective than the WST. Both studies were quantitative and involved factor analyses, ANOVA, MANOVA, and structural equation modeling to provide robust results.

Lodewyk's (2007) focus was on the relationships between task structure and epistemological beliefs. Results suggested that students who view knowledge as simple and fixed tended to have more difficulty with the IST. Additionally, findings suggested that non-availing beliefs, especially quick, fixed and simple views of knowing, predict lower student achievement. Lodewyk suggested that teachers should provide students with both types of problem structures, as too many well-structured tasks tend to promote a simple-knowledge belief system in students and too many ill-structured tasks tend to promote anxiety and withdrawal. Finally, Lodewyk suggested that studies of epistemological beliefs should focus more on domain-specific beliefs and be more fine-grained in nature.

Lodewyk, Winne, and Jamieson-Noel (2009) suggested that teachers should consider student achievement when assigning tasks. Their study investigated aspects of self-regulation exhibited by students while working on the WST and the IST discussed above. They found that lower-level achievers regulated their efforts and performed better on the WST but had difficulty scaffolding their knowledge to regulate and perform well on the IST due in large part to a lack of setting appropriate subgoals. In contrast, higher-level achievement students tended to be more motivated, demonstrated higher levels of self-regulation, and performed better on the IST due to the high cognitive demand

inherent in such problems. Thus, teachers should vary the cognitive load of tasks assigned to students to simultaneously promote self-efficacy and critical thinking.

*Schoenfeld's Mathematical Problem-Solving Model*

Schoenfeld (1985) stated, "By definition, problem situations are those in which the individual does not have ready access to a (more or less) prepackaged means of solution" (p. 54). This definition differentiates *problem solving* from routine mathematical exercises and was the operational definition for problem solving throughout this study. Schoenfeld's definition is also congruent with the *doing mathematics* cognitive demand category described above (Stein et al., 2000). The majority of Schoenfeld's empirical studies involved undergraduate university students (1982, 1983, 1985) and high school students (1988, 1989). His work produced a framework for examining mathematical problem solving. The components of his framework will serve as the categorical subheadings for the ensuing discussion: (1) resources, (2) heuristics, (3) control, and (4) belief systems (Schoenfeld, 1985).

*Resources.* To undertake any endeavor, one must have the necessary resources to complete the task. Mathematical problem solving is no different and educators must take resource accessibility into account when infusing problem solving into any mathematics agenda. The most basic resource available to the prospective problem solver is "domain-specific knowledge" (Schoenfeld, 1983, p. 332). For instance, a student attempting to determine two functions whose intersections form an enclosed region with area equivalent to some real number must either recall from elementary calculus that the area between curves is obtained via an integral expression or utilize an approximation technique from calculus or geometry. However, simply knowing a mathematical fact will

not be sufficient unless a student accesses the appropriate knowledge from long-term memory (LTM) while working a problem (Schoenfeld, 1983). For example, even though it is intuitively obvious to a calculus student that a non-horizontal, linear function cannot be positive on its entire domain, will the student access this knowledge when attempting to prove, or disprove, whether a general cubic function can be concave up on its entire domain?

According to Schoenfeld (1985), expert mathematicians access and apply relevant knowledge routinely in problem situations. Expert mathematicians develop a system of *chunks* of mathematical scenarios, have a schema prepared for each familiar chunk, and are able to perform even in complex, unfamiliar problem-solving situations.

Unfortunately, novice problem solvers may fail to recognize a problem type, apply flawed resources to a recognized scenario, or simply fold under the pressure of a difficult, unfamiliar problem despite having the vision and resources necessary to adequately solve the problem. Since a student may fail to solve a problem for various reasons, Schoenfeld (1985) suggested that researchers keep “an inventory of resources,” which “should include not only the pieces of knowledge accessible to the individual, but the kinds of access that the individual has to them” (p. 57). The inventory may aid the researcher in determining the cause for the incomplete solution and provide an impetus for follow-up interview discussions.

More specifically, resources are domain-specific within the overall mathematical discipline (Schoenfeld, 1985). For instance, students who have only studied first-year high school algebra may find difficulty in determining an equation of the line tangent to a circle in the plane without significant support. Although they have learned about average

rates of change and may have a vague notion of the meaning of tangent, they have not learned about the geometric properties of tangents, nor have they discussed instantaneous rates of change. Schoenfeld points out that a broad range of resources exists for each mathematical domain and range from “informal and intuitive knowledge” to “algorithmic” and “routine procedures” (pp. 54–55). Students will demonstrate varying mastery of these resources, which will heavily impact the degree of success that may be attained in problem solving within the domain. Finally, researchers need to build a repertoire of common student mistakes and be prepared to identify them based solely on the nature of students’ solution attempts. The fact that this can be done implies that teachers need to become more adept at identifying common student errors and develop pedagogical means for remedying them that are more robust than simply reiterating the same procedure taught in class.

Schoenfeld (1992) provided further insights into the importance of researchers’ understanding and identification of students’ resource capacity:

Did they fail to pursue particular options because they overlooked them, or because they didn’t know of their existence? In the former case, the difficulty might be metacognitive or not seeing the right “connections;” in the latter case, it is a matter of not having the right tools. From the point of view of the observer or experimenter trying to understand problem-solving behavior, then, a major task is the delineation of the knowledge base of individuals who confront the given problem solving tasks. It is important to note that in this context, the knowledge base may contain things that are not true. Individuals bring misconceptions and

misunderstood facts to problem situations; it is essential to understand that *those* are the tools they work with. (p. 349)

*Heuristics*. Schoenfeld (1985) distinguished heuristics from routine problem solving strategies, as follows: “The use of a general problem-solving strategy is *heuristic* [italics added] if the problem solver is having difficulty, and there is reason to suspect that taking this particular approach might help” (p. 60). Expert problem solvers share a fairly common set of heuristic strategies and employ them during problem solving episodes (Schoenfeld, 1985). Thus, at the time of Schoenfeld’s (1985) work, mathematics educators had devoted large amounts of energy to teaching heuristic strategies but with few positive results. Schoenfeld suggested that the lack of success may be partially attributable to the lack of detail inherent in most heuristics-based pedagogy.

In his seminal problem-solving work *How to Solve It*, Polya (1957) described a large number of heuristics that both teachers and learners may draw upon. Many of Polya’s heuristic strategies are referenced by NCTM’s (2000) reform suggestions for teaching through problem solving. Polya’s (1957) heuristic list includes, but is not limited to, draw a picture, set up an equation, introduce proper notation, introduce auxiliary elements, work backwards, special cases, and develop subgoals. Schoenfeld (1982, 1985) specifically cited two of Polya’s heuristics strategies: *special cases* and *subgoals*. To utilize the *special cases* heuristic, one must recognize that the problem can be simplified without loss of generality, determine an appropriate simplified form, utilize resources to solve the simpler problem, and then use the result to determine the solution for the general-case problem. Applying the *subgoals* heuristic requires a similar number of steps and is generally considered more difficult to employ than the special cases heuristic

(Schoenfeld, 1985). The subgoals established for the problem should be in partial fulfillment of the given conditions. Once established, the problem-solver would then work to complete each subgoal until the original problem is solved. Expert problem solvers are capable of applying these strategies with ease and precision, but naïve problem-solvers demonstrate great difficulty with them (Schoenfeld, 1983, 1985).

Schoenfeld (1985) suggested two major causes for students' lack of ability to apply heuristics strategies. First, simply teaching, or demonstrating, general heuristics is not enough. As shown above, each heuristic method comes with a large quantity of sub-steps that students must be taught and then allowed the opportunity to explore. Otherwise, students will have the general notion of the heuristic but lack the intuition to apply it. Second, students' successful implementation of heuristics is heavily dependent on their possession of and access to resources. Mastery of a heuristic strategy will not help a student who cannot access appropriate subject matter knowledge in the given domain.

*Control.* Schoenfeld's (1985) notions of control are closely related to Zimmerman's (2000) performance control and self-reflection phases. Considering resources and heuristics as key factors in planning and goal-setting for solving problems, Schoenfeld's framework may be seen as a *self-regulatory* problem-solving framework, and has been cited as such (De Corte, Verschaffel, & Op 't Eynde, 2000; Muis, 2008). To aid students in his university problem-solving courses taught in the 1970s, Schoenfeld (1985) developed a detailed strategy integrating heuristics and control, which was developed by him as he worked problems. Schoenfeld (1983, 1985) suggested that there was no need for control if a problem was routine to the solver. Thus, he told his students

to work problems as they normally would, but refer to the detailed strategy integrating heuristics and control when they were at an impasse in a particular problem. His problem-solving strategy proceeds as follows: (1) analysis, (2) design/exploration, (3) implementation, and (4) verification.

During *analysis*, a careful examination of the problem is made and common heuristics, such as drawing a diagram or simplifying the problem without loss of generality, may be applied (Polya, 1957; Schoenfeld, 1985). Once a preliminary decision is made, one moves to the *design* phase, which entails determining an optimal plan of attack that may provide a path to success. During this phase, *exploration* is taking place, which is the most active heuristics stage involving such practices as replacing conditions with equivalent ones, developing subgoals, and constructing analogous, simpler problems (Polya, 1957; Schoenfeld, 1985). At any point during exploration, one may wish to go back and re-analyze the problem or alter facets of the current design given that navigation of the problem space is not going well. Once a schematic solution plan is obtained from this stage, one moves to *implementation* and completes the work required by the chosen methods. Finally, during *verification*, the solution is tested to ensure accuracy. For the purposes of protocol analysis, Schoenfeld parsed problem-solving sessions into timed episodes, as follows: read, analyze, explore, plan, implement, and verify.

Overall, Schoenfeld's (1982, 1983, 1985) results were favorable in that students who utilized his strategy were typically able to more readily apply heuristics and correctly solve problems as a result of his course. However, Schoenfeld's (1985) myriad observations and videotaped sessions suggested enough variation that he developed a continuum of "effects of control decisions on problem-solving success," as follows:

Type A. Bad decisions guarantee failure: Wild goose chases waste resources, and potentially useful directions are ignored.

Type B. Executive behavior is neutral: Wild goose chases are curtailed before they cause disasters, but resources are not exploited as they might be.

Type C. Control decisions are a positive force in a solution: Resources are chosen carefully and exploited or abandoned appropriately as a result of careful monitoring.

Type D. There is (virtually) no need for control behavior: The appropriate facts and procedures for problem solution are accessed in long-term memory (LTM).

(p. 116)

Of these four levels, Schoenfeld asserted that Type C control demonstrates “true problem-solving skill” (p. 127). The problem has aspects that the potential solver is unable to readily access, yet via control and continuous monitoring, the solution is still obtained.

Lerch (2004) challenged Schoenfeld’s (1985) assertions that control was at the heart of students’ failures, citing a significant lack of resources amongst most undergraduate students. Lerch suggested that Schoenfeld may have disproportionately focused on control issues, and failed to give proper credence to the immense supply of problem-solving *resources* available to the expert professors that he compared with his student participants. She further suggested that students’ lack of such *resources* may explain their wild goose chases better than issues of control. Lerch’s (2004) small-scale qualitative study investigated the differences inherent in undergraduate mathematics students’ problem-solving capacity with respect to well-structured (textbook) exercises



versus ill-structured problems. Her findings were that the students successfully applied and adapted strategies for solving the textbook problems, but were unable to successfully complete the ill-structured problems. Lerch suggested that students' success on textbook problems may be attributed to the accessibility of a procedural model; students' lack of a mathematical processing model (due to insufficient resources) for unfamiliar problems may explain the failure to solve the ill-structured problems. Although not empirical, Lerch's study supported the assertion that students need to develop a more broad-based, conceptual approach to problem solving, which steers away from the algorithmic procedures that made the students successful on the textbook problems. From a mathematical problem-solving perspective, such success is artificial anyway. It only indicates the ability to follow a recipe, not that any mathematical problem-solving prowess has been attained (Schoenfeld, 1992). Finally, Lerch suggested that student attainment of mathematical problem solving skills is a multi-faceted enigma and the compelling evidence presented for using SRL as a framework only tells part of the story.

Perels, Gürtler, and Schmitz (2005) presented some promising results concerning SRL and problem-solving training for students from their study of 249 eighth-grade German gymnasium students. Using an SRL problem-solving framework based on Zimmerman (2000) and de Corte et al. (2000), Perels et al. found that providing students with six weeks of SRL and problem-solving training resulted in improvements in students for both constructs. Training sessions focused on self-regulatory components consistent with Zimmerman's model and problem-solving training focused on applying heuristic methods. To ensure quality training sessions, the researchers kept group sizes to no more than 19 students per training session. The study implemented a pretest-posttest design

with each test having an SRL questionnaire component and a mathematical problem-solving component.

The results of Perels et al.'s (2005) study revealed that the students trained in both SRL processing and problem-solving skills improved from pretest to posttest for both constructs. Unfortunately, not every aspect of SRL (including goal-setting and learning strategies) was improved to a statistically significant level, which led Perels et al. to request further research to determine more appropriate means of teaching these skills. A surprising result was that students who only received SRL training (but no problem-solving training) showed improvements in both SRL processing *and* problem-solving ability. Thus, although Perels et al. suggested difficulties in developing self-regulatory capacity, the benefits inherent to improving problem-solving capacity indicate that the endeavor should be of paramount importance to constructivist-minded mathematics educators.

*Belief systems.* Researchers have examined the relationships between epistemological beliefs and mathematical problem solving (Kloosterman & Stage, 1992; Muis, 2004, 2008; Royce & Mos, 1980; Schoenfeld, 1983, 1985, 1988, 1989, 1992). Much of Schoenfeld's research was devoted to investigating students and experts while solving mathematical problems. Based on his videotaped problem-solving sessions, Schoenfeld (1985) suggested that three nonavailing beliefs typically pervade students' attempts at solving mathematical problems:

1. Formal mathematics has little or nothing to do with real thinking or problem solving.

2. Mathematics problems are always solved in less than ten minutes, if they are solved at all.
3. Only geniuses are capable of discovering or creating mathematics. (p. 43)

In a synthesis of problem solving literature, Schoenfeld (1992) provided additional non-availing student beliefs:

1. Mathematics problems have one and only one right answer.
2. There is only one correct way to solve any mathematics problem—usually the rule the teacher has most recently demonstrated to the class.
3. Ordinary students cannot expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanically and without understanding. (p. 359)

Although obviously couched in a mathematical problem-solving perspective, these non-availing beliefs are fairly congruent with low-level, multi-dimensional epistemological beliefs described above (Hofer, 2000; Hofer & Pintrich, 1997; Schommer, 1990). Specifically, the beliefs listed above are congruent with Schommer's (1990) quick learning belief and the low-level beliefs from Hofer and Pintrich's (1997) source of knowledge and simplicity of knowledge dimensions. Along a more contemporary vein, Muis (2004) stated, "In the context of *mathematics epistemological beliefs*, beliefs include perspectives on the nature of mathematics knowledge, justifications of mathematics knowledge, sources of mathematics knowledge, and acquisition of mathematics knowledge" (p. 326).

Romberg (1992) described mathematical epistemology from a process perspective, as opposed to an acquisition perspective:

When many nonmathematicians look at mathematics, they see a bounded and static set of concepts and skills to be mastered. This is perhaps a reflection of the mathematics they studied in school or college rather than insight into the discipline itself. For many, *to know* means to identify the artifacts of the discipline—its basic concepts and procedures. For others more familiar with the discipline, *to know* mathematics is *to do* mathematics. A person gathers, discovers, or creates knowledge in the course of some activity having a purpose. Only if the emphasis is put on the process of *doing* is mathematics likely to make sense to students. (pp. 60–61)

Partially based on the works of Schoenfeld, Kloosterman and Stage (1992) developed the Indiana Mathematics Belief Scales (IMBS) to measure “secondary school and college students’ beliefs about mathematics as a subject and how mathematics is learned” (p. 109). Through a series of studies, the five scales were validated and include measurements of the following topics of mathematical beliefs: duration of problem-solving engagement, solutions of problems via procedural means, importance of conceptual understanding, importance of word problems, and attributing mathematical ability to effort versus innate skill. An additional scale measuring students’ beliefs about the usefulness of mathematics was also included with the other five scales. A more complete assessment of the IMBS may be found in *Chapter III: Methodology*.

Schommer-Aikins, Duell, and Hutter (2005) “tested the hypothesis that general epistemological beliefs are linked to the mathematical problem-solving beliefs” in a study of 1269 middle school students from the American Midwest (p. 292). Additionally, Schommer-Aikins et al. explored the developmental nature of the structure of middle

school students' beliefs, and examined relationships and possible causality between epistemological beliefs, mathematical problem solving beliefs, and performance. Data were collected from a middle school version of Schommer's (1990) epistemological belief questionnaire, the IMBS, students' performance on a standardized exam, and students' overall GPA. To test the hypotheses, data analyses included exploratory factor analysis, regression calculations, and path analysis.

Results indicated that the structure of middle school students' beliefs tend to be less developed than secondary and college students' beliefs. Exploratory factor analysis revealed that quick/fixed learning and studying aimlessly were the two strongest general epistemological beliefs and effortful math, useful math, math confidence, and understand math concepts were the four strongest mathematical problem-solving beliefs. Step-wise regression indicated that the less students believe in quick/fixed learning, the more likely they will believe that problem solving is effortful and useful, requires understanding of concepts, and have confidence in their ability to solve problems. An additional step-wise regression analysis revealed that beliefs in quick/fixed learning and that mathematics is useful predicted mathematical problem-solving performance. Finally, path analysis indicated that beliefs in quick/fixed learning, useful mathematics, math confidence, and understand math concepts had a significant effect on overall academic performance. In sum, Schommer-Aikins et al. (2005) stated, "The results of our study also suggest that both general epistemological beliefs and mathematical beliefs may play a role in students' problem-solving performance" (p. 301).

Mason (2003) used an Italian interpretation of the IMBS to study 599 Italian high school students' mathematical beliefs. Participants included students from two school,

representative of each of the five grades of Italian high schools. The mixed methods study included analyses of the Italian version of the IMBS and students' achievement in mathematics, and individual interviews with select students. All but one scale, *importance of word problems*, of the Italian version of the IMBS demonstrated moderate to high reliability. It is not surprising that the *importance of word problems* scale demonstrated low reliability since Kloosterman and Stage (1992) reported difficulty in generating reliable ratings for this scale due to students' confusion with the term *word problem*. The results of the study indicated that as students progress through high school, their beliefs that all problems may be solved via routine means begin to diminish. Unfortunately, findings indicated that non-availing beliefs emerge during the high school years as students' beliefs that they can solve difficult problems and their beliefs in the usefulness of mathematics decrease with time. Additionally, the findings indicated that four scales predicted student achievement to varying degrees, as follows in order from strongest to weakest: duration of problem-solving engagement, solutions of problems via procedural means, usefulness of mathematics, and importance of conceptual understanding. The findings of this study indicated the importance of considering mathematical beliefs as factors in students' mathematics education. Mason suggested that educators develop interventions for fostering availing mathematical beliefs in students and also suggested that teachers design instruction, tasks, and assessments in alignment with such availing mathematical beliefs.

Royce and Mos' (1980) and Schoenfeld's (1983, 1985) findings suggested that mathematics *professors* exhibit rationalist-based approaches to mathematical problem-solving. Expert mathematicians and professors develop a rational sense of problem

solving through graduate school experiences and are able to derive mathematical information even if they cannot recall it (Schoenfeld, 1983, 1985). However, Schoenfeld's (1983, 1985) findings suggested that college mathematics *students* tend to solve problems from an empirical perspective. In Schoenfeld's studies, multiple students, with little idea as to how to progress, simply tried every plausible solution by visual inspection, rather than utilizing a logical chain of reasoning or proof. Schoenfeld (1985) provided the title *naïve empiricism* to such a belief system. In contrast, when Schoenfeld (1983) gave the same problems to mathematicians, they derived necessary results, monitored their performance, and verified their solution processes. In other words, mathematicians modeled Royce and Mos' (1980) rationalist epistemic profile and demonstrated many of the phases of the SRL processing model used in this study (Winne & Hadwin, 1998; Zimmerman, 2000).

Schoenfeld (1985) also described how high school classroom experiences promote empirical beliefs in students and stymie the use of proof and discovery in problem solving:

Such experience is abstracted as part of the students' mathematical world view as follows: Mathematical argumentation only serves to verify established knowledge, and argumentation (proof) has nothing to do with the processes of discovery or understanding. As a result, students who are perfectly capable of deriving the answers to given problems do not do so, because it does not occur to them that this kind of approach would be of value. (p. 186)

On a positive note, Schoenfeld (1983) suggested that his problem solving courses promoted heuristics-based control approaches and a more rational belief system amongst

students in his university problem-solving courses. These changes resulted in improved problem-solving performance, which shows that success and progress can be made in a classroom setting.

Muis (2004) suggested that, to a certain extent, students' beliefs concerning mathematical knowledge are dependent upon classroom practices. Based on her extensive synthesis of mathematics-based epistemology literature, Muis suggested that teacher-centered classrooms focused on rote-memorization for the purpose of passing standardized exams are detrimental to students' active pursuit of mathematical knowledge as constructed schemas for solving problems. She further suggested that teacher-centered classrooms foster non-availing epistemological beliefs amongst students. She suggested that teachers develop classrooms that foster student construction of mathematics and place teachers in the role of facilitators of learning. In line with Muis' views, de Corte et al. (2000) shunned the overuse of standardized assessments and suggested that teachers assume a theory of assessment devoted to authentic, complex, real-life alternative forms of assessment. Alternative assessments may include performance tasks, journals, and portfolios (NCTM, 2000). However, it is not enough to develop such assessments and then mark them in a traditional manner with a grade ranging from zero to one hundred. From both the SRL and constructivist perspectives, teachers must provide constructive, cognitive feedback (Butler & Winne, 1995; Nicol & Macfarlane-Dick, 2006) and "stimulate in students the development of attitudes toward and skills in assessing their own mathematical learning processes and performances" (de Corte et al., 2000).



Schoenfeld's (1985, 1988, 1989) findings were among the evidence that led to Muis' (2004) and de Corte et al.'s (2000) assertions. Schoenfeld studied high school geometry students' epistemological beliefs via a year-long observational period (Schoenfeld, 1988) and a questionnaire containing both Likert-scale and open-ended items (Schoenfeld, 1989). Students reported that learning mathematics requires memorization of formulas and proofs, homework problems should be solved in a matter of minutes or not at all, and that quality teaching involves showing students how to use rules. Problem solving was discussed in classrooms, but rarely did teachers engage students in mathematical tasks beyond the scope of rote-memorized material. Thus, although performance on standardized exams was high, the observed instruction and student engagement was sub-par from a conceptual perspective. Schoenfeld (1988) continually referenced instances in which instruction focused on mechanics and form (e.g., two-column proofs and procedural constructions) at the expense of understanding. Additionally, concepts were segmented to the point that students could not make connections or apply prior knowledge to newly discussed material. In total, Schoenfeld's findings painted a bleak picture of non-availing student belief profiles for learning mathematics. I choose to close this section with a quote from Schoenfeld (1989) that still resonates in my mind as I reflect on my own mathematics teaching practice:

Perhaps the most troubling aspect of the present study is the suggestion that these students have come to separate school mathematics—the mathematics they know and experience in their classrooms—from abstract mathematics, the discipline of creativity, problem solving, and discovery, about which they are told but which they have not experienced. (p. 349)

*Issues of Group Dynamics in Mathematical Problem Solving*

Theoretical and empirical work from educational psychology has provided mounting evidence that problem-solving endeavors in small group settings can be successful (Johnson & Johnson, 2009). Additionally, the social constructivist viewpoint used in most reform mathematics literature (e.g., NCTM, 2000) has suggested that students should be critically thinking about and communicating mathematical ideas with their peers. However, simply placing students in heterogeneous groups of two to three members will not guarantee successful problem solving experiences (Johnson & Johnson, 2009). A multitude of factors must be considered by researchers investigating and educators implementing group problem solving. This section will review two frameworks for analyzing group problem-solving (Artzt & Armour-Thomas, 1992; Goos, Galbraith, & Renshaw, 2002).

Expanding Schoenfeld's (1985) episodic framework for analyzing problem-solving protocols, Artzt and Armour-Thomas (1992) developed a framework for analyzing cognitive and metacognitive behaviors exhibited by individual students engaged in group problem solving. In addition to Schoenfeld's *read, analyze, explore, plan, implement, and verify* episodes, Artzt and Armour-Thomas considered *understanding the problem* and *watching and listening* important instances in group problem-solving progression. Within these episodes, an individual member of the group may exhibit overt evidence of either cognitive or metacognitive actions. The authors distinguished these behaviors as follows:

Metacognitive behaviors can be exhibited by statements made about the problem or statements made about the problem-solving process. Cognitive behaviors can

be exhibited by verbal or nonverbal actions that indicate actual processing of information. (p. 141)

Testing their framework, Artzt and Armour-Thomas (1992) investigated the group problem-solving practices of 27 seventh-grade students from an urban middle school in Queens, New York. Six groups containing four to five students each were randomly selected to participate. The study involved the six groups working on a problem together in the classroom setting. All sessions were videotaped and students were interviewed individually while watching particular portions of the video. Coding of the videotapes was consistent with the framework developed by the authors and interrater reliability percentages were above 90% for all coders.

In terms of overall group statistics, the results of the study revealed that the highest percentage of metacognitive behaviors occurred during the *exploring* and *understanding* episodes. By a very large margin, the *exploring* episodes contained the highest percentage of students' cognitive behaviors. In most cases, the metacognitive verbalizations of individual students redirected group work and curtailed erroneous ideas.

Additionally, results indicated that *watching-and-listening* is an important factor in group problem-solving success. The only group that failed to solve the problem contained members who did not listen to one another or communicate well. In fact, the group that was unable to solve the problem had the lowest percentage of metacognitive behaviors, the highest percentage of cognitive behaviors, and the lowest percentage of watch-and-listen behaviors. On the other hand, one of the groups to successfully solve the problem provided evidence indicating negative aspects of *watching-and-listening*. In this group, one member dominated the problem-solving session while the others watched and

listened, participating only intermittently and showing signs of not understanding the evolution of the solution process. Artzt and Armour-Thomas (1992) described the level of interaction among group members on a continuum from students who work independently and do not contribute to students who actively pursue problem solutions in tandem with their fellow group members.

Finally, the study indicated that having high-ability students is not sufficient for proper group functioning or problem-solving success. The group that failed to solve the problem contained a very high achieving student who admittedly never works well with other students in groups. Another group contained a high achieving student who independently solved the problem, but failed to see the need in ensuring that others contributed to group success or learned from the experience. Artzt and Armour-Thomas (1992) concluded that group work has the potential for fostering student learning and metacognitive development, but many factors are involved that may confound group problem-solving endeavors.

Goos, Galbraith, and Renshaw (2002) pointed out that Artzt and Armour-Thomas's (1992) framework failed to differentiate between an individual student's monitoring of their own actions and thinking and monitoring of their partners' actions and thinking. Thus, Goos et al. suggested an alternative framework for analyzing metacognitive discourse during group problem-solving:

- *Self-disclosure* – Self-oriented statements and responses that clarify, elaborate, evaluate, or justify one's own thinking.
- *Feedback Request* – Self-oriented questions that invite a partner to critique one's own thinking.

- *Other-monitoring* – Other-oriented statements, questions and responses that represent an attempt to understand a partner's thinking. (p. 199)

The authors conducted a three-year study of senior secondary students in Queensland, Australia working novel problems in the classroom setting. One lesson per week was observed, videotaped, and audiotaped over the three-year duration. The problems worked by the participants were novel in that they represented new content and were presented as mathematical problems. The topics included projectile motion, combinatorics, and compound interest.

Goos et al. (2002) reported and analyzed the results of one successful and one unsuccessful problem solving session. The successful problem session, which involved students solving a projectile motion problem, indicated that group members were actively engaged in metacognitive assessments of their actions and thoughts, as well as those of others. Surprisingly, the unsuccessful problem-solving session, which involved multiple combinatorics problems, contained a similar percentage of metacognitive activities as the successful session. However, further analysis revealed that the main difference between the sessions involved the *transactive* nature of the metacognitive activities. In this study, the difference between successful and unsuccessful group problem solving was attributable to the degree that individual group members engaged in assessing and monitoring the thinking and actions of the group as a whole. In other words, a group was successful when all members engaged in the serious consideration and analysis of every thought and action posed by the group. In summary, Goos et al. suggested that cognitive and metacognitive analyses of group problem-solving sessions should include interactive,

dynamic facets of group discourse, which is an extension of Artzt and Armour-Thomas's (1992) framework that focused on individual students' metacognitive behaviors.

*Muis's (2008) Study of Mathematical Problem-Solving, Epistemology, and SRL*

Muis (2008) conducted a two-part study investigating the relationships between student epistemic profiles and SRL processing while engaged in mathematical problem solving. She framed SRL from the perspective of Schoenfeld's (1985) view of problem-solving control and epistemology from the perspective of Royce and Mos' (1980) *empirical* and *rational* epistemic styles. In the first part of the study, 268 undergraduate university students enrolled in mathematics and statistics courses were given the Psycho-Epistemological Profile (PEP; Royce & Mos, 1980) and the *metacognitive self-regulation* scale from the Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich, Smith, Garcia, & McKeachie, 1991), both described in detail in the *Chapter III: Methodology*. Royce and Mos designed the PEP to classify students as predominantly empirical, rational, or metaphorical. Muis also developed a scaled scoring method to determine students' *epistemic profiles*, as follows: predominantly rational, both rational and empirical, and predominantly empirical. In Muis' study, rational and empirical epistemic profiles were assumed to have the same connotation as that intended by Schoenfeld (1985). The number of students classified as metaphorical, a third classification of epistemic style from the PEP, was so low that Muis did not include them in the study. Using MANOVA, the results indicated that rational students scored significantly higher on the metacognitive self-regulation subscale of the MSLQ than either group, with students classified as both rational and empirical scoring significantly higher on the same subscale as empirical students.

For the second half of the study, a sub-sample of 24 students majoring in mathematics was taken from the original sample of participants. Students completed six problems (two from geometry, one from algebra, and three on the binomial distribution) outside of the classroom environment. The two geometry problems and one algebraic problem were taken from Schoenfeld's (1982) study and students worked these problems with no prior exposure to content. For the three binomial distribution problems, students studied a brief excerpt on the binomial distribution and then worked the problems with the option of referring back to the material just read. In completing all six problems, students adhered to the think-aloud protocol methodology (Ericsson & Simon, 1993) and sessions were audiotaped. Once completed, student work was coded for evidence of SRL processing and application of either a rational or empirical beliefs orientation. The SRL codes used in the study were "planning, metacognitive monitoring, and metacognitive control" (Muis, 2008, p. 190). Validation of coding and interpretations was obtained via retrospective interviews (Ericsson & Simon, 1993), member-checking, and inter-rater agreement. Inter-rater agreement percentages all exceeded Miles and Huberman's (1994) 80% suggestion.

Findings from the second half of the study indicated that students categorized as *rational* by the PEP demonstrated statistically higher usage of planning, monitoring, and metacognitive control than the other two groups. Additionally, students categorized as rational correctly solved more problems than the other two groups. These findings are in line with Schoenfeld's (1982, 1985) assertions regarding approaches to problem solving made by students exhibiting rational beliefs. However, Muis stated, "What has yet to be explored is why, in the context of mathematics problem solving, individuals profiled as

predominantly rational engage in more regulation of cognition” (p. 200). The qualitative results of Muis’s study also indicated that students’ epistemological beliefs may be enacted during the definition of the task phase (Winne & Hadwin, 1998). Once enacted, Muis suggested that students’ epistemological beliefs carry through other phases of SRL based mainly on the relationship between beliefs and learning standards. This finding provided support for Muis’s (2007) model infusing epistemological beliefs into Winne and Hadwin’s model of SRL. Finally, Muis cautioned researchers that, despite the advantages, sole use of think-aloud protocols may not provide a rich description of students’ engagement in a task. Multiple forms of data collection are certainly preferable.

#### Theoretical Framework: An Integrated Model

Three major theoretical constructs were considered in this study: self-regulated learning (SRL), epistemology, and mathematical problem solving. Thus, the design of the study necessitated a framework integrating these constructs for the purposes of data collection, data analysis, and discussion. The SRL model used in this study provided the most overarching tool for describing students’ navigation through complex problem-solving tasks. Steeped in the tenets of social cognitive theory, Zimmerman’s (2000) cyclic model involves the following three phases: (1) forethought, (2) performance control, and (3) self-reflection. Preceding these three phases is Winne and Hadwin’s (1998) definition of the task phase.

A survey of relevant literature (Hofer, 1999, 2000, 2004a, 2004b; Hofer & Pintrich, 1997; Kloosterman & Stage, 1992; Mos & Royce, 1980; Muis, 2004, 2007, 2008; Muis & Franco, 2009; Schoenfeld, 1983, 1985, 1988, 1989, 1992) informed the integration of epistemological beliefs into the model. Muis and colleagues have made



significant progress in determining relationships between SRL and epistemological beliefs both theoretically (Muis, 2004, 2007) and empirically (Muis, 2008; Muis & Franco, 2009). Muis's (2007) model that integrated epistemological beliefs into Winne and Hadwin's (1998) recursive model of SRL initiated the work of theorizing possible phases of SRL at which beliefs may be enacted. Additionally, Muis (2007, 2008) suggested further influences that beliefs may have on each phase of SRL. Finally, Schoenfeld's (1982, 1983, 1985, 1988, 1989, 1992) problem-solving contributions that incorporated cognitive control strategies and mathematical beliefs were infused into the model. The result was an integrated framework that guided all facets of the design and implementation of the study.

#### *An Integrated Model*

As students navigate a mathematical problem space, issues of control and beliefs arise that affect both current and future functioning (Muis, 2008; Schoenfeld, 1982, 1983, 1985, 1988, 1989, 1992). Issues of control may be assessed by evaluating students' application of SRL processing, as exemplified by this study, which employed a combination of Winne and Hadwin's (1998) and Zimmerman's (2000) models of SRL. Recent theoretical and empirical work has revealed that significant relationships exist between epistemological beliefs and SRL processing (Hofer, 2004a; Hofer & Pintrich, 1997; Muis, 2007, 2008; Muis & Franco, 2009; Schommer-Aikins, 2004). In particular, Muis (2008) suggested students' epistemic profiles are related to their performance and SRL processing while engaged in mathematical problems, and epistemological beliefs may be enacted during the task definition phase of SRL and then affect subsequent phases of SRL.

Based on a thorough review of relevant literature, a theoretical framework encompassing epistemological beliefs and SRL theory was developed to guide the study. Figure 1 below provides a visual representation of the model that is the basis of the framework. To begin, SRL is displayed as a cyclic, recursive process (Winne & Hadwin, 1998; Zimmerman, 2000) that students may utilize to guide their actions through a problem space. Ideally, students who practice SRL processing navigate through a task in an A-B-C-D progression (see Figure 1) and positively alter subsequent SRL processing based on self-reflections from D. However, students tend to be unpredictable and do not necessarily follow this pattern verbatim (Boekaerts & Niemivirta, 2000; Pintrich, 2000; Winne & Hadwin, 1998; Zimmerman, 2000). Thus, the multitude of two-way arrows indicate that students may navigate through the phases in virtually any combination of ways, with events from a current phase prompting motion to subsequent, but not necessarily sequential, phases.

Additionally, the model depicts students' epistemological beliefs, both general and mathematics-specific, as being enacted at any phase and affecting all aspects of SRL processing. In theory, epistemological beliefs are enacted at the *definition of the task* phase, affect the goals and plans developed during *forethought*, influence the strategies used during *performance control*, and influence standards set during *definition of the task* that are used to assess efforts during the *self-reflection* phase (Muis, 2007, 2008; Muis & Franco, 2009). However, students often enter SRL processing at various phases (not always beginning with task definition and then progressing sequentially), but they will bring their beliefs with them regardless of the entry point.

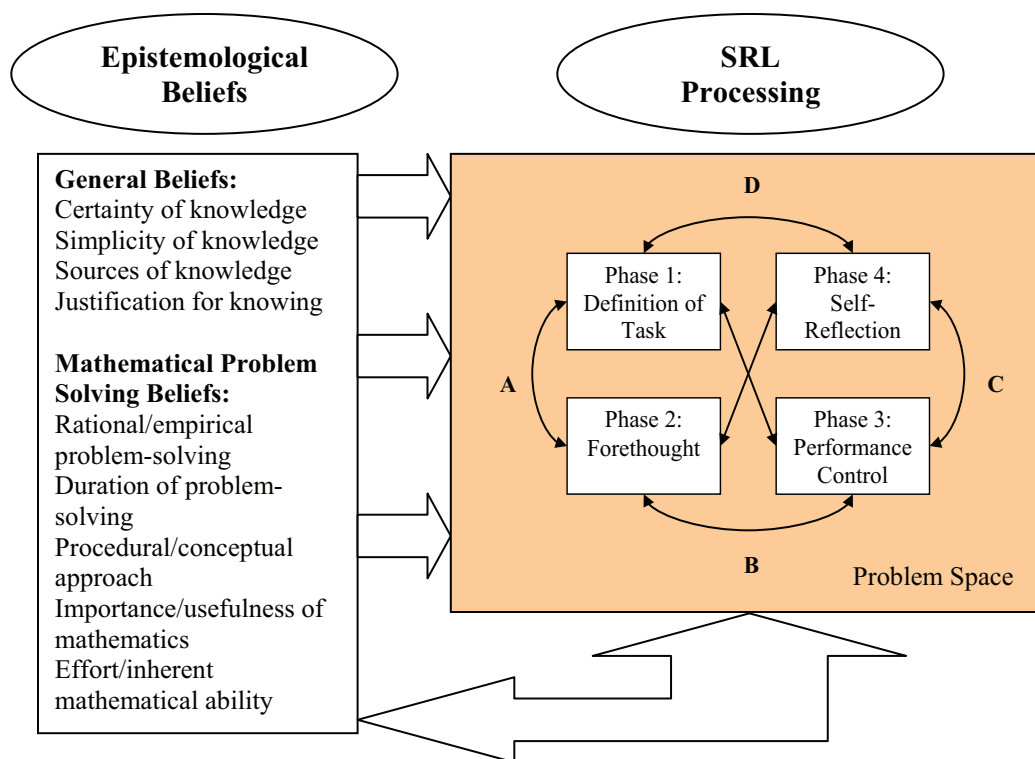


Figure 1. Model of Epistemology and SRL for Mathematical Problem Solving

Thus, the researcher suggests that the enactment and influence of epistemological beliefs may follow the recursive, cyclic, and unpredictable pattern found in the actual practice of SRL. For instance, a student with a non-availing belief in problem-solving duration may jump blindly into implementing a seemingly appropriate, but erroneous, strategy without curtail. When unsuccessful, the student may stop working and move on to other tasks because of the belief that the problem should be solved quickly or not at all (Schoenfeld, 1985). In this case, the student's belief that problems should be solved quickly has been enacted at the performance control phase and subsequently stifled monitoring and reflecting processes. Finally, the two-way arrow at the bottom of the model depicts the reciprocal nature of the relationship between epistemological beliefs and SRL, as suggested by Muis (2007, 2008).

### *Definition of the Task*

In terms of navigation of the problem space, students will likely engage in the definition of the task phase of SRL during the *reading* and *analysis* episodes (Schoenfeld, 1985). Students will read the problem, and possibly later re-read the problem, then rely on their mathematical resources in developing an appropriate definition of the task. Experts will often jump to implementation and remain on auto-pilot as they navigate a familiar problem space. The particularly well-attuned problem solver may envision a chunk of the problem resembling prior work, a schema based on categorizing the problem, or a more elaborate functional response to the problem situation (Schoenfeld, 1985). Novice problem solvers typically must engage in *analysis* in an attempt to understand the problem more fully. Often, initial heuristic strategies are employed, such as drawing a picture, to gain further insight before developing a plan. Any of these progressively more sophisticated initial responses has the potential to develop into a rational approach to the problem at hand (Schoenfeld, 1983, 1985). It should be noted that information gleaned from this phase is subject to change as the problem-solver delves into problem specifics, but all or part of the task definition may remain intact dependent upon the circumstances (Winne & Hadwin, 1998). Extremely novice problem solvers may fail to tap into their available mathematical resources and thus, have little or no logical vision of a possible solution technique, which implies that this phase may be skipped by naïve problem solvers. As discussed in detail above, this may result in the student engaging in empirical *wild goose chases* (Schoenfeld, 1983, 1985).

### *Forethought*

In terms of mathematical problem solving, students exhibit evidence of forethought processing during the *analysis*, *exploration*, and *planning* episodes of problem execution (Schoenfeld, 1985). As a student navigates through these episodes, each epistemological belief espoused by the student may influence the standards on which goals are based (Hofer & Pintrich, 1997; Muis, 2007). More constructivist-based beliefs tend to produce mastery-oriented goals, which are more conducive to self-regulatory processing (Pintrich, 2000; Muis, 2008; Muis & Franco, 2009). For instance, if a student believes that effort can yield increases in mathematical ability, as opposed to mathematical prowess being innate (Kloosterman & Stage, 1992; Schoenfeld, 1985), then the student may adopt a mastery-oriented approach to a task rather than simply focusing on their performance.

Additionally, students' arsenal of heuristics heavily influences the forethought phase. An experienced, or particularly adept, problem solver includes heuristics in their problem-solving plans. As mentioned above, heuristic planning must include the multitude of sub-strategies that fall under the heuristic of choice (Schoenfeld, 1985). For instance, if a student chooses to use the sub-goal heuristic approach, then the student must plan for each mathematical exercise that has been deemed an appropriate sub-problem to build a solution for the original problem. Naïve, or inexperienced, problem solvers may not have advanced heuristic strategies at their disposal and may simply plan to apply a seemingly appropriate mathematical operation to the problem. If this fails, other mathematical operations will be applied, in a manner congruent with the empiricist approach. This may not be entirely the students' fault—many mathematics teachers, with

the best intentions, *demonstrate* appropriate heuristics, but “bypass all of [the] complexity” and “leave the students completely unprepared to use the strategy” (Schoenfeld, 1985, p. 91).

### *Performance Control*

Whether planning is overt or implied, students will begin applying strategies in an effort to solve the given problem. Schoenfeld (1985) called this set of actions the *implementation* episode. Self-regulating students will apply their plan during this phase and monitor progress based on standards set when establishing goals (Zimmerman, 2000). Recent theoretical and empirical works have suggested that students’ epistemological beliefs influence the standards they set and thus, may affect strategy use and monitoring (Muis, 2007, 2008; Muis & Franco, 2009; Schommer-Aikins, 2004). Specifically, Schoenfeld’s (1985) problem-solving control continuum, Types A–D, provides a means of assessing students’ abilities to self-evaluate and make on-line checks of progress based on their goals. Briefly, Type A-controlled problem solvers apply strategies without considering consequences and often fail to solve problems. Type B-controlled problem solvers apply some level of control but fail to tap all necessary resources. Type C-controlled problem solvers, the optimum level, monitor solution processes such that regulation results in positive actions toward success. Finally, Type D-controlled problem solvers easily access solution methods and need virtually no control interventions.

The results of self-monitoring are inputs into internal feedback loops that may result in the need to alter strategy use, re-assess goals and plans, or re-read the problem to better define the task (Butler & Winne, 1995; Winne & Hadwin, 1998; Zimmerman,

2000). In fact, Schoenfeld (1985) stated, “One of the hallmarks of good problem solvers’ control behavior is that, while they are in the midst of working problems, such individuals seem to maintain an internal dialogue regarding the way that their solutions evolve” (p. 140). External feedback from peer and teachers may also serve as inputs into internal feedback loops that may alter students’ solution paths (Butler & Winne, 1995; Nicol & Macfarlane-Dick, 2006). Such interruptions in cognitive processing often occur when students are engaged in group problem solving and may curtail wild goose chases or have adverse effects on solution paths (Artzt & Armour-Thomas, 1992; Goos et al., 2002).

### *Self-reflection*

From a mathematical problem-solving perspective, the *verification* episode involves checking a solution for either accuracy or agreement to a set of conditions, depending on the nature of the task (Schoenfeld, 1985). For instance, an ill-defined task will not typically have a *correct* answer, rather students would simply need to make sure that their solution did not oversimplify the conditions, completely ignore a condition, etc. However, a well-defined task will have a particular unique solution or set of solutions that may be checked for accuracy. Schoenfeld (1985) and Polya (1957) suggested that adept problem solvers assess their solution paths and reflect on such factors as whether alternative solutions may exist or whether solutions may be generalized for further application. This deeper assessment of the navigation of a problem space is more synonymous to the *self-reflection* phase than the mere checking step described above. One consequence of students’ participation in self-reflection is the assessment of causal attributions to task results (Zimmerman, 2000). Epistemological beliefs may emerge

during this phase, as suggested by Zimmerman, who stated that “attributions of errors to a fixed ability prompt learners to react negatively and discourage efforts to improve” (p. 22).

### Summary

This review of relevant literature delved into SRL, epistemology, and problem solving to inform and frame this study. Educational aspects of SRL emerged from the development of social cognitive theory (Bandura, 1986). Much of the early literature was dedicated to describing and defining general aspects of the construct (e.g., Zimmerman, 1989). Then studies began appearing tying SRL to multiple aspects of education, including student learning and achievement, pedagogical practices, and even specific disciplines (Butler & Winne, 1995; Hadwin et al., 2004; Muis, 2008; Muis & Franco, 2009; Perels et al., 2005; Schunk, 1996; Usher, 2009). Additionally, theoretical and empirical advancements have provided researchers with a plethora of general SRL models (e.g., Pintrich, 2000; Winne & Hadwin, 1998; Zimmerman, 2000) and a model integrating SRL and epistemological beliefs (Muis, 2007).

Perry’s (1970) study is generally cited as the origin of educational epistemology. His work investigated the development of epistemology amongst college students. Shortly after Perry’s work, Royce and colleagues suggested that students could be assigned epistemic profiles based on three ways of knowing: rational, empirical, or metaphorical (Diamond & Royce, 1980; Royce & Mos, 1980; Wardell & Royce, 1978). Schoenfeld (1983, 1985) applied the empiricist and rationalist titles to his descriptions of mathematical beliefs with respect to problem solving. A major turning point in the study of epistemology was Schommer’s (1990) study introducing the conception of



epistemological beliefs as a multitude of independent dimensions. Several studies and theoretical reviews have suggested the nature of these epistemological beliefs and developed various integrated models (Hofer, 2000; Hofer & Pintrich, 1997; Kloosterman & Stage, 1992; Muis, 2007; Muis et al., 2006; Schommer-Aikins, 2004).

Winne and Perry's (2000) work prompted researchers to consider either augmenting self-report questionnaires with other types of data or developing more robust tools to analyze fine-grained instances of SRL processing. Recently, research has been both conducted and requested utilizing think-aloud protocols and qualitative methods (Greene & Azevedo, 2009; Hadwin et al., 2004; Muis, 2007, 2008; Usher, 2009; Zimmerman, 2008). Studies utilizing qualitative methods and think-aloud protocols are also common in both epistemological beliefs (Hofer, 2004a, 2004b; Muis, 2008) and problem-solving (Muis, 2008; Schoenfeld, 1983, 1985) disciplines. We now turn to a detailed description of the methods employed in this case study, which explored the relations between students' epistemological beliefs and SRL while engaged in mathematical problem solving.

## CHAPTER III

### METHODOLOGY

This study investigated the relationships that exist between advanced mathematics students' epistemological beliefs and SRL processing while engaged in mathematical problem solving. Specifically, the current study sought to answer the following research questions:

1. How are students' epistemological beliefs related to self-regulatory processing practices during engagement in mathematical problem-solving tasks?
2. What self-regulation strategies do students employ while preparing for the AP Calculus exam and engaging in problem-solving episodes?
3. What epistemological beliefs influence students' choice and use of heuristic strategies to solve mathematical problems?
4. How are self-regulated learning strategies and epistemological beliefs related to student performance on problem-solving tasks?

The researcher selected the case study methodology to obtain in-depth, exploratory descriptions and explanations of the above phenomena. Yin (2008) provided confirmation of the appropriateness of this methodological decision:

The more that your questions seek to explain some present circumstance (e.g., “how” or “why” some social phenomenon works), the more that the case study will be relevant. The method also is relevant the more that your questions require an extensive and “in-depth” description of some social phenomenon. (p. 4)

Additionally, Winne and colleagues questioned the use of self-report questionnaires as the sole, or predominant, form of data collection in SRL studies (Muis, 2008; Muis, Winne, & Jamieson-Noel, 2007; Winne & Perry, 2000; Winne & Jamieson-Noel, 2003). Winne and Perry further suggested that studies investigate SRL at a finer grain size to accumulate extensive descriptions of SRL processing. Such studies have the potential to produce more dynamic and accurate models of SRL. Thus, recent studies have sought to investigate SRL processing more closely, such as Hadwin, Boutara, Knoetzke, and Thompson's (2004) case study and Greene and Azevedo's (2009) quantitative study applying think-aloud protocols.

Expressing similar skepticism for self-report questionnaire use in epistemological studies, Hofer and Pintrich (1997) stated, "Although each of the dimensions is conceptualized as a continuum, it may be difficult to assume that a continuum of epistemological beliefs can be represented or measured by simply stating extreme positions and registering degrees of agreement" (p. 110). Muis and colleagues provided more recent criticism of self-report instruments and requested more fine-grained, qualitative studies, which utilize think-aloud protocols, and mixed-methods designs (Muis, 2007, 2008; Muis, Bendixen, & Haerle, 2006). These recommendations are in line with design suggestions from current self-regulated learning literature (Muis, 2007; Zimmerman, 2008). Hofer (2004a, 2004b) utilized think-aloud protocols and case study methods recently to investigate epistemological beliefs. Muis and colleagues have begun to tie epistemological beliefs to facets of SRL via mixed methods studies (Muis, 2008; Muis & Franco, 2009). Suggestions and studies from both SRL and epistemological beliefs literature provide a rationale for the design of the proposed study.

Due mainly to his mathematical and engineering background, the researcher adhered to the postpositivist qualitative paradigm, described by Creswell (2007), an admitted postpositivist himself:

In terms of practice, postpositivist researchers will likely view inquiry as a series of logically related steps, believe in multiple perspectives from participants rather than a single reality, and espouse rigorous methods of qualitative data collection and analysis. They will use multiple levels of data analysis for rigor, employ computer programs to assist in their analysis, encourage the use of validity approaches, and write their qualitative studies in the form of scientific reports, with a structure resembling quantitative approaches (e.g., problem, questions, data collection, results, conclusions). (p. 20)

Evidence of the postpositivist paradigm pervaded this study, which began with a “thorough review of literature” (Yin, 2008, p.3) that established “a priori theories” (Creswell, 2007, p.20) that were utilized to develop codes, analyze data, and interpret results. A thorough description of the methods employed in this study is presented below and organized as follows: (1) the setting, (2) selection of participants, (3) instrumentation and protocols, (4) data collection procedures, (5) data analysis, (6) validity and reliability, and (7) ethical issues.

### The Setting

This case study was conducted at Pine Valley High School, located in the suburbs of a large metropolitan city in southeastern USA, during the spring semester of the 2009–2010 school year. At the time of the study, Pine Valley High School had a population of 2168 students. Demographic data for the 2009–2010 school year were unavailable at the

time of this writing, so remaining data will correspond to the 2008–2009 school year. The breakdown of students at Pine Valley High School by race was as follows: 65% Caucasian, 19% African American, 8% Hispanic, 4% Asian, and 3% Multiracial. Additionally, 19% of the student body was eligible for free or reduced meals, 9% of the students had various disabilities, and 2% were limited in their English language proficiency. Pine Valley High School had a mathematics and science magnet program, which means that students from all over the district applied for admission into the rigorous program and, if accepted, were provided transportation to the school. The school was also both locally and globally renowned for its character education program, student government involvement, and academic rigor. In fact, it was not uncommon for visitors from other states and even other nations to travel for the purposes of observing various facets of the school.

#### Selection of Participants

At the time of this study, the researcher taught Advanced Placement (AP) Calculus AB and BC at Pine Valley High School and purposefully selected one of his AP Calculus BC classes to participate in the study. AP Calculus AB is a course designed by College Board to provide advanced high school students an opportunity to experience first-semester college calculus. AP Calculus BC is an extension of AP Calculus AB and introduces students to topics such as power series, two-dimensional vector calculus, and the calculus of parametric- and polar-defined relations. Both courses culminate in a standardized exam that gives students the opportunity to earn college credit, dependent upon their exam scores and individual college's entrance requirements. The selected class contained 30 students, who represented a more disparate group of students with respect to

achievement in AP Calculus AB, the course prerequisite for AP Calculus BC, when compared to the researcher's other classes. With respect to gender, the selected class contained 22 male and 8 female students. With respect to race, the class contained 24 Caucasian, 3 Hispanic, 1 African American, 1 Pacific Asian, and 1 Middle-Eastern Asian student. The 30 students were amongst the highest mathematics achievers in the school and were generally motivated either intrinsically to learn or extrinsically to perform. Additionally, these students were heavily involved in extracurricular and community service activities (e.g., sports, academic clubs, band, student government) that placed added demands on them before and after school.

Before data collection, the researcher obtained Institutional Review Board (IRB) approval from both the participating school district and the university supervising the study. Additionally, participant and parental consent (see Appendix A) were obtained prior to data collection. Then, all students completed an abbreviated version of the Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich, Smith, Garcia, & McKeachie, 1991), the Psycho-epistemological Profile (PEP; Royce & Mos, 1980), and the Indiana Mathematics Belief Scales (IMBS; Kloosterman & Stage, 1992). See Appendix B for full reproductions of the IMBS and PEP. Permission to use each questionnaire was obtained either from the first author or the university owning the copyright (see Appendix C). Full reproduction of the MSLQ questions used in this study could not be reproduced due to copyright restrictions. Full-length versions of the PEP and IMBS were administered, but only select, intact subscales of the MSLQ were used in this study. Each questionnaire was selected based on the theoretical framework and research questions. Pintrich et al. (1991) reported validity and reliability statistics for each

subscale and stated explicitly that subscales could be used separately or as a complete questionnaire. The PEP provided domain-general insights into students' *rational* and *empirical* beliefs, as identified by Mos and Royce (1980) and Schoenfeld (1983, 1985). The IMBS quantified various mathematics-specific beliefs, including students' beliefs in the importance of *conceptual* and *procedural* approaches to problem-solving (Kloosterman & Stage, 1992; Schoenfeld, 1992). The subscales selected from the MSLQ generated data consistent with the four-phase model of SRL in the theoretical framework (Pintrich et al., 1991; Winne & Hadwin, 1998; Zimmerman, 2000). As part of the course verification process at the beginning of each semester, the researcher had access to students' transcripts, from which grades were obtained for all students from AP Calculus AB, the prerequisite for the course involved in this study. From these data, six students were selected as participants for the purposes of individual case and cross-case analyses.

Using SPSS Version 15, descriptive statistics were generated to analyze the above data and determine appropriate categorizations for the purpose of applying a mixture of *quota* and *maximum variation* sampling (Miles & Huberman, 1994). For mathematics achievement, the class mean and standard deviation for grades from AP Calculus AB were determined. Students scoring more than two standard deviations above the mean were considered *Category I achievers*. Students scoring between one and two standard deviations above the mean were considered *Category II achievers*. Similarly, students were considered *Category III* and *Category IV achievers* for scores having standard deviations below the mean. Using the PEP, a researcher may determine an individual's epistemological profile as either *rational*, *empirical*, or *metaphorical* by summing scores for each subscale and assigning the appropriate predominant profile based on the highest

score. Thus, students were assigned a predominant profile based on their highest score obtained from the three subscales of the PEP.

Difficulties arose in determining quantifying values for students based on the abbreviated MSLQ and the IMBS, which have seven scales and six scales, respectively. For both questionnaires, individual students' scores for each scale were assigned an integer score from the interval  $[-3, 3]$  based on the score's distance from the mean,  $M$ . More specifically, six subintervals of width equal to one standard deviation,  $SD$ , were obtained for each scale from the continuous interval  $[M - 3SD, M + 3SD]$ . For instance, a student may have scored 2.2 standard deviations above the mean for the Critical Thinking scale of the MSLQ. This student would receive a score of 3 for that scale since their score fell in the interval  $(M + 2SD, M + 3SD)$ . All students' scores were summed to obtain a combined score for the MSLQ and IMBS allowing for comparison to the class population. Ultimately, the researcher considered six quantifiers when applying the sampling strategy: achievement, MSLQ combined score, IMBS combined score, and three individual PEP scores. Distributions for all 16 scales from the three questionnaires used in this study can be found in Appendix D. Data from the scales were relatively normal since skewness and kurtosis values for most measures fell between  $\pm 2$  (George & Mallery, 2008). The only exception was the Belief 6 scale of the IMBS, which had a kurtosis value of 2.12. This is not surprising as advanced mathematics students would generally be expected to find mathematics useful.

A matrix was developed to aid in selecting participants that appropriately represented the variation in scores from the class. Figure 2 represents the effectiveness of the sampling strategy in obtaining a valid *quota* and *maximum variation* sample based on



the factors considered in this study. To obtain a single scale for graphing purposes,  $z$ -scores were calculated to standardize all relevant data. For simplicity, a single value was computed for the IMBS and MSLQ since the questionnaires together had 13 scales. Participants' IMBS and MSLQ values were obtained by calculating the mean of participants'  $z$ -scores for the respective questionnaires. Despite data being categorical, a line graph was chosen to aid the reader in tracking the scores of individual participants. Each participant was assigned a pseudonym to aid in maintaining confidentiality. Similar categorical sampling techniques have produced robust results in both SRL and epistemological research (Hadwin et al., 2004; Muis, 2008).

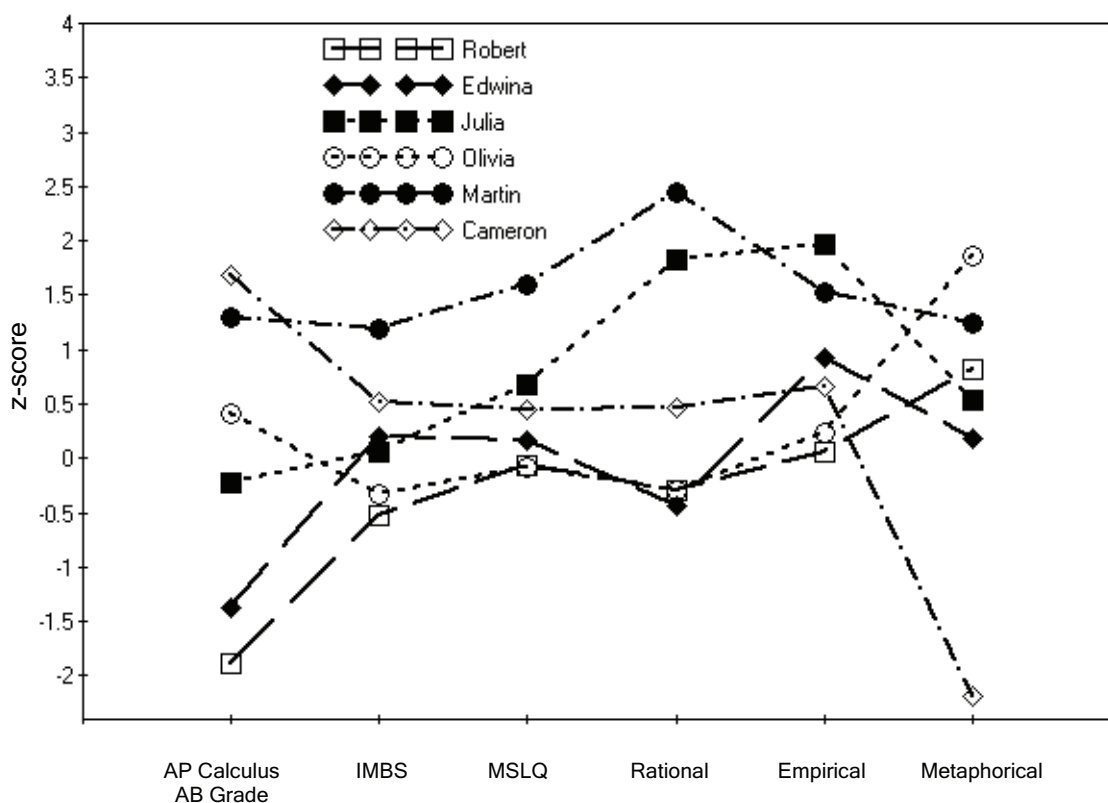


Figure 2. Distribution of Measures Used for Sampling

Ultimately, six participants were selected to represent the diversity of the overall class based on the prescribed categories—this is the essence of quota sampling (Miles & Huberman, 1994). To begin the quota sampling process, students were chosen who provided the most accurate representation of the spread of all 30 students' *mathematics achievement* categories. Simultaneously, various combinations of *mathematics achievement* and questionnaire categorizations were considered until a sample was obtained that represented the diversity of the class (see Figure 2). Quota sampling added richness to the study by providing in-depth descriptions of representatives from the various categorizations. Quota sampling also allowed for cross-cases analysis that made this study more compelling. Additionally, two of the participants represented the extremes, or outliers, from the class with respect to mathematical achievement—this is the essence of maximum variation (Miles & Huberman, 1994). Participants representing extreme cases based on questionnaire data were merely coincidences from other sampling strategies. For instance, the selection of Martin added an additional *Category I* mathematics achiever to build the quota sample and provided an extreme case since he had the highest Rational score on the PEP (see Figure 2). The inclusion of “outlier cases” helped to determine “whether main patterns still hold” (p. 28). Main patterns which emerge from the data may be compared and contrasted with specific data from outlier cases. Since the outlier cases came from the higher and lower performance subgroups developed for quota sampling, they also served to provide rich, cross-case analysis. Additionally, the outlier cases provided both confirmation of some findings and prompted the consideration of alternative explanations for other findings. Both of these contributions added richness to the study. On the one hand, confirmation increased the

validity of the study. On the other hand, alternative explanations prompted further investigation within this study.

### Instrumentation and Protocols

The sampling procedures, described in detail above, utilized questionnaire data to determine case-study participants. The questionnaires also provided self-reported quantitative data indicating students' epistemological beliefs and their intentions to self-regulate. However, Winne and colleagues have pointed out that students' self-reported use of SRL processing may not suffice as accurate or reliable sole sources of data (Muis, 2008; Muis, Winne, & Jamieson-Noel, 2007; Winne & Perry, 2000; Winne & Jamieson-Noel, 2003). Along the same vein, epistemological beliefs literature has questioned the validity and plausibility of examining such complex, multi-dimensional constructs using self-report questionnaires (Hofer & Pintrich, 1997; Muis, 2008; Muis, Bendixen, & Haerle, 2006). Thus, the self-report instruments utilized in this study identified participants and served as a starting point for data collection. To closely examine *how* students' epistemological beliefs and SRL processing related the self-report questionnaire instruments were augmented heavily by qualitative data.

All instruments and protocols are described below and the discussion is partitioned into the phases of data collection. Under Phase I: Self-report questionnaire administration, the following instruments will be discussed: (1) the MSLQ (Pintrich, Smith, Garcia & McKeachie, 1991), (2) the PEP (Royce & Mos, 1980), and (3) the IMBS (Kloosterman & Stage, 1992). Then, Phase II: AP calculus exam preparation instruments and protocols will be discussed: (4) AP Calculus AB free-response exam questions, (5) observational protocols, (6) reflective journals. Phase III: Think-aloud problem solving

data collection features the following sources of data: (7) problem solving tasks, (8) student solutions to problem-solving tasks, (9) think-aloud transcriptions. Finally, during Phase IV: Confirmation of findings, the single source of data will be (10) follow-up interview transcriptions.

*Phase I: Self-reported questionnaire administration*

*MSLQ*. The MSLQ (Pintrich et al., 1991) is an 81-question, 7-point Likert scale, self-report questionnaire designed to measure multiple aspects of students' self-regulatory attributes and processing and is divided into two separate sets of scales: Motivation and Learning Strategies. Pintrich et al. performed a confirmatory factor analysis on the MSLQ, which yielded factor validity for six subscales from the Motivation category and nine subscales from the Learning Strategies category. Additionally, "Chronbach's alphas are robust, ranging from .52 to .93" for the 15 subscales (p. 4). Due to the length of the questionnaire, individual subscales were chosen for administration based on alignment with the literature reviewed. From the Motivation category, the following subscales were used: (1) Task Value, (2) Intrinsic Goal Orientation, and (3) Extrinsic Goal Orientation. From the Learning Strategies category, the following subscales were used: (4) Critical Thinking, (5) Metacognitive Self-regulation, (6) Peer Learning, and (7) Help Seeking. Particular attention was given to assuring that the MSLQ subscales were congruent with the theoretical framework developed in the previous chapter. Thus, the definition of the task, forethought, performance control, and self-reflection phases will be related to each subscale, as appropriate, in the ensuing discussion.

*Task Value.* The Task Value subscale ( $\alpha = .90$ ) is most closely related to Winne and Hadwin's (1998) definition of the task phase, but only requires students to consider motivational aspects of their assessment of a task. None of the cognitive or metacognitive behaviors so crucial to navigation through a problem space is assessed via this scale. Pintrich, et al. (1991) stated, "On the MSLQ, task value refers to students' perceptions of the course material in terms of interest, importance, and utility" (p. 11). This subscale contains five items, including: "I am very interested in the content area of this course" (p. 11).

*Intrinsic and Extrinsic Goal Orientation.* The Intrinsic Goal Orientation subscale ( $\alpha = .74$ ) is closely related to the mastery-goal orientation from Pintrich's (2000) SRL model. In fact, Pintrich et al. (1991) stated, "Intrinsic goal orientation concerns the degree to which the student perceives herself to be participating in a task for reasons such as challenge, curiosity, *mastery* [italics added]" (p. 9). This subscale contains four items, including: "When I have the opportunity in this class, I choose course assignments that I can learn from even if they don't guarantee me a good grade" (p. 9).

The Extrinsic Goal Orientation subscale ( $\alpha = .62$ ) is the antithesis of the previous subscale and is, thus, closely related to the performance-goal orientation of Pintrich's (2000) SRL model. Pintrich et al. (1991) defined extrinsic goal orientation as "the degree to which a student perceives herself to be participating in a task for reasons such as grades, rewards, *performance* [italics added], evaluation by others, and competition" (p. 10). This subscale contains four items, including: "The most important thing for me right now is improving my overall grade point average, so my main concern in this class is getting a good grade" (p. 10).

*Critical Thinking.* The Critical Thinking subscale ( $\alpha = .80$ ) is most closely related to the performance control phase of SRL and is directly related to aspects of mathematical problem solving. Pintrich et al. (1991) stated, “Critical thinking refers to the degree to which students report applying previous knowledge to new situations in order to *solve problems* [italics added], reach decisions, or make critical evaluations with respect to standards of excellence” (p. 22). This scale contains five items, including: “When a theory, interpretation, or conclusion is presented in class or in the readings, I try to decide if there is good supporting evidence” (p. 22).

*Metacognitive Self-regulation.* The metacognitive self-regulation subscale ( $\alpha = .79$ ) is the most all-encompassing of the subscales and includes aspects of the forethought, performance control, and self-reflection phases of SRL. The scale includes 12 items on subjects such as goal-setting, self-monitoring, and making adjustments to cognitive activities.

Although some goal-setting items are in this scale, the metacognitive self-regulation scale is most closely related to the performance control and self-reflection phases of the theoretical framework. Unfortunately, the scale does not fit well with Schoenfeld’s (1985) control aspect of mathematical problem solving. The scale instead focuses more on study habits than on the cognitive aspects of learning inherent to problem solving. This misalignment must be considered as data are analyzed and interpreted. Overall, this scale provided a very general, broad-ranged quantifier of student SRL processing.

*Peer Learning and Help Seeking.* The final two subscales assess students’ abilities to manage resources. The Peer Learning subscale ( $\alpha = .76$ ) contains three items that

measure the degree to which students utilize peers to aid in the learning process. A sample question is as follows: “When studying for this course, I often set aside time to discuss the course material with a group of students from the class” (Pintrich et al., 1991, p. 28). The Help Seeking subscale ( $\alpha = .52$ ) contains four items that measure the degree to which students utilize peer and instructor assistance when they need help with an academic task. A sample item is: “I ask the instructor to clarify concepts I don’t understand well” (p. 29). Both subscales measure students’ use of external resources to control cognitive activities.

*PEP*. The Psycho-Epistemological Profile (PEP, Royce & Mos, 1980) is a 90-question, 5-point Likert-scale questionnaire that provides a hierarchical assessment of an individual’s epistemological profile. Specifically, the PEP delineates the degree to which individuals self-report adhering to rational, empirical, and metaphorical epistemological beliefs. Thirty questions from the questionnaire apply to each type of profile. Thus, the maximum score per scale is 150. Although Royce and Mos suggested that an individual may exhibit behaviors consistent with all three ways of knowing, results from the PEP indicate an individual’s predominant adherence to one of the three beliefs based on the individual’s highest score.

Royce and Mos (1980) reported that multiple factor analyses have been performed on the PEP and items consistently loaded onto the aforementioned three epistemological beliefs. In a recent study, Muis (2008), concerned with the datedness of the questionnaire, performed a confirmatory factor analysis on the PEP with 268 university students and obtained a CFI value of .86. Her findings concurred with Royce and Mos’ assertion that the PEP is a good fit to the three factors. In terms of internal

validity, Royce and Mos reported adjusted Spearman Brown split-half correlation coefficients ranging from .75 to .77 for the Rational scale, .76 to .77 for the Empirical scale, and .85 to .88 for the Metaphorical scale. The three ways of knowing presented by Royce and Mos (1980) are steeped in philosophical literature contemporary to the time of PEP development. Table 2 provides a summary of the “three ways of knowing,” as defined by Royce and Mos (p. 3). Expert mathematicians are generally associated with the rational epistemic style (Royce & Mos, 1980; Schoenfeld, 1985). Additionally, research has suggested that rational problem-solvers tend to perform better and demonstrate more SRL processing than their empirical peers while problem solving (Muis, 2008; Schoenfeld, 1982, 1983, 1985).

Table 2

*Royce and Mos's (1980) Three Ways of Knowing*

Profile	Assessment of Reality	Cognitive Processing
Rationalism	Logical consistency	Clear thinking, rational analysis, synthesis of ideas
Empiricism	Observational	Active perception, sensory experience
Metaphorism	Insight and awareness	Symbolizing



*IMBS*. The Indiana Mathematics Belief Scales (IMBS) is a 36-item, 5-point Likert scale questionnaire that measures “secondary school and college students’ beliefs about mathematics as a subject and about how mathematics is learned” (Kloosterman & Stage, 1992, p. 109). Students’ beliefs about mathematical problem solving are measured by the following six scales, with Kloosterman and Stage’s reported Chronbach’s alpha values:

- (1) I can solve time-consuming mathematics problems. ( $\alpha = .77$ )
- (2) There are word problems that cannot be solved with simple, step-by-step procedures. ( $\alpha = .67$ )
- (3) Understanding concepts is important in mathematics. ( $\alpha = .76$ )
- (4) Word problems are important in mathematics. ( $\alpha = .54$ )
- (5) Effort can increase mathematical ability. ( $\alpha = .84$ )
- (6) Mathematics is useful in daily life. ( $\alpha = .86$ ) (pp. 112, 115)

The first five scales were developed by Kloosterman and Stage and originally contained ten items. However, multiple stages of testing for scale validity reduced each scale to six items. The sixth scale contains six items from Fennema-Sherman’s (1976) Usefulness of Mathematics Scale. Scales (1) through (4) and scale (6) contain three positively-oriented items and three negatively-oriented items. All six items from Scale (5) are positively-oriented.

Scale (1) is based on Schoenfeld’s (1985) assertion that many students feel that mathematics problems should be solved quickly or not at all. A positively-oriented sample item from this scale is: “Math problems that take a long time don’t bother me” (p. 115). The theoretical basis for scale (2) is that good problem solvers tend to be motivated

to solve problems even when no apparent algorithm applies; whereas poor problem solvers either give up or apply incorrect algorithms to such problems. A negatively-oriented sample item from this scale is: “Any word problem can be solved if you know the right steps to follow” (p. 115). Scale (3) measures the degree to which students have the availing belief that conceptual understanding is important, as opposed to the non-availing belief that solely applying algorithmic procedures leads to successful problem-solving. A positively-oriented sample item from this scale is: “Time used to investigate why a solution to a math problem works is time well spent” (p. 115).

The belief measured by scale (4) is the degree to which students relate mathematical acuity to the attainment of computational skills, a non-availing belief, or problem-solving skills, an availing belief. A negatively-oriented sample item from this scale is: “Math classes should not emphasize word problems” (p. 115). Scale (5) measures the degree to which students have availing beliefs about effort yielding positive results in obtaining mathematical skills. All items were positively-oriented in this scale and a sample item is: “Ability in math increases when one studies hard” (p. 115).

Kloosterman and Stage (1992) included six items from the Fennema-Sherman (1976) Usefulness of Mathematics scale because of relations between availing beliefs about the usefulness of mathematics in daily life and motivation to learn. A negatively-oriented sample item from this scale is: “Studying mathematics is a waste of time” (p. 115). In sum, the IMBS provided an appropriate fit to the theoretical framework developed for this study. Additionally, the IMBS provided data for sampling procedures and a springboard for more in-depth analyses of students’ mathematical beliefs via qualitative data.

*Phase II: AP Calculus exam preparation*

*AP Calculus AB free-response exam questions.* During all classroom observations, students were engaged in completing AP Calculus AB free-response exam questions from previous years' exam administrations (see Appendix E) in small heterogeneous groups. Permission was obtained to use the questions prior to the initiation of this study (see Appendix C). Care was taken to fully abide by the legal specifications detailed by the College Board with regard to citations and exclusion from the views expressed by this study. Use of these questions provided students with an authentic sense of the difficulty level of their ensuing sitting for the AP exam. Additionally, research has suggested that on-line observations of student engagement having an impact on performance may be more conducive to determining actual student functioning than solely relying on assessments of student engagement on contrived tasks (Hadwin, et al., 2004). Assessment of student behaviors during observations was dependent on detailed field notes taken by the researcher and student journal entries.

*Observational protocols.* Researcher-generated products from AP exam preparation sessions were classroom observation protocol forms (see Appendix E) completed during each AP exam preparation session. Since all classroom-observation sessions involved students working in groups, participants' actions were recorded that demonstrated aspects of group-regulation, as identified in the literature review (Artzt & Armour-Thomas, 1992; Goos, Galbraith, & Renshaw, 2002). Each of the six participants was observed twice during the study, resulting in twelve total sessions of approximately 15-minute duration. The researcher sat in relatively close proximity to the appropriate group and recorded detailed notes focused on the behaviors and actions of the *participant*

with respect to group navigation of each problem space. For clarity, the term *participant* will refer to individuals from the class selected from the sampling strategy and the term *student* will refer to individuals from the class not selected from the sampling strategy.

*Reflective journals.* Student-generated products from AP exam preparation sessions were problem-solving journal entries. Using the AP Exam Preparation Journal Format (see Appendix E), students recorded their solution to a given AP exam problem on the left side of the journal entry form. Then, students recorded their plans, thoughts, and resource use for particular stages of the problem solution on the right side of the form, directly across from the applicable mathematical work recorded on the left side. Care was taken not to *lead* students' right-side responses and questions were answered with the statement: *Please provide as much detail as possible.* Each student in the class maintained and submitted a journal, regardless of whether they were selected as a participant. The journals completed the triangulation of data with self-report survey results and researcher observation notes for the AP exam preparation phase of the study.

### *Phase III: Think Aloud Problem-Solving*

*Problem-solving tasks.* The purpose of the think-aloud problem-solving sessions was to produce a vast amount of data for analyzing the six participants' SRL processing and epistemological beliefs during mathematical problem-solving episodes. Two problems were developed by the researcher that reflect single-variable calculus concepts learned by students during the previous semester (see Appendix F). Both problems fit Polya's (1957) definition of a "problem to find" by requiring students to find, or develop, unknown functions or quantities in consideration of given constraints (p. 154).

Additionally, both problems were aligned with Schoenfeld's (1985) distinction between a mathematical problem and a mathematical task:

The difficulty with defining the term *problem* is that problem solving is relative. The same tasks that call for significant efforts from some students may well be routine exercises for others, and answering them may just be a matter of recall for a given mathematician. Thus being a "problem" is not a property inherent in a mathematical task. Rather, it is a particular relationship between the individual and the task that makes the task a problem for that person. The word *problem* is used here in this relative sense, as a task that is difficult for the individual who is trying to solve it. (p. 74)

Thus, problems were developed that drew on topics discussed in AP Calculus AB; however, solution paths to the particular problems were never explicitly discussed. So, the problems were based on material that students had not seen in several months and asked questions that stretched that content knowledge, which is in line with Schoenfeld's definition of a problem described above.

*Student work from problem-solving tasks.* During the problem solving sessions, each individual case participant worked on the problems described above during two 30-minute sessions, thinking-aloud as they worked. Participants were allowed to write on the paper containing the problems and were given extra paper if needed. All work was collected. Participants were instructed to write down all work and, rather than erasing, to draw a line through any work they deemed incorrect. Collection of work provided a complete, written account of participants' efforts in solving the problems. This method

for preserving student work was employed successfully by Schoenfeld (1982, 1985) in his problem-solving queries.

*Think-aloud session transcriptions.* The bulk of data collected during this phase came from transcriptions of participants' think-aloud sessions. Students were asked to think aloud while working the problems and provide a retrospective report of their work. The think-aloud sessions were conducted in strict adherence to Ericsson and Simon's (1993) and Schoenfeld's (1985) methodological prescriptions. See Appendix F for the think-aloud scripts used by the researcher. The retrospective report that followed each session involved participants' recall of their thinking as they reviewed their work. Ericsson and Simon (1993) recommended the use of both think-aloud and retrospective reports when analyzing problem-solving behaviors. They rationalized the usage of retrospective reports following think-aloud sessions as follows:

Even for cognitive processes of long duration, where we know that the retrospective report will be incomplete, it will be quite useful. In this case, it will more clearly convey the general structure of the process, as most of the detailed information will not be retrieved, and retrieval will use the higher-level organizational cues, like subgoals, or recall cues. (p. 379)

All sessions were recorded by a digital recording device and, in case of data loss, a traditional tape-recorder. Each recorded session was transcribed verbatim and supplemented with notes that were written during the researcher's observations of the think-aloud session. Transcriptions were completed immediately following sessions to ensure the most accurate rendition of events.

*Phase IV: Confirmation of findings*

*Follow-up interview transcriptions.* Semi-structured, follow-up interviews with the six participants served as member-checking sessions and provided a final opportunity for the researcher to obtain additional information pertinent to this study (see Appendix G for the interview protocol). Students reviewed the initial findings of their individual case narratives and commented as to the degree of accuracy of events and behaviors reported. Additional questions were asked of each participant based upon their behaviors and verbalizations during this study. A protocol was not developed because questions were unique to each participant's experiences. All interviews were recorded by a digital recording device and, in case of data loss, a traditional tape-recorder. Each interview was transcribed and the researcher applied any necessary changes to the findings based on student comments. Additional findings and clarifications gleaned from final interview questions were included in the results and discussion of the findings.

#### Data Collection Procedures

During this study, multiple sources of data were collected from each participant to help achieve triangulation of findings, a crucial element to validity in case studies and qualitative inquiries in general (Creswell, 2007; Marshall & Rossman, 2006; Yin, 2008). Prior to data collection, IRB approval, participant consent, and parental consent were obtained (see Appendix A). Then, the four phases of data collection commenced: (1) self-report questionnaire administration, (2) AP Calculus exam preparation, (3) think aloud problem-solving sessions, and (4) follow-up interview sessions.

*Phase I: Self-Report Questionnaire Administration*

The MSLQ, PEP, and IMBS (see Appendix B) were administered to serve as participant sampling criteria and initial data for participants' narratives. Due to the length of the questionnaires, administration was conducted in two days. The abbreviated MSLQ and IMBS have a combined 74 items and took approximately 20 minutes to administer. The PEP contains 90 items and administration took approximately 20 to 30 minutes, in congruence with Royce and Mos' (1980) estimation. Instructions for administration described in the MSLQ and PEP manuals and the IMBS article were followed precisely to obtain reliable and valid results from participants (Kloosterman & Stage, 1992; Pintrich et al., 1991; Royce & Mos, 1980).

*Phase II: AP Calculus Exam Preparation*

A key component of the AP Calculus BC course involved preparation for the ensuing AP exam, which is the gateway to receiving college credit. During the early part of the semester, exam preparation consisted of reviewing material learned in the previous course, AP Calculus AB. Based on College Board's approval (see Appendix C), this goal was achieved by having students work actual AP Calculus AB free-response exam questions from prior years' administrations (see Appendix E). During these activities, participants were observed for data collection purposes.

Once the six participants were confirmed, the class was divided into ten heterogeneous groups of three students. To achieve heterogeneity, the researcher ordered students from lowest to highest based on their grade from the prior course, AP Calculus AB. Then the researcher subdivided the class into three groups of ten, which naturally formed high, average, and low student groupings. Heterogeneous groups of three were



then formed by selecting a student from each group and checking the average AP Calculus AB grade for each group to ensure that differences between groups were not excessive. The group AP Calculus AB grade means ranged from 85.00 to 88.67, which are relatively consistent with the overall class mean of 86.80. Additionally, six of the ten groups contained one participant for the purposes of individual observation.

Students worked all of the AP Calculus exam preparation problems while in assigned groups. During each session, students recorded work and reflections in their AP exam preparation journals and the researcher took detailed observational notes on one of the six groups containing a participant (see Appendix E). Each AP exam preparation session was conducted during approximately fifteen to twenty minutes of regular class time. There were two to three AP exam preparation sessions each week for the duration of the study, ultimately providing a total of twelve opportunities for data collection. The twelve sessions allowed for two observations of each participant collaboratively preparing for a high-stakes exam. Data were coded and analyzed simultaneously with data collection to determine any necessary adjustments in the design of the study and to begin preparing individual case narratives.

### *Phase III: Think-Aloud Problem-Solving Sessions*

Phase III data collection was conducted concurrently with Phase II outside of the classroom environment. The six participants worked on two calculus-based problem-solving tasks (see Appendix F) during two sessions, thinking aloud as they worked. Participants were presented with the *application of differentiation* problem during the first session and then returned to continue working on the problem during a second session if the task had not been completed. Participants were allowed to work on or think

about the problem during the interim between sessions but were required to submit all work from the first session to the researcher for the purposes of analysis. If a participant solved the *application of differentiation* problem during either session, then they would receive the *application of integration* problem and could work on it for the remainder of their allotted time. To avoid revealing possible problem-solving solution methods, the *application of differentiation* and *application of integration* problems were re-named as *problem solving task #1* and *problem-solving task #2*, respectively, when presented to participants during the think-aloud sessions.

Think-aloud problem-solving sessions were conducted in the classroom immediately following school for approximately one hour. The researcher coordinated schedules with the six participants and scheduled sessions well in advance. At the beginning of the first session, participants were trained to think aloud, as prescribed by Ericsson and Simon (1993). Each participant was instructed to verbalize their in-line thinking while solving problems. Ericsson and Simon cautioned that having participants expound on or explain their thoughts may affect problem solving performance, so students were trained to simply speak exactly what thoughts were on their minds without explanations or justifications. Participants were given a computational practice problem and asked to practice the think-aloud and retrospective report procedures. Another practice problem was available to participants if needed. Once questions were answered, participants were given approximately thirty minutes to complete the appropriate problem. If the researcher felt that the participant was engaged in a productive problem-solving activity at the 30-minute mark, then a few additional minutes were allowed for the participant to complete the line of reasoning. The researcher prompted participants to

“keep talking” when instances of silence exceeded ten to fifteen seconds (Ericsson & Simon, 1993, p. 83). Upon completion of each session, participants were asked to provide a retrospective report of their thinking as they reviewed their work. Additionally, the researcher provided general cognitive feedback for the purposes of exploring SRL processing but did not discuss specifics concerning the problem unless a student had solved it (Nicol & Macfarlane-Dick, 2006; Butler & Winne, 1995). For instance, Julia considered the parent function  $f(x) = x^3$  while working part (a) of the *application of differentiation* problem (see Appendix F). Unable to use the parent function to solve part (a) during her first think aloud session, the researcher prompted her to consider how the parent function may be used to develop a solution.

The second session was conducted similarly, however participants were given the option of doing a practice think-aloud problem or immediately commencing the timed session. An additional difference was that after the retrospective report, an informal interview was conducted to discuss and clarify specific aspects of each participant’s individual problem-solving approaches and manifested epistemological beliefs. It should be noted that no interview protocols were developed for the informal interviews since the content of each interview was specifically related to participants’ navigation through the problem space. Questions were developed by the researcher during the sessions. Finally, each think-aloud problem-solving session was recorded by digital recording device and, in case of data loss, traditional tape recorder. All participant work, including scratch work and work completed during the interim between sessions, was collected for the purposes of data analysis.

*Phase IV: Follow-up interview sessions*

Since initial data analysis was conducted concurrently with data collection, all open coding and some axial coding had been completed shortly after the final think-aloud and classroom observation sessions. Then, draft narrative reports were written for each of the six participants. Once the draft reports were completed, an individual, semi-structured follow-up interview (see Appendix G) was scheduled with each of the six participants. The purpose of this interview was to provide member-checking validation of the findings and further insight into discrepancies and inconsistencies that emerged during initial data analysis. Each participant was presented their narrative report, asked to read it carefully, and determine whether or to what degree they agreed with the findings. This created a dilemma. On one hand, the reports needed to be ready for participants soon after data collection while experiences were fresh in their minds. On the other hand, hurried data analysis may yield poor results at best. So, the researcher was diligent to analyze data during data collection and kept a running draft report for each participant that simply required editing and finalizing at the end of data collection. This strategy allowed the researcher to produce quality narratives for participants to review in a timely manner. Each follow-up interview session was recorded.

#### Data Analysis

The final product of data analysis consisted of six individual, narrative case reports and an extensive cross-case analysis. Quantitative data from the MSLQ, PEP, and IMBS provided a small amount of data to begin developing narrative reports. The remainder of each narrative report was developed via deep analysis of qualitative data from observations, journals, think-aloud and interview transcriptions, and quantitative

results from student performance on problem-solving and AP exam preparation tasks. Finally, pattern and theme development via even deeper analysis yielded findings used in the cross-case analysis. The overall results of data analysis provided a rich, thick description of the significant findings of the study. Data analysis procedures are described below and subdivided as follows: (1) quantitative analyses, (2) coding, (3) problem-solving task assessment, (4) development and analyses of matrices, and (5) technology use.

### *Quantitative Analyses*

Quantitative data collected from the MSLQ, PEP, and IMBS mainly served to delineate six participants from the intact AP Calculus BC course involved in this study (see Figure 2). However, the quantitative data also served as initial building blocks for each participant's narrative report. Using SPSS Version 15, descriptive statistics were calculated for the entire class with respect to each scale of the MSLQ and IMBS and the three ways of knowing scores from the PEP. Then, each participant's scores were analyzed and compared to whole-class means and standard deviations to determine a quantitative, self-reported categorization (*low*, *average*, or *high*) of specific aspects of SRL and epistemological beliefs. For instance, the overall class' mean score for the *problem-solving duration* scale of the IMBS was 22 with a standard deviation of approximately 3.97. A participant who scored 30 on the *problem-solving duration* scale, which is more than 2 standard deviations above the mean, was categorized as having a *high* belief that mathematical problems may require time to solve. Each participant's self-reported results were heavily augmented by qualitative data to provide contextually rich narrative reports and cross-case analysis.

### *Coding*

The qualitative data from the AP exam preparation and think-aloud problem-solving sessions were exhaustively coded based on the SRL, epistemological beliefs, and problem solving constructs. The data sources coded were AP exam preparation journal entries, observational protocol transcriptions, student solutions to problem-solving tasks, think-aloud with retrospective interview transcriptions, and member-checking interview transcriptions. The first phase of the coding process involved *open coding*, defined by Creswell (2007) as a process that “involves taking data (e.g., interview transcriptions) and segmenting them into categories of information” (p. 240). In this study, data were initially coded at the micro-level for specific instances of the construct being analyzed. Micro-level coding was repeated in an iterative fashion until all data were saturated of information. Creswell (2007) stated, “In this process, I finally come to a point at which the categories are ‘saturated’; I no longer find new information that adds to my understanding of the category” (p. 240). Then, data were collapsed into macro-level nodes that served as general categories for the study’s findings (Miles & Huberman, 1994).

Detailed micro- and macro-level code lists were developed for each construct based on the literature review presented in the prior chapter (see Appendix H). For SRL, micro-level codes were based primarily on Zimmerman’s (2000) SRL framework, Greene and Azevedo’s (2009) expanded coding scheme, and Schoenfeld’s (1985) episodic problem-solving framework. Greene and Azevedo’s coding scheme was used on think-aloud protocol data and contained 35 separate SRL categories. For example, Greene and Azevedo subdivided “monitoring” into “judgment of learning, feeling of

knowing, self-questioning, content evaluation, identify adequacy of information, monitor progress towards goals, and monitor use of strategies” (pp. 25–26). Although Greene and Azevedo’s codes provided an excellent initial list, the researcher augmented them with more appropriate mathematical problem-solving based codes, based primarily on Schoenfeld’s work, and deleted others that did not fit this study. For example, the researcher included specific heuristic strategies, such as *solve a simpler problem*, in the performance control phase, providing a mathematics-based problem-solving context for students’ strategy use (Polya, 1957; Schoenfeld, 1985). Additionally, the researcher deleted items such as “free search” (p. 26), which is a code that applied specifically to student use of the hypermedia environment used in Greene and Azevedo’s study. Once micro-level coding was completed, data were collapsed into macro-level categories, or nodes, based on the four phases of SRL inherent to my theoretical framework, which were definition of the task, forethought, performance control, and self-reflection (Winne & Hadwin, 1998; Zimmerman, 2000).

Data were also coded for specific evidence of epistemological beliefs that manifested as participants worked problems. Rather than an extensive list of micro-level codes, operational definitions were developed for both general and mathematics-specific epistemological beliefs (Hofer, 2000; Hofer & Pintrich, 1997; Kloosterman & Stage, 1992; Muis, 2004; Royce & Mos, 1980; Schoenfeld, 1983, 1985, 1992). Overt participant behaviors that exemplified specific beliefs were coded as evidence of adherence to that belief. General epistemological beliefs included certainty/simplicity of knowledge, source of knowledge, justification of knowledge, attainability of truth (Hofer, 2000; Hofer & Pintrich, 1997). Mathematics-specific problem-solving epistemological beliefs included

unique/arbitrary nature of problem solutions, rational/empirical approach to problem solving, duration of problem solving, procedural/conceptual approach to problem solving, importance/usefulness of mathematics, and effort/inherent mathematical ability (Kloosterman & Stage, 1992; Royce & Mos, 1980; Schoenfeld, 1983, 1985, 1992).

Upon completion of open coding by hand, data sources and their respective codes were input into NVivo Version 8 for the purpose of deeper analysis. Axial coding was the next phase in the coding process, defined by Creswell (2007):

The researcher takes the categories of open coding, identifies one as a central phenomenon, and then returns to the database to identify (a) what caused this phenomenon to occur, (b) what strategies or actions actors employed in response to it, (c) what context (specific context) and intervening conditions (broad context) influenced the strategies, and (d) what consequences resulted from these strategies. (p. 237)

In this study, NVivo Version 8 expedited the axial coding process by generating matrices that related aspects of the study (e.g., participants, actions, beliefs) to each major phenomenon, or theme that emerged from the data. Examination of matrices revealed multiple categories of information converging on each phenomena.

Further exploration of the data via NVivo Version 8 facilitated selective coding. This final phase of coding was described by Creswell (2007): “The researcher takes the central phenomenon and systematically relates it to other categories, validating relationships and filling in categories that need further refinement and development” (p. 240). For example, monitoring emerged as a major theme and matrices were generated in NVivo Version 8 that allowed for the examination of this major construct with respect to



all other categories, including but not limited to individual, categorized participants and various aspects of participants' beliefs.

### *Problem-Solving Task Assessment*

Student performance on the think-aloud problem-solving tasks were scored using a modified version of Schoenfeld's (1982) scoring scheme, which provided for multiple levels of analysis based on both full and partial problem solutions. Recall that students submitted all work for the four parts of the *application of differentiation* and *application of integration* problems. All solution attempts were graded using the following scheme: 0 points for "an approach that is not pursued," 1 to 5 points for an approach "making 'little' progress," 6 to 10 points for "'some' progress," 11 to 15 points for "'almost' solutions," and 16 to 20 points for "'solved'" problems (Schoenfeld, 1982, pp. 39–40). Individual scores obtained from the multiple-count scoring categories were determined based on the amount of progress attained. For example, the range given for solved problems above was applied if students correctly solved a problem but failed to adequately justify their solution. Specifically, a participant working the application of differentiation problem solved for  $a$ ,  $b$ ,  $c$ , and  $d$  using purely empirical means by guessing values based on flawed logic. Upon graphing the guess using a calculator, the participant noted that problem conditions were met. However, the only justification provided was that the graph of the function met the conditions. A score of 17 was applied to this participant's work for the accurate solution and partial justification. Schoenfeld (1982) reported that "reliability with the researcher's grading was better than 90%" (p. 40).

For the AP exam preparation problems, College Board provides specific scoring guidelines for previous exams. Participants' solutions from journal entries were graded

using the appropriate scoring guidelines and produced integer scores ranging from zero to nine. Due to the specificity of the scoring guidelines, little subjectivity was involved in scoring participants' AP exam preparation problems.

#### *Development and Analyses of Matrices*

To answer the research questions, multiple matrices were assembled for individual and cross-case analyses. Preliminary matrices contained general categories such as the four phases of SRL, general epistemological beliefs, and mathematical problem-solving beliefs. For example, an initial matrix had the six participants as rows and accumulated codes from the four phases of SRL as columns. All matrices were generated with NVivo Version 8 and thus, each cell was hyperlinked to corresponding text, providing access to particular quotations corresponding to the relations being analyzed. From an individual case analysis perspective, matrices of this type provided a means for analyzing a participant's overall utilization of SRL processing. Matrices with numeric values provided frequencies of occurrence for phenomena. The hyperlinked access provided by NVivo Version 8 to text corresponding to matrix cells allowed for the development of thick descriptions and text-based analysis of phenomena.

As data analysis progressed, matrices became increasingly more specific in nature. An example of a more specific matrix included availing and non-availing beliefs as rows and specific instances of monitoring as columns. Yet again, NVivo Version 8 provided access to both frequencies and corresponding text for all cells. From a cross-case analysis perspective, this matrix provided a means for comparing and contrasting epistemological beliefs with performance control phase processing. Both quantitative and thick descriptive data obtained from matrices were analyzed based on the multitude of

SRL and epistemological beliefs descriptors identified and reported in each participant's narrative report. Data analysis continued as multiple general, specific, individual-case, and cross-case matrices were developed. These matrices provided rich, detailed, and accurate interpretations of actual events.

### *Technology Use*

Descriptive statistics for quantitative data were obtained via SPSS Version 15. Due to the volume of qualitative data collected, NVivo Version 8 was used for all data coding organization and matrix development. NVivo Version 8 is essentially a powerful organizational software program and will only organize coded data and develop matrices based on information input by the researcher. Thus, the researcher conducted open coding line-by-line by hand over multiple iterations to obtain data saturation. However, once finished, NVivo Version 8's organizational and categorical capacity allowed axial coding, selective coding, and matrix development via mouse clicks, as opposed to the arduous development of such analytical tools using bulky stacks of data and simple spreadsheet software.

### Validity and Reliability

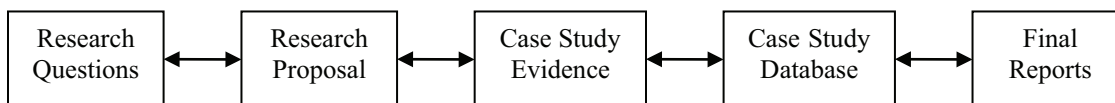
Issues of validity arise in any research endeavor and must, therefore, be addressed at the design phase. For case study research, validity essentially refers to the quality, trustworthiness, and credibility of a study. Considering validation a mutual process, Creswell (2007) stated, "I consider 'validation' in qualitative research to be an attempt to assess the 'accuracy' of the findings, as best described by the researcher and the participants" (pp. 206–207). The researcher employed data triangulation, thick, rich descriptions, and member checking to bolster validity for this study (Creswell, 2007; Yin,

2008). Data triangulation was achieved by collecting multiple sources of data from all participants and then carefully analyzing every source to increase accuracy and depth of descriptive and inductive findings. Creswell (2007) stated, “In triangulation, researchers make use of multiple and different sources, methods, investigators, and theories to provide corroborating evidence” (p. 208). In this study, the multitude of data sources, variety of data collection methods, and diversity of the individual participants provided the means for assessing the degree of convergence of the findings. Well-developed, research-based codes and categories ensured that the appropriate operationalized construct was being measured. Thick, rich descriptions of experiences allow readers to determine the degree to which results are transferable to other contexts. Finally, follow-up interviews with case participants provided member checking of initial findings and themes. Participants were presented with draft narrative reports. To bolster credibility in the findings of the study, participants were encouraged to identify any inconsistencies, inaccuracies, or omissions in the text provided to them (Creswell, 2007).

Reliability refers to the repeatability of a study’s procedures and the “stability of responses to multiple coders of data sets” (Creswell, 2007, p. 210; Yin, 2008). Reliability for this study was enhanced by creating a case study database, establishing a chain of evidence, and conducting peer review of the coding scheme (Creswell, 2007; Miles & Huberman, 1994; Yin, 2008). Creating an exhaustive case study database will provide an outside auditor, or curious academic, access to all evidence (i.e., transcriptions, student work) so that the degree of consistency between raw data and final reports may be assessed. The case study database was kept digitally, organized in folders, on my

computer's hard drive, external hard drive, and CD-ROMs and physically in multiple binders organized by participant.

Yin (2008) used the analogy of forensic investigations to explain the maintenance of a chain of evidence. Basically, the purpose of a chain of evidence is to provide a clear path from the origins of the case (or *crime*) to the final presentation of findings (or *court case*). My chain of evidence (see Figure 3) was established by clear, focused research questions. Then, the development of my proposal (or prospectus) established a rationale for the study, operational constructs to be measured, and appropriate methodology for collecting and measuring evidence of the phenomena. During the study, evidence was collected, analyzed, and organized in the case study database. Finally, the results of the study were recorded in *Chapter IV: Individual Case Results* and *Chapter V: Cross-Case Results* of this dissertation and may be summarized for one or more scholarly journals. If appropriately maintained, the chain of evidence will allow bi-directionality of the above progression, thus allowing for assessments of accuracy at all stages of the research process.



*Figure 3.* Chain of Evidence.

### Ethical Issues

Common to most studies of human subjects are ethical issues involving the welfare of the participants. Hence, the researcher would be remiss to fail to address any

foreseeable ethical issues inherent in this study. Throughout this section the titles *researcher* and *teacher* will be used to clarify my role in the given context. The reader is reminded that the researcher and teacher are one and the same person. First, issues of informed consent were considered. As consent forms were distributed, the researcher explained in detail the procedures involved in conducting the study and the general goals and aims of the study. This enhanced students' understanding of the ramifications of signing the consent form and avoided ambiguity (Creswell, 2007; Miles & Huberman, 1994). In reference to implications for analysis, Miles and Huberman (1994) stated, "Weak consent usually leads to poorer data: Respondents will try to protect themselves in a mistrusted relationship, or one formed with the researchers by superiors only" (p.291). Students may also have felt pressured to sign the consent form since the researcher was also their teacher. To counter this, the researcher assured students that there was no pressure to participate in the study and there would be no repercussions or ill will if they decided to withdraw for any reason. Additionally, students were ensured that withdrawal would be confidential. Full exclusion from the study was ensured for any student who wished to withdraw (Miles & Huberman, 1994).

Second, student confidentiality was enhanced by assigning letters to students and having them use those for all documents submitted to the researcher (Miles & Huberman, 1994). Participants were assigned pseudonyms for use in narrative reports and cross-case analysis (Creswell, 2007). Files referring to students by their actual names were locked securely in a closet in the classroom and the researcher's home. Digital files were saved onto the researcher's laptop, external hard drive, and back-up CD-ROMs. This ensured that no data were lost but also that sensitive data were not saved on high traffic computers

at the researcher's school. Computers holding any sensitive data were password-protected to further enhance data security. Finally, all forms of data (e.g., transcriptions, tape-recordings, journal entries) will be kept for two years and then destroyed.

Third, it may have appeared that the teacher was ignoring students' needs by simply observing while they struggled through AP practice problems in the classroom. Miles and Huberman (1994) referred to this ethical dilemma as "*detached inquiry versus help*" and identified two extremes: (1) focusing solely on understanding and (2) offering assistance to the point of losing intellectual objectivity and *going native* (p. 296). However, the teacher's philosophy has always been that students need opportunities to struggle with such problems before intervention is provided. There is a 45-minute time limit imposed on students to answer 3 questions during their actual sitting for the exam. Giving students time to work on their own provides opportunities for solution development independently of the instructor, which is a more authentic scenario with respect to standardized exam experiences. So, in the past, the teacher has allowed students 15–20 minutes to *do as much as they can and struggle* with each problem. However, the teacher always discussed every problem solution in great depth once time was called; the same process was applied to the problems worked during this study. Therefore students did not lose any instructional time due to the researcher's needs to obtain observational data. Additionally, the dual roles of teacher and researcher were appropriately separated to avoid the extremes of ignoring students' needs or *going native*.

Finally, participants' emotional and psychological states were potentially in jeopardy should certain wording appear in the draft narrative reports given to them during member checking (Miles & Huberman, 1994). Care was made to omit terminology such

as *low* and *poor*. The reports simply described the facts of each participant's experiences and results from data analysis. When possible, participants were asked if they agreed with certain categorizations, such as rational or empirical problem-solver, as long as the categories did not contain potentially demeaning wording or titles. In conclusion, extensive effort was expended to ensure that students were treated fairly and without harm.

### Summary

This chapter has provided a detailed account of the methods employed in this case study investigation. Participants were selected based on a mixture of quota and maximum variation sampling with prior achievement and questionnaire responses as parameters (Miles & Huberman, 1994). The validity and reliability of quantitative instruments were discussed in depth, as well as the development of qualitative instrumentation. Logistical and chronological details for collecting data were presented and means of overcoming barriers discussed. Then, coding schemes and matrix development were discussed in relation to analysis of the data. Finally, issues of validity, reliability, and ethical concerns inherent in this qualitative study were addressed. The next chapter provides descriptive results of the study in the form of individual case narratives.



## CHAPTER IV

### INDIVIDUAL CASE RESULTS

This study examined the relationships between students' epistemological beliefs and self-regulated learning (SRL) processing while engaged in mathematical problem solving tasks. Applying a multiple-case study design, six students purposefully selected from an Advanced Placement (AP) Calculus BC course engaged in individual mathematical problem-solving while thinking aloud and group AP Calculus exam preparation. Narrative accounts of each participant's experiences based on self-report questionnaire data, think-aloud transcriptions, observational field notes, participant journal entries, and individual interviews are presented in this chapter. Quantitative data were analyzed via descriptive statistics obtained from SPSS Version 15 software. Qualitative data were analyzed by open, axial, and selective coding, matrix development, and extensive thematic analysis using NVivo Version 8 software.

The main purpose of this chapter is to provide rich, thick descriptions of participants' experiences. In many cases, inferences and interpretations were necessary to bridge the gap between the theoretical framework (see *Chapter II: Review of Relevant Literature*) and the reality of the lived events described below. Thus, member checking was employed to enhance credibility (Creswell, 2007; Miles & Huberman, 1994; Yin, 2008). The findings are broadly subdivided into two main categories: whole-class results and participant narratives. Accessibility to whole-class results facilitates the comparison of individual participant results to a given population, which, in this case, encompassed

the 30 students in the AP Calculus BC class. Participant narratives are presented in a time-elapse manner to re-create accurate and detailed events.

### Whole-Class Results

#### *Calculus Achievement*

For the calculus achievement category, student grades from the previous course, AP Calculus AB, were obtained for the 30 students involved in this study. The AP Calculus AB grades yielded a mean score of 86.80 ( $SD = 7.85$ ), of which the lowest grade was 72 and the highest was 100. Both of these extreme cases, Robert and Cameron respectively, were included as participants in this study. Ensuring that extreme cases are included as participants, known as maximum variation sampling, was employed to ensure that data were obtained from outliers and to assess whether main patterns in the data still hold (Miles & Huberman, 1994). From the data, intervals of width equivalent to one standard deviation were computed and categorized for calculus achievement: *Category I* (94.65, 102.50), *Category II* (86.80, 94.65), *Category III* (78.95, 86.80), and *Category IV* (71.10, 78.95). For the purposes of *quota sampling*, four additional individual cases were selected to represent the diversity of the class (Miles & Huberman, 1994). A more complete and detailed account of sampling procedures may be found in *Chapter III: Methodology*.

#### *Self-Report Questionnaires*

The three self-report questionnaires used in the study were given to all 30 students in the AP Calculus BC course. Questionnaire data were subdivided categorically using descriptive statistics. For each scale, the mean and standard deviation were calculated. Then, three subdivisions of width equivalent to two standard deviations were obtained

and categorized: *high* ( $M + SD, M + 3SD$ ), *average* ( $M - SD, M + SD$ ), and *low* ( $M - 3SD, M - SD$ ).

The Indiana Mathematics Belief Scales (IMBS; Kloosterman & Stage, 1992) is a five-point Likert scale questionnaire and all six scales contain six questions. Thus, a minimum score of 5 and a maximum score of 30 may be obtained for each scale. Table 3 summarizes the data obtained from all 30 students in the AP Calculus BC course. These scores are higher than those reported by Kloosterman and Stage (1992), who validated the scales using a sample of remedial college students and college students majoring in early education. However, higher scores are not surprising as Kloosterman and Stage predicted that “one might expect highly able students to score higher” (p. 114).

Additionally, it should be noted that student scores for Belief 6 were so high that it was impossible to obtain the *high* categorization, implying that the majority of students in this course self-reported a belief that mathematics is useful. Thus, in the ensuing analysis, only students who scored in the *low* range will be discussed.

Students’ scores on the IMBS informed *quota sampling* by capturing a broad range of self-reported mathematical beliefs. Due to the preponderance of data, an integer quantifier was calculated for each student’s overall IMBS scale scores. The quantifier was determined by establishing six intervals of width equivalent to one standard deviation from the mean for each scale, assigning an integer value on the interval  $[-3, 3]$  based on the relative positioning of each student’s scale scores, and summing the results. For instance, Edwina scored 21 points for Belief 3, which fell in the interval ( $M - 2SD, M - SD$ ) and thus, was assigned a score of  $-2$ . Her other scale scores were similarly calculated and the sum resulted in an overall score of 1 for mathematical beliefs. With

respect to the whole class, mathematical beliefs scores ranged from  $-10$  to  $10$ . For sampling, student performance data were merged with mathematical beliefs data and multiple students were considered as potential participants. The six students who were selected and consented to participate in the study had mathematical beliefs scores ranging from  $-4$  to  $9$ .

Table 3

*Descriptive Statistics for the Indiana Mathematics Belief Scales (N = 30)*

Scale	Minimum	Maximum	<i>M</i>	<i>SD</i>
Belief 1	14	30	22.00	3.97
Belief 2	14	27	20.80	3.18
Belief 3	15	30	24.83	3.52
Belief 4	12	29	20.07	4.95
Belief 5	16	30	24.27	4.28
Belief 6	12	30	25.67	4.41

*Note.* Belief 1 = *I can solve time-consuming mathematics problems*;  
 Belief 2 = *There are word problems that cannot be solved with simple, step-by-step procedures*; Belief 3 = *Understanding concepts is important in mathematics*; Belief 4 = *Word problems are important in mathematics*; Belief 5: *Effort can increase mathematical ability*;  
 Belief 6 = *Mathematics is useful in daily life.* (Kloosterman & Stage, 1992, p. 115)

The Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich, et al., 1991) is a seven-point Likert scale questionnaire, which is divided into two main subsets of scales: Motivation and Learning Strategies. Since the scales contain varying amounts of questions, an overall mean is calculated for each scale for the purposes of data analysis. Thus, a minimum score of one and a maximum score of seven may be obtained for each scale. Based on the theoretical framework and purposes of the study, an abbreviated version of the MSLQ was used in this study. The Motivation scales used were Intrinsic Goal Orientation, Extrinsic Goal Orientation, and Task Value. The Learning Strategies scales used were Critical Thinking, Metacognitive Self-regulation, Peer Learning, and Help Seeking. Table 4 summarizes the results obtained from the MSLQ for all 30 students in the AP Calculus BC course.

The same technique described above for the IMBS was utilized to obtain integer scores from students' MSLQ results, thus providing an SRL score. The resulting range of scores for the whole class was -12 to 14. The SRL score provided a third indicator for participant selection. The six individual students that participated in the study had scores ranging from -3 to 14.

Finally, the Psycho-Epistemological Profile (PEP; Royce & Mos, 1980) is a 90-question, 5-point Likert scale questionnaire that determines whether a person's epistemic style is predominantly *rational*, *empirical*, or *metaphorical*. Traits of a rational perspective include the use of logic, rigid analysis, and synthesis of ideas. The empirical perspective is largely based on observational phenomena, perception, and the senses. The metaphorical perspective is synonymous to symbolism and insightfulness (Royce & Mos, 1980). Since each scale contains 30 questions, the minimum score per scale is 30 and the

maximum is 150. Table 5 summarizes the results obtained from the PEP for the 30 students in the AP Calculus BC course.

Table 4

*Descriptive Statistics the for Motivated Strategies for Learning*

*Questionnaire (N = 30)*

Scale	Minimum	Maximum	<i>M</i>	<i>SD</i>
IGO	1.75	7.00	4.79	1.35
EGO	3.00	6.50	5.28	0.86
TV	1.50	7.00	5.42	1.28
CT	1.00	7.00	3.79	1.53
MSR	2.83	6.50	4.49	0.72
PL	1.33	7.00	3.96	1.54
HS	1.00	7.00	4.63	1.55

*Note.* IGO = Intrinsic Goal Orientation; EGO = Extrinsic Goal

Orientation; TV = Task Value; CT = Critical Thinking; MSR =

Metacognitive Self-regulation; PL = Peer Learning; HS = Help Seeking

Overall, 18 students profiled as predominantly *rational*, 12 as predominantly *empirical*, 1 as predominantly *metaphorical*, and 2 students obtained the same score for Rational and Empirical scales. Continuing to meet purposeful sampling goals, the researcher chose the participants with the highest scores on each scale: Martin with a Rational scale score of 123, Julia with an Empirical scale score of 125, and Olivia, the

only predominantly metaphorical student, with a Metaphorical scale score of 113. Further sampling enhancement was obtained as Cameron, the upper-bound *maximum variation* case with respect to performance, obtained the exact same Rational and Empirical scale score, 105, and had the lowest Metaphorical scale score, 67, in the class.

Table 5

*Descriptive Statistics for the Psycho-Epistemological Profile (N = 30)*

Scale	Minimum	Maximum	<i>M</i>	<i>SD</i>
Rational	96	123	106.93	6.60
Empirical	77	125	102.37	11.54
Metaphorical	67	113	91.83	11.37

*AP Calculus AB Exam Practice*

All 30 students in the AP Calculus BC course also participated in the in-class, group AP Calculus exam preparation sessions. Despite working in groups, some within-group variation in scores occurred as students submitted individual work via journal entries. Additionally, due to absenteeism, all 30 students were not present for every session. Each student's journal entries were graded by the researcher using the College Board Scoring Guidelines for the appropriate problem. The maximum score for each problem is 9 points. Table 6 summarizes the descriptive statistics obtained from the AP Calculus exam preparation sessions for the students in the AP Calculus BC course. AP Calculus exam preparation results are discussed extensively below within each participant's narrative.

Table 6

*Descriptive Statistics for AP Calculus Exam Preparation*

Question	<i>N</i>	Minimum	Maximum	<i>M</i>	<i>SD</i>
AB 2003B #2	30	3.00	7.00	4.53	1.04
AB 2004B #4	26	1.00	9.00	5.12	2.14
AB 2004B #6	29	1.00	8.00	2.66	1.45
AB 2005B #5	26	3.00	9.00	5.85	1.99
AB 2006B #3	30	0.00	9.00	4.83	2.26
AB 2006B #4	30	2.00	8.00	4.90	1.67
AB 2007B #2	19	2.00	7.00	4.42	1.74
AB 2007B #5	30	2.00	9.00	4.97	1.90
AB 2007B #6	28	0.00	7.00	3.21	2.06
AB 2008B #3	28	2.00	9.00	6.43	2.41
AB 2009B #2	30	1.00	9.00	4.73	1.91
AB 2009 #1	30	4.00	9.00	7.33	1.52
AB 2009 #1	30	4.00	9.00	7.33	1.52

*Note.* Question titles are abbreviated as follows: AB 2003B #2 = 2003 AP Calculus AB (Form B) Free Response Question 2. The remaining questions follow the same format of abbreviation. The question in the last row was not from a Form B exam. The variation in *N* is due to student absenteeism on days in which the class engaged in AP exam preparation.



## Participant Narratives

The purpose of this section is to provide detailed narratives of participants' experiences. Each narrative is subdivided into three main categories: (1) achievement and questionnaire data, (2) think-aloud problem-solving sessions, and (3) AP exam preparation sessions. Subdivision (1) of each narrative provides a detailed report with interpretations of a participant's categorizations based on achievement and self-report questionnaire data. Subdivision (2) of each narrative contains rich, thick descriptions of a participant's experiences during the think-aloud problem-solving sessions based on think-aloud transcriptions, student work, and interview transcriptions. Finally, subdivision (3) of each narrative describes a participant's experiences within their group while working on two AP exam practice problems based on AP exam preparation journal entries and researcher observational field notes. In the interest of space and alleviation of redundancy, the following abbreviations were developed and used throughout the narratives: TA1 = first think-aloud problem solving session, TA2 = second think-aloud problem solving session, RRI1 = first retrospective report and interview, RRI2 = second retrospective report and interview, and MCI = member-checking interview.

### *Robert's Narrative*

#### *Mathematical Achievement and Questionnaire Data*

Robert was categorized as a *Category IV* calculus achiever compared to his peers because he scored a 72 in AP Calculus AB. Since his grade was lowest in the class, Robert constituted the lower-bound *maximum variation* participant. For the IMBS, Robert was categorized as having *average* beliefs for all scales except Belief 2, for which he received a *low* rating. A low rating indicates that he self-reported a belief that word

problems can be solved by following *procedural*, algorithmic steps (Kloosterman & Stage, 1992). The remainder of his self-reported mathematical beliefs was consistent with his peers.

For all MSLQ scales, Robert was categorized as *average*. One point of interest is that Robert scored higher on Extrinsic Goal Orientation (5.5) than on Intrinsic Goal Orientation (4.25). Thus, based on self-report data, Robert may tend to participate in tasks for reasons such as grades or other performance indicators instead of being driven by the desire to master content (Pintrich et al., 1991). Finally, Robert's PEP categorizations for each belief dimension were all *average*. For the Rational, Empirical, and Metaphorical scales, he scored 105, 103, and 101, respectively. These results indicated that Robert was predominantly *rational*, but due to the proximity of the scores, the results were deemed inconclusive. Qualitative methods were employed to derive further categorization of Robert's predominant adherence to either the rational, empirical, or metaphorical epistemic style.

#### *Think-Aloud Sessions*

*Session 1.* Robert began TA1 by spending 15 seconds reading part (a) of the application of differentiation problem (see Appendix F). This was the extent of his cognitive activity in the definition of the task phase. Then, fixated on the *exactly one root* condition of part (a) of the problem, he spent 28 min 25 sec on a fruitless exploration. During this exploratory episode of Robert's navigation through the problem space, evidence of the forethought phase was restricted to the first minute. His plan to use substitution consumed him and he never diverted from it. When probed later during his

RRI2, Robert cited problem recognition from a prior course, Algebra 2, as the reason for trying to use substitution to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ .

A significant number of overt instances of metacognitive monitoring were coded during this exploratory excursion, potentially indicative of activity in the performance control phase of SRL. Despite 11 instances of locally assessing his problem strategy and 2 instances of globally assessing his progress toward his *perception* of the goals of the problem, no corrective actions or problem-solving transitions were employed by Robert. In each instance of monitoring, Robert's references to his current solution state were vague and did not focus on the specific conditions and goals of the problem. Rather, his assessments were focused on his substitution strategy or were general statements such as the following: "As of right now, I don't really know if I'm doing this right, but I'm getting rid of variables, so I guess that's a good thing."

His inability to focus on the particular goals of the problem may be explained by a persistent confusion between the goals and conditions of the application of differentiation problem that was still present during his RRI2. In fact, during that interview Robert stated, "Maybe I wasn't supposed to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ . I was supposed to just deal with those and try to find tangent lines or the roots in terms of them." Since the problem clearly asks the solver to find  $a$ ,  $b$ ,  $c$ , and  $d$  using conditions involving roots and tangents, Robert seems to have struggled with appropriately defining the task. His lack of attention to the preliminary phases of SRL processing may also explain his inability to convert his monitoring to problem-solving control aimed at progression toward a solution during the performance control phase. During his RRI2, I asked Robert whether mathematical resources, strategies, or control were his most significant barriers. Although he cited lack

of control as the most significant barrier for the application of differentiation problem, a statement later in the interview referring to the same problem seemed to contradict his assessment and imply that lack of mathematical resources was an issue. “I really just couldn’t think of anything else to do. That’s really all I could come up with,” he stated.

At 28 min 40 sec into TA1, Robert concluded that part (a) was not possible due to the failure of his substitution method to produce a solution, instead yielding  $0 = 0$ . He then made a statement indicative of the self-reflection phase of SRL: “I feel like if I did this right and I get zero equals zero, that might mean that the *if possible* just means it’s not possible.” His reference to doing the problem *right* indicated a causal attribution that was strategy-focused, which is less likely to cause learning deficiencies than ability-focused causal attributions (Zimmerman, 2000). At this point, Robert moved on to part (b) of the application of differentiation problem. He spent 1 min 27 sec reading and re-reading the problem, then moved on to promising analysis of the problem conditions. For 43 seconds, Robert worked with the second derivative, tying its roots to possible points of inflection. If only he had considered the type of function that he was working with, the fact that no solution was possible would have fallen into his lap! However, the arbitrary constants, which he claimed in subsequent interviews plagued him throughout both sessions, confused him to the point of reverting to his substitution strategy, and he spent his final 1 min 21 sec on this fruitless exploration. Additionally, during TA1, Robert’s only instances of heuristic strategy use were *setting up the equations* for finding the roots of the function and its first two derivatives.

Robert’s work and verbalizations from TA1 exhibited three distinct epistemological beliefs. First, Robert’s work during TA1 represented a continuous

application of trial-and-error strategies, devoid of reason and logic, which demonstrates an *empirical* belief in mathematical problem solving. With respect to questionnaire data, this finding is not surprising since his PEP results yielded virtually the same score for all three scales of beliefs. The following quote from TA1 provides an example of Robert's manifestations of an empirical belief in problem solving: "I'm going to plug it into the modified equation that I just got—see if that will get me anywhere." Second, his devotion to the substitution strategy and his citation of this strategy as a causal attribution to his success or failure imply a belief that applying the right *procedures* to a mathematical problem will result in a solution. This finding is triangulated by questionnaire data since Robert received a low categorization for Belief 2 on the IMBS. Finally, Robert's lack of contextual consideration for his substitution technique and failure to relate the conditions of the problem in some meaningful manner indicate a *straightforward* belief in the simplicity of knowledge (Hofer, 2000; Hofer & Pintrich, 1997). The following quote from his RRI2 is indicative of Robert's manifestations of all three beliefs:

So, I just went through this whole process of a few pages of work trying to solve for  $a$ ,  $b$ ,  $c$ , and  $d$  and initially I thought it was a pretty good idea. But, then I started to get these pages upon pages of work, which really didn't make sense to me. But, I just kept going with it because I was in the thick of it.

*Session 2.* At the beginning of TA2, Robert continued to work on the application of differentiation problem. It should be noted that he was given the opportunity to work on the problem at home, if desired, during the interim between sessions, which amounted to two evenings since he had school during daytime hours. Robert reported that he simply thought about the problem during the interim, but wished that he would have done some

work on it. He spent the first 29 seconds of TA1 engaged in *analysis* of the problem space, relating the roots of the first derivative to horizontal tangents. Then, just as in TA1, the arbitrary constants  $a$ ,  $b$ ,  $c$ , and  $d$  diverted Robert's attention from his promising work and eventually led him back to his substitution technique, which amounted to a 7 min 2 sec fruitless *exploration*. During this portion of TA2, six occurrences of either monitoring strategy or goal state were coded but did not lead to any meaningful transitions in Robert's navigation of the problem space. At one point during his exploration, Robert stated, "See if there's an easier way to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ . Hmm, the function,  $f$ , equals zero—it could be any number of values." This quote represents a belief that *all mathematical problems do not have a unique solution*, and thus, certain liberties in solution path development may exist. However, he continued to assume that a single, algebraic method existed and attempted to use the computer algebra system (CAS) built into his calculator to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ .

Robert then turned his attention to part (b) of the application of differentiation problem. He spent 18 seconds *reading* the problem and 48 seconds engaged in *analysis*, working with the second derivative. Then, 2 min 16 sec of exploration ensued, producing little progress toward the solution. This portion of the session ended when Robert stated, "Neither one of them [referring to parts (a) and (b) of the problem] are possible, because it does give me that option to say that they're not possible, which I know good and well they most likely are. I'm just not thinking of the right way to do it." This self-evaluation is evidence of the *self-reflection* phase of SRL. Robert's assignment of causal attributions to his own ability may eventually prove detrimental to future learning endeavors, as

“attributions of errors to a fixed ability prompt learners to react negatively and discourage efforts to improve” (Zimmerman, 2000, p. 22).

At this point, a brief transition period occurred while the researcher retrieved the application of integration problem (see Appendix F) for Robert. He spent approximately 30 seconds *reading* the problem and then engaged in 55 seconds of *analysis*. During the analysis, Robert employed the *draw a picture* heuristic by sketching a graph of the function along with its bounds and then shading the appropriate region. This analysis led directly to a productive *exploration* lasting 3 min 46 sec, which resulted in expressions for the entire area and half the area of the enclosed region in terms of  $m$  (with a minor error). At this point, Robert transitioned back to the forethought phase of SRL and recycled the goal in working memory, stating, “Now, I just have to find the line that bounds that.” Despite this plan, Robert was unable to develop a strategy to find an equation for the vertical line.

With no immediate strategies available for completing part (a), Robert decided to move to part (b). He spent 11 seconds *reading* the problem, followed by 50 seconds engaged in *analysis*. Yet again, the analysis phase consisted mainly of Robert applying the *draw a picture* heuristic by sketching the graph of the curve, bounds, and shaded region. This analysis appeared to lead to productive exploration. However, Robert made a costly error when calculating the area of the enclosed region. His integral expression did not account for the boundary  $y = 1/2$ . So, despite three occurrences of monitoring either strategy use or goal state, Robert allowed this major error to go unchecked. Additionally, even if he had caught his mistake, the goal of part (b) was to find a horizontal line, which would have required a completely different approach. Calculating the area using

integration with respect to  $x$  lends itself to finding an equation for a vertical line, but not a horizontal line. At any rate, Robert then moved into a fruitless exploration that would last the final 13 min 32 sec of the session.

Congruent with his earlier perusals of problem spaces, Robert's actions during this exploration logged multiple instances of monitoring either strategy use or goal state—12 instances, in fact. The culprit of his lack of attaining control, yet again, was the arbitrary constant  $m$ . A significant amount of metacognitive monitoring occurred as Robert grappled with the significance of  $m$ . To begin, he quickly dismissed solving for  $m$ , recalling his earlier problems solving for  $a$ ,  $b$ ,  $c$ , and  $d$ , and decided to keep the  $m$  in the function. A good example of monitoring during this exploratory episode presented itself when Robert stated, “I know it's not as easy as just saying it's three-fourths because the halves wouldn't be equal.” A very lengthy exploration of the effects that  $m$  may have on the graph of  $h$  followed but was never resolved. He did consider making the substitution  $m = 1$ , which would have had some heuristic promise if he had considered solving this *simpler* problem. Instead, he merely used the substitution to view a graph. Then, Robert ended the session by trying a proportionality approach to solving for the necessary lines, but to no avail.

From an epistemological standpoint, Robert's overall navigation of the problem space during TA2 was coded as predominantly *empirical*, despite some flashes of *rational* logic mainly occurring during the application of integration problem. A sample quote supporting this assessment is as follows: “Well for the sake of I don't really know what, I'm just going to factor out an  $m$ .” Similarly, despite some glimpses of a *conceptual* belief in mathematical problem-solving, most notably connecting integration to area



under the curve, this session was coded as demonstrative of a predominantly *procedural* stance with respect to mathematical problem solving. Evidence of this assessment is Robert integrating part (b) with respect to  $x$  just as he did in part (a) without considering the conceptual ramifications of this decision with respect to the goal of finding an equation for a horizontal line. In fact, after reading part (b), he stated, “So, it’s basically the same concept, it looks like. I’ve just got to figure out what that concept is.” Yet again, questionnaire results provided triangulation for both the *empirical* and *procedural* epistemological classifications.

Interestingly, Robert’s MCI contradicted the *procedural* coding of his problem-solving activities. Despite approving of the accuracy of his narrative which included the *procedural* coding, Robert made the following statement when asked whether procedural or conceptual understanding is more important to problem solving success:

There have been plenty of people who don’t follow conventional steps and are still able to solve the problem in a round-about way. If you solve everything procedurally, some hiccup comes up and you don’t know how to handle it because you’re just focused on doing something you think just has to go step by step. . . If you know calculus, you can get through, well not everything, but you can sometimes formulate your own pathway, I guess.

With respect to his problem-solving actions, this statement provides compelling evidence of a disconnect between Robert’s idealized belief in a *conceptual* approach and his realized manifestation of a *procedural* approach to mathematical problem solving.

Robert’s RRI2 and MCI revealed an additional epistemological belief that manifested during the study. Robert’s confusion regarding the arbitrary constants  $a$ ,  $b$ ,  $c$ ,

$d$ , and  $m$  had a rather deep-seeded, mathematical-beliefs basis. Schoenfeld (1992) suggested that many students have the non-availing belief that all mathematical problems have a unique solution. To situate this belief in the current context, consider the following statements. A *unique* belief regarding problem solutions would imply that a student would expect a unique solution for  $a$ ,  $b$ ,  $c$ , and  $d$  in the application of differentiation problem and require a unique value for  $m$  in the application of integration problem. An *arbitrary* belief regarding problem solutions would imply that a student is open to the possibility of infinite or no solutions for  $a$ ,  $b$ ,  $c$ , and  $d$  in the application of differentiation problem and comfortable with  $m$  representing a family of exponential functions from which a general solution with respect to  $m$  may be obtained.

Based on his problem-solving actions and interview responses, Robert's belief regarding problem solutions was coded as predominantly *unique*. It should be noted that Robert stated that  $a$ ,  $b$ ,  $c$ , and  $d$  could be any number of values, which is demonstrative of an *arbitrary* belief regarding problem solutions. Based on the theoretical framework for this study, beliefs lie on a continuum and thus, may manifest in a contradictory fashion based upon context (Hofer & Pintrich, 1997; Schommer, 1990). Then, the above finding implies that Robert lies somewhere between the extremes of this belief's continuum, but is closer to the *unique* classification. An alternative hypothesis may be that Robert held an *arbitrary* belief for the duration of the application of differentiation problem but lacked the mathematical resources to develop additional strategies. Based on this data, the researcher asserts that Robert's non-availing, *unique* belief and a lack of mathematical resources worked in tandem to produce his unsuccessful problem-solving attempt. The arbitrary/unique belief regarding problem solutions dimension seems closely related to

Hofer and Pintrich's (1997) *fixed* and *fluid* beliefs in the *certainty of knowledge*. Further discussion of this suggestion may be found in *Chapter V: Cross-Case Results*.

In a manifestation of his *unique* belief, Robert expressed his confusion regarding the role that the constant  $m$  played in the application of integration problem during his RRI2:

I still have the whole  $m$  problem, which I feel more confident about the value of  $m$  being irrelevant, just because it tells me it's a nonzero positive integer. And I guess that could be a guideline, so when you solve it and you get negative one, you're just wrong. But I feel like you just have to know it's positive, more so than figuring out the actual value for it. But, it still just messed with my head a little bit. I'm not good with having a variable that I don't know the answer for.

When probed further as to whether substituting an arbitrary value for  $m$  would produce an acceptable solution for the problem, Robert responded:

It would have been, because as long as I keep my  $m$  consistent, it really shouldn't matter, I wouldn't think. Because if I plugged in a seventy-two, I have a seventy-two all the way through; so, I don't think it would matter because in the end, using given values it would give me, I don't know if it would give me a different answer. It would give me different work, certainly. But, I think the answer would probably still be the same.

Finally, during his MCI, the researcher asked Robert to discuss his confusions with the arbitrary constants and, if possible, indicate a source for his confusions. His response indicated that his *unique* belief continued to persist as of this writing and provided a possible source for its inception:

Well, just having variables there. I've always been taught you need to solve for your variables. Because other than calculus there has never really been a problem where you've had variables, but that's not really the focus of the problem.

As will be discussed further below, Robert was not alone in this confusion with arbitrary constants and variables.

*Think-aloud problem-solving session performance.* Robert made progress on both problems, but was unable to fully solve either of the problems given during the think-aloud sessions. For part (a) of the application of differentiation problem, the researcher deemed that his problem-solving approaches led to *little* progress and applying Schoenfeld's (1982) scoring range of 1–5 points (see *Chapter III: Methodology* for details), assigned him a score of 4 points for part (a) of the application of differentiation problem. The rationale for assigning four points is that Robert's recognition of the calculus connections between the conditions and the first derivative merited some recognition, but no progress was made toward connecting the conditions to the goal of the problem. For part (b) of the application of differentiation problem, Robert was awarded 10 points based on Schoenfeld's (1982) 6–10 point range given for approaches yielding *some* progress. The rationale for this score is that accurate calculation of the second derivative was a significant step in the solution of this problem. In fact, recognizing that a linear function with nonzero, defined slope may never be strictly positive was all that was left to obtain full credit. So, Robert's total score for the application of differentiation problem was 14 out of 40 points.

For part (a) of the application of integration problem, Robert made *little* progress and, factoring in a minor calculation error, a score of four points was assigned for part (a)

of the application of integration problem. For part (b) of the application of integration problem, he made *little* progress and, factoring in a major integral setup error, a score of two points was assigned for part (b) of the application of integration problem. So, Robert's total score for the application of integration problem was 6 out of 40 points. His total combined score for both problems was 20 out of 80 points.

#### *AP Calculus Exam Preparation*

Each participant was asked to describe the differences in the problem-solving tasks used during AP Calculus exam preparation and think-aloud sessions. Regarding the AP Calculus exam preparation problems, Robert made the following statement:

The ones in class were difficult calculus but they were straightforward calculus.

There weren't too many that were just out there. They were difficult concepts, you know, *a lot of steps* [emphasis added] and that sort of stuff.

From the researcher's perspective, the AP Calculus exam preparation problems generally provided either an explicit or implicit solution path. However, mindless application of procedures would not have yielded a successful solution; connections to various concepts based on multiple representations of information were required to successfully solve the problems (Stein, Smith, Henningsen, & Silver, 2000).

In contrast, Robert indicated significant differences in the think-aloud session problems:

The ones outside of class seemed to be a lot more abstract, you know, you had the variables and it was just kind of weird. . . The ones after school seems like you needed a little bit more practice, you needed to be a more seasoned calculus student, or user, I guess.

The researcher concluded that the think-aloud problems required participants to exert significant cognitive load, develop non-algorithmic solution paths, and apply multiple calculus concepts learned at various times throughout the course (Stein et al., 2000). Overall, the exam preparation problems were categorized as “procedures with connections tasks” and the think-aloud problems as “doing mathematics tasks” (Stein et al., 2000, p. 16).

*Classroom observation 1.* Robert was observed working on question 3 of the AP Calculus AB 2006 (Form B) exam with his partners, Dan and Tom (see Appendix E). Students were provided a graph that modeled the height of a skateboard ramp and conditions that the graph met and given four problems, (a)–(d), to solve. For part (a), the group had to show that the general quadratic  $f(x) = ax^2$  did not meet one of the given conditions. The researcher got the impression very early in the session that Tom had become the *de facto* leader of the group. He developed a plan for solving part (a) and shared his plan with the group. Dan and Tom embraced the plan and began working. However, Robert was unable to move forward due to the abstract nature of the function. He *self-disclosed* his problems and *requested feedback* from his partners. Ironically, as Dan and Tom helped Robert, they caught an error in their work. Due in large part to Robert’s request for help and the ensuing group monitoring, all three group members received the full two points for part (a).

For part (b), students had to find a coefficient for a family of cubic functions that met one of the given conditions. There was virtually no discussion about this part; all three seemed to understand what needed to be done. Possibly based on the monitoring lesson learned above, the group actually engaged in two separate instances of monitoring

for part (b). The result was a verified solution whose accuracy earned all three members the one point designated for part (b).

For part (c), the group had to show that the function they developed in part (b) did not meet one of the conditions given in the problem. Tom had to assume his leader role again, as Dan and Robert were unable to contribute any ideas. Upon presenting his plan, Dan began working, but Robert stated that he had “no idea what is going on.” This was another example of group monitoring by Robert, as *self-disclosure* of current state and *feedback requests* ensured that all group members understood the problem. So, aided by direction provided by Tom, Robert calculated the required derivative and provided an explanation based on his work. The result was that all three group members received the full two points for part (c).

For part (d), a new function,  $h(x) = x^n/k$ , was introduced and students were required to find values for  $n$  and  $k$  such that all conditions were met. Upon reading this part, Robert said that “his head was starting to hurt.” His verbalizations indicated that he understood the general connections between the conditions of the problem and the goal. However, as in his think-aloud sessions, the arbitrary constants  $k$  and  $n$  confounded the development of a solution path. Despite requesting and receiving feedback from Tom, Robert was unable to rectify part (d) and finished the session *watching and listening*. Dan and Tom continued working on part (d) until time was called. Thus, Robert received zero points, Dan received two points, and Tom received three points out of four possible points. In sum, Robert received 5 points, Dan received 7 points, and Tom received 8 points, resulting in a mean group score of 6.67 out of 9 possible points. The group outperformed the class, which averaged only 4.87 points.

*Classroom observation 2.* For his second classroom observation, Robert worked with his group on question 2 of the AP Calculus AB 2007 (Form B) exam (see Appendix E). This question provided students with a velocity function and its graph on a closed interval and four subsequent problems, (a)–(d). For part (a), Robert suggested they use the second derivative to calculate the acceleration and asked for feedback from the group. Tom pointed out that the given function and graph represented velocity and thus, only one derivative was needed. This instance of monitoring and discussion resulted in all group members receiving the full one point for part (a).

Unfortunately, the group was unable to do much after their early success on part (a). For part (b), Tom erroneously informed the group that total distance is the definite integral of the velocity function over a domain. Dan and Robert assented without argument and began working. The group moved on upon obtaining a solution but returned to part (b) later and discussed whether the solution was accurate. During this discussion, Tom recalled the need for absolute value, but wanted to use the position function. Dan and Robert applied *group monitoring* and convinced Tom that velocity was the correct function. Unfortunately, no one considered the conceptual significance of their arguments. They were all trying to recall procedures and never considered examining the role of the integral and the physical aspects of the problem. Thus, all three students left their answers as before and received zero out of two possible points.

For part (c), students were to find the final position of the object given an initial position and an interval of time. No one was able to even provide an idea for part (c) and all three left the problem blank, earning zero out of three possible points. For part (d), students were to determine the point at which the object is farthest right over a given time



interval. At the beginning, Dan and Tom discussed the problem and seemed to be on track to provide a full solution. Robert was *watching and listening* and at one point asked which part the group was working on. At this point, Tom described the current state of his solution to Robert, who assented but did not indicate whether he fully understood the problem. Despite having an accurate solution and engaging in some group monitoring, no one received full credit because their analyses were incomplete. Robert and Tom each received one point and Dan received two out of a possible three points. In sum, Robert and Tom received 2 points and Dan received 3 points, resulting in a mean group score of 2.33 out of 9 possible points. The group was outperformed by the class, which averaged 4.42 points.

*AP Calculus exam preparation performance.* All three members of Robert's group were present for all twelve AP practice sessions. Group statistics for the twelve problems were as follows: Robert averaged 4.50 points, Dan averaged 4.75 points, and Tom averaged 5.92 points. Thus, the group's overall average score for the 12 problems was 5.06 points, resulting in a difference between Robert's and his group's average score of  $-0.56$  points. The overall class average for the 12 problems was 4.92, resulting in a difference between Robert's average score and the overall class' average score of  $-0.42$  points.

#### *Edwina's Narrative*

##### *Mathematical Achievement and Questionnaire Data*

With respect to her peers, Edwina was categorized as a *Category IV calculus achiever* with a grade of 76 in AP Calculus AB. Her inclusion in the study was for the purposes of *quota sampling*, as she placed slightly higher in the achievement interval than

Robert (Miles & Huberman, 1994). For the IMBS, Edwina was categorized as *average* for all scales except Belief 3, for which she received a *low* categorization. Thus, based on self-reported data, Edwina tended to believe that understanding mathematical concepts is not important as long as performance can be maintained to a satisfactory level (Kloosterman & Stage, 1992).

For the MSLQ, Edwina was categorized as *average* for all scales except for the Peer Learning scale, for which she received a *high* categorization. So, Edwina self-reported that she found value in collaborating with peers while working on assignments and studying for this course (Pintrich, et al., 1991). Additionally, like Robert, Edwina scored higher on Extrinsic Goal Orientation (5.75) than on Intrinsic Goal Orientation (3.75). Thus, based on self-report data, Edwina tended to participate in tasks for reasons such as grades or other performance indicators instead of being driven by the desire to master content (Pintrich et al., 1991).

Edwina's PEP scores for the Rational, Empirical, and Metaphorical scales were 104, 113, and 94, respectively. All three scores categorized Edwina as *average* and indicated that she was predominantly empirical in her views on knowledge construction and development. Based on her self-reported empirical epistemological preference, Edwina indicated a tendency to apply perceptual cognitive processing and to justify knowledge claims via observational criteria (Royce & Mos, 1980).

### *Think-Aloud Sessions*

*Session 1.* Edwina began TA1 by spending 27 seconds *reading* part (a) of the application of differentiation problem and spending 1 min 58 sec engaged in *analysis* of the problem space. Unfortunately, two issues emerged during her analysis that plagued

her future work on the problem. First, Edwina exhibited conceptual confusion by trying to tie the first derivative of  $f$  to the roots of  $f$ . When questioned later during her RRI1, Edwina was unable to identify the meaning of the root of a function. Second, Edwina stated, “This is weird because I am working mostly with variables and constants,” indicating confusion with the arbitrary constants  $a$ ,  $b$ ,  $c$ , and  $d$ . Her interviews provided confirmation of this assertion and will be discussed further below. So, despite making some progress with the conditions, Edwina entered part (a) with a poor *definition of the task*, due in large part to her misconceptions.

After this analysis of part (a), Edwina spent 17 seconds reading part (b) of the application of differentiation problem. Then, the remaining 23 min 2 sec were spent engaging in a series of fruitless explorations. Edwina began her exploration by applying the heuristic *draw a picture* by graphing the function with her calculator. During the session, I was confused by how she produced a graph since  $f$  represented a family of curves. In her retrospective interview, my probes revealed that she had fixed  $a = 1$ ,  $b = 1$ ,  $c = 1$ , and  $d = 0$ . Her substitutions provided a means for viewing a sample graph and a special case of  $f$  to begin building a function. However, she had no plans to use her special case in this manner. Rather, she used this special case in place of  $f$  in an attempt to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ , not realizing the circular nature of her logic. When questioned as to the ramifications of substituting for  $a$ ,  $b$ ,  $c$ , and  $d$  during the retrospective report and interview, Edwina’s responses were indicative of continued confusion with the role of the arbitrary constants and the final goal of the problem. Further probing during the interview presented evidence of a *unique* belief regarding problem solutions as described in Robert’s narrative above and as indicated by the following statement:

I knew I had to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ . I just didn't know how so I just took circular routes, doing stuff I thought I knew how to do. I think that's it. I'm just really bad when, it sounds bad, but the less information, when you do have more constants and more coefficients that aren't given confuses me—that's my one weakness.

During her exploration, Edwina showed some signs of *forethought*, as her actions produced five codes for planning. Her plans were generally *empirical* in nature and did not result in significant progress toward the overall goal of the problem. Early in the exploration Edwina lost sight of the goal of the problem and made a plan early to solve for  $x$ , seemingly to find the locations of the horizontal tangents and the root. Upon solving for  $x$  using the first derivative of her special case for  $f$ , Edwina continued to show signs of conceptual confusion, particularly with respect to the role of the first derivative:

$x$  equals negative two thirds, but this can't be right because I'm solving for tangents. And I just solved for the slope. But the derivative is the tangent line, but not the horizontal tangent, so this doesn't make sense.

Other examples of plans included constructing sign lines for both derivatives, writing equations of tangent lines, and substituting the value of  $x$  obtained above into the original function. All plans were indicative of both *empirical* and *procedural* beliefs in problem solving.

Evidence of actions in the *performance control* phase of SRL processing included six instances of local assessments of strategy use and two instances of global assessments of goal state. Each of her assessments focused on surface qualities of current strategies and general statements of an inability to move forward, producing no information to alter

current strategies or transition to other ideas. For example, she stated, “I graphed the second derivative and that didn’t do anything for me because it just went straight to the origin as a line. It has nothing to do with concavity right here.” Her linear graph of the second derivative had *everything* to do with the concavity of  $f$  and analysis of the graph’s qualities leads directly to a solution for part (b). However, her conceptual confusions mentioned above stifled her ability to monitor her current state, assess the graph, and produce transitional metacognitive feedback necessary for productive action during the *performance control* phase.

Edwina’s session ended with her giving up on the problem after 26 min 14 sec, rather than completing the thirty-minute session. Just before she stopped working, Edwina made a statement that shed some light on her conceptual confusions: “I’m not used to finding  $a$ ,  $b$ ,  $c$ , and  $d$ . This is probably really easy if I saw how to do it once.” Her statement was indicative of a *procedural* belief in mathematical problem solving, which is in direct contradiction to a *conceptual* stance. This result is consistent with her *low* score on the Belief 3 scale of the IMBS, which measures the degree to which students feel conceptual knowledge is important to mathematics. Further evidence and discussion of this belief will be presented below.

*Interim between sessions.* Despite stopping before the session ended, the researcher encouraged Edwina to take the application of differentiation problem home and continue working on or thinking about it during the interim between sessions, which consisted of three evenings since she was in school during the day. By a stroke of luck, a calculus teacher was visiting next door to Edwina. Demonstrating the *help-seeking* qualities of SRL, Edwina asked this teacher for help and, thus, was able to solve part (a)

of the problem, providing the function  $f(x) = x^3 + 2x^2 + 10$ . Unfortunately, she was only able to partially explain and justify the solution path. Additionally, Edwina stated that part (b) was not possible but was unable to provide any justification for her answer.

*Session 2.* Satisfied with her progress on the application of differentiation problem, Edwina spent all of her time during TA2 working on the application of integration problem. She spent the first 52 seconds of the session *reading* part (a) of the problem, noting the conditions and identifying the goal. Then, she spent the next 5 min 5 sec engaged in *analysis* of the graphical properties of the function and the accompanying shaded regions (see Figure 4). During this period she used her graphing calculator to view two special cases such that  $m = 1$  and  $m = 5$  to determine the effects that  $m$  may have on the graph. She also recycled the goal in working memory, indicating that she had not lost sight of the desired end state. So, Edwina entered the remainder of the problem session with an accurate and adequate *definition of the task* for part (a).

Upon properly defining the problem space for (a), Edwina spent 19 seconds *reading* part (b), noting the conditions and identifying the goal state. Then, she spent 1 min 51 sec engaged in *analysis* similar to that in part (a), which resulted in an erroneous sketch of the shaded region since she bounded the region by the  $x$ -axis, instead of the  $y$ -axis. Additionally, Edwina stated, “This is the same kind of problem,” which implied that she failed to note the significant alteration inherent in the requirement of finding a horizontal line, rather than a vertical line. Thus, Edwina did not enter the problem space with a sufficient *definition of the task* for part (b).

Edwina made virtually no progress on either part after this point. She spent her remaining 17 min 13 sec in a series of fruitless explorations. Her first exploratory action

was prefaced with the following statement: “Maybe I’ll take the derivative, just for the heck of it, and see what it does.” This statement is indicative of the *empirical* mindset that pervaded the remainder of the session. Finding no application for the derivative, Edwina examined her graphs again and discovered a problem with the shaded region for part (b)—it was boundless. Unable to resolve this issue, Edwina developed a plan to find the area of the enclosed region and then divide it in half. This sole instance of activity in the *forethought* phase showed that Edwina had a general notion of the demands of the problem. However, Edwina was unable to recall the difference between the integral-defined formulas for area and volume:

OK, we’ll try that, we will solve for area right now. And, area is different from volume; I’ll have to remember that. Volume is right over left and bottom over the top.

Edwina had truly confused the calculus of area and volume. She partially recalled the mnemonics *right minus left* and *top minus bottom*, which many novice calculus students apply when trying to determine the order of subtraction for *area* problems, not volume problems as indicated by Edwina.

Unable to develop the integral expression for area, Edwina tried to approximate the area. Her first attempt involved using the Pythagorean Theorem to estimate the area of the region, which she assumed to be approximately triangular. Unfortunately, she used a height value, or  $y$ -value of 4, which should have been 1 and obtained a large value for the hypotenuse. Unable to rectify this, she turned to part (b) and discovered that the unbounded nature of her shaded region was due to an error in her sketch. Based on this local assessment of her current strategy, she corrected the problem and thus, obtained an

accurate sketch of the shaded region (see Figure 5). Deciding the region was too small to estimate, Edwina made one last attempt to recall calculus-based area. When this was unsuccessful, Edwina decided to stop working after 25 min 20 sec instead of waiting for the thirty-minute session to expire. Over the duration of the session, only six instances of *self-monitoring* were coded and led to no productive transitions or alterations in strategy, providing little evidence of the *performance control* phase.

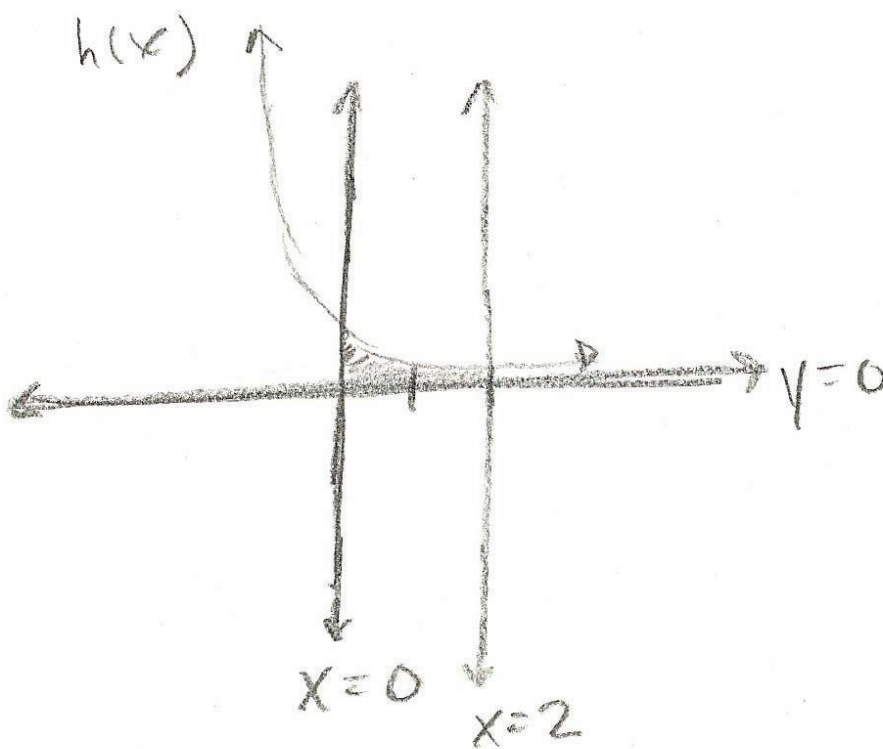


Figure 4. Edwina's Sketch for Part (a) of the Application of Integration Problem-Solving Task.



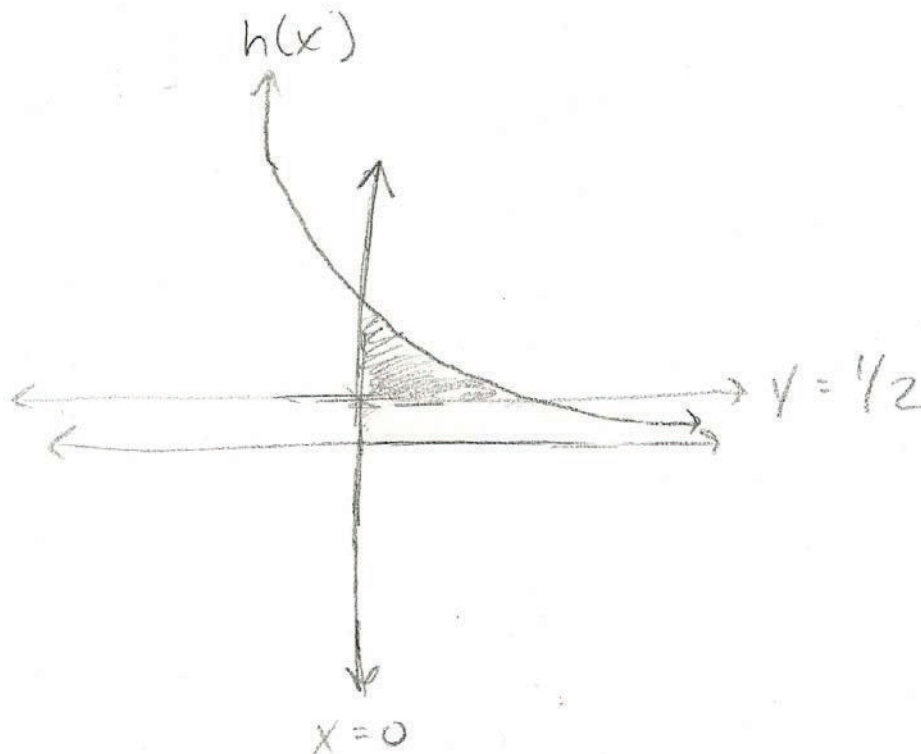


Figure 5. Edwina's Corrected Sketch for Part (b) of the Application of Integration Problem-Solving Task.

At one point during her exploration, Edwina recalled the goal of the problem and made the following statement, indicating an underlying reliance on procedures: “I don’t remember *how to solve* [emphasis added] for a vertical line and I don’t know how we’re going to find out where it divides it exactly in half.” Edwina expected a procedural method for finding equations of vertical lines. For this problem, there is no such algorithmic procedure; one must conceptualize the meaning of enclosed area and the role of bounds to develop a means for finding the vertical line. This interpretation of Edwina’s *procedural* belief in mathematical problem solving is supported by her *low* score on the Belief 3 scale of the IMBS. Additional support for this finding is her response to a query concerning barriers to problem solving from her RRI2: “Having memorized the equations

in the past and not really knowing how to apply them, so therefore you forget the equations and it's not good because you need to remember them." Edwina also cited lack of access to mathematical resources as an issue that prevented her from considering multiple problem-solving strategies.

Edwina shed some light onto the source of her procedural belief and lack of mathematical problem-solving resources:

My experience in the past—and not to hate on the teachers I've had—but they've never really encouraged you to think. It's all been cookie-cutter questions, even with word problems. I remember my Algebra One teacher, she had a little trick for everything. So, of course, I don't remember the trick now and I don't remember why I was doing it. So, I felt like there were a lot of short cuts, and I was never really taught why you were using it. So, I memorized everything, which is what I've been doing ever since.

So, Edwina attributed her beliefs in *procedural* mathematical problem-solving and her lack of mathematical resources to a consistent focus on memorization and cookie-cutter mathematical problems presented in past courses. In a manner similar to Robert, Edwina's responses during the MCI indicated a disconnect between desired beliefs and manifested beliefs. Her comments to my query regarding the importance of procedural or conceptual understanding to problem solving were as follows:

Understanding the underlying concepts because you can't get very far without them, like I was unable to do. I didn't understand all the concepts, so I wasn't able to solve any parts of the problem. Even if I understood part of it, I couldn't get very far.

So, Edwina realized that her lack of conceptual understanding was her demise, but her prior experiences in mathematics courses appears to have resulted in a manifestation of *procedural* dependence while solving mathematical problems.

Finally, Edwina indicated having difficulty with the arbitrary constant  $m$ , which was consistent with her issues with  $a$ ,  $b$ ,  $c$ , and  $d$  during the first session. The inability to reconcile these arbitrary constants may be attributed to a *unique* belief regarding problem solutions. When asked to comment on her confusions and their possible source, Edwina made the following statement:

So I wasn't really sure *how to* [emphasis added] solve the problem when you're just given constants because that requires you to *think conceptually* [emphasis added] and think, well what's going on in this equation, rather than oh, let's see what's going on once I plug these numbers in. So that's where I had the problem, it goes back to the whole conceptual knowledge part, I think.

True to her deeply ingrained belief in procedural problem solving, Edwina ironically used the words *how to* in the same sentence that ended with *think conceptually*. Edwina's comments also introduce a possible link between the non-availing *unique* belief regarding problem solutions and the non-availing *procedural* belief in problem solving.

*Think-aloud problem-solving session performance.* For part (a) of the application of differentiation problem, Edwina made little progress during her first session. Despite having the help of a calculus teacher during the interim between sessions, Edwina was unable to adequately justify each step leading to her answer for part (a). Thus, based on Schoenfeld's (1982) scoring guidelines, Edwina was awarded 16 points out of 20. For part (b) of the application of differentiation problem, Edwina made significant progress

by calculating the second derivative but could not recall its connection to concavity. Then, during her interim interview, she stated, “I wasn’t sure how it could be concave up from negative infinity to infinity, so I don’t think it’s possible.” Her uncertainty was still apparent during RRI2, as she stated that the problem confused her and she was uncertain of her answer. Based on the above, Edwina was determined to have made *some* progress on part (b) and was awarded 10 points out of 20. In total, Edwina scored 26 points out of 40 on the application of differentiation problem.

For the application of integration problem, Edwina made very little progress on both parts (a) and (b). By the end of TA2, Edwina had made accurate sketches of the regions for both parts, but made no progress toward connecting conditions to goals. Thus, Edwina received 1 point for each part of the application of integration problem, resulting in a total score of 2 out of 20 points. Overall, Edwina scored 28 out of 80 points for the think-aloud sessions.

#### *AP Calculus Exam Preparation*

During each MCI, participants were asked to comment on the differences in the tasks given during the AP Calculus exam preparation sessions held during class and the think-aloud sessions held after school. With respect to the think-aloud problems, Edwina stated, “The after school problems were definitely more abstract and required me to think all on my own.” This description is consistent with categorizing the think-aloud problems as “doing mathematics tasks” (Stein et al., 2000). However, Edwina’s comments regarding the AP exam practice problems were too vague to assign a classification.

*Classroom observation 1.* Edwina was observed while working on question 6 from AP Calculus AB 2004 (Form B) exam with her partners, Bob and Ken (see

Appendix E). The question provided a graph of the general power function  $y = x^n$  and its tangent at the point  $(1, 1)$  and posed three problems, (a)–(c). For part (a), students were asked to calculate the integral of the function  $y = x^n$  with respect to  $x$  from zero to one. Bob, who appeared to be the *de facto* leader, integrated the expression without problem, but Ken and Edwina seemed confused. Bob and Ken discussed the problem, resolved the issue of uncertainty as to whether bounds are substituted in for  $n$  or  $x$ , and both were satisfied with their work. Edwina assented to their plan and *followed along* procedurally. For part (a), all three group members earned the full two points.

For part (b), students were required to calculate the area under the tangent line. After lengthy discussions, Bob and Ken successfully determined a method for finding the slope of the tangent and chose to integrate with respect to  $y$  to find the area. Edwina was unable to follow along, but assented and carried out the established plan. The method for finding the slope of the tangent was successful, but integrating with respect to  $y$  proved too difficult. Finding the area using geometric formulae or integrating with respect to  $x$  would have been simpler. Due to a minor arithmetic error, Bob and Ken earned two points out of three. Unfortunately, despite group verification, Edwina's answer contained the arithmetic error and the integral was not properly set up. So, she only received one point out of three for part (b).

Finally, part (c) required students to find the area contained between the general power function,  $y = x^n$ , and the tangent, then determine the value of  $n$  that maximizes the area. Upon recognizing the need for optimization, Bob provided a general plan for solving the problem but could not adapt his plan to the current context. Ken and Edwina could not follow his logic and were unable to contribute. None of the group members

made any significant progress on part (c), so all three received zero points out of four. In sum, Edwina received 3 points and Bob and Ken received 4 points, resulting in a mean group score of 3.67 out of 9 possible points. Despite the low score, the group outperformed the class, which averaged 2.72 points. This was the lowest average score for the class of all twelve problems.

*Classroom observation 2.* For her second observation, Edwina was observed working on question 3 from the AP Calculus AB 2008 (Form B) exam with her group members (see Appendix E). The question provided a table of water depth values at various distances from the shore of a river and a function describing the river's velocity. Based on the given information, four problems, (a)–(d), were posed. For part (a), students were required to use the trapezoidal rule to approximate the cross-sectional area of the river. After a brief conversation, all three students appeared to understand the concept and began working. Unfortunately, despite group verification, only Bob and Ken received the one point for part (a). Edwina had the correct answer, but her Trapezoidal rule setup was incorrect and did not lead to her answer. So, she received zero points for part (a). The reason for this and other instances of point discrepancies between Edwina's and her group's scores may be explained by her lack of questioning, as she indicated during her MCI, stating, "I didn't want to keep asking questions. I felt bad for interrupting their thinking process." So, group dynamics definitely affected the number of group monitoring codes recorded for each group.

For part (b), volumetric flow was defined as the product of cross-sectional area and velocity. Students were required to calculate the average volumetric flow over a given time period. Bob immediately recognized that average value was needed and

conceptually explained his reasoning to Ken. Edwina assented to their plan and all three began working. Edwina had calculator problems and upon asking for help, Ken appeared to rectify the issue. Unfortunately, Edwina's final answer was still inaccurate, so she received two points out of three. Bob also received two points out of three for having an inaccurate setup. Finally, Ken received the full three points for part (b).

For part (c), a function was proposed to model the depth of the river in lieu of the table of values given. Students were required to find the cross-sectional area using the newly-defined function for depth. After initial confusion with wording, Bob yet again developed the group's solution path, suggesting that simple integration was sufficient. Both Ken and Edwina assented and all three began working. Just as in part (b), Edwina had problems inputting the necessary commands into her calculator. She again relied on her partners to monitor her problems, as Bob was called upon to assess her calculator issues. In the end, all three students earned the full two points for part (c).

Finally, part (d) required students to decide if water must be diverted based upon a constraint given for the volumetric flow. Bob developed a plan involving the use of results from (c) and information from (b) to generate a solution. Edwina did not follow his plan and upon request, Bob clarified for her. Once satisfied, all three worked the problem and earned the full three points for part (d). Throughout the observation, Edwina remained a passive observer and assented to plans developed by Bob and Ken. In sum, Edwina received 7 points, Bob received 8 points, and Ken received 9 points, resulting in a mean group score of 8 out of 9 possible points. Once again, the group outperformed the class, which averaged 6.43 points.

*AP Calculus exam preparation performance.* Since Ken missed two AP exam practice sessions, all group data contain only the ten sessions for which the group was intact. Group statistics for the ten problems were as follows: Edwina averaged 4.0 points, Bob averaged 5.8 points, and Ken averaged 5.2 points. Thus, the group's overall average score for the 10 problems was 5.0 points, resulting in a difference between Edwina's average score and her group's average score of  $-1.0$ . Edwina's overall average performance on all 12 in-class AP Calculus AB practice problems was 4.17 out of 9 possible points. The overall class average for all 12 problems was 4.92, resulting in a difference between Edwina's average score and the overall class' average score of  $-0.75$  points.

#### *Julia's Narrative*

##### *Mathematical Achievement and Questionnaire Data*

With an AP Calculus AB grade of 85, Julia was very close to the class average ( $M = 86.8$ ) and was categorized as a *Category III* calculus achiever. Her inclusion in the study was a result of continuing to build the *quota* sample (Miles & Huberman, 1994) and a preponderance of *high* scale categorizations from the questionnaires. For the IMBS, Julia was categorized as *average* for every scale except Belief 5, for which she was categorized as *high* with a maximum score of 30. Thus, Julia self-reported a strong, availing belief that students can increase mathematical proficiency via effort (Kloosterman & Stage, 1992).

For the MSLQ, Julia was categorized as *average* on four scales and *high* on three scales. For the Motivation scales, Julia's only *high* categorization was for the Extrinsic Goal Orientation subscale. Additionally, like Robert and Edwina, Julia scored higher on



Extrinsic Goal Orientation (6.5) than on Intrinsic Goal Orientation (5.5). Thus, based on self-report data, Julia tends to participate in tasks for reasons such as grades or other performance indicators instead of being driven by the desire to master content (Pintrich et al., 1991). For the Learning Strategies scales, Julia was categorized as *high* for Peer Learning and Help Seeking. Thus, Julia self-reported the ability to manage resources and use peers, teachers, and other sources to facilitate her learning and achievement (Pintrich et al., 1991).

Finally, Julia was categorized as *high* for both rational and empirical epistemological persuasions from the PEP. She was identified as predominantly empirical, producing the highest score for this scale (125) in the class. However, with a *high* categorization for the Rational scale as well, Julia self-reported the tendency to apply cognitive processing that may exhibit a mixture of perception and analysis, and justifications that may be based on a combination of observations and logic (Royce & Mos, 1980).

### *Think-Aloud Sessions*

*Session 1.* Julia began TA1 by spending 47 seconds *reading* both parts of the application of differentiation problem. Then, she engaged in *analysis* for 1 min 45 sec, establishing a relationship between the conditions and the goal of part (a) of the application of differentiation problem by setting up two equations—one to solve for the roots of the function  $f$  and one to solve for the roots of the derivative of  $f$ . So, Julia appeared to enter part (a) of the application of differentiation problem with a well established *definition of the task*. However, Julia did not recognize the arbitrary nature of the problem and subsequently, set off on a fruitless exploration of the problem space that

lasted 9 min 45 sec. This exploratory phase of her problem solving effort was peppered with four instances of planning, indicative of processing in the *forethought* phase of SRL. However, the planning instances were not precluded by an *assessment of the current state* of the problem and thus, promoted further distancing from the conditions of the problem and the goal state (Schoenfeld, 1985). For example, one of her plans was to set the function equal to the derivative and try to use the equation to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ . Clearly, this equation has little, if any, mathematical meaning and indicates that Julia was simply trying to accomplish something.

Particularly lacking from Julia's exploration were instances of monitoring. In fact, only three instances of locally monitoring strategy use and no instances of globally monitoring goal state were coded. Additionally, all four instances focused on deficiencies in the current state of her efforts, as follows: "That's pointless;" and "That one didn't take out the  $a$ ." The instances of monitoring also yielded no fruitful transitions or modifications to the strategies being employed.

Julia spent 5 min 9 sec alternately re-reading and analyzing parts (a) and (b) of the problem. During this period, 50 seconds were spent re-reading and the remaining 4 min 19 sec were spent in analysis. Julia employed some rather productive heuristics during this period of analysis. She drew a few graphs and began considering the simpler case of the function  $y = x^3$ . During this analysis, she stated that a cubic will be concave up if the value of  $a$  is positive. This statement baffled me until she explained, during RRI2 that she was confusing the concept of a quadratic graph *opening up* with a general polynomial being *concave up*. Two very enlightening, yet simultaneously frustrating, statements were made during this phase. The first, which regards part (b), is as follows: "It couldn't be

cubic—if it were going to be concave up the whole time, so  $a$  would have to be zero, but it can't be. (Exasperated) So, I don't know.” The problem clearly contained the wording *if possible*, implying that no solution was a possibility. Julia had developed a sufficient and appropriate argument that part (b) is not possible, but doubted her assertion. The second statement, which regards part (a), is as follows: “If it has two horizontal tangents and one root, that means one horizontal tangent is below the  $x$ -axis and one's above.” Clearly this is a false statement, but this direction would have been fruitful to explore; however, Julia left it unchecked and moved to other explorations.

Despite this promising analysis of the problem space, Julia was unable to make the necessary critical connections, and instead reverted to her substitution methods and spent the remaining 15 min 12 sec on this fruitless exploration. This phase of her problem solving endeavor began with the statement, “OK, let's try. . . (loud sigh) I have to solve for something.” Then, despite having just developed a rational argument for part (b) having no solution, Julia attempted to solve for  $a$ ,  $b$ ,  $c$ , and  $d$  by setting the second derivative equal to zero. This led her to the equation  $6ax + 2b = 0$  and having just stated that  $a \neq 0$ , she was once again primed to show that part (b) was not possible. Instead, she spent the remainder of the session trying to find solutions for  $a$ ,  $b$ ,  $c$ , and  $d$  for the roots of the second derivative and for the conditions she had developed from part (a).

During this exploration, the extent of Julia's actions in the *forethought* phase encompassed two instances of planning. Each plan resulted in a *transition* to a new problem-solving strategy, but Julia never considered the consequences of the transitions in terms of progression toward the goal state of the problem. Only two instances of monitoring were coded, indicating little evidence of *performance control* during this

exploration. Additionally, all three instances of monitoring were negative assessments of her current strategy, as the following example indicates: “ $c$  equals  $x$  over negative three  $ax$  plus  $2x$ —that’s not helping.” As time was called, Julia was continuing to randomly try substitution, factoring, and other strategies to find the solution. Her work had led her to the equation  $x^2 = -1$ , which unfortunately has no real solutions.

Evidence of three epistemological beliefs was noted during TA1. First, Julia’s verbalizations during TA1 indicated conceptual understanding of roots, tangents, and concavity. However, Julia’s actions during TA1 indicated a predominantly *empirical* stance on mathematical problem solving. Support for this assertion includes her Empirical score on the PEP, which was highest in the class, and her own words during RRI1. Wording like “I tried” and “I thought I would” can be found throughout her retrospective report, with little, or no, discussion of the conceptual support for her strategies.

Second, Julia’s actions were representative of a *procedural*, rather than conceptual view of mathematical problem solving. Overt evidence of this belief includes her development of the implausibility of part (b), only to attempt to solve for  $a$ ,  $b$ ,  $c$ , and  $d$  anyway. Julia discussed this event during her RRI2:

I didn’t see how it could always be concave up, because that would make it a quadratic and if  $a$  can’t equal zero, then it has to be cubic, no matter what. So, I didn’t really think that was possible, but I don’t know. And, um, but then, I knew that to solve for the concave up intervals, you could do the second derivative to find those intervals.

So, despite having a conceptual argument as a solution for (b), Julia felt the need to attempt to build a procedural solution. This finding contradicts her IMBS score for Belief 3, which indicated that Julia did *not* feel that all problems have an algorithmic procedure.

Finally, Julia's actions provided evidence of a belief in knowledge as *straightforward*, as opposed to interrelated. Evidence of this belief was elicited every time Julia engaged in a strategy that disregarded the conceptual context established by the conditions and goals of the problem. A specific example is when Julia developed a plan early in the session to set the derivative function equal to the original cubic function. This plan exhibits a disregard for conceptual aspects of the problem since any solutions derived from the established equation would have been devoid of conceptual importance for the problem.

*Interim between sessions.* Of the six participants, Julia accomplished the most during the interim between sessions, which amounted to two evenings since she was in school during the day. During an interview about her efforts in the interim, Julia stated that she had worked for a couple of hours on the problem at home. The extra time allowed her to make a very important realization about the direction of her TA1 explorations: "I realized that, like, the main goal was to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ . I was getting further and further away from that." This assessment led her to more productive exploration and ultimately, to a successful solution. She used the Internet to look up the definition of horizontal tangent and access an interactive grapher. By looking up the definition of horizontal tangent, she was able to correct a misconception—during the first session, she was confusing the term *horizontal tangent* with *horizontal asymptote*. Julia's confusion explains her conjecture from session one that a cubic with one root should have

one horizontal tangent below and one above the  $x$ -axis. Then, equipped with a more appropriate image of her goal state, she used an online interactive grapher to explore the effects of different values for  $a$ ,  $b$ ,  $c$ , and  $d$  until she had built a cubic function that met all conditions of the problem. Her final answer was  $f(x) = 3x^3 - 2x^2 - x + 2$ . She also provided the values of the roots and the relative extrema.

She then explained that with her improved *definition of the task*, the fact that part (b) had no solution became obvious. Despite some rather awkward statements for her justification, Julia's explanation indicated conceptual understanding of the reasoning for the *not possible* solution for part (b). Her main justifications were graphical reasoning and the fact that  $a \neq 0$  and thus, reduce  $f$  to a quadratic function.

Julia's discussion of her work during the interim provided additional information regarding her epistemological beliefs and SRL processing. To begin, her work on part (a) revealed that Julia can successfully employ both *rational* and *empirical* problem-solving strategies. She established an accurate *definition of the task* that connected conditions to goals, which is indicative of a *rational* approach to problem solving. Then, armed with her logically-based understanding of the problem, she applied *empirical* means to test her assertions. Additionally, she attributed time and access to mathematical resources as the most important contributors to her success. By spending a couple of hours of her spare time working the problem and attributing time to successful mathematical problem-solving, Julia certainly demonstrated a belief that difficult mathematical problems may require a *long duration* of time to solve. Her work during the interim between sessions also affirmed her *high* categorization for Belief 5 on the IMBS, which measures the degree to which an individual attributes increases in mathematical proficiency to effort

(Kloosterman & Stage, 1992). Julia also made a revealing statement regarding the relationship between mathematical resources and SRL processing: “Yeah, I realized I worked on one path that I knew was not working for me, but I just couldn’t think of anything else to do.” In essence, students cannot control strategy use unless they can access alternative strategies.

*Session 2.* With the application of differentiation problem solved to her satisfaction over the interim, Julia moved on to the application of integration problem for TA2. She began the session by *reading* part (a) of the problem for 33 seconds. Then, she spent 2 min 33 sec engaged in *analysis* of the problem conditions and goals. Most of this time was spent applying an appropriate heuristic—sketching a graph with a shaded region that defined the area in question. She then spent 15 seconds re-reading the problem and stated that she did not understand what  $A$  meant. The researcher interjected and 22 seconds were spent making sure that Julia understood that  $A$  simply represented the bounded area for both parts (a) and (b). Julia then spent 5 min 38 sec engaged in productive *exploration* of the problem space. With the aid of her graphing calculator, this phase began with an examination of the effects that various values of  $m$  had on the graph of the function  $h$ . Then, Julia decided to let  $m = 1$  and solve part (a) for this special case. In sum, the first 9 min 21 sec were spent developing an appropriate definition of the task, establishing goals for the purposes of analyzing and exploring the problem space, and transitioning to the next phase of her plan based on informed assessments and conceptual justification.

This rational-based demonstration of controlled problem-solving prowess came to an abrupt halt and Julia spent the next 14 min 16 sec on a lengthy *empirical* search for a

solution for her special case of part (a) for  $m = 1$ . Upon obtaining an accurate area for the enclosed region and dividing the area in half, Julia fixed the lower bound of her integral to zero and spent the remainder of her time substituting guesses in for an upper bound that yielded half of the original area. Although seven instances of monitoring were coded during this period, all were related to assessments of the validity of her guesses. So, from a purely frequency-based analysis of this portion of her session, it appeared that Julia was engaged in *performance control* processing but a more qualitative assessment revealed that she was merely controlling the surface features of a flawed plan. This phase ended with her solution of  $x = 0.5662036$ , which is fairly accurate for the special case  $m = 1$ , but the solution did not provide her with a means of generalizing a solution with respect to  $m$ .

Satisfied with her work, Julia decided to work on part (b) of the application of integration problem. She spent 21 seconds *reading* the problem and then 35 seconds engaged in *analysis*, consisting mainly of sketching the shaded region to be considered. Then, failing to note the conceptual significance of the goal of obtaining a *horizontal* line, as opposed to the vertical line for part (a), Julia procedurally mimicked her work from part (a) while working on part (b). Thus, the final 8 min 2 sec of her session were spent trying to find the equation of a horizontal line by testing  $x$ -values, which can only represent vertical lines. Additionally, her integral expression for the bounded region was not properly set up, so the area that she obtained was inaccurate. With respect to *forethought*, all goal-setting was done with little attention to her current problem-solving state. Additionally, this final phase of her session contained only two instances of monitoring and both were simply assessing the validity of a proposed upper bound. The



session ended with Julia in the middle of guessing an upper bound. Then, she stated, “OK, I didn’t like that. I know I was doing something wrong.” This statement is indicative of the *self-reflection* phase of SRL and although the statement is deficit-focused, the causal attribution was based on her strategy, as opposed to the more academically debilitating attributions to ability (Zimmerman, 2000).

Overall, Julia’s TA2 was coded as indicative of a belief in both *rational* and *empirical* approaches to mathematical problem-solving. Her work logically established an appropriate *definition of the task* for part (a) and thus, was coded as a *rational* belief in problem solving. However, the remainder of her session was deemed an *empirical* perusal of the problem space. These results are supported by her *high* categorizations for both the Rational and Empirical scales of the PEP.

Despite some glimpses of *conceptual* insight, Julia’s TA2 was coded as indicative of a predominantly *procedural* belief in mathematical problem solving. For example, her use of the exact same plan for part (b) that was used in (a), regardless of changes in problem conditions, provided a concrete example of this assertion. However, keeping with the trend set by Robert and Edwina, Julia reported an idealized *conceptual* belief in mathematical problem solving during her MCI. However, manifestations of a *procedural* belief proliferated in her work. Julia verbalized her idealized *conceptual* belief in her MCI, as follows:

I think it’s more important to have a conceptual knowledge of it even if you know the procedures, but if you have the conceptual knowledge, you can reinvent the procedures. Like, if you know what’s going on, you can still solve for it.

So, there appears to be a fundamental disconnect between Julia's desired problem-solving belief system and her practiced problem-solving belief system.

Additionally, Julia's confusion regarding  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $m$ , resulted in a coding of a predominantly *unique* belief regarding problem solutions. In fact, Julia made the following statement during RRI2 when asked what her most significant barrier was to success:

The  $m$  thing. The variables are the hardest part, I guess. I know math was easy until you get to letters and alphabets that are not supposed to be there. But, I just didn't see any way at all to *solve* for  $m$ . They didn't give you any information about  $m$  to be able to *solve* for it. [emphasis added]

Julia's expectation that a solution existed for  $m$  is indicative of a *unique* belief. When probed to comment further, Julia made a statement that hinted at the source her *unique* and *procedural* beliefs:

Usually, I'm used to directions giving you, like, here's the information you need to know about it, so then you use that to solve for it. And I'm sure if I was given that, then that would have made more sense, but critically thinking . . .

Julia made a similar statement during her MCI that blamed prior mathematics courses for her *procedural* belief:

Until calculus it didn't really matter because you would learn something, then you would take a test on it, and then you would just drop it. I mean, you would build upon certain basic knowledge and that's what I understood and that kind of thing. But then specific things you just learned for a test and then you can just drop it;

whereas in calculus, you build on everything you learn which I think has kind of helped me though, because you still have to recall back at things you've learned. So, Julia seemed to indicate that past experiences have given her an expectation of a *unique* solution for variables and sufficient information to develop a *procedure* to solve given problems. Julia cited the calculus as a course that seems to be improving her ability to work with abstraction. However, as the following quote from her RRI2 indicates, Julia's confusions continued to persist:

I think that's what messed me up in the beginning was that the first day I was thinking, OK, there's a specific answer for  $a$ ,  $b$ ,  $c$ , and  $d$ ; that's the only way it works. And then I think once I realized that there were an infinite number of answers, it's not as hard as I thought it was. But, I was surprised because you used to, just always, as you're solving for a variable and usually the variable has a given number to it. And that's why I wasn't sure if  $m$  is a variable and that's just supposed to stay like that and so it has different solutions or if it had a specific answer.

In this single quote, Julia seemed to go full circle from a *unique* to an *arbitrary* belief, then back to a *unique* belief.

*Think-aloud problem-solving session performance.* Julia received nearly full credit for the application of differentiation problem, but made little progress on the application of integration problem. During TA1, she made little progress on parts (a) and (b), but took full advantage of the interim between sessions, developing a full solution for both parts. For her efforts, she was awarded the full 20 points for part (a) of the application of differentiation problem. However, her justification for part (b) was rather

vague and incomplete. She rightly stated that as long as  $a$  is not zero, making the function quadratic, then a cubic can never be concave up on the continuum of real numbers. Then, a wild excursion into taking the cubed root of the function and other comments that danced around the issue constituted the remainder of her justification. Hence, Julia was awarded 17 points for solving part (b) with a partial justification for the answer and thus, received a combined score of 37 out of 40 points for the application of differentiation problem.

For part (a) of the application of integration problem, Julia showed early promise, but her work only produced an approximation for the special case of  $m = 1$ . So, Julia was awarded 8 out of 20 points for her efforts. The rationale for the score is that solving the area in terms of  $m$  and solving a simpler problem with  $m = 1$  is indicative of making *some progress*, but failure to use the simpler problem as a heuristic to move to the general case or make other connections to problem conditions hampered further solution efforts. Very little progress was made with respect to solving part (b) and the path chosen was procedurally identical to part (a), despite the change in goal state. Thus, Julia was awarded a score of 1 out of 20 for part (b). Her total score for the application of integration problem was 9 out of 40, resulting in a combined score of 46 out of 80 points for both problems.

#### *AP Calculus Exam Preparation*

During the MCI, all participants were asked to comment on the differences in the AP Calculus exam preparation problems and the think-aloud problems. Julia responded as follows:

The after-school sessions were definitely more conceptual and you had to kind of like, I had never seen this stuff before. I had a clue of the basic knowledge but I had no clue how to deal with it, and whereas the in-school group was all stuff we had been working on this year. So, I knew what to do and it was more procedural; you could find stuff from what you knew from the procedure you learned earlier in the year.

Julia's response provided further justification for categorizing the AP exam practice problems as "procedures with connections tasks" and the think-aloud problems as "doing mathematics tasks" (Stein et al., 2000, p. 16).

*Classroom observation 1.* Julia was observed working question 4 from the 2004 AP Calculus AB (Form B) exam with her partners, Ron and Lee (see Appendix E). This problem provided the graph of a derivative for a function  $f$  and then presented three questions, (a)–(c), about the function  $f$ . For part (a), the group had to determine  $x$ -coordinates for the points of inflection of  $f$  and provide an explanation for their solution. The group worked quickly and quietly, with very little noted interaction. This is unfortunate, because this was the only part for which group members had a discrepancy in score. Julia received the full two points, but her fellow group members only received one point because their explanations were insufficient. Had group monitoring occurred, a discussion may have helped to clear up Ron and Lee's confusions in developing their justifications.

For part (b), the group had to find the absolute extrema for  $f$  on a closed interval. This part really confused the group; in fact, Julia at one point claimed that it could not be solved and that the problem writers should have provided more information. Her

requirement of a unique, well-defined function,  $f$ , represented a further manifestation of a *unique* belief regarding problem solutions, as described above to explain Julia's trouble dealing with the arbitrary constants in the think-aloud problems. Fortunately for Julia, Ron and Lee engaged in a nice mathematical discussion about the problem and developed an accurate solution. However, the three group members could not develop a sufficient justification, so all got two points out of four.

Finally, part (c) defined a function  $g(x) = x f(x)$  and required the group to write the equation of a tangent line for  $g$  at a given point. For this part, the group demonstrated much more control. Julia's actions were coded three times for group monitoring during part (c). She was on the receiving end of the most notable monitoring action. The whole group had disclosed their progress and upon review, Ron reminded Julia and Lee that the product rule was required for calculating the derivative of  $g$ . Due in part to their monitoring, all group members received the full two points for part (c). In sum, Julia received 7 points and Ron and Lee received 6 points, resulting in a mean group score of 6.33 out of 9 possible points. The group outperformed the class, which averaged only 5.12 points.

*Classroom observation 2.* Julia's second classroom observation occurred while working on question 6 from the 2007 AP Calculus AB (Form B) exam with her group members (see Appendix E). This question provided students with two ordered pairs for a function,  $f$ , and then posed four problems, (a)–(d), regarding the function,  $f$ , and another function,  $g(x) = f(f(x))$ . For part (a), the group had to apply the Mean Value Theorem (MVT) to the function  $f$ . Although he could not recall the name of the MVT, Lee was able to adequately explain the concept to his group members. Then, silent work ensued,

with the only further discussion concerning whether continuity had to be explicitly shown. Despite a lack of group monitoring and verification, all three group members earned the two points for part (a).

Part (b) was similar to part (a), but the difficulty level increased because the MVT had to be applied to the derivative of  $g$ . Part (b) provided the only evidence of group monitoring during the observational period. Julia was the primary instigator, with two instances of group monitoring coded during this part of the problem. In the first instance, Julia asked Ron how he was working the problem. She received some assistance from him in understanding the similarities between parts (a) and (b). Soon after, however, she was asking for help again when she disclosed that she was struggling with the explanation. The group interactions resulted in a score of two points out of three for all members. Each student's work was accurate, but no one was able to develop an adequate explanation using the MVT.

Once satisfied with part (b), the group moved on to part (c), which dealt with concavity of  $f$  and  $g$ , and worked silently until the end of my observation. This was unfortunate for Julia and Lee since they did not earn the full two points. Both students' calculations of the second derivative omitted the product rule. (This is ironic because Ron had to remind them of the product rule during their first observation.) Had group monitoring occurred, this error would have been noted and Julia and Lee could have made corrections. Due to time constraints, the group was unable to work on part (d). In sum, Julia and Lee received 5 points and Ron received 6 points, resulting in a mean group score of 5.33 out of 9 possible points. Yet again, the group outperformed the class, which averaged only 3.14 points.

*AP Calculus exam preparation performance.* Julia was only present for ten of the twelve AP Calculus preparation sessions. Additionally, Lee was absent during another session. So, the group statistics below contain data from the nine sessions in which her group was intact and the overall class statistics contain data from the ten sessions for which she was present. Group statistics for the nine problems were as follows: Julia averaged 5.00 points, Ron averaged 5.33 points, and Lee averaged 5.67 points. Thus, the group's overall average score for the 9 problems was 5.33 points, resulting in a difference between Julia's average score and her group's average score of  $-0.33$  points. Julia's overall average performance on the 10 in-class AP Calculus AB practice problems was 5.30 points. The overall class average for the 10 problems that Julia completed was 4.70, resulting in a difference between Julia's average score and the overall class' average score of  $+0.60$  points.

#### *Olivia's Narrative*

##### *Mathematical Achievement and Questionnaire Data*

Olivia was categorized as a *Category II* calculus achiever, with an AP Calculus AB grade of 90. She was included in the study for her contribution to the *quota* sample (Miles & Huberman, 1994), for multiple *low* scale categorizations, and for being the only predominantly metaphorical participant. For the IMBS, Olivia was categorized as *average* for four scales and *low* for the other two. Her *low* categorizations were for the Belief 3 and Belief 6 scales. So, Olivia self-reported non-availing beliefs that conceptual understanding is not important as long as performance is maintained and that mathematics is not useful in daily life (Kloosterman & Stage, 1992). For the MSLQ, Olivia was categorized as *average* for all scales except Critical Thinking, for which she



received a *low* categorization. Thus, Olivia self-reported a lack of applying prior knowledge to new problems and a lack of critically evaluating newly presented content (Pintrich et al., 1991).

Finally, for the PEP, Olivia's scores for the Rational, Empirical, and Metaphorical were 105, 105, and 113, respectively. She was categorized as *average* for the Rational and Empirical scales and *high* for the Metaphorical. As mentioned above, Olivia was the only student in the class who was predominantly metaphorical. So, Olivia self-reported that she predominantly utilizes symbolism in her cognitive processing and justifies knowledge claims via insight and awareness (Royce & Mos, 1980).

#### *Think-Aloud Sessions*

*Session 1.* Olivia worked on the application of differentiation problem for the duration of TA1. She began the session by spending 33 seconds *reading* part (a) of the problem, noting conditions, and identifying the goal. Then, she spent 1 min 27 sec engaged in *analysis* of the problem space. During this time, she connected the term *root* to the zeroes of the function and the term *horizontal tangent* to the zeros of the first derivative. So, Olivia developed an adequate *definition of the task* for part (a) of the application of differentiation problem.

The remainder of her work with part (a) amounted to a 21 min 5 sec fruitless exploration of the problem space. Early in the exploration, she applied the heuristic *draw a picture* and sketched a graph of the function and its derivative. Not realizing the rich connections between her graphs and the problem goal, Olivia then began trying to work with the slope intercept and point-slope forms of linear equations, hoping to make some connections to the horizontal tangent lines. Shortly after this effort, Olivia turned to the

researcher and stated that she had tried everything. The researcher suggested that she could move to part (b). After the conversation, she began working with the second derivative, which seemed to indicate that she had moved on to part (b). However, Olivia's verbalizations indicated that she was still working on part (a). During her RRI1, Olivia clarified my confusions—she had noted that part (b) dealt with concavity and decided to use the second derivative to eliminate variables, in this case  $c$ . This decision to use the second derivative without establishing conceptual connections to part (a), was indicative of Olivia's belief that mathematical problems can be solved by *procedural* means. This finding is supported by her *low* rating on Belief 3 of the IMBS and her work that followed.

Upon deciding to use the second derivative, Olivia applied a system of equations approach that produced no promising leads for solving for  $a$ ,  $b$ ,  $c$ , and  $d$ . During this period, Olivia set no overt goals and she only logged three instances of *monitoring*. In each case, monitoring involved local assessments of her strategy use and indicated that she did not feel that her plan was working. Thus, little evidence of the *forethought* and *performance control* phases of SRL were present during her exploration.

Near the end of her exploratory excursion, Olivia turned and spoke to me twice, voicing her struggles and lamenting her inability to “get rid of  $d$ .” She also stated that she wished the problem merely asked for the horizontal tangents, not “all the letters.” Her statements further supported the interpretation that her problem-solving belief system was mainly *procedural*-based. Additionally, the confusion with *all the letters* indicated a *unique* belief regarding problem solutions. Additional triangulation came from her RRI1.

When asked about alternative strategies that she considered, Olivia made the following statement:

The only other thing I could do that I guess would make sense is to take a derivative and find the max and min and that would be the minimum and that would be the maximum [pointing to her sketch]. But when I tried that you just end up with variables and letters and it doesn't simplify anything.

In this statement, Olivia continued to hold strong to the belief that a *unique* solution could be attained via *procedural* means.

At 23 min 25 sec into the session, Olivia moved to part (b) of the application of differentiation problem. Oddly enough, there was no overt indication that she read the problem or noted the conditions or goal. She engaged in *analysis* lasting 4 min 11 sec and produced a complete solution. The solution was the result of the *heuristic* draw a picture, which in this case, was a sketch of the function and its first two derivatives on the same Cartesian plane. With this visual, Olivia easily determined that part (b) was not possible and developed a well-written justification for her claim. Thus, despite her lack of success on part (a), Olivia ended the session in a positive manner.

Overall, Olivia's actions during TA1 were coded as a predominantly *empirical* approach to problem solving. Her work with part (b) during the last several minutes of the session was purely *rational*, based on conceptual insight and logical reasoning and justification. However, the first 23 min 25 sec of the session were filled with brute force algebra and attempts to solve for  $a$ ,  $b$ ,  $c$ , and  $d$  without regard to context or conditions. In fact, Olivia confirmed this assessment of her efforts during the RRI1, stating the following:

Well, I don't think that's really legal because I had to make the derivatives equal 0. So, it's really not going to be accurate, you know. It was just something to do.

This statement indicates that Olivia had run out of options and was simply trying to do *something* since she was in the middle of the problem. Finally, Olivia's verbalizations were also indicative of her classification as predominantly *metaphorical* for the PEP. Her references to functions and expressions as *you* provided a subtle indication that a portion of her cognitive processing consisted of constructing "internally generated forms," which symbolized various aspects of the problem space (Royce & Mos, 1980, p. 6). Further and more direct evidence of her *metaphorical* beliefs is discussed below.

*Session 2.* Olivia did not work on part (a) of the application of differentiation problem during the interim between sessions, which only amounted to one evening since she was in school during the day. She had a lot of homework to complete, including studying for an AP Calculus BC power series quiz. Despite this fact, she felt that no more progress would be made and thus, worked on the application of integration problem for the duration of TA2.

Realizing that the application of integration problem involved the area bounded by multiple curves, Olivia requested the use of her colored pencils. Then, she spent 3 min 46 sec engaged in a combination of *reading* and *analyzing* part (a). Her *analysis* of the problem space involved sketching and color-coding each function to produce a sketch of the bounded region (see Figure 6). Upon completion of her sketch, Olivia engaged in an additional 4 min 15 sec of analysis, attempting to establish relationships between the problem conditions and goal. During this analysis, two instances of monitoring were coded, including the discovery that her graph representing  $h(x) = e^{-mx}$  was incorrect. In

terms of developing a *definition of the task*, Olivia's sketches provided a clear understanding of the conditions of the problem.

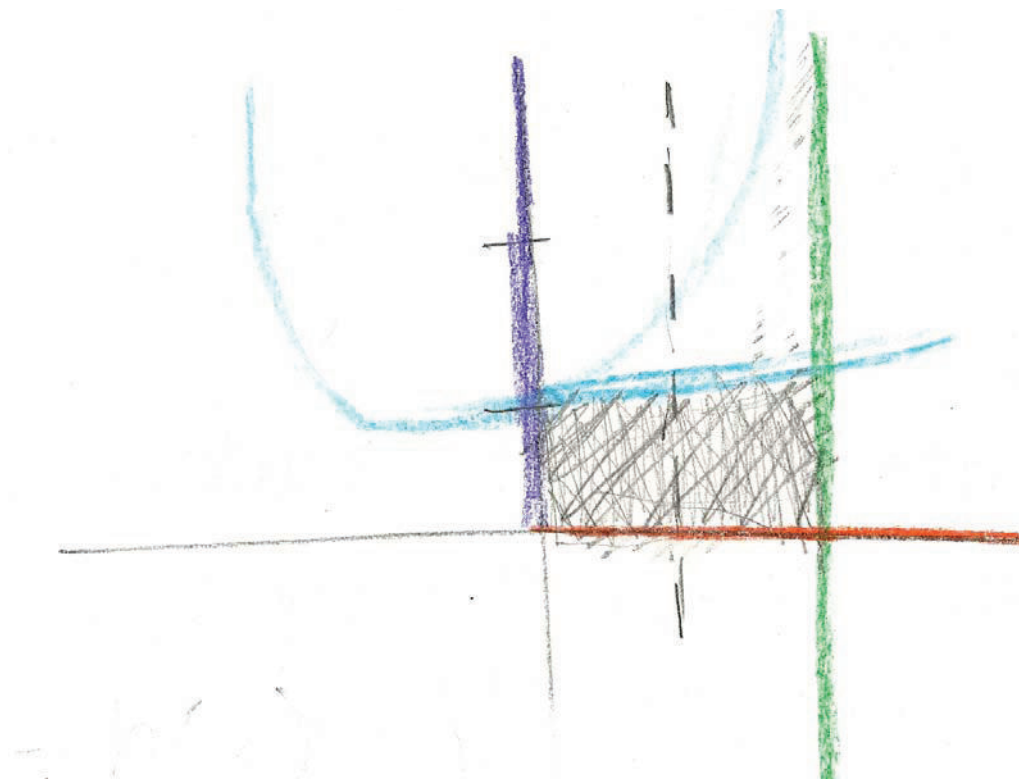


Figure 6. Olivia's Sketch for Part (a) of the Application of Integration Problem-Solving Task.

However, Olivia was unable to tie conditions to the overall goal of the problem, as the product of her analysis was an inferred plan (i.e., participants' actions rather than verbalizations provided evidence of a plan) to find the value of  $c$  via the equation

$$\int_0^2 [h(x)]^2 dx = \int_0^c [h(x)]^2 dx + \int_c^2 [h(x)]^2 dx.$$

Her problem-solving plan did not adequately connect problem conditions to desired goal since an integral with a squared integrand does not produce area and this expression would merely find an arbitrary value for  $c$ , rather than the value of  $c$  corresponding to half of the area. Thus, her actions continued to indicate an *empirical* approach to problem solving and an adherence to *procedural* means for navigating a problem space.

Olivia then spent 9 min 22 sec in fruitless exploration since she was implementing a flawed plan that took her farther from the desired goal state. During this exploration, no instances of monitoring were coded and the result of her work was the solution  $c = 1$ , which makes sense with her setup, but certainly does not produce a bound for half the area of  $A_1$ . Her lack of overt monitoring indicated that Olivia engaged very little in the *performance control* phase of SRL. At this point, which was 17 min 6 sec into the session, the researcher noted that Olivia did not have her calculator on her desk and reminded her that she could use one. She replied that she did not see the relevance of the calculator since it does not “do letters,” but she retrieved it anyway.

Upon returning with her calculator, Olivia moved to part (b) and engaged in *reading* and *analysis* similar to that conducted in part (a) for 3 min 19 sec. Thus, yet again, she had a color-coded sketch of the bounded region and entered the problem space with an adequate and appropriate representation of problem conditions for her *definition of the task* (see Figure 7). Then, Olivia engaged in a 3 min 14 sec productive exploration of the problem space. Her first efforts during this exploration involved integral expressions with respect to  $x$ , which were not conducive for finding equations of horizontal lines. Global monitoring of the goal of part (b) indicated to Olivia that

integration with respect to  $y$  would yield a more desirable integral equation. Thus, at the end of this exploratory period, Olivia developed a plan to solve for  $b$  using the equation

$$\int_{1/2}^b \left[ -\frac{\ln y}{m} \right]^2 dy = \int_b^1 \left[ -\frac{\ln y}{m} \right]^2 dy.$$

If the squares are taken from the integrands, this equation produces the correct solution for part (b). The presence of the squared integrands provided further indication of Olivia's *procedural* belief in problem solving, as she was confusing procedures for applications of integration involving area and volume. Thus, Olivia navigated more productively between the *forethought* and *performance control* phases of SRL processing for part (b) than part (a).

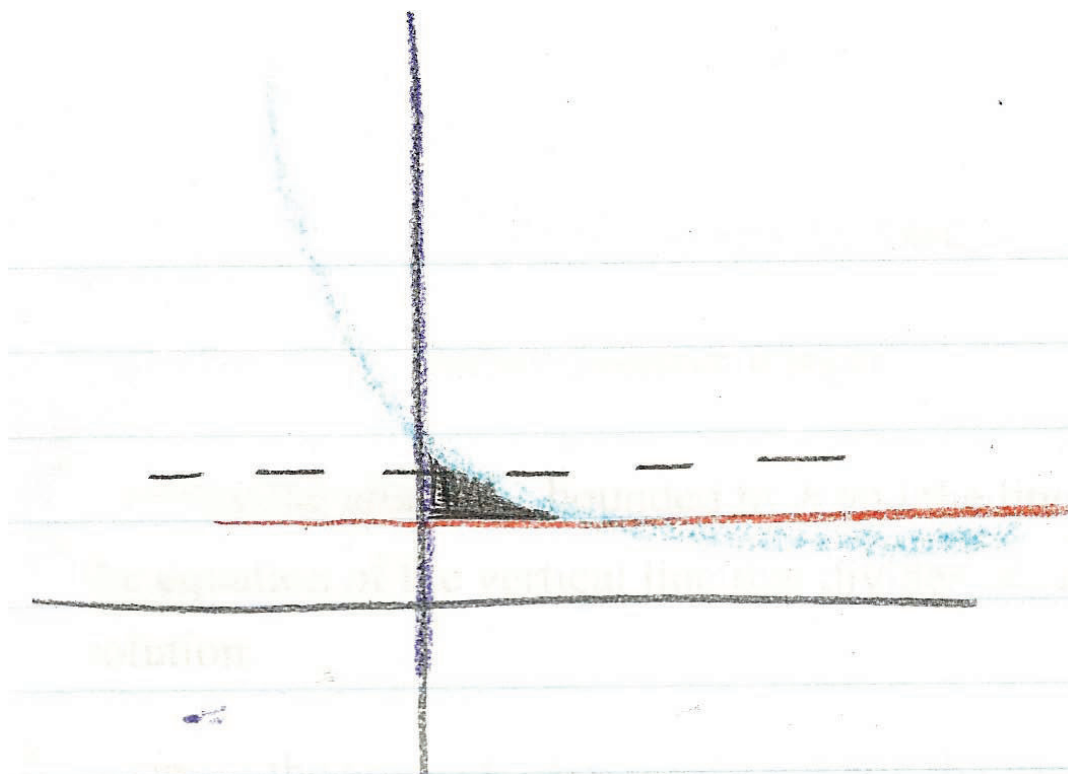
Troubled by the difficulty of the above integral equation, Olivia turned to the researcher after 25 min 54 sec of work and made the following statement:

If I ask you a question, can you tell me the answer? OK, I know that when you're revolving, you square; if you're not, you don't. OK.

Having apparently answered her own question, she turned and spent 1 min 47 sec correcting all of her erroneous work for part (a). The result of her corrections was a perfectly set-up equation for finding the solution to part (a):

$$\int_0^c h(x) dx = \int_c^2 h(x) dx.$$

Despite having a calculator with a built-in CAS, she spent her final 7 min 35 sec trying to solve the equation by hand. Her work contained a multitude of errors, her verbal report indicated no overt instances of monitoring, and she never achieved a final solution. Additionally, she never corrected her work for part (b), leaving the squared integrands in her submitted work. After 34 min 16 sec of work, the researcher asked Olivia if she needed more time to complete her current work, but she declined and stated, “I just need to stop.”



*Figure 7.* Olivia's Sketch for Part (b) of the Application of Integration Problem-Solving Task.



Overall, Olivia's navigation through the problem space for the application of integration problem was coded as both *rational* and *empirical*. Her early work was empirical in nature, resulting in unchecked strings of algebraic expressions that carried her farther from the goals of the problems. However, her empirical meanderings eventually led to her more *rational* latter work, which was indicative of a logical progression from problem conditions to goal state. As a possible explanation of the source of her capacity to work from both epistemological stances, Olivia provided the following statement during her RRI2:

I believe that the public education system teaches students to regurgitate and I have never been taught how to think by the school. Fortunate for me, I have pretty amazing parents; they've really instilled that in me. I was very fortunate to be born with some intelligence and a natural curiosity and a bit of common sense. Like I've said before, my dad pretty much taught me math—how to do it and you know if you don't know what to do draw a picture, write everything you know, just keep doing whatever can possibly relate to it until something clicks. So, that's pretty much what I did.

So, Olivia cited her parents as her sole source for developing thinking skills and her father as the source of her ability to adapt to a problem and utilize heuristic strategies. Finally, Olivia's TA2 provided further support for her *metaphorical* classification from the PEP. The main source of evidence came from her verbalizations while sketching. Rather than simply producing a sketch, Olivia assigned a color-code to each one, with statements like "You are blue, you are red," etc. Olivia's color assignments provided further evidence of the subtle cognitive activity of assigning symbols to structures, such

as the graphs produced by the functions in the application of integration task (Royce & Mos, 1980).

Olivia's work during TA2 provided further support that she held a *procedural* belief in mathematical problem solving. The majority of her work indicated that she was searching for a familiar process to map onto the problem, rather than considering the conceptual ramifications of problem conditions. Her continued use of a squared integrand could have been curtailed by examining the dimensionality of the integral, noting that solutions would be cubed and thus, not provide the desired area. Additionally, her first problem-solving plan applied a property of integration that produces solutions for interior bounds and was applied during the previous semester. Her use of this equation to solve part (a) provided further indication of a *procedural* mindset since no conceptual ties can be made between problem conditions and her plan.

Olivia's response to whether procedures or concepts are more important to problem solving during her MCI provides further insight as to possible sources of her adherence to a *procedural* belief in mathematical problem solving:

I have to learn the procedures first because I can't understand the concepts until I actually do the problem. And once I do a couple of them and let it sit in the back of my mind I can start to get the concepts. It helps me to know why I'm doing what I'm doing but I need to do whatever it is first before I understand why.

Since the think-aloud problems were novel with respect to Olivia's current calculus functioning level, she was unable to frame them conceptually and thus, reverted to her procedural approach. Additionally, Olivia's stance may be counterproductive to a constructivist philosophy of education, which implies that students actively construct

conceptual knowledge, as opposed to applying rote-memorized rules and procedures (NCTM, 2000).

During her RRI2 and MCI, Olivia's comments indicated conflicting interpretations with respect to her beliefs regarding problem solutions. For example, when asked how many solutions were possible for part (a) of the application of differentiation problem during her RRI2, Olivia stated, "At least one, maybe more. Part (a) was not very friendly to me, but there might be more than one cubic that would satisfy those conditions." Her comments indicated openness to an *arbitrary* belief regarding problem solutions and a sophisticated understanding of the role of the arbitrary constants  $a$ ,  $b$ ,  $c$ , and  $d$  in producing a family of cubic functions.

Then, later in the RRI2, Olivia made the following statement when asked a similar question about the application of integration problem:

Depending on what  $m$  is, well, it wouldn't have any effect on  $c$ . So, I guess in that particular problem, I would say that the variables were pretty much independent, obviously  $m$  and  $x$  are related, but  $c$  and  $m$ , I didn't necessarily see a relationship. Although her last statement is false,  $c$  and  $m$  are related, this statement provided further evidence that Olivia may hold a more *arbitrary* belief regarding problem solutions than indicated by her TA1.

During her MCI, Olivia seemed to revert to a more *unique* belief, with statements indicative of the certainty of a solution path:

Well, often with problems that have some symbolic representation for a numerical value, you can use algebra to isolate them and figure out what the letters or whatever actually stands for. And so I tried that because I am an algebraic person,

I like that, it works, it makes sense, and it didn't pan out for me in this, so I went to a graph and I tried to make some sense of it . . . It was difficult for me to have to deal with some conceptual, ideological letter that represented something else that I could not apply algebra to.

So, Olivia seemed to rely on algebra to obtain a solution and when the algebraic procedures failed to produce a *unique* solution, she was unable to adapt to the *arbitrary* nature of the problem space. It is possible that she is currently developing a more *arbitrary* belief regarding problem solutions, but further examination of such development is beyond the scope of the current study. Thus, the contradiction will be interpreted as a disconnect between beliefs and practice since Olivia's actions indicated a *unique* belief but some of her interview quotes indicated an *arbitrary* belief.

*Think-aloud problem-solving session performance.* Olivia made progress on both the application of differentiation and application of integration problems. For part (a) of the application of differentiation problem, she was unable to make significant progress beyond establishing that roots are the zeros of a function and horizontal tangents are related to the zeros of the derivative of a cubic function. Applying Schoenfeld's (1982) grading scheme, Olivia made *little progress* on part (a) and was awarded 5 points out of 20. For part (b) of the application of differentiation problem, Olivia provided a correct answer with an appropriate and sophisticated justification, thus earning the full 20 points. Overall, Olivia earned 25 out of 40 points for the application of differentiation problem.

For part (a) of the application of integration problem, Olivia eventually developed an equation whose solution would have yielded the correct answer. Unfortunately, she was unable to solve the equation and did not provide overt justification for her method.

Thus, her work indicated an *almost solution*, which merits a score between 11 and 15 points. Since her work failed to yield a final answer, she was awarded 12 out of 20 points. For part (b), Olivia was on the right track, but never corrected the squared integrands and did not make any progress beyond setting up the integral equation. So, Olivia made *some progress* and was awarded a score of 8 out of 20 points. In sum, Olivia earned 20 out of 40 points for the application of integration problem, resulting in a total of 45 out of 80 points for her two think-aloud problem-solving sessions.

#### *AP Calculus Exam Preparation*

During each MCI, participants were asked to comment on the differences in the group AP Calculus exam practice session problems and the individual think-aloud session problems. Olivia's comments were as follows:

The problems in the group sessions made more sense. There was more algebra and more just applying principles and using procedures that you had to find before, like you have this kind of problem this is what you need to do for it, versus the problems that I did alone were very conceptual. We hadn't really done anything like that before so it was very new versus the other ones we had done before were similar enough.

Olivia's comments provided further justification for the categorization of the AP exam practice problems as "procedures with connections tasks" and the think-aloud session problems as "doing mathematics tasks" (Stein et al., 2000, p. 16).

*Classroom observation 1.* Olivia was observed working on question 4 from the AP Calculus AB 2006 (Form B) exam with her group members, Tim and Jon (see Appendix E). This problem provided students with a piecewise-defined, graphical

representation for a function describing the rate calories are burned as a person uses an exercise machine and posed four problems, (a)–(d). For part (a), students were required to calculate the derivative of the function at a given point and indicate units of measure. All three group members applied the slope formula since the graph was linear at the given point. Unfortunately, no one realized that their answers did not include units. So, despite having an accurate numeric solution, all three group members received zero points since part (a) was only worth one point.

For part (b), students were required to find the time that the function obtained its greatest rate on the given interval and provide reasoning for their answer. Based on empirically analyzing the graph, Jon made a conjecture as to the answer but then was confused since he could not “derive a graph.” He then developed an answer that his group members agreed with but over-complicated the problem by trying to use the second derivative. To this point, Jon had dominated group planning and interaction, but Olivia interceded and both group members developed a plan for obtaining a solution. Upon completion, the group members verified their solutions, which were numerically accurate. However, this problem required a deep level of analysis and, despite checking final solutions, it was obvious that analytical work was not verified by the group. Thus, due to incomplete analyses, Tim and Jon received three points, and Olivia received two points out of four for part (b).

For part (c), students were required to find the total number of calories burned over a specified time interval. All three group members recognized that this part required combining integration and geometric area formulae to calculate total function change. Content with conceptual understanding, the group worked silently while calculating the

appropriate area. Upon completion, Olivia realized that she misread the problem and thought that the problem was asking for average value. Jon examined her work and helped her understand the conceptual aspects of the problem. As he was doing so, Tim and Jon realized that Olivia had a computational error in her work and were able to resolve it. At some point, Tim, who I thought had the correct answer, recorded the average value solution as his final answer. Thus, Olivia and Jon received the full two points and Tim received one point for part (c).

Part (d) introduced a vertical shift to the given function and required students to find the value of the vertical shift that would maintain a specified average value. Olivia was able to draw on her average value approach that was erroneous for part (c) to help Jon with his confusion. Once a plan was established, the group worked until time was called and thus, had no means for verifying solutions. Thus, Olivia received one point out of two for a perfect setup but no solution, Jon received the full two points for his setup with correct solution, and Tim received one point for recording the correct solution with an insufficient setup. In sum, Olivia received 5 points, Tim received 6 points, and Jon received 7 points, resulting in a mean group score of 6 out of 9 possible points. The group outperformed the class, which averaged only 4.90 points.

*Classroom observation 2.* For her second observation, Olivia worked on question 2 from the AP Calculus 2009 (Form B) exam with her group members (see Appendix E). The question provided a function that modeled the rate of change of the distance from a road to the edge of water during a storm, as well as various other given information, and posed four problems, (a)–(d). For part (a), students were required to compute the distance between the road and the edge of the water at the end of the storm. Jon suggested this

problem was an application of integration. The group agreed and worked silently. Upon completion, the members of the group monitored each other's solutions and verified numerical accuracy. However, Jon had not set up his integral expression properly, so he only received one point out of two. Olivia and Tim each received the full two points for part (a).

For part (b), students were required to provide an interpretation with proper units for the derivative of the function at a given time. Jon immediately complained of issues with the necessary units. Once he had finished his interpretation, Jon requested feedback from Olivia, who did not respond. Rather than following up with his request, Jon merely moved on to the next problem. This is unfortunate, because Olivia crafted an accurate interpretation with units and received the full two points for part (b). Jon received only one point due to incorrect units and Tim received one point because he had an inaccurate interpretation with correct units.

Part (c) required students to determine when the distance between the road and the water is decreasing most rapidly during the storm. Jon suggested that the problem was an extrema application for the first derivative. Despite its conceptual accuracy, Jon questioned his own plan and began arguing for the need of the next derivative. In an application of group monitoring, Olivia argued the group back to optimizing the first derivative. The group members then worked quietly, but were only calculating relative extrema, rather than absolute extrema. Upon completion, Jon identified the relative maximum, having misread the problem. This began a lengthy debate between Olivia and Jon as to whether the answer should be the relative maximum or minimum. (Neither realized that they were only finding relative extrema, instead of absolute extrema.)



Despite an animated discussion, the group decided on the relative maximum, which was not the correct answer. Since the group was not applying an absolute extrema analysis and decided on the wrong solution, each group member received only one point out of four for part (c).

Due to spending a long time on part (c), the group did not attempt part (d) and all members received zero points out of two. In sum, Olivia received 5 points, Tim received 4 points, and Jon received 3 points, resulting in a mean group score of 4 out of 9 possible points. Due mainly to the group's issues with part (c), the class, which averaged 4.73 points, outperformed the group.

*AP Calculus exam preparation performance.* Since Olivia missed five AP exam preparation sessions and Tim and Jon missed one AP practice session on the same day, all group data contain only the six sessions for which the group was intact. Group statistics for the 6 problems were as follows: Olivia averaged 4.67 points, Tim averaged 4.67 points, and Jon averaged 5.00 points. Thus, the group's overall average score for the 6 problems was 4.78 points, resulting in a difference between Olivia's average score and her group's average score of  $-0.11$  points. Olivia's overall average performance on the 7 in-class AP Calculus AB practice problems for which she was present was 4.86 points. The overall class average for the 7 problems that Olivia completed was 5.11, resulting in a difference between Olivia's average score and the overall class' average score of  $-0.25$  points.

*Martin's Narrative**Mathematical Achievement and Questionnaire Data*

Martin was classified as a *Category I* calculus achiever since his AP Calculus AB grade was 97. He was included in this study for his contribution to the *quota sample* (Miles & Huberman, 1994) and the preponderance of *high* scale categorizations he received. For the IMBS, Martin received three *average* and three *high* categorizations. The *high* categorizations were for the Belief 1, Belief 3, and Belief 4 scales. Thus, Martin self-reported highly availing beliefs that he can solve difficult mathematics problems regardless of duration, conceptual understanding is important in mathematical problem solving, and word problems are important in mathematics (Kloosterman & Stage, 1992).

For the MSLQ, Martin was categorized as *high* for all scales except for Motivation: Extrinsic Goal Orientation and Learning Strategies: Help Seeking. These high categorizations imply that he self-reported participation in a multitude of self-regulatory practices and processes. Unlike the previous four participants, Martin had a higher Intrinsic Goal Orientation score (7.0) than Extrinsic Goal Orientation score (6.0), indicating the tendency to engage in tasks to be challenged or obtain conceptual mastery. His *high* rating on the Motivation: Task Value scale suggests that Martin was interested in tasks in this course and found them important and useful. Martin's remaining *high* categorizations were in the Learning Strategies section of the questionnaire and included the Critical Thinking, Metacognitive Self-Regulation, and Peer Learning scales. Thus, Martin self-reported that he applies prior knowledge to problems and evaluates new content, regulates his cognitive and metacognitive processes, and places value on providing and receiving contributions from peers. The Metacognitive Self-Regulation

scale is the most comprehensive on the MSLQ and includes twelve items that delve into planning, monitoring, and regulating (Pintrich et al., 1991). Thus, Martin's categorizations indicated that he may participate in SRL processing to a higher degree than his peers.

Finally, for the PEP, Martin's Rational, Empirical, and Metaphorical scale scores were 123, 120, and 106, respectively. All of his scores were categorized as *high*; in fact, Martin had the highest score in the class for the Rational scale and the next highest score for Empirical scale. Overall, he was profiled as predominantly rational, but with the proximity of the scores, a more appropriate categorization may be rational/empirical. So, Martin self-reported the tendency to apply logic when justifying knowledge claims and use analysis and synthesis for cognitive processing. However, observational and perceptual inputs may be utilized by Martin for cognitive processing or justification when necessary (Royce & Mos, 1980).

### *Think-Aloud Sessions*

*Session 1.* For the entirety of TA1, Martin worked on the application of differentiation problem. He began by spending 32 seconds *reading* part (a), noting the conditions and the goal of the problem. Then, he spent 3 min 53 sec engaged in *analysis*, connecting the first derivative to horizontal tangents and establishing relationships between the conditions and the goal. Thus, Martin developed an adequate *definition of the task* to aid in developing solution strategies. This analysis portion of his session was also indicative of early signs of processing from the *forethought* phase of SRL, with one instance of prior knowledge activation coded, one code for recycling the goal in working memory, and one code for an overt plan.

The overt plan coded during the analysis phase applied the quadratic formula to solve for  $x$ -coordinates of horizontal tangents, and was a foreshadowing of Martin's strict adherence to a *procedural* approach to mathematical problem solving during TA1. Making little progress with the quadratic formula, Martin embarked on a 26 min 55 sec *exploration*, producing relatively fruitless expressions and equations from the conditions in various forms of preparation for a system of equations approach to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ . The exploratory phase opened with Martin planning to set the derivative equal to the function, which has little mathematical meaning, especially in this context. He later informed me during his MCI that he recognized that both functions were equal to zero and thus, could be set equal to one another. Fortunately, Martin exercised *performance control* over his actions throughout the session. Unique to other participants, he applied the monitoring technique *self-questioning* for most of his local and global assessments, with five instances coded during this phase. For example, when assessing the validity of setting the derivative and function equal, Martin made the following statement:

I know that zero is equal to the original function and it produces one root. But I don't know if one of those roots is also the horizontal tangents. So if I back track, what would I be able to solve down to, anyway? If I continued working that out assuming that there is a root actually equal to a horizontal tangent, I mean, that's not going to tell you anything.

Unfortunately, Martin's monitoring skills proved insufficient to *procedurally* solve the application of differentiation problem.

After making little progress solving the system by hand, Martin spent the remainder of this exploration attempting to use the computer algebra system (CAS)

capabilities of his calculator to solve for  $a$ ,  $b$ ,  $c$ , and  $d$ . Ideas ranged from simply solving the equations *as is* to storing the expressions for  $x$  obtained from the quadratic formula, then substituting these values into the cubic and trying to solve. Despite logging six instances of local and global assessments of strategy use and goal state, Martin was unable to alter his solution path or transition to another strategy more conducive to solution development.

Finally, Martin moved to part (b) and spent 23 seconds *reading* the problem, noting conditions and identifying goals. Despite only spending 4 min 35 sec on part (b), he had already developed an adequate *definition of the task* and was beginning to work with linear functions. However, time was called before he could advance further.

With respect to epistemology, Martin's behavior was erratic, directly contrasting his questionnaire results that were highest in the class for all scales. For example, despite demonstrating an *arbitrary* belief regarding problem solutions by understanding the infinitude of possible values for  $a$ ,  $b$ ,  $c$ , and  $d$ , he remained fixated on applying *procedural* techniques that generally produce a single solution. Interview data from his RRI1 triangulated this finding:

I was assuming more than one [solution] for sure. But, to be honest, I thought if I'd just start working through the math, I mean, I left it as an open blank whether I don't really know how many solutions there will be. So, I decided not even to think about that and rather just work out the math and see if I can start making conclusions without the various variables I had.

Later in the interview, after expressing disdain for guess-and-check as a mathematical problem-solving method, Martin bluntly described his mindset during TA1:

Looking back at my thought process, I think that I was so determined that there wasn't going to be any guess-and-checking to it. There was going to be a straight algorithmic way to approach it rather than, oh well, now that I know this and know this, let's try a couple of values here or here or a range of values. I mean, maybe looking back now, that would have been a better idea if I were still working on that problem.

As his comments indicate, Martin realized late in the interview that a mixture of logically-based mathematical problem-solving techniques may be advantageous depending on the context. Despite overt instances of monitoring, Martin's TA1 would be best described as a series of applications of fruitless *empirical* explorations.

*Session 2.* Martin made up for his lack of success in TA1 by displaying a flurry of mathematical problem-solving prowess during TA2. With extracurricular demands, homework, and only one evening between sessions, he was unable to even look at the problem during the interim between sessions. So, Martin requested to have both problems on the table simultaneously and work intermittently between the problems. He began TA2 by *reading* part (a) of the application of integration problem for 48 seconds, noting conditions and identifying the goal. Then, Martin spent 36 seconds engaged in *analysis* of the problem space, establishing relationships between the conditions and the goal. Based on his analysis, Martin developed a plan to find the area, divide it by two, and use the upper bound of the integral to find an equation for the vertical line. He spent 6 min 53 sec implementing this plan. Martin required only one instance of monitoring during the implementation and seemed perfectly comfortable with the technique that he was applying. Except for two minor errors that did cost him accuracy, he employed his

method flawlessly. Had he spent some time in *verification*, Martin may have received full credit.

Upon completion of part (a), Martin spent 19 seconds *reading* part (b) of the application of integration problem. From reading, Martin moved to a 4 min 8 sec *analysis* episode and made significant progress, finding the area of the bounded region with precision. During this phase, he demonstrated excellent problem-solving control by catching an error in his original graph. Martin had inadvertently misplaced one of the boundaries and was prepared to set up an improper integral, or integral over an unbounded region, when he decided to graph the region on his calculator. Upon typing the boundaries into the calculator, Martin caught his error and was able to move forward with a bounded region.

Martin failed to read the problem carefully and assumed that part (b) was similar to part (a). Thus, he set up an integral with respect to  $x$  and attempted to find an equation of a *vertical* line separating the area into two parts, despite the directions requesting a *horizontal* line. This error led to a 7 min 16 sec fruitless *exploration* ending with an unsolvable equation since the area of the region increases without bound horizontally as  $m$  approaches zero. The most troublesome aspect of this phase was the lack of self-monitoring; Martin only monitored his performance two times during the implementation, never re-read the problem, and moved through transitional phases with no regard to their effect on the goal of the problem. For this reason, his problem-solving practices were coded as *rational* and *conceptual* for part (a) since he demonstrated understanding of the underlying principles of the problem and translated them to mathematical output. However, his blind perception that part (b) was the *same type* of

problem resulted in *empirical* and *procedural* coding for his problem-solving efforts for part (b).

Stumped by the lack of a solution for part (b) of the application of integration problem, Martin spent 19 seconds *reading* part (b) of the application of differentiation problem. Then, he spent 2 min 59 sec engaged in *analysis* of the conditions of the problem, tying the second derivative to concavity and brainstorming ways to make linear functions always positive, which is the crux of the problem.

Realizing that time was not on his side, Martin moved on to part (a) of the application of differentiation problem without re-reading the instructions. He spent 4 min 33 sec in productive exploration that resulted in a solution. During this phase, he attempted to use the CAS on his calculator one more time, but without success. Monitoring kept him from continuing down this erroneous path. Then, he attempted to build a cubic function based on conceptual understanding and developed the function  $f(x) = -3x^3 + 4x^2 + 2x + 1$ , which meets all of the conditions of the problem. Some of the logic leading to his solution was flawed, resulting in a lower score for justification.

Satisfied with his result for part (a) of the application of differentiation problem, Martin spent 2 min 41 sec in *analysis* and *verification* of part (b) of the application of differentiation problem. His analysis of the problem space led him to the conclusion that a linear function with non-zero, defined slope cannot be positive on its entire domain. Upon verification, Martin reported his answer and received full credit for this solution.

For his final 4 min 31 sec, Martin engaged in *exploration* and *analysis* of part (b) of the application of integration problem, since he still had no solution to report. Despite analyzing his sketches and making one more global assessment of his goal state, Martin



failed to note that he was not even answering the correct question. Thus, his session ended with him still pondering this final, incomplete problem.

In terms of epistemological beliefs, Martin's received an overall categorization of *rational/empirical* for his beliefs in mathematical problem solving. His navigation of the problem space included both logically-based problem solving plans and perceptually-based explorations. Martin's RRI2 and MCI elicited findings regarding two beliefs. First, despite appearing to be comfortable with the arbitrary constants in both problems, Martin cited them as a major barrier to problem-solving success during his RRI2:

Conceptually [the problems] are not easy by any means but I think even harder than that is just working with all the variables and all the arbitrary  $m$  and I guess in this case the arbitrary  $a$ ,  $b$ ,  $c$ , and  $d$ . Really just trying to keep up with it is the most difficult.

Earlier in the interview, he made some statements that indicated a possible source of these troubles:

Well, originally when I did 1 (a) [application of differentiation problem], especially yesterday, I remember thinking, and this is bad thinking on me, but I remember thinking you wouldn't give me a question in which there is more than one answer. But I think that was more of me thinking of what kind of problem you would give me rather than looking at the actual problem itself.

Probed as to what gave him the impression that the problem may have only one solution, Martin stated:

Oh, I don't know, I just always feel a lot of times in math they try and focus and give you a systematic way to do it which would mean there is only one answer.

Martin's expressions indicate, despite a certain level of comfort in the *arbitrary* nature of problem solutions, a *unique* belief may manifest in students' mathematical problem-solving practices based on past experiences in mathematics courses.

Second, Martin's final comment above indicated that an overt focus on *procedural* methods of navigating problem spaces may be the culprit for his confusions. His RCI comments provided triangulation for such an assertion. When asked whether knowing procedures or understanding concepts is more important to problem-solving success, he made the following statement:

I would say understanding underlying concepts which would lead you hopefully to a certain set of steps; that would be my guess. . . conceptual knowledge, I think, leads to the procedural knowledge, or the procedural steps generally to solve the problem.

So, despite emphasizing the conceptual, Martin continued to indicate a desire for procedures. When probed as to the sources of his desire to have a more procedural, or direct, approach, Martin responded:

Because I felt that if I had just done the guess-and-check thing I wasn't really justifying anything, I really was just kind of guess-and-checking. Where if I had shown work for it at least I would have work behind it to explain how I got to that solution, which I like better.

So, Martin indicated the need for a procedurally-based, as opposed to a conceptually-based, justification for his mathematical work. It appears that a disconnect between an ideally conceptual belief in mathematical problem-solving and the realized procedural

manifestation of problem-solving habits exists in high-achieving, advanced mathematics students.

*Think-aloud problem-solving session performance.* Martin received nearly full credit for both parts of the application of differentiation problem and part (a) of the application of integration problem. During the second session, Martin was able to adequately solve part (a) of the application of differentiation problem. However, his justification was insufficient; thus, based on Schoenfeld's (1982) grading scheme, he was awarded 17 points for part (a) of the application of differentiation problem. Martin's solution with justification for part (b) of the application of differentiation problem was perfect. Hence, Martin was awarded the full 20 points for solving part (b) and earned a combined score of 37 out of 40 points for the application of differentiation problem.

For part (a) of the application of integration problem, Martin worked like an expert. Very little control was applied and his methods were flawless. Unfortunately, he made two careless errors that affected the final solution. Thus, Martin was awarded 18 out of 20 points for his efforts. Finally, Martin made *some* progress on part (b) but the solution path chosen was procedurally identical to part (a) despite the change in goal state. However, his area was accurate and with respect to  $m$ . If he had simply noted the need for a horizontal line, he may have been able to solve the problem. Thus, Martin was awarded a score of 10 out of 20 points for part (b). His total score for the application of integration problem was 28 out of 40, resulting in a combined total of 65 out of 80 points for both problems.

*AP Calculus Exam Preparation*

During the MCI, each participant was asked to comment on the differences in the AP Calculus exam preparation session problems and the think-aloud session problems. Martin stated that the think-aloud session problems were “more difficult” and that he could not “remember any AP problems either in groups or in class that I didn’t really understand what I was doing.” Unfortunately, despite noting the difficulty-level differences and citing the procedural aspects of the AP exam practice problems, Martin provided insufficient detail for providing additional justification for categorizations of the tasks based on Stein et al.’s (2000) system.

*Classroom observation 1.* Martin was observed working on question 5 from the AP Calculus AB 2005 (Form B) exam with his partners, Roy and Mae (see Appendix E). The question required students to work with an implicitly-defined relation and solve four problems, (a)–(d). For part (a), students were to show that the derivative of the implicit relation was equal to a given expression. The group finished this part so quickly that they were working on part (b) when the researcher began the observation. Upon inspecting solutions, things must have gone well because all three group members earned the full two points.

For part (b), students had to find all points such that the slope of the curve equals one-half. Mae presented an idea to the group, but Martin pointed out a flaw in her plan and suggested that the group set the derivative from part (a) equal to one-half and solve for one of the variables. Martin and Mae employed this plan but soon realized the cumbersome algebra in store for them. At this point, Martin *requested feedback* from Roy, who demonstrated an alternative solution path. All three agreed with the new plan,

which was similar to Martin's plan but with fewer algebraic steps. All three group members successfully solved for  $x$ , but made an error in solving for  $y$ . Thus, each group member received one out of the possible two points for part (b).

For part (c), students were to show that the curve had no points such that the tangent was horizontal. Mae was unable to recall the definition of a horizontal tangent but Martin was able to provide a conceptual explanation for her. Then, all group members appeared to be working on the problem. Providing an overt example of group monitoring, Roy noted that Martin had set the denominator equal to zero instead of the numerator and saved him a costly error. Despite the discussions engaged by Martin and Roy, Mae was still unable to make any progress with part (c). Eventually, Martin simply explained verbatim how to show that the curve can have no horizontal tangents so the group could move on. All three group members received the full two points for part (c).

For part (d),  $x$  and  $y$  were defined as functions of  $t$ , additional information was provided, and students were asked to find the derivative of  $x$  with respect to  $t$  at a given point. The entire group was stumped for several minutes and Roy was literally scratching his head. None of them saw the connection to related rates. Fortunately, Martin, in an expression of critical-thinking genius, developed a plan to differentiate all variables with respect to  $t$  at the given point. (He had essentially developed the concept of related rates without realizing it!) His fellow group members had difficulty following him. Roy decided to work independently and this decision cost him as he only earned one point out of three. Mae probed Martin, who provided a sufficient explanation for her to earn two points out of three. Despite attempting to verify his solution, Martin's stroke of genius only yielded him two points out of three, as he had two arithmetic errors and thus, an

incorrect final solution. In sum, Martin and Mae received 7 points and Roy received 6 points, resulting in a mean group score of 6.67 out of 9 possible points. The group outperformed the class, which averaged 5.85 points.

*Classroom observation 2.* During his second observation, Martin worked on question 1 from the AP Calculus AB 2009 exam with his group members (see Appendix E). In direct contrast to the first observation, this observation is best described as three students working independently on a problem and occasionally discussing their thoughts. The question provided a piecewise-linear graph representing the velocity of a student's bicycle as she rides to school and posed four problems, (a)–(d). For part (a), students were required to find the acceleration of the bike at a given time with appropriate units of measure. Martin and Roy immediately recognized that differentiation was necessary and set to work calculating the slope of the linear piece containing the given time. Mae walked in a little late and had to catch up with her partners. In the first example of the lack of group unity, each group member received one point out of two for part (a) but all for different reasons. Martin and Roy indicated incorrect units with differing errors. If they had participated in group monitoring, they may have reconciled the issue. Mae had correct units, which she did not share with the group but made a calculation error that was not monitored.

For part (b), students were required to interpret the meaning of the integral of the absolute value of the velocity function over a give interval and then find the value of the integral. Yet again, the group failed to monitor or verify each other's interpretations and this cost Martin, as he excluded the time interval from his answer. Ironically, Mae could not recall whether the integral produced total distance or displacement and upon asking

for help, received a good explanation of the conceptual differences in distance and displacement from Martin, and earned a point that Martin himself did not receive. For the calculation, Martin presented the plan to use geometric formulas to calculate the integral. Everyone assented and worked quietly calculating the area. Upon completion, Martin asked the group if they got 1.8 as an answer, but everyone had obtained different results. Upon inspection, Martin and Roy determined that the error involved the scale of the graph; the horizontal scaling was one unit, but the vertical scaling was 0.1 units. So, Roy and Mae re-calculated their areas. Mae checked her solution with Martin and found that she was still slightly inaccurate. She mumbled something under her breath about *just leaving it*. Upon checking her work, it was confirmed that she had not corrected the error. So, for part (b), Martin earned one point out of two due to his incomplete interpretation of the integral, Roy earned the full two points, and Mae earned one point since she did not correct her erroneous calculation of the integral.

Part (c) required students to determine the time at which the student returned home to retrieve her calculus homework and provide a reason for their answer. At time  $t = 2$ , the graph indicated a change in velocity from positive to negative, demonstrating a change in direction. The group had no noticeable discussions of this part and simply worked quietly. As a result, all group members determined the correct answer but Mae's reasoning was inaccurate. So, Martin and Roy received the full two points and Mae received one point for part (c).

Part (d) introduced another student riding his bike to school with a specific function, rather than a piecewise graphical representation, describing the velocity of the bike. Students were required to determine which student lived closer to the school.

Martin presented the plan to compare both students' displacements, rather than distances, which was an accurate conceptualization of the problem. Students then struggled with rectifying the change in direction that resulted from the first student's bike trip. Eventually, Martin and Roy figured out a technique to find the first students' distance and were able to complete the problem. Despite receiving help from Martin, Mae was unsuccessful in her solution for part (d) and only received one point out of three. Roy received the full three points since his integrals were properly set up, calculated, and the results interpreted. Martin received two points out of three since he made all the calculations appropriately but misinterpreted his results and made the wrong conclusion. So, the session ended as it began, with lack of group monitoring resulting in three different scores for a group of students supposedly working together. In sum, Martin received 6 points, Roy received 8 points, and Mae received 4 points, resulting in a mean group score of 6 out of 9 possible points. Due in large part to a lack of group monitoring, the class, which averaged 7.33 points, outperformed the group.

*AP Calculus exam preparation performance.* Since Martin and Mae missed the same two AP practice sessions, all group and overall class data contain only the ten sessions for which the group was intact. Group statistics for the ten problems were as follows: Martin averaged 6.10 points, Roy averaged 6.60 points, and Mae averaged 4.50 points. Thus, the group's overall average score for the 10 problems was 5.73 points, resulting in a difference between Martin's average score and his group's average score of +0.37. The overall class average for the 10 problems that Martin completed was 4.95, resulting in a difference between Martin's average score and the overall class' average score of +1.15 points.



*Cameron's Narrative**Mathematical Achievement and Questionnaire Data*

Cameron was the upper-bound *maximum variation* participant (Miles & Huberman, 1994) based on his grade of 100 in AP Calculus AB. His grade was the highest among students from all sections of AP Calculus AB offered at the school, obviously making him a *Category I* calculus achiever. Cameron also had a very diverse array of categorizations from the questionnaires. For the IMBS, Cameron had one *low*, three *average*, and two *high* categorizations. His low categorization was for the scale Belief 5, implying that he self-reported the non-availing belief that mathematical knowledge is innate and hard work may not produce mathematical proficiency. His two high categorizations were for the Belief 1 and Belief 4 scales, indicating that he self-reported the availing belief that he could solve difficult mathematical problems regardless of duration, and word problems are important to mathematics, respectively (Kloosterman & Stage, 1992).

For the MSLQ, Cameron was categorized as *low* for one scale, *average* for three, and *high* for three. His low categorization was for the scale Learning Strategies: Help Seeking, indicating that Cameron self-reported placing little value on seeking help from outside sources. His three high categorizations were for Motivation: Intrinsic Goal Orientation, Motivation: Task Value, and Learning Strategies: Critical Thinking. Congruent with Martin and in opposition to the other participants, Cameron scored higher on Intrinsic Goal Orientation (6.5) than Extrinsic Goal Orientation (5.5), indicating the tendency to engage in tasks for the purposes of mastering and understanding concepts. His other high categorizations, Task Value and Critical Thinking, indicate that Cameron

self-reported being interested and seeing importance in tasks for this course, and in applying prior knowledge to problems and critically evaluating content presented in this course, respectively (Pintrich et al., 1991).

Finally, for the PEP, Cameron's scores for the Rational, Empirical, and Metaphorical scales were 110, 110, and 67, respectively. He was categorized as *average* for the Rational and Empirical scales and *low* for the metaphorical scale. In fact, Cameron's metaphorical score was lowest in the class. Due to obtaining the exact same score for both scales, Cameron was categorized as *rational/empirical*. This categorization implies that Cameron self-reported giving equal credence to analysis, synthesis, and logic (rational), and observation and perception (empirical) for cognitive processing and justifying knowledge claims. The low categorization assigned to the Metaphorical scale indicates that Cameron gives little credence to insight, awareness, and symbolism as means for cognitive processing and justifying knowledge (Royce & Mos, 1980).

#### *Think-Aloud Sessions*

*Session 1.* During TA1, Cameron spent all of his time working on the application of differentiation problem. He began TA1 by *reading* both parts (a) and (b) for 42 seconds and then spending 1 min 30 sec engaged in *analysis* of the conditions and goals of part (a) of the problem. During his analysis, Cameron established accurate and appropriate relationships between problem conditions and goals by applying the *restate the problem* and *setting up equations* heuristics, enacting prior knowledge, and recycling the goal in working memory. Thus, Cameron developed a *definition of the task* that allowed for establishment of initial goals that would inform the *forethought* phase of SRL processing. From this analysis phase, Cameron shifted to a rather fruitless exploration of

the quadratic formula as a means for finding the  $x$ -coordinates of the horizontal tangents that lasted 1 min 18 sec. An earlier global assessment indicated that he doubted the usefulness of this idea. The exploration resulted in an expression for  $x$  dependent upon the variables  $a$ ,  $b$ , and  $c$ .

Then Cameron assessed the current state of the problem and established a plan to “create values” for  $a$ ,  $b$ ,  $c$ , and  $d$ . This decision made in the *forethought* phase of SRL processing proved to be the most important to his successful navigation through the problem space. The ensuing 14 min 2 sec was comprised of an elegantly developed and successfully *implemented* solution and *verification* for part (a). To use his own words, Cameron was “working backwards,” which is a useful heuristic, by “creating arbitrary points.” He began this process by assigning an  $x$ -coordinate to the single point of inflection that he had logically deduced must exist. Since the second derivative still contained two unknowns,  $a$  and  $b$ , he arbitrarily assigned  $a = 1$  so he could solve for  $b$ . Then, using a combination of *rational* logic and *empirical* substitutions, Cameron used the first derivative to find an appropriate value for  $c$  and then assigned a value to  $d$  that lifted the function above the  $x$ -axis.

During this implementation, six instances of local assessment of strategy use and two instances of global assessment toward goal state were coded and led to transitions that were purposeful and productive. It should be noted that, although successful, Cameron’s problem-solving path was not seamless; his ability to control erroneous and unforeseen results during the *performance control* phase of SRL was crucial to his success. For example, when solving for  $c$ , Cameron had established two  $x$ -values for his horizontal tangents, but substituting the  $x$ -values resulted in two different  $c$  values. The

reason for this rather strange occurrence is that he had correctly set his horizontal tangents equidistant from the point of inflection but made a minor arithmetic error when solving an equation. Spending only seconds contemplating this issue, Cameron decided that the best option would be to accept one of the  $c$ -values and continue working the problem. Thus, control kept him from spending too much time focusing on issues that had no bearing on the overall goal state of the problem. During *verification*, Cameron used his graphing calculator to check that his solution, which was  $f(x) = x^3 - 3x^2 - 9x + 28$ , met the conditions of the problem.

Satisfied with his solution for part (a), Cameron spent 16 seconds *re-reading* part (b) and then spent 1 min 8 sec engaged in *analysis* of the second derivative. Then, Cameron developed an argument that part (b) was not possible based on the linear qualities of the second derivative and *verified* his result by re-reading his answer. The solution and verification of part (b) lasted 3 min 31 sec. No control was needed because Cameron conceptually understood the relationship between goals and conditions to the point that strategy use was unnecessary.

In terms of epistemological beliefs, Cameron's navigation through TA1 was coded as predominantly *rational* but with evidence that an *empirical* belief in mathematical problem solving is important as well. This result is supported by Cameron's scores on the Rational and Empirical scales of the PEP. Additional support includes his solution strategy for part (a), which was based simultaneously on conceptual understanding and arbitrary assignments of values to variables. Additionally, a strict adherence to the belief in *conceptually* solving mathematical problems pervaded the session. There was little evidence of prescription or procedure during the plan and

implementation phase of part (a). Rather, Cameron's navigation of the problem space consisted of decisions based on conceptual insights and transitions applied in light of local and global assessments of his strategy.

Cameron's work during TA1 was also indicative of a belief that knowledge is *interrelated*. For example, he was the only participant to consider the second derivative and points of inflection while developing a strategy to solve part (a). His consideration of the information inherent in the second derivative provided him with another equation to work with, a point (of inflection) from which to develop a function, and the Second Derivative Test to justify his extrema. Finally, Cameron's work and RRI1 provided evidence of an *arbitrary* belief regarding problem solutions. Within this specific context, this belief dimension is representative of the degree of acceptance that multiple representations may exist for the constants contained in both problems. Overt evidence of Cameron's *arbitrary* belief is represented by the following quotes from his RRI1: "When I first read the question I saw all the variables,  $a$ ,  $b$ ,  $c$ , and  $d$ , that left it up for grabs;" and "I realized that there were a lot of possibilities here so I could start making arbitrary assumptions." Summarily, Cameron demonstrated an *availing* belief in mathematical problem solving.

*Session 2.* Having successfully solved the application of differentiation problem during TA1, Cameron spent all of TA2 working on the application of integration problem. Cameron began TA2 by spending 52 seconds reading the problem and then 58 seconds engaged in *analysis*. The analysis portion of this phase of the session involved sketching a graph of the shaded region and establishing relationships between conditions and goals. So, as in TA1, Cameron entered the *forethought* and *performance control*

phases of SRL processing with an accurate *definition of the task*. Following analysis, Cameron embarked on a 58-second, productive exploration that resulted in obtaining the area of the bounded region for part (a). However, upon global assessment, Cameron dismissed this result as unnecessary based on the plan he had in mind.

Recognizing the conceptual significance of the bounds of an integral in solving for area, Cameron determined the following equation to be the most appropriate means for solving the problem:

$$\int_0^a h(x) dx = \int_a^2 h(x) dx .$$

He followed this *plan* to success over a 5 min 2 sec *implementation* phase. The execution of the plan was so precise that little monitoring was required; hence, little evidence of *performance control* was coded. In fact, only three instances of locally assessing strategy use and three instances of globally assessing progress toward goals were coded. In the end, Cameron solved the integral equation completely by hand and got an answer of

$$x = -\frac{1}{m} \ln \left( \frac{1}{2} + \frac{1}{2} e^{-2m} \right) .$$

He then used the CAS capabilities of his calculator to verify his result by setting both definite integrals equal to each other and obtaining the word *true*.

Cameron's work for part (b) began by *re-reading* the problem for 17 seconds, then spending 1 min 20 sec engaged in *analysis*. During this analysis phase, he sketched

the appropriate region and more importantly noted the importance of re-writing  $h(x)$  as a function of  $y$  to facilitate finding an equation for the horizontal line. Then, appropriately using the heuristic of *recalling a similar problem*, he applied his plan for part (a), but with respect to  $y$ , and developed a solution in 7 min 32 sec. It should be noted that his plan would not have been successful had he not re-written  $h$  as a function of  $y$ .

Additionally, Cameron monitored his actions, logging seven instances of locally assessing his strategy and three instances of globally assessing his goal state. The most intriguing assessment occurred as Cameron neared the end of implementing his problem-solving plan. He had been solving by hand but reached an unsolvable equation and used the CAS to find the solution. As he did so, he realized that the solution was to be unique (i.e., not related to  $m$ ) and this caused him to consider the plausibility of his solution.

Once Cameron had a solution,  $y \approx 0.6348108$ , he spent his final 3 min 51 sec *verifying* it in the same manner as part (a). Up to the end, Cameron was trying to convince himself that one unique solution was plausible for part (b).

From an epistemological perspective, multiple findings emerged from Cameron's TA2 and RRI2. To begin, Cameron's navigation of the problem space was coded as completely *rational* throughout. Every thought and action was logically-based and grounded in the integral calculus. Consequently, the problem-solving session was also coded as exhibiting a predominantly *conceptual* belief in mathematical problem-solving and the *interrelated* nature of knowledge. These beliefs were evident when Cameron transitioned from using the area in part (a) to set up an integral equation. This action demonstrated a deviation from procedure, which would typically lead to a numeric value for the area, to a more sophisticated and contextual understanding of conceptual aspects

of the conditions and goals of the problem. Cameron confirmed this interpretation during his MCI, as demonstrated by the following excerpt of his answer to whether knowing procedures or understanding concepts is more important to problem solving:

In a conceptual understanding, you can not only apply it to any of the cases but also apply it to cases that you haven't been taught a procedure for. You can at least give it a try and have a better shot at going into the unknown. And if you've just known procedures, then you will not be able to take that outside.

Finally, Cameron's session also indicated an *arbitrary* belief regarding problem solutions. The most informative evidence of this *arbitrary* belief was Cameron's reaction to the unique solution obtained for part (b) during TA2: "Am I going to get a value here? That's odd. Uh, well let me keep going and I will ponder that later." Later in the session while verifying his solution, he was still perplexed by the unique solution: "I still think it's weird that I'm getting that number answer for something when not everything is defined." These statements indicated that Cameron understood the effect that the constant  $m$  typically has on a problem of this nature. Thus, he was not expecting a unique solution and upon receiving one, questioned its validity.

During his RRI2, Cameron made the following statement:

And then with 2 (a) and (b), because there was this  $m$ , I was expecting a solution that would have  $ms$  in it. So that it wouldn't be an exact value; it would be a variable answer kind of thing. So with (a) my expectations were met, and then with (b) it surprised me that I got a numerical answer as opposed to some expression involving  $m$ .



Thus, Cameron seemed to hold a strong belief in the *arbitrary* nature of the constants and their differing roles in the problems analyzed in this study. When probed during his MCI, Cameron was unable to identify the source of his *arbitrary* belief, but he did concur that previous mathematics courses did not prepare him adequately for such abstractions by stating, “Well, I have actually noticed, now that you’ve mentioned it, but all the previous math classes when we have a variable we solve for it and it eventually becomes a number and never any arbitrary constants and general solutions.”

*Think-aloud problem-solving session performance.* Cameron was the only participant to earn the full 80 points for the given problems. Additionally, he successfully solved both problems within the time constraints of the sessions, solving the application of differentiation problem in 22 min 27 sec and the application of integration problem in 24 min 19 sec. Despite two minor errors, all four final solutions were accurate and adequately and conceptually justified.

#### *AP Calculus Exam Preparation*

During the MCI, each participant was asked to comment on the differences between the in-class, AP Calculus exam preparation session problems and the after-school, think-aloud session problems. Cameron responded with the following:

The practice problems we did in class, even though they were AP questions, were just more stuff we had learned, just taking our knowledge and applying it to just a little problem; whereas the afternoon sessions were a little more outside the box, let’s think a little more. And it has to be the AP questions that they need to focus on the knowledge you currently have and test the knowledge we have, as opposed to trying to build new knowledge.

So, Cameron's response provides further justification for categorizing the AP Calculus exam preparation session problems as "procedures with connections tasks" and the think-aloud session problems as "doing mathematics tasks" (Stein et al., 2000, p.16).

*Classroom observation 1.* Cameron was observed working on question 2 of the AP Calculus AB 2003 (Form B) exam with his partners, Jim and Ned (see Appendix E). The question provided functions describing the rates heating oil is pumped into and removed from a tank. Four problems, (a)–(d), were posed based on the given information. For part (a), students had to find the total amount of oil pumped into the tank over a give time period. The group simply began working quietly on the problem and once finished demonstrated *group verification* by checking one another's answers. During verification, Jim pointed out to Cameron that he was missing part of the function in his calculation and thus, had an incorrect answer. However, no one realized that Ned had only recorded two decimal places of accuracy, thus he lost a point. For part (a), Jim and Cameron received the full two points and Ned received one point.

For part (b), the group determined whether the level of oil was increasing or decreasing at a certain time. For this part, Cameron's *group monitoring* practices came to Jim's aid. Both began by assuming a derivative was required, but upon re-reading, Cameron realized that the functions were already providing rates. After some calculations, the group, yet again, took time to verify solutions and the result was that all three received the full one point.

As soon as Jim and Cameron read part (c), both students knew that an initial condition integral expression was required. As with the first two parts, Ned provided little input and simply assented and applied Jim and Cameron's plan. All worked quietly, until

Ned informed the group that he could not input the integral expression into his calculator. Jim and Cameron helped him clear up the calculator problems. Then Jim discussed the conceptual meaning behind the initial condition and integration performed. Despite this showing of group monitoring, Ned did not set up the integral properly. Thus, Jim and Cameron received the full three points and Ned received two points for part (c).

None of the students recalled the necessary conceptual knowledge to solve the absolute extrema problem presented in part (d). In fact, Cameron's journal even indicated a plan to apply *relative* extrema techniques. The group spent a lot of time quietly reading and discussing this problem but was never able to get past considering relative extrema. For this reason and because time was called, Ned and Cameron received one point and Jim received zero points out of three. In sum, Cameron received 7 points, Jim received 6 points, and Ned received 5 points, resulting in a mean group score of 6 out of 9 possible points. The group outperformed the class, which averaged only 4.53 points.

*Classroom observation 2.* During his second observation, Cameron worked on question 5 from the AP Calculus AB 2007 (Form B) exam (see Appendix E). The question posed four problems, (a)–(d), based on a given, non-separable differential equation. For part (a), students simply had to sketch a slope field for the differential equation. The group worked silently on this part and no one showed signs of misunderstanding the concept. However, thanks to *group monitoring*, Cameron caught an error made by Ned while the group was verifying solutions. All three group members earned the full two points for part (a).

Part (b) required students to find the second derivative and discuss the concavity of the solution curves to the differential equation. Yet again, Jim and Cameron

formulated a plan for solution and Ned merely assented and followed along. However, during implementation, Jim was unable to rectify the  $dy/dx$  remaining in the expression for the second derivative. In another example of *group monitoring*, Cameron pointed out the substitution provided by the given information. At this point, the group got very confused, as Jim thought that (b) was finished and moved on to (c) and Ned attempted to continue following along. Cameron straightened the group out by reminding them that part (b) had a second question. Thus, Cameron's focus on the *definition of the task* and goal state had a heavy influence on group performance. For part (b), all three group members earned the full three points.

Since he had moved on earlier, Jim knew that part (c) required the use of the Second Derivative Test to determine whether a given solution to the differential equation contained a relative extrema value at a given point. Based on this plan, Jim and Cameron began working but Ned could not recall how to apply the Second Derivative Test. Thus, Cameron explained the concept of the Second Derivative Test and its application to the given context. After completing his work, Cameron began work on part (d), but Jim made him stop so the group could verify solutions. All three group members earned the full two points for part (c).

The group was truly stumped by part (d), which required them to find values for  $m$  and  $b$  such that  $y = mx + b$  would be a solution for the differential equation. Jim and Cameron discussed the problem extensively, with the most important instance of group monitoring occurring when Cameron stifled Jim's desire to solve using separation of variables. Ned interrupted the deep discussion to ask which part the group was working on, clearly showing that he was unable to follow along. Then, Ned realized that his

solution for part (c) was incorrect and Cameron re-explained the concept to him. As the researcher stood to leave, Cameron had a revelation and began building an argument for using the result from part (b), which provided a solution for zero concavity, or linearity. An inspection of his work showed that he had developed a solution. He must have shared his solution with the group since all three received the full two points for part (d). In sum, all three group members received the full 9 points for this question, obviously resulting in a mean group score of 9 points. The group outperformed the class by a large margin, as the class averaged only 4.97 points.

*AP Calculus exam preparation performance.* Since Jim missed one AP practice session, all group data contains only the eleven sessions that the group was intact. Group statistics for the eleven problems were as follows: Cameron averaged 7.64 points, Jim averaged 6.91 points, and Ned averaged 5.73 points. Thus, the group's overall average score was 6.76 points, resulting in a difference between Cameron's average score and his group's average score of +0.88. Cameron's overall average performance on the 12 in-class AP Calculus AB practice problems was 7.50 out of 9 possible points. The overall class average for the 12 problems was 4.92, resulting in a difference between Cameron's average score and the overall class' average of +2.58 points.

### Conclusion

This chapter has provided rich, thick descriptive narratives of the experiences of the six participants. In addition, theoretical interpretations were provided via triangulated data sources with respect to epistemological beliefs, SRL, and mathematical problem-solving based on the theoretical framework developed in *Chapter II: Review of Relevant Literature*. Each participant engaged in a member-checking interview shortly after the

study to review initial findings, augment results, and answer follow-up questions to ensure credibility of the findings. Overall, the results suggest the following:

- SRL processing and mathematical problem-solving prowess are related to students' beliefs in a *procedural* or *conceptual* approach to mathematical problem solving and additionally a *simple* or *interrelated* belief in the *simplicity of knowledge*;
- SRL processing and mathematical problem-solving prowess are related to students' *unique* or *arbitrary* beliefs regarding problem solutions; and
- SRL processing and mathematical problem-solving prowess are related to students' abilities to appropriately apply both an *empirical* and *rational* approach to mathematical problem-solving.

Further qualitative exploration of these assertions were addressed via cross-case analysis and is the subject of the ensuing chapter.

## CHAPTER V

### CROSS-CASE RESULTS

The purpose of this study was to explore relationships between students' epistemological beliefs and self-regulatory processing in the context of mathematical problem solving. To achieve this goal, the researcher applied *quota* and *maximum variation* sampling to obtain six individual participants from an intact Advanced Placement (AP) Calculus BC class (Miles & Huberman, 1994). Then, participants were provided opportunities to work mathematical problems both individually and in small groups. Data were collected from *think-aloud* transcriptions, student work samples, journal entries, observational field notes, and individual interviews. Finally, the researcher provided students with draft narrative accounts of their experiences to review for the purposes of member checking.

In the previous chapter, rich, thick descriptions of participants' experiences during the eleven-week study presented initial interpretations that emerged from open-coding analyses (Creswell, 2007; Miles & Huberman, 1994). Then, using NVivo Version 8, data were analyzed via axial and selective coding to determine patterns, themes, and relationships (Bazeley, 2007; Creswell, 2007). Matrices and models were developed to provide both final analyses and visualizations of the findings. This chapter provides a culminating report of cross-case data analyses with respect to categorizations developed for participants.

## General Overview

The previous chapter provided narratives reports of the analysis of each participant's experiences. This chapter will present cross-case analysis that revealed major themes and patterns which emerged from the data. A broad overview of participants' experiences will be provided. During the think-aloud sessions, virtually all students demonstrated some degree of processing in all four phases of the SRL model developed in the theoretical framework (see Table 7). Differences and similarities between participants and their attributes will be discussed at length below. It should be noted that the *performance control* phase was further subdivided into five subcategories, as follows: *self-control and self-observation, problem-solving explorations and implementations, general and heuristic strategy use, monitoring, and transitions*. Additionally, participants' capacity to appropriately monitor their progress and transition to more purposeful problem-solving plans emerged as a major finding during analysis and thus, pervades much of the discussion.

Due to the nature of group problem solving, credit for problem interpretations and development of problem solving plans during the AP exam preparation sessions was often blurred. Thus, coding was done only for the *performance control* and *self-reflection* phases and only if the *participant* was actively involved in the action. The 6 individuals identified via sampling are referred to as *participants*; whereas the remaining 24 individuals in the class are referred to as *students*. In other words, participants' actions were coded at an individual level for their active participation in monitoring group problem solving during the AP exam preparation sessions. Classroom observations and, to a lesser degree, participants' journals provided data from the AP exam preparation



sessions. Yet again, all participants demonstrated some level of self-regulatory (or group-regulatory) activity during their two classroom observations (see Table 8). Further discussion of similarities and differences amongst participants and their attributes may be found below.

Table 7

*Coding Frequencies for SRL Processing During Think-Aloud Problem-Solving Sessions*

Participant	DT	FO	PC	SR
Robert	16	30	98	7
Edwina	10	14	8	2
Julia	18	34	64	2
Olivia	6	8	13	0
Martin	15	25	29	4
Cameron	9	33	41	7

*Note.* DT = definition of the task; FO = forethought; PC = performance control; SR = self-reflection

### Categorizations

Each participant's performance on the think-aloud problems was quantified using Schoenfeld's (1982) grading scheme. Performance on the AP Calculus exam preparation problems was assessed via scoring guidelines developed by College Board. (See *Chapter IV: Individual Case Results* for further details.) Participants were categorized based on

their performance during the think-aloud problem solving sessions and the AP Calculus exam preparation sessions. Students were placed into categories based on their performance from both sessions as indicated by the following intervals: *low* ( $M - 1.5SD$ ,  $M - 0.5SD$ ), *average* ( $M - 0.5SD$ ,  $M + 0.5SD$ ), and *high* ( $M + 0.5SD$ ,  $M + 1.5SD$ ). For the think-aloud sessions, the mean score was  $M = 47.33$  ( $SD = 22.39$ ), resulting in the following respective performance intervals: (13.75, 36.14), (36.14, 58.53), and (58.53, 80.92). For the AP exam preparation sessions, the mean score was  $M = 5.40$  ( $SD = 1.23$ ), resulting in the following respective performance intervals: (3.56, 4.79), (4.79, 6.02), and (6.02, 7.25).

Table 8

*Coding Frequencies for Group Problem-Solving Behaviors During AP Exam Preparation*

Participant	FR	SD	OM	VS	WL	FA	WQ
Robert	5	2	2	4	2	2	2
Edwina	2	3	0	1	4	7	0
Julia	1	2	1	1	0	0	4
Olivia	1	2	4	2	0	1	3
Martin	1	1	6	2	0	0	3
Cameron	0	1	8	3	0	0	3

*Note.* FR = feedback request; SD = self-disclosure; OM = other-monitoring;

VS = verifying solutions; WL = watching and listening; FA = following

along; WQ = working quietly

The performance results for both sessions yielded the same categorization for each participant and were relatively consistent with each participant's original *calculus achievement* categorization obtained for sampling purposes. Although some ordinal variation occurred within the three performance indicators, all six participants' overall categorizations were consistent across all three indicators. Thus, for the purposes of cross-case analysis, Robert and Edwina were categorized as *low* achievers, Julia and Olivia as *average* achievers, and Martin and Cameron as *high* achievers with respect to calculus performance prior to and during the study.

Throughout the study, participants' actions and verbalizations suggested adherence to particular epistemological beliefs. Based on Muis's (2008) cautions, participants' epistemological beliefs were not determined based on a single data source. Instead, participants' verbalizations and actions during the four parts of the think-aloud session were given overall codes for beliefs that manifested as participants worked. Then participants' interviews were coded for overt references to particular epistemological beliefs. Based on these manifested and self-reported beliefs, participants were categorized as predominantly adhering to either an availing or non-availing persuasion for each belief emerging during this study (See Table 9). Lacking a high degree of confidence in assigning categorizations based on a close proximity of coding frequencies, participant categorizations were assigned only if the difference between the frequencies for a given belief exceeded two. For example, Olivia was described as predominantly *straightforward* in terms of *simplicity of knowledge* since the difference in frequencies was three but was described as *procedural/conceptual* for *conceptual mathematical problem-solving* since the difference in frequencies was only two. When possible,

participants' categorizations were compared to self-report questionnaire findings to either triangulate or call to question the assigned categorization. Such comparisons were presented in participants' narratives (see *Chapter IV: Individual Case Results*). Additionally, participants were probed during member checking to further inform categorizations. None of the participants disagreed with the categorizations.

Only two issues were discussed during member checking regarding findings. To maintain confidentiality, participants' pseudonyms will not be identified during this discussion. One participant described in detail the reasoning for exploring a solution strategy during the application of differentiation problem. The wording in the original draft narrative indicated that the strategy lacked conceptual reasoning. The explanation provided by the participant was inserted into the narrative to more accurately depict the cognitive reasoning involved in exploring the strategy. Another participant agreed with the accuracy of the findings but was displeased with the overall tone of the narrative. This participant felt that the wording in the narrative was overly negative. This reaction prompted a re-evaluation of the wording for all participants' narratives. Wherever possible, changes were made to eliminate both explicit and implicit negative connotations while maintaining accuracy of participants' experiences.

Finally, based on the literature, the following belief codes (see Table 9) were considered *non-availing* for their respective beliefs dimension: *fixed*, *straightforward*, *procedural*, and *empirical* (Hofer, 2000; Hofer & Pintrich, 1997; Kloosterman & Stage, 1992; Muis, 2004, 2008; Royce & Mos, 1980; Schoenfeld, 1983, 1985). The remaining codes were considered *availing* for their respective beliefs dimension. Thus, for the purposes of cross-case analysis, Robert, Edwina, and Julia were assigned a *novice*

mathematical problem-solving beliefs (MPB) categorization since they had non-availing beliefs for all dimensions. Olivia and Martin were assigned an *emerging* MPB categorization since the majority of their beliefs dimensions were indeterminate. Finally, Cameron was the only participant to attain the *advanced* MPB beliefs categorization because he was assigned availing beliefs for all dimensions.

Table 9

*Coding Frequencies for Epistemological Beliefs*

Participant	Nature of solutions		Simplicity of knowledge		Approaches to problem solving			
	UN	AM	ST	IN	PR	CO	EM	RA
Robert	9*	0	8*	0	11*	1	7*	1
Edwina	6*	0	4*	0	5*	1	4*	0
Julia	10*	2	6*	1	9*	1	7*	1
Olivia	4	2	5*	2	6	5	3	4
Martin	0	3*	4	4	7	5	5	5
Cameron	0	9*	0	7*	0	9*	1	8*

*Note.* Numbers with asterisks indicate that the participant was categorized as predominantly adhering to the corresponding belief. UN = unique; AM = ambiguous; ST = straightforward; IN = interrelated; PR = procedural; CO = conceptual; EM = empirical; RA = rational

### Cross-Case Analysis

Based on individual participant narratives and categorizations, patterns and themes emerged from the data. In most cases, overall quantitative tabulations of open coding provided insufficient descriptions of actual events and thus, deeper qualitative analyses were required to accurately depict the emerging patterns and themes. In fact, initial results of summed open codes of SRL processing seemed to indicate a surprising result. Robert and Julia, two participants attributed with non-availing MPB, registered the most codes for SRL processing in this study (see Table 7). However, as discussed in depth below, a more accurate depiction of participants' experiences was obtained from a finer-grained, qualitative interpretation of the data. Using NVivo Version 8 for axial coding, selective coding, textual analysis, and matrix development, the nature of patterns and themes was revealed (Bazeley, 2007).

#### *Beliefs Affected Definition of the Task and Forethought*

Only think-aloud session data were analyzed for this section since AP Calculus exam preparation sessions provided insufficient evidence for individual participants. From a purely quantitative perspective, the results pertaining to the *definition of the task* and *forethought* phases appeared inconclusive (see Table 7). Thus, deeper qualitative analyses were conducted to gain a clearer rendition of lived events and reveal findings hiding behind the numbers. To begin, participants' frequencies for the three finer-grained codes under the *definition of the task* node were analyzed and revealed distinct patterns (see Table 10). Despite logging the majority of codes for *noting conditions* and *identifying goal state*, the three novice MPB participants logged fewer instances of *establishing relationships between conditions and goals* than the other three participants.

Participants were assigned the *establishing relationships* code if they made mathematical connections between conditions and goals that were conducive to plan development (Schoenfeld, 1985). Thus, despite referring to the given information multiple times for each part of the two problems, novice participants were relatively unsuccessful at establishing an adequate *definition of the task* upon entering the *forethought* and *performance control* phases. In contrast, the *emerging* and *advanced* MPB participants were more successful at establishing relationships and developing a more adequate and appropriate *definition of the task*. This finding is congruent with prior research suggesting that epistemological beliefs may be activated during the *definition of the task* phase and subsequently affect planning in the *forethought* phase (Muis, 2008; Muis & Franco, 2009).

A finer-grained analysis of participants' frequencies of *forethought* phase codes also revealed further insights (see Table 11). Participants' coding frequencies for *prior knowledge activation* provided evidence, along with prior course completion, of the availability of appropriate calculus resources for completing the given problems. Participants received the *prior knowledge activation* code if they activated *accurate* mathematical knowledge that had potential for aiding in the solution of the given problem. This code in no way implies that the participant properly utilized the knowledge, only that the knowledge was available for use. Edwina appeared to be the only notable exception, demonstrating insufficient prior knowledge to complete the problems. Although she passed the previous course, Edwina genuinely appeared to lack the necessary resources to complete both problems and would have received a very low overall think-aloud score had she not consulted an expert during the interim between

sessions. Thus, conclusions drawn with respect to Edwina's data must be tempered with the fact that she lacked the appropriate resources to complete the problems. All other participants were able to activate the appropriate calculus knowledge but had varying success in applying the knowledge.

Table 10

*Total Frequencies for Definition of the Task Behaviors*

Participant	NC	IG	ER
Robert	14	6	1
Edwina	8	4	0
Julia	11	4	2
Olivia	3	3	2
Martin	9	5	3
Cameron	4	4	4

*Note.* NC = noting conditions of the problem; IG = identifying goal state; ER = establish relationships between conditions and goal

According to Zimmerman (2000), an important product of the *forethought* phase is a plan for successfully completing the learning task. With the exception of Olivia, all participants verbalized overt plans for developing problem-solving strategies (see Table 11). Although Olivia's work indicated significant planning, her verbalizations during the think-aloud sessions were vague and skipped from idea to idea with little indication of the



planning involved. This occurred despite providing her with multiple practice think-aloud problems (Ericsson & Simon, 1993). Since little data from the forethought phase was obtained from Olivia, only the five other case participants are included in the following discussion. To assess these five participants' planning acuity, a comprehensive, qualitative analysis of participants' verbalized *overt* plans was conducted based on their beliefs (see Tables 12 and 13). Using NVivo Version 8, participants were placed into sets based on their beliefs (i.e., unique or arbitrary, procedural or conceptual, empirical or rational) and codes for *definition of the task* and *forethought* verbalizations were analyzed qualitatively via axial coding (Bazeley, 2007; Creswell, 2007). Patterns and themes emerging from the data suggest that availing beliefs are related to more productive processing during the definition of the task and forethought phases of SRL.

#### *Unique Versus Arbitrary*

The *unique* belief regarding problem solutions was negatively related to the *definition of the task* and *forethought* phases. For example, Julia, categorized as a participant holding a *unique* belief regarding problem solutions, exhibited a major misconception concerning  $m$  in part (a) of the application of integration problem. She believed a unique value needed to be assigned to  $m$  and as long as  $m$  was any positive integer, then an accurate solution could be obtained. Assigning a unique value to  $m$  would have been consistent with applying Polya's (1957) *solving a simpler problem* heuristic but solution paths for both parts of the application of integration problem had to be involve  $m$ . This inaccurate assessment of the task led to a flawed plan being developed in the *forethought* phase, involving serial testing of upper bounds to obtain a solution for her special case (see Table 13). Her plan was flawed because the solution of her simpler

problem did not lead to a solution path which could be generalized (Schoenfeld, 1985). Thus, Julia's *unique* belief in the value of  $m$  in the problem led to a poor definition of the task and subsequently, to an unproductive plan from the forethought phase. The finding from this study suggesting a relationship between a unique belief regarding problem solutions and academic deficiencies is consistent with prior research (Neber & Schommer-Aikins, 2002; Schoenfeld, 1992). Furthermore, Neber and Schommer-Aikins suggested that the lack of ambiguity in high school physics courses is particularly detrimental to motivation and SRL strategy use in gifted students. These results also lend support to Muis' assertions that beliefs are activated in the definition of the task phase of SRL (Muis, 2007, 2008; Muis & Franco, 2009).

Table 11

*Total Frequencies for Forethought Behaviors*

Participant	OP	IP	PK	RG
Robert	13	3	8	7
Edwina	10	1	1	2
Julia	11	7	8	7
Olivia	0	5	4	0
Martin	14	3	5	3
Cameron	16	6	10	3

*Note.* OP = overt planning; IP = inferred planning; PK = prior knowledge activation; RG = recycle goal in working memory

In contrast to Julia's efforts, Martin's work with  $m$  in part (a) of the application of integration problem demonstrated that an *arbitrary* belief regarding problem solutions may lead to an appropriate definition of the task. He was fully aware that his solution should be in terms of  $m$  and thus, developed a plan that incorporated  $m$  into the solution (see Table 13). His plan led to a solution receiving almost full credit for part (a) but lack of attention to detail stymied his solution of part (b).

*Procedural Versus Conceptual and Straightforward Versus Interrelated*

Patterns were uncovered indicating that procedural versus conceptual and straightforward versus interrelated beliefs affected the *forethought* phase. In general, participants categorized as availing for both sets of beliefs developed more productive plans than their non-availing beliefs counterparts (see Tables 12 and 13). For example, Robert's fixation on finding a unique solution for  $a$ ,  $b$ ,  $c$ , and  $d$  led to his decision to apply systems of equations to the application of differentiation problem. Despite receiving feedback from monitoring that his plan was not working, Robert continued to revert to his procedural technique and cited problem recognition from prior courses as his reasoning. This pattern continued during the application of integration problem, for which Robert was able to apply the procedure of calculating area but failed to develop a conceptual plan to find the necessary bounds. Additionally, once Robert realized that a procedure had failed, his planning based on monitoring generally consisted of considerations of other fully-intact procedures without regard to conceptual implications, which took him farther from the problem goals (see Table 13). Thus, Robert's planning consisted of random procedures inconsistent with the conditions and goals of the problem. Findings based on Robert's experiences lend support to Hofer's (2004a)

Table 12

*Participants' Verbalizations of Overt Planning During the Application of Differentiation Problem*

Participant	Overt Planning
Novice	
Robert	<p>Part (a)</p> <ul style="list-style-type: none"> <li>- go ahead and solve for <math>x</math>, I would guess</li> <li>- solve things in terms of things and substitute that back in</li> <li>- going to be setting [derivative] equal to zero</li> <li>- try to plug this into my calculator</li> </ul> <p>Part (b)</p> <ul style="list-style-type: none"> <li>- take the second derivative of this</li> <li>- need to find values for <math>x</math> that are positive</li> </ul>
Edwina	<p>Part (a)</p> <ul style="list-style-type: none"> <li>- take the derivative . . . set it equal to, to something, to zero, and that can give me one root</li> </ul> <p>Part (b)</p> <ul style="list-style-type: none"> <li>- maybe I'll try solving for <math>x</math>. . . put my function into the calculator</li> <li>- just try taking the second derivative . . . see what I can get from that</li> <li>- I'll try a sign line</li> <li>- could plug negative two-thirds back into the original equation and see what I get</li> </ul>

Table 12 (continued)

Participant	Overt Planning
Novice	
Julia	<p>Part (a)</p> <ul style="list-style-type: none"> <li>- gonna set and find what <math>x</math> is when it's equal to zero, or when <math>x</math> is zero</li> <li>- set the derivative equation and use it as two functions</li> <li>- find what <math>x</math> is at zero</li> </ul> <p>Part (b)</p> <ul style="list-style-type: none"> <li>- use the derivative I already have . . . derive that again</li> </ul>
Emerging	
Olivia	No <i>overt</i> planning coded.
Martin	<p>Part (a)</p> <ul style="list-style-type: none"> <li>- Maybe I could try and use quadratic [formula]</li> <li>- I can say those two functions are equal</li> <li>- I plug that back into the original function and set equal to zero</li> <li>- I have everything but <math>a</math>, I think, can I use a calculator?</li> <li>- if I were to solve for <math>x</math></li> <li>- make the first one negative and the rest positive, I can try that</li> </ul> <p>Part (b)</p> <ul style="list-style-type: none"> <li>- have to prove that <math>6ax + 2b</math> is always going to be greater than zero</li> </ul>

Table 12 (continued)

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Participant	Overt Planning
Advanced	
Cameron	Part (a) <ul style="list-style-type: none"><li>- have to create <math>a</math>, <math>b</math>, <math>c</math>, and <math>d</math> so that there are three distinct values for that</li><li>- that would just simplify everything if I set <math>a</math> equal to one</li><li>- make the point of inflection at point zero</li><li>- I can now find <math>c</math></li><li>- I guess I'll need points for <math>x</math></li><li>- I've got to find the . . . relative minimum</li><li>- need to somehow get two roots out of that . . . let's just plug it into the calculator</li><li>- to find out which one was the minimum, I plug in negative one and three into the double prime</li></ul>
	Part (b) <ul style="list-style-type: none"><li>- have to prove that somehow</li></ul>

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Table 13

*Participants' Verbalizations of Overt Planning During the Application of Integration**Problem*

Participant	Overt Planning
Novice	
Robert	Part (a) <ul style="list-style-type: none"> <li>- need to probably find the area</li> <li>- have to find the line that bounds that</li> </ul> Part (b) <ul style="list-style-type: none"> <li>- have to figure out where the other two intersect</li> <li>- have to find the horizontal line that divides it in half</li> <li>- graph this on my calculator</li> <li>- do just some, I don't know, proportions</li> </ul>
Edwina	Part (a) <ul style="list-style-type: none"> <li>- go ahead and graph this, the lines</li> <li>- let's see what happens when I put in a number besides one</li> </ul> Part (b) <ul style="list-style-type: none"> <li>- draw this graph out as well</li> <li>- we will solve for area right now</li> <li>- maybe I can estimate the area in my graph</li> </ul>

Table 13 (continued)

Participant	Overt Planning
Novice	
Julia	Part (a) <ul style="list-style-type: none"> <li>- going to graph that real fast</li> <li>- guess I'll solve for <math>m</math></li> <li>- guess I'll solve it without finding <math>m</math> . . . plug it into my calculator</li> <li>- set 0.4323 equal to the integral of <math>h(x)</math></li> </ul> Part (b) <ul style="list-style-type: none"> <li>- find the horizontal line that divides <math>A</math> exactly in half</li> </ul>
Emerging	
Olivia	No <i>overt</i> planning coded.
Martin	Part (a) <ul style="list-style-type: none"> <li>- I'm going to do two first</li> <li>- let's figure out what <math>A_1</math> is equal to</li> </ul> Part (b) <ul style="list-style-type: none"> <li>- let's graph this</li> <li>- just plug that into my calculator</li> <li>- going to have to half that, define where my <math>z</math> is</li> <li>- what if I try to solve this by hand</li> </ul>



Table 13 (continued)

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Participant	Overt Planning
Advanced	
Cameron	Part (a) <ul style="list-style-type: none"><li>- so to integrate that</li><li>- the integral from zero to <math>a</math> . . . will equal the integral from <math>a</math> to two</li><li>- just have to find a way to solve for <math>a</math></li><li>- in order to justify, I'll have to plug that entire value back into the bound</li></ul>
	Part (b) <ul style="list-style-type: none"><li>- have to write <math>h(x)</math> . . . as a function of <math>y</math></li><li>- just have to . . . the integral now from one-half to <math>a</math></li><li>- going to the solve command</li><li>- plug in that value</li></ul>

---

suggestions that epistemological beliefs are not independent, but rather form an interconnected theory of epistemology since Robert's *unique* belief regarding problem solutions yielded *procedural* problem-solving planning.

On the other hand, Cameron's plans for both problems were based on conceptualization of the conditions and focused attention to goals. Thus, upon receiving feedback from monitoring, he was able to cycle back to forethought and alter plans to meet problem criteria. In contrast, Robert's procedural plans were not malleable since they were based on prescription. In sum, as feedback was taken in, conceptualization led to purposeful planning; whereas a procedural approach implied the need for a fully-established procedure as a new plan. For the mathematical problem-solving suggested by Schoenfeld (1985) and implied by NCTM (2000), procedural beliefs would be insufficient since students are applying problem-solving to learn, or construct, new knowledge.

#### *Empirical Versus Rational*

In line with prior studies, differences were noted between problem-solving behaviors for participants categorized as empirical or rational (Muis, 2008; Schoenfeld, 1985). The results of this study indicate differences in the *forethought* phase for participants with predominantly empirical or rational beliefs. For instance, Edwina, a *novice* MPB participant, read the application of differentiation problem and then recalled that roots of the first derivative produce horizontal tangents. Unable to progress from this point, Edwina applied a series of empirically-based plans for the remainder of her first session. In terms of SRL, Edwina received internal feedback from monitoring that her current plan was not working, which she verbalized during the think-aloud. Then, rather

than building logically from her current state, Edwina simply *jumped into* a new plan, thus reducing the productivity of planning that occurred from cycling back to the *forethought* phase. The interpretations gleaned from Edwina's empirical beliefs must be tempered with the fact that she did not appear to have the appropriate mathematical resources to solve the problems. In fact, during this session she could not recall whether the first or second derivative described concavity. Serial testing of mathematically-based plans that lacked logical significance to the current problem was prolific amongst all participants while empirical beliefs were being manifested. In fact, Julia spent the majority of her second session engaged in serial testing of decimal upper bounds for an integral expression that did not even provide the desired solution.

Cameron, the only predominantly rational participant based on actual performance, produced plans that were highly productive and based on logical connections between conditions and goals. For instance, at one point during part (a) of the application of differentiation problem, Cameron's assessment of the problem space revealed that he needed to determine two  $x$ -values for the horizontal tangents. Cameron developed a logically based plan to place the  $x$ -values equidistant from the point of inflection that he had determined, which is a property of cubic functions with two extrema. This property of cubic functions had never been addressed in class; Cameron's conceptualization of the cubic function allowed him to develop this justification. Thus, he was building a justification for his solution as he solved the problem since his work provided a logical mapping of the problem space.

### *Beliefs Affected Performance Control*

#### *Think-Aloud Problem-Solving Sessions*

*General overview.* Based solely on the performance control data from Table 7, findings appear mixed and inconsistent. Thus, deeper qualitative analyses necessitated grouping codes into sets using NVivo Version 8 for the purposes of axial and selective coding (Bazeley, 2007; Creswell, 2007). All beliefs coded as *non-availing* and *availing* were included in two sets, regardless of the overall MPB categorization of individual participants. Thus, availing and non-availing beliefs became the units of analysis. Then, monitoring and transitioning activities were further subdivided into *advanced* and *low-level* categories. An example of *advanced* monitoring is monitoring progress based on implementation of a plausible plan; whereas an example of *low-level* monitoring is monitoring strategy use while embarking on a fruitless exploration of the problem space. An example of *advanced transitions* is assessing the appropriateness of the new direction; whereas an example of *low-level* transitions is jumping into a transition without considering ramifications. Finally, all deficit-focused self-observations were included as a set in NVivo Version 8. The results indicate that a relationship existed between the *quality* of performance control processing and participants' beliefs (see Table 14). This finding provides support for Muis' hypothesis that epistemological beliefs enacted in earlier phases of SRL affect SRL processing in latter phases (Muis, 2007, 2008; Muis & Franco, 2009).

More specifically, manifestations of participants' beliefs affected their ability to control problem-solving performance (see Table 14). Cameron, the only *advanced* MPB participant, accounted for virtually all instances of advanced monitoring and transitions.

One may question whether other factors, such as prior performance and accessibility of mathematical resources, may provide alternative hypotheses for Cameron's findings. Martin's results provided a negative case for the prior performance and mathematical resource hypotheses. Despite his *high* achieving categorization and evidence of sufficient prior knowledge activation, Martin, an *emerging* MPB participant, produced mainly low-level monitoring and transitioning codes. Martin's activities in the performance control phase coupled with his *emerging* MPB categorization provided a compelling counterexample for alternative hypotheses. The researcher fully acknowledges that SRL processing is related to multiple factors. The purpose of this study was to examine relationships between SRL and a single factor, epistemological beliefs. The results of this study indicate that epistemological beliefs are related to SRL processing. Further research is warranted to assess the *degree* to which epistemological beliefs (and other factors) are related to students' regulation of mathematical problem-solving tasks.

On the opposite end of the spectrum, Robert and Julia, while they appeared to dominate SRL processing in the study, were actually involved in only low-level performance control activities and deficit-focused verbalizations. Finally, Edwina and Olivia provided the fewest codes for performance control processing in the study. The performance control codes noted for Edwina and Olivia were almost exclusively low-level. Future investigations of these phenomena will require even deeper qualitative analyses into specific beliefs dimensions.

Table 14

*Coding Frequencies for Advanced and Low-level Performance Control**Behaviors*

Source	AM	AT	DF	LM	LT
By belief categorization					
Availing	34	10	1	6	5
Non-availing	0	6	40	90	39
By participant					
Robert	0	2	23	35	13
Edwina	0	1	12	15	2
Julia	0	2	4	18	18
Olivia	1	1	1	10	2
Martin	2	1	0	17	5
Cameron	31	8	1	1	3

*Note.* AM = advanced monitoring; AT = advanced transitions; DF = deficit- focused verbalizations; LM = low-level monitoring; LT = low-level transitions

*Unique versus arbitrary.* As mentioned above, participants' *unique* and *arbitrary* beliefs affected their actions in the *definition of the task* and *forethought* phases. Since activities in both phases directly affect the performance control phase, participants' *unique* and *arbitrary* beliefs regarding problem solutions also affected their strategy use and control as they navigated the problem space (Muis, 2007, 2008; Winne & Hadwin,

1998; Zimmerman, 2000). For example, Robert, a *unique* belief participant, became so fixated on finding a unique solution for part (a) of the application of differentiation problem that he spent his entire first session engaged in a fruitless attempt to solve for  $a$ ,  $b$ ,  $c$ , and  $d$  via a system of equations. He monitored this plan fairly closely, checking for errors and assessing the validity of his substitutions. However, despite doubts and deficit-focused statements regarding his plan, Robert never transitioned to other strategies. He later stated in an interview that he expected a unique solution for  $a$ ,  $b$ ,  $c$ , and  $d$  but did not know of any other technique to find it. So, his belief that a single, unique solution existed negatively impacted his ability to make executive, transitional decisions based on his monitoring. Lerch (2004) suggested an alternative explanation to the one presented above. She argued that Schoenfeld (1985) disproportionately attributed students' control issues to their lack of success at mathematical problem solving. Lerch suggested that Schoenfeld's *mathematical resources* category in tandem with students' beliefs provided a more accurate explanation for students' struggles with mathematical problem solving during her case study. However, the findings in this study show that students were able to access the appropriate mathematical resources but failed to make controlled executive decisions regarding how to apply these resources in the given context.

In direct contrast, Cameron, who maintained an *arbitrary* belief throughout both think-aloud sessions, engaged in a well-controlled plan for part (a) of the application of differentiation problem. His plan consisted of building a cubic function based simultaneously on arbitrary assignments and conceptualizations. His belief that multiple solutions existed and that he could develop *a* solution (rather than *the* solution) as knowledge of the conditions and goals of the task unfolded allowed him to make

executive decisions which led to prosperous transitions. Initially, however, Cameron was no different from other participants. He was visibly and audibly shocked when first faced with the application of differentiation problem, which involved finding a solution for four variables with two equations. In his first solution attempt, he applied the quadratic formula to the equation formed by setting the first derivative function equal to zero. The difference in Cameron's work within the problem space was his ability to abandon this procedural method and seek non-standard techniques of solution, which eventually led to the plan outlined above and ultimately to a successful solution.

Robert and Cameron were on opposite ends of a continuum of beliefs regarding the nature of problem solutions. Olivia was closer to the middle of the continuum and provided a different set of experiences to examine (see Table 9). For the application of differentiation problem, Olivia's actions and verbalizations indicated both *unique* and *arbitrary* manifestations of beliefs. For instance, she reacted to part (a) just as Robert and tried to apply algebraic systems of equations to find unique values for  $a$ ,  $b$ ,  $c$ , and  $d$ . Then, for part (b), she seemed completely comfortable with the arbitrary constants and developed and executed an argument for the lack of a solution with little effort. Her interview responses provided triangulation for these findings. For instance, when probed to expound on her actions during session one, her responses indicated a clear understanding that  $a$ ,  $b$ ,  $c$ , and  $d$  generated a family of cubic functions but simultaneously indicated the expectation of a unique solution for part (a). In sum, her actions and verbalizations indicated the belief that an infinite family of cubic functions existed but did not carry over to developing an arbitrary member of the family based on the problem conditions. Finally, the findings discussed in this section indicate that students' beliefs



regarding the nature of problem solutions affected actions in the *performance control* phase while solving mathematical problems.

*Procedural versus conceptual and straightforward versus interrelated.* Due to similarities, discussion regarding manifestations of procedural versus conceptual problem solving beliefs and straightforward versus interrelated beliefs in the simplicity of knowledge are conducted together. Edwina exhibited non-availing beliefs for both sets of beliefs. In both sessions, Edwina's actions and verbalizations indicated that she was expecting a *procedural* method for solving the problems. Additionally, her problem-solving strategies always consisted of *straightforward* applications of one concept at a time, never attempting to *interrelate* topics or find connections. For example, Edwina was able to apply all the procedural steps necessary for success on part (b) of the application of differentiation problem. She calculated the first and second derivatives of  $f$  and even attempted to apply sign line procedures, indicating that she had sufficient mathematical resources for this part of the problem despite her overall lack of resources demonstrated in other parts of the study. When her procedural efforts failed, Edwina applied a useful heuristic by graphing the second derivative, which produced a line passing through the origin. Lacking a *conceptual* or *interrelated* mindset, Edwina determined the graph to be useless for conclusions about the concavity of the function  $f$ . Thus, her beliefs stifled the development of productive transitions from her assessment of the current state of the problem. In other words, she was unable to interrelate the properties of linear functions with the conceptual aspects of concavity and the second derivative.

As mentioned above, Lerch (2004) advocated for lack of mathematical resources as the primary source of students' failures at problem solving. She further argued for a "mathematical process model" that would provide "universal procedures" to be applicable for a variety of problem-solving situations (p. 34). Thus, her argument for students' success is grounded in specific problem-solving strategies that transcend problem types. Based on the findings in this study and especially with regard to Edwina's experiences above, I advocate a more *conceptual*-beliefs approach to mathematical problem-solving that would foster a more controlled navigation of the problem space based on conditions and goals, rather than predetermined *procedures* that may or may not be accessible in long term memory.

Edwina's procedural mindset carried through the entire problem, as indicated by her lamentations at the end of the session of not having seen *how to solve* the problem before it was presented to her. As indicated by Schoenfeld (1985), however, if one has seen how to work a mathematical *exercise*, it is no longer a mathematical *problem*. Finally, her inability to develop a procedural method for either problem led to a significant number of deficit-focused verbalizations (see Table 14), which may explain why she gave up before either session had ended (Zimmerman, 2000).

Cameron's work indicated purely *conceptual, interrelated* beliefs throughout both sessions and painted a very different picture than Edwina's. As mentioned previously, Cameron was initially stumped by the situation of having two equations and four variables for part (a) of the application of differentiation problem but was able to navigate through the problem space due largely to monitoring which led to productive transitions. An example of his belief that concepts are interrelated emerged as he considered the

second derivative and applied it to his solution path. Other participants considered the second derivative but only Cameron monitored its introduction and transitioned to a plan involving a single point of inflection for  $f$  and the application of the second derivative test. Every other participant who introduced the second derivative expected the extra equation to aid in eliminating variables for the purposes of their systems of equations procedure. Thus, their assessment, monitoring, and subsequent transitions based on the introduction of the second derivative were fruitless. There were no previously learned algorithmic procedures for solving part (a) of the application of differentiation problem, which provided the novelty necessary for true *problem* solving (Schoenfeld, 1985). Thus, Cameron's conceptual, interrelated belief system provided the perfect engine for controlled performance for this problem.

Martin, whose MPB categorization was indeterminate, provided a unique perspective. Despite maintaining an *arbitrary* belief and indicating that part (a) of the application of differentiation problem may have infinite solutions, Martin *procedurally* applied system of equations techniques during the majority of his first session, which indicated the expectation of obtaining a single solution. Thus, like Robert, his strict adherence to a system of equations approach made purposeful transitions impossible, as he considered all assessments of monitoring from his procedural perspective. However, unlike Robert, his logic for applying procedures did not evolve from a fixed belief, rather he felt that justifications of mathematical work must come from rigorous, algebraic procedures. Thus, his close-minded belief that mathematics problems must be solved systematically led to the demise of any productive performance control activities during his first think-aloud session (Schoenfeld, 1992).

During his second session, Martin demonstrated the dual nature of his beliefs via productive work grounded mainly in *conceptual, interrelated* decision-making, which produced the majority of his high-level performance control codes. Despite these manifestations of his *conceptual* and *interrelated* beliefs, his procedural mindset did emerge during the second session and hampered part of his work. Upon obtaining a solution with only minor errors for part (a) of the application of integration problem, Martin assumed that part (b) was procedurally the same and thus, worked part (b) in exactly the same manner as part (a). When he was unable to solve part (b), Martin spent all of his efforts in the performance control phase monitoring his procedures for errors but never considered to monitor the more global, conceptual aspects of the problem. Thus, Martin's lack of conceptual focus and attention to procedures inhibited his transition to a more productive plan and ultimately resulted in an unsuccessful solution attempt.

*Rational and empirical problem-solving beliefs.* Participants' rational and empirical approaches to problem solving also affected performance control. Julia's actions and verbalizations indicated an *empirical* belief in problem solving, which was consistent with her high score on the Empirical scale of the PEP. Virtually all activities during her think-aloud sessions were coded as empirical (see Table 9). The most overt example of her empirical beliefs occurred as she worked part (a) of the application of integration problem during session two. Having found the area of the bounded region using a special case for  $m = 1$ , she was unable to develop a solution strategy to find an equation of the vertical line that divided the area in half. Thus, she began arbitrarily substituting values in for the upper bound of her integral in an attempt to find the

numerical value that corresponded to half the area. This empirical method of solution would eventually lead to a fairly accurate answer but no generalizations conducive for finding a solution with respect to  $m$ . At any point, she could have monitored her progress and *rationally* determined that her serial testing of values was equivalent to finding an unknown value and that a variable could be assigned to the upper bound. However, her overt empiricism led her to merely check her serial tests for accuracy, a very low level of monitoring, until she had an acceptable value. Thus, Julia's lack of logical reasoning led to low-level monitoring that produced assessments of a flawed plan, rather than generating internal feedback to foster advancement toward the goal state.

Virtually the opposite result occurred for Cameron, who demonstrated rational beliefs throughout the sessions. While working on part (a) of the application of integration problem, he began by determining the area just as Julia had, except his area was in terms of  $m$ . Then, based on a logical assessment of the area with respect to the overall goal of the problem, Cameron determined that his area calculation was unnecessary and transitioned to an integral equation approach that yielded an accurate solution. Despite having far fewer codes for monitoring than Julia during this part of the problem, Cameron's performance control processing was logically-based and productive. Cameron's lower frequency of monitoring may not be surprising, as Zimmerman (2000) suggested that students who attain a higher level of *self-regulatory skill* for a given task will require fewer instances of monitoring to obtain successful results. Further discussion of *self-regulatory skill* is provided below. Both Cameron and Julia's findings are supported by previous literature examining rational and empirical beliefs with respect to mathematical problem solving (Muis, 2008; Schoenfeld, 1983, 1985, 1988, 1989).

Despite having the highest Rational scale score in the class on the PEP, Martin's actual performance during the problem-solving sessions indicated even adherence to both empiricism and rationalism. This may be explained by his insistence on applying procedural techniques as described above. This insistence led him to many illogically-based decisions, which resulted in a serial testing of procedures. Hence, like Julia, the empirical side of Martin's beliefs led to low-level monitoring that produced a quantity of monitoring codes with little quality. However, when Martin applied more rational beliefs to his endeavors, mainly during the application of integration problem, his monitoring focused more consistently on the goal of the problem and led to more productive transitions.

#### *AP Exam Preparation Sessions*

*Overview.* Since AP exam preparation sessions involved small group collaboration, the unit of analysis became participants' interactions with their respective group members. As discussed above, this section will focus mainly on participants' behaviors in the *performance control* and *self-reflection* phases while working in groups. Thus, participants' behaviors were coded with respect to participation, or lack thereof, in assessments of both personal and group progress toward developing solutions for the problem. With increased focus on applying social constructivist ideologies in mathematics classrooms, assessments of students' abilities to work with others in a group setting are becoming more important.

As indicated by Table 8, all participants engaged in some level of group-monitoring and group-verification, which are synonymous to performance control and self-reflection, respectively. To provide deeper analysis, activities were classified as *non-*

*participatory*, *participatory*, or *highly participatory* group-regulation and were grouped into sets in NVivo Version 8. *Non-participatory* group-monitoring activities included *watching and listening* and *following along* since these codes indicated that the participant either stopped working or was merely following a plan without participating in its development (Artzt & Armour-Thomas, 1992; Goos, Galbraith, & Renshaw, 2002). *Participatory* group-monitoring activities included *self-disclosure* and *feedback request* since these codes indicated assessments focused solely on personal progression toward problem solutions (Goos, Galbraith, & Renshaw, 2002). Finally, *highly participatory* group monitoring included *other-monitoring* and *verifying solutions* since these codes indicated assessments focused on whole-group attainment of problem goals (Goos, Galbraith, & Renshaw, 2002). Participants' actions for all other group problem-solving codes had varying effects (e.g., both positive and negative) on group problem-solving productivity and thus, could not be grouped into sets.

Overall, the results indicated that beliefs do affect group monitoring and verification (see Table 15). Robert, Edwina, and Julia, the three *novice* MPB participants, accounted for the lowest mean for *highly participatory* codes ( $M = 2.67$ ) and the highest mean for *non-participatory* codes ( $M = 5.33$ ). Cameron, the only *advanced* MPB participant, had the highest frequency of *highly participatory* codes with ten codes and zero *non-participatory* codes. Then, Martin and Olivia had mean *highly participatory* codes ( $M = 6.50$ ) and mean *non-participatory* codes ( $M = 0.50$ ) which were between the *novice* and *advanced* MPB categorizations but skewed toward Cameron's figures.

*Novice MPB.* The three *novice* participants engaged in group activities differently based on multiple factors. Edwina, who demonstrated the least productive group

monitoring, *followed along* with every idea suggested by her group members without monitoring the validity of the assertions. She was on the receiving end of all monitoring activities with which she was involved based on *self-disclosures* and *feedback requests* verbalized to her other group members. Finally, she rarely engaged in group assessments of goal state. Robert was more involved in his group, which actively engaged in verifying solutions. However, Robert was prone to shutting down when he got stumped and reducing his actions to *watching and listening*. Robert rarely engaged in *other-monitoring*, but was on the receiving end of many instances of group monitoring based on *self-disclosures* and *feedback requests*. It should be noted that Edwina and Robert were both in groups in which another group member emerged as a *de facto* leader.

Table 15

*Coding Frequencies for Advanced and Low-Level Group Monitoring and Verification Behaviors*

Participant	Non-		Highly
	Participatory	Participatory	Participatory
Robert	6	7	5
Edwina	8	5	1
Julia	0	3	2
Olivia	1	3	5
Martin	0	2	8
Cameron	0	1	10



In stark contrast, Julia was in a group that did not have a clearly-defined leader and she did not volunteer herself as such. Her group spent much of its time engaged in silent work and only occasionally engaged in group monitoring. Julia's initiations of monitoring mainly involved *self-monitoring* and *group verification* of solutions. Due to her group's dynamics, Julia registered the fewest group monitoring codes. Thus, little more may be discussed concerning her participation in this portion of the study. Overall, participants with novice MPB tended to be very passive and assumed a role as followers, rather than actively monitoring group performance.

*Emerging MPB.* Olivia and Martin played a more active role in their groups' performance control. In fact, Martin assumed the role of *de facto* leader for his group and instigated the monitoring of others' work on six occasions. His participation in the group led to productive discussions, monitoring, and transitions based on conceptualizations of the problems. As his narrative indicates, Martin's availing beliefs emerged during this portion of the study. He seemed more comfortable with the problem types and was able to develop more logically-based, conceptual ideas based on his monitoring of the problem space. Recall that participants' member-checking interviews indicated that the AP exam preparation problems more closely resembled *procedures with connections* than the *doing mathematics* think-aloud problems. Thus, Martin's increased comfort level may be explained by the change in cognitive demand. This finding is congruent with Lodewyk, Winne, and Jamieson-Noel's (2009) suggestions that high achieving students do not require *self-regulatory* processing to complete a well-structured task, which may have resulted in Martin having sufficient time and resources to monitor *group* performance during the AP exam preparation problems. Despite suggesting that

educators provide a mixture of well- and ill-structured tasks for students, the authors warned that a preponderance of well-structured tasks may lead to boredom and apathy amongst high-achievers. This may explain the apathetic appearance of Martin's group during their second observation since it was the last of the twelve in-class AP Calculus exam preparation sessions.

Olivia was involved in a group with no overt leader, although one of her group members dominated much of the group interaction. She was very active in all group interactions and was on both the receiving and giving ends of group-monitoring. Just like Martin, her more availing beliefs emerged during this portion of the study and provided conceptual tools by which she could convert group-monitoring into productive group transitions.

*Advanced MPB.* Like Martin, Cameron assumed the role of *de facto* leader for his group. Cameron's availing beliefs were evident in both the AP Calculus exam practice sessions and think-aloud problem-solving sessions. He logged the most instances of monitoring others of all participants and attempted to provide group members with conceptual understanding as well as performance control. Despite providing significant feedback, he rarely received feedback since he was able to conceptualize problem demands and required little self-monitoring during the sessions. In many cases, Cameron single-handedly solved problems and group members merely followed along, content to accept his assertions.

*Summary.* Participants with availing beliefs displayed more productive group-monitoring behaviors. This may be explained by a significant decline in *self*-regulatory processing for students with availing beliefs compared to the more ill-structured, doing

mathematics think-aloud session problems. All AP exam problems used in this study contained multiple parts (see Appendix E). For the first parts of AP exam problems, the general pattern of engagement was prior knowledge activation, set-up equations, apply procedures, and verify results. Then, the latter parts of the problems required significant metacognitive monitoring and critical thinking.

Cameron, whose beliefs and self-regulatory skills remained at an optimal level for the duration of the study, was a productive and active monitor for the group. For Olivia and Martin, the lower cognitive demand of the AP questions appeared to foster the more availing aspects of their beliefs than the higher cognitive demand think-aloud problems. With the decrease in abstraction, Olivia and Martin more readily connected problems to concepts and engaged in logically-based discussions. Then, with a more manageable problem to monitor, Olivia and Martin's self-regulatory skills could be applied in the group setting as needed. Unfortunately, the decrease in cognitive demand was insufficient in eliciting availing beliefs and productive regulatory behaviors from Robert, Edwina, and Julia. Although some variation existed between their individual results, an overall qualitative assessment revealed that all three participants generated limited behaviors indicative of collaborative group monitoring.

Group dynamics provide an alternative perspective for analyzing participants' group-monitoring behaviors. When asked to comment on the overall group-work experience, all participants except Robert made some reference to either disliking group work altogether or having problems with one or more group members. Specifically, Edwina did not feel comfortable asking a preponderance of questions of her group members because she felt that her questions would interrupt their development of a

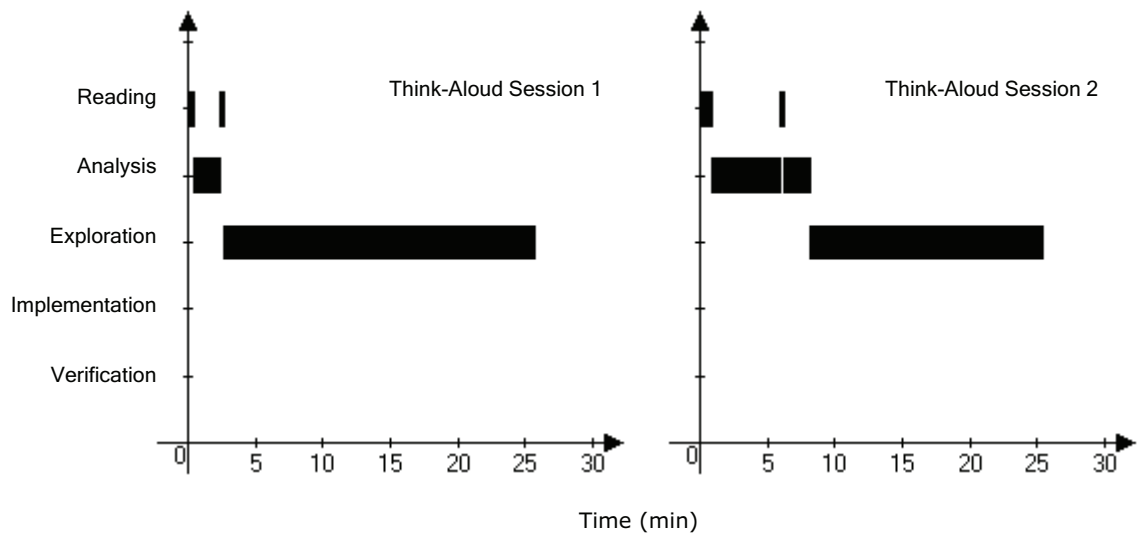
problem solution. Julia was not used to working in groups that do not interact but did not feel comfortable taking the leadership role since she was not the highest performer in her group. Martin felt that one of his group members could have interacted more. Olivia and Cameron stated that they have never enjoyed group work. Additionally, Olivia stated that she had never worked well with one of her group members. From a social cognitive perspective, participants' statements merged with their actions provided further evidence of the degree of their self-regulatory prowess. Zimmerman (2000), discussing the self-control aspect of performance control, stated, "*Attention focusing* is designed to improve one's concentration and screen out other covert processes or external events" (p. 19). Thus, at the theoretical level, group dynamics provide self-regulating students further opportunities to apply their adaptive skills.

#### *Analysis of Time Allocation During Think-Aloud Sessions*

To further illustrate the differences in overall SRL processing, participants' actions in each phase of Schoenfeld's (1985) problem-solving framework will be analyzed. Participants engaged in *definition of the task* and *forethought* phase behaviors predominantly during reading and analysis episodes. *Performance control* phase behaviors were distributed throughout analysis, exploration, and implementation episodes. The highest frequencies of low-level monitoring occurred during the exploration stage. All *self-reflection* phase behaviors occurred during verification episodes.

Differences in time allocation during the think-aloud sessions were noted among the three MPB categorizations. Edwina, a *novice* MPB participant, spent all of her time engaged in the reading, analysis, and exploration episodes (see Figure 8). Since most of

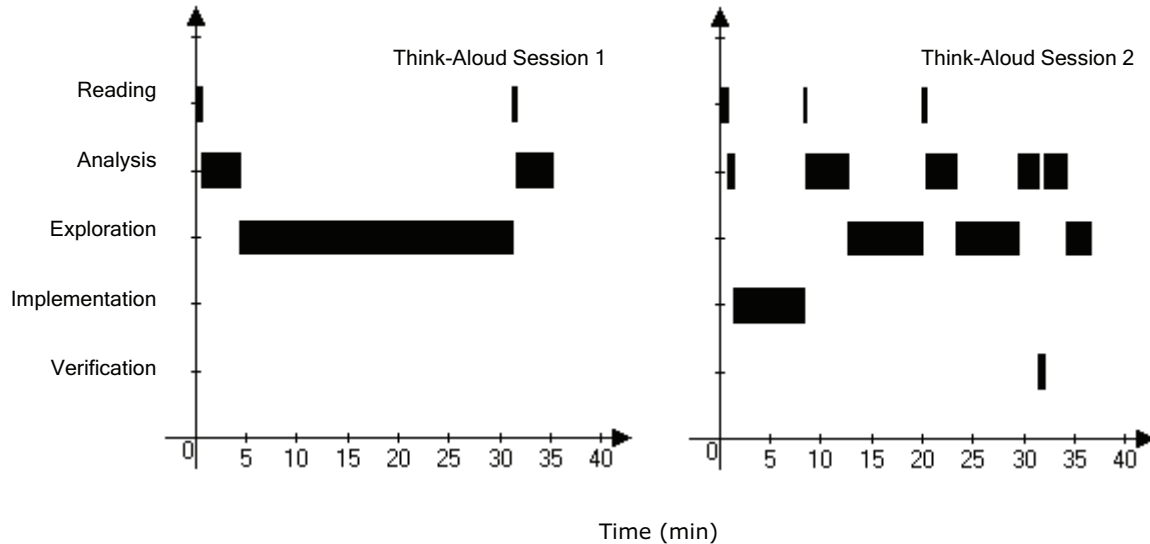
her time was spent in exploration, she did not engage in advanced monitoring and transitions that became the mark of more successful problem-solving endeavors. Findings suggest that Edwina's struggles with self-regulatory behaviors are derived from low-level MPB and a lack of necessary mathematical resources.



*Figure 8.* Time Line Representations for Edwina's Think-Aloud Problem-Solving Sessions.

Martin was categorized as an emerging MPB participant based on his actions and verbalizations during this study. Taken separately, his think-aloud sessions presented two completely divergent interpretations of Martin's problem-solving prowess (see Figure 9). Taken as a whole, Martin's mathematical problem-solving and self-regulatory skills are revealed. Based on the expectation of a procedural problem with a unique solution, Martin spent the majority of the first session engaged in an exploration of the problem space that led to very little progress toward the desired solution. Very little self-

regulatory skill was demonstrated as multiple assessments of monitoring progress went unheeded. Thus, Martin entered the second session with the daunting task of needing solutions for both tasks. In a flurry of nonlinear-SRL activity, Martin engaged in well-managed solution paths, heeded monitoring, and transitioned to new plans based on conceptual assessments of problem conditions and goals. The only negative aspect of Martin's second session was that lack of verification cost him points that could have easily been salvaged.



*Figure 9.* Time Line Representations for Martin's Think-Aloud Problem-Solving Sessions.

Cameron, the only *advanced* MPB participant, successfully applied Schoenfeld's (1985) framework during both sessions. Cameron had not been taught the framework directly; his problem-solving techniques manifested naturally in the manner visually

described by Figure 10. During the first session, he had initial difficulty with part (a) of the application of differentiation task (see Appendix F). However, unlike participants who held non-availing beliefs, Cameron heeded the internal feedback provided by monitoring and applied conceptual, interrelated plans to solve the problem. Cameron's second session was more indicative of an expert solving a problem. However, he still relied on self-regulatory processing to guide his efforts. Cameron also differed from the other participants in time spent verifying solutions. *Self-reflection* is a critical phase in the SRL process and Cameron was the only participant who regularly applied it. Overall, this time analysis provides further evidence of the relationships between epistemological beliefs, SRL processing, and mathematical problem solving.

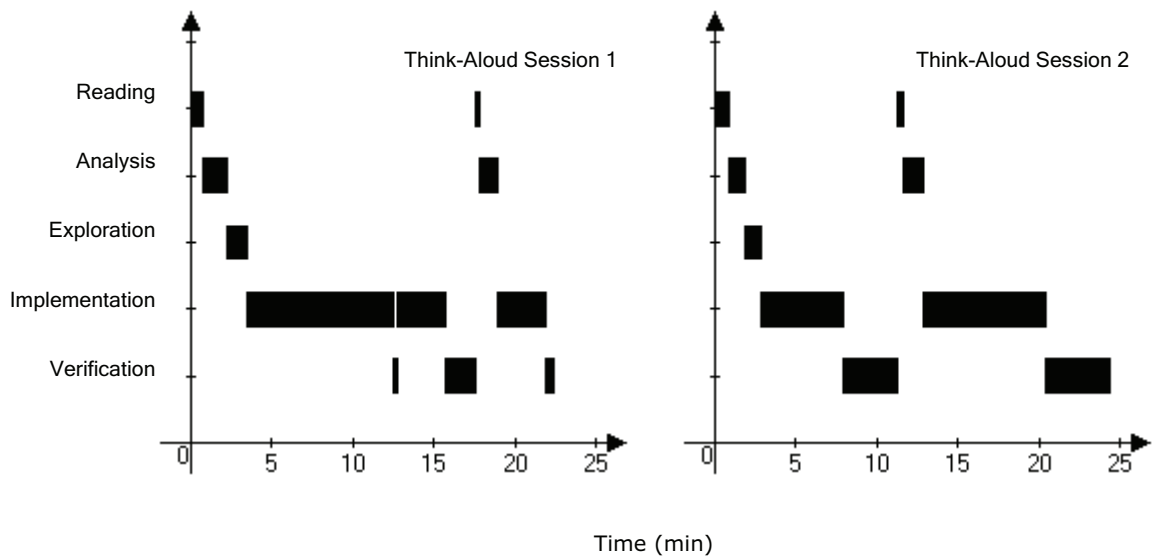


Figure 10. Time Line Representations for Cameron's Think-Aloud Problem-Solving Sessions.

### *Beliefs Related to Heuristic Strategy Use*

During this study, participants applied five heuristic strategies while engaged in mathematical problem solving. The heuristics applied were draw a picture, recall a similar problem, restate the problem, set up equations, and establish subgoals. All participants used heuristic strategies to a certain degree during this study (see Table 16). Robert, Julia, Martin, and Cameron registered higher frequencies of codes for heuristics than Edwina and Olivia. Thus, deeper analysis was required to determine whether relationships existed between epistemological beliefs and heuristic strategy use.

Using NVivo Version 8, heuristic strategy use was analyzed with respect to availing and non-availing beliefs sets (see Table 16). Recall that the availing and non-availing beliefs sets contained all instances coded for the respective belief regardless of participants' overall MPB categorization. Since participants demonstrated a combination of availing and non-availing beliefs during this study (e.g., exhibiting a conceptual approach to problem solving while empirically testing conjectures), discrepancies may be noted between frequency totals for *by belief categorization* values and *by participant* values in Table 16. Rather surprisingly, frequencies of heuristic strategy use were higher for participants demonstrating non-availing beliefs than availing beliefs.

Upon deeper qualitative analysis, differences in the application of heuristic strategies were noted based on participants' MPB categorizations. Specifically, Robert and Martin, novice and emerging MPB participants respectively, each applied the heuristic *setting up an equation* on seven instances. The majority of these codes were registered while Robert and Martin were attempting to apply systems of equations to solve for  $a$ ,  $b$ ,  $c$ , and  $d$  in part (a) of the application of differentiation problem (see



Appendix F). Neither participant was able to utilize the development of equations to solve the given problem. Both participants cited beliefs in *unique* problem solutions and *procedural* problem-solving approaches as sources of their lack of success but Robert also cited the inability to access additional mathematical resources as a barrier to success. The relationship between heuristic strategy use and accessibility to mathematical resources is consistent with Schoenfeld's (1985) findings:

Often the successful implementation of a heuristic strategy depends heavily on a firm foundation of domain-specific resources. It is unrealistic to expect too much of these strategies. (pp. 73–74)

Cameron, the only advanced MPB participant, also applied the *setting up an equation* heuristic seven times. The difference in his application of the heuristic was that equations were not developed to find a final solution for the think-aloud problems but were developed as the problem unfolded and assessments of the current state necessitated further conditions. In fact, his equations were generally developed to further his application of the *establishing subgoals* heuristic. The ability to apply multiple heuristics while engaged in problem solving is demonstrative of a sophisticated level of functioning (Schoenfeld, 1985). During his second think-aloud session, Martin also demonstrated this high level of sophistication by incorporating three of his *setting up an equation* heuristic codes into an *establishing subgoals* heuristic strategy. For both Martin and Cameron, applications of multiple heuristic strategies led to productive advancement toward problem goals. In sum, participants who held non-availing beliefs applied heuristic strategies more frequently but participants who held availing beliefs tended to be more productive in their application of heuristic strategies. Access to mathematical resources

and adherence to epistemological beliefs appear to be jointly related to the application of heuristic strategies.

Table 16

*Coding Frequencies for Heuristic Strategy Use*

Source	DP	RP	RS	EQ	ES
By belief categorization					
Availing	6	2	1	12	4
Non-availing	14	4	1	18	4
By participant					
Robert	3	1	0	7	2
Edwina	4	0	0	1	0
Julia	6	3	0	3	2
Olivia	2	0	0	2	0
Martin	2	1	1	7	1
Cameron	3	1	1	7	3

*Note.* DP = draw a picture; RP = recall a similar problem; RS = restate the problem; EQ = set up equations; ES = establish subgoals

*Beliefs and SRL Related to Overall Performance*

The results of this study indicated that epistemological beliefs and SRL are interrelated. Additionally, participants' beliefs categorizations were related to their overall performance (see Table 17). This finding is congruent with Cano's (2005)

structural equation modeling results, which suggested that students' performance was directly affected by epistemological beliefs. For both AP Calculus exam preparation and think-aloud sessions, participants' performance increased progressively from novice to advanced beliefs categorizations. Although certainly not conclusive, findings from this study indicate that availing beliefs and quality SRL processing affect students' performance in solving mathematical problems.

*A Disconnect Existed Between Idealized Beliefs and Realized Practice*

While analyzing member-checking transcriptions, a discrepancy was discovered between *novice* MPB participants' responses concerning the importance of a conceptual approach to problem solving, other interview responses, and their categorizations from behaviors exhibited during this study. Specifically, when Julia was asked directly whether conceptual or procedural knowledge was more important to problem solving, she stated that conceptual knowledge was more important. However, later in the interview when asked about her struggles with the arbitrary constants, Julia lamented a lack of *procedural* means for solving for the constants and further stated that the source of her woes originated from always learning procedural steps when studying for exams. Similarly, Robert stated that a conceptual approach to problem solving would be more appropriate but later stated that his approach to the arbitrary variables was based on *procedures* he applied in previous courses. Finally, Edwina believed that a conceptual approach would be more conducive to solving mathematical problems but blamed the educational system itself for instilling a *procedural*, cookie-cutter approach to mathematical problem-solving into her. Her previous mathematics teachers had always taught her shortcuts and step-by-step means for solving problems until Algebra 2. Not

armed with any other skills, she had applied the procedural approach and memorized steps for all high school mathematics courses.

Table 17

*Participant Performance By Mathematical  
Problem-Solving Beliefs Categorization*

Participant	Mean	
	AP Exam Preparation	Think- Aloud
Novice		
Robert	4.50	20
Edwina	4.17	28
Julia	5.30	46
Emerging		
Olivia	4.86	45
Martin	6.10	65
Advanced		
Cameron	7.50	80

Two results may be derived from these interview data. First, the *novice* beliefs participants may be farther from the procedural end of the continuum than reported here. Certainly their beliefs continue to manifest in their *realized* problem solving approaches as procedural but acknowledgement that conceptual approaches are more *ideal* may indicate a progression toward a realized conceptual belief. Second, participants' perceptions of their epistemological beliefs did not always coincide with their practice due to both personally- and institutionally-based habits of the mind. Edwina's responses imply that mathematics teachers can play a major role in the development of students' beliefs and subsequently, their ability to apply SRL processing to mathematical problem-solving.

### Conclusion

Since this study involved advanced mathematics students, all participants were motivated to perform and engaged in some form of SRL processing. Thus, participants who demonstrated little *purposeful* control received codes indicative of SRL processing. It became apparent that an accurate rendition of events necessitated analysis and discussion of the quality of SRL processing. This finding was not a complete surprise, as Goos, Galbraith, and Renshaw (2002) encountered a similar issue in their study investigating group metacognition. Focusing on quality rather than quantity, the current study became an investigation into participants' levels of self-regulatory skills. Zimmerman (2000) suggested that an individual's level of self-regulatory skill is based on "social as well as self sources of influence" (p. 29). Hence, educators may have an influence on developing students' self-regulatory mathematical problem-solving skill level.

Upon careful consideration of the findings, Zimmerman's (2000) levels of self-regulatory skill may be applied to participants' behaviors with respect to the skill required to solve the problems. Zimmerman identified four levels of self-regulatory skill, as follows: (1) observation, (2) emulation, (3) self-control, and (4) self-regulation. Since they were able to "systematically adapt their performance to changing . . . contextual conditions" of the problems, Martin and Cameron were identified as demonstrating *self-regulated skill* during the study (p. 30). Robert, Julia, and Olivia were identified as demonstrating the *self-controlled* level of skill development since they were able to apply self-regulatory processing of their skills "in structured settings outside the presence of models" but had difficulty adapting their skills to the problems in the study (p. 30). It should be noted that Martin would have been categorized as demonstrating the self-control level based solely on his first think-aloud. His behaviors during the second think-aloud session earned him the next level for this study. Finally, Edwina's behaviors demonstrated an *emulation* level of skill for the problems in the study. She was unable to navigate any of the problem spaces alone, lamented the lack of a procedural model to follow, and was only successful when an expert explained a problem to her during the interim between think-aloud sessions. As will be discussed further in the following chapter, educators may have a significant influence over students' self-regulatory skill development and the underlying epistemological beliefs that seem to partially determine self-regulatory development.

## CHAPTER VI

### DISCUSSION, CONCLUSIONS, AND IMPLICATIONS

The purpose of this multiple-case study was to explore relationships between students' epistemological beliefs and self-regulated learning (SRL) processing while engaged in mathematical problem-solving. Digressing from conventional SRL studies involving students with academic struggles (e.g., Cleary & Zimmerman, 2004), this study focused on advanced students in an effort to inform the gifted education community. The specific research questions addressed by this study were:

1. How are students' epistemological beliefs related to self-regulatory processing practices during engagement in mathematical problem-solving tasks?
2. What self-regulation strategies do AP Calculus students employ while preparing for the AP exam and engaging in problem-solving episodes?
3. What epistemological beliefs influence students' choice and use of heuristic strategies to solve mathematical problems?
4. How are self-regulated learning strategies and epistemological beliefs related to student performance on problem-solving tasks?

Based on the findings from this study, conclusions are drawn for each research question. A discussion of each conclusion ensues and is grounded in the findings of this study and the current literature. The discussion follows the order of the research questions and is laid out as follows: (1) relationships between SRL and epistemological beliefs, (2) SRL processing strategies based on task demand, (3) relationships between epistemological

beliefs and heuristic strategies, (4) effects of SRL and epistemological beliefs on performance, and (5) disconnect between beliefs and practice. Discussion point (5) was not included in the research questions but emerged as a theme from the findings. Finally, the chapter concludes with a discussion of the limitations of the study, implications for practice and further research, and researcher reflections.

## Discussion of Findings

### *Relationships Between SRL and Epistemological Beliefs*

#### *Definition of the Task*

Findings from this study indicate that participants' *unique* and *arbitrary* beliefs regarding problem solutions manifested during think-aloud sessions affect their actions in the definition of the task phase. When faced with a task involving arbitrary constants or variables, students with *unique* beliefs may struggle with the role of the arbitrary constants, whereas students with *arbitrary* beliefs will tend to develop a more appropriate definition of the task. The expectation that such problems should have unique solutions places a barrier between the student and appropriate understanding of the problem. As suggested previously, "students who . . . seek single answers, avoid ambiguity . . . tend to experience more difficulty with the ambiguous features of tasks that call for reflective judgments, perseverance, and appropriate self-regulated learning" (Lodewyk, 2007, p. 324). Although Lodewyk has examined some facets of SRL, he has not analyzed students' actions in the definition of the task phase. The finding that *unique* and *arbitrary* beliefs affect the definition of the task phase provides an extension to Lodewyk's general assertion.



### *Forethought*

Based on the results of this study, all three sets of beliefs affect participants' actions in the forethought phase, particularly with respect to problem-solving planning. A commonly-recurring theme was the difference in planning based on procedures learned in prior courses and planning based on conceptual aspects of the particular problem being presented. Participants who developed plans based solely on procedural recall were unable to work past a certain point in each problem. Thus, it can be concluded that sole reliance on a *procedural* belief in mathematical problem solving may be insufficient in effective processing through the forethought phase while engaged in ill-structured, *doing mathematics* tasks.

Schoenfeld (1992) identified students' expectations of procedural methods for solving problems as a non-availing belief. Despite a national focus on mathematical problem solving suggested by NCTM (2000) a decade ago, five of the six advanced mathematics students in this study expected a procedurally-based task when they approached the think-aloud problems and subsequently developed procedural problem-solving plans. When probed as to the source of their procedural allegiance, participants cited experiences in prior mathematics courses as the primary source. Thus, a further conclusion to be drawn from this study is that mathematics educators may not effectively facilitate problem-solving habits of mind (Schoenfeld, 1985) in students to the degree recommended by the NCTM (2000).

### *Performance Control*

Participants' actions in the performance control phase became a major focus of this study. The results of this study indicated that all three sets of epistemological beliefs

are related to participants' processing in the performance control phase. The belief that problems with arbitrary constants had *unique* solutions led participants through fruitless explorations devoid of purposeful transitions or elicited low-level monitoring of flawed plans for procedural accuracy. In contrast, participants who exhibited *arbitrary* beliefs regarding problem solutions in this study tended to make more purposeful transitions as problem navigation informed the changing nature of the constants. Since Cameron was the only participant in the advanced mathematics course to consistently demonstrate *arbitrary* beliefs, one might conclude that secondary students are not receiving sufficient exposure to arbitrary variables and constants or may be developing *unique* beliefs from patterns within the secondary mathematics curriculum itself.

In this study, performance control was related to participants' beliefs regarding *conceptual* or *procedural* approaches to problem solving. Yet again, availing beliefs that concepts are interrelated typically led to meaningful transitions based on problem assessments; whereas the non-availing, simple belief that procedures are available for all problems typically led to dead-ends when the procedures had to be modified to account for problem assessments. Edwina's overt assertion as well as other participants' implications that mathematics teachers use too many cookie-cutter problems led to the following conclusion: secondary mathematics educators may be focusing too much time and energy developing procedures for mathematical exercises and not enough time and energy developing conceptual understanding. In that that the nature of reform mathematics and any constructivist-oriented curricula implies that mathematical problem solving is one medium for learning (NCTM, 2000), continued focus on procedural aspects of problems at the expense of conceptual understanding seems counterproductive

to the constructivist-oriented cause. Based on the findings of this study, a further conclusion can be that the development of beliefs that foster SRL processing should supersede, or at least coincide with, the implementation of constructivist-oriented mathematics initiatives.

Finally, the results of this study indicated that students' *rational* and *empirical* approaches to mathematical problem solving are related to processing during the performance control phase. As shown by the example above, participants demonstrating the more availing, logic-based rational beliefs tended to yield more productive self-monitoring than their non-availing, perceptual-based empirical counterparts. The findings from this study also indicated that epistemological beliefs are interrelated, as suggested by Hofer (2004a). As evidenced in previous chapters, novice MPB participants implemented and monitored *procedural* plans based on the expectation of a *unique* solution and upon the failure of the initial plan, began serially testing alternate procedures in an *empirical* fashion. In contrast, Cameron, basing his plans on the possibility of *ambiguous* solutions to the mathematical problems, assessed problem progress at a more *conceptual* level and transitioned logically and *rationally* as consideration of conditions moved toward problem goals. Although not consistent throughout the study, the presence of this progression indicates that epistemological beliefs may be interrelated and may also explain the preponderance of evidence obtained via axial coding of beliefs and SRL processing. Additionally, the myriad empirical codes generated by the advanced mathematics students in this study suggest that logic has been omitted or deemphasized in secondary mathematics curricula. Prior research has suggested that overt emphasis on

surface features of problems and pattern recognition may lead to empirical, as opposed to logical and rational, navigation of a problem space (Schoenfeld, 1988, 1989).

### *Self-Reflection*

Since most participants were unable to finish the problems, it was not possible to assess the self-reflection phase of SRL. The few codes that were obtained appeared to indicate that participants who held non-availing beliefs assigned ability-focused causal attributions to their failures, which promote further negative attitudes toward learning (Zimmerman, 2000). Future research is needed to investigate the relationships between epistemological beliefs and the self-reflection phase of SRL.

### *Mathematical Resources*

Findings from this study indicate that mathematical resources and epistemological beliefs work in tandem in relation to students' SRL processing of problem-solving tasks. All participants passed the prerequisite course, AP Calculus AB, which dealt with topics presented in both the think-aloud and AP Calculus exam preparation problem-solving sessions. However, exposure to mathematical content does not imply accessibility (Schoenfeld, 1985, 1992). I assert that epistemological beliefs are indirectly related to the accessibility of mathematical resources. Edwina had the most notable deficiency in accessing appropriate mathematical resources to solve the think-aloud problems. She also stated that she had been converting mathematical knowledge to prescriptive procedures since entering high school. Thus, my assertion is that certain non-availing beliefs exacerbate students' inability to conceptualize mathematical content, which in turn leads to deficiencies in recalling necessary content. Recall that Julia, who seemed more comfortable recalling appropriate calculus content for the application of integration

problem (see Appendix F), continuously applied procedures that carried her closer to a solution. However, she ultimately failed to make conceptual connections and thus, was unable to complete the problems. I further assert that developing students' beliefs in conceptual approaches to mathematical problems will lead to improved access to mathematical resources. Further research is needed to investigate relationships between epistemological beliefs and students' access to mathematical resources.

*A Model for SRL, Epistemological Beliefs, and Mathematical Problem-Solving*

Based on the findings of this study, the model suggested in the theoretical framework should be updated to include relational connections between epistemological beliefs and SRL processing (see Figure 11). Based on the findings in this study, unidirectional connections were utilized representing the hypothesis that epistemological beliefs affect certain phases of SRL. Future research is needed to determine whether Muis' (2007) hypothesized reciprocal relationship between SRL and epistemological beliefs may be confirmed for the mathematical problem-solving domain. Prior research and literature has indicated that mathematics students enter a problem space with certain beliefs (Muis, 2004, 2008; Schoenfeld, 1983, 1985, 1992). Based on the findings of this study, I suggest that these mathematical problem-solving beliefs (MPB) are *variant attributes* that affect SRL processing during problem solution attempts. The term *variant attributes* indicates that each set of beliefs lies on a continuum from availing to non-availing and student expressions of beliefs may be inconsistent based on a plethora of factors, possibly including social and motivational issues (Muis, 2004).

Additionally, it should be noted that Muis and Franco (2009) have applied structural equation modeling to determine relationships between Hofer and Pintrich's

(1997) epistemological beliefs dimensions and college students' self-reported SRL processing and achievement in a college-level educational psychology course. Assuming a *domain-specific* theory of epistemological beliefs, a goal of this study was to produce compelling evidence for relationships between possible mathematics-specific epistemological beliefs and SRL processes to inform the development of a study involving such methods as grounded theory or structural equation modeling (Hofer & Pintrich, 1997; Muis, 2004; Muis, Bendixen, & Haerle, 2006). The findings of this study suggest that relationships exist between epistemological beliefs and SRL processing. Thus, the researcher recommends that future studies continue this work by employing such methods as grounded theory or structural equation modeling.

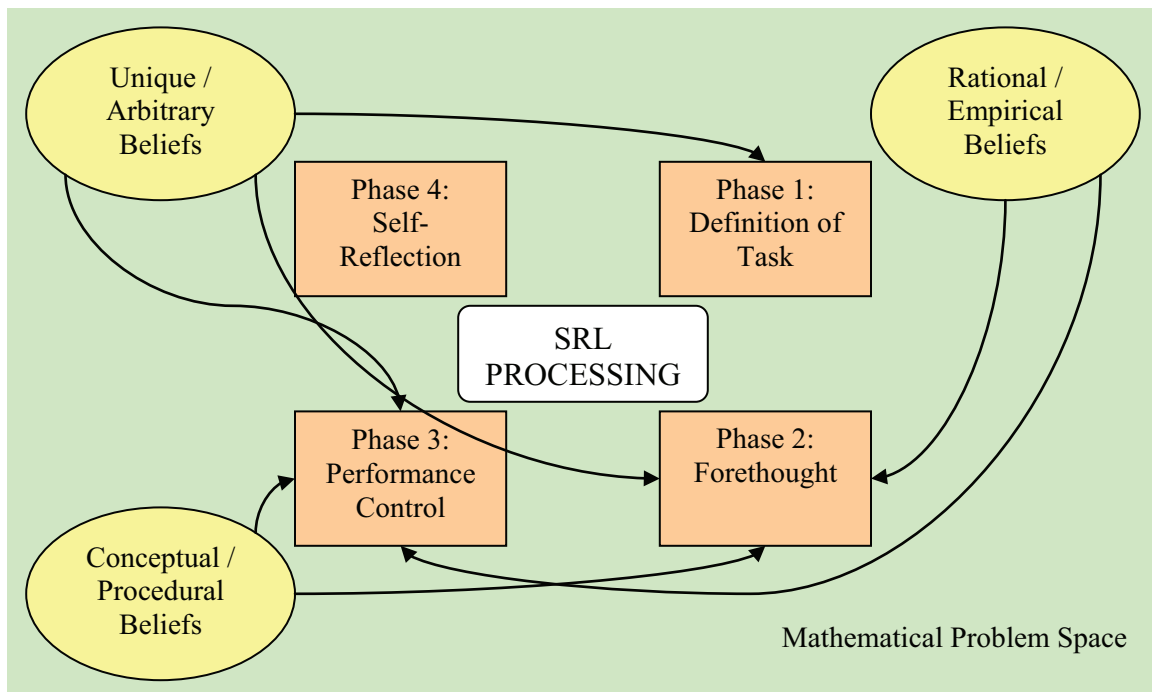


Figure 11. Updated Model of Epistemology and SRL for Mathematical Problem Solving.

*SRL Processing Strategies Based on Task Demand*

In this study, participants who had emerging and availing MPBs required less cognitive and metacognitive regulation for the well-structured AP exam preparation problems than the ill-structured think-aloud problems. The lower cognitive demand of the AP exam preparation tasks allowed the availing MPB participants to engage in more productive group-monitoring than their novice MPB counterparts. This finding suggests that the inclusion of more ill-structured tasks into curricula promotes self-regulatory practices for well-attuned advanced mathematics students. However, for advanced students with novice MPB, more scaffolding may be required before a preponderance of ill-structured tasks are introduced. In fact, Lodewyk, Winne, and Jamieson-Noel (2009) suggested that teachers be cognizant of the cognitive level of tasks since tasks with too little cognitive demand promote individual work and tasks with too much cognitive demand promote reliance on others and loss of peer-learning opportunities. Hence, mathematics teachers should heed the cognitive developmental level of individual students and the collective level of groups when designing tasks. Professional development will likely be required to achieve this goal as mathematics teachers may lack the necessary skills to assess students' cognitive developmental levels and the cognitive levels of tasks.

*Relationships Between Epistemological Beliefs and Heuristic Strategies*

In this study, participants who held non-availing beliefs applied more heuristic strategies while solving mathematical problems, but participants who held availing beliefs tended to be more productive in their application of heuristic strategies. This finding may be explained by considering participants' prior experiences and the

definition of *heuristic strategies*. First, all participants had passed the previous course, AP Calculus AB. This course introduced participants to all of the heuristics which were needed in this study. Additionally, successful navigation through assessments in the AP Calculus AB course required students to apply heuristics. Second, Schoenfeld (1985) defined heuristic strategies as “techniques used by problem solvers when they run into difficulty” (p. 74). Participants in this study who held non-availing beliefs *ran into difficulty* more often than participants who held availing beliefs. Thus, it follows that participants who held non-availing beliefs would require the use of heuristics at a higher frequency than participants who held availing beliefs. Research may be warranted to investigate whether this pattern holds for mathematics students who are not on an advanced track.

Findings regarding the productivity of heuristic strategy use for participants who held availing beliefs further suggest that epistemological beliefs and access to mathematical resources are both related to SRL processing. The successful adaptation of heuristic strategies to mathematical problem-solving tasks is evidence of the performance control phase of SRL. The existence of a framework relating beliefs and resources to successful heuristic strategy usage may explain the lack of successful implementation of heuristics into American mathematics classrooms (Schoenfeld, 1985, 2007). It may be that the introduction of heuristic strategy usage into mathematics curricula and teachers’ pedagogical practices is more difficult than Schoenfeld (2007) has indicated.

#### *Effects of SRL and Epistemological Beliefs on Performance*

The results of this case study indicate that problem solving performance is related to participants’ epistemological beliefs and SRL processing. This finding is congruent



with other studies that investigated SRL, beliefs, and performance (Cano, 2005; Lerch, 2004; Lodewyk, 2007; Muis, 2008; Schoenfeld, 1982, 1985). A conclusion that may be drawn from this finding is that students with availing mathematical problem-solving beliefs (MPB) and, subsequently, either a *self-control* or *self-regulated* level of SRL mathematical problem-solving skill may be more successful in adapting to constructivist-oriented mathematics curricula. The ability to make purposeful transitions based on logical, conceptual assessments of task progression appears to be an important component to successful navigation of mathematical problems, which is one component of constructivist-oriented mathematics curricula. As this study has indicated, such beliefs and skills may not come naturally to advanced mathematics students and the system itself may be promoting non-availing beliefs and then indirectly, reducing self-regulatory skill. As will be discussed further below, mathematics educators may potentially play a crucial role in students' development of more availing beliefs and higher levels of self-regulatory skill.

#### *Disconnect Between Beliefs and Practice*

A further finding that emerged from this study is that participants' verbalized, idealized problem-solving beliefs are not always consistent with their problem-solving practices. This finding was most prominent among novice MPB participants, who professed that conceptual problem-solving beliefs were preferable to procedural beliefs during their member-checking interviews, yet adhered to procedural approaches throughout the study. Additionally, the novice MPB participants made both explicit and implicit statements during interviews indicating procedural beliefs in problem solving.

One conclusion from this finding is that mere awareness that an epistemological belief is preferable may not imply adherence to that belief. A ramification of this conclusion is that students may need more than exposure to availing mathematical problem-solving beliefs. The participants in this study were taught in a calculus course that emphasized the conceptual over the procedural, yet this exposure was insufficient in breaking the cycle of procedural adherence which had been developed in previous mathematics courses. Thus, mathematics educators may need to develop alternative methods for scaffolding conceptual beliefs in problem-solving through creative and research-based pedagogical interventions.

An additional conclusion that may be gleaned from this finding is that self-regulatory problem-solving skill may be more dependent upon the manifestation of students' internally held epistemological beliefs than the external, verbalized expressions of their beliefs. This relationship explicates the findings from this study. Applying the assumption that each set of beliefs in this study lies on a continuum, we may assume that novice MPB participants were near the procedural end of the continuum (Hofer, 2000; Hofer & Pintrich, 1997; Muis, 2004; Schommer, 1990). Their idealized beliefs in a conceptual approach to mathematical problem-solving emerged when probed during interviews but the overwhelming power of ingrained procedurally-based mathematical practices may have hampered conceptualization of the problem space and thus, led to their lack of control demonstrated during the study. Mathematics educators and researchers need to be cognizant that expressed, self-reported availing beliefs should be tempered with actual students' behaviors and alternative means of measuring the manifestations of epistemological beliefs may need to be considered.

Uncovering a disconnect between *students'* beliefs and practice may not be surprising to the mathematics education community because studies indicating a disconnect between *teachers'* beliefs and practice proliferate the body of literature (Philipp, 2007; Thompson, 1992). Similar to findings regarding sources of non-availing beliefs, research suggests that the source of teachers' disconnect between beliefs and practice is the system itself. Specifically, time constraints, classroom-management issues, and political, social, and parental pressures cause teachers to abandon their beliefs about mathematics teaching and learning and assume a more constrained, traditional teaching role.

#### Limitations

The design of this study necessitated a small sample size and the establishment of specific, closed boundaries, which provided thick, rich descriptive results for the purposes of exploration. This design was effective since the study sought a deeper understanding of specific phenomena than may be obtained by more general studies of larger samples. Thus, the findings reported in this study cannot be generalized beyond the bounded case setting presented. Additionally, an unfortunate side effect to the sampling strategy was that only one participant, Cameron, was found to display advanced MPB throughout the study. Thus, findings involving Cameron's data must be tempered with the fact that no corroborating cases were available. Although Olivia and Martin provided some availing beliefs data for the purposes of triangulation, no participants demonstrated availing beliefs throughout the entire duration of the study to directly and globally compare with Cameron's achievements.

An additional limitation to this study was the use of the think-aloud methodology. Although successfully applied to studies of epistemological beliefs (Hofer, 2004a; Muis, 2008) and SRL (Greene & Azevedo, 2009; Muis, 2008), the think-aloud technique does not produce a complete depiction of participants' cognitive processing while engaged in a task (Ericsson & Simon, 1993). According to Ericsson and Simon (1993), the Type II think-aloud and retrospective report technique that was applied to the study only incurs a small effect on problem-solving processes but may not provide a fully consistent or complete representation of participants' cognitive and metacognitive activities. Additionally, Olivia struggled with the think-aloud process despite working every practice problem available to her. As mentioned in the previous chapter, her verbalizations were often vague and somewhat incoherent. She was coaxed during the think-loud sessions to *keep talking* but still was unable to produce a complete cognitive report. Consequently, findings became more inferential based on a combination of her written work, think-aloud verbalizations, and retrospective reports.

An argument may be developed asserting that issues of group dynamics limited the findings of the study during the AP exam preparation sessions. However, as discussed in the previous chapter, issues of group dynamics provided an additional opportunity to assess participants' abilities to regulate their learning environment. Zimmerman (2000) cited "attention focusing" as a *self-control* mechanism that can be invoked during the performance control phase to overcome both covert and overt distractions (p. 19). Participants who mentioned having problems with group dynamics but whose results indicated they had overcome them demonstrated application of *attention focusing* processing. However, participants who allowed issues of group dynamics to affect their

group-regulatory participation and performance were focusing on deficits in their learning process, which Zimmerman claims eventually may lead to adverse effects on learning.

Finally, researcher bias based on personal, underlying assumptions may have influenced interpretations of the results in this study. I do not issue an apology for this bias but rather embrace it as a necessary mechanism for conducting qualitative research. I have intentionally provided an interpretation of the findings of this study based on a constructivist approach to learning mathematics grounded in social cognitive theory. Additionally, methodological tools (i.e., maximum variation sampling, multiple sources of data, member-checking interviews) were employed in order to enhance internal validity. Researchers who approach my data from differing philosophical orientations may draw different conclusions from the findings. Hence, the findings of this exploratory case study should be assessed from the viewpoint that mathematical knowledge may be developed by students of their own volition in tandem with peers. In my opinion, this viewpoint on learning mathematics necessitated further understanding of students' abilities to self-regulate their learning in authentic mathematical problem-solving settings and provided a justification for the study.

#### Implications for Practice

After the NCTM (2000) introduced recommendations for reforming the teaching of mathematics, school districts across America altered curricula to adhere to a more constructivist, problem-solving-based approach to teaching and learning mathematics. Unfortunately, many states have already abandoned constructivist-oriented curricula and reverted to traditional, back-to-basics mathematics instruction due to political pressures

that seem to ignore the failures of the most recent national back-to-basics movement (Schoenfeld, 2007). The results of this exploratory multiple-case study inform problem-solving-based pedagogy. These insights into problem-solving-based pedagogy may help mathematics educators implement constructivist-oriented curricula and shed light on future constructivist-oriented movements, which Schoenfeld (2007) predicted will eventually return in the typical cyclic fashion of educational reform.

All participants in this case study were on a highly-advanced mathematics track. Specifically, five of the six participants registered for the multivariable calculus course offered at the school during their senior year. Edwina was in her senior year while taking the course involved in this study and thus, did not have the option of taking the multivariable calculus course. The multivariable calculus course is aligned with course descriptions from local universities' third-semester calculus courses. Despite their successful navigation through advanced secondary mathematics courses, five of the six participants struggled when presented with a novel problem to solve during the think-aloud sessions. Although not generalizable, this finding provides compelling evidence that even successful secondary mathematics students are not learning with conceptual understanding. The findings of this study suggest that the development of availing MPB in conjunction with self-regulatory processing skills influence successful problem solving and thus, influence implementation of constructivist-oriented mathematics curricula.

Based on the discussion above, recommendations for practicing mathematics educators, school administrators, and curriculum developers follow:

1. Mathematics educators should evaluate students' MPB early in the course. Such assessment may be done by self-report instrument or qualitative probing.

Educators should assess qualitatively since instrument completion consumes valuable class time and findings from self-report instruments may be unreliable (Hofer & Pintrich, 1997; Muis, Winne, & Jamieson-Noel, 2007; Winne & Perry, 2000). Qualitative assessments may be made via individual conversations with students as they complete class work and assessments of student work.

Curriculum developers and school administrators should consider providing mathematics teachers appropriate training regarding epistemological beliefs so that assessments may be developed and implemented appropriately.

2. Based on student evaluations of beliefs, mathematics educators should incorporate the development of availing MPB into their pedagogy. Specifically, mathematics educators should align learning goals, instructional habits, and authentic assessments with the availing beliefs discussed in this study. Cano (2005) suggested that merely informing students of availing beliefs is insufficient; rather, pedagogy should be in constant and consistent agreement with espoused beliefs. To facilitate sustainability of any implemented initiatives, assessment of the effectiveness of developing students' beliefs may be distributed to teachers via collaborative communities of professional learning and growth (Hargreaves & Fink, 2006; Harris, 2006; Katzenmeyer & Moller, 2001).
3. Despite the findings of this study, mathematics educators should not assume that developing availing MPB in students will naturally lead to the development of *self-regulatory skill* in mathematical problem solving (Zimmerman, 2000). Zimmerman suggested that self-regulatory skill development depends on significant social influences during the *observation* and *emulation* levels.

However, social influences should taper off as student behaviors indicate attainment of the *self-control* and *self-regulation* levels. Thus, it is recommended that mathematics educators incorporate self-regulatory skill development into their pedagogy and differentiate pedagogical decision making (e.g., teacher-based assistance, group assignments, cognitive level of tasks) based on assessments of individual students' self-regulatory prowess. Initially, school administrators should provide sufficient professional development to teachers based on their level of understanding of SRL. Then, assessments of the implementation of initiatives may be monitored via professional learning communities.

4. Finally, Perels, Gürtler, and Schmitz (2005) suggested that combining formal SRL training with formal problem-solving skill training for students may provide the optimal means for developing students' mastery of both skills. It is further recommended that students' MPB be added to any such developmental program. Based on the findings of this study and Perels, Gürtler, and Schmitz's study, it is recommended that school systems adopting, or currently implementing, constructivist-oriented curricula use the combined resources of mathematics teachers and curriculum developers to infuse a combination of SRL, MPB, and problem-solving skills training into secondary mathematics courses. This addition would not *add* to the curriculum but serve as a way to introduce process standards to *complement* the curriculum and provide an alternative engine for pedagogical practice to the current, more direct approach that is once again prevailing in many secondary mathematics classrooms (Schoenfeld, 2007).



### Implications for Future Research

Based on the results of this study, recommendations are made for future research. This multiple-case study explored the relationships between epistemological beliefs and SRL processing during mathematical problem-solving in an effort to inform possible extensions on research along this vein (Schoenfeld, 1985; Muis, 2008). Upon deep reflection of the results and conclusions derived from this study, the following recommendations for future research are provided:

1. The majority of findings in this study converged on the performance control phase of SRL. Future research should investigate deeper the effects of epistemological beliefs on the definition of the task, forethought, and self-reflection phases of SRL during mathematical problem solving.
2. Similar to the suggestion above, further research should investigate the effects of source of knowledge and justification of knowing on SRL processing during mathematical problem-solving, as little data emerged from the current study to inform these sets of beliefs. Qualitative or mixed methods approaches seem most appropriate to this task as we still know little about *how* beliefs affect self-regulatory skill in problem-solving and self-report instruments continue to demonstrate insufficient reliability.
3. Since this study investigated advanced mathematics students, it is recommend that future researchers replicate or adapt this study to investigate mathematics students at other levels of achievement. To better inform productive change in the development of mathematics education initiatives, input is needed as to which

results from this study converge and which results diverge with respect to other populations of mathematics students.

4. To address issues of researcher bias and additional limitations inherent in the design of this study, a call for research on larger populations is suggested to assess the transferability and generalizability of findings to other populations of advanced mathematics students. Similar studies may also be merited on other populations once qualitative studies, as advised above, are conducted. I suggest that two types of studies may sufficiently address such a request: (a) grounded theory study on a larger population to determine if similar findings emerge, and (b) mixed methods study with a large population combining a case study similar to the current study with a subset of the sample and structural equation modeling for the entire sample.

My final call for research is presented in prose form as it is nearer to my heart and based on a deeper level of meditation and reflection on the findings of this study. I call for a new generation of researchers, a generation that conducts studies *in the trenches*. I call for research to emerge from those who sit side-by-side with the *participants* daily and know them better than anyone else. It is time to truly merge theory and practice and who better suited to do so than the practitioners! Why do my findings echo those of other researchers preceding me by three decades? Are the engines of change in education truly that rusty or are we (teachers) failing to provide the necessary propulsion? It is our classroom. It is our hour. It is our semester. Are we willing to *construct* the knowledge necessary to implement positive change in the mathematics classroom or are we merely

hypocrites who cannot practice what we preach? It is time to merge theory with practice and the responsibility is on us.

### Researcher Reflections

As I reflect on this dissertation research journey, I do so with solemnity and awe. I have attempted through this study to provide an accurate and revealing glimpse into the lived experiences of select advanced secondary mathematics students as they solved problems. The qualitative research paradigm has, in my opinion, provided an appropriate medium for analyzing this phenomenon. My students have voluntarily spoken for themselves and I have attempted to capture the essence of their voices, both the overt and the covert. I am hopeful that each participant took away from his or her personal experiences further insights into his or her role as a learner and crafter of mathematical knowledge.

In conducting this research and reporting my findings, my goal was to provide the larger research community with a clear depiction of the relationships that exist amongst the three converging constructs studied: self-regulated learning, epistemological beliefs, and mathematical problem solving. I feel confident that this study will place another brick in the wall that represents mathematics-education research and hope that future researchers, possibly even teacher researchers, are able to continue building onto my portion of the wall. I am a firm believer that mathematics education needs to continue on the reform path explicated by NCTM (2000) and that the constructivist philosophy needs to be infused into more curricula and pedagogy. I have conducted research that informs pedagogical and institutional changes necessary for the furtherance of the mathematics education reform. In doing so, I have gained significant insights into the needs of my own

students that will inform and enhance my practice as a secondary mathematics teacher.

Locally, I plan to place added focus on students' beliefs as I assess progress and promote availing beliefs practices amongst my students. Globally, I plan to share my findings via communities of learning at my school and larger forums, such as conferences and convocations. I also intend to continue providing research from the *trenches* and provide an outlet for student voices in the hopes that someone is listening.

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Appendix A: Consent Forms



## Participant Consent Form

My signature below indicates that I have read the information provided and have decided to participate in the study titled Epistemological Beliefs and Self-Regulated Learning: A Case Study of AP Calculus Test Preparation and Advanced Problem Solving to be conducted at school between the dates of January 5, 2010 and March 31, 2010. I understand that the signature of the principal and classroom teacher indicates they have agreed to allow student participation in this research project.

I understand the purpose of the research project will be to determine why and how mathematical beliefs affect self-regulation of problem-solving tasks and that I will participate in the following manner:

1. All students will complete AP Calculus exam practice problems in a journal.
2. If you are one of six students chosen for the case study, two additional problem solving sessions of one-hour duration will be completed after school.
3. For confirmation of data, you may be asked to participate in an interview after school of approximately 30-minute duration.

Potential benefits of the study are: increased performance on AP exam, increased performance in the course, increased performance in mathematical problem solving, development of self-regulatory skills, and development of problem-solving strategies.

I agree to the following conditions with the understanding that I can withdraw from the study at any time should I choose to discontinue participation.

- The identity of participants will be protected. No student names will appear in any of the data collected or in the final report of the findings. Additionally, all data will be destroyed within five years of the completion of the study.
- Information gathered during the course of the project will become part of the data analysis and may contribute to published research reports and presentations.
- There are no foreseeable inconveniences or risks involved in participating in the study.
- Participation in the study is voluntary and will not affect either student grades or placement decisions. Students who do not participate in the study will continue to participate in classroom activities and receive the same AP exam practice as their peers, but no data will be collected as a result of their work. If I decide to withdraw permission after the study begins, I will notify the school of my decision.

If further information is needed regarding the research study, I can contact James Clinton Stockton at [james.stockton@cobbk12.org](mailto:james.stockton@cobbk12.org) or (678) 594 – 8190, ext. 469.

Signature \_\_\_\_\_  
Participant Date

Signature \_\_\_\_\_  
Principal Date

Signature \_\_\_\_\_  
Classroom Teacher/Researcher Date

## Parental Consent Form

My signature below indicates that I have read the information provided and have decided to allow my child to participate in the study titled Epistemological Beliefs and Self-Regulated Learning: A Case Study of AP Calculus Test Preparation and Advanced Problem Solving to be conducted at my child's school between the dates of January 5, 2010 and March 31, 2010. I understand that the signature of the principal and classroom teacher indicates they have agreed to allow student participation in this research project.

I understand the purpose of the research project will be to determine why and how mathematical beliefs affect self-regulation of problem-solving tasks and that my child will participate in the following manner:

1. All students will complete AP Calculus exam practice problems in a journal.
2. If your child is one of six students chosen for the case study, two additional problem solving sessions of one-hour duration will be completed after school.
3. For confirmation of data, your child may be asked to participate in an interview after school of approximately 30-minute duration.

Potential benefits of the study are: increased performance on AP exam, increased performance in the course, increased performance in mathematical problem solving, development of self-regulatory skills, and development of problem-solving strategies.

I agree to the following conditions with the understanding that I can withdraw my child from the study at any time should I choose to discontinue participation.

- The identity of participants will be protected. No student names will appear in any of the data collected or in the final report of the findings. Additionally, all data will be destroyed within five years of the completion of the study.
- Information gathered during the course of the project will become part of the data analysis and may contribute to published research reports and presentations.
- There are no foreseeable inconveniences or risks involved to my child participating in the study.
- Participation in the study is voluntary and will not affect either student grades or placement decisions. Students who do not participate in the study will continue to participate in classroom activities and receive the same AP exam practice as their peers, but no data will be collected as a result of their work. If I decide to withdraw permission after the study begins, I will notify the school of my decision.

If further information is needed regarding the research study, I can contact James Clinton Stockton at [james.stockton@cobbk12.org](mailto:james.stockton@cobbk12.org) or (678) 594 – 8190, ext. 469.

Signature \_\_\_\_\_  
Parent Date

Signature \_\_\_\_\_  
Principal Date

Signature \_\_\_\_\_  
Classroom Teacher/Researcher Date

## Appendix B: Questionnaires

## Indiana Mathematics Beliefs Scale (IMBS; Kloosterman &amp; Stage, 1992)

1. Memorizing steps is not that useful for learning to solve word problems.
2. Learning computational skills is more important than learning to solve word problems.
3. There are word problems that just can't be solved by following a predetermined sequence of steps.
4. If I can't do a math problem in a few minutes, I probably can't do it at all.
5. Computational skills are of little value if you can't use them to solve word problems.
6. Mathematics will not be important to me in my life's work.
7. Most word problems can be solved by using the correct step-by-step procedure.
8. I find I can do hard math problems if I just hang in there.
9. By trying hard, one can become smarter in math.
10. A person who can't solve word problems really can't do math.
11. Working can improve one's ability in mathematics.
12. I study mathematics because I know how useful it is.
13. Ability in math increases when one studies hard.
14. It's not important to understand why a mathematical procedure works as long as it gives a correct answer.
15. Math classes should not emphasize word problems.
16. Studying mathematics is a waste of time.
17. Word problems can be solved without remembering formulas.

18. A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem.
19. Any word problem can be solved if you know the right steps to follow.
20. I'm not very good at solving math problems that take a while to figure out.
21. It doesn't really matter if you understand a math problem if you can get the right answer.
22. Hard work can increase one's ability to do math.
23. Learning to do word problems is mostly a matter of memorizing the right steps to follow.
24. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.
25. I can get smarter in math if I try.
26. Getting a right answer in math is more important than understanding why the answer works.
27. Math problems that take a long time don't bother me.
28. Knowing mathematics will help me earn a living.
29. Word problems are not a very important part of mathematics.
30. Time used to investigate why a solution to a math problem works is time well spent.
31. I can get smarter in math by trying hard.
32. If I can't solve a math problem quickly, I quit trying.
33. Mathematics is of no relevance to my life.
34. Computational skills are useless if you can't apply them to real life situations.

35. I feel I can do math problems that take a long time to complete.
36. Mathematics is a worthwhile and necessary subject.

## Psycho-Epistemic Profile (PEP; Mos &amp; Royce, 1980)

1. A good teacher is primarily one who has a sparkling entertaining delivery.
2. The thing most responsible for a child's fear of the dark is thinking of all sorts of things that could be "out there".
3. Most people who read a lot, know a lot because they come to know of the nature and function of the world around them.
4. Higher education should place a greater emphasis on fine arts and literature.
5. I would like to be a philosopher.
6. A subject I would like to study is biology.
7. In choosing a job I would look for one which offered opportunity for experimentation and observation.
8. The Bible is still a best seller today because it provides meaningful accounts of several important eras in religious history.
9. Our understanding of the meaning of life has been furthered most by art and literature.
10. More people are in church today than ever before because they want to see and hear for themselves what ministers have to say.
11. It is of primary importance for parents to be consistent in their ideas and plans regarding their children.
12. I would choose the following topic for an essay: The Artist in an Age of Science.
13. I feel most at home in a culture in which people can freely discuss their philosophy of life.
14. Responsibility among men requires an honest appraisal of situations where irresponsibility has transpired.
15. A good driver is observant.
16. When people are arguing a question from two different points of view, I would say that the argument should be resolved by actual observation of the debated situation.
17. I would like to visit a library.
18. If I were visiting India, I would be primarily interested in understanding the basis for their way of life.
19. Human morality is molded primarily by an individual's conscious analysis of right and wrong.
20. A good indicator of decay in a nation is a decline of interest in the arts.
21. My intellect has been developed most by learning methods of observation and experimentation.

22. The prime function of a university is to teach principles of research and discovery.
23. A good driver is even tempered.
24. If I am in a contest, I try to win by following a pre-determined plan.
25. I would like to have been Shakespeare.
26. Our understanding of the meaning of life has been furthered most by mathematics.
27. I like to think of myself as a considerate person.
28. I would very much like to have written Darwin's "The Origin of Species".
29. When visiting a new area, I first try to see as much as I possibly can.
30. My intellect has been developed most by gaining insightful self knowledge.
31. I would be very disturbed if accused of being insensitive to the needs of others.
32. The kind of reading which interests me most is that which creates new insights.
33. The greatest evil inherent in a totalitarian regime is alienation of human relationships.
34. Most atheists are disturbed by the absence of factual proof of the existence of God.
35. In choosing a job I would look for one which offered the opportunity to use imagination.
36. In my leisure I would most often like to enjoy some form of art, music, or literature.
37. The kind of reading which interests me most is that which stimulates critical thought.
38. I prefer to associate with people who are spontaneous.
39. In my leisure I would like to play chess or bridge.
40. Most people who read a lot, know a lot because they develop an awareness and sensitivity through their reading.
41. When visiting a new area, I first pause to try to get a "feel" for the place.
42. Many T.V. programs lack sensitivity.
43. I like to think of myself as observant.
44. Happiness is largely due to sensitivity.



45. I would be very disturbed if accused of being inaccurate or biased in my observations.
46. A good teacher is primarily one who helps his students develop their powers of reasoning.
47. I would like to be a novelist.
48. The greatest evil inherent in a totalitarian regime are restrictions of thought and criticism.
49. Most people are in church today than ever before because theologians are beginning to meet the minds of the educated people.
50. The most valuable person on a scientific research team is one who is gifted at critical analysis.
51. Many T.V. programs lack organization and coherence.
52. I like country living because it gives you a chance to see nature first hand.
53. Upon election to Parliament I would endorse steps to encourage an interest in the arts.
54. It is important for parents to be familiar with theories of child psychology.
55. The prime function of a university is to train the minds of the capable.
56. I would like to have written Hamlet.
57. Higher education should place a greater emphasis on mathematics and logic.
58. The kind of reading which interests me most is that which is essentially true to life.
59. A subject I would like to study is art.
60. I feel most at home in a culture in which realism and objectivity are highly valued.
61. The prime function of a university is to develop a sensitivity to life.
62. When playing bridge or similar games I try to think my strategy through before playing.
63. If I were visiting India, I would be primarily interested in noting the actual evidence of cultural change.
64. When buying new clothes I look for the best possible buy.
65. I would like to visit an art gallery.
66. When a child is seriously ill, a good mother will remain calm and reasonable.

67. I prefer to associate with people who stay in close contact with the facts of life.
68. Many T.V. programs are based on inadequate background research.
69. Higher education should place greater emphasis on natural science.
70. I like to think of myself as logical.
71. When people are arguing a question from two different points of view, I would say that each should endeavor to assess honestly his own attitude and bias before arguing further.
72. When reading an historical novel, I am most interested in the factual accuracy found in the novel.
73. The greatest evil inherent in a totalitarian regime is distortion of the facts.
74. A good driver is considerate.
75. Our understanding of the meaning of life has been furthered most by biology.
76. I would like to have been Galileo.
77. My children must possess the characteristics of sensitivity.
78. I would like to be a Geologist.
79. A good indicator of decay in a nation is an increase in the sale of movie magazines over news publications.
80. I would be very disturbed if accused of being illogical in my beliefs.
81. Most great scientific discoveries come about by thinking about a phenomenon in a new way.
82. I feel most at home in a culture in which the expression of creative talent is encouraged.
83. In choosing a job I would look for one which offered a specific intellectual challenge.
84. When visiting a new area, I first plan a course of action to guide my visit.
85. A good teacher is primarily one who is able to discover what works in class and is able to use it.
86. Most great scientific discoveries come about by careful observation of the phenomena in question.
87. Most people who read a lot, know a lot because they acquire an intellectual proficiency through the sifting of ideas.
88. I would like to visit a botanical garden or zoo.

89. When reading an historical novel, I am most interested in the subtleties of the personalities described.
90. When playing bridge or similar games I play the game by following spontaneous cues.

Appendix C: Approval Correspondences

## College Board Approval to Use AP Questions

For the purposes of confidentiality, personal information has been omitted.

09/30/09

Name : James Stockton

Email: james.stockton@cobbk12.org

RE: 2000 - 2009 Calculus AB and BC free response questions

Dear James Stockton:

Thank you for your request to reproduce the aforementioned AP Material for the purposes indicated below:

Title of Your Work: Self-regulation, epistemology, problem solving, gifted students

Author: James C. Stockton, AP Calculus AB/BC teacher

Distribution/Audience: Dissertation committee, possible inclusion in educational research journal

Distribution date: TBD

Quantity: TBD

Price: N/A

Permission to use the aforementioned Items is granted and is contingent upon the following:

- 1)Permission is granted on a one-time, non-exclusive, and non-transferable basis.
- 2)Please include the following credit line, exactly as written below, in each instance where the Items appear:

Source: Copyright © 2009. The College Board. Reproduced with permission.  
<http://apcentral.collegeboard.com>.

Please refer to the above contract number in any further correspondence.

Thank you,

Kelly Fitzsimmons  
Assistant Director, AP Policy and Publications

## Email Approval to Use the Indiana Mathematics Belief Scales

For the purposes of confidentiality, personal information has been omitted.

Clint,

The only restriction on use of the Indiana Scales involves selling them for profit and I can't imagine that's your aim — you are quite welcome to use them and to include them in your appendices. You do not need to get permission for the Fennema-Sherman Usefulness scale. Good luck with your dissertation.

Peter Kloosterman  
Professor of Mathematics Education  
School of Education 3274  
Indiana University  
Bloomington, IN 47405

## Email Approval the Motivated Strategies for Learning Questionnaire

For the purposes of confidentiality, personal information has been omitted.

I mail out the MSLQ for a fee of \$20. Make your check payable to the University of Michigan. With this payment, you are allowed to use the MSLQ for your needs but making sure you give the authors credit. You can copy the MSLQ for your needs and also put it on a password protected website for your people but do not distribute it outside of your group.

Also, I am willing to send it out before I receive your check so you can get it as soon as possible. Please send me back your complete address and I will use that as my label. ...Marie

---

Marie-Anne Bien, Secretary  
The University of Michigan  
Combined Program in Education & Psychology (CPEP)  
610 East University, 1413 School of Education  
Ann Arbor, MI 48109-1259

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## Email Approval to Use the Psycho-Epistemic Profile

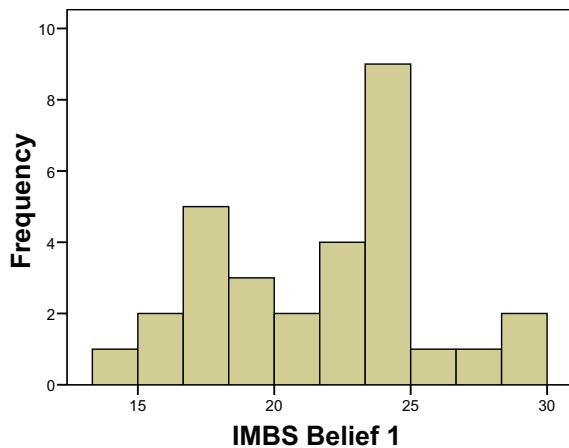
For the purposes of confidentiality, personal information has been omitted.

From Dr. Leo Mos  
To James Clinton Stockton  
Date Monday, September 28  
Subject Re: Request to purchase and use PEP

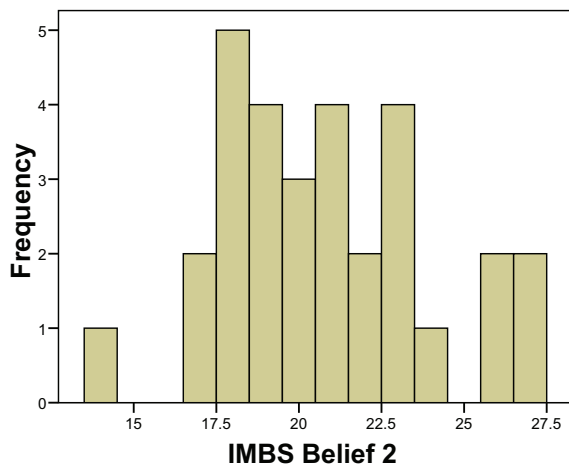
Hi Clint, I will forward a copy of the PEP if you send me your address. I haven't worked with the PEP since the late-60s and certainly have not kept up with the literature. The Manual (to which I have copyright) also contains the questionnaire, use it as you deem fit. Best wishes on the doctoral research. Leo



Appendix D: Distributions of Scores for Questionnaires



*Figure 12.* Distribution of Scores for the Indiana Mathematics Belief Scales (IMBS):  
Belief 1 Scale ( $N = 30$ ).



*Figure 13.* Distribution of Scores for the IMBS: Belief 2 Scale ( $N = 30$ ).

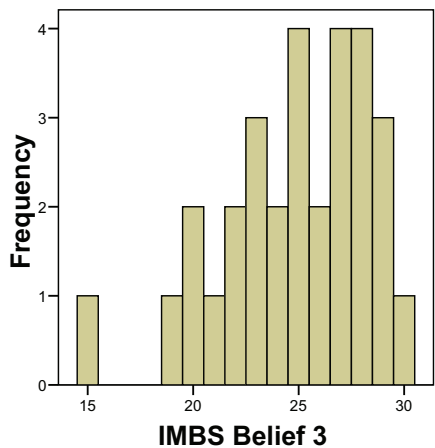


Figure 14. Distribution of Scores for the IMBS: Belief 3 Scale ( $N = 30$ ).

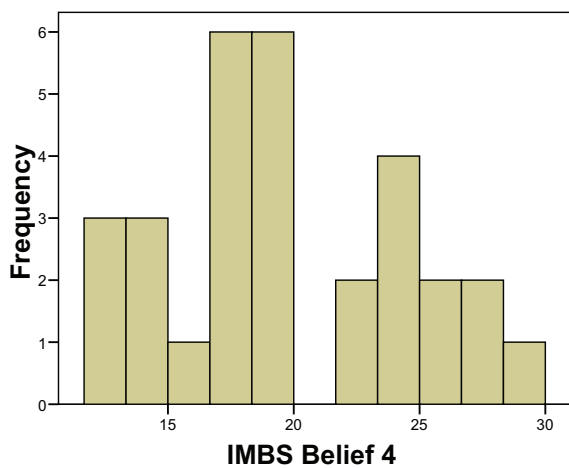


Figure 15. Distribution of Scores for the IMBS: Belief 4 Scale ( $N = 30$ ).

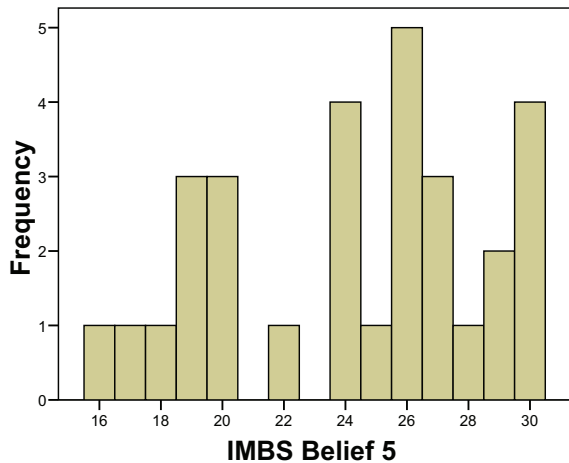


Figure 16. Distribution of Scores for the IMBS: Belief 5 Scale ( $N = 30$ ).

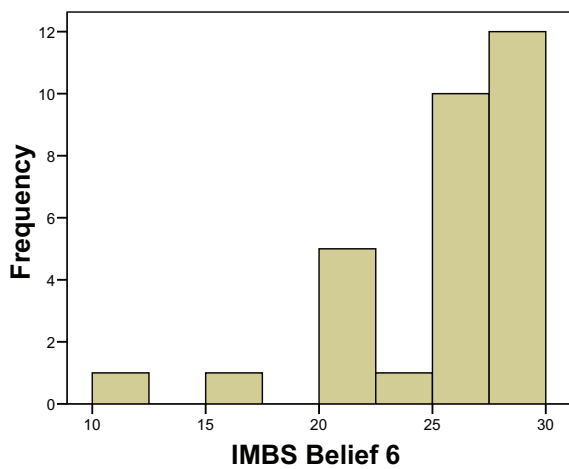
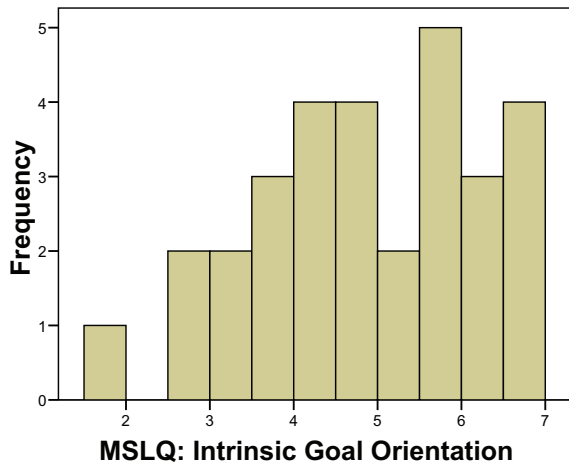
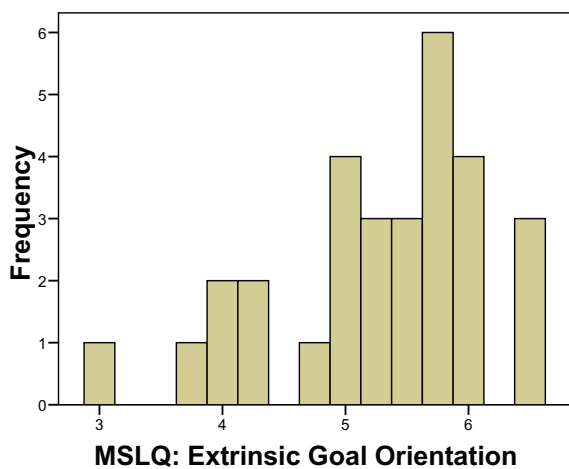


Figure 17. Distribution of Scores for the IMBS: Belief 6 Scale ( $N = 30$ ).



*Figure 18.* Distribution of Scores for the Motivated Strategies for Learning (MSLQ): Intrinsic Goal Orientation Scale ( $N = 30$ ).



*Figure 19.* Distribution of Scores for the MSLQ: Extrinsic Goal Orientation Scale ( $N = 30$ ).

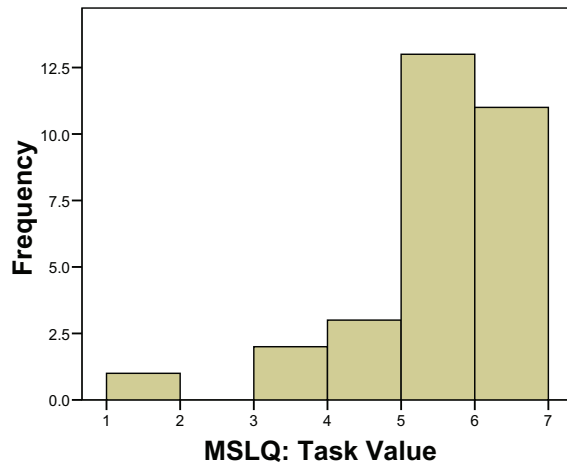


Figure 20. Distribution of Scores for the MSLQ: Task Value Scale ( $N = 30$ ).

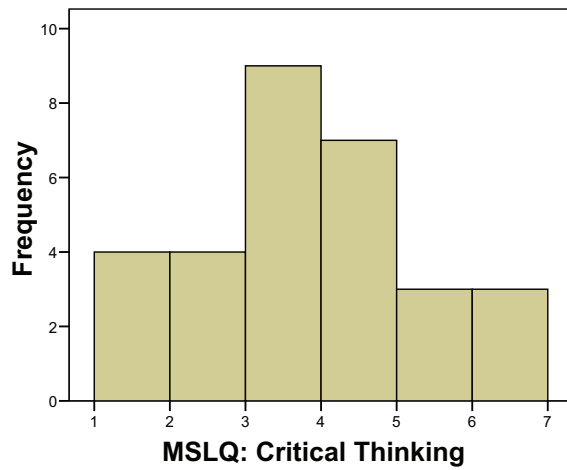


Figure 21. Distribution of Scores for the MSLQ: Critical Thinking Scale ( $N = 30$ ).

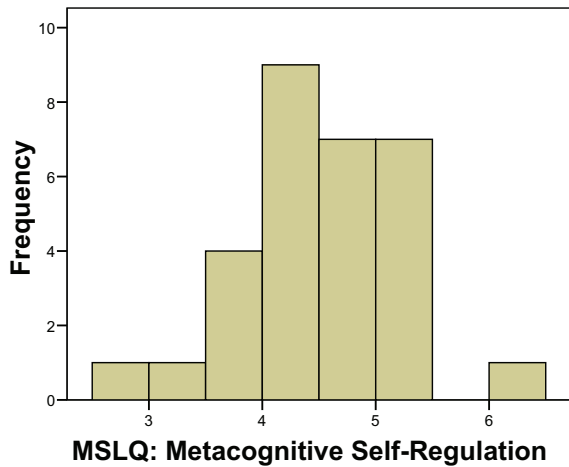


Figure 22. Distribution of Scores for the MSLQ: Metacognitive Self-Regulation Scale ( $N = 30$ ).

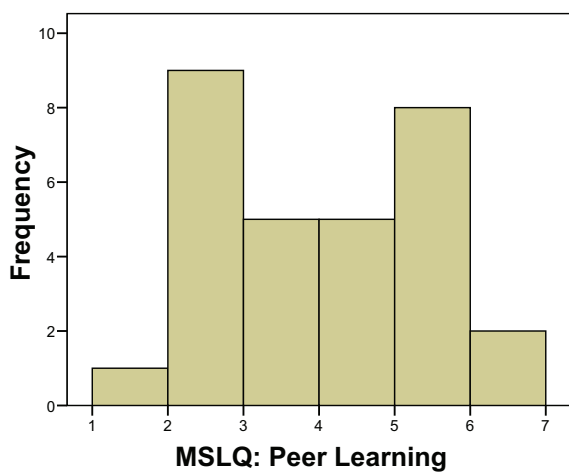


Figure 23. Distribution of Scores for the MSLQ: Peer Learning Scale ( $N = 30$ ).

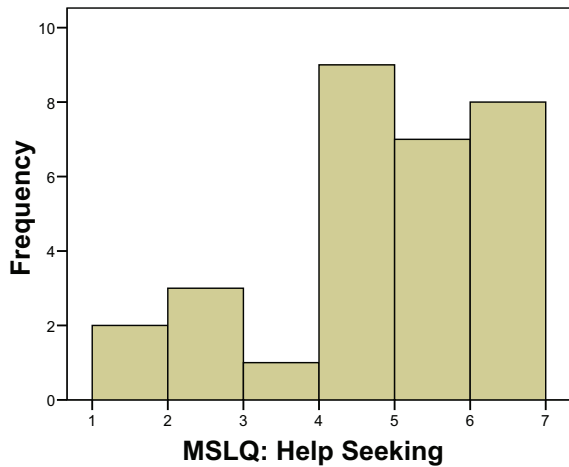


Figure 24. Distribution of Scores for the MSLQ: Help Seeking Scale ( $N = 30$ ).

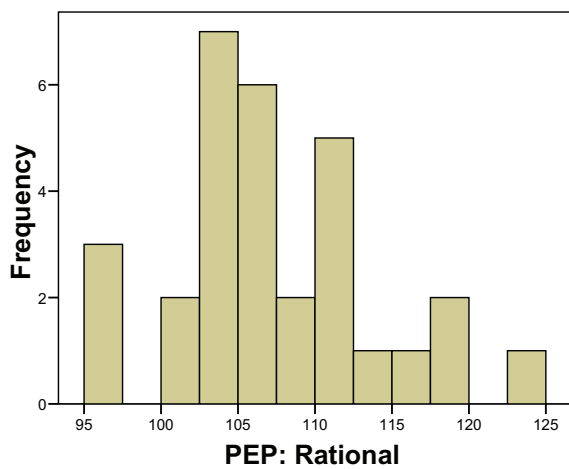


Figure 25. Distribution of Scores for the Psycho-Epistemological Profile (PEP): Rational Scale ( $N = 30$ ).



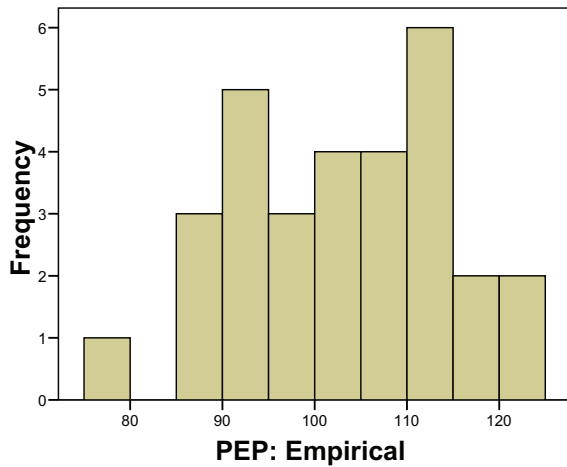


Figure 26. Distribution of Scores for the PEP: Empirical Scale ( $N = 30$ ).

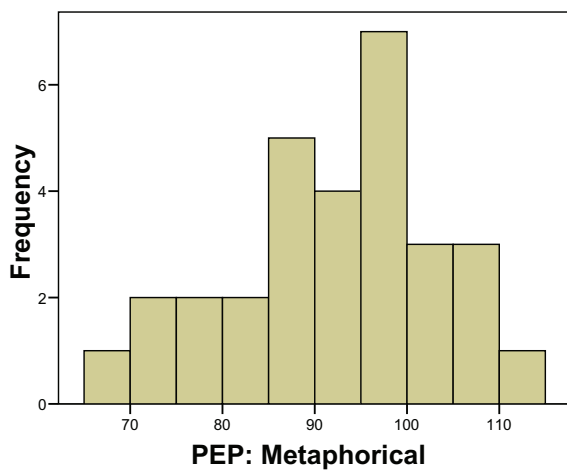


Figure 27. Distribution of Scores for the PEP: Metaphorical Scale ( $N = 30$ ).

Appendix E: Advanced Placement (AP) Exam Preparation  
Instrumentation and Protocols

AP<sup>®</sup> Calculus AB Free-Response Questions\* Used During This Study2003 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

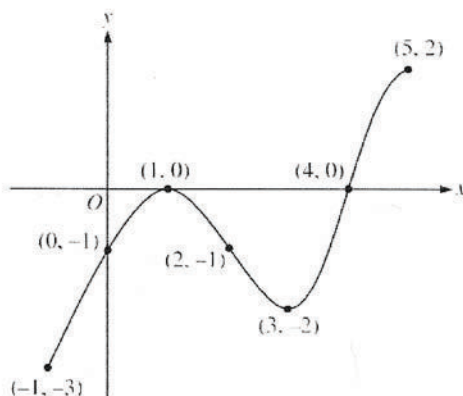
2. A tank contains 125 gallons of heating oil at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

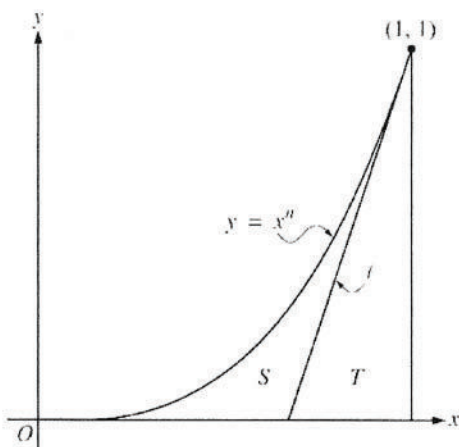
$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

- How many gallons of heating oil are pumped into the tank during the time interval  $0 \leq t \leq 12$  hours?
- Is the level of heating oil in the tank rising or falling at time  $t = 6$  hours? Give a reason for your answer.
- How many gallons of heating oil are in the tank at time  $t = 12$  hours?
- At what time  $t$ , for  $0 \leq t \leq 12$ , is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

2004 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)Graph of  $f'$ 

4. The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , on the closed interval  $-1 \leq x \leq 5$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$  and  $x = 3$ . The function  $f$  is twice differentiable with  $f(2) = 6$ .
- Find the  $x$ -coordinate of each of the points of inflection of the graph of  $f$ . Give a reason for your answer.
  - At what value of  $x$  does  $f$  attain its absolute minimum value on the closed interval  $-1 \leq x \leq 5$ ? At what value of  $x$  does  $f$  attain its absolute maximum value on the closed interval  $-1 \leq x \leq 5$ ? Show the analysis that leads to your answers.
  - Let  $g$  be the function defined by  $g(x) = xf(x)$ . Find an equation for the line tangent to the graph of  $g$  at  $x = 2$ .

**2004 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

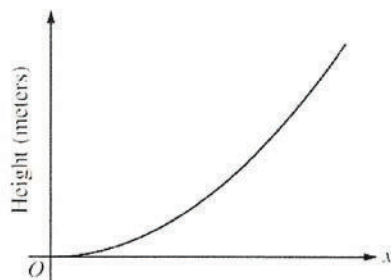


6. Let  $\ell$  be the line tangent to the graph of  $y = x^n$  at the point  $(1, 1)$ , where  $n > 1$ , as shown above.
- Find  $\int_0^1 x^n dx$  in terms of  $n$ .
  - Let  $T$  be the triangular region bounded by  $\ell$ , the x-axis, and the line  $x = 1$ . Show that the area of  $T$  is  $\frac{1}{2n}$ .
  - Let  $S$  be the region bounded by the graph of  $y = x^n$ , the line  $\ell$ , and the x-axis. Express the area of  $S$  in terms of  $n$  and determine the value of  $n$  that maximizes the area of  $S$ .
- 

**2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

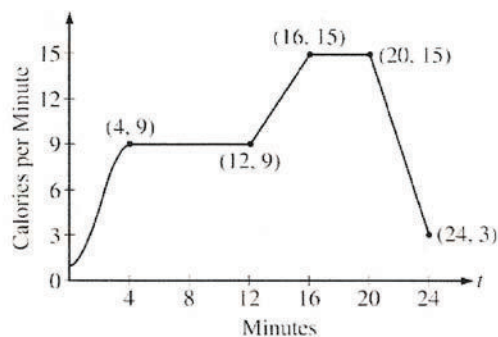
5. Consider the curve given by  $y^2 = 2 + xy$ .
- Show that  $\frac{dy}{dx} = \frac{y}{2y - x}$ .
  - Find all points  $(x, y)$  on the curve where the line tangent to the curve has slope  $\frac{1}{2}$ .
  - Show that there are no points  $(x, y)$  on the curve where the line tangent to the curve is horizontal.
  - Let  $x$  and  $y$  be functions of time  $t$  that are related by the equation  $y^2 = 2 + xy$ . At time  $t = 5$ , the value of  $y$  is 3 and  $\frac{dy}{dt} = 6$ . Find the value of  $\frac{dx}{dt}$  at time  $t = 5$ .
-

**2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**



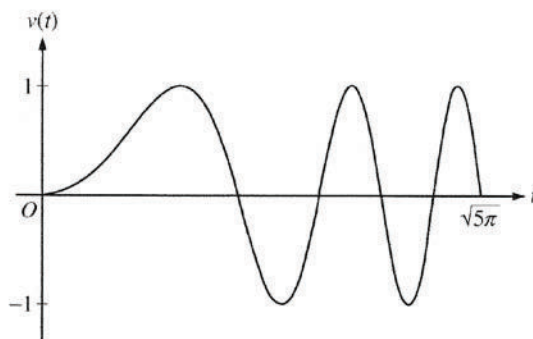
3. The figure above is the graph of a function of  $x$ , which models the height of a skateboard ramp. The function meets the following requirements.
- (i) At  $x = 0$ , the value of the function is 0, and the slope of the graph of the function is 0.
  - (ii) At  $x = 4$ , the value of the function is 1, and the slope of the graph of the function is 1.
  - (iii) Between  $x = 0$  and  $x = 4$ , the function is increasing.
- (a) Let  $f(x) = ax^2$ , where  $a$  is a nonzero constant. Show that it is not possible to find a value for  $a$  so that  $f$  meets requirement (ii) above.
  - (b) Let  $g(x) = cx^3 - \frac{x^2}{16}$ , where  $c$  is a nonzero constant. Find the value of  $c$  so that  $g$  meets requirement (ii) above. Show the work that leads to your answer.
  - (c) Using the function  $g$  and your value of  $c$  from part (b), show that  $g$  does not meet requirement (iii) above.
  - (d) Let  $h(x) = \frac{x^n}{k}$ , where  $k$  is a nonzero constant and  $n$  is a positive integer. Find the values of  $k$  and  $n$  so that  $h$  meets requirement (ii) above. Show that  $h$  also meets requirements (i) and (iii) above.
-

**2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**



4. The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function  $f$ . In the figure above,  $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$  for  $0 \leq t \leq 4$  and  $f$  is piecewise linear for  $4 \leq t \leq 24$ .
- Find  $f'(22)$ . Indicate units of measure.
  - For the time interval  $0 \leq t \leq 24$ , at what time  $t$  is  $f$  increasing at its greatest rate? Show the reasoning that supports your answer.
  - Find the total number of calories burned over the time interval  $6 \leq t \leq 18$  minutes.
  - The setting on the machine is now changed so that the person burns  $f(t) + c$  calories per minute. For this setting, find  $c$  so that an average of 15 calories per minute is burned during the time interval  $6 \leq t \leq 18$ .
- 

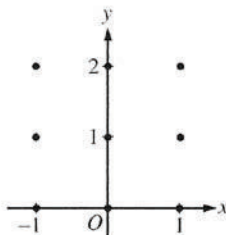
**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**



2. A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t \geq 0$  is given by  $v(t) = \sin(t^2)$ . The graph of  $v$  is shown above for  $0 \leq t \leq \sqrt{5\pi}$ . The position of the particle at time  $t$  is  $x(t)$  and its position at time  $t = 0$  is  $x(0) = 5$ .
- Find the acceleration of the particle at time  $t = 3$ .
  - Find the total distance traveled by the particle from time  $t = 0$  to  $t = 3$ .
  - Find the position of the particle at time  $t = 3$ .
  - For  $0 \leq t \leq \sqrt{5\pi}$ , find the time  $t$  at which the particle is farthest to the right. Explain your answer.
-

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

5. Consider the differential equation  $\frac{dy}{dx} = \frac{1}{2}x + y - 1$ .
- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.  
(Note: Use the axes provided in the exam booklet.)



- (b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Describe the region in the  $xy$ -plane in which all solution curves to the differential equation are concave up.
- (c) Let  $y = f(x)$  be a particular solution to the differential equation with the initial condition  $f(0) = 1$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 0$ ? Justify your answer.
- (d) Find the values of the constants  $m$  and  $b$ , for which  $y = mx + b$  is a solution to the differential equation.
- 

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

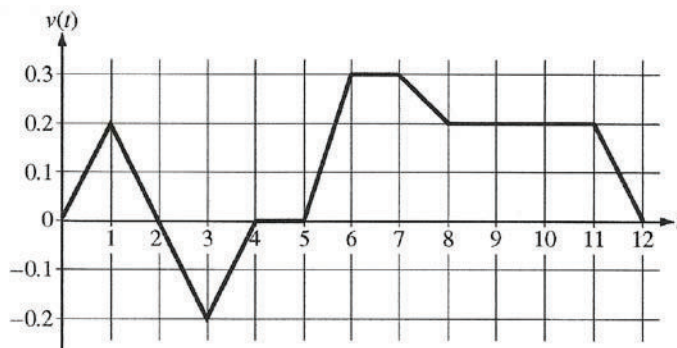
6. Let  $f$  be a twice-differentiable function such that  $f(2) = 5$  and  $f(5) = 2$ . Let  $g$  be the function given by  $g(x) = f(f(x))$ .
- (a) Explain why there must be a value  $c$  for  $2 < c < 5$  such that  $f'(c) = -1$ .
- (b) Show that  $g'(2) = g'(5)$ . Use this result to explain why there must be a value  $k$  for  $2 < k < 5$  such that  $g''(k) = 0$ .
- (c) Show that if  $f''(x) = 0$  for all  $x$ , then the graph of  $g$  does not have a point of inflection.
- (d) Let  $h(x) = f(x) - x$ . Explain why there must be a value  $r$  for  $2 < r < 5$  such that  $h(r) = 0$ .
-

2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by  $v(t) = 16 + 2\sin(\sqrt{t+10})$  for  $0 \leq t \leq 120$  minutes.
- Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.
  - The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from  $t = 0$  to  $t = 120$  minutes.
  - The scientist proposes the function  $f$ , given by  $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$ , as a model for the depth of the water, in feet, at Picnic Point  $x$  feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.
  - Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval  $40 \leq t \leq 60$  minutes. Does this value indicate that the water must be diverted?
-



2009 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

1. Caren rides her bicycle along a straight road from home to school, starting at home at time  $t = 0$  minutes and arriving at school at time  $t = 12$  minutes. During the time interval  $0 \leq t \leq 12$  minutes, her velocity  $v(t)$ , in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.
- (a) Find the acceleration of Caren's bicycle at time  $t = 7.5$  minutes. Indicate units of measure.
- (b) Using correct units, explain the meaning of  $\int_0^{12} |v(t)| dt$  in terms of Caren's trip. Find the value of  $\int_0^{12} |v(t)| dt$ .
- (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.
- (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function  $w$  given by  $w(t) = \frac{\pi}{15} \sin\left(\frac{\pi}{12}t\right)$ , where  $w(t)$  is in miles per minute for  $0 \leq t \leq 12$  minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

2009 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

2. A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by  $f(t) = \sqrt{t} + \cos t - 3$  meters per hour,  $t$  hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of  $f(t)$  is  $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$ .
- (a) What was the distance between the road and the edge of the water at the end of the storm?
- (b) Using correct units, interpret the value  $f'(4) = 1.007$  in terms of the distance between the road and the edge of the water.
- (c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.
- (d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of  $g(p)$  meters per day, where  $p$  is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

## AP Exam Preparation Journal Entry Format

Student: Date: AP Question Year and Number:	As you work, record your thinking, planning, and strategy use on this side. Be very detailed and record ALL actions involved in solution development.
<u>Solution:</u>  a)          b)          c)          d)	<u>Outline of solution development:</u>  a)          b)          c)          d)

## Classroom Observation Protocol

Students: Date and Time: AP Question Year and Number:	
Sketch of group with relative positioning:	
Descriptive Notes:	Reflective Notes:

## Appendix F: Think-Aloud Session Instrumentation and Protocols

## Application of Differentiation Problem-Solving Task

Suppose  $f(x) = ax^3 + bx^2 + cx + d$ ,  $a \neq 0$ , represents a family of cubic functions.

- a) If possible, find  $a$ ,  $b$ ,  $c$ , and  $d$  such that  $f$  has exactly two horizontal tangents and exactly one root. Justify your solution.
- b) If possible, find  $a$ ,  $b$ ,  $c$ , and  $d$  such that  $f$  is concave up on  $(-\infty, \infty)$ . Justify your solution.

## Application of Integration Problem-Solving Task

Let  $h(x) = e^{-mx}$ , where  $m$  is a nonzero, positive integer.

- a) Suppose the area,  $A_1$ , bounded by  $h$  and the lines  $y = 0$ ,  $x = 0$ , and  $x = 2$ . Find the equation of the vertical line that divides  $A_1$  exactly in half. Justify your solution.
- b) Suppose the area,  $A_2$ , bounded by  $h$  and the lines  $y = 1/2$  and  $x = 0$ . Find the equation of the horizontal line that divides  $A_2$  exactly in half. Justify your solution.

### Think-Aloud Problem-Solving Session #1 Protocol (Ericsson & Simon, 1993)

In this experiment I am interested in what you think about when you are working on a calculus problem. To explore this, I am going to ask you to THINK ALOUD as you work on given problems. What I mean by think aloud is that I want you to tell me EVERYTHING you are thinking from the time you first see the problem until you give me an answer. I would like you talk aloud CONSTANTLY from the time I present each problem until you have given your final answer. I don't want you to try to plan out what you say or try to explain to me what you are saying. Just act as if you are alone in the room speaking to yourself. It is most important that you keep talking. If you are silent for any long period of time I will ask you to talk. Do you understand what I want you to do?

#### **PRACTICE PROBLEM # 1**

Good, now we will begin with a practice problem. Please solve the following problem and think aloud as you do so.

**Give student Practice Problem #1. Student will work the problem and think aloud.**

**If the student is silent for 10 – 15 seconds, prompt them to “KEEP TALKING.”**

Good, now I want to see how much you can remember about what you were thinking from the time you read the problem until you gave the answer. I am interested in what you actually can REMEMBER rather than what you think you must have thought. If possible I would like you to tell about your memories in the sequence in which they occurred while working on the problem. Please tell me if you are uncertain about any of your memories. I don't want you to work on solving the problem again, just report what

you can remember thinking about when solving the problem. Now tell me what you can remember.

**Student will respond.**

**PRACTICE PROBLEM # 2**

Good. Now I will give you one more practice problem before we proceed to the problems I will be analyzing. I want you to do the same thing for this problem. I want you to think aloud as before as you think about the problem, and after you have answered it I will ask you to report all that you can remember about your thinking. Any questions? Here is your next problem.

**Give student Practice Problem #2. Student will work the problem and think aloud.**

**If the student is silent for 10 – 15 seconds, prompt them to “KEEP TALKING.”**

Now tell me all that you can remember about your thinking.

**Student will respond.**

**PROBLEMS FOR THE STUDY**

Good, now I have two problems that I will be analyzing for the study. You will receive an application of differentiation problem first. If you complete it, you may receive an application of integration problem if time allows. You will think aloud as you work on the problem(s) for thirty minutes.

**Give student the Application of Differentiation problem. Student will work the problem and think aloud. If the student is silent for 10 – 15 seconds, prompt them to “KEEP TALKING.” If student finishes the problem with sufficient time remaining, give them the Application of Integration problem.**

Now tell me all that you can remember about your thinking.



**Student will respond. I will provide cognitive feedback for the purposes of SRL processing.**

Thank you very much for participating. When you return for your second session, you will continue working on the current problem. The second session will be similar to this one in terms of structure. You may think about the problem during the interval between sessions if you wish.

### Think-Aloud Problem-Solving Session #2 Protocol (Ericsson & Simon, 1993)

In this experiment I am interested in what you think about when you are working on a calculus problem. To explore this, I am going to ask you to THINK ALOUD as you work on given problems. What I mean by think aloud is that I want you to tell me EVERYTHING you are thinking from the time you first see the problem until you give me an answer. I would like you talk aloud CONSTANTLY from the time I present each problem until you have given your final answer. I don't want you to try to plan out what you say or try to explain to me what you are saying. Just act as if you are alone in the room speaking to yourself. It is most important that you keep talking. If you are silent for any long period of time I will ask you to talk. Do you understand what I want you to do? Good, do you need a practice problem for this session?

#### **PRACTICE PROBLEM # 3 (IF NEEDED)**

**If yes:** Please solve the following problem and think aloud as you do so.

**Give student Practice Problem #3. Student will work the problem and think aloud.**

**If the student is silent for 10 – 15 seconds, prompt them to “KEEP TALKING.”**

Good, now I want to see how much you can remember about what you were thinking from the time you read the problem until you gave the answer. I am interested in what you actually can REMEMBER rather than what you think you must have thought. If possible I would like you to tell about your memories in the sequence in which they occurred while working on the problem. Please tell me if you are uncertain about any of your memories. I don't want you to work on solving the problem again, just report what

you can remember thinking about when solving the problem. Now tell me what you can remember.

**Student will respond.**

Good. Now, do you need another practice problem?

**PRACTICE PROBLEM # 4 (IF NEEDED)**

**If yes:** I want you to do the same thing for this problem. I want you to think aloud as before as you think about the problem, and after you have answered it I will ask you to report all that you can remember about your thinking. Any questions? Here is your next problem.

**Give student Practice Problem #4. Student will work the problem and think aloud.**

**If the student is silent for 10 – 15 seconds, prompt them to “KEEP TALKING.”**

Now tell me all that you can remember about your thinking.

**Student will respond.**

**PROBLEMS FOR THE STUDY**

Good, now let's return to the problems that I will be analyzing for the study. You will think aloud as you work on the problems for thirty minutes.

**Give student appropriate problem solving task. Student will work the problem and think aloud. If the student is silent for 10 – 15 seconds, prompt them to “KEEP TALKING.” If student finishes the Application of Differentiation problem with sufficient time remaining, give them the Application of Integration problem.**

Good, now I want to see how much you can remember about what you were thinking from the time you read the problem until you gave the answer. I am interested in what you actually can REMEMBER rather than what you think you must have thought. If

possible I would like you to tell about your memories in the sequence in which they occurred while working on the problem. Please tell me if you are uncertain about any of your memories. I don't want you to work on solving the problem again, just report what you can remember thinking about when solving the problem. Now tell me what you can remember.

**Student will respond.** Thank you very much for participating.

## Appendix G: Follow-Up Interview Protocol

Research Project: Self-regulation and epistemology in complex problem solving

Student:

Date:

Time:

“The purpose of this interview is to provide you an opportunity to review my initial findings and ensure that my final report will be an accurate representation of your experiences.”

Questions:

1. Please review the following document which summarizes the results of your participation in this research project.

Notes:

2. Do you concur with the findings of the study in relation to your participation?

Notes:

3. Is there a better way these findings could clarify or better describe your participation in this study? If so, how?

Notes:

4. Do you think the results of this study could be used to improve students' abilities to solve mathematical problems? If so, how?

Notes:

“Thank you very much for your participation in this study.”

## Appendix H: Code Book

### **PHASES OF SRL** (Winne & Hadwin, 1998; Zimmerman, 2000)

**\*\* Roman numerated items indicate macro-level nodes. Bulleted items indicate micro-level codes with suggestions of even more specific, finer-grained codes for specific student behaviors. \*\***

#### **I. Definition of Task**

- **Reading and re-reading the problem (RP) (Schoenfeld, 1985)** – noting conditions of the problem (NC), identifying goal state (IG), assessment of content knowledge based on task (CK)
- **External Feedback (Nicol & Macfarlane-Dick, 2006; Winne & Hadwin, 1995)** – reaction to peer feedback (PF), reaction to teacher feedback (TF)
- **Analysis of the Problem (AP) (Schoenfeld, 1985)** – establish relationship(s) between conditions and goal (ER)

#### **II. Forethought**

- **General Planning (GP) (Greene & Azevedo, 2009; Pintrich, 2000)** – overt evidence of mastery-approach goal orientation (MGAP), overt evidence of mastery-avoidance goal orientation (MGAV), overt evidence of performance-approach goal orientation (PGAP), overt evidence of performance-avoidance goal orientation (MGAV), prior knowledge activation (PK), recycle goal in working memory (RG)

- **Problem-Solving Planning (PSP) (Schoenfeld, 1985)** – assessment of plan (AP), inferred planning (IP), overt planning (OP)

### III. Performance Control

- **Self-control (SC) (Zimmerman, 2000)** – self-instruction (SI), imagery (IM), attention focusing (AF), task strategies (TS)
- **Self-Observation (SO) (Zimmerman, 2000)** – evidence of internal feedback (IF), performance-focused self-observations (PF), deficit-focused self-observation (DF)
- **Regulating Motivation (RM) (Garcia & Pintrich, 1994)** – self-handicapping (SH), defensive pessimism (DP), self-affirmation (SA), attributional style (AT), self-consequating (SC), environmental control (EC), interest enhancement (IE), performance self-talk (PST), mastery self-talk (MST)
- **Exploration of problem space (EXP) (Schoenfeld, 1985)** – continuing on “wild goose chase” (WGC), monitoring progress (MP), purposeful exploration (PU)
- **Implementation of Problem-Solving Plan (IMP) (Schoenfeld, 1985)** – global assessment of implementation (GA), local assessment of implementation (LA)
- **Strategy Use (SU) (Greene & Azevedo, 2009)** – accessing memory (AM), re-reading problem (RP), inferences (IN), hypothesizing (HY), knowledge elaboration (KE)
- **Heuristic Strategies (HS) (Polya, 1957; Schoenfeld, 1985)** – draw a picture (DP), recall a similar problem (RSP), solve a simpler/similar problem (SP), subgoals (decomposing and recombining) (DR), introduce appropriate notation (NO), restate the problem (RP), set up equations (EQ)

- **Monitoring (MO) (Greene & Azevedo, 2009)** – judgment of learning (JL), feeling of knowing (FK), self-questioning (SQ), monitor progress toward goals (MG), monitor use of strategies (MS)
- **Problem-Solving Transitions Based on New Information (TR) (Schoenfeld, 1985)** – assessment of current solution state (AC), attempt to salvage work (SW), assessment of appropriateness of new direction (AD), assessment of short- or long-term effects of new direction (ED), “jumping in” to new direction (JI)
- **Task Difficulty and Demands (TD) (Greene & Azevedo, 2009)** – time and effort planning (TE), assessment of task difficulty (TD), expectation of adequacy of information (AI)

#### IV. Self-reflection

- **Self-evaluation (SE) (Zimmerman, 2000)** – mastery criteria (MC), current functioning (CF), past performance (PP), normative criteria (NC)
- **Causal Attributions (CA) (Zimmerman, 2000)** – ability-focused (AF), strategy-focused (SF)
- **Verification of Solution (VS) (Schoenfeld, 1985)** – review of solution (RS), testing of solution (TS), assessment of confidence in solution (AC)
- **Assessment of Future Use of Methods (FM) (Polya, 1957)** – global assessment of methods used (GA), discovery of new or better solution for current problem (NBS), expression of usefulness of result for future work (UF)



**GROUP REGULATION** (Artzt & Armour-Thomas, 1992; Goos, Galbraith, & Renshaw, 2002)

**\*\* When solving problems in groups, Schoenfeld's problem-solving framework, as coded above, will suffice for the majority of student behaviors. However, this section of the codebook provides codes for group-problem-solving specific behaviors. \*\***

- **Group-specific problem-solving behaviors (GPS)** – watching and listening (WL)
- **Group problem-solving monitoring (GMO)** – self-disclosure (SD), feedback request (FR), other-monitoring (OM)
- **Group problem-solving verification (GPV)** – testing solutions (TS)

**DIMENSIONS OF GENERAL EPISTEMOLOGICAL BELIEFS** (Hofer, 2000;

Hofer & Pintrich, 1997; Muis & Franco, 2009)

**\*\* Roman numerated items indicate macro-level nodes. Bulleted items indicate micro-level codes with operational definitions to help identify specific instances of students expressing each belief. It should be noted that a dimensionality stance on beliefs implies a continuum exists for each dimension. Thus, individuals will express differing degrees of beliefs for each dimension, often exhibiting contradictions, and typically context-dependent.\*\***

**I. Certainty of Knowledge (CE)**

- **Overt evidence of *fixed* belief (FI) – def.:** knowledge cannot be doubted, everyone will develop the same conclusions
- **Overt evidence of *fluid* belief (FL)– def.:** knowledge is not certain, knowledge evolves as more info is gathered

**II. Simplicity of Knowledge (SI)**

- **Overt evidence of *straightforward* belief (ST)– def.:** knowledge is one fact after another and unrelated
- **Overt evidence of *interrelated* belief (IN) – def.:** conceptual meanings are complex, relative to others, and contextually dependent

**III. Source of Knowledge (SO)**

- **Overt evidence of *external* belief (EX) – def.:** knowledge is handed down from authority; authority should not be questioned
- **Overt evidence of *internal* belief (IN) – def.:** knowledge constructed by interactions, logic, and/or evidence; experts should be questioned

#### IV. Justification of Knowledge (JU)

- **Overt evidence of *authoritative* belief (AU) – def.:** knowledge claims are accepted if authorities come to consensus
- **Overt evidence of *personal* belief (PE) – def.:** knowledge claims are accepted based on experience-based logic and/or evidence

#### V. Attainability of Truth (ATT)

- **Overt evidence in belief of *attainability* of truth (AT) – def.:** every question or problem has a solution
- **Overt evidence of belief in *non-attainability* of truth (NA) – def.:** some questions and problems have no solution, but should still be explored

## **DIMENSIONS OF MATHEMATICAL PROBLEM-SOLVING BELIEFS**

(Kloosterman & Stage, 1992; Muis, 2008; Royce & Mos, 1980; Schoenfeld, 1985)

**\*\* Roman numerated items indicate macro-level nodes. Bulleted items indicate micro-level codes with operational definitions to help identify specific instances of students expressing each belief. It should be noted that a dimensionality stance on beliefs implies a continuum exists for each dimension. Thus, individuals will express differing degrees of beliefs for each dimension, often exhibiting contradictions, and typically context-dependent.\*\***

### **I. Empirical/rational problem solving (ER)**

- **Overt evidence of *rational* belief (RA) – def.:** knowledge is derived and justified via reason and logic; use of mathematical argumentation, derived proofs, theorems, or facts to solve/justify problems
- **Overt evidence of *empirical* belief (EM) – def.:** knowledge is derived and justified via direct observation; use of trial-and-error, serial testing of hypotheses, perceptual information to solve/justify problems

### **II. Problem-solving duration (DU)**

- **Overt evidence of *duration-conscious* belief (DC) – def.:** if problems cannot be solved quickly, they probably cannot be solved at all
- **Overt evidence of *duration-oblivious* belief (DO) – def.:** some problems take a long time to solve, but should still be worked to completion

### **III. Procedural/conceptual problem solving (PC)**

- **Overt evidence of *procedural* belief (PR) – def.:** problems can be solved if the right steps are applied

- **Overt evidence of *conceptual* belief (CO) – def.:** problems can be solved by understanding the underlying principles; conceptual understanding is important even when procedural means are applied

#### **IV. Effort/innate attainment of ability (EI)**

- **Overt evidence of *effort* belief (EF) – def.:** mathematical ability is attainable via effort and hard work
- **Overt evidence of *innate* belief (IN) – def.:** mathematical ability is inherent, mathematical geniuses are “born with it”

#### **V. Importance of problem solving (IPS)**

- **Overt evidence of the degree of importance assigned to problem solving versus computational skills**

#### **VI. Usefulness of mathematics (UM)**

- **Overt evidence of the degree of belief that mathematics is useful to one’s endeavors**