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The Direct and Indirect Paths Impacting Geometry Student Achievement

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THE DIRECT AND INDIRECT PATHS IMPACTING

GEOMETRY STUDENT ACHIEVEMENT

by

MarLynn Bailey

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DEDICATION

This dissertation is dedicated to none other than my inspiration, motivation, the person who believes in me more than I believe in myself, my husband.

I truly believe that I would not be "Dr. Bailey" without your love, support, encouragement, and guidance. People say to "reach for the stars," but from where I'm from, you can't reach for something you cannot see. Not only did you show me the stars, you raised me on your shoulders so that I could be that much closer.

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God is great!

ABSTRACT

THE DIRECT AND INDIRECT PATHS IMPACTING

MATHEMATICS STUDENT ACHIEVEMENT

by

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Previous studies have shown that several key variables influence student achievement in geometry, but more research needs to be conducted to determine how these variables interact. A model of achievement in geometry was tested on a sample of 102 high school students. Structural equation modeling was used to test hypothesized relationships among variables linked to successful problem solving in geometry. These variables, including motivation, achievement emotions, pictorial representation, and categorization skills were examined for their influence on geometry achievement. Results indicated that the model fit well. Achievement emotions, specifically boredom and enjoyment, had a significant influence on student motivation. Student motivation influenced students' use of pictorial representations and achievement. Pictorial representation also directly influenced achievement. Categorization skills had a significant influence on pictorial representations and student achievement. The implications of these findings for geometry instruction and for future research are discussed.

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CHAPTER 1

INTRODUCTION

Background of the Study

Mathematics integrates the skills of data collection, measurement, analysis, induction, deduction, problem solving, proofs, and mathematical modeling of real-world phenomena (National Council of Teachers of Mathematics, 1989, 2000; National Research Council, 1989). A strand of mathematics, geometry, requires students to utilize many of these skills (e.g. measurement, induction, deduction, problem solving, proofs, and modeling of real-world phenomena). Geometry is recognized by the National Council of Teachers of Mathematics (2000) as one of two strands in mathematics, the other being algebra, in which students spend the most time learning in their middle and high school mathematics courses. An understanding of geometric concepts is critically important for representing and solving problems in other mathematics and in science (Barndorff-Nielsen & Jensen, 1999; Herr, 2008; National Council of Teachers of Mathematics, 2000; Sherard, 1981). For example, high school geometry is a prerequisite for successive mathematics courses, such as advanced algebra, trigonometry, and calculus (NCTM, 2000; Sherard, 1981). It is also a necessary prerequisite for sciences such as chemistry and physics (Sherard, 1981). For instance, understanding that integrals represent the area under a curve can be instrumental when problem solving in calculus and chemistry.

Despite the importance of mathematics and geometry, students in the United States are underachieving when compared to other nations (Mullis et.al, 2000; OECD, 2009; Wilkins & Xin, 2002). The Organization for Economic Co-Operation and Development's (OECD) (2010) 2009 study ranked the mathematics proficiency of 15-year-old students in the United States as 32nd out of 65 countries. In the most recent Trends in International Mathematics and Science Study (TIMSS) (2011), geometry was the strand of mathematics in which students scored the lowest and the only strand that students performed below the average scale score. Specifically, United States eighth-graders scored 24 scale score points lower than their overall mathematics average scale score (Mullis, Martin, Foy, & Arora, 2012).

Statement of the Problem

A number of critical variables contribute to student success in geometry. The research in mathematics education focuses primarily on the impact of cognitive-related variables on students' success in geometry. Conceptual knowledge of concepts and proofs, visualization skills, and the ability to correctly set up and solve geometry problems are among those variables (NCTM, 2000). Visualization skills include students' ability to create diagrams that depict the important elements of geometry problems as a strategy to problem-solve (NCTM, 2000).

Existing research, however, has not considered how these variables interact to impact geometry achievement, which of these variables is most important, and how motivation and affect impact cognitive variables in explaining geometry achievement. The research in educational psychology indicates that motivation plays a critical role in impacting cognition and achievement in most any domain (Abuhamdeh &

Csikszentmihalyi, 2009; Awan, Noureen, & Naz, 2011; Glynn, Aultman, & Owens, 2005; Tella, 2007; Wigfield, Eccles, Schiefele, & Roeser, 2008). In addition, activity-related achievement emotions have been studied and are thought to be critical for impacting student learning and performance across a variety of domains (Pekrun, Elliot, & Maier, 2009). Given the importance of geometry for (a) more advanced math and science and (b) the fact that students in the United States are underachieving in geometry, it is important to understand how variables interact to contribute to success in geometry. Also, because of their interactions, it is important to understand which variables are most essential for geometry achievement. This information will help mathematics educators and researchers intervene to support geometry achievement.

Research Questions

The following research questions were developed based on existing research that identifies a need for improvement in geometry achievement, and on research that identifies variables influencing geometry achievement.

> How do motivation, achievement emotions (boredom and enjoyment), pictorial representations, and problem categorization interact to influence achievement in geometry?
>
> Which of these variables are most influential?
>
> Which variables impact achievement in geometry indirectly through other variables?

Purpose of the Study

The primary goal of the present study was to test and validate a model of the variables that contribute to achievement in geometry. The tested model identifies key variables contributing to achievement in geometry, describes how these variables influence each other, and quantifies the relative contributions of each variable. Specifically, the model will examine the influences of motivation, achievement emotions, pictorial representations, and categorization skills on achievement in geometry. The knowledge that results from studying these variables and how they interact can be used to improve the teaching of geometry.

Conceptual/Theoretical Framework

The tested model of geometry achievement was developed with three theories in mind. The first focuses on modeling cognition such as problem solving in academic domains (Anderson & Schunn, 2000; Anderson et al., 2004), like in science and mathematics. The second provides researchers with a framework for analyzing motivation and emotions experienced during achievement related activities (Pekrun, 2006). Lastly, social cognitive theory establishes the role of motivation in the context of learning (Schunk, 2012).

Anderson and colleagues' *Adaptive Control of Thought-Rational* (ACT-R) theory of learning and cognition provides a general system to use when modeling cognitive processes (Anderson, Matessa, & Lebiere, 1997). Models built upon the ACT-R theory acknowledge that several modules, or variables, of learning and cognition exist and illustrate how several variables function independently and dependently in achieving a goal (Anderson et al., 2004). The model present in this study suggests that several variables influence geometry achievement. Some of the variables are representative of the problem solving process, such as pictorial representations and categorization skills.

According to the ACT-R theory, cognitive processes, like problem solving, involve both declarative and procedural knowledge (Anderson, 1996; Anderson & Schunn, 2000). Declarative knowledge includes facts that have been stored in an individual's bank of knowledge, and procedural knowledge refers to the knowledge of knowing how to use declarative knowledge to perform cognitive tasks (Anderson & Schunn, 2000). Both types of knowledge is thought to be a network of what ACT-R theorists refer to as chunks (e.g. Anderson & Schunn, 2000). Understanding requires a large amount of both declarative and procedural knowledge chunks. The level of understanding and learning is reliant on the speed of the activation process, which refers to the successful retrieval of declarative knowledge. The speed of activation depends on the fluency in performance which consists of the level of activation of the chunks of knowledge being retrieved, as well as the strength of the cognitive resources doing the retrieving (Anderson & Schunn, 2000). Relative to the variables in the present model, categorization skills, the quality of pictorial representations, and student achievement in geometry all require the retrieval of both declarative and procedural knowledge.

Pekrun's (2006) *Control-Value Theory of Achievement Emotions* provides a framework for analyzing achievement emotions. This theory suggests that achievement emotions are reliant on appraisals of control and values. Achievement emotions can be situational, or domain-specific, such as a student who experiences enjoyment during science-related activities, but boredom during mathematics-related activities. Achievement emotions can be experienced momentary or can be reoccurring. The control-value theory recognizes that an individual's subjective control and subjective values of achievement activities are specifically relevant to achievement emotions.

Subjective control refers to an individual's perceived control during the achievement activities and the outcomes that result from participation in the activities. Subjective values refer to the valences that an individual associates with achievement activities and outcomes.

While achievement emotions include both emotions experienced during an activity (e.g. boredom, enjoyment, and frustration) and emotions experienced after (e.g. shame, pride, anxiety, and relief) (Pekrun, 2006), this study focuses only on achievement emotions experienced during the activity, specifically enjoyment and boredom. Pekrun's (2006) control-value theory posits that enjoyment is experienced when an individual concurrently perceives adequate control of the activity and values the achievement activity in a positive way. Regardless of the perceived control, if an individual lacks incentive value for the activity, boredom is experienced. Boredom can occur if the incentive value is lowered due to the demands of the activity being beyond the capabilities of the individual, or if the activity is drastically below the individual's capabilities, failing to provide a challenge. Changing negative reoccurring achievement emotions is important for improving success in any educational domain (Pekrun, 2006), especially in mathematics where United States students repeatedly fall short of success compared to other countries (Mullis et.al, 2000; OECD, 2009; Wilkins & Xin, 2002). Reducing negative emotions by positively influencing appraisals of control and value is assumed to be critical to the success of educational intervention programs (Pekrun, 2006).

Lastly, Bandura's social cognitive theory suggests that motivation and behavior result from a triadic reciprocal process that includes interactions among personal, environmental, and behavioral influences (Figure 1) (Bandura, 1989; Bandura, 1997;

Schunk, 2012). This posits that human functioning is more than a product of personal influence, or more than simply a product of external influences from the environment (Bandura, 1989; Schunk, 2012). A person's motivation and actions are determined by the interplay between factors representative of all three variables, as shown in Figure 1, including personal contributions like cognition and affect, environmental contributions like observations, and behavioral contributions such as engaging in tasks (Bandura, 1989; Schunk, 2012).

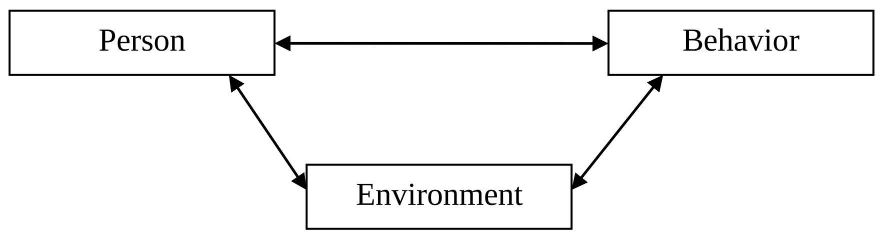

Figure 1. Social cognitive model of learning.

Social cognitive theory identifies motivation as an important influence in the learning process (Schunk, 2012). Self-efficacy, an individual's beliefs about their ability to perform on a specific task, provides a foundation for motivation (Bandura, 1997). A person's level of motivation, emotion, and their behaviors are not necessarily constructed from what is objectively true, but rather what they believe to be true (Bandura, 1997). When people do not believe they can perform well enough on a specific task to produce a desired outcome, they will not possess the motivation required to excel through challenges that may arise (Pajares, 2002). Consequently, a person's internal standards and self-reflection of their actions tend to motivate and regulate their future behaviors (Schunk, 2012).

Review of Relevant Terms

- motivation – measured by the Geometry Motivation Questionnaire (see Appendix A). Motivation can be defined as an internal state that arouses, directs, and sustains behavior towards a goal (Awan et al., 2011; Glynn et al., 2005).
- achievement emotions – measured by the Achievement Emotions in Geometry Questionnaire (see Appendix B). Achievement emotions are emotions linked not only to achievement outcomes, but are also emotions experienced during achievement activities (Pekrun, 2006).
- pictorial representation – measured by the quality of diagrams drawn while solving twelve geometry problems (see Appendix C & D). Pictorial representations refer to diagrams used during the problem-solving process to interpret, represent, and visualize the important elements of geometry problems (NCTM, 2000).
- categorization skills – measured by the Categorization Task (see Appendix E & F), that like typical categorization tasks used by researchers, require students to categorize problems and provide an explanation for the categorizations (e.g., Heyworth, 1999).
- geometry achievement – high school students' success in the mathematics strand of geometry; measured using the mean score of four unit tests that covered a variety of geometry topics.

CHAPTER 2

LITERATURE REVIEW

The following review of literature is organized into sections based upon the variables from the model in the present study. These include: motivation, emotions, pictorial representations, and conceptual knowledge and problem categorization. The influence of motivation on achievement is discussed through the discussion of several variables of motivation. Next, the research on achievement emotions, specifically boredom and enjoyment, is reviewed. The importance of the use of diagrams in problem solving in mathematics follows. Finally, the influence of conceptual knowledge on student achievement is discussed, along with the use of how categorization tasks have been used to assess conceptual knowledge, and how conceptual knowledge relates to the use of diagrams in problem solving.

Motivation

Ample research has linked motivation to achievement (Abuhamdeh & Csikszentmihalyi, 2009; Awan et al., 2011; Glynn et al., 2005; Tella, 2007; Wigfield et al., 2008). An individual's motivation is what determines the effort and behaviors one will put forth towards achieving their goals (Schunk, 2012). Research indicates that the variables of motivation that should be taken into account when considering students' motivation to learn include: intrinsic motivation, extrinsic motivation, self-determination, task relevancy, self-efficacy, and test anxiety (Pintrich, Smith, Garcia, & McKeachie, 1991; Wigfield et al., 2008; Wolters & Pintrich, 2001).

Intrinsic motivation is one of the central constructs of achievement motivation research (Wigfield et al., 2008), and is the motivation to participate in an activity simply for the enjoyment and interest of the activity. When individuals are intrinsically motivated to participate in a given task they usually excel (Abuhamdeh & Csikszentmihalyi, 2009; Husman & Lens, 1999)

Extrinsic motivation involves an individual's participation in an activity for the sake of what is to come at the end, such as earning a high letter grade (Abuhamdeh & Csikszentmihalyi, 2009; Halawah, 2006; Wigfield et al., 2008). Although intrinsic and extrinsic motivation have been presented as contrasting variables of motivation, research suggests that a combination of both is particularly beneficial for achievement in a domain (Abuhamdeh & Csikszentmihalyi, 2009; Husman & Lens, 1999).

Self-determination, another important variable of motivation, involves students intentionally engaging in the learning process with a full sense of volition, where one's internal self is the cause of the engagement (Deci, Vallerand, Pelletier, & Ryan, 1991; Wigfield et al., 2008). Studies connecting this variable of motivation to student achievement have demonstrated that students with more self-determination for completing school related activities are more likely to succeed (Deci et al., 1991; Wigfield et al., 2008).

Two other important variables of motivation are task relevancy and self-efficacy. Task relevancy refers to how important, useful, or interesting a student finds a task, which consequently determines whether the task is worth pursuing (Liem, Lau, & Nie, 2008; Pintrich & Schunk, 2002). Students who find a task to be more relevant to their personal goals are expected to be more involved in the learning of the task (Pintrich et al.,

1991) and more motivated to achieve (Wigfield et al., 2008). Self-efficacy is an individual's beliefs about their ability to perform on a particular task, and can influence a student's choice and performance in a specific domain (Pajares & Miller, 1994; Wolters & Pintrich, 2001). Bandura and Locke (2003) go as far to say that self-efficacy is the most prevalent mechanism of human agency, in which all other variables of motivation are rooted. Students' self-efficacy in mathematics has been linked to their problem-solving success (Pajares, 1996; Pajares & Kranzler, 1995; Pajares & Miller, 1994; Pajares & Miller, 1995). The level of a student's self-efficacy can predict a student's ability to mathematically problem-solve just as much as their general mental ability (Pajares, 1996; Pajares & Kranzler, 1995; Pajares & Miller, 1994; Pajares & Miller, 1995). Higher levels of self-efficacy in mathematics result in higher achievement (Pajares, 1996).

Test anxiety is a variable of motivation that has been found to negatively influence academic achievement. It occurs when students experience anxiety like worry or negative disruptive thoughts while taking assessments (Cassady & Johnson, 2002; Pintrich et al., 1991). When a student's level of test anxiety is low they perform better (Cassady & Johnson, 2002).

Extensive research has demonstrated the importance of motivation in mathematics education for secondary students (e.g., Awan et al., 2011; Stevens, Olivarez, Lan, & Tallent-Runnels, 2004). This is important given that mathematics is a domain that students historically have poor motivation towards, which can affect their achievement (Middleton & Spanias, 1999). However, there is a dearth of research on how motivation impacts mathematics achievement when emotional, motivational, and cognitive variables are considered simultaneously.

Emotions

Emotions are important because they can influence the energy and efforts needed to arouse, direct, and sustain behaviors necessary to achieve a particular goal (Hannula, 2006; Pekrun, Goetz, Titz, & Perry, 2002). For this reason, emotions have been directly linked to motivation and achievement (Frenzel, Pekrun, & Goetz, 2007; Pekrun, 2006; Pekrun et al., 2002; Pekrun, Molfenter, Titz, and Perry, 2000; Pekrun, & Stephens, 2009). In an academic context, achievement emotions are emotions experienced in relation to a particular academic activity.

The achievement emotions that have been explored in the research include positive emotions such as enjoyment, relief, pride, and hope, and negative emotions such as shame, hopelessness, anxiety, boredom, and anger. These emotions have been found to be domain specific (Pekrun, 2006), and influence motivation as well as cognitive processes, such as memory storage and retrieval, attention, perception, decision making, and problem solving (Frenzel et al., 2007; Pekrun et al., 2002; Pekrun, & Stephens, 2009). For example, positive emotions that create favorable moods during cognitive problem solving have been found to support creative and flexible ways of problem solving (Frenzel et al., 2007; Pekrun, 2006; Pekrun et al., 2002; Pekrun, & Stephens, 2009).

Enjoyment, a positive achievement emotion often reported by students, has been positively linked to student motivation and performance (Pekrun et al. 2002; Pekrun & Stephens 2009). The research on achievement emotions indicates that students who experience enjoyment during a task should allocate more cognitive resources and attention to the given task (Pekrun et al. 2002; Pekrun & Stephens, 2009). Research has determined significant correlations between domain specific levels of enjoyment, and specific motivational variables such as control and value. This research indicates that when domain specific enjoyment is high, so are levels of control and value. Inversely, when domain specific enjoyment is low, so are control and value (Pekrun et al., 2002).

Boredom, another achievement emotion often reported by students, has been negatively linked to student motivation and performance (Pekrun, Goetz, Daniels, Stupnisky, & Perry, 2010; Pekrun et al., 2002; Pekrun & Stephens, 2009). When negative emotions arise, such as boredom, more attention and cognitive resources are allocated to the emotion itself, rather than the activity at hand (Frenzel et al., 2007; Pekrun et al., 2010; Pekrun et al., 2002; Pekrun, & Stephens, 2009). For instance, if a student is problem solving, but is feeling bored, cognitive resources are being used by the feeling of boredom, and consequently, less cognitive resources are being applied to the problem-solving process (Pekrun et al., 2002; Pekrun, & Stephens, 2009). A study on boredom with university students found that boredom was negatively correlated with intrinsic motivation, effort, self-regulation, academic performance, and the use of sophisticated cognitive and metacognitive strategies (Pekrun et al., 2010).

There is limited research in mathematics examining the role of emotions on motivation and achievement. It is important for researchers to explore the extent to which emotions impact motivation directly, and the extent to which emotions impact conceptual knowledge, problem solving, and achievement indirectly through motivation.

Pictorial Representations

Geometry is a strand of mathematics where conceptual understanding and successful problem solving is reliant on the understanding of the shape, size, and properties of different figures (NCTM, 2000). For this reason, the use of diagrams is essential for successfully setting up and solving geometry problems (NCTM, 2000). These diagrams allow students to illustrate and interpret the important elements of geometry problems during the problem-solving process (NCTM, 2000). One of the goals of geometry instruction identified by NCTM (2000) is to enable students to use geometric modeling when problem solving.

Problems in geometry often involve determining the area, perimeter, or volume of a variety of objects including quadrilaterals, triangles, circles, and spheres (Larson, Boswell, & Stiff, 2004). Drawing a diagram of the objects prior to problem solving allows the students to visualize and depict the objects, angles, radii, side lengths, heights, diagonals, and other important key elements.

Geometry textbooks, high school geometry instructors, and mathematics education researchers emphasize the importance of pictorial representations in geometry (e.g., Larson, Boswell, & Stiff, 2004; Zodik & Zaslavsky, 2007). For example, students are encouraged to use and draw diagrams when learning about and solving geometry problems (Zodik & Zaslavsky, 2007). In addition, most geometric concepts presented in class or in textbooks are accompanied by a diagram. The use and manipulation of diagrams is more important for geometry than any other mathematics, and research has even shown that students' general spatial abilities improve after geometry instruction (e.g., Baki, Kosa, & Guven, 2009; Gittler & Gluck, 1998).

Although research has shown that scores on spatial visualization tests are positively correlated with geometry problem solving and achievement (Battista, 1990), there is a lack of empirical research examining the relationship between the use of diagrams during problem solving and students' geometry achievement. One goal of the present study was to examine the link between pictorial representations, conceptual knowledge, and achievement in geometry.

Conceptual Knowledge and Problem Categorization

Conceptual knowledge is critical for achievement in geometry (NCTM, 2000). Problem categorization tasks are one common method that researchers have used to assess students' conceptual knowledge across a variety of domains (e.g., Heyworth, 1999). These tasks typically require individuals to categorize problems and explain the reasoning behind their categorizations. For example, in Chi, Feltovich, and Glaser's seminal 1981 study, experts and novices were compared and asked to sort physics problems in any manner they chose. The researchers found that the experts sorted the problems based on deeper underlying features, such as by the theorems or principles needed to solve the problems. Novices sorted the problems based on superficial surface level features, such as the type of objects presented in the problems. The way the problems were sorted provided insight into the conceptual knowledge of the experts and novices.

Research has shown that a relationship exists between the way students organize their mathematical content knowledge and their problem solving success. Lawson and Chinnappan (2000) assessed high-achieving and low-achieving students' performances on geometry tasks requiring students to recall and identify well known geometry forms, relationships, theorems, and formulas. Students in the high-achieving group scored significantly higher than students in the low-achieving group on a problem solving task. Lim (2013) compared students' organization of knowledge with their success on chapter tests in a college algebra course. Organization of knowledge was measured by the information students selected to include on a single page “cheat sheet,” and the organization of that information. The researcher found that the quality of cheat sheets could be used to predict student achievement.

Research in physics education indicates that individuals with greater conceptual knowledge are more likely to draw a picture when solving problems (e.g., Dhillon, 1998; Stylianou & Silver, 2004). There has not yet been a study that determines the extent to which conceptual knowledge impacts students’ diagrams in geometry, and the extent to which these diagrams in turn impact successful problem solving. In addition, students’ diagrams in geometry need to be examined to determine the quality or complexity of the diagrams that students are drawing during the problem solving process.

Present Study

The present study used structural equation modeling, specifically path analysis, to test a model of geometry achievement. The model examines the impact of motivation, achievement emotions (boredom and enjoyment), pictorial representations, and problem categorization on achievement in geometry. The study is innovative in that it simultaneously tests the influences of these variables on achievement and will help determine the contributions of individual variables when other variables are considered.

The model was developed based on the existing research on achievement emotions, motivation, and problem solving in mathematics, particularly in geometry. As displayed in the model shown in Figure 2, achievement emotions were expected to influence motivation. It was expected that students experiencing greater levels of enjoyment when studying geometry would be more motivated to learn geometry, and students experiencing boredom when studying geometry would be less motivated to learn geometry. Motivation was expected to directly impact students' pictorial representations in that more motivated students would be more likely to draw complex diagrams when solving geometry problems. Consistent with the research on motivation, it was expected that motivation would directly impact achievement. Pictorial representations were expected to directly impact achievement in that students who drew more complex pictures would be more likely to correctly set up and solve geometry problems. Categorization skills were expected to impact achievement both directly and indirectly through pictorial representations. Therefore, it was expected that students with greater conceptual knowledge would be more likely to draw diagrams to help them successfully solve geometry problems. Furthermore, conceptual knowledge was expected to be directly linked to higher achievement.

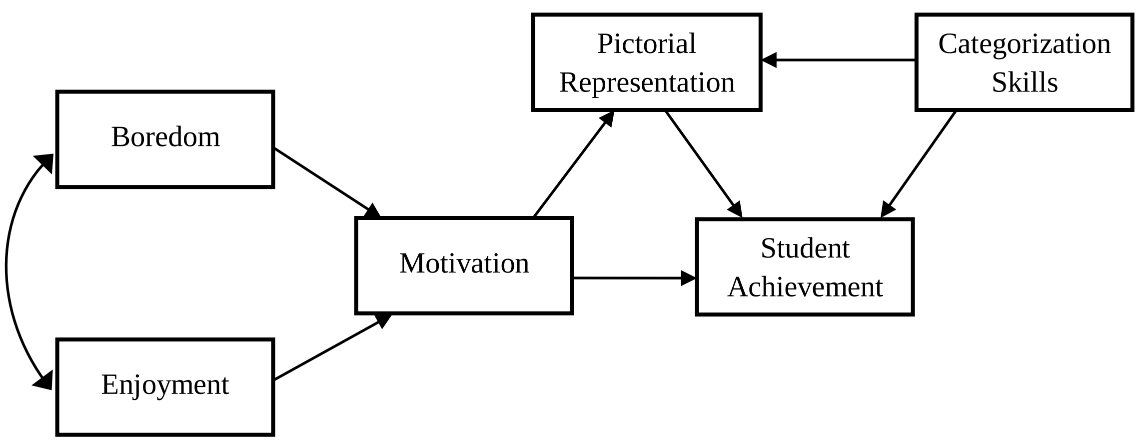

Figure 2. Theoretical model of student achievement in geometry.

CHAPTER 3

METHODOLOGY

Research Questions

How do motivation, achievement emotions (boredom and enjoyment), pictorial representations, and problem categorization interact to influence achievement in geometry?

Which of these variables are most influential?

Which impact achievement in geometry indirectly through other variables?

Participants

Participants included 102 high school students (50 males and 52 females) from nine sections of a tenth grade required mathematics course with a major geometry variable that high school students take in preparation for college in the state of Georgia. The classes were taught by four different teachers in a high school approximately twenty miles outside of the Atlanta area. Overall, a large portion of the students enrolled in the year-long course participated in the study (60%). The ethnicities of the participants are as follows: 62.7% Black, 17.6% Hispanic, 10.8% Multiracial, 6.9% White, less than 1% American Indian/Alaskan Native, and less than 1% Asian. The sample is representative of the overall school population. Students' participation was voluntary, and students were not penalized if they chose not to participate. Following the guidelines for research with human subjects identified by the institutional review board, informed consent forms were signed by a parent or legal guardian of the participants, and assent forms were signed by the participants.

Procedure and Materials

All students who participated in the study were administered a packet that included a geometry motivation questionnaire; achievement emotions questionnaire; twelve geometry problems designed to assess students' use of pictorial representations when problem solving; and a categorization task used to assess conceptual knowledge by examining whether students focus on conceptual or surface features of geometry problems. Demographics information was collected as well as information on students' geometry achievement using teacher access to grading and information software systems. The packet was administered during the spring semester after the completion of four consecutive geometry units that required approximately three and a half months to complete. The geometry content covered during these four months included major topics such as special right triangle patterns, trigonometric ratios, measurements of circles, and properties of circles. Students spent approximately 60-90 minutes completing the packet, and were not allowed to use their notes or textbook. Students were required to complete the packet independently, without influence or assistance from their instructors or peers. The packets were scored by a high school mathematics teacher, and a copy can be obtained by contacting the author of the study.

Motivation. The Geometry Motivation Questionnaire (GMQ) was used to assess student motivation. The GMQ was derived from the Science Motivation Questionnaire (SMQ), an instrument used to assess motivation in science (Glynn & Koballa, 2006; Glynn, Taasoobshirazi, & Brickman, 2007), but was modified in the current study to assess motivation in geometry. Specifically, the word science in the questionnaire was replaced with the word geometry, and is therefore referred to as the GMQ (see Appendix A). The questionnaire includes 30 items that measure six important variables of student motivation in geometry. These variables include intrinsic motivation in geometry (e.g., “I enjoy learning geometry”), extrinsic motivation in geometry (e.g., “I like to do better than other students on geometry tests”), relevance of learning geometry to personal goals (e.g., “The geometry I learn relates to my personal goals”), self-determination for learning geometry (e.g., “It is my fault if I do not understand geometry”), self-efficacy in learning geometry (e.g., “I believe I can master the knowledge and skills in geometry courses”), and anxiety about geometry assessment (e.g., “I become anxious when it is time to take a geometry test”). The 30 assessment items were randomly ordered, and student responses were measured on a 5-point Likert scale ranging from 1 (*never*) to 5 (*always*) from the perspective of “When learning geometry….” The items that assessed anxiety about geometry assessments were reverse scored when added to the total, so that high scores on this variable reflected low levels of anxiety. Composite scores on the 30 items have been used to assess students’ overall motivation (Glynn & Koballa, 2006, Glynn, Taasoobshirazi, & Brickman, 2007).

One student left one item on the questionnaire unanswered, so the item was scored by using the mean substitution method based on responses on the other items for that variable. In addition, two students each marked two responses on a single item. In this case, the average of the two responses was taken. Previous findings (Glynn & Koballa, 2006) indicate that the SMQ is reliable as measured by coefficient alpha (α =

.93) and valid as indicated by concurrent validity (Glynn, Taasoobshirazi, & Brickman, 2007). For the present study, internal consistency for the GMQ was found to be (α = .88).

Achievement emotions. Variables of the Achievement Emotions Questionnaire (AEQ) were used to assess students' emotions when studying geometry (Pekrun, Goetz, Frenzel, Barchfeld, & Perry, 2011). The questionnaire was modified for use in geometry, and is therefore referred to as the AEQ-G (see Appendix B). The items were revised so that they were specific to geometry. For example, an enjoyment item "I enjoy acquiring new knowledge" was revised to "I enjoy acquiring new geometry knowledge." Students were administered 21 items that assess two important variables of student achievement emotions including enjoyment when studying geometry (e.g., "I look forward to studying geometry"), and boredom when studying geometry (e.g., "Studying for my geometry class bores me"). Students responded to each of the 21 items (10 enjoyment items and 11 boredom items) that were grouped by emotion on a 5-point Likert scale ranging from 1 (*never*) to 5 (*always*) from the perspective of "When studying geometry…."

Two students each left one item on the questionnaire unanswered, so the item was scored by using the mean substitution method based on responses on the other items for that variable. The two variables were scored separately. Construct validity for the AEQ has been established as has the reliability of the instrument and its variables (Pekrun et al., 2011). For the present study, internal consistency for the enjoyment items was found to be (α = .71). Internal consistency for the boredom items was found to be (α = .93).

Pictorial representations. Twelve geometry problems were administered to students and were used to assess students' pictorial representations (see Appendix C). Although students were asked to solve the problems, only their pictures were scored. The quality of students' pictorial representations was examined because great emphasis in geometry instruction is placed on the importance of students pictorially representing the problems they are solving (Breslow, 2001; NCTM, 2000; Polya, 1957; Schoenfeld, 1992). Therefore, it is expected that students with little understanding of the geometry concepts would either not draw a sketch of the problem at all, or draw one that lacked important key elements required to solve the problem. On the other hand, students with a deeper understanding of the geometry concepts would include those key elements necessary to solve the problem.

The twelve problems were multiple-choice items and required a single solved mathematical solution. Students were asked to show all of their work when solving the problems. Students were not prompted to draw a picture or diagram; however, all of the problems described geometric objects and relations, so the decision to draw a diagram would aide successful problem solving. Given the emphasis in geometry instruction on the importance of diagrams when solving geometry problems, students were intentionally not prompted to draw diagrams to determine whether students would draw a diagram on their own and how detailed the diagram would be. In addition, drawing a diagram was viewed as a problem solving strategy, therefore, drawing a diagram needed to be the students' own approach to solving the problems.

Each of the twelve problems was based on major geometry concepts including special right triangle patterns, trigonometric ratios, measurements of circles, properties of circles, and properties of quadrilaterals[1]. All twelve problems were released items from the county-wide standardized Mathematics 2 Benchmark Assessment.

[1] Properties of quadrilaterals is a prerequisite concept.

The pictures were scored by comparing the students' pictures to a target sketch of each problem. The target sketches were created by a high school mathematics teacher, and were compared to sketches created by a second high school mathematics teacher to verify the key elements that needed to be included in the target sketch of each problem (see Appendix D). Students' sketches were scored so that students earned 1 point for each key element pictorially represented in each target sketch with the range of possible total scores being from 0 to 57. To assess reliability, the sketches were scored by two raters, with an intra-class correlation coefficient of .99.

Problem categorization. Students were administered a problem categorization task based on major topics in geometry including special right triangle patterns, trigonometric ratios, measurements of circles, and properties of circles (see Appendix E). The task included eight geometry problems, two problems from each major topic. Students were instructed to categorize the problems by putting them into pairs and then provide an explanation of why they paired the two problems the way they did. Students were not required to solve the problems. The specific underlying concepts for the problems included the use of the Pythagorean Theorem, trigonometric ratios, the area formula of a circle, or the formula used to calculate the circumference of a circle.

The problems for the task were from the Mathematics 2 state adopted textbook (Georgia High School: Mathematics 2, 2007). Students' categorizations and explanations were used to determine whether students focused on surface level features or underlying concepts when pairing the problems. The problems were selected so that students focusing on surface level features, such as visual similarities, rather than the underlying concepts needed to solve the problems would incorrectly pair a set of problems.

One of the correct pairings involved two problems that asked students to find the value of a side length of a right triangle that could both be solved using the Pythagorean Theorem. In this pairing, two side lengths, an acute angle measure, and markings on a second angle indicating 90 degrees were given. Another correct pairing included two right triangle problems similar to those described above, with the same given elements except with only one side length given. Both problems in this pairing required the use of either special right triangle patterns or right triangle trigonometry to find the value of the designated side length. The Pythagorean Theorem could not be used to solve the problems in this pairing. All four right triangle problems could have been solved using trigonometry; however no student paired them in this manner. If this were to happen the students would have received credit for the pairings. Students who would match problems based on surface level characteristics would match a problem from the first pair to a problem in the second pair because the size and orientation of a right triangle in the first pair was exactly the same as a triangle in the second pair. However, this would be a superficial pairing because the problems would require different underlying concepts necessary to correctly solve each problem.

The four problems from the last two pairings all included a diagram of a circle, with diagrams from two problems including a radius and its length, and the other two problems including a diameter and its length. Although the measurements were all different, the circle diagrams in all four problems were the same size. A student who would pair the problems based on surface level features would pair the two problems whose diagrams provided a radius and the two problems whose diagrams provided a diameter. A student with a greater conceptual understanding of geometry would recognize that one problem with a radius and another with a diameter requires the area of the circle, without using straightforward terminology in both problems. For instance, while one area problem simply asks for the area of the circle in the diagram, the other explains that a circular play-area is to be carpeted, and asks the student to determine the amount of carpet needed. Likewise, one of the problems whose diagram includes a radius and one that includes a diameter require students to find the circumference, where once again, one of the two problems leaves the student to determine which measurement they are to find.

To receive full credit for a pairing, students had to provide a correct explanation, eliminating the possibility of making correct pairings by chance. Students could receive a total of eight points on the task, where one point was awarded for correctly pairing two problems, and an additional point was awarded for a correct explanation of why those particular problems should be paired. Higher scores represented a focus on and understanding of the essential underlying concepts of the geometry problems. Three high school mathematics teachers completed the task to verify the correct pairings when using underlying key concepts. The same three teachers collaborated to determine acceptable explanations for each correct pairing (see Appendix F). To assess the reliability, the task was scored by two raters, with an intra-class correlation coefficient of .99. See Appendix G for examples of student work for both the pictorial representations and categorization tasks.

Student achievement. Student achievement in geometry was measured using the mean score on the four unit tests that covered the geometry topics discussed in the course. The unit tests were created by the collaborative team of teachers who taught the course at the participating high school. The team of teachers all administered the same four unit assessments.

At the end of the course, students are required to take a state mandated standardized assessment, called the Mathematics 2 End of Course Test (EOCT). Teachers have access to a study guide and practice assessment for the EOCT that the state releases through an electronic website. Problems on the unit tests were selected directly from the EOCT study guide and practice assessment. Each of the unit assessments included 10 to 17 multiple-choice and short answer problems, accompanied by one or two bonus problems designed by the collaborative team to challenge students' depth of understanding. The multiple choice problems were scored as correct or incorrect and the collaborative team of teachers together created common and detailed rubrics for scoring each open-ended problem on the unit assessments. In addition to the rubrics used to score the open-ended items, to ensure that the problems were being scored the same for all students, three student tests were selected and scored by all of the teachers on the collaborative team. The teachers then met to ensure that scores were the same and the rubric was being used properly. The unit tests of each participating student in the study were scored by their corresponding teacher.

CHAPTER 4

FINDINGS

The PASW (Predictive Analytics Software) program, version 18.0 was used to run descriptive statistics, mean comparisons, and correlations among the variables. LISREL Version 8.52 (Jöreskog & Sörbom, 2002) with a covariance matrix generated by PRELIS Version 2.52 (Jöreskog & Sörbom, 1996) was used to test the model.

Mean Comparisons and Correlations

Mean comparisons indicated that there were no differences across ethnic groups for the model variables. The only gender difference was in pictorial representations with females (M = 1.36, SD = 1.27) being more likely than males (M = .88, SD = 1.11) to draw a complex picture $t(100) = 2.066$, $p = .04$, Cohen's d =.40 (these means and standard deviations are based on transformed data as described below). As illustrated in Table 1, there were significant correlations among the model variables. There was a significant and large negative correlation between enjoyment and boredom, indicating that boredom when studying geometry was negatively related to enjoyment. Boredom was negatively correlated with motivation; enjoyment was positively correlated with motivation. Both of these correlations were significant and large in size. There was a positive and small correlation between motivation and pictorial representations, suggesting that higher motivation was related to the depiction of more substantial diagrams. Pictorial representations were significantly and moderately correlated with achievement. Motivation was also significantly correlated with achievement; this medium sized correlation indicated that higher motivation was related to higher achievement. Categorization skills were significantly correlated with pictorial representations. This medium sized correlation indicated that greater conceptual knowledge was related to more complex pictorial representations. Finally, there was a medium sized and significant correlation between categorization skills and achievement, indicating that greater conceptual knowledge of geometry was related to higher achievement.

Table 1

Correlation Matrix, Means, Standard Deviations, Skewness, and Kurtosis

Variable	1	2	3	4	5	6
1. Enjoyment	--					
2. Boredom	-.58**	--				
3. Motivation	.76**	- .54**	--			
4. Pictorial	.14	-.10	.27**	--		
5. Categorization	.10	-.04	.12	.43**	--	
6. Achievement	.26**	-.10	.36**	.40**	.36**	--
M	1.47	34.98	98.15	1.12	1.50	67.27
SD	.13	10.24	15.27	1.21	2.41	19.16
Skewness	-.71	-.20	-.07	.52	1.58	-.60
Kurtosis	2.19	-.45	.19	-1.20	1.37	.07

**$p < .01$.

Model Testing

Prior to empirically testing the model, the data were examined for univariate and multivariate normality. Two of the variables, pictorial representations and enjoyment, had kurtosis values greater than the absolute value of two. For this reason, the data for these two variables were transformed by taking the log of the values. This led to a kurtosis value less than the absolute value of 2 for pictorial representations and a kurtosis only slightly above 2 for enjoyment. Mardia's coefficient was 1.07, meeting the assumption of multivariate normality. For this reason, the model was tested by means of the maximum likelihood method estimation for normally distributed data.

To evaluate the fit of the model, several fit indices were considered. The chi-square statistic was: $\chi^2(5) = 3.91$, $p = .56$. The Steiger-Lind Root Mean Square Error of Approximation (RMSEA) was 0.0. The standardized root-mean-square residual (SRMR) was .03. Finally, the Bentler comparative fit index (CFI) was 1.00. These fit indices were all below recommended cutoff values (e.g., Bentler, 1990; Browne & Cudeck, 1993; Hoyle & Panter, 1995; Hu & Bentler, 1999), indicating that the model had an excellent fit.

Decomposition of Effects

The standardized path values for the model and their associated *t*-values are reported in Table 2. All of the direct path values were statistically significant based on a cutoff value of $t = 1.96$ for a two-tailed test. The criterion R^2 (proportion of variance explained) by motivation was .59, by pictorial representations was .22, and by achievement was .26. Figure 3 illustrates the model and the standardized path values. In interpreting the size and influence of the standardized path values, which range from 0 to

1, criteria similar to those of Keith (1993) were adopted. Path values in the .05 to .10 were considered small in size and influence; path values in the .11 to .25 range were considered medium in size and influence; path values larger than .25 were considered large in size and influence.

Table 2

Decomposition of Effects in the Model

Predictor and criterion	Direct effect		Indirect effect	
	PC	*t*	PC	*t*
Enjoyment				
Motivation	.67	8.43		
Pictorial			.15	2.42
Achievement			.22	3.34
Boredom				
Motivation	-.16	-1.96		
Pictorial			-.03	-1.55
Achievement			-.05	-1.72
Motivation				
Pictorial	.23	2.52		
Achievement	.32	3.64	.05	1.73
Pictorial				
Achievement	.23	2.37		
Categorization				
Pictorial	.40	4.47		
Achievement	.32	3.58	.09	2.09

Note. A cutoff value of t = 1.96 was used to determine whether paths were statistically significant. In terms of the relative size and influence of the standardized path coefficients, paths ranging from .05 to .10 are considered small in size and influence. Paths ranging from .11 to .25 are moderate in size and influence, and paths above .25 may be considered large in size and influence (Keith, 1993). PC = standardized path coefficient.

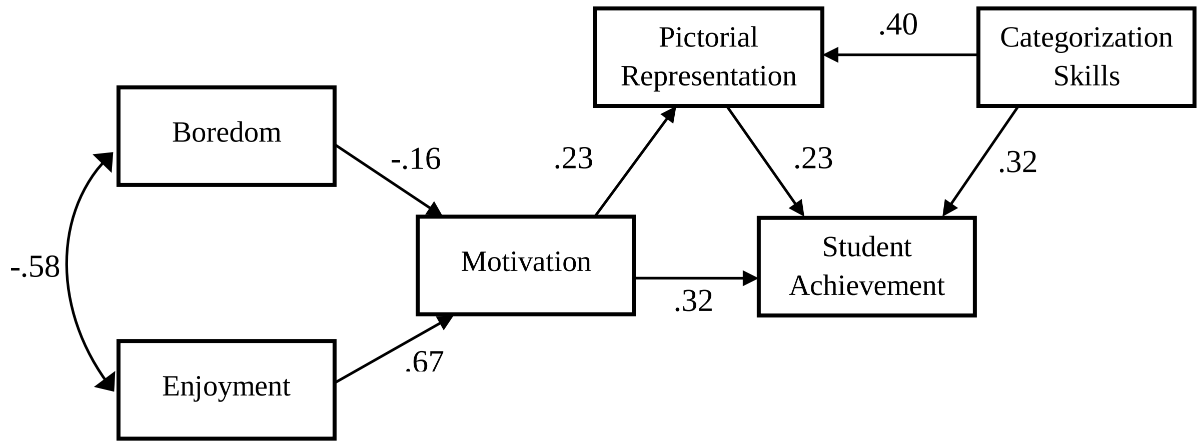

Figure 3. Tested model of student achievement in geometry. All path values are statistically significant.

The path from boredom to motivation was medium in size (-.16), indicating that students who felt bored when studying geometry had lower motivation to learn geometry. The path from enjoyment to motivation was large in size (.67), indicating that students who enjoyed studying geometry had higher motivation. This path was the largest in the model, emphasizing the importance of the emotional variable enjoyment on motivation. Motivation had a medium sized (.23) influence on pictorial representations, indicating that students who were more motivated to learn geometry were more likely to draw a complex picture. Categorization skills had a large (.40) impact on pictorial representations, indicating that students with a greater conceptual understanding of geometry were more likely to draw complex pictures. Categorization skills had a large (.32) influence on achievement; pictorial representations had a medium sized (.23) influence on achievement. These two paths indicated the importance of conceptual knowledge and diagrams on geometry achievement. Finally, motivation had a large (.32) influence on achievement. This is consistent with the research emphasizing the importance of motivation on achievement in any domain.

In terms of indirect paths, the path from enjoyment—motivation—pictorial representations was significant and medium in size (.15), indicating that students who enjoy studying geometry have greater motivation, which in turn leads to better problem solving in the form of more complex pictures. The path from enjoyment—motivation—achievement was significant and medium in size (.22), indicating that higher enjoyment supports higher motivation, which in turn impacts higher achievement. Finally, the path from categorization—pictorial representation—achievement was significant and small in size (.09), indicating that students with greater conceptual knowledge were more likely to draw complex pictures, which in turn impacted achievement.

CHAPTER 5

DISCUSSION, IMPLICATIONS, & CONCLUSIONS

Discussion of Findings

The purpose of this study was to test a model to determine how several key variables interact to impact student achievement in geometry. This study contributes to the existing body of research because it is one in which the influences of motivation, achievement emotions (boredom and enjoyment), pictorial representations, and problem categorization on achievement in geometry were simultaneously tested in a model. The current study examined the influences of individual variables when other variables were taken into account. It also addressed the role of both motivation and achievement emotions on problem solving and achievement in geometry.

Results indicated that the model fit well and all of the direct paths were statistically significant. The paths from both boredom and enjoyment to motivation indicated a significant relationship between achievement emotions and motivation. Motivation significantly impacted both pictorial representations and student achievement. Pictorial representations had a significant influence on student achievement. Categorization skills influenced the use of pictures as well as student achievement.

The direct path from enjoyment to motivation was the largest path in the model, emphasizing the importance of enjoyment on motivation, which in turn, as indicated by the indirect paths, impacts diagrams and achievement.

The results indicate that enjoyment has a much stronger influence than boredom on motivation. Pekrun et al., (2002) found boredom and enjoyment to be linked to motivation; however, this relationship was not tested within a larger framework including motivation, conceptual knowledge, and problem-solving skills. These data indicate that when boredom and enjoyment are used to predict motivation directly, and knowledge, pictorial representations, and achievement indirectly, it is enjoyment that appears to be the most important variable.

Boredom and enjoyment differ in two ways. Boredom has a negative valance and is passive (negative deactivating emotions) and enjoyment has a positive valance and is considered by Pekrun et al., (2002) to be active. Within the framework of the model tested, it is the active, positive emotion that appears to influence problem solving and achievement through motivation. Boredom, in contrast, did not have as strong an influence on motivation, problem solving, and achievement.

One way to make geometry more enjoyable, less boring, and to increase student achievement in geometry is to contextualize the geometry concepts that students are learning. Geometry is often viewed by students as being abstract and irrelevant to their everyday lives (Duatepe-Paksu & Ubuz, 2009). Contextualizing geometry involves integrating real-world contexts and scenarios into the geometry concepts students are learning (Sheppard, 2009). For instance, the contexts of construction, tool-making, architecture, and engineering can be used to teach students major geometry concepts (Burke & Moore, 2009; Sheppard, 2009). A large scale example of contextualized geometry includes the yearly, integrated contextualized geometry and construction program in a Colorado high school where students use geometry to build a house for a family in need (NRCCTE, 2011). Such efforts to contextualize geometry have been found to have a positive influence on students' geometry enrollment, enjoyment, motivation, and achievement (e.g, NRCCTE, 2011).

Motivation also played a central role in the model, influencing both pictorial representations and achievement. Therefore, when geometry educators use research-based strategies to motivate students, other variables, like those present in the model, are likely to be impacted. This finding for geometry students is consistent with the research emphasizing the importance of motivation across a variety of domains.

Conceptual knowledge played a major role in the model as indicated by the direct paths from categorization skills to pictorial representations and achievement, and the indirect path from categorization to pictorial representations to achievement. Pictorial representations also significantly impacted achievement. Efforts to increase students' conceptual knowledge and use of diagrams should have a major impact on students' geometry achievement. With the advancement of new technologies designed for mathematics instruction (Yu, Barrett, & Presmeg, 2009), the use of computer software is one way to attend to both conceptual knowledge and pictorial representations (Battista, 2009; Contreras & Martinez-Cruz, 2009; Yu et al., 2009). One program that high school geometry instructors can integrate into their curriculum is the Geometer's Sketchpad. Geometer's Sketchpad is an interactive geometry software program that allows students to create, manipulate, measure, and animate various geometric figures in order to solve geometry problems (Battista, 2009; Contreras & Martinez-Cruz, 2009; Guven, Baki, & Cekmez, 2012). This in turn, assists with students' understanding of various geometric figures (Battista, 2009; Meng & Sam, 2011; Yu et al., 2009) and problem solving (Alba, 1998; Contreras & Martinez-Cruz, 2009). The use of Geometer's Sketchpad during instruction has been shown to improve conceptual understanding (Garofallo, 2004; Meng & Sam, 2011), engagement and motivation (Contreras & Martinez-Cruz, 2009; Sinclair, 2006), and student achievement (Battista, 2002; Battista, 2009; Hollebrands, 2007).

Limitations and Future Research

To measure student achievement, the average assessment score of four geometry unit tests was used. Although problems on each unit assessment were developed based on a standardized state-wide assessment, actual state administered test scores would have served as a better assessment of students' overall achievement in geometry. With averaging four unit assessments, student attendance could have impacted achievement. For instance, if a student was absent on one of the assessment dates, and never took the assessment thereafter, the student received a score of zero[2]. Finally, the study packet was administered approximately one month after the completion of the last geometry unit. Although the fit of the model is excellent, the administration of these items closer to the end of geometry instruction may have provided more accurate results.

The present study tested a preliminary model that can be expanded to further investigate the contributions of emotional, motivational, and cognitive variables. This is one of the first studies to examine how emotions influence motivation and achievement in mathematics. A longitudinally designed future study might provide a better picture of how emotions drive motivation and achievement over time.

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[2] Eight students in the study scored a zero on one of the unit tests due to an absence. These students were given the opportunity to take the missed test any time before the end of the semester, but did not do so. With the eight students dropped from analysis, results (correlation matrix, model relations, fit indices, and path values) remained the same with only very minor differences in path and correlation values. This was also true when mean substitution was used to replace the eight zero scores.

Future research needs to expand this model to examine more closely the relative value of the different motivation constructs assessed in this study. While a substantial number of motivational constructs have been proposed to influence achievement, there has been little research comparing the relative value of these constructs as predictors of achievement. The current study indicates that motivation has a significant impact and future research can examine which forms of motivation are most important. Finally, the model can also be expanded to include additional emotional variables such as hopelessness and relief, and additional cognitive variables, such as specific problem solving strategies (e.g., means ends strategy) to better explain the variability in geometry achievement.

Conclusion

The results of this study have important implications for geometry instruction. With U.S. students underperforming in geometry compared to other nations (Mullis et.al, 2000; OECD, 2009; Wilkins & Xin, 2002) it is necessary for researchers and instructors to understand what variables are most important in influencing student achievement in geometry before they are able to effectively intervene. Findings suggested that geometry teachers should focus on students' conceptual knowledge when planning instruction, engage students in activities that positively influence motivation and enjoyment, and encourage the use of complex pictures during geometry problem solving.

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Appendix A
Geometry Motivation Questionnaire

In order to better understand what you think and how you feel about your high school geometry courses, please respond to each of the following statements from the perspective of: ‘‘When I am in a high school geometry course. . .’’

01. I enjoy learning the geometry.
 Never Rarely Sometimes Usually Always

02. The geometry I learn relates to my personal goals.
 Never Rarely Sometimes Usually Always

03. I like to do better than the other students on the geometry tests.
 Never Rarely Sometimes Usually Always

04. I am nervous about how I will do on the geometry tests.
 Never Rarely Sometimes Usually Always

05. If I am having trouble learning the geometry, I try to figure out why.
 Never Rarely Sometimes Usually Always

06. I become anxious when it is time to take a geometry test.
 Never Rarely Sometimes Usually Always

07. Earning a good geometry grade is important to me.
 Never Rarely Sometimes Usually Always

08. I put enough effort into learning the geometry.
 Never Rarely Sometimes Usually Always

09. I use strategies that ensure I learn the geometry well.
 Never Rarely Sometimes Usually Always

10. I think about how learning the geometry can help me get a good job.
 Never Rarely Sometimes Usually Always

11. I think about how the geometry I learn will be helpful to me.
 Never Rarely Sometimes Usually Always

12. I expect to do as well as or better than other students in the geometry course.
 Never Rarely Sometimes Usually Always

13. I worry about failing the geometry tests.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

14. I am concerned that the other students are better in geometry.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

15. I think about how my geometry grade will affect my overall grade point average.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

16. The geometry I learn is more important to me than the grade I receive.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

17. I think about how learning the geometry can help my career.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

18. I hate taking the geometry tests.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

19. I think about how I will use the geometry I learn.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

20. It is my fault, if I do not understand the geometry.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

21. I am confident I will do well on the geometry assignments and projects.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

22. I find learning the geometry interesting.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

23. The geometry I learn is relevant to my life.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

24. I believe I can master the knowledge and skills in the geometry course.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

25. The geometry I learn has practical value for me.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

26. I prepare well for the geometry tests and quizzes.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

27. I like geometry that challenges me.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

28. I am confident I will do well on the geometry tests.
☐ Never ☐ Rarely ☐ Sometimes ☐ Usually ☐ Always

29. I believe I can earn a grade of “A” in the geometry course.
 Never Rarely Sometimes Usually Always

30. Understanding the geometry gives me a sense of accomplishment.
 Never Rarely Sometimes Usually Always

Appendix B
Achievement Emotions in Geometry

Studying geometry can induce different feelings. This questionnaire refers to emotions you may experience when studying geometry. Before answering the questions below, please recall some typical situations of studying geometry which you have experienced during the course of your studies.

1. I look forward to studying geometry.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

2. I enjoy the challenge of learning the geometry material.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

3. I enjoy acquiring new geometry knowledge.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

4. I enjoy dealing with the geometry material.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

5. Reflecting on my progress in my geometry coursework makes me happy.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

6. I study geometry more than required because I enjoy it so much.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

7. I am so happy about the progress I made that I am motivated to continue studying geometry.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

8. Certain geometry subjects are so enjoyable that I am motivated to do extra readings about them.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

9. When my geometry studies are going well, it gives me a rush.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

10. I get physically excited when my geometry studies are going well.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

11. The geometry material bores me to death.

O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

12. Studying for my geometry class bores me.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

13. Studying geometry is dull and monotonous.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

14. While studying this boring geometry material, I spend my time thinking of how time stands still.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

15. Geometry is so boring that I find myself daydreaming.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

16. I find my mind wandering while I study geometry.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

17. Because I'm bored I have no desire to learn geometry.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

18. I would rather put off this boring geometry work till tomorrow.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

19. Because I'm bored I get tired sitting at my desk and studying geometry.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

20. Geometry bores me so much that I feel depleted.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

21. While studying geometry I seem to drift off because it's so boring.
O Strongly Disagree O Disagree O Undecided O Agree O Strongly Agree

Appendix C
Geometry Problems Used to Assess Pictorial Representation

Circle your answer choice for each problem below.

1.

A 15-foot tree makes an 8-foot shadow. What is the sine of the angle between the ground and the ray of sunlight creating the shadow?

A. 8/15 **B.** 15/8

C. 8/17 **D.** 15/17

2.

A lighthouse on the shore of Point Loma stands 75 feet high. From the observation deck on top of the lighthouse a watchman determines that a ship is located at 17° angle of depression from the horizontal. How far is the boat from the lighthouse? Round your answer to the nearest foot.

angle	sin	cos	tan
73°	.9563	.2924	3.2709
17°	.2924	.9563	.3057

A. 23 feet **B.** 245 feet

C. 72 feet **D.** 257 feet

3.

$\overline{AE}$ and $\overline{BE}$ are two tangent segments to the same circle from point E with points of tangency A and B respectively. AE = 5 km, what is BE?

A. 2.5 km **B.** 5 km

C. 10 km **D.** 15 km

4.

An equilateral triangle is inscribed in a circle. What is the measure of the central angle formed by two radii going to two adjacent vertices of the triangle?

A. 30° **B.** 60°

C. 90° **D.** 120°

5.

A tangent and a chord intersect to form a 75° angle. What is the measure of the major arc formed by the chord?

A. 285°
B. 105°
C. 210°
D. 185°

6.

What is the perimeter of a 45-45 right triangle with a hypotenuse length of 28 feet?

A. 56 ft
B. 64.5 ft
C. 67.6 ft
D. 84 ft

7.

Beatriz entered her collie in a dog show. During the main event, she will walk in a 129° arc in front of the judges. If the arc were to continue in a circle, its radius would be 3.5 feet. What is the distance Beatriz and her dog will need to walk (find the arc length)? Round your answer to the nearest hundredth. Use $\pi = 3.14$.

A. 7.88 feet
B. 15.75 feet
C. 3.50 feet
D. 7.00 feet

8.

What is the length of an altitude of an equilateral triangle with sides of length 24 in. rounded to the nearest tenth.

A. 41.6 in.
B. 20.8 in.
C. 33.9 in.
D. 17.0 in.

9.

You are standing 16 feet from a circular swimming pool. The distance from you to a point of tangency on the pool is 32 feet. What is the radius of the swimming pool?

A. 24 ft
B. 22.6 ft
C. 64 ft
D. 48 ft

10.

Two tangents form an angle outside of a circle such that the measure of one of the arcs formed is 138°, what is the measure of the angle?

A. 138°
B. 111°
C. 42°
D. 84°

11.

If $\tan M = \frac{3}{4}$, then $\cos M = ?$

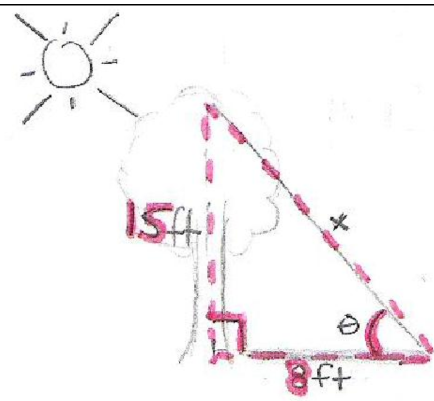
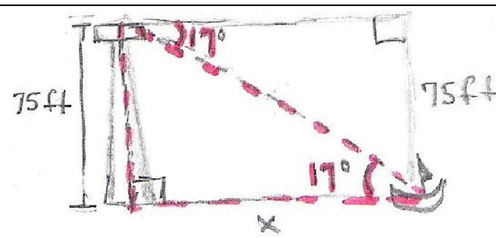
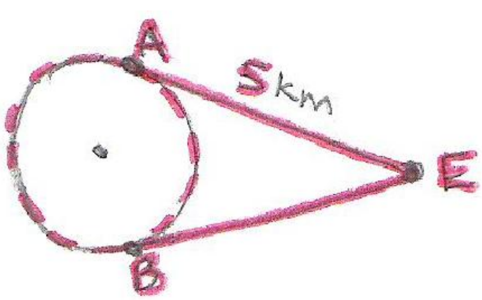
A. $\frac{3}{4}$
B. $\frac{4}{3}$
C. $\frac{3}{5}$
D. $\frac{4}{5}$

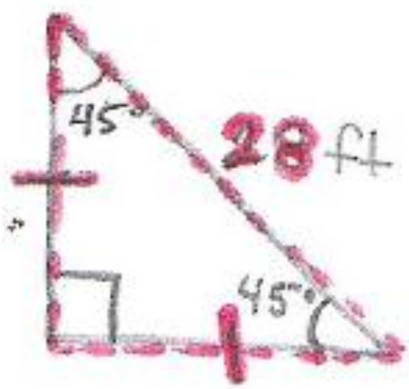
12.

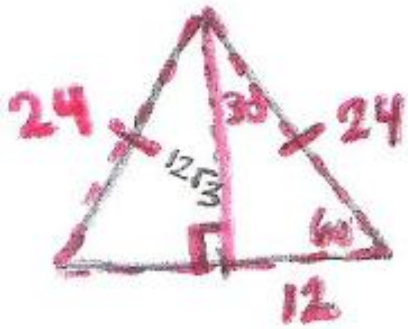
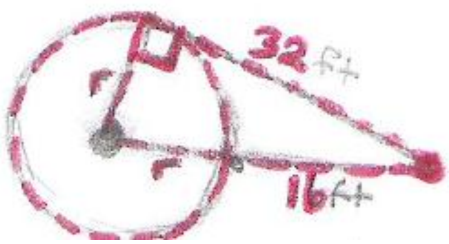
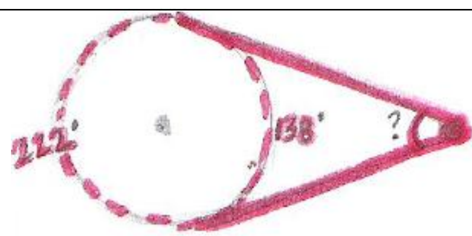
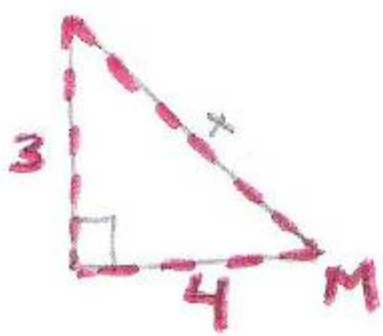
A band must know the dimensions of the stage where they are having a concert. The area of the rectangular stage is 85 ft^2 and the width is x. The length of the stage is $(4x - 3)$ feet. What is the length of the stage?

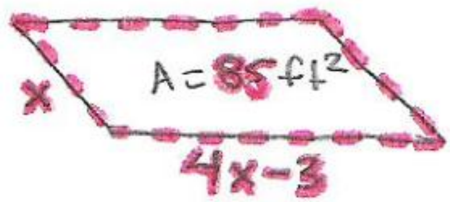
A. 17 ft
B. 50 ft
C. 21 ft
D. 43 ft

Appendix D
Pictorial Representation Scoring Rubric

Problem #	Target Sketch	Award one point for each of the following:
1		• **triangle** • **15 on the vertical leg** • **8 on the horizontal leg** • **angle mark** • **right angle mark**
2		• **right triangle** • **one angle mark** • **17 by angle mark** • **75 on vertical leg**
3		• **circle** • **two tangents** • **labels: A, B, & E** • **5 on one tangent**

4		• **circle** • **triangle** • **two radii drawn from center to two vertices** • **angle mark**
5		• **circle** • **chord** • **tangent intersecting the chord** • **angle mark** • **75 next to angle mark** • **150 next to intercepted arc**
6		• **right triangle** • **28 on the hypotenuse** • **either: congruent marks on legs, OR angle marks to show congruence, OR 45 in at least one angle**
7		• **circle** • **two radii** • **3.5 on at least one radii** • **129 on minor arc**

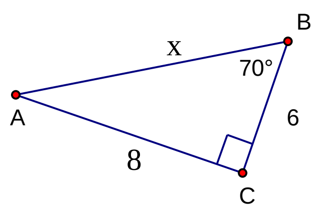
8		• equilateral triangle • 24 on at least one side • altitude • right angle mark • at least a 30 or 60 in one angle • 12 on half of one side
9		• circle • radius to tangent point • tangent • right angle mark • segment connecting exterior point on tangent to the center of the circle • 32 on tangent length • "r" on both radii • 16 on the outside portion of the segment connecting the tangent & the center
10		• circle • two tangents from the same exterior point • angle mark • 138 on minor arc • 222 on major arc
11		• right triangle • M label on one acute angle • 3 on leg opposite M • 4 on leg adjacent to M

12		• **rectangle** • **85 representing the area** • **"x" on at least one side** • **"4x - 3" on at least one side adjacent to the side labeled x**

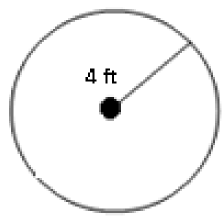
Appendix E
Categorization Task

Cut out each problem below. Without having to solve the problems, group the problems in pairs however you choose. Glue or tape the pairs in the table provided, and describe why you grouped the pairs in the way you did.

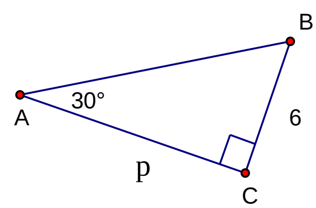
Find the value of x



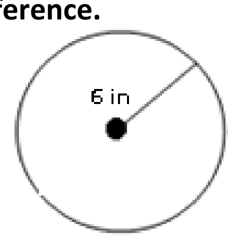
The radius of the circle is given below. Find the area.



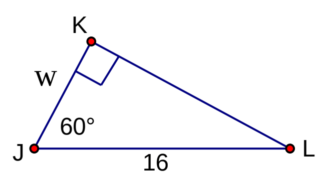
Find the value of p.



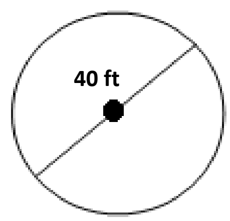
If the radius of the circle is 6 in., find the circumference.



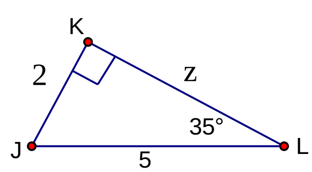
Find the value of w.



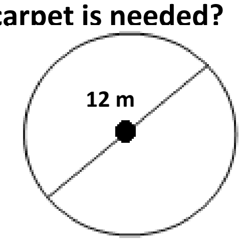
Find the distance around a circular swimming pool if the diameter is 40 ft.



Find the value of z.

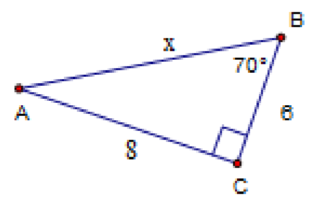
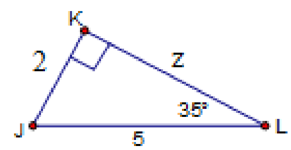
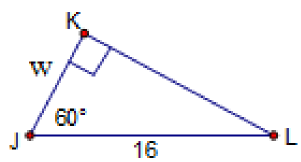
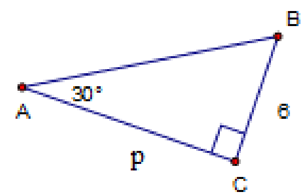
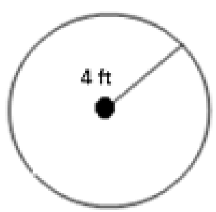
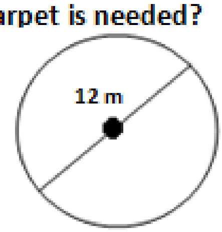
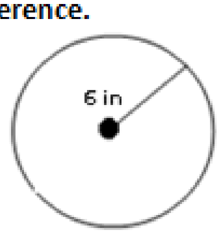
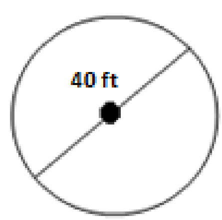


A circular play area will need to be carpeted. If the diameter is 12 m, how much carpet is needed?



	Pairs		Why I Paired Them
Pair 1			
Pair 2			
Pair 3			
Pair 4			

Appendix F
Categorization Task Rubric

	Pairs		Why I Paired Them
Pair 1	Find the value of x 	Find the value of z. 	• can be solved with the Pythagorean Theorem • other right triangles
Pair 2	Find the value of w. 	Find the value of p. 	• special right triangles • can be solved using trigonometry
Pair 3	The radius of the circle is given below. Find the area. 	A circular play area will need to be carpeted. If the diameter is 12 m, how much carpet is needed? 	• must use the area formula of a circle to solve
Pair 4	If the radius of the circle is 6 in., find the circumference. 	Find the distance around a circular swimming pool if the diameter is 40 ft. 	• must use the circumference formula to solve

Appendix G
Examples of Student Work

Pictorial Representations Task Example:
Pictures drawn by the student include many key elements

Geometry Tasks

Circle your answer choice for each problem below.

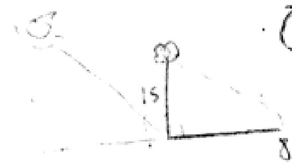
40

1.

A 15-foot tree makes an 8-foot shadow. What is the sine of the angle between the ground and the ray of sunlight creating the shadow?

A. 8/15
B. 15/8
C. 8/17
D. 15/17

SOH
CAH
TOA



4

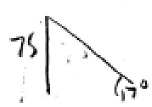
2.

A lighthouse on the shore of Point Loma stands 75 feet high. From the observation deck on top of the lighthouse a watchman determines that a ship is located at 17° angle of depression from the horizontal. How far is the boat from the lighthouse? Round your answer to the nearest foot.

angle	sin	cos	tan
73°	.9563	.2924	3.2709
17°	.2924	.9563	.3057

A. 23 feet
B. 245 feet
C. 72 feet
D. 257 feet

Tan 17 = 0.3057 • 75
= 22.9268

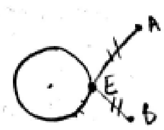


4

3.

$\overline{AE}$ and $\overline{BE}$ are two tangent segments to the same circle from point E with points of tangency A and B respectively. AE = 5 km, what is BE?

A. 2.5 km
B. 5 km
C. 10 km
D. 15 km



2

4.

An equilateral triangle is inscribed in a circle. What is the measure of the central angle formed by two radii going to two adjacent vertices of the triangle?

A. 30°
B. 60°
C. 90°
D. 120°

360 ÷ 3 = 60°

2

5.

A tangent and a chord intersect to form a 75° angle. What is the measure of the major arc formed by the chord?

A. 285°
C. 210°
B. 105°
D. 185°

$$\begin{array}{r} 360° \\ -\ 150° \\ \hline 210° \end{array}$$

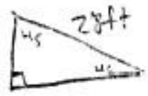
5

$75 \cdot 2 = 150°$

6.

What is the perimeter of a 45-45 right triangle with a hypotenuse length of 28 feet?

A 56 ft
B. 64.5 ft
C. 67.6 ft
D. 84 ft



3

$28 \cdot 2 = 56ft$

7.

Beatriz entered her collie in a dog show. During the main event, she will walk in a 129° arc in front of the judges. If the arc were to continue in a circle, its radius would be 3.5 feet. What is the distance Beatriz and her dog will need to walk (find the arc length)? Round your answer to the nearest hundredth. Use $\pi = 3.14$.

A 7.88 feet
B. 15.75 feet
C. 3.50 feet
D. 7.00 feet

$d = 2r = 7$

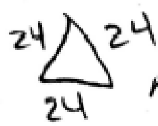
4

$\frac{129}{360} = 0.3583$

8.

What is the length of an altitude of an equilateral triangle with sides of length 24 in. rounded to the nearest tenth.

A. 41.6 in.
B. 20.8 in.
C. 33.9 in.
D. 17.0 in.



2

9.

You are standing 16 feet from a circular swimming pool. The distance from you to a point of tangency on the pool is 32 feet. What is the radius of the swimming pool?

A. 24 ft
B. 22.6 ft
C. 64 ft
D. 48 ft

$48 \div 2 = 24$

5

10.

Two tangents form an angle outside of a circle such that the measure of one of the arcs formed is 138°. what is the measure of the angle?

A. 138°
B. 111°
C. 42°
D. 84°

$222 \div 2 = 111$

$\frac{(360^\circ - 138^\circ)}{2} = 111^\circ$

4

11.

If $\tan M = \frac{3}{4}$, then $\cos M = ?$

A. $\frac{3}{4}$
B. $\frac{4}{3}$
C. $\frac{3}{5}$
D. $\frac{4}{5}$

SOH
CAH
TOA

A

$\frac{4}{5}$

12.

A band must know the dimensions of the stage where they are having a concert. The area of the rectangular stage is 85 ft^2 and the width is x. The length of the stage is $(4x - 3)$ feet. What is the length of the stage?

A. 17 ft
B. 50 ft
C. 21 ft
D. 43 ft

$\frac{x(4x)}{4} = \frac{88}{4}$

$x(x) = 22$

$\boxed{85ft^2}\,(4x-3)$

x

$x(4x-3) = 85$
$\quad +3 \quad +3$

4

Pictorial Representations Task Example:
Pictures drawn by the student do not include many key elements or the student failed to draw a picture when problem solving

12

Geometry Tasks

Circle your answer choice for each problem below.

1.

A 15-foot tree makes an 8-foot shadow. What is the sine of the angle between the ground and the ray of sunlight creating the shadow?

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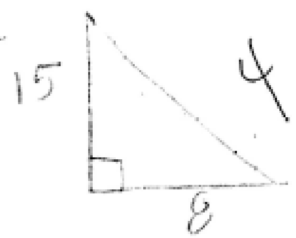
B. 15/8
~~C. 8/17~~
~~D. 15/17~~



2.

A lighthouse on the shore of Point Loma stands 75 feet high. From the observation deck on top of the lighthouse a watchman determines that a ship is located at 17° angle of depression from the horizontal. How far is the boat from the lighthouse? Round your answer to the nearest foot.

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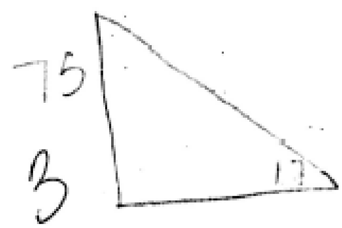

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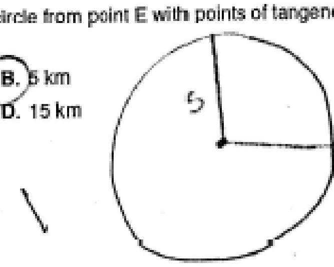

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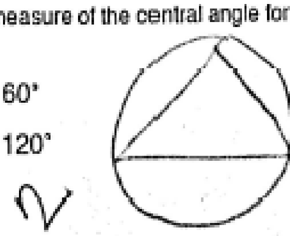

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C. 67.6 ft

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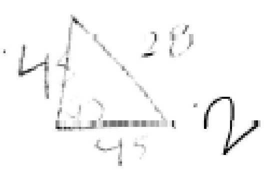

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Beatriz entered her collie in a dog show. During the main event, she will walk in a 129° arc in front of the judges. If the arc were to continue in a circle, its radius would be 3.5 feet. What is the distance Beatriz and her dog will need to walk (find the arc length)? Round your answer to the nearest hundredth. Use $\pi = 3.14$.

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You are standing 16 feet from a circular swimming pool. The distance from you to a point of tangency on the pool is 32 feet. What is the radius of the swimming pool?

A. 24 ft

B. 22.6 ft

C. 64 ft

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Two tangents form an angle outside of a circle such that the measure of one of the arcs formed is 138°, what is the measure of the angle?

A. 138°

B. 111°

C. 42°

D. 84°

11.

If $\tan M = \frac{3}{4}$, then $\cos M =?$

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

$S \frac{o}{h} \frac{3}{5}$

$C \frac{a}{h} \frac{4}{5}$

$T \frac{o}{a} = \frac{3}{4}$

12.

A band must know the dimensions of the stage where they are having a concert. The area of the rectangular stage is 85 ft^2 and the width is x. The length of the stage is $(4x - 3)$ feet. What is the length of the stage?

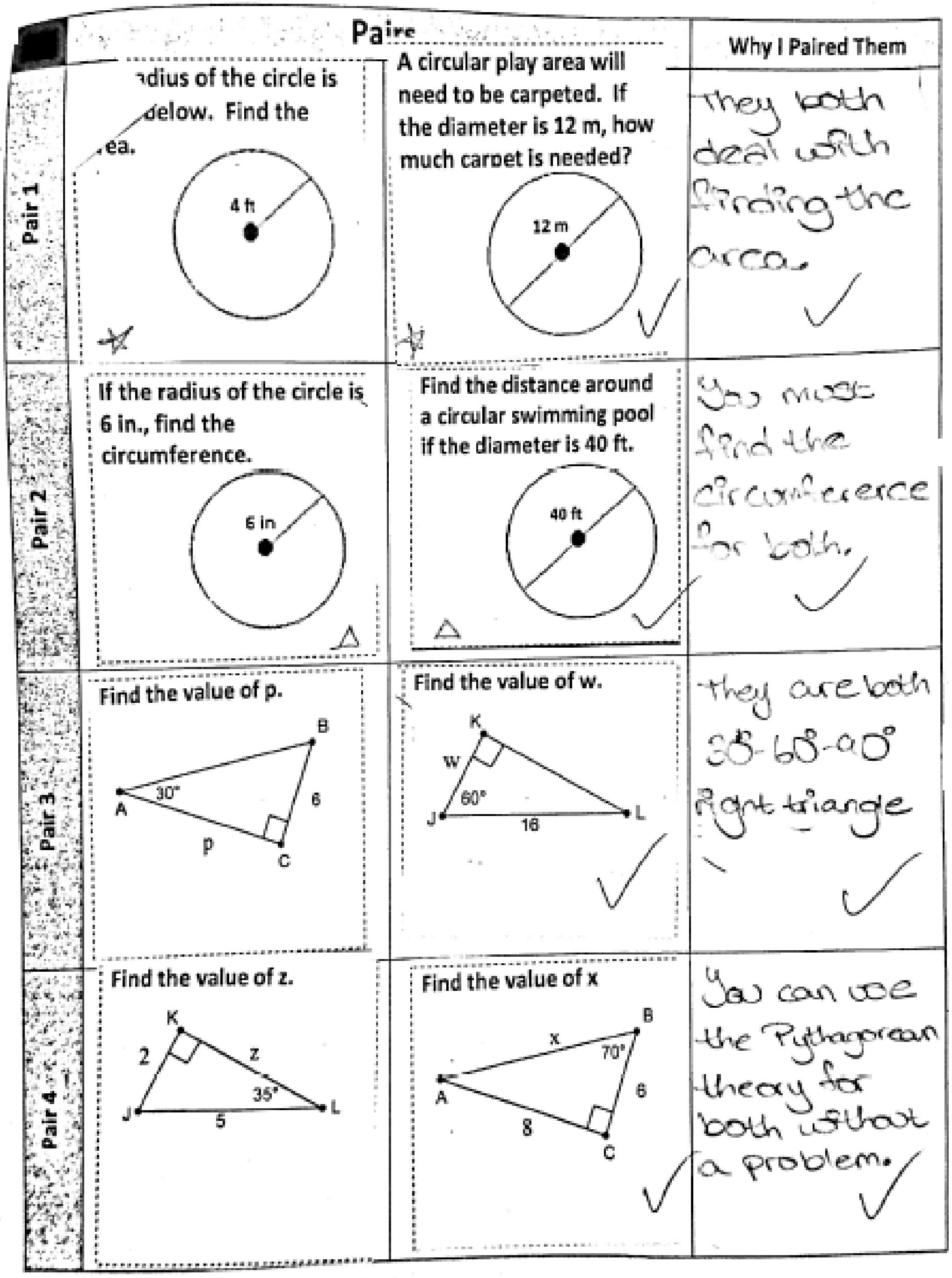
A. 17 ft

B. 50 ft

C. 21 ft

D. 43 ft

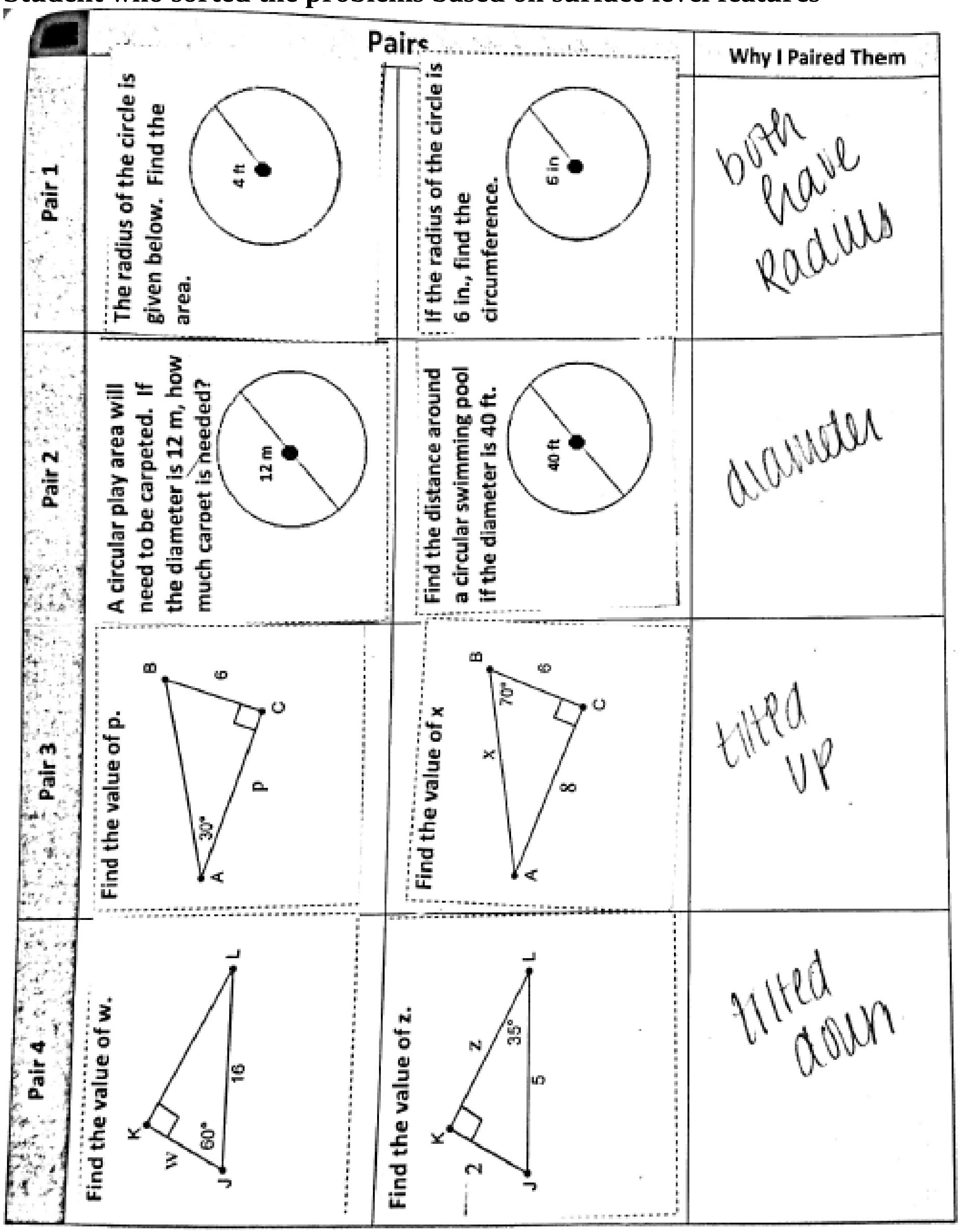
Categorization Task Example:
Student who sorted the problems based on underlying features



	Pairs		Why I Paired Them
Pair 1	…adius of the circle is …below. Find the …ea. (4 ft)	A circular play area will need to be carpeted. If the diameter is 12 m, how much carpet is needed? (12 m)	They both deal with finding the area.
Pair 2	If the radius of the circle is 6 in., find the circumference. (6 in)	Find the distance around a circular swimming pool if the diameter is 40 ft. (40 ft)	You must find the circumference for both.
Pair 3	Find the value of p. (A, B, C; 30°; 6; p)	Find the value of w. (K, J, L; w; 60°; 16)	they are both 30-60-90° right triangle
Pair 4	Find the value of z. (K, J, L; 2; z; 35°; 5)	Find the value of x (A, B, C; x; 70°; 6; 8)	You can use the Pythagorean theory for both without a problem.

8

Categorization Task Example:
Student who sorted the problems based on surface level features



	Pairs		Why I Paired Them
Pair 1	The radius of the circle is given below. Find the area. (4 ft)	If the radius of the circle is 6 in., find the circumference. (6 in)	both have Radius
Pair 2	A circular play area will need to be carpeted. If the diameter is 12 m, how much carpet is needed? (12 m)	Find the distance around a circular swimming pool if the diameter is 40 ft. (40 ft)	diameter
Pair 3	Find the value of p. (B, C, A, 6, p, 30°)	Find the value of x (B, C, A, 70°, 6, x, 8)	tilted up
Pair 4	Find the value of w. (K, L, J, w, 16, 60°)	Find the value of z. (K, L, J, z, 35°, 5, 2)	tilted down