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Development and Validation of the Mathematical Problem-solving Dispositions and Beliefs Scale

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DEVELOPMENT AND VALIDATION OF THE MATHEMATICAL PROBLEM-SOLVING DISPOSITIONS AND BELIEFS SCALE

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Dedication

I dedicate this work to the memory of my grandmother, Bonnie Glenn, the most selfless person I have ever known, who loved God and others. I also dedicate this dissertation to my future children. I love you more than you know.

Abstract

Mathematical problem solving has received recent attention and been recognized as central to analysis and application in everyday life. Mathematical problem solving has often been characterized by traditional word problems. From the models-and-modeling perspective, students problem solve mathematically by engaging in conceptual development through interaction with communities of practice that produce artifacts that are continually under design. Productive problem-solving dispositions and beliefs mold students who are confident and willing to take on new tasks. Attitudes, feelings, dispositions, and beliefs are manipulatable, and thus individuals' problem-solving identity is complex. To date, there are no empirical studies that have measured students' levels of mathematical problem-solving dispositions and beliefs. This study describes the development and validation of a measure of mathematical problem-solving dispositions and beliefs (MPSDB), based on the models-and-modeling perspective of problem solving. An initial pool of 72 items represented six different dimensions of the model. Data were collected from 575 middle grade students to validate and examine the MPSDB scale. Through a series of phases including a pilot study, expert panel, and exploratory factor analysis, a final 40-item MPSDB scale was validated with strong reliability. The validation study showed that scores on the 40-item measure: (a) established construct validity as the MPSDB scores correlated with two of the theoretically related constructs, including math anxiety and self-efficacy and the usefulness of mathematics; (b) established content validity as there was a high degree of agreement between the expert panel's review of items; (c) established criterion validity as MPSDB scores were positively correlated with GPA and mathematics class average; and (d) established incremental validity as the MPSDB added significant predictive capacity to the model.

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Chapter 1: Introduction

Statement of the Problem

In recent years, the No Child Left Behind Act of 2001 (NCLB) has mandated policies and practices related to national content standards, teacher qualifications, and legal responsibilities of systems within American schools. Lesh and Zawojewski (2007) claimed that current K–12 teaching in the United States has not placed as much emphasis on problem-solving standards as on high-stakes testing and basic skills. While mathematics education research has progressed, teaching practices have not developed since the early problem-solving research. As a result of emphasis being placed on basic skills, problem solving in the classroom has been placed on the backburner. New common core state standards push for students to engage in the problem-solving process and to persevere in solving real-life problems by analyzing relationships (NGA, 2012).

Congress has recently ended NCLB, as the Education Department called for less standardized testing and less punishment for failing schools. The Every Student Succeeds Act of 2015 (ESSA) was signed into law, and one rationale behind Congress' decision to end NCLB. Mathematics teachers hope the new measure of accountability will once again allow schools to place focus on mathematics problem solving and other areas that prepare students to be productive members of society. The ESSA provides states more control while still safeguarding underserved students by maintaining state accountability.

Gange (1980) acknowledged that the fundamental idea of education is to teach individuals to think, to use their critical judgments, and to become better problem solvers. Still today, most psychologists, teachers, and mathematics education researchers view problem solving as one of the foundational outcomes of education. The reason for this is every person in their personal and professional lives has to solve problems repeatedly (Jonassen, 2000). President Obama's administration addressed the concern of students being able to thrive in society by stating:

> Because economic progress and educational achievement go hand in hand, educating every American student to graduate prepared for college and success in a new work force is a national imperative. Meeting this challenge requires that state standards reflect a level of teaching and learning needed for students to graduate ready for success in college and careers. (Office of the Press Secretary, 2010)

Basic skills do not prepare students for the challenges of life. The majority of instructional models in schools today emphasize basic skills and, yet, the literature suggests different models of mathematics instruction that are less formulated directly increase achievement in mathematics (Cobb & McClain, 2006).

Students need to experience problem solving where they can express, test, revise, discuss, and refine their ideas. "Such uses of mathematics require that mathematical knowledge be reconstituted or created for the local problem situation, and that useful content knowledge involves the integration of ideas and abilities related to a variety of mathematics topics and other disciplines" (Lesh & Zawojewski, 2007, p. 781). Studies show that student beliefs about problem solving become more unproductive each year the traditional model of instruction is used (Schoenfeld, 2004; Stein & Lane, 1996). Traditional teaching of mathematics is characterized by a culture in which mathematics specialists attempt to impart knowledge of different mathematical procedures and students inadvertently gain complete understanding of these mathematical methods (Boaler, 2002). In addition, society holds tight to the belief that practicing step-by-step procedures repeatedly will result in understanding mathematics.

Many students today believe mathematics problems should be completed in a few minutes or less (Schoenfeld, 2011). Students' beliefs about persistence affect their success in problem solving. Students' willingness or unwillingness to deal with difficulties becomes a problem-solving disposition they adopt (Dweck, 2006). Thus,

> Investigations on beliefs and dispositions would benefit from studies that investigate the development of relevant beliefs, feelings, values, and dispositions by involving students in activities where they express, test, and revise their own attributes during post-hoc reflections in problem-solving experiences. (Lesh & Zawojewski, 2007, p. 778)

Early on mathematics education researched emphasized the importance of using a problem-solving model rather than a procedural model so students develop a mindset for continued capacity of growth (Erlwanger, 1973). Erlwanger (1973) suggested that a reason for low mathematical thought lies in the methodological approaches used in schools. School leaders would be wise to stop viewing education as a cast in a mold approach and place emphasis on problem solving. Thus, "a fresh perspective of problem-solving is needed—one that goes beyond current school curricula and state standards" (Lesh & Zawojewski, 2007, p. 780).

Conceptual Framework

The models-and-modeling perspective is associated with philosophies of constructivism, and views education through a sociocultural lens. Lesh and Doerr (2003) proposed a models-and-modeling perspective for conducting research and interpreting results. Lesh and Zawojewski (2007) claimed, "The development of problem-solving abilities are highly interdependent and far more socially constructed and contextually situated than traditional theories have supposed" (p. 779).

In a models-and-modeling approach to problem solving, students are able to adopt greater understanding of mathematical concepts as they participate in, revise, differentiate, and improve their thinking through interactions with others (Lesh & Zawojewski, 2007). The models-and-modeling approach to problem solving views learning as multidimensional and, thus, factors such as beliefs and depositions arise as relevant to learning. Six themes that continually arise as important in the models-and-modeling approach are *mathematical mindset, problem-solving perseverance, mathematical revision and refinement, mathematical communities of practice, problem-solving processes*, and *problem-solving utility*. When defining the models-and-modeling perspective, these six themes repeatedly arise in studies and articles by multiple mathematics education researchers (Doerr & English, 2003; Lesh, Carmona, & Post, 2002; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007; Schoenfeld, 2011).

The models-and-modeling approach encourages the use of model-eliciting activities (MEAs) as these problem-solving tasks promote six important principles: (1) the model construction principle, (2) the reality principle, (3) the self-assessment principle, (4) the construct documentation principle, (5) the construct shareability and reusability principle, and (6) the effective prototype principle (Lesh, Hoover, Hole, Kelly, & Post, 2000). As an example, consider a model-eliciting activity, "The Big Foot Problem." This activity requires middle school students to investigate photographs, footprints, newspaper articles, and accurate data charts developed by experts (Lesh & Doerr, 2003). Using this information allows students to develop a "how to" toolkit that police can use to make accurate estimates on peoples' size just by looking at footprints. The Big Foot Problem involves proportional reasoning and linear relationships, as well as scale factors. Model-eliciting activities (MEAs) are open-ended and require mathematical reasoning through revising, extending, and altering initial interpretations of mathematical situations. Traditional methods of mathematical instruction emphasize basic skills and do not foster productive beliefs and dispositions towards problem solving to create successful interpretations. In contrast, MEAs allow opportunities for students to develop adaptable and reusable theoretical tools, called models, for creating, explaining, and using mathematical methods. According to Chamberlin and Moon (2005), the primary objective of MEAs is to get students to build mathematical models and to solve complex problems. Unlike traditional word problems, these MEAs require students to use higher order thinking and take ownership of their learning.

Through a models-and-modeling perspective, students are able to experience a process of revision and analysis as they create products for the real world. Problem solving within the models-and-modeling framework requires students to act in communities of practice as they create knowledge through investigation and discourse. Engaging in problem solving allows students to participate in collaborative learning. According to Lesh and Zawojewski (2007),

> Models-and-modeling perspectives adopt more sophisticated conceptions of development based on the observation that when students (or groups) go through a series of modeling cycles in which they integrate, differentiate, revise, and refine their existing relevant ways of thinking development seldom occurs along a single, one-dimensional, ladder-like sequence. Instead development occurs along a variety of dimensions and students' final interpretation often inherits characteristics from a variety of problem solvers' early interpretations (p. 795).

Problem solving is a process in which students develop tools for use in the everyday context of the mathematical world. Teams of students must learn to perform research that informs their decisions while making sense of the mathematics. An important characteristic of a models-and-modeling perspective is that student research is planned around the construction of tools that are then tested in the classroom (Lesh & Zawojewski, 2007). The models-and-modeling approach is different from a traditional view on problem solving, as this new perspective requires hands-on experience. The process of problem solving is no longer a prescriptive list of steps, such as Polya's (1945) suggestion to: (1) understand the problem, (2) devise a plan, (3) carry out the plan, and (4) look back. Now when looking for solutions to problem-solving tasks, the process is cyclic in nature. Lesh and Zawojewski (2007) suggested that the mathematical process is "a full process of modeling as problem solving" and can only be learned through experience (p. 785).

A models-and-modeling perspective is based in the creation of tangible tools and artifacts for school-based use by employing the principles portrayed by a suggested conceptual model of learning and teaching (Lesh & Zawojewski, 2007). These tools and artifacts should be relevant to students' local context. Students should see the functionality of problem solving within their community. In addition, the solutions students develop should be generalizable in nature. Students need to be comfortable developing artifacts that can be used within their community as well as outside for a broader purpose. Students problem solve as they participate in creating a powerful design for society. As students design these tools for schools, businesses, governments, etc., they challenge themselves to engage in meaningful learning. Solutions to mathematical problem-solving tasks become reusable not only to the students, but also to other people. These solutions require refinement, revision, and testing. The process of problem solving allows students to refine and deepen their understanding. For instance, "knowledge (or the solution to the problem) that emerges is viewed as developing rather than being in a state of learned verses not learned" (Lesh & Zawojewski, 2007, p. 790).

Additionally, the models-and-modeling perspective contains characteristics of situated cognition and communities of practice (Lesh & Zawojewski, 2007). As stated, “Recent themes of research—such as those related to situated cognition, communities of practice, and representational fluency—seem to be converging to a models-and-modeling perspective on mathematical learning and problem-solving” (Lesh & Zawojewski, 2007, p. 793). The reason for this shift is that knowledge is socially situated, and through the models-and-modeling approach, students engage with others as they develop conceptual tools and mathematical concepts for a problem-solving context. Furthermore, students interact with people and their cognitive systems evolve based on the learning context (Greeno, 2003).

Basic mathematical skills and problem-solving skills develop in the context of students’ beliefs and dispositions. Feelings, dispositions, values, and other various traits work simultaneously to form students’ complete problem-solving persona (Lesh & Zawojewski, 2007). Consequently, beliefs play a role in problem solving as they impact interpretation of situations. The effects of these beliefs cannot be ignored. According to Lesh and Zawojewski (2007), “When students interpret situations mathematically, they do a great deal more than simply engage in concepts that are purely logical or mathematical in nature; their interpretations also involve feelings, beliefs, and dispositions” (p. 777). The process of problem solving allows opportunities for students to develop their beliefs through revision, refinement, and testing of their solutions to mathematical tasks. The models-and modeling perspective has the potential to foster productive beliefs as students engage in MEAs that promote belief development as they engage with communities of practice. Students as well as teachers must be aware of beliefs that impact students’ problem-solving identities (Schoenfeld, 1985; Silver, 1985).

The models-and-modeling perspective creates an environment where the class is a safe place to develop these beliefs about mathematical problem solving. According to Lesh and Zawojewski (2007):

> The goal, therefore, should be to help students recognize the difference and to use such beliefs in ways that are appropriate. The notion of stable trait like beliefs and dispositions should be abandoned in favor of the notion of developing a productive problem-solving persona or identity that involves a complex, flexible and manipulatable profile of attitudes, feelings, disposition, and beliefs. (p. 776)

Silver (1985) recommended creating a safe atmosphere so the classroom is conducive in creating good problem solvers who acknowledge their feelings and beliefs while problem solving. In a model-and modeling approach to problem solving, both the student and teacher have a responsibility to participate and share ideas about mathematical tasks that promote various philosophies, thoughts, concepts, and designs. These various ideas foster productive dispositions as students learn that beliefs should be elastic and multidimensional as knowledge is gained. Students need to learn their beliefs are complex and flexible. Beliefs impact students' success in the mathematics classroom (McLeod, 1992). Therefore, students need to not only be aware of these beliefs, but also understand that their beliefs can change. McLeod (1989) recognized students' emotional states range from positive to negative during problem solving. Consistently viewing problem solving negatively could affect students' permanent views. For this reason, the models-and-modeling perspective does not discount the importance of beliefs. In a models-and modeling approach, emphasis is placed on valuing beliefs and using them to communicate with others, while at the same time being open to other beliefs as they may be more industrious.

Productive beliefs and dispositions allow students to recognize the value of new information as it could alter their original immature beliefs.

Although it appears to the general public that student success should be based on standardized tests, the success of students is not solely based on their achievement on standardized tests. The National Council of Teachers of Mathematics (NCTM) defined the learning of mathematics to include the development of productive beliefs and dispositions (NCTM, 2014). Students who prove to be successful problem solvers are often not those who have high scores on traditional tests. This is because the factors that contribute to success are often quite different than those that have been emphasized in traditional tests (Lesh & Doerr, 2003). For this reason, schools should focus on beliefs. “Research from a models-and-modeling perspective involves the development of specific methodologies, theoretical models, and tools that are designed in response to the problem being investigated” (Lesh & Zawojewski, 2007, p. 780). Beliefs are complex in nature. Thus, there are limited scales that measure student beliefs about problem solving. Scale development in this area is needed, as “few tools exist for measuring constructs claimed to be important in mathematical problem solving” (Lesh & Zawojewski, 2007, p. 795).

Purpose

The purpose of this study was to develop a reliable and valid instrument to measure mathematical problem-solving dispositions and beliefs (MPSDB) within the framework of a models-and-modeling perspective among secondary mathematics students. This involved three stages: (a) the identification and operationalization of scale items that conceptually reflect a models-and-modeling approach, (b) the establishment of reliability, and (c) the validation of the developed scale items with other related measures of MPSDB.

Research Objectives

There were five research objectives addressed in this study:

1. To develop a reliable measure of MPSDB.
2. To establish content validation using a panel of experts with positive agreement and high inter-rater reliability as to the accurate representation of item samples, appropriateness of content, and appropriateness of item format.
3. To explore the construct validity of the measure MPSDB and the relationship between scores from related mathematics scales correlations (i.e., Fennema & Sherman, 1976; May, 2009).
4. To determine criterion validity by examining the relationship between scores on the MPSDB scale and logically related concurrent behavioral criteria, including grade point average (GPA) and course performance.
5. To conduct item analysis (i.e., factor analysis and reliability analysis) in order to explore the factor structures of the scale and examine the reliability of the scale.

Significance of the Study

The lack of empirical, research-driven investigations supporting the development of problem-solving dispositions and beliefs through a models-and-modeling perspective to problem solving has limited the usefulness of this approach in theory and practice. Current conceptualizations of mathematics teaching and learning are shifting from simple isolated principles of constructivism towards the sociocultural development of problem-solving dispositions and beliefs (Cobb, 1994). Therefore, many problem-solving models developed in previous years solely based on constructivism no longer fit this principle. For example, Polya's (1945), "How to Solve It," framework based following a traditional list of steps did not involve the sociocultural aspect of learning as it focused more on the individual. The lack of research into students' beliefs about problem solving and the limited sociocultural lens through which mathematics education has been viewed have limited further investigation into mathematical problem solving. This study contributes to the empirical evidence concerning the reliability and validity of a scale developed to measure mathematical problem-solving dispositions and beliefs based on Lesh and Zawojewski's (2007) models-and-modeling approach to problem solving. This study offers empirical evidence to validate the scale developed in the current study.

Relevant Terms

Validity is generally referred to as " the extent to which any measuring instrument measures what it is intended to measure" (Carmines & Woods, 2005, p. 1172). Cronbach (1951) suggests that individuals can differentiate between the categories of validity by observing that each encompasses a different importance on the criterion. These are seen below.

Construct validity. According to Peng and Muller (2004), "construct validity is the extent to which the test is shown to measure a theoretical construct or trait" (p. 183). It refers to the degree to which interpretations can justifiably be made from the theoretical paradigms on which operationalization within the study has been defined.

Content validity. According to Sireci (1998), “Content validity is the degree to which an assessment represents the content domain it is designed to measure” (p. 1076). This requires expert opinion.

Criterion validity. This shows that two constructs that were thought to be related are in fact related.

Reliability measures are also important when developing a scale. Scales prove reliable when they offer stable and consistent responses over administration of the scale (Santos, 1999).

Cronbach’s alpha. According to Santos (1999), Cronbach’s alpha is “an index of reliability associated with the variation accounted for by the true score of the underlying construct” (p. 3).

Reliability coefficient. According to Kelley (1942), this value “demonstrates whether the test designer was correct in expecting a certain collection of terms to yield interpretable statements about individual differences” (p. 76). Nunnally and Bernstein (1978) stated 0.7 is an acceptable reliability coefficient.

Chapter 2: Literature Review

Mathematical problem solving has continued to gain increasing recognition due to the National Council of Teachers of Mathematics (NCTM; 2014) and other state and national education departments encouraging mathematics teachers to implement tasks that promote reasoning and problem solving. Many theorists have proposed to explain what dispositions and beliefs motivate students to engage in mathematical problem solving. Although the literature covers a wide variety of such theories, this review will focus on themes related to problem solving, students, teachers, and beliefs. Although the literature presents these items in a variety of contexts, this review will primarily focus on their application to problem solving as defined by a models-and-modeling perspective.

Theoretical Framework

Early in mathematics education research, Piaget's (1954) theories of constructivism began to play a critical role in the understanding of "mathematical content, problem-solving and metacognitive processes, the role of internal and external representations in mathematical sense making and learning, and the reorganization of knowledge structures in conceptual growth" (Edwards, Esmonde, & Wagner, 2011, p. 57). This new perspective brought the idea of mathematical inquiry and mathematical exploration into the school. Piaget (1954) argued against the rules and skills of mathematical procedure and emphasized ideas of modeling. This led to schools adopting a more experiential learning concept of mathematics education. Constructivism proposed two main principles: (a) "knowledge is not passively received but actively built upon by the cognizing subject"; and (b) "the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality" (von Glasersfeld, 1989, p. 114). Students create their own knowledge, but not in the absence of their peers. Cobb (1994) noted that “this constructive activity occurs as the cognizing individual interacts with other members of the community” (p. 14).

For this reason, Vygotsky (1978) claimed the development of the mind of the adolescent is both separate and social at the same time. Therefore, true mathematical learning takes place within students’ social and cultural context. According to Saxe (1991), accomplishments in mathematical activity correlate to a sociocultural tradition. Researchers suggested that the societal perspective arose as importance was placed on conversation, gesticulation, and writing (Hall & Stevens, 1995). Sociocultural theory focuses less on the individual alone, but rather on the individual in relation to peers. Cobb (2001) analyzed social interactions and practices in elementary classes, and suggested that discourse and the language of mathematics plays a role in learning. According to Cobb (1994), “Learning is the process of both self-organization and a process of enculturation that occurs while participating in cultural practices, frequently while interacting with others” (p. 18).

Cobb (1994) stated that “sociocultural and constructivist theorist both highlight the crucial role that activity play in mathematical learning and development” (p. 14). Each of these theories emphasizes the importance of cognitive activities as well as social activities. Thus, a models-and-modeling approach to learning is needed in a mathematics classroom. Lesh and Zawojewski (2007) suggested that a models-and-modeling perspective “can serve as a framework to encourage the integration of ways of thinking drawn from a variety of practical and theoretical perspectives” (p. 779). As students engage in mathematical problem solving via a models-and-modeling approach, they have the opportunity to engage in challenging tasks where they become skilled practitioners of their community (Steiner & Mahn, 1996). Problem solving provides opportunities of transaction and transformation instead of knowledge transmission.

According to Steiner and Mahn (1996), learning must provide an "opportunity for discussion and problem solving in the context of shared activities, in which meaning and action are collaboratively constructed and negotiated" (p. 197). The models-and-modeling approach allows students to engage in MEAs. These activities allow classes to "become learning communities— communities in which each participant makes significant contributions to emergent understandings of all members, despite having unequal knowledge concerning the topic under study" (Palincsar, Brown, & Campione, 1993, p. 43). For this reason, this study used a models-and-modeling perspective to problem solving influenced both by constructivist and sociocultural theories.

Review of the Literature

Mathematical Problem Solving

Life requires problem solving. Therefore, one reason for teaching mathematics in school is to empower students to solve problems (DiMatteo & Lester, 2010). Thinking mathematically is about creating, describing, and explaining. Model-eliciting problem solving involves "quantifying, dimensionalizing, coordinatizing, catergorizing, algebratizing, systemizing relevant objects, relationships, actions, patterns, and regularities" (Lesh & Doerr, 2003, p. 5). Because of its complexity, mathematics cannot progress in the absence of beliefs and dispositions concerning problem solving. Thus, adopting a models-and-modeling perspective to problem solving requires individuals to embrace a social and developmental viewpoint when examining problem solving in a secondary mathematics classroom.

According to Lesh and Zawojewski (2007), traditional perspectives on problem solving have focused on simple identification of task variables, such as heuristics. The early research of Polya (1945) and Schoenfeld (1985) focused on studying specific tools and strategies used that distinguished students from expert and novice problem solvers. For example, expert problem solvers were able to use Polya's strategy of using diagrams, looking for patterns, listing possible solutions, trying special cases, working backwards, guessing and checking, and creating a simpler problem. Polya (1945) described general strategies needed for mathematical problem solving, while Schoenfeld (1992) described more specific. A review of the literature shows cycles in these two schools of thought (Lesh & Zawojewski, 2007). Schoenfeld (1992) expanded on Polya's research and provided descriptive heuristics versus prescriptive strategies.

Lester (1994) discussed five important aspects identifying a "good" problem solver:

1. Good problem solvers know more than poor problems solvers, and what they know, they know differently—their knowledge is well connected and composed of rich schema;
2. Good problem solvers tend to focus their attention on structural features of problems while poor solvers on the surface;
3. Good problem solvers are more aware than poor problem solvers of their strengths and weaknesses as problem solvers;
4. Good problem solvers are better than poor problem solvers at monitoring and regulating their solving efforts; and
5. Good problem solvers tend to be more concerned than poor problem solvers about obtaining "elegant" solutions to problems (p. 665).

Charles and Silver's (1988) as well as Schoenfeld's (1987) ideas were supported by the qualities Lester identified. Additional studies have established that true learning through problem solving is linked to the context of situations (Elstein, Shulman, & Sprafka, 1978). Problem solving does not need to be taught as a stand-alone process or skill, as Polya and Schoenfeld tried (Lesh & Zawojewski, 2003).

Current research needs to involve more complex conceptual systems. For example, as Zawojewski and Lesh (2003) explained:

> Research on the development of expertise needs to go beyond an assumption that experts first learn the content, then learn the problem-solving strategies, and then learn ways to appropriately select and apply already learned mathematics. Rather, the development of expertise seems to involve holistic co-development of content, problem-solving strategies, higher order thinking, and affect—all to varying degrees and situated in particular context. (p. 768)

In order for students to develop the expertise described by Zawojewski and Lesh, a shift from a traditional mathematics education is necessary. Research has revealed students' problem-solving failures are often due not to a lack of mathematical knowledge or their use of strategies, but their inability to apply the strategies and mathematical knowledge to new situations (Schoenfeld, 1987). DiMatteo and Lester (2010) claimed that "even though most mathematics educators agree that the development of students' problem-solving abilities and expertise is the primary objective of instruction, determining how this goal is to be reached involves a wide range of factors" (p. 7). These views are widespread because problem solving involves not only individuals' knowledge base and problem-solving strategies, but also their mathematical beliefs and dispositions. Thus, problem solving has been redefined.

Traditional researchers' views on mathematical problem solving are in opposition to more modern ones. Lester (1994) acknowledged this divide, stating that "problem solving has been the most written about, but possibly the least understood topic in mathematics curriculum in the United States" (p. 661). Problem solving, according to Reitman (1966), is when an individual has been given a depiction of an issue but has not established a solution to fulfill an all-encompassing interpretation of the question. Reitman (1966) further defined problem solving by labeling a problem solver as a person recognizing an objective without an instantaneous means of accomplishing the goal. The NCTM (2000) defined problem solving as "engaging in as task for which the solution method is not known in advance" (p. 52). Lester (1983) defined problem solving as a task where (a): " the individual or group confronting it wants or needs to find a solution," and (b) "there is not a readily assessable procedure that guarantees or completely determines the solution" (p. 231). Heuristics are at the heart of each of these traditional definitions.

Traditionally, in mathematics education research and development, problem solving has been defined as getting from givens to goals when the path is not immediately obvious or blocked, whereas heuristics has been conceived as answers to the question, "what can you do when you are stuck?" However, when attention shifts towards MEAs, in which a series of interpretation cycles are required to produce adequate ways of thinking about givens and goals, then the essence of problem solving involves finding ways to interpret these situations mathematically (Lesh & Harel, 2003, p. 160). Lesh and Doerr (2003) defined problem solving as the extension of initially inadequate conceptual models in order to create successful interpretations. Students develop adaptable and reusable theoretical tools, called models, for creating, explaining, and using mathematical methods. Lesh and Doerr (2003) advocated for this models-and-modeling perspective as it involves "the discovery and/or making of new meanings through construction of new representations and inferential moves" (p. 513). There is a broad consensus in the field of mathematics that becoming adaptively competent in mathematics can be conceived of as acquiring a mathematical disposition. Adopting a models-and-modeling approach to problem solving develops mathematical disposition. In these well-designed eliciting activities, problem solving leads to significant forms of learning (Lesh & Doerr, 2003). These forms of learning not only fall under domain-specific knowledge and development of heuristic methods, but also mathematical affect. Therefore, in this study, I used Lesh and Zawojewski's (2007) definition of problem solving:

> The process of interpreting a situation mathematically, which usually involves several iterative cycles of expression, testing, and revising mathematical interpretations—and of sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics. (p. 782).

Attributes of Problem Solving

Problem solving involves connections, communication, and reasoning. The NCTM (NCTM; 2000) explained that authentic problem solving allows students to "acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom" (p. 52). The NCTM places emphasis not only on the application of content knowledge but also on the mathematical affect of students. In the models-and-modeling perspective,

> The assumption is that an essential mechanism for moving the learner (group or individual) beyond current ways of thinking is through the interaction of a variety of

alternative conceptual systems that are potentially relevant to the interpretation of a given situation. (Lesh & Zawojewski, 2007, p. 789)

Students initiate their learning involvement by developing abstract organizations for making sense of tangible everyday situations where it is essential to produce, enhance, or adapt a mathematical way of thinking. According to Lesh and Zawojewski(2007), the process of problem solving can be referred to as modeling. Greeno (1998) described the problem solving space as "emerging in the process of working on the problem" (p. 7).

Representations and tools to produce them are among the most essential objects students encounter in the world (Steffe & Thompson, 2000). Thus, problem solving must involve students working to create, modify, and apply these artifacts in the classroom. In the models-and-modeling perspective, problem solving involves creating models "developed for specific purposes, for specific people, and for specific situations" (Lesh & Zawojewski, 2007, p. 789). Problem solving should require the process of refining and reformulating, as real-life situations are not "neatly packed" (NCTM, 2000). Problem solving is often painful for students. Yet, Dewey (1933) predicted that the "attitude of suspended conclusion [is] likely to be somewhat painful when involved in reflective thinking," as students are constructing ideas about which methods of representations to use when solving problems or reflecting and justifying their thinking; not merely providing an answer (p. 13). Frustration is involved because students do not know what to expect and must explore various situations to arrive at a logical solution. The NCTM (2014) highlighted the importance of struggle. The NCTM suggested using tasks that have multiple points of entry and multiple solutions. This struggle promotes reasoning and problem solving. The concept of struggle is also supported in the mathematics education literature: Wilson, Fernandez, and Hadaway (1993) claimed that problem solving must challenge students’ curiosity and bring into play their inventive faculties. Thus, “if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery” (Wilson et al., 1993, p. 57).

The models-and-modeling perspective encourages group activity and participation. According to Lesh and Zawojewski (2007), “group-based discourse sets up opportunities for individuals’ ideas to be challenged within their zone of proximal development, leading to further development of those ideas” (p. 790). Therefore, a class where group work is utilized fosters mathematical thought. Discourse allows students to engage in cooperative learning by sharing skills and strategies that help find solutions when problem solving. Working in this zone of proximal development increases students’ individual growth as they receive feedback from their peers. Based on a models-and-modeling perspective, “a task or goal-directed activity becomes a problem when the problem solver (which may be a group of collaborating specialists) needs to develop a more productive way of thinking about the given situation” (Lesh & Zawojewski, 2007, p. 782).

Thus, the product of mathematical problem solving is an artifact created by students. For instance, Lesh, Post, and Behr (1987) claimed that an answer to a mathematical problem-solving task should require a solution (i.e., product or conceptual tool) to be established that includes a variety of media for the resolution of explanation, simplification, justification, or production. For example, the Big Foot Problem used at a variety of universities for instructional purposes requires students to create a tool that police could use to predict how big people are just by looking at a footprint. This is a situation police encounter on a daily basis and, thus, the tool is beneficial in real life. In addition, the students use mathematical relationships to create their solution product. As Lesh and Doerr (2003) observed, the first tool students create typically has flaws and is immature. Therefore, successful solutions are attained when the group goes through cycles of expressing, testing, and revising their solutions (Lesh & Doerr, 2003). Ultimately, it is critical to create activities that allow students to go through the problem-solving cycle of revising their solutions. Activities need “to ensure that the solution (artifact, tool) problem solvers create embodies the mathematical process they constructed for the situation, and thus these types of problems are called model-eliciting activities” (Lesh & Zawojewski, 2007, p. 784). The use of MEAs is a key attribute of teaching through mathematical problem solving.

The Role of Students in Problem Solving

In a models-and-modeling perspective, the student plays an active role. Problem solvers are defined as “a group, where a diversity of powerful technological and conceptual tools is brought to the solution process, and the trial solutions posed go through cycles of testing and revision” (Lesh & Zawojewski, 2007, p. 789). Key attributes of problem solving for learners include sophisticated thoughts, determination, inquisitiveness, and self-assurance. In addition to displaying specific attributes, students must develop a productive disposition to problem solving if they are to be successful. According to the NCTM (2000), “a problem solving disposition includes the confidence and willingness to take on new and difficult tasks” (p. 334). According to the National Research Council (2001), this productive disposition is useful in helping students learn mathematics. In a models-and-modeling perspective on problem solving, it is important that students have a natural inclination to see mathematics as practical and useful. In addition, students must develop persistence and efficacy as they solve problems.

Solutions are complex artifacts. Students need to develop tools that describe, explain, justify, and construct. Students are expected to bring their own particular perspective to light on a problem, and to test and revise their understanding over a series of modeling cycles. Therefore, students' initial interpretations in MEAs often tend to be immature, primitive, or unstable compared with their final products (Lesh & Harel, 2003). It is important that students learn to create artifacts and tools that are "useful for a given client in a given situation and those artifacts need to be sharable and reusable in other situations, for other data sets, or by other people" (Lesh & Zawojewski, 2007, p. 784). Successful problem solvers consider multiple approaches if their first few approaches fail (NCTM, 2000). Students must gain conceptual understanding and procedural fluency as they create solutions to problem-solving tasks. In order to create an operative artifact, students have to learn the skill of carrying out procedures flexibly, accurately, efficiently, and appropriately (NRC, 2001).

In the models-and-modeling approach, students have the ability to transform their perspective and increase their level of positive mathematical affect through meaningful classroom discourse. According to Lesh and Zawojewski (2007), "learners are thought to bring some understanding to the table. Then interactions among group members provide opportunities for individuals' understandings to be tested, integrated, differentiated, extended, revised, or rejected" (p. 790). For instance, students should listen carefully to their peers while also critiquing the reasoning of their peers. Examples to support or counterexamples to refute their peers' arguments should be presented (NCTM, 2014). This idea is further supported as one of the problem solving standards and one of the communication standards for mathematical practice highlight the importance of students constructing viable arguments and critiquing the reasoning of others. Additionally, seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing others' approaches promotes engagement in the problem-solving process. Students are able to move from a "lack of involvement in the classroom to an active enjoyment" in the class through discourse (Fennema & Sherman, 1976).

Students learn to act as a community of practice as they engage in tasks that are too challenging for one student alone. The idea behind MEAs is that knowledge is socially situated (Lesh & Zawojewski, 2007). Ultimately, students' interpretations go beyond logic and mathematics to include feelings, dispositions, values, and beliefs (Goldin, 2002). Discourse has the ability to not only expand individuals' mathematical knowledge but also to bring beliefs, values, and particular dispositions to the table.

Principles of the models-and-modeling perspective on problem solving require students to reflect on their mistakes and misconceptions to improve their mathematical understanding. Reflection requires students to think about their mathematical beliefs as they engage in MEAs. Students are then able to respond to their peers by giving suggestions and supporting the learning of their classmates. Students' confidence in their ability to learn and do mathematics increases when they develop processes of metacognition and self-assessment (NCTM, 2014). In becoming proficient problem solvers, students are able to use their interactions with others to further their understanding not only of the mathematics, but also of themselves. According to the National Research Council (2001), "students learn how to form mental representations of problems, detect mathematical relationships, and devise novel solution methods when needed" as they play an active role in a community of practice (p. 126).

Problem solving provides students with opportunities to face productive struggle: "Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle, encourage one's confidence doing mathematics" (NCTM, 2014, p. 52). The models-and-modeling perspective fosters the development of students' self-efficacy, and consequently, productive beliefs in students. Another attribute that MEAs foster is an understanding about how much time to devote to mathematics problems. Students should persevere in problem solving and believe it is okay to say, “I don’t know how to proceed from here.” However, students need to also believe it is not acceptable to give up. These self-efficacy beliefs are vital to success and failure (De Corte, Mason, Depaepe, & Verschaffel, 2011). According to Lesh and Doerr (2003), MEAs require multiple class periods to complete, and therefore students need to develop the belief that their artifacts (solutions) make take longer than five minutes to solve. Students must value time and effort.

In addition, students need to recognize that trial solutions tend to be primitive and need refining. Therefore, believing that hard work fosters the necessary skills for problem solving is beneficial. The models-and-modeling perspective allows students to develop a growth mindset as opposed to a fixed mindset. A growth mindset is based on the belief that an individual’s basic qualities are things they can cultivate through their own efforts (Dweck, 2006). A key attribute to successful problem solving is developing the belief that through effort, ability in mathematics increases. The goal is that as students engage in problem solving, they will reveal aspects of their own thinking and beliefs not only to teachers, but also to themselves (Lesh & Zawojewski, 2007). Students need to learn to stretch themselves and believe that it is about becoming smarter, not merely being smart. The Mathematics Learning Study Committee (MLSC) stated that “mathematical proficiency cannot be characterized as simply present or absent” (MLSC, 2001, p. 135). There are many levels of ideas, and the goal should be to foster growth from students’ current levels.

Mathematics educators agree that the ability to think about and to solve challenging problems is a learning objective that all students should master. Research shows that students who have the ability to solve difficult mathematical problems effectively tend to exhibit specific characteristics. Kantowski (1997) found that “the use of heuristics was consistently more evident in solutions with scores above the median, that is, the percentage was higher for each subject of solutions with scores above the mean" (p. 165). This same theme was found in research done by Silver (1985) and Wilson (1967). Additional research showed that "analysis and synthesis (deduction inferred from hypothesis followed by a synthesis, then by further inferences from the new synthesis and so forth) was noted in the solutions of problems with score above the median" (Kantowski, 1977, p. 166). This finding suggests that problem solving promotes higher order thinking in mathematical thought, as a feature of thought is judgment. According to Dewey (1933), judgment involves: (a) "a controversy, consisting of the opposite claims regarding the same objective situation"; (b) "a process of defining and elaborating these claims and of sifting the facts adduced to support them"; and (c) "a final decision, or sentence, closing the particular matter in dispute and also serving as a rule of principle for deciding future cases" (p. 74).

Kantowski (1997) observed this process when investigating solving non-routine mathematics problems. Determining whether specific approaches generalize to a broad class of problems is an important principle in the models-and-modeling perspective, as problem solving is a process and an individual cycles through different understandings in different contexts (Lesh & Doerr, 2003). Adaptive reasoning is needed to succeed in problem solving. Students must learn to develop logical thought, reflection, explanation, and justification. The National Research Council (2001) identified adaptive reasoning as one of the strands of mathematical proficiency important to solving problems.

Another common theme in the literature emphasizes that students with a certain level of mathematical knowledge are more persistent when solving complex problems. Developing mathematical thought requires time, and patience is required when solving problems that do not require use of a specific process. Kantowski (1997) found that as students spent more time on seeking a solution once, “more rational methods of approach were used” (p. 169). Beliefs about how much time an individual should spend on mathematical problems is essential in the models-and-modeling perspective as it emphasizes situations “in which the problem solver is expected to create, refine, or adapt mathematical interpretations or ways of thinking” (Lesh & Zawojewski, 2007, p. 782).

The Role of Teachers in Problem Solving

Teachers’ instructional decisions and actions shape students’ mathematical dispositions (NCTM, 2000). The teacher’s role is crucial in guiding the class experience. Teachers can choose more interesting problems to incorporate into their classroom. According to the NCTM (2000), teachers “can motivate students by encouraging communication and collaboration” (p. 259). Adopting a models-and-modeling approach will provide students with opportunities to engage in MEAs, which increase productive beliefs (Lesh & Zawojewski, 2007). These models are conceptual systems that “reveal important aspects about how students are interpreting the problem-solving situation” to the teacher (Lesh and Doerr, 2003, p. 9). Because models are evident in internal and external systems, these models can be observed in student thought and action. For example, spoken language, written symbols, diagrams, pictures, computer programs, all act as external models (Lesh and Doerr, 2003). Teachers must consider not only the written mathematical symbols and language that students present but also drawings, images, and figures. According to the NCTM (2014), there are eight teaching practices that must be used to foster productive dispositions in students. Several of these practices place emphasis on problem solving. For example, the NCTM proposed that teachers should implement tasks that promote reasoning and problem solving. The MEAs used in a models-and-modeling framework promote reasoning as students are encouraged by teachers to develop metacognitive abilities, which come naturally from these MEAs. Teachers use MEAs to promote reflection. MEAs encourage students to reflect on changes in their metacognitive strategies as they move through the process of problem solving (Lesh & Doerr, 2003). The models-and-modeling approach highlights the importance of using MEAs as they provide opportunities for students to engage in real life situations, which prepare students to act as effective members in society. According to Lesh and Zawojewski (2007), it is important that teachers use these problem-solving tasks to enhance communication capability and conceptual flexibility, as these are central to the construction of solutions to everyday problems. In addition, the NCTM (2014) argued, “effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow for multiple entry points and varied solution strategies” (p. 10). Communication and flexibility are key.

Once again, the NCTM (2014) recognized that the models-and-modeling approach to problem solving is needed to successfully teach students in that it does not simply a focus on solution strategies but also stresses communication and student thinking. For example, another principle of teaching involves facilitating meaningful discourse. The NCTM (2014) explained that teachers are encouraged to “facilitate discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments” (p.10). The specific role that the teacher would play in discourse is ensuring that the learning community develops productive sociomathematical norms (Yackel & Cobb, 1996). These sociomathematical norms are different and important in a mathematics class as they differ from regular norms because they allows students to develop understanding of what acceptable mathematical explanation is and how to present these ideas to others. The teacher has the opportunity to create a safe environment in which students can communicate clearly with others and synthesize the information of group members (Lesh and Doerr, 2003). This is important because problem solvers are expected to design complex models and artifacts with a team. Social norms and discourse thus become necessary for effective communication.

Mathematical affect is complex and includes many dimensions like attitude, beliefs, emotion, and anxiety. Another way teachers can foster mathematical affect is through the use of student discourse. Teachers should “facilitate discourse among students by positioning them as authors of ideas, who explain and defend their approaches” (NCTM, 2014, p. 35). By publicly recognizing individual students’ ideas, or the ideas produced by a student group, students begin to see themselves and their peers as mathematicians, capable of developing their own thoughts. Productive beliefs are fostered through discussion and experience. Hackett and Betz (1989) suggested that mathematics teachers should pay attention to mathematical affect as one’s mathematical disposition is developed through beliefs. Therefore, the teacher’s role includes that of guide and supporter whose “guidance is purposely mediated, almost hidden, embedded in the activities” (Moll, Tapia, & Whitmore, 1993, p. 38).

Beliefs develop and morph throughout mathematical problem solving task. Students must use the mathematical problem-solving task to develop their beliefs within the learning community. The lens through which they experience and view problem solving impacts students’ mathematical affect. The teacher’s primary responsibility when negotiating mathematical meaning with students is to appropriate their actions into this wider system of mathematical practices (Cobb, 1994, p. 15).

Student Beliefs in Mathematics

There are many definitions for beliefs; Colby (1973) defined beliefs as creditability of conceptualizations. McLeod (1989) explained that credibility of conceptualization “has to do with whether one accepts, rejects, or suspends judgment concerning a set of concepts and the interrelationships among those concepts" (p. 41). Consequently, beliefs carry varying degrees of magnitude depending on the circumstance and nature of the problem-solving task. Silver (1985) recognized that student beliefs affect the process of problem solving, but also suggested that more research needed to be done to investigate this complex concept. Schoenfeld (1989) proposed that classroom community affected the development of beliefs. Current mathematics education researchers assume that the cultural setting of the classroom heavily influences the development of beliefs about mathematics. Thus, the models-and-modeling approach, which emphasizes beliefs as well as peer influence, on problem solving is logically related as this new direction in problem solving extends from the sociocultural philosophies.

Lesh and Zawojewski (2007) proposed that the development of problem-solving abilities and the development of beliefs cannot be studied as separate phenomena. Schoenfeld's (1992) review of the literature revealed that students "abstract their beliefs about formal mathematics-their sense of their discipline-in large measure from their experiences in the classroom" (p. 359). Experiencing real life-situations and being held responsible for their beliefs by learning to think about and to develop their beliefs among their community of practice students are able to develop skills needed to think critically about their beliefs as well as problem solving. Consequently, through the models-and-modeling approach, students' beliefs can help them form positive problem-solving strategies.

Beliefs about both self and mathematics impact student problem solving (McLeod 1989). Beliefs about mathematics, which are void of emotion, are "central to the development of attitudinal and emotional responses to mathematics" (McLeod, 1992, p. 579). Additionally, problem-solving beliefs, whether viewed as a success or failure, may impact an individual's “capacity for doing mathematical problems, leading to an increase in confidence” (Fennema, 1989). This view can also increase anxiety and thus must be considered in mathematics education research.

Schoenfeld (1985) and Silver (1985) stressed the importance of students’ beliefs about mathematics, as these beliefs can potentially weaken or strengthen their ability to solve non-routine problems. Both researchers suggested a curriculum reform that would encourage productive beliefs and foster growth in students in relation to development of their beliefs about mathematics. However, the National Research Council observed that “as children become socialized by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory (NRC, 1989, p. 7).

Kloosterman and Stage (1992) created a scale to measure students’ beliefs. They found that beliefs about mathematics problem solving affect one’s willingness to engage in it, as well as the decisions one makes during the process. Kloosterman and Stage’s findings were supported by Schoenfeld’s (1985) study, in which the author observed that students who did not engage in problem solving believed that mathematics problems should be solved in ten minutes or fewer. Additionally, students develop beliefs about the usefulness of mathematics. A national assessment exposed that 48% of students felt mathematics was about memorizing, and Schoenfeld’s study supported this as he found students believed “they should accept procedures without trying to understand how they work” (as cited in Kloosterman & Stage, 1992, p. 110).

De Corte et al. (2011) recognized that problem solving was not only impacted by epistemological beliefs about mathematics but also beliefs about the self. Dweck (2006) observed that many students believed their abilities in mathematics were not based on their efforts. More recently, Dweck (2008) studied the mindset of students, and found that many students believe that their mathematical ability is fixed. The models-and-modeling approach on problem solving does not emphasize getting the “right” answer the first time, but rather a process where students “go through a series of modeling cycles in which they integrate, differentiate, revise, and refine their existing ways of thinking” as growth infrequently transpires along a one-dimensional sequence (Lesh and Zawojewski, 2007, p. 795). Consequently, beliefs have the potential to influence students’ actions and their opportunity for learning, depending on what mindset they adopt. Reyes (1984) found that beliefs about confidence have the ability to affect achievement in mathematics, as his studies showed a positive correlation of greater than .40. This means that students who are confident in their mathematical ability tend to achieve at higher rates.

Mathematical Affect Scales

In the last forty years, researchers have developed many instruments to assess mathematical affect on problem solving. For the purpose of this study, four of the most widely used scales in mathematics education were selected for review as they are most related to beliefs and problem solving. These include the Fennema-Sherman Attitude Scale (Fennema & Sherman, 1976), the Math Self-Scale (Opachich & Kadijevich, 1997), the Indiana Mathematics Belief Scales (Kloosterman & Stage, 1992), and the Mathematics Self-Efficacy and Anxiety Questionnaire (MSEAQ) (May, 2009).

The Fennema-Sherman Mathematics Attitude Scale was developed to gain understanding concerning females’ learning of mathematics. The Fennema-Sherman scale consists of “nine domain specific Likert scales that measure important attitudes related to mathematics learning” (Fennema & Sherman, 1976, p.1). These are: the attitude toward success in mathematics scale, mathematics as a male domain scale, the mother/father scale, the teacher scale, the confidence in learning mathematics scale, the mathematics anxiety scale, the effectance motivation scale in mathematics, and the mathematics usefulness scale. The scales can be administered as a group, individually, or in any desired combination. Huck (2003) has cautioned individuals against using these scales as they have only undergone a one-time reliability and validity measure. In addition, the Fennema-Sherman Mathematics Attitude Scale applies to the general domain of mathematics and not specifically the problem-solving process.

This scale is widely used in mathematics education research. For example, Betz & Hackett's (1983) used these attitude scales in their study of the relationship of mathematics self-efficacy expectations to the selection of science-based college majors. More recently, Kahveci (2010) utilized the scales in a study on students' perceptions to use technology for learning, and Schommer- Aikins, Duell, & Hutter (2005) also used them in a study on student perceptions. However, to date no further validation and reliability analysis has been performed in reference to problem solving.

Opachich & Kadijevich (1997) designed the Math Self-Scale to determine if a sufficient number of factors could address self-concept. The authors assessed the subjects' generalized self-efficacy, intellectual self-efficacy, external locus of control, and non-verbal IQ scores. Opachich and Kadijevich found that "the mathematical self may be primarily influenced by the global self-esteem and mathematical achievement" (p. 405). Thus, it appeared that self-concept relates to self-efficacy and the ability to do mathematics successfully. This scale applies to the general discipline of mathematics and not specifically the problem-solving process.

Kloosterman and Stage (1992) created the Indiana Mathematics Beliefs Scale to validate what Kloosterman and Stage proposed as the five beliefs about mathematical problem solving.

- I can solve time-consuming mathematics problems.

- There are word problems that cannot be solved with simple step by step procedures."
- Understanding concepts is important in mathematics.
- Word Problems are important in mathematics.
- Effort can increase mathematical ability.

This scale is intended for use by secondary and college level students. Kloosterman and Stage (1992) emphasized that before administering the scale it should be used in a class where the term word problem has been explained, as their reliability coefficient was lower on this particular scale than the other four. Although this scale is most closely related to the problem-solving process, the definition of problem solving in Kloosterman and Stage's study is quite different than the models-and-modeling perspective, which differentiates between word problems and problem solving. For instance, word problems are no longer an important belief to be examined in the models-and-modeling perspective because they focus on a one solution answer and no artifact design (Lesh and Zawojewski 2007). This scale is also widely used in mathematics education. Researchers continue to use or adapt this scale to gather information on students' beliefs. For example, Mason (2003) and Schommer- Aikins et al. (2005) used the Indiana Mathematics Beliefs Scale to examine achievement in mathematics and development of beliefs in mathematical problem solving.

May (2009) designed the MSEAQ to explore how mathematics self-efficacy and anxiety are related. Although designed as a college questionnaire, this scale has application in secondary schools as mathematics problem-solving achievement is often influenced by self-efficacy and anxiety in that they affect beliefs. The scale was found to be reliable, valid, and efficient to administer, which is important if it is to be used by a classroom teachers and students. Unfortunately, even this more recent scale is applicable to the general discipline of mathematics rather than specifically to the problem-solving process as defined by a models-and-modeling approach. Although May's (2009) scale is relatively new, there are a number of studies that have used the student questionnaire to study anxiety along with student self-efficacy. For example, Jain and Dowson (2009) used the MSEAQ to examine mathematics anxiety as a function of multidimensional self-regulation and self-efficacy.

Chamberlin (2010) argued that three needs have gone unmet in the development of mathematical affect instruments. The first unmet need is that early instruments only assess one component of affect. Mathematical beliefs are far too complex to explore only one component. Secondly, the classroom teacher cannot use many instruments, as they require a psychologist or psychometrics to interpret the results. Classroom teachers need to be able to use an assessment instrument in a models-and-modeling perspective on problem solving. It is important that students investigate their development of beliefs, as they are involved in activities "where they express, test, and revise their own attributes during post-hoc reflections" (Lesh and Zawojewski, 2007, p. 778). Finally, Chamberlin argued that "all of the current instruments assess students' affect regarding the discipline of mathematics in general as opposed to assessing students during or after the process of mathematical problem solving" (2010, p. 177). This claim supports the argument for a new perspective on problem solving as suggested by the model-and-modeling perspective, as the goal of such an instrument could be used by both teacher and student.

The purpose of the present study was to develop a reliable and valid scale that measures students' beliefs on problem solving as defined through a models-and-modeling approach. Although the No Child Left Behind Act (NCLB) is no longer in place, its effects can still be felt. Under the Every Student Succeeds Act (2015), educators today face a reality in which standardized assessments play a major role in mathematics education. Chamberlin (2010) recognized that mathematical dispositions “are the components of education that are potentially the item most frequently neglected as a result of increased attention to standardized assessment” (p. 167). As a result, teachers neglect the importance of beliefs in the learning process. Therefore, it is important for both the teacher and student to be aware of these beliefs throughout the problem-solving process. For this reason, students must engage in the process of problem solving through a MEA before being administered the scale. After students have engaged in the MEA the assessment should be administered. The procedure for scale validation will then be employed.

Value of Specific Methodology

After reviewing existing scales related to mathematical affect and problem solving it was apparent that a new scale should be created for reasons related to reliability and validity. For example, the Indiana Mathematics Belief Scale places emphasis on the belief that word problems are important in mathematics (Kloosterman & Stage, 1992). Although the intention of the word problem scale items is to see if students feel that computation skills are more important than problem-solving skills, the wording of the scale is no longer applicable to students. The models-and-modeling perspective does not consider a word problem by definition to be a problem-solving task. Therefore, the scale items would need to be revised in order to reflect current mathematical problem-solving literature. In addition, the Fennema-Sherman Mathematics Attitudes Scales, although still used by some researchers today, may present problems of validity and reliability as “word meanings change over a period of nearly four decades” (Chamberlin, 2010, p.173; Fennema & Sherman, 1976). The wording of the scale needs to be reconsidered, along with estimates of reliability and validity, as a one-time validation no longer provides compelling evidence for assessments of reliability and validity. Additionally, the only scale related to problem solving specifically, the Indiana Mathematics Belief Scales, was validated with a sample of college students. Validation and reliability measures were never performed again with a different sample. Kloosterman and Stage (1992) advised that these attitude scales were appropriate for middle school and high school students as well. Thus, along with new wordings and items, a problem- solving scale needs to be validated with a sample of middle grade students.

Item Generation and Scale Construction

The researcher generated an initial set of items through a review of mathematics education literature on problem solving. The researcher examined historical studies and articles. For example, the researcher examined Polya's (1957) steps for successful problem solving as well as Schoenfelds's (1985) list of strategies. Furthermore recent studies and articles about mathematical problem solving were examined (Chan 2008, Lesh and Zawojewski 2007, Chamberlin & Moon 2005, Lesh and Doerr, 2003). The researcher generated an initial list of items based on the themes that ran through the literature and an examination of other scales. The literature review revealed six common themes across historical and recent mathematics education articles as well as studies addressing mathematical problem solving. These six themes are *mathematical mindset, mathematical problem-solving perseverance*, *mathematical revision and refinement, mathematical communities of practice, problem-solving processes, and problem-solving utility*. Each of these themes was continually referenced in studies as important to problem solving, and thus help define the models-and-modeling perspective described by Leah and Zawojewski (2007). The following sections provide an overview of these themes.

Items (see Appendix A) are aligned with the productive dispositions and beliefs students should have when engaged in problem solving through a models-and-modeling perspective. For example, some of the items measure the level of belief that testing and revising solutions to complex problems is important in mathematics. Lesh and Zawojewski (2007) emphasized the importance that in a models-and-modeling design of problem solving, the production of tangible tools must go through “cycles of expressing, testing, and revising their solutions” (p. 779). Thus, the belief that problem solving cannot be solved by following a set of memorized set of procedures is applicable. For instance, there is not a set rule to follow when designing a tool for use by using some mathematical conceptual system. The process of developing this artifact might involve trial and error in order to achieve a product worth using not only in but also outside of class, as it applies to a larger population. According to Lesh and Zawojewski (2007), students bring their own personal meaning to bear on a problem by going through the problem-solving process of testing and revising their interpretations. Thus, the belief that testing, revising, modifying, integrating, and refining concepts and tools is productive. This belief is necessary for students who engage in problem solving.

Another aspect of the models-and-modeling perspective is the need for solutions to be complex artifacts instead of the traditional conventional story problems about premathematized situations. Thus, the MEAs in which students are involved in relation to problem solving tend to take more time than traditional word problems. More time is spent when students are engaged in a MEA (Lesh & Zawojewski, 2007). Schoenfeld (1988) suggested that students who believe that problems should take no more than five minutes are often not willing to persist to create a solution to the assigned mathematical task. Measuring the value in spending time solving problems and the willingness to persist in solving a problem is a belief that impacts problem solving (Kloosterman and Stage, 1992). Thus the belief that perseverance is important solving problems is a productive belief, as it motivates an individual to move forward in the problem-solving process.

The models-and-modeling approach highlights the need for the class to act as a community of practice because interactions among peers often aids in the learning process and increases discourse that challenges the individual (Wenger, 1999). Valuing learning communities over independent learning is productive because learning is social. Research indicates the models-and-modeling perspective on problem solving can enable the learner to move “beyond current ways of thinking through the interaction of a variety of alternative conceptual systems that are potentially relevant to the interpretation of a given situation” (Lesh & Zawojewski, 2007, p. 789). Thus, interacting with peers allows students’ ideas to be challenged through testing, revision, and refinement. The belief that problem solving is more effective when a team works together in the development of solutions to complex problems is beneficial results in value being placed on student-to-student interactions.

Additionally, some of the items address the concept of mindset in reference to problem solving. Mindset refers to the students’ beliefs about their ability to successfully solve a problem. Some students have a fixed mindset and some students have a growth mindset (Dweck, 2006). The growth mindset involves believing that one’s ability to solve mathematics problems can increase as one puts forth the effort. This belief in ability growth is productive in the models-and-modeling perspective, as the process of mathematical problem solving is expected to be continually under development (Lesh & Zawojewski, 2007). For instance, the thought that if one has to work at it, it was not meant to be, is unproductive; problem solving requires revision, restructuring, adapting, and editing based on new knowledge that is gained in the act of problem solving. Thus, the researcher examined the belief that one’s mathematical problem-solving ability can be cultivated through hard work and effort.

There is a growing acknowledgement that a discrepancy exists between the low-level skills emphasized in test-driven curricula and the kind of understanding and skills that are necessary to thrive beyond school (Lesh & Zawojewski, 2007). The models-and-modeling perspective places emphasis on problem solving in a real life context. This perspective on problem solving requires students to use “real world, local community, and even individualized examples” in which students analyze and interpret these situations in the hope that they see mathematics as a way to understand reality (Boaler, 1993). The models-and-modeling perspective allows students to become involved in mathematical problem solving by breaking down their “perceptions of a remote body of knowledge” (Boaler, 1993, p. 13). They are able to see the usefulness of mathematics not only in the classroom, but also their everyday life. Lastly, students must place value on understanding the mathematics if they are to engage in problem solving. Although students can carry out a procedure to get an answer to a problem, they cannot use this procedure to interpret and to understand new and dynamic situations (Lesh & Zawojewski, 2007). Consequently, it is important that students understand relationships mathematically. The belief that it is necessary to understand the solution to a mathematical situation—as opposed to just getting an answer—is a valuable one.

The view and individual adopts for himself/herself profoundly marks his/her beliefs and decisions in regards to their mathematical journey in school. Each student has a certain mindset when he/she solves problems, which greatly affects his/her success. The fixed mindset is evident in students who believe that their mathematical abilities are unchangeable, while the growth mindset is the "belief that your basic qualities are things you can cultivate through your efforts (Dweck, 2008, p. 7).

Mathematical problem solving involves risk and effort and therefore mindset is one construct that affects problem solving. The models-and-modeling perspective on problem solving requires effort as one continually refines their ideas; thus, mathematical mindset has the ability to turn beliefs into actual accomplishment. In empirical studies, there has been no clear consensus on the underlying factor structure of mathematical mindset (Chamberlin, 2010).

Mathematical perseverance refers to the ways in which students persevere in problem solving despite the difficulties, obstacles, and discouragement they may experience. Lesh and Harel (2003) observed that students' initial interpretations of mathematical situations are juvenile, elementary, or ambiguous compared with interpretations that underlie their final solutions. Schoenfeld (1985) was also concerned that students were often not willing to work on a mathematical problem for more than five minutes. Valuing time spent problem solving leads students to develop more sophisticated solutions that apply to the real world. Again, in empirical studies, there has been no clear consensus on the underlying factor structure of mathematical perseverance (Chamberlin, 2010).

Mathematical revision refers to the way students reexamine their thinking, improve their current models, and amend their solutions to create refined solutions that prove fruitful. Revision is vital in a models-and-modeling perspective on problem solving. Students' mathematical thinking ranges from immature to effective. Higher magnitudes of revision enable one to produce higher quality solutions. Lesh and Doerr's (2003) study revealed that students' successful solutions are attained when the group goes through cycles of expression, testing, and revising. Thus, valuing the revision process leads productive dispositions in the problem-solving process.

Exploring more about how mathematical ideas evolve is needed and currently there are no empirical studies that measure attitudes of individuals towards revision (Chamberlin, 2010).

Learning is social. Vygotsky (1978) argued that learning occurs on both the social and individual levels. People are expected to collaborate and function efficiently in the workplace. The models-and-modeling perspective takes this into account and encourages teachers to use MEAs in which students must collaborate and function as a community of practice to accomplish tasks. The problem solver is no longer one individual, but a group of students with a common goal. Each student brings different conceptual understandings and skills to the classroom. Social learning as opposed to individual understanding leads to a more productive attitude in mathematical problem solving. Lesh and Zawojewski (2007) accepted this shift to a more social approach to learning and suggested that an “essential mechanism for moving students beyond current ways of thinking is through the interaction of a variety of alternative conceptual systems” (p. 789). When students collaborate interaction between different abstract organizations and patterns occurs. Consequently, valuing working in a community of practice is worthwhile. Chamberlin (2010) suggested that there are a few scales that measure one’s belief in working as teams. Thus, valuing one’s peers as a community of practice is of particular interest.

Mathematical problem solving can be applied in an individual’s adult life. While students may be cognitively capable, they often lack awareness of how mathematical processes can help in other contexts. Fennema-Sherman (1967) developed an attitude scale to measure the usefulness of mathematics, but this scale needs revising as the wording is outdated and it does not specifically apply to the problem-solving process as defined by a models-and-modeling perspective (Chamberlin, 2010). This current study is particularly designed to measure beliefs about the utility of mathematical problem solving.

Mathematical problem solving requires students to experience MEAs. These MEAs are open-ended, real-world, and client-driven problems (Diefes-Dux, Moore, Zawojewski, Imbrie, & Follman, 2004). Therefore the process by which students engage in problem-solving tasks is important. Students' beliefs about their role as a problem solver become paramount. Students' beliefs about their teacher's role become meaningful. In addition, students' understandings of problem solving become influential. Therefore measuring beliefs about the problem-solving process is of interest in the current study. Chamberlin (2010) argued that there are few scales that exist to measure mathematical affect and even fewer that measure affect while engaged in the process of problem solving. Thus, the proposed factor of *mathematical problem-solving processes* is essential in this study.

Thus, items generated centered on the six themes: *mathematical mindsets, problem-solving perseverance, mathematical revision, mathematical communities of practice, problem-solving utility,* and *problem-solving processes*. Problem solving is complex and this study focused on identifying only one factor affecting these six constructs.

Chapter 3: Methodology

Purpose of the Study

The aim of this study was to develop a reliable and valid instrument to measure students' mathematical problem-solving dispositions and beliefs. The present study had both quantitative and qualitative components. The development process for the Mathematical Problem Solving Dispositions and Beliefs Scale (MPSDB) involved item construction, reliability analysis, and establishing the validity of mathematics problem-solving dispositions and beliefs items through exploratory factor analysis (EFA). The researcher used student feedback and comments to improve items and interpret the results of factor analysis.

The concept of this study is based on the idea that "there is a single underlying characteristic that an instrument is designed to measure" (Wilson, 2004, p. 5). In the current study, this characteristic was mathematical problem-solving dispositions and beliefs as they relate to the models-and-modeling perspective. This construct is not observable by direct means and therefore this study involved developing and validating a scale to measure this phenomenon. One student's belief in his or her ability to solve mathematical problems can be very strong while another student's belief could be very weak. Wilson (2004) described constructs as continuous that include any point in between high and low. DeVellis (2012) described these constructs as variable, where the "strength and magnitude" change (p. 17). Student beliefs can change from productive to unproductive and from being weak to strong, thus having both strength and magnitude.

McLeod (1989) described mathematical affect in problem solving as having magnitude and direction. In order to measure a student's mathematical problem-solving dispositions and beliefs, which cannot be directly observed, the researcher developed a scale to measure psychological and social phenomena (DeVellis, 2012). Utilizing a scale, a measurement instrument reveals "levels of theoretical variables not readily observable by direct means" (DeVellis, 2012, p. 11). The research design section that follows outlines development and validation of the MPSDB.

Chamberlin (2010), in his review of instruments to assess affect in mathematics, found that "all the current instruments assess students' affect regarding the discipline of mathematics in general as opposed to assessing students' affect during or after the process of problem solving" (p. 177). Ma and Kosher (1997) also emphasized the importance of giving the assessment during or after the problem-solving task. For this reason, the MPSDB was administered towards the end of problem-solving process. The first phase consisted of concept clarification, description of the intended population, and initial item generation. The second phase featured completion of the item generation and revisions based on expert review, focus groups, and other validity measures. Finally the third phase addressed the final administration for assessment of psychometrics.

Research Objectives

There were five research objectives addressed in this study:

1. To develop a reliable measure of MPSDB.
2. To establish content validation using a panel of experts with positive agreement and high inter-rater reliability as to the accurate representation of item samples, appropriateness of content, and appropriateness of item format.
3. To explore the construct validity of the measure MPSDB and the relationship between scores from related mathematics scales correlations (i.e., Fennema & Sherman, 1976; May, 2009).

4. To determine criterion validity by examining the relationship between scores on the MPSDB scale and logically related concurrent behavioral criteria, including grade point average (GPA) and course performance.
5. To conduct item analysis (i.e., factor analysis and reliability analysis) in order to explore the factor structures of the scale and examine the reliability of the scale.

Research Design

The researcher implemented this study in three phases that included eight steps, and utilized both quantitative and qualitative methodology. The researcher used qualitative data collected through feedback sessions and expert review in item generation and scale revision. The researcher collected quantitative data during the pilot and final administration to address the main research objectives presented above.

Phase I was conceptual and included concept clarification, description of the envisioned population and initial item generation based on the literature review. Phase II included completion of item revisions based upon expert panel review and feedback session. Phase III involved final testing of the MPBSD instrument. The design of the MPSDB scale development procedures began with DeVellis' suggestion from *Scale Development: Theory and Applications* (2012). DeVellis prescribed eight steps in Classical test theory (CTT) scale development. These eight steps are:

1. Determine clearly what is to be measured;
2. Generate an item pool;
3. Determine the format for measurement;
4. Have initial item pool reviewed by experts;

5. Consider inclusion of validation items;
6. Administer items to a development sample;
7. Evaluate the items; and
8. Optimize scale length.

This process is almost identical to Churchill's (1979) eight steps of better measures. The steps below lay out the procedures involved in the development of the mathematical problem-solving dispositions and beliefs scale based on the suggested steps by DeVellis (2012).

Phase I

Step 1: Item Generation → Literature Review

Step 2: Expert Review of Measures → Content Validity

Phase II

Step 3: Collect Data → Initial Survey

Step 4: Purify Measures→ Factor Analysis

Phase III

Step 5: Collect Data → Factor Analysis

Step 6: Assess Reliability → Coefficient Alpha

Step 7: Assess Validity → Criterion (self-reported GPA), Construct (Fennema-Sherman Correlation & May Correlation), Content (Expert Panel & Ratings), and Incremental (GPA and math GPA)

Step 8: Conclusion of Statistics → Summarize Distribution of Scores

Setting

This study took place in a large urban public school district located near Atlanta, Georgia during the spring semester of 2016. The sample population included participants from sixth, seventh, and eighth grades. The public school is located in one of the largest urban public school districts in Georgia. The study took place in middle grades mathematics courses during normal school hours.

Participants

The researcher conducted the study with 575 middle school students and 13 middle school teachers. According to Costello & Osborne (2005), subject to item ratios of 10:1 are acceptable. This "early and still-prevalent rule-of-thumb" is still suggested by researchers for determining *a priori* sample size (Costello & Osborne, 2005, p.137). This researcher ensured the 10:1 ratio was satisfied before data analysis. The students were enrolled in sixth, seventh or eighth grade mathematics. The demographics of the large urban public school were: 43% of participants White, 31% African American, 16% Hispanic, 8% Asian, 3% Multiracial, and less than 1 % Native American/Hawaii Pacific Islander. 37 % of participants qualified for free/reduced meals. The teachers taught either sixth, seventh or eighth grade mathematics. All 575 students took a paper version of the MPSDB survey. All teachers took a paper version of the models-and-modeling questionnaire. During the recruitment phase, an email invitation, along with a written letter, provided all sixth, seventh and eighth grade teachers and parents with a brief overview of the study, guidelines of data collection procedures and letter of consent. Students were asked to read and to sign an assent form directly before data were collected. To maximize the validity of self-reports, the confidentiality and anonymity of responses were emphasized to participants. The researcher gained access to the site as she taught eighth grade mathematics at this school and out of convenience selected the sixth, seventh and eighth grade students and teachers to participate. The principal of the school acted as the gatekeeper between the school district and the researcher giving permission to the researcher to conduct the study at this public school.

Instrumentation

MPSDB

The mathematical problem-solving dispositions and belief scale(MPSDBS) is a 40 item self-reported Likert-type scale that measures students' dispositions and beliefs towards mathematical problem solving as defined in a models-and-modeling approach. The scale measures six constructs: *mathematical mindset, mathematical problem-solving perseverance, mathematical revision, mathematical communities of practice, problem-solving utility*, and *problem-solving processes*.

Mathematics Self-Efficacy and Anxiety Questionnaire (MSEAQ)

Seven items from MSEAQ (May, 2009) were adopted to measure students' self-efficacy and anxiety towards mathematics. The sample item of this scale includes "As an adult I will use mathematics," and the Cronbach's alpha of this scale is .96 for this study. The researcher used MPSDB scale scores and correlated them with items from the MSEAQ because mathematics self-efficacy and anxiety has been associated with achievement and persistence to problem solve. Higher scores on the MSEAQ mean that students have high self-efficacy and low anxiety.

Attitude Scale-Usefulness of Mathematics

The researcher adopted five items from the Attitude Scale-Usefulness of Mathematics (Fennema & Sherman, 1976) to measure students' perceived usefulness of mathematics. The sample item of this scale includes "I get nervous when asking questions in my math class," and the Cronbach's alpha of this scale is .88 for this study. The researcher used MPSDB scale scores and correlated them with items from the Attitude Scale-Usefulness of Mathematics as "perceived usefulness of mathematics is an important component of motivation" and problem solving (Kloosterman & Stage, 1992. P.111). Higher scores on the attitude scale indicate that students perceive mathematics to be useful in their everyday life.

The researcher used Evans (1996) recommended values of correlation—.0-.19 very weak, .2-.39 weak, .4-.59 moderate, .6-.79 strong, and .8-1 very strong—to determine the strength of correlation between the MPSDB scale scores with each established measure mentioned above.

Achievement and Demographic Information

575 middle school students (N = 575; 275 females and 300 males) ranging in age from 11 to 15 years participated in this study. Thus, 47.7% of participants were male and 52.3% of participants were female. 418 Caucasian students, 71 African American students, 31 Asian /Pacific Islander students, 2 Native American students, 26 Hispanic/Latino students and 27 students of other ethnic backgrounds participated in this study. The average GPA reported of participants was 3.7 and the average math class average of participants reported was 90.4.

Data Collection Procedures

Expert Panel

The researcher invited 29 experts in the field of mathematics education to be judges of the MPSDB initial items (see Appendix B). Nine of 29 invited experts agreed to participate in the study by receiving the initial pool of items presented in MPSDB scale. At least two individuals participated in the panel from each area of expertise including: measurement/scale development (n= 2); secondary education (n=2); mathematics teaching (n=2); and mathematics problem solving (n=3). In particular, two mathematics education professors and researchers from the University of Georgia provided email communication as well as filling out the expert rating form. The researcher sent a rating form designed to evaluate potential MPSDB items was sent to each expert via email (see Appendix C). This form included four sections that asked experts to judge the following: relevancy of each item to the conceptual definition of mathematical problem solving as approached in a models-and-modeling; realistic beliefs related to mathematical problem; word choice with respect to its appropriateness to the target audience and response format with respect to its relevance to the items.

Feedback Session

The researcher conducted a feedback session with sixth, seventh, and eighth grade students (n=16) to obtain feedback on initial scale items. Kitzinger (1995) described feedback sessions as involving carefully planned and documented discussions among homogenous individuals around specified topics of interest. The discussions delved into perceptions and interpretations of the scale items as well as other beliefs and dispositions students might have about mathematical problem solving. The researcher recorded notes during the session to provide documentation in addition to what the students physically recorded on their initial pilot survey.

To understand how students interpreted the items on the MPSDB, a group of sixteen sixth, seventh and eighth grade students were given the MPSDB scale for review and discussion. These sixteen participants had not seen the MPSDB scale prior to the feedback session. The researcher asked the students to respond to the items and explain their responses. The feedback session guide included follow-up items such as, “Why did you respond to that item that way?” and “what situation makes you feel that way?” (see Appendix D).

Preliminary Scale Administration

The researcher prepared the preliminary scale for administration following the expert panel review and analysis of feedback session data. The preliminary scale was administered to a group of sixth, seventh and eighth grade students (n=64) in three public school mathematics classes. During class time in suburban public schools middle schools classrooms, the researcher administered the anonymous paper/pencil self-report instrument. The researcher followed the established protocol they developed by advising students to choose the best answer for each question, and if the respondent were unsure or unclear about a question, they were asked to leave it blank or write in their own thoughts on that particular topic (see Appendix E). Upon completion of the survey, students were told to place the survey in the envelope on the front table. The preliminary scale is presented in Appendix F. The lead researcher collected the completed surveys from the table for data entry and subsequent analysis. The researcher then grouped the participants’ responses by item and analyzed for common themes with respect to each factor found in the EFA. For example, EFA was performed in order to identify the number of factors as well as the items’ loadings.

Final Scale Administration

After conducting preliminary analyses on the preliminary scale (see results reported in Chapter 4), the final scale included a total of 40 items. The final self-report scale instrument included the following measures: 1) an eight item Mathematical Mindset Scale; 2) a six item Mathematical Problem-solving Perseverance Scale; 3) an eleven item Mathematical Revision Scale; 4) a five item Mathematical Communities of Practice Scale; 5) a five item Mathematical Problem-solving Utility Scale; and 6) a five item Mathematical Problem-solving Process Scale (see Appendix G). To run validity measures, the researcher also asked students to report both their overall and mathematics grade point average (GPA), in addition to their mathematics teacher. In addition the MPSDB scale, for validity purposes, included items from the Fennema-Sherman Mathematics-Usefulness of Mathematics Scale (1976) and items from May's (2009) MSEAQ. These items helped establish construct validity. The 13 teachers who participated in this study completed a paper and pencil copy of the model-eliciting activities (MEAs) and Teaching Practice Beliefs Questionnaire at the same time as the final administration of the MPSDB (see Appendix H).

Data Analysis Procedures

Expert Panel

The researcher carefully examined experts' ratings and open-ended suggestions to determine item inclusion and revisions of the preliminary scale (DeVellis, 2012). The researcher proposed decision criteria for retaining, deleting, and rewriting items consistent with the expert panel review. Items that received a rating of 1 on the rating scales by more than half of the experts were eliminated; items that received a rating of 2 on both scales were revised; and items that received a 3 by more than half of the experts on the ratings scales remained unchanged.

Feedback Session

Based on the feedback after the preliminary administration of the MPSDB scale from the feedback session, the researcher eliminated items that generated discussion from half of the participants about unclear wording or interpretation, and items that generated discussion from a third of the feedback session were revised based on open-ended suggestions recorded by the participants. All other items remained unchanged.

Factor Analysis

The researcher used EFA to determine which factors accounted for the most variance. Factor analysis involved the last recommended procedure presented by DeVellis (2012) as he encouraged the optimization of scale length. Factor analysis should be inspired Churchill's (1979) emphasis for the researcher to provide guidance on the interpretation of the results, as this statistical procedure provides a frame of reference to describe relations among the variables by defining the number of variables and allowing for interpretation.

Researchers commonly use factor analysis as a statistical tool for identifying how many latent variables motivate a set of items (DeVellis, 2012). In recent decades, factor analysis is used because of the development of statistical software such as SAS, SPSS, BMD, and DATATEXT. Software, like SPSS, makes the statistical analysis required in factor analysis easier and faster to perform. This researcher used factor analysis in developing the scale not only to identify latent variables, but also to support the validity. It is important to recognize that factor analysis "assumes that the observed (measured) variables are linear combinations of some underlying source variables (or factors)" (Kim & Mueller, 1978, p. 8). The purpose of factor analysis is to embody a fixed number of variables in terms of a smaller number of hypothetical variables. More specifically, researchers use explanatory factor analysis as a way of finding out which factors load to the construct of mathematical problem-solving dispositions and beliefs. Kim and Mueller (1978) proposed that factor analysis "can be used as an expedient way of ascertaining the minimum number of hypothetical factors that can account for the observed co-variation, and as a means of the data for possible reduction" (p. 9).

Using The Statistical Package for the Social Sciences (SPSS) version 22.0, the researcher performed EFA to identify the factor structure of the MPSDB scale. More specifically, the researcher used principal axis factoring for extraction. This statistical analysis was convenient as the researcher suspected that a measure designed to assess mathematical problem-solving dispositions and beliefs among secondary mathematics students contains a dimensional structure, and that measuring the separate dimensions would lead to a better understanding of the construct. According to Fabrigar, Wegener, MacCallum, & Strahan (1999),

> The primary purpose of EFA is to arrive at a more parsimonious conceptual understanding of a set of measured variables by determining the number and nature of common factors needed to account for the pattern of correlations among the measured variables. (p. 275)

After extraction, the researcher decided how many factors to retain for rotation. Cattell's (1996) scree test along with reliability was used to help determine how many factors to retain. Costello & Osborne (2005) claimed that the scree test is the "best choice for researchers" because it is contained in most statistical software packages and commonly used (p. 134). In the scree test, the eigenvalues are given in decreasing order and linked with a line. The researcher examined the eigenvalues of the graph created in SPSS to determine the point at which the last significant drop or break took place. The researcher then created a scree plot that plots the eigenvalues against the corresponding factor numbers. This graph provides insight into the number of factors to extract as one can examine when the rate of decline tends to become almost horizontal. The elbow in the graph indicated that each successive factor accounted for smaller and smaller amounts of variance. According to Ledesma & Valero-Mora (2007), this "point divides the important or major factors from the minor or trivial factors" (p. 3).

The researcher performed rotation to simplify and to clarify the data structure. According to Costello & Osborne (2005), educational fields generally anticipate some correlation among factors, because human feelings and beliefs are rarely segregated into boxed units that function independently of one another. For this reason, the researcher used oblique rotation. After performing oblique direct oblimin rotation in SPSS, the researcher examined both the pattern and structure matrix for item loadings, in addition to the factor correlation matrix, which revealed correlation between the factors. The researcher also examined factor matrices to determine the communalities. Generally, communalities are considered high if their value is greater than or equal to .8. However, common magnitudes in the social sciences tend to be more moderate with values of .40 to .70 (Costello & Osborne, 2005). If the item had a magnitude of less than .4 that item was dropped. Also, it should be noted that factors with fewer than three items are generally weak and unstable; five or more strongly loading items (.50 or better) are desirable and indicate a solid factor (Costello & Osborne, 2005). Thus factors in this study had five or more items.

Establishments of Reliability and Validity of the MPSDB Scale

Principles of reliability and validity are needed to develop a good scale. The principles of reliability and validity assess the degree to which scores are an accurate measure of a characteristic. This researcher performed measures of both reliability and validity to ensure that the MPSDB scale measures mathematical problem-solving dispositions and beliefs. It is important to recognize that reliability analysis indicates the capacity of a test to yield consistent scores and validity analysis specifies which stable characteristics test scores measure (Furr & Bacharach, 2013). In the case of this study, the researcher performed reliability analysis before validity analysis because Nunnally, Bernstein, & Berge (1967) suggests reliability is a prerequisite for validity. The researcher determined the results of reliability and validity using SPSS.

Reliability. After the scale items were generated, the researcher considered the degree of reliability. A reliable instrument is one “that performs in consistent, and predictable ways” (DeVellis, 2012, p. 31). According to Friedenberg (1995), a reliable scale “can be depended on to generate scores that are realistic estimates of test takers’ actual knowledge or characteristics” (p. 178). This measure of reliability can be represented statistically, which in the literature is referred to as the reliability coefficient. Classical Test Theory (CTT) bases reliability analysis on two factors: stable characteristic of the individual, called the true characteristic of the individual; and chance features of the individual, called random measurement error (Friedenberg, 1995). Thus, it follows that using the formula, X= T+E, a reliable test, is one where “the value of E should be close to 0 and the value of T should be close to the actual test score, X” (Friedenberg, 1995, p. 181).

Friedenberg (1995) defined the reliability coefficient as “the proportion or percent of test score variance due to true score differences” (p. 182). The formula used calculate the reliability coefficient in this study can be seen below, where r_{xy} is the reliability coefficient, ${\sigma_t}^2$ is the true score variance, and ${\sigma_x}^2$ is the observed score variance:

$$r_{xy} = \frac{{\sigma_t}^2}{{\sigma_x}^2}$$

The ratio should be close to 1 if there is little error, and hence, a high reliability. Conversely, if the ratio is close to 0, it implies no correlation, and no reliability. A ratio between .7 to .9 is adequate to establish reliability (Nunnally, et al., 1967). The researcher calculated the ratio using reliability analysis in SPSS.

Researhcers typically use Cronbach's (1951) alpha to examine reliability. According to Churchill (1979) and DeVellis (2012), the recommended measure of internal consistency is provided by coefficient alpha. Nunnally and Bernstein (1978) also recognized that coefficient alpha provides a worthy estimate of reliability. DeVellis (2012) defined alpha as "the proportion of a scale's total variance that is attributable to a common source, presumably the true score of a latent variable underling the items" (p. 37). The formula the researcher used in this study to calculate alpha is below where α is the coefficient alpha, k is the number of items, and $\frac{\sum \sigma_i^2}{\sigma_{yi}^2}$ is the total proportion of total variance:

$$\alpha = \frac{k}{k-1} (1 - \frac{\sum \sigma_i^2}{\sigma_{yi}^2})$$

The researcher calculated alpha using reliability analysis in SPSS.

Test length and test score variability are two other considerations when developing a scale in terms of the reliability. Longer tests usually provide a more representative sample of reliability. The Spearman-Brown formula can be used to determine if increasing or decreasing the number of items on the MPSDB scale results in more reliable results (Spearman, 1910; Brown, 1910). In fact, the researcher used the following equation to estimate the number of items needed to obtain highly reliable results where k is the number of items the test would have to be lengthened to, r_{kk} is the desired reliability, and r_{11} is the reliability of the existing test:

$$k = \frac{r_{kk}(1 - r_{11})}{r_{11}(1 - r_{kk})}$$

In addition, the scale needs to truly test the characteristic it intends. Friedenberg (1995) proposed that "the most reliable tests are those that include a representative sample from this set of possible test items" (p. 185). In order to ensure reliability with optimal test length, the researcher purified items using reliability formulas, as well as performed factor analysis procedures to determine grouping and clusters of variables. This is why the reducing the final scale to 40 items while maintaining 575 responses ensured reliability.

Validity. In this study, the researcher used three different types of validity in conjunction to establish overall validity. The theory behind validity is substantiated by Classical Test Theory in that X= T+E, where X is the total instrument score, T is the true score, and E is the error. Friedenberg (1995) proposed that true score has 2 components, which are: stable characteristics of the individual relevant to the purpose of the test; and stable characteristics of the individual irrelevant to the purpose of the test. This relationship can be represented as the systematic measurement error. This is expressed as X= R+I+E, where X is the test score, R is the relevant characteristics, I is the effect of stable characteristics irrelevant, and E is the random measurement error or effect of chance events.

Just as in reliability, one needs to examine the performance of the sample and thus the researcher used variance. According to Friedenberg (1995), the test score equation should be written as $\sigma_X^2 = \sigma_R^2 + \sigma_I^2 + \sigma_E^2$, where σ_X^2 is test score variance, σ_R^2 is relevant score variance, σ_I^2 is systematic error variance, and σ_E^2 is variance due to chance factors. A valid test is one "that (1) predicts future performance on appropriate variables (criterion validity), (2) means an appropriate domain (content validity), or (3) measures appropriate characteristics of test takers (construct validity)" (Friedenberg, 1995, p. 221). Thus this researcher employed three types of validity measures: content, criterion-related, and construct. The researcher also used SPSS to examine validity.

Criterion validity. According to Friedenberg (1995), criterion validity "is the ability of a test to predict performance on another measure" (p. 94). This type of validity is important when making decisions about future performance. In the case of this study, if the MPSDB scale designed has criterion validity, the scale would predict a relevant criterion measure, such as grade point average (GPA). This type of validity is sometimes referred to as concurrent validity (DeVellis, 2012). This name is given based on the approach used to obtain the criterion validity. The criterion validity coefficient can be calculated, which represents the relationship between scores: the predictor and the criterion. This statistic is known as r_{xy}. r_{xy} indicates the relationship between predictor and criteria. In this concurrent validity study, the researcher determined the correlation between test scores and a current criterion measure using SPSS. Friedenberg (1995) stated that, theoretically, the proportion of interest in criterion validity is $\frac{\sigma_R^2}{\sigma_X^2}$ (p. 227). The square of the coefficient, ($r_{xy}{}^2$), is the coefficient of determination. This statistic indicates the proportion of variance in criterion scores predicted by test scores. The researcher used SPSS to correlate the scale scores with scores of overall GPA and mathematics' GPA, as those students who have productive dispositions and beliefs on problem solving potentially have correlations with these measures. The researcher ran correlation as this particular scale and correlation has not been explored before.

Content validity. Content validity consists of detailed domains of items included on the scale. A higher degree of interrelated reliability implies consensus and thus establishes content validity. According to DeVellis (2012), "a scale has content validity when its items are a randomly chosen subset of the universe of appropriate items" (p. 60). An expert in the field is required to judge whether the subset reflects the specific domain. This is why this researcher employed an expert panel when constructing items to determine if the items on the MPSDB scale were appropriate to mathematical problem-solving dispositions and beliefs. For this study the expert panel consisted of mathematics education professors as well as one experienced mathematics teacher with a specialist degree. The mathematics education professors that responded and provided feedback were from the University of Georgia and the University of Indiana. Each of the professors has published multiple influential studies in the mathematics education literature. In addition, the experienced teacher has presented at multiple state and national mathematics education conferences, such as GCTM. It is essential that differences in test scores, σ_X^2, reflect differences in domain relevant characteristics, σ_R^2. Although content validity seems to involve qualitative measures, Brown (1983) and Cronbach & Thorndike (1971) have suggested using statistical measures to support any conclusions made by the expert judges. This researcher used SPSS to analyze the content validity.

Brown's (1983) suggested creating a scale that judges would use to rate a particular scale. The degree of agreement among different judges would be viewed from a statistical perspective to determine the content validity of the test. Following the suggestion of Brown, the researcher created a rating form (see Appendix C). This form included four sections that asked experts to judge the following: relevancy of each item to the conceptual definition of mathematical problem solving as approached in a models-and-modeling perspective (high relevance = 3, moderate relevance = 2, low relevance = 1); realistic beliefs related to mathematical problem solving (very realistic = 3, realistic = 2, not realistic = 1); word choice with respect to its appropriateness to the target audience (very appropriate = 3, appropriate = 2, not appropriate = 1); and response format with respect to its relevance to the items (very relevant = 3, relevant = 2, not relevant = 1). The researcher then analyzed the ratings and compared them using SPSS.

Construct validity. According to Friedenberg (1995), a common procedure is "to correlate scores on the test with scores on another established test measuring the same construct" (p. 254). The correlation coefficient should be positive, establishing congruent validity. Performing measures of construct validity determined whether the MPSDB scale accurately measured mathematical problem-solving dispositions and beliefs. The researcher used congruent validity measures to determine whether the MPSDB scale measured what it intended to measure.

Incremental validity. According to Haynes and Lench (2003), incremental validity is the degree to which a measure explains or predicts some phenomena, relative to other measures" (p. 457). The main purpose in establishing incremental validity is to estimate the relative proportions of variance in the criterion variable that can be associated with variance in the new measures. The researcher established incremental validity to supplement the three traditional forms of validity described above. The analysis hierarchical linear regression is a data analytic strategy important to establishing incremental validity. The researcher performed hierarchical linear regression analysis to observe the degree to which the addition of a measure to one or more other measures increased predictive efficacy. Using SPSS, the researcher examined the coefficient of determination, R-square, using the f-test to examine the significance. The coefficient of determination highlighted the percent of variance explained by the variable added to the model. The transformation in R-square is more suitable than simply observing the raw correlation values because the raw correlations do not account for the intersection of the newly introduced measure and the existing measures (Haynes & Lench, 2003). In this study the measures consisted of prediction of mathematics class average, where GPA and MPSDB scale scores accounted for a large proportion of variance in mathematics class average. The use of the MPSDB scale was supported by incremental validity evidence. For example, the MPSDB scale was thought to be correlated with mathematics class average while GPA was also thought to be correlated with mathematics class average. Both measures appear as predictors of mathematics class average, but in fact GPA and MPSDB scores are correlated, so the researcher tested for how much predictive power came from the MPSDB scale when accounting for GPA. The incremental validity is indicated by the change in R-square, coefficient of determination, when GPA is included in the model. In this case, GPA accounted for 39.9% of the variance in mathematics class average and the combination of GPA plus MPSDB accounted for 43.1% of the variance in mathematics class average. Therefore, the MPSDB added 3.2% to our predictive power. The researcher used SPSS to calculate the significance of R-square by using the f-test. The value of the f-test determined that the MPSDB scale has incremental validity over using mathematics GPA alone to predict overall GPA.

Ethical Considerations

All research methods and necessary consent forms were approved by Kennesaw State University Institutional Review Board (IRB). IRB applications including student assent, parent consent forms, and teacher consent forms are included in Appendices I-K. In addition all county approval forms including county application, parent guardian permission form, and principal support form are included in Appendices L-N.

There are no known risks in this study. Individuals might benefit indirectly from participation in this study from sharing information about their mathematics dispositions and beliefs through the survey, or gaining personal satisfaction from participating in the study. This study may benefit society in that the knowledge gained could impact the local community as well as local institutions. Institutions benefit, for example, by receiving information to improve a preservice mathematics education program as a result of this study. The local community could benefit through improved public educational programs arising based on data about students' dispositions and beliefs involved in mathematics problem solving.

The parental consent form was sent home by the researcher to obtain signatures of parents, guardians, and authorized representatives. The consent form included the child assent statement. In addition, the first page of the survey included an additional assent statement and note of voluntary participation. The researcher stored signed parental consent forms in a locked cabinet in a locked office in a locked building. The students' instructor was not the one administering the survey, therefore reducing any perception of coercion. The survey for students did not ask for any identifiable information. Signed informed consents were not used as the identifying link to the research data and did not contain participant ID numbers nor were they filed with other research data files. The survey for teachers asked for given ID numbers that were linked to that teacher. The signed consent forms as well as the surveys were held in a locked file cabinet, in a locked room, in a locked building. To ensure confidentiality, these ID numbers will not be given out at any time.

The researcher stored data on a computer and encrypted the data to prevent unintentional breaches of security. Digital files were password protected. Sensitive data was also encrypted, stored, and securely erased at the appropriate time.

Chapter 4: Findings

The aim of this study was to develop an instrument to measure mathematical problem-solving beliefs and dispositions (MPSBD) among young adolescents and to assess the initial reliability and validity of the instrument. The researcher developed a 40-item instrument and tested it using a sample of sixth, seventh, and eighth graders (n=575). Chapter 4 describes the results of the scale development study, including: qualitative data collected through expert review and a student feedback group; pilot testing of the initial items; and final results corresponding to the study research objectives.

The results are organized by research phases. Phase I included construct clarification, description of the reference population, and an explanation of how the preliminary specifications for initial item generation was derived. This phase also included completion of item generation/modification based on expert panel review and student feedback group. Phases II and III included the pilot testing and final scale administration, respectively. Statistical analyses included EFA to identify the underlying dimensions of mathematical problem-solving beliefs and dispositions as assessed by the MPSBD instrument. The researcher examined reliability was using Cronbach alpha, and examined validity by correlating the MPSBD scores with other related constructs and predictive measures.

Phase I Item Development

Initial Item Generation

Initial item generation included a review of the literature in an effort to obtain background information on mathematical problem-solving beliefs and dispositions among adolescents and to identify existing instruments designed to measure these types of dispositions and beliefs. As described in Chapters 2 and 3, the researcher used a models-and-modeling perspective on problem solving to guide the structure and generation of the initial MPSDB items.

Expert Panel Review

Nine of 29 invited experts agreed to participate in the study and provided feedback through Survey Monkey and email communication. Experts were asked to provide feedback on the scales with regards to: 1) how relevant each item was to the intended construct presented; 2) how realistic each situations was to the intended population of middle school students; 3) the word choice for the scale and its appropriateness to the target population; and 4) response category. In general, four experts felt that items were confusing and needed clarification with regards to definition of each construct. Through email communication with the experts it was determined that items including "math" and "mathematics" were mainly considered the source of confusion when referencing mathematical problem solving. Experts suggested using terms consistently on each scale. Based on this initial assessment, the researcher revised all items to better reflect and define the intended constructs of *mathematical mindset*, *problem-solving perseverance*, *mathematical revision*, *mathematical communities of practice, utility of problem-solving*, and *problem-solving processes*. For example, an original item from the problem solving perseverance scale stated "When doing a math problem, I stay committed until I can develop a solution to the problem." After a professor at Indiana State University said the items needs to stay consistent with problem solving to ensure the items are measuring the same construct the item was changed to "When problem solving, I stay committed until I can develop a solution to the problem." Items better measured the constructs when terminology remained consistent.

Relevance. To judge the relevance of the items to the constructs, response options included high relevance, relevant, and low relevance. Most experts judged items in each of the six scales as highly or moderately relevant. One expert did not consider item 12 (“If I can’t seem to solve a math problem, I feel upset because it reminds me how I was not born smart at math”), which involved mathematical mindset, as relevant and the researcher removed this item from the scale on the MPSDB prior to the preliminary administration of the MPSDB. In general, relevancy ratings for items were the same or very similar across all items and thus retained for the preliminary scale. Thus, there was a high degree of agreement among the experts.

Realistic. The researcher asked the experts how realistic each situation would be to the intended population of middle school students. Response options included very realistic, realistic, and not realistic. For the most part, comments and suggestions made in response to the relevancy questions were reiterated and/or referenced when experts rated how realistic items were. Most experts considered the items to be realistic for the population of students. However, one expert considered item 2 (“ In order to problem solve, a list of steps needs to be given to me”), and item 6 (“When assigned mathematical problem-solving tasks, I wait to be told how to start the problem”), which involved process, not realistic and the researcher therefore removed these items prior to the preliminary scale administration. Again, realistic ratings were the same or very similar across all items, and thus there was a high degree of agreement among the expert panel.

Word choices. The researcher asked experts to rate the word choice for the scale and its appropriateness to the target population. Although most experts felt the word choice was very appropriate, one expert suggested changing the word “persevere” to “keep working” as this would be better understood by a sixth grader. In addition, another expert suggested that the statement “I evaluate my solutions” also contain the word “refine” so that sixth- and seventh-grade students would have a clear picture of what “evaluation” means in the context of this particular statement. Thus, the researcher adjusted item 11 involving problem-solving perseverance and item 1 involving mathematical revision. All experts rated word choice as very appropriate, thus establishing a high degree of agreement.

Response category. Although most of the experts rated the proposed six-point response format as appropriate, one of the experts felt that this format was not the best choice for measurement among sixth graders. This expert felt that a smaller number of labeled options for young respondents would result in more accurate findings. Eight experts suggested using a six-point scale, and one expert suggested using a four-point scale. The researcher used a six-point scale (strongly disagree, somewhat disagree, slightly disagree, slightly agree, somewhat agree and strongly agree) for the final set of pilot items. The final number of response options and labels were largely based on expert feedback. Eight out of nine experts rated the format as very relevant, again establishing a high degree of agreement among the judges.

Feedback Group

The feedback group involved a qualitative discussion with sixth-, seventh-, and eighth-grade students (n=16). The researcher identified common themes by reviewing the observer notes. One third of students expressed concerns and questions about two of the items during the feedback session. In response, the researcher revised these two items before the pilot administration of the MPSDB. The first item that generated discussion was item number 8 involving problem-solving perseverance. Students were confused that the item only specified a five-minute time limit to problem solve. One common theme that arose involved students asking what they should put if they were willing to work for ten minutes rather than five minutes. Based on the conceptual definition of a models-and-modeling perspective and to account for student confusion, the researcher revised this item to say “in a single setting,” which better reflected a models-and-modeling perspective on problem solving and accounted for student clarification. Another theme that arose with the sixth- and seventh-grade students was what GPA represented. Although eighth graders knew what their overall GPA was, sixth and seventh graders had to be reminded of the idea of averages. Thus, before administering the pilot and final MPSDB scale, teachers taught students how to find their GPA using all their grades in their individual classes. In response to the follow-up questions (see Appendix D), respondents offered replies that indicated students were clear about what individual items were asking.

Phase II EFA Results From the Pilot Administration

The researcher conducted exploratory factor analysis (EFA) to analyze the pilot scale administration (n=150). To confirm the factor structures of each preliminary scale, the researcher also conducted separate factor analysis for each of the individual scales. Several well-recognized criteria for the factorability were used in each of the EFA: correlations between each pair of the items($r > .3$); Kaiser-Meyer-Olkin measure of sampling adequacy (>.6), and Bartlett's test of sphericity ($p<.05$). After checking these criteria, the researcher conducted EFA with principle-axis factor extraction to determine the factor structure of the MPSDB. Table 1 summarizes the EFA results for the six scales created to measure dispositions and beliefs in the models-and-modeling mathematical problem-solving context.

Mathematical Mindsets

Initially, the researcher examined the factorability of the 11 mathematical mindset items and conducted an EFA using the principle-axis factor extraction to determine the factor structure of the MPSDB. Because beliefs do not function independently of one another, the researcher used oblique direct oblimin rotation to clarify the structure of the MPSDB. The initial eigenvalues showed that the first factor explained 40.474% of the variance, and the second factor explained 7.630 % of the variance.

Table 1

Summary Results of EFA for Pilot Study (n=150)

Scales	Number of Original Items	Number of Factors Emerged from EFA	Amount of Variance Explained by Largest Factor	Number of Items Kept for Final Scale Administration	Mean	SD	Cronbach's Alpha
Mathematical mindsets	11	2	40.474	8	5.025	6.770	.878
Problem-solving perseverance	12	1	46.302	12	4.117	12.355	.907
Mathematical revision	16	2	32.432	11	4.240	10.721	.879
Mathematical communities of practice	12	1	33.256	7	4.348	7.062	.783
Problem-solving utility	10	2	44.317	8	4.800	8.723	.874
Problem-solving processes	11	3	24.323	6	3.935	5.789	.738

The one factor solution was preferred because of its previous theoretical support, and the insufficient number or primary loadings and difficulty of interpreting subsequent factors. Eight items met the minimum criterion of having a primary factor loading of .4 or above, and no cross loading of .4 or above. A total of three items were eliminated because they did not contribute to a simple factor structure or had an insufficient number or primary loadings. The factor loadings of each item are reported in Appendix O. The researcher used Cronbach's alpha to examine internal consistency for the mathematical mindset scale. The alpha was strong at .847. Reliability and scale statistics are presented in Table 1.

Mathematical Problem-Solving Perseverance

Initially, the researcher examined the factorability of the 12 perseverance items and conducted an EFA using the principle-axis factor extraction to explain one factor relating to mathematical problem-solving perseverance for the MPSDB. The initial eigenvalue showed that one factor explained 46.302% of the variance. A total of twelve items were retained because they each contributed to a simple factor structure and met the minimum criterion of having a primary loading of .4 or above. The factor loadings of each item are reported in Appendix O.

The researcher used Cronbach's alpha to examine internal consistency for the mathematical problem-solving perseverance scale. The alpha was strong at .907. Reliability and scale statistics are presented in Table 1.

Mathematical Revision

Initially, the researcher examined the factorability of the 16 mathematical revision items and conducted an EFA using the principle-axis factor extraction to determine the factor structure of the MPSDB. Because beliefs do not function independently of one another, the researcher used oblique direct oblimin rotation to clarify the structure of the MPSDB. The initial eigenvalues showed that the first factor explained 32.432% of the variance, and the second factor explained 6.483 % of the variance.

The one factor solution was preferred because of its previous theoretical support, and the insufficient number or primary loadings and difficulty of interpreting subsequent factors. Eleven items met the minimum criterion of having a primary factor loading of .4 or above, and no cross loading of .4 or above. A total of five items were eliminated because they did not contribute to a simple factor structure or had an insufficient number of primary loadings. The factor loadings of each item are reported in Appendix O. The researcher used Cronbach's alpha to examine internal consistency for the mathematical revision scale. The alpha was strong at .879. Reliability and scale statistics are presented in Table 1.

Mathematical Communities of Practice

Initially, the researcher examined the factorability of the 12 communities of practice items and conducted an EFA using the principle-axis factor extraction was to explain one factor relating to mathematical communities of practice for the MPSDB. The initial eigenvalue showed that one factor explained 33.256% of the variance. Seven items met the minimum criterion of having a primary factor loading of .4 or above. A total of five items were eliminated because they did not contribute to a simple factor structure and had an insufficient number of primary loadings. The factor loadings of each item are reported in Appendix O.

The researcher used Cronbach's alpha to examine internal consistency for the mathematical communities of practice scale. The alpha was strong at .783. Reliability and scale statistics are presented in Table 1.

Problem-Solving Utility

Initially, the researcher examined the factorability of the 10 mathematical problem-solving utility items and conducted an EFA using the principle-axis factor extraction to determine the factor structure of the MPSDB. Because beliefs do not function independently of one another, the researcher used oblique direct oblimin rotation to clarify the structure of the MPSDB. The initial eigenvalues showed that the first factor explained 44.317% of the variance, and the second factor explained 10.465 % of the variance.

The one factor solution was preferred because of its previous theoretical support, and the insufficient number or primary loadings and difficulty of interpreting subsequent factors. Eight items met the minimum criterion of having a primary factor loading of .4 or above, and no cross loading of .4 or above. A total of two items were eliminated because they did not contribute to a simple factor structure and had insufficient number of primary loadings. The factor loadings of each item are reported in Appendix O. The research used Cronbach's alpha to examine internal consistency for the utility of problem-solving scale. The alpha was strong at .874. Reliability and scale statistics are presented in Table 1.

Problem-Solving Processes

Initially, the researcher examined the factorability of the 11 mathematical problem-solving process items and conducted an EFA using the principle-axis factor extraction to determine the factor structure of the MPSDB. Because beliefs do not function independently of one another, the researcher used oblique direct oblimin rotation to clarify the structure of the MPSDB. The initial eigenvalues showed that the first factor explained 24.323% of the variance, and the second factor explained 11.389 % of the variance and the third factor explained 4.765% of the variance.

The one factor solution was preferred because of its previous theoretical support, and the insufficient number or primary loadings and difficulty of interpreting subsequent factors. Six items met the minimum criterion of having a primary factor loading of .4 or above, and no cross loading of .4 or above. A total of five items were eliminated because they did not contribute to a simple factor structure and had insufficient number of primary loadings. The factor loadings of each item are reported in Appendix O. The researcher used Cronbach's alpha to examine internal consistency for the mathematical process scale. The alpha was strong at .738. Reliability and scale statistics are presented in Table 1.

Phase III EFA Results from the Final Scale Administration

The researcher conducted EFA again to analyze the data from the final scale administration (n=575). The researcher conducted separate factor analysis for each of the individual scales to confirm the factor structures of each scale. The researcher also used several well-recognized criteria for the factorability in each of the EFA: correlations between each pair of the items($r > .3$); Kaiser-Meyer-Olkin measure of sampling adequacy (>.6); and Bartlett's test of sphericity ($p<.05$). After checking these criteria, the researcher conducted EFA with principle-axis factor extraction to determine the factor structure of the MPSDB. Table 2 summarizes the EFA results for the six scales created to measure dispositions and beliefs in the models-and-modeling mathematical problem-solving context.

Mathematical Mindset

Initially, the factorability of the eight mathematical mindset items was examined. An EFA using the principle-axis factor extraction was conducted to determine the factor structure of the MPSDB. Because beliefs do not function independently of one another, oblique direct oblimin rotation was used to clarify the structure of the MPSDB. The initial eigenvalues showed that the first factor explained 45.485% of the variance, and the second factor explained 7.277% of the variance.

The one factor solution was preferred because of its previous theoretical support, and the insufficient number or primary loadings and difficulty of interpreting subsequent factors. All items met the minimum criterion of having a primary factor loading of .4 or above, and no cross loading of .4 or above. A total of eight items were retained because they contributed to a simple factor structure and met the minimum criterion of having a primary loading if .4 or above. The factor loadings of each item are reported in Table 3.

Table 2

Summary Results of EFA for Final Scale Administration

Scales	Number of Items in Final Scale Administration	Number of Factors Emerged from EFA	Amount of Variance Explained by Largest Factor	Number of Items for Further Analysis	Mean	SD	Cronbach's Alpha
Mathematical mindsets	8	2	45.485	8	5.057	6.309	.847
Problem-solving perseverance	12	2	44.998	6	4.21	6.245	.852
Mathematical revision	11	1	42.794	11	4.306	10.791	.889
Mathematical communities of practice	7	2	35.273	5	4.502	5.033	.757
Problem-solving utility	8	2	46.598	5	4.706	5.602	.820
Problem-solving processes	6	1	35.051	5	4.250	5.187	.749

Table 3

Factor loadings and Communalities Based on EFA with Direct Oblimin Rotation for 8 items from the Mathematical Mindset Scale (N=575)

Item	Factor 1 Mathematical Mindset
By trying hard, I can become better at math.	.568
Hard work can increase my ability in math.	.550
The more mathematics I learn, the more my math ability grows.	.590
The harder I try, the better I can be at math.	.592
I learn from making mistakes in math, which pushes me to work harder next time.	.668
I will never be good at math.	.497
I get better in math because I learn more every year.	.785
If I can't seem to solve a math problem, I work harder and try new strategies.	.683

Note: Factor loadings <.4 are suppressed

Internal consistency for the mathematical mindset scale was examined using Cronbach's alpha. The alpha was strong at .847. Reliability and scale statistics are presented in Table 2.

Mathematical Problem-Solving Perseverance

Initially, the researcher examined the factorability of the 12 mathematical perseverance items. The researcher also conducted an EFA using the principle-axis factor extraction to determine the factor structure of the MPSDB. Because beliefs do not function independently of one another, the researcher used oblique direct oblimin rotation to clarify the structure of the MPSDB. The initial eigenvalues showed that the first factor explained 44.998% of the variance, and the second factor explained 6.992% of the variance.

Table 4

Factor Loadings and Communalities Based on EFA with Direct Oblimin Rotation for 12 Items from the Mathematical Problem-solving Perseverance Scale (N=575)

Item	Factor 1 Mathematical Problem-solving Perseverance	Factor 2 Mathematical Problem-solving Perseverance
If I have difficulty problem solving, I keep working and do my own research to figure a solution out.		.765
When problem solving, I stay committed until I can develop a solution to the problem.		.765
After being assigned a challenging math task, I keep working to find solutions.		.728
Even if I don't know how to solve the problem or feel like I don't have enough information, I stick with it to develop a solution.		.613
I am willing to try several times before I find solutions to math tasks.		.652
I am willing to work as long as it takes when problem solving.		.626
If I become frustrated while problem solving, I usually stop trying.	.887	
If I can't find a solution to a problem-solving task in a single setting I stop looking for a solution.	.765	
I give up after my first few attempts to find solutions to math tasks.	.599	
If I can't develop a solution to a math tasks in a few minutes I usually stop trying	.763	
Problem solving takes too long to complete	.571	
I am unwilling to spend more than five minutes finding solutions to math tasks	.693	

Note: Factor loadings <.4 are suppressed

The one factor solution was preferred because of its previous theoretical support and the primary loadings. Six items met the minimum criterion of having a primary factor loading of .4 or above, and no cross loading of .4 or above. A total of six items were eliminated because they did not contribute to a simple factor structure and many recoded items that caused difficulty interpreting subsequent factors. The factor loadings of each item are reported in Table 4.

The researcher used Cronbach's alpha to examine internal consistency for the mathematical problem-solving perseverance scale. The alpha was strong at .852. Reliability and scale statistics are presented in Table 2.

Mathematical Revision

The researcher examined the factorability of the 11 mathematical revision items. The researcher also conducted an EFA using the principle-axis factor extraction to explain one factor relating to mathematical revision for the MPSDB. The initial eigenvalue showed that one factor explained 42.794.432% of the variance.

All items met the minimum criterion of having a primary factor loading of .4 or above. A total of 11 items were retained because they contributed to a simple factor structure and met the minimum criterion of having a primary loading if .4 or above. The factor loadings of each item are reported in Table 5.

The researcher used Cronbach's alpha to examine internal consistency for the mathematical revision scale. The alpha was strong at .889. Reliability and scale statistics are presented in Table 2.

Table 5

Factor Loadings and Communalities Based on EFA for 11 Items from the Mathematical Revision Scale (N=575)

Item	Factor 1 Mathematical Revision
When completing a problem-solving task, I evaluate and refine my solutions.	.723
I reflect on the appropriateness of my solutions.	.657
It is important to find alternative solutions when problem solving.	.599
When creating solutions to problem-solving tasks, I think about whether or not my solution can be used in a similar situation.	.550
If my solution is not working I am willing to revise my thinking.	.680
I find value in testing out my solution.	.719
Once I have solved a problem, I evaluate how it is working.	.723
When problem solving, revising my solutions creates a better model that applies to the real world.	.678
When solving real life problems, I improve my solutions as I gain additional knowledge, even if I have already found an answer.	.644
When problem solving, understanding how I developed a solution is more important than the fact that I actually have a solution.	.528
In addition to creating a solution, it is important to know why the solution works.	.661

Note: factor loadings <.4 are suppressed

Mathematical Communities of Practice

Initially, the researcher examined the factorability of the seven mathematical communities of practice items. The researcher also conducted an EFA using the principle-axis factor extraction to determine the factor structure of the MPSDB. Because beliefs do not function independently of one another, the researcher used oblique direct oblimin rotation to clarify the structure of the MPSDB. The initial eigenvalues showed that the first factor explained 35.273% of the variance, and the second factor explained 11.690% of the variance.

The one factor solution was preferred because of its previous theoretical support, and the insufficient number or primary loadings and difficulty of interpreting subsequent factors. Five items met the minimum criterion of having a primary factor loading of .4 or above. Although one item ("It's better to work with a team of people than alone") had a cross loading of .4 or above, it was retained based on the theoretical support and expert feedback. A total of two items were eliminated because they did not contribute to a simple factor structure and had insufficient number or primary loadings. The factor loadings of each item are reported in Table 6.

Table 6

Factor Loadings and Communalities Based on EFA with Direct Oblimin Rotation for 7 Items from the Mathematical Communities of Practice Scale (N=575)

Item	Factor 1 Mathematical Communities of Practice	Factor 2 Mathematical Communities of Practice
When problem solving, I value other people's input when creating solutions.	.753	
When problem solving, I find my peers' input to be helpful.	.862	
When comparing solutions, I compare each possible solution with my peers' solutions to find the best one.	.577	
It's better to work with a team of people than alone.	.477	.744
When working on a problem-solving task, it is important to describe my thinking to others.	.469	

Note: Factor loadings <.4 are suppressed

The researcher used Cronbach's alpha to examine internal consistency for the mathematical communities of practice scale. The alpha was strong at .757. Reliability and scale statistics are presented in Table 2.

Problem-Solving Utility

The researcher examined factorability of the eight mathematical problem-solving utility items. The researcher also conducted an EFA using the principle-axis factor extraction to determine the factor structure of the MPSDB. Because beliefs do not function independently of one another, the researcher used oblique direct oblimin rotation to clarify the structure of the MPSDB. The initial eigenvalues showed that the first factor explained 46.598% of the variance, and the second factor explained 8.770 % of the variance.

The one factor solution was preferred because of its previous theoretical support, and the insufficient number or primary loadings and difficulty of interpreting subsequent factors. Four items met the minimum criterion of having a primary factor loading of .4 or above. One item ("When I am older, I don't plan on having a job that requires mathematical problem solving") had a cross loading of .4 or above but was retained based on theoretical support and expert feedback. A total of three items were eliminated because they did not contribute to a simple factor structure and had insufficient number of primary loadings. The factor loadings of each item are reported in Table 7.

The researcher used Cronbach's alpha to examine internal consistency for the mathematical problem-solving utility scale. The alpha was strong at .820. Reliability and scale statistics are presented in Table 2.

Table 7

Factor Loadings and Communalities Based on EFA with Direct Oblimin Rotation for 8 Items from the Mathematical Problem-solving Utility (N=575)

Item	Factor 1 Mathematical Problem-solving Utility	Factor 2 Mathematical Problem-solving Utility
I'll need to know problem-solving skills for my future job.	.551	
When I am older, I don't plan on having a job that requires mathematical problem solving.	.186	.403
Working on problem-solving tasks in math class will help me in the future.	.932	
Math is a worthwhile subject to learn because it teaches me problem solving.	.828	
I will use mathematical problem solving as an adult.	.690	
My job one day will not involve problem solving.		.632
I will never use mathematical problem solving after I graduate high school.		.791
Problem solving will not be important for my life.		.804

Note: Factor loadings <.4 are suppressed

Problem-Solving Processes

The researcher examined factorability of the six mathematical problem-solving process items, and conducted an EFA using the principle-axis factor extraction to one factor relating to *problem-solving processes* for the MPSDB. The initial eigenvalues showed that one factor explained 35.051% of the variance.

Five items met the minimum criterion of having a primary factor loading of .4 or above. A total of one item was eliminated because it did not contribute to a simple factor structure and failed to meet the minimum criterion of having a primary loading of .4 or above. The factor loadings of each item are reported in Table 8.

The researcher used Cronbach's alpha to examine internal consistency for the mathematical problem-solving process scale. The alpha was strong at .749. Reliability and scale statistics are presented in Table 2.

Table 8

Factor Loadings and Communalities Based on EFA for 6 Items from the Mathematical Problem-solving Process (N=575)

Item	Factor 1 Mathematical Problem-solving Process
An important part of problem solving is developing my own steps to find answers.	.609
I develop my own procedures when problem solving.	.757
When problem solving, I often create a formula for myself.	.630
When given a problem-solving task, I first identify what the goal is.	.567
Being creative is important when problem solving.	.512

Note: factor loadings <.4 are suppressed

Content Validity

In addition to the expert panel's qualitative comments and feedback, the researcher performed correlation between the expert judges' ratings was done to establish content validity.

A high degree of reliability was found between expert judge's relevance measurements. The average measure ICC was .931 with a 95% confidence interval from .905 to .952 (F (74,592) = 14.559, p<.001). In addition, a high degree of reliability was found between expert judge's realistic measurements. The average measure ICC was .895 with a 95% confidence interval from .855 to .927 ($F (74,592) = 9.534$, $p<.001$).

Construct Validity

The researcher correlated MPSDB mean scale scores with May's MSEAQ mean scale scores as well as with Fennema-Sherman *Usefulness of mathematics* mean scale score to establish construct validity. *Mathematical mindset* was strongly correlated to high self-efficacy and low anxiety, while *mathematical mindset* was moderately correlated to the *usefulness of mathematics*. There was a significant correlation between these variables. *Mathematical problem-solving perseverance* was both moderately correlated to *self-efficacy and anxiety* and the *usefulness of mathematics*. There was a significant correlation between these variables. *Mathematical revision* was both moderately correlated to *self-efficacy and anxiety* and the *usefulness of mathematics*. There was a significant correlation between these variables. Mathematical communities of practice produced weak correlations to *self-efficacy and anxiety* and the *usefulness of mathematics*. There was a significant correlation between these variables. Mathematical *problem-solving utility* was moderately correlated to *self-efficacy and anxiety*, while mathematical *problem-solving utility* was strongly correlated to the *usefulness of mathematics*. There was a significant correlation between these variables. Mathematical *problem-solving processes* was moderately correlated to *self-efficacy and anxiety*, while mathematical *problem-solving processes* was weakly correlated to the *usefulness of mathematics*. There was a significant correlation between these variables. Correlations are presented in table 9. Overall MPSDB mean scores correlated positively with May's (2009) MSEAQ and Fennema-Sherman's (1976) Mathematical Usefulness Scale. According to Evans (1996) correlations were strong as they were between .6-.79.

Table 9

Summary of Correlation Coefficients for MPSDB, MSEAQ, Fennema-Sherman Mathematics Attitudes Scale, GPA, and Math Class Average

V	M	P	R	CP	U	PR	MSE	FS	GPA	MA	O
M											
P	.656										
R	.642	.725									
CP	.387	.360	.509								
U	.531	.491	.523	.325							
PR	.469	.547	.601	.402	.397						
MSE	.632	.540	.490	.248	.448	.431					
FS	.568	.497	.512	.281	.748	.375	.501				
GPA	.188	.253	.222	.104*	.214	.246	.312	.243			
MA	.269	.301	.262	.083*	.245	.255	.389	.293	.631		
O	.805	.835	.908	.622	.676	.725	.606	.631	.260	.305	
Means	5.057	4.212	4.289	4.504	4.697	4.245	4.581	4.926	3.233	5.204	4.491
SDs	.785	1.038	.997	1.005	1.120	1.037	1.144	1.107	.606	1.451	.73

*Note. M = Mathematical mindset, P = Mathematical problem-solving perseverance, R = Mathematical revision, CP = Mathematical communities of practice, U = Problem-solving utility, PR = Problem-solving processes, O= Overall MPSDB mean score, MSE = May's MSEAQ, FS = Fennema-Sherman Attitude Scale, GPA= Grade point average, MA= Math class average. Significance of correlation is noted as *p <.05, all others significant at the p <.01.*

Criterion Validity

The researcher correlated MPSDB mean scale scores to overall GPA as well as mathematics class average in order to establish criterion validity. All correlations between the MPSDB mean scores and both GPA and mathematics class average were positive. Using Evans (1996) criteria, the researcher determined all correlations of mean scale scores were weak except mathematical communities of practice that produced very weak correlations. Correlations are presented in Table 9. Overall MPSDB scores had positive correlations with GPA and mathematics class average. According to Evans (1996) criteria, correlations were weak as they were between .2-.39.

Incremental Validity

To examine the question of incremental validity of the student dispositions and beliefs in relation to the GPA in predicting mathematics class average, the researcher performed a hierarchical regression analysis in which the GPA was allowed to enter in a set, followed by the six factors of the MPSDB measures which were allowed to enter stepwise in the second set. As can be seen in Table 10, the results of this analysis revealed that GPA significantly entered the equation to predict mathematics class average with a multiple correlation of $R = 0.631$ ($p < 0.01$). In the second step, MPSDB significantly entered the equation producing a multiple correlation of $R = 0.657$ ($p < 0.01$). The GPA accounted for 39.9% of the variance in mathematics class average, with MPSDB adding an additional 3.2% of the variance in mathematics class average.

Table 10

Results of Hierarchical Linear Regression Analysis with Six MPSDB Constructs Predicting GPA

			Dependent Variable: Math Class Average	
Step	Variable	Multiple R	R squared/Cohen's Effect size for multiple Regression	R squared change
1	GPA	.631	.399	.399*
2	Six MPSDB constructs	.657	.431	.032*

*Note: n= 575, *p<.001*

As can be seen again through ANOVA testing, the six MPSDB constructs significantly added predictive capacity. The ANOVA shows that this regression model is significantly better when MPSDB scores are added to the model than GPA alone. Table 11 presents the results of ANOVA.

Table 11

Results of ANOVA for GPA and Six Constructs of MPSDB Predicting for Math Class Average

	df	*F*	*p*
Model 1			
Regression	1	379.792	.000
Residual	573		
Model2			
Regression	7	61.365	.000
Residual	567		

Note: n =575, significant at the p<.001 level.

The researcher employed a sequential multiple regression analysis to predict mathematics class average. On the first step GPA was entered into the model. It was significantly correlated with mathematics class average, as shown in Table 11. On the second step all of the remaining predictors were entered simultaneously, resulting in a significant increase in R^2, F (1, 573) = 379.792, p < .001. The full model R^2 was significantly greater than zero, F (7, 567) = 61.365, p < .001.

Student ANOVA Results

The researcher collected student MPSDB scale scores from three different groups (sixth, seventh, and eighth graders). The mean MPSDB score for the students in sixth grade group was 4.644(SD= .769), the seventh grade group was 4.471(SD= .798), and the eighth grade group was 4.399(SD=.710). The researcher conducted one-way ANOVA in SPSS software (version 22.0) to test the mean difference among these three groups, and the results revealed grade level does not have a significant effect on student scale scores, F(2,572)= 3.764, p>.001. The Games-Howell post hoc test revealed that the differences between each pairs of either two of the groups (sixth vs. seventh, seventh vs. eighth, and sixth vs. eighth) are all not significantly different (p>.001).

The effect size of the difference is .013, which indicates the grade level of students has small effects on student MPSDB scores.

Teacher Data

The researcher computed the correlation coefficient to assess the relationship between the student MPSDB mean scores and their mathematics teacher beliefs and frequency of MEA exposure in their mathematics class. There was a positive correlation between the MPSDB mean scores and teacher beliefs, $r = 0.018$, $n = 575$, $p < 0.05$. Table 12 summarizes the results. There was again a positive correlation between the MPSDB mean scores and teacher frequency of using MEAs, $r = .038$, $n=575$, $p<.05$.

Table 12

Summary of Correlation Coefficients for MPSDB scores, Teacher Beliefs Scores, and Frequency of MEAs Used Based on Teacher Survey Data

	MPSDB mean score	Teacher Beliefs	Frequency of MEAs
MPSDB mean score	1		
Teacher Beliefs	.018	1	
Frequency of MEAs	.038	.462	1

Note: n= 575, p<.05

The correlations found between MPSDB scores and teachers' beliefs as well as frequency of MEAs are very weak and consistent with Evans (1996) criteria.

Chapter 5: Discussion, Conclusions, and Implications

As stated previously, if problem solving is going to remain an important, viable aspect of mathematics, then its various components must be rigorously and thoroughly examined and more adequately understood. The current study has endeavored to do just that, by developing a self-reporting MPSDB scale that is theoretically sound, and has demonstrated its reliability and validity. This study has offered a way to explicitly assess aspects of mathematical problem solving with a self-report scale. The original MPSDB scale had 75 items. The revised 40-item scale showed acceptable reliability and some indication of being a valid assessment of mathematical problem-solving dispositions and beliefs.

Expert Panel Review and Feedback Session

Through expert panel review, the researcher was able to delete three items from the original MPSDB scale. One expert rated the item, "If I can't seem to solve a math problem, I feel upset because it reminds me how I was not born smart in math," as non-relevant when it came to mathematical mindset, and argued it was more related to ideas in literature about mathematical concept. The expert cautioned against using this item as a math concept as it has been confused with numerous definitions of mindset. This confirmed the findings of Marsh, Walker, & Debus (1991), who argued that the math concept is confounded by varying imprecise factors. Therefore, the researcher deleted this item. In addition, two items, "In order to problem solve, a list of steps needs to be given to me" and "When assigned mathematical problem-solving tasks, I wait to be told how to start the problem," which related to mathematical problem-solving process, were rated as non-realistic based on the idea that these items were more related to the theories of monitoring and self-regulation. The item, "when assigned problem-solving tasks, I wait to be told how to start the problem," involved aspects of metacognition and thus is more related to ideas of monitoring and self-regulation (Schoenfeld, 2011). Based on expert review, the researcher removed both of these items prior to any scale administration.

The researcher revised all items based on qualitative expert feedback. For instance, a professor at both the University of Georgia and University of Indiana at Bloomington pointed out that the wording of each scale needed to be consistent. For instance, items in the mathematical mindset scale referred to "math" and "math problem solving." The researcher revised these and similar items to reflect accurate wording based on construct definitions. This type of revision is supported by Clark & Watson (1995), who proposed that a key in scale development lies in conceptualizing target definitions for constructs by ensuring precise and consistent meanings. Items were also revised prior to administering the preliminary scale based on qualitative student feedback. A third of students during feedback sessions discussed item 8, as they were unclear about the five-minute time limit explained in the item. The new item used the word "single setting" to clarify what the item was indeed referring to. In addition, sixth and seventh grade students received instruction on how to calculate GPA prior to the final scale administration, as a third of students were unclear about the last question on the MPSDB.

Factor Analysis

After preliminary MPSDB scale administration, factor analysis techniques enabled an additional 20 items to be eliminated. One reason for elimination was that items did not meet the minimum requirements of having a magnitude loading of .4 or higher for each factor (Costello& Osborne, 2005). There was one exception to this rule of thumb in the mathematical problem-solving utility scale, as the item that stated "when I am older I don't plan on having a job that requires mathematical problem solving" had a loading of .186, and cross loading to factor 2 of .403. However, based on theoretical underpinnings, and the fact that strong factors need to retain a minimum of five items, this item was retained in the final scale. In addition, Kloosterman & Stage (1992) argued that problem-solving skills are often more important than computation skills later in life. Another reason for elimination was the scope of this study, as only one factor was examined for each construct. Although each construct may have contained additional factors, this study focused on examining one factor for each of the six scales making up the MPSDB scale. After final MPSDB scale administration, factor analysis techniques, the researcher reduced the final version of the MPSDB to 40 items.

The very nature of exploratory factor analysis (EFA) formulating and validating a scale, is at best, a method that allows for a preliminary outcome. There may be other factors that affect the outcome. Tucker and MacCallum (1997) suggested that researchers should anticipate issues and potentially more factors than fits the scope of one study. Tucker and MacCallum (1997) further stated,

> The achievement of the objective of factor analytic research requires a series of studies, proceeding from initial studies where hypotheses are only loosely formed and analyses are exploratory, to final studies where confirmatory analyses are conducted to test well-developed hypotheses. (p.132)

To guard against this, the researcher employed an organized and guarded procedure for item generation and analysis. This included a comprehensive review of the literature (including related instruments), consultation with both mathematics education and secondary education experts (including professors at multiple universities), structured factor analysis sessions, and pilot testing, with attention to mathematical beliefs and problem-solving theories. However, future researchers might consider confirmatory analyses to test what has already been done in this study.

Although the 6-factor MPSDB scale that emerged from this study is theoretically and empirically plausible, additional studies are required to further explore, and possibly to confirm this structure and its psychometric properties. In future studies, researchers might consider performing confirmatory factor analysis to further examine the factor structure within the MPSDB scale.

Reliability

After initial factor analysis, the researcher used Cronbach's alpha to determine the reliability of each scale. Following Cronbach (1951), all six scales, except *mathematical problem-solving processes* and *mathematical communities of practice*, had "good" internal consistencies as their alpha value was between .8-.9, and *mathematical problem-solving process* and *mathematical communities of practice* still had alpha values (a= .749 and a= .757) that are acceptable. This is further supported by George & Mallery (2003), in that the alpha values found in this study fell into the "good" range. In fact, in the social sciences, it is accepted that anything higher than .9 would be unrealistic. Given the reliability results for each subscale, each of the six scales should be included in the overall MPSDB scale.

Validity

The researcher collected four types of validity evidence to answer the major research objectives that were raised to determine if the construct of mathematical problem-solving dispositions and beliefs was measurable. The validity framework presented in Chapter 3 guided the researcher in reaching general conclusions about the developed measure and identifying needs to further develop the theory of mathematical problem-solving dispositions and beliefs. Although four types of validity were used to improve the validity of the measure, further analysis should be explored. All belief and problem-solving practice data were self-reported. The exact validity of these is not known. Future researchers should further explore the validity of this scale by gathering further information, such as actual GPA and math class averages, as these were self-reported.

Content Validity

Content validity was established not only through qualitative data from comments made by an expert panel of judges, but also by correlating scores from the different judges. The judges' ratings and comments were both very similar and consistent. One common theme that arose was the idea of consistency in wording. For example, eight out of nine judges felt like the wording of the MPSDB scale was at times inconsistent and suggested this be changed prior to scale administration. This is expected as Smith and McCarthy (1995) advised that content validation inevitably involves "refinement." Analyzing comments established high level of agreement among the judges. The researcher revised items to better reflect construct definitions. For example, in many instances, "math" was changed to "mathematical problem solving," to better focus on problem solving and ideas centered on the theory. Beliefs and dispositions can be "fuzzy," meaning that content validation is challenging (Murphy & Davidshofer, 1994). Therefore, it is important to have multiple ways to establish validation. Under the suggestion of Brown (1983) and Cronbach & Thorndike (1971) this study further found the degree of agreement between the judges statistically through correlation measures. The degree of agreement was reported to be .931 in reference to relevancy ratings and .895 in reference to realistic ratings, thus establishing content validity. Establish content validity is vital as now inferences carry meaning. Using an invalid instrument would degrade any inferences made. This study is confident in inferences and implications based on established validity measures.

Criterion Validity

Although criterion validity was established by correlating scores on the MPSDB scale with GPA and mathematics class average, it is entirely possible that accurate GPA and mathematics class averages were not reported, as these were self-reported measures. The positive correlations produced suggests that MPSDB is related to GPA and mathematics class average. However, the strength of the correlation being weak suggests that accurate GPA and mathematics class average were not reported. Herman (2003) found that students' self-reports of GPA can often taint the data and thus caution using this as the only variable to establish validity. Students often do not want to report accurate grades as they feel judged and desire to think highly of themselves (Pajares, 1996). This is more likely to occur when stakes are high or at younger ages (Baird 1976). Examining the data, it would appear that the participants in this study may not have accurately reported their GPA and mathematics class averages as the correlation between scale score and GPAs were either weak positive or very weak positive correlation. The researcher chose to have the scale be anonymous, so as to make students feel less pressured to answer certain ways. In the future, it would benefit researchers to have access to grades and to collect general background information to address validity concerns. As identified in the pilot study, sixth and seventh-grade students were confused about GPA, although organized instruction did occur prior to the final scale administration, there are concerns as to whether or not these students accurately calculated and reported their true GPA. The Georgia Milestones assessment is a relatively new measure of student achievement, so this might be a better more accurate portrait of student achievement. In the future, once Georgia Milestones is an established assessment, these scores could be used to determine correlations with the MPSDB scores to further establish criterion validity.

Construct Validity

Construct validity was established by correlating mean MPSDB scale scores to items from May's MSEAQ mean scores and items from Fennema-Sherman's Mathematics Usefulness mean scores. All six scales, except *mathematical communities of practice*, have moderate to strong correlations, which established construct validity. It makes sense that valuing communities of practice would have weak correlation as those who tend to have higher scores on the self-efficacy and anxiety items would not feel they need to depend on others when problem solving. Bandura (1994) suggested the social cognitive theory explained how the beliefs of students will impact how students act when problem solving. Beliefs regarding self-ability will influence the actions of students (Bandura, 1994). However, in a models-and-modeling perspective, mathematical problem solving cannot be separated from working with teams of people (Lesh & Zawojewski, 2007). Students must learn to value community regardless of their mathematics self-efficacy or mathematics anxiety. Real life problems require teams of people.

Incremental Validity

Incremental validity was also established in order to supplement the previous three forms of validity. Nunnally and Bernstein (1978), along with many other researchers such as Sechrest (1963) and Haynes and Lench (2003), have recommended any new measure to establish incremental validity, yet many studies do not. This study established incremental validity through statistical measures. There was a 3.2 percent increase in predictive capacity, which was statistically significant. This shows that adding mathematical mindset, mathematical revision, mathematical problem-solving utility, mathematical problem-solving process, mathematical communities of practice, and mathematical perseverance to the model increases the model's predictive power at predicting mathematics class average. Despite the fact this number does not appear to be significant, it shows these six factors are increasing the percentage of variance accounted for. Although 3.2 percent appears to be a small amount, the value is viewed as significant. In addition, students may not have accurately reported their GPA and mathematics class average. If these values were more accurately reported, the predictive capacity might increase. This should be examined in future studies.

Student ANOVA Results

Although the researcher found no significant differences between sixth, seventh, and eighth graders, MPSDB mean scale scores were found, the researcher speculated as to whether each grade level received opportunities to engage in problem solving based on a models-and-modeling perspective. If there are classroom that were exposed more frequently to a models-and-modeling approach to problem solving it is speculated differences between grade levels would arise. There is no way to be sure that each grade level did experience MEAs prior to scale administration, and thus future researchers need to examine score differences between grade levels for significant differences and why this finding could occur.

Teacher Data

Correlations between MPSDB scores and teacher beliefs were found to be positive but very weak. The correlation between MPSDB scores and teacher frequency of MEAs was also found to be positive but very weak. The MPSDB survey as well as the teacher questionnaire was self-reported, thus the extent to which responses were accurate and truly reflect individual beliefs or frequency in using MEAs is unknown. In addition, the extent to which the mathematics teachers engaged their students in a MEA prior to taking the MSPDB survey is also unknown.

Additional studies need to be conducted to further explore the teacher data in relation to student mean scores.

Implications for Future Research

Evidence suggests that the six underlying constructs of the MPSDB each consist of multiple factors. However, this study focused on identifying a single factor to measure each construct: *mathematical mindset, mathematical problem-solving perseverance, mathematical revision, mathematical communities of practice, mathematical problem-solving utility, and mathematical problem-solving processes*. Thus, the research is limited to exploring a single factor structure for each of the six constructs. Future researchers should explore the concept of multiple factors. This can be accomplished through further EFA. Additionally, confirmatory factor analysis can be performed to further explore the structure of the MPSDB scale.

Although both reliability and validity measures were established, it is important to note that measure calibration and validation is an ongoing process. This study represents just the first step in this process to fully understand problem-solving dispositions and beliefs. Students need to be exposed to a models-and-modeling perspective on problem solving as it leads to more productive dispositions and beliefs, but many students are still not exposed to these types of curricula. Further research needs to be conducted to validate the scale under different circumstances with different populations of students and teachers. This study only begins to reveal the factors underlying mathematical problem-solving dispositions and beliefs based on a models-and-modeling perspective. Some unanswered questions have been exposed in this endeavor. For example, if this scale prove reliable and valid for a different population.

The researcher collected teacher data about beliefs regarding mathematical problem solving as well as to the frequency they use MEAs. It should be noted this questionnaire was extensive and results were not generalized to form any conclusions. The researcher did not perform any validity measures to determine if students who were in classes in which teachers had more productive beliefs as defined by a models-and-modeling perspective would result in students who also had more productive problem-solving beliefs and dispositions. The researcher ran initial correlations to determine the relationship between student scale scores and teacher beliefs as well as frequency of MEAs. However, future research is needed to continue to explore this idea.

Limitations and Delimitations

There were numerous limitations to this study. First, there were several limits to the degree to which these findings can be generalized to all sixth, seventh, and eighth graders.
If the study were conducted among students from different Fulton County schools as well as different counties in Georgia, in terms of size, location, mathematics curriculum, and experience of mathematics teachers, the results may have been substantially different. Additionally, the sample may have under-represented several groups of students from the study such as those (1) who were absent on the day the MPSDB was administered; (2) who did not return the parental consent form; and (3) who were not able to read English well. These three circumstances may have affected the results. For example, students who did not return parental permission slips may have had parents who are less involved in their mathematics influences at home and this parental influence may be different between these two groups, and, thus, may have influenced beliefs and behaviors. Students who received more attention at home and help with mathematics may have

had more positive beliefs and dispositions towards problem solving. Second, due to the nature of the data, levels of mathematical problem-solving dispositions and beliefs were only measured at one point in time, and changes related to curriculum and experience were not determined. Third, self-report data can result in several biases. Despite the use of anonymous measures, assurance of confidentiality and requests for honesty, a number of students may have been inclined to give misleading answers, either overestimating or underestimating their beliefs and dispositions, or even GPA.

Another limitation of this study involved the researcher's decisions in applying the rules for EFA. Rules were modified because of limitations in the distribution of MPSDB scores. Using the proposed decision rule (i.e., loadings greater than .4), there was an occasion when an item cross-loaded (> .35) on two or more factors and thus, an alternate, less conservative, criterion was applied based on theoretical underpinnings.

There were also several delimitations in this study. The researcher conducted only one feedback session, and the feedback session data was only examined by the researcher, which may have introduced some bias into the reported results. Furthermore, comprehensive scale development requires numerous validation studies (Spector, 1992). This study only assessed primary psychometric properties of the MPSDB scale using a comparatively similar, sample of sixth, seventh and eighth grade students.

Implications for Future Practice

The current study presents the development and validation of the Mathematical Problem-solving Dispositions and Beliefs Scale (MPSDB). Seventy-five potential items were examined by means of EFA, reliability analysis, and validity analysis. The final scale was comprised of 40 items measuring productive dispositions and beliefs related to mathematical problem solving. This brief measure demonstrated good reliability as well as content, construct, and incremental validity in a sample of middle school students.

There are a few possibilities of how data from this scale can be utilized in teacher instruction. The key is that mathematics teachers must plan to include some goals for explicitly fostering the development of dispositions and beliefs within their regular instruction of problem solving. Mathematics teachers should continue to engage students in problem-solving tasks that encourage revision, refinement, team work, real life situations, perseverance, and hard work. These characteristics are needed in jobs today. Productive beliefs are established in classrooms through MEAs and thus these tasks need to be implemented more frequently. Teachers and schools can use the MPSDB scale to measure students' current dispositions and beliefs about problem solving, so that they may recognize students' current magnitude and strength of beliefs and foster the continued growth of these.

In conclusion, mathematics extends beyond the classroom to real life. Hence, mathematics will always serve the student well. Whether students decide to become a mathematician, a manager, a marketing executive, a custodian an attorney, or a doctor, mathematical problem solving is necessary. Productive problem-solving dispositions and beliefs are helpful as they link the theoretical and practical. This study highlights that productive dispositions and beliefs are positively related to achievement and that though this process is theoretical in nature the implementation is also practical.

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Appendix A: Proposed Items for Initial MPSDB Scale

Thank you for participating in a research project by completing this questionnaire. Through this questionnaire, I would like to understand your feelings and beliefs about mathematical problem-solving. There is **NO right or wrong answer** to each item on this questionnaire. It is very important that you answer each item with **HONEST feelings about yourself and your beliefs.** There are eight sections in the questionnaire, and it may take you 30 minutes to complete the whole survey. To ensure the integrity of the data collected from you, we ask you to take time to

1. **read each item carefully,**
2. **respond to each item individually,**
3. **and complete the whole survey.**

In order to ensure the responses are anonymous on the survey, please don't put down your name on the survey.

If you see something you do not understand or are not clear about the instructions or items on the survey, you may ask questions to your advisement teacher. After you complete the survey, please place it in the envelope located on the front table.

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	By trying hard, I can become better at math	1	2	3	4	5	6
2	Hard work can increase my ability in math	1	2	3	4	5	6
3	The more mathematics I learn, the more my math ability grows	1	2	3	4	5	6
4	The harder I try, the better I can be at math	1	2	3	4	5	6
5	I am afraid to give an incorrect solution to a math problem because I feel judged	1	2	3	4	5	6
6	I learn from making mistakes in math, which pushes me to work harder next time	1	2	3	4	5	6
7	Some people are born smarter in math than others.	1	2	3	4	5	6
8	There is nothing I can do to increase my math ability	1	2	3	4	5	6
9	I will never be good at math	1	2	3	4	5	6
10	I get better in math because I learn more every year	1	2	3	4	5	6
11	If I can't seem to solve a math problem, I work harder and try new strategies	1	2	3	4	5	6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	If I have difficulty problem-solving, I keep working and do my own research to figure a solution out	1 2 3 4 5 6
2	When problem-solving, I stay committed until I can develop a solution to the problem	1 2 3 4 5 6
3	If I cannot develop a solution for a math task in a few minutes, I usually stop trying	1 2 3 4 5 6
4	I am unwilling to spend more than five minutes finding solutions to math tasks	1 2 3 4 5 6
5	Problem-solving takes too long to complete	1 2 3 4 5 6
6	I am willing to work as long as it takes when problem-solving	1 2 3 4 5 6
7	If I become frustrated while problem-solving , I usually stop trying	1 2 3 4 5 6
8	If I can't find a solution to a problem-solving task in a single setting, I stop looking for a solution	1 2 3 4 5 6
9	I am willing to try several times before I find solutions to math tasks	1 2 3 4 5 6
10	I give up after my first few attempts to find solutions to math tasks	1 2 3 4 5 6
11	After being assigned a challenging math task, I keep working to find solutions	1 2 3 4 5 6
12	Even if I don't know how to solve the problem or feel like I don't have enough information, I stick with it to develop a solution	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	When completing a problem-solving task, I evaluate and refine my solutions	1 2 3 4 5 6
2	I reflect on the appropriateness of my solutions	1 2 3 4 5 6
3	When problem-solving, once I create one solution, I feel I am done with the task	1 2 3 4 5 6
4	It is important to find alternative solutions when problem-solving	1 2 3 4 5 6
5	When creating solutions to problem-solving task, I think about whether or not my solution could be used in a similar situation	1 2 3 4 5 6
6	When problem-solving, it does not matter if other people can interpret my solution as long as it is correct	1 2 3 4 5 6
7	If my solution is not working, I am willing to revise my thinking	1 2 3 4 5 6
8	When creating a solution, I pick the first design I create	1 2 3 4 5 6
9	I find value in testing out my solution	1 2 3 4 5 6
10	After I find a solution that works, I never look back to refine my solution	1 2 3 4 5 6
11	Once I have solved a problem, I evaluate how it is working	1 2 3 4 5 6
12	When problem-solving, revising my solutions creates a better model that applies to the real world	1 2 3 4 5 6
13	Revising solutions, when problem-solving, takes too much time	1 2 3 4 5 6
14	When solving real life problems, I improve my solutions as I gain additional knowledge, even if I have already found an answer	1 2 3 4 5 6
15	When problem-solving, understanding how I developed a solution is more important than the fact that actually have a solution	1 2 3 4 5 6

16	In addition to creating a solution, it is important to know why the solution works	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	When faced with a difficult math task, it is better to find a solution myself than to get advice from my peers	1 2 3 4 5 6
2	When problem-solving, I value other people's input when creating solutions	1 2 3 4 5 6
3	When problem-solving, I find my peers' input to be helpful	1 2 3 4 5 6
4	I do not like to depend on my peers to help create solutions to problem-solving tasks	1 2 3 4 5 6
5	When comparing solutions, I compare each possible solution with my peers' solutions to find the best one	1 2 3 4 5 6
6	If my partner has a different solution than me, but both work for the problem, it doesn't matter which one we pick	1 2 3 4 5 6
7	It's better to work with a team of people than alone	1 2 3 4 5 6
8	I like working on problem-solving tasks alone	1 2 3 4 5 6
9	When working on a problem-solving task, it is important to describe my thinking to others	1 2 3 4 5 6
10	As long as I understand the mathematical idea, it's not important to be able to describe it to a peer	1 2 3 4 5 6
11	If my solution was not correct , I make an argument for it anyway	1 2 3 4 5 6
12	If my solution is correct, I refine my ideas to make it better	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	I'll need to know problem-solving skills for my future job	1 2 3 4 5 6
2	When I am older, I don't plan on having a job that requires mathematical problem-solving	1 2 3 4 5 6
3	Working on problem-solving tasks in math class will help me in the future	1 2 3 4 5 6
4	Math is a worthwhile subject to learn because it teaches me problem-solving	1 2 3 4 5 6
5	I will use mathematical problem-solving as an adult	1 2 3 4 5 6
6	My job one day will not involve problem-solving	1 2 3 4 5 6
7	I will never use mathematical problem-solving after I graduate high school	1 2 3 4 5 6
8	Problem-solving will not be important for my life	1 2 3 4 5 6
9	Once I create solutions to a problem, I think about how others can use my solutions in solving future problems	1 2 3 4 5 6
10	When developing solutions to a math task, I ensure they can be used by others in solving future problems	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	An important part of problem-solving is developing my own steps to find answers	1 2 3 4 5 6
2	I develop my own procedures when problem-solving	1 2 3 4 5 6
3	When problem-solving, I often create a formula for myself	1 2 3 4 5 6
4	When given a problem-solving task, I first identify what the goal is	1 2 3 4 5 6
5	Mathematical problem-solving is a process without specific procedures	1 2 3 4 5 6
6	Mathematical problem-solving is done by following the steps the teacher gives me	1 2 3 4 5 6
7	Memorizing steps is one of the best strategies to use when problem-solving	1 2 3 4 5 6
8	Being creative is important when problem-solving	1 2 3 4 5 6
9	There is not always a list of steps to follow when problem-solving	1 2 3 4 5 6
10	Memorizing specific procedures is not helpful when problem-solving	1 2 3 4 5 6
11	Mathematical problem-solving is not following a set of steps, but rather discovering what steps need to be taken to find a solution	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	**2**	**3**	**4**	**5**	**6**
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	I study mathematics because I know how useful it is	1 2 3 4 5 6
2	As an adult I will use mathematics	1 2 3 4 5 6
3	Taking mathematics is a waste of time	1 2 3 4 5 6
4	In terms of my adult life it is not important for me to do well in mathematics in high school	1 2 3 4 5 6
5	I will use mathematics later in life	1 2 3 4 5 6

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	I believe I am the type of person who can do mathematics	1 2 3 4 5 6
2	I get nervous when I have to do mathematics outside of school	1 2 3 4 5 6
3	I feel confident when using mathematics outside of school	1 2 3 4 5 6
4	I feel confident to ask questions in my mathematics class	1 2 3 4 5 6
5	I get nervous when asking questions in my mathematics class	1 2 3 4 5 6
6	I believe I can complete all of the assignments in a mathematics course	1 2 3 4 5 6

7	I believe I am the kind of person who is good at mathematics	1 2 3 4 5 6

Directions: Circle your selection(s) below.

Gender (circle one): 1- Male or 2-Female

Ethnicity (circle one):
1-White 2-Hispanic/Latino 3-African American

4-Native American/ American Indian 5- Asian 6-Pacific Islander 7-Other

What teacher do you have for math class (circle one)?
1-Mr. Diaz 2-Ms. Barrett 3-Mr. Aubrey 4-Ms. Tieles

5-Ms. Hires/Sullivan 6-Mr. Sarris 7- Ms. Howell 8. Ms. Hayes

9- Ms. Hatchett 10- Ms. Merritt 11-Ms. Ferenczy 12- Ms. Isabell 13- Mrs. King

What was your grade in math class on your report card from last quarter (circle one)?

A+ (100-95)
A (94-90)
B+ (89-85)
B (84-80)
C+ (79-75)
C (74-70)
F (69-below)

What was your overall GPA last quarter (circle one)?

A (4.0+) B (3.0-3.9) C (2.0-2.9) F (0-1.9)

Appendix B: Expert Panel Invitation

2/8/16

Dear Expert Name,

I am developing a scale to measure mathematical problem-solving dispositions and beliefs for middle school students. This research is to fulfill my requirements for my doctoral dissertation at Kennesaw State University.

I would like to invite you to participate in an expert panel to review and evaluate the initial item pool developed for the proposed scale on mathematical problem-solving dispositions and beliefs. The panel will be asked to rate the relevancy and clarity of each item to the definition of the construct. Expert reviewers will also be invited to evaluate individual items with open ended comments. Your participation will include online communication through Survey Monkey. The survey is anonymous and your IP address will not be recorded. The link to the rating form is below. The form will be sent back to me electronically upon your submission.
https://www.surveymonkey.com/r/MPDBBARRETT

The information you provide will help to maximize the content validity of my scale. I hope you will assist me in this research effort. I appreciate your help. If you have any questions, please call me at (770-833-7209) or email me at lleduc123@gmail.com.

Sincerely,

Laura Barrett

Appendix C: Rating Form for Expert Panel Evaluating Potential Items for a Mathematical Problem-Solving Dispositions and Beliefs (MPSDB) Scale

Expert Instructions:

In a models-and-modeling approach on problem-solving students are able to adopt greater understanding of mathematical concepts as they participate in, revise, differentiate, and improve their thinking (Lesh and Zawojewski, 2007) through interactions with others. The models-and-modeling- approach to problem-solving views learning as multidimensional and thus factors such as beliefs and depositions arise as relevant to learning.

The models-and-modeling approach encourages the use of model-eliciting activities as these problem-solving task promote six important principles: the model construction principle, the reality principle, the self-assessment principle, the construct documentation principle, the construct shareability and reusability principle, and the effective prototype principle (Lesh, Hoover, Hole, Kelly, & Post, 2000). For instance the infamous model-eliciting activity, The Big Foot Problem, requires middle school students to investigate photographs, footprints, newspaper articles, and accurate data charts developed by experts, to develop a “How to” tool kit that police can use to make accurate estimates on peoples size just by looking at footprints. The Big Foot problem involves proportional reasoning and linear relationships as well as scale factors. Model-eliciting activities shift from traditional mathematical problem-solving task as these new task are open ended and require mathematical reasoning through revising, extending, and altering initial interpretations of mathematical situations. Traditional methods of mathematical instruction does not foster productive beliefs and dispositions towards problem-solving as only basic skills are used and word problems tend to be the definition of problem-solving as opposed to a models-and-modeling perspective where Lesh and Doerr (2003) defined problem-solving as the extension of initially inadequate conceptual models in order to create successful interpretations. Model-eliciting activities allow opportunities for students to develop adaptable and reusable theoretical tools, called models, for creating, explaining, and using mathematical methods.

Instructions to Expert: This section of scale measures students’ mathematical problem-solving dispositions and beliefs, **particularly measuring their mathematical mindset: self-beliefs about their mathematical ability**. Please rate 1) how each item is relevant to the construct of self-beliefs about their mathematical ability, and 2) how each item is realistic for grades 6th, 7th, and 8th students. In the first rating column, please rate each item with respect to its relevance to the defined construct: mathematical problem-solving dispositions and beliefs (3=high relevance, 2=moderate relevance, 1-low relevance). In the second rating column, please rate how realistic each belief is to mathematical problem-solving for the intended population (6th, 7th, and 8th grade students) (3=very realistic, 2=realistic, 1=not very realistic). Please feel free to provide feedback on wording, content, and make suggestions.

3	2	1
High Relevance	Moderate Relevance	Low Relevance

		Relevance to mathematical problem-solving dispositions and beliefs	Realistic beliefs for intended population
1	By trying hard, I can become better at math	3 2 1	3 2 1
2	Hard work can increase my ability in math	3 2 1	3 2 1
3	The more I learn, the better I will be in math.	3 2 1	3 2 1
4	I know if I do more work and try harder, I can get better at math	3 2 1	3 2 1
5	I get good grades in math.	3 2 1	3 2 1
6	I never get good grades in math even when I put forth a lot of effort.	3 2 1	3 2 1
7	I am afraid to give an incorrect solution to a math problem	3 2 1	3 2 1
8	I learn from making mistakes in math , which pushes me to work harder next time	3 2 1	3 2 1
9	Some people are born smarter in math than others.	3 2 1	3 2 1
10	There is nothing I can do to increase my math intelligence	3 2 1	3 2 1
11	I have never been good at math	3 2 1	3 2 1
12	I get better in math and learn more every year	3 2 1	3 2 1
13	If I get a bad grade in math I am not upset, I just know I need to work harder next time	3 2 1	3 2 1
14	If I do not pass a math test I feel upset and it reminds me how I was not born smart at math	3 2 1	3 2 1

COMMENTS:

Instructions to Expert: This section of scale measures students' mathematical problem-solving dispositions and beliefs, **particularly measuring their persistence to problem solve: beliefs about willingness to persevere in the problem-solving process**. Please rate 1) how each item is relevant to the construct of self-beliefs about their perseverance, and 2) how each item is realistic for grades 6^{th}, 7^{th}, and 8^{th} students. In the first rating column, please rate each item with respect to its relevance to the defined construct: mathematical problem-solving dispositions and beliefs (3=high relevance, 2=moderate relevance, 1-low relevance). In the second rating column, please rate how realistic each belief is to mathematical problem-solving for the intended population (6^{th}, 7^{th}, and 8^{th} grade students) (3=very realistic, 2=realistic, 1=not very realistic). Please feel free to provide feedback on wording, content, and make suggestions.

3	2	1
High Relevance	Moderate Relevance	Low Relevance

		Relevance to mathematical problem-solving dispositions and beliefs	Realistic beliefs for intended population
1	I believe that any math task can be solved in five minutes or less	3 2 1	3 2 1
2	Even if the teacher gives me more time, it does not help me solve math problems	3 2 1	3 2 1
3	If I cannot answer a math problem in a few minutes, I usually stop trying	3 2 1	3 2 1
4	I am unwilling to spend more than about five minutes finding a solution to a math problem	3 2 1	3 2 1
5	I am not good at problem-solving because it takes a long time to complete	3 2 1	3 2 1
6	I am good at problem-solving because I am persistent	3 2 1	3 2 1
7	It does not bother me if it takes a long time to complete a problems solving task	3 2 1	3 2 1
8	I enjoy working on problem-solving task because I know I will do well if I hang in there	3 2 1	3 2 1
9	I am willing to try many times before I find a solution to a math problem	3 2 1	3 2 1
10	I give up after my first two attempts to find a solution to a problem don't work	3 2 1	3 2 1
11	Despite being assigned a challenging math task, I keep working to find a solution	3 2 1	3 2 1

12	I tend to wait for the teacher or one of my peers to help me get started on a solution	3 2 1	3 2 1
13	If I can't find a solution to a problem-solving task in a single setting(twenty minutes or more), then I won't be able to find a solution at all	3 2 1	3 2 1
14	If I can't find a solution to a problem-solving task in a single setting(twenty minutes or more), I will stop looking for a solution	3 2 1	3 2 1

COMMENTS:

Instructions to Expert: This section of scale measures students' mathematical problem-solving dispositions and beliefs, **particularly measuring their belief about the revision process of problem-solving**. Please rate 1) how each item is relevant to the construct of beliefs about revision, and 2) how each item is realistic for grades 6th, 7th, and 8th students. In the first rating column, please rate each item with respect to its relevance to the defined construct: mathematical problem-solving dispositions and beliefs (3=high relevance, 2=moderate relevance, 1-low relevance). In the second rating column, please rate how realistic each belief is to mathematical problem-solving for the intended population (6th, 7th, and 8th grade students) (3=very realistic, 2=realistic, 1=not very realistic). Please feel free to provide feedback on wording, content, and make any suggestions.

3	2	1
High Relevance	Moderate Relevance	Low Relevance

		Relevance to mathematical problem-solving dispositions and beliefs	Realistic beliefs for intended population
1	When completing a problem-solving task, I evaluate and refine my solutions	3 2 1	3 2 1
2	I reflect on the appropriateness of my solutions	3 2 1	3 2 1
3	When problem-solving, once I create one solution, I feel I am done with the task	3 2 1	3 2 1
4	I tend to look for many solutions when problem-solving	3 2 1	3 2 1
5	When creating solutions to problem-solving task, it is important to think about whether or not my solution could be used in a similar situation	3 2 1	3 2 1
6	When problem-solving, it does not matter if other people can interpret my solution as long as its correct	3 2 1	3 2 1

7	There are problem-solving task where there are no procedures for finding the solution	3	2	1	3	2	1
8	Sometimes there is no right and wrong answer to a problem	3	2	1	3	2	1
9	If my solution is not working, I am willing to revise my thinking	3	2	1	3	2	1
10	When creating a solution, I pick the first design I create	3	2	1	3	2	1
11	I find value in testing out my solution	3	2	1	3	2	1
12	After I find a solution that works, I never look back to refine my solution	3	2	1	3	2	1
13	Once I have solved a problem I evaluate how it is working	3	2	1	3	2	1
14	When problem-solving, revising my solutions creates a better model that applies to the real world	3	2	1	3	2	1
15	Revising solutions, when problem-solving, takes too much time	3	2	1	3	2	1
16	When solving these real life problems, it is important to improve my solution as I gain new knowledge, even if I have already found an answer	3	2	1	3	2	1

COMMENTS:

Instructions to Expert: This section of scale measures students' mathematical problem-solving dispositions and beliefs, **particularly measuring their belief in communities of practice: beliefs about working and communicating with peers**. Please rate 1) how each item is relevant to the construct of beliefs about communities of practice, and 2) how each item is realistic for grades 6th, 7th, and 8th students. In the first rating column, please rate each item with respect to its relevance to the defined construct: mathematical problem-solving dispositions and beliefs (3=high relevance, 2=moderate relevance, 1-low relevance). In the second rating column, please rate how realistic each belief is to mathematical problem-solving for the intended population (6th, 7th, and 8th grade students) (3=very realistic, 2=realistic, 1=not very realistic). Please feel free to provide feedback on wording, content, and make suggestions.

3	2	1
High Relevance	Moderate Relevance	Low Relevance

		Relevance to mathematical problem-solving dispositions and beliefs	Realistic beliefs for intended population
1	When faced with a difficult math task, it is better to find a solution myself rather than to get advice from my peers	3 2 1	3 2 1
2	I value other people's input when creating solution for a problem-solving task	3 2 1	3 2 1
3	I usually find my peers ideas to be most helpful when finding a solution for a problem-solving task	3 2 1	3 2 1
4	I do not like to depend on my peers to help solve a math problem	3 2 1	3 2 1
5	I work with my peers to pick the solution that best solves the problem	3 2 1	3 2 1
6	If my partner has a different solution than me, but both work for the problem, it doesn't matter which one we pick	3 2 1	3 2 1
7	It's better to work with a team of people than alone	3 2 1	3 2 1
8	I like working on problem-solving task alone	3 2 1	3 2 1
9	When working on a problem-solving task, it is important to describe my mathematical ideas to others	3 2 1	3 2 1
10	As long as I understand the mathematical idea, it's not important to be able to describe it to a peer	3 2 1	3 2 1

COMMENTS:

Instructions to Expert: This section of scale measures students' mathematical problem-solving dispositions and beliefs, **particularly measuring their belief in the utility of mathematics: beliefs about how useful mathematics is for themselves.** Please rate 1) how each item is relevant to the construct of beliefs about the utility of mathematics, and 2) how each item is realistic for grades 6^{th}, 7^{th}, and 8^{th} students. In the first rating column, please rate each item with respect to its relevance to the defined construct: mathematical problem-solving dispositions and beliefs (3=high relevance, 2=moderate relevance, 1-low relevance). In the second rating column, please rate how realistic each belief is to mathematical problem-solving for the intended population (6^{th}, 7^{th}, and 8^{th} grade students) (3=very realistic, 2=realistic, 1=not very realistic). Please feel free to provide feedback on wording, content, and make any suggestions.

3	2	1
High Relevance	Moderate Relevance	Low Relevance

		Relevance to mathematical problem-solving dispositions and beliefs	Realistic beliefs for intended population
1	I'll need to know problem-solving skills for my future job	3 2 1	3 2 1
2	When I am older, I don't plan on having a job that requires mathematical problem-solving	3 2 1	3 2 1
3	Working on problem-solving task in math class will help me in the future	3 2 1	3 2 1
4	Math is a worthwhile subject to learn because it teaches me problem-solving	3 2 1	3 2 1
5	I will use math is many ways as an adult	3 2 1	3 2 1
6	My job one day will not involve problem-solving	3 2 1	3 2 1
7	I will never use math again after I graduate	3 2 1	3 2 1
8	Problem-solving will not be important for my life	3 2 1	3 2 1
9	Once I create a solution to a problem, I think about how other people can use my solution on future problems	3 2 1	3 2 1
10	When developing a solution to a math task, I ensure it can be used by other people in the future	3 2 1	3 2 1

COMMENTS:

Instructions to Expert: This section of scale measures students' mathematical problem-solving dispositions and beliefs, **particularly measuring their belief in valuing understanding mathematics: beliefs about the relationships in mathematics** as opposed to viewing it as a step of procedures to follow. Please rate 1) how each item is relevant to the construct of beliefs about valuing relationships within mathematics, and 2) how each item is realistic for grades 6^{th}, 7^{th}, and 8^{th} students. In the first rating column, please rate each item with respect to its relevance to the defined construct: mathematical problem-solving dispositions and beliefs (3=high relevance, 2=moderate relevance, 1-low relevance). In the second rating column, please rate how realistic each belief is to mathematical problem-solving for the intended population (6^{th}, 7^{th}, and 8^{th} grade students) (3=very realistic, 2=realistic, 1=not very realistic). Please feel free to provide feedback on wording, content, and make any suggestions.

3	2	1
High Relevance	Moderate Relevance	Low Relevance

		Relevance to mathematical problem-solving dispositions and beliefs	Realistic beliefs for intended population
1	Getting the right answer in math is more important than understanding why the right answer works	3 2 1	3 2 1
2	In addition to getting the right answer, it is important to know why the answer is correct	3 2 1	3 2 1
3	If my solution was not correct , I make an argument for it anyway	3 2 1	3 2 1
4	If my solution is correct, I refine my ideas to make it better	3 2 1	3 2 1
5	When I get a task, I try to figure out what the problem is	3 2 1	3 2 1
6	When assigned a task, I wait to be told what I need to do	3 2 1	3 2 1
7	Mathematical problem-solving is a process	3 2 1	3 2 1
8	Mathematical problem-solving can be done by following steps the teacher gives me	3 2 1	3 2 1
9	I am confident I can find alternative solutions for problems, when my initial solution does not work	3 2 1	3 2 1
10	Being creative is important when solving math task, as there are often more than one correct answer	3 2 1	3 2 1

COMMENTS:

Finally please rate the scale overall in reference to the word choice with respect to its appropriateness to the target audience (6th, 7th, and 8th grade students) (very appropriate = 3, appropriate = 2, not appropriate = 1); and please rate the response format with respect to its relevance to the items (very relevant = 3, relevant = 2, not relevant = 1).

Please rate the scale overall in reference to the word choice with respect to its appropriateness to the target audience (6th, 7th, and 8th grade students)

3-Very Appropriate ***2- Appropriate*** ***1- Not Appropriate***

Comments:

Please rate the response format with respect to its relevance to the items. Students have a selection of numbers 1-6, ranging from strongly disagree to strongly agree. An example is included below:

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	By trying hard, I can become better at math	1 2 3 4 5 6
2	Hard work can increase my ability in math	1 2 3 4 5 6

3-Very Relevant ***2- Relevant*** ***1- Not Relevant***

Comments:

Appendix D: Feedback Session Guide

Hello and welcome!

My name is Laura Barrett and I will be leading this feedback session today. At times I will be taking notes during our discussion. I would like to get your ideas about feelings and beliefs students your age experiences when engaging in mathematical problem-solving. I am developing a questionnaire to understand students' mathematical problem-solving dispositions and beliefs. . But before any final questionnaires are developed it's important to find out about the types of feelings seventh and eighth graders experience. I would like to hear your thoughts about the questionnaire as you see it for the first time. Please respond to the items on your paper copy in addition to writing down any comments you wish to further explain your responses. Occasionally I may ask you "Why did you respond to that item that way?" and "what situation makes you feel that way?" or "why do you believe that?"
Also, if there is any question you don't want to answer, you certainly don't have to, and you are of course are free to stop participating at any time. If you do wish to stop participating you can sit quietly. As I just mentioned, the purpose of this focus group is to find out your thoughts and beliefs about realistic situations when engaged in mathematical problem-solving. As we discuss your thoughts, please do not include any individual names or other information that could identify people.

So let's get started:
Ice Breakers:
What's your favorite part of middle school?
What do you like about your mathematics class this year?
Are students your age filled with lots of feelings and thoughts during a problem-solving task?

Questions that I asked to students about their responses that lead to further discussion:
Why did you respond to that item that way?
What situation makes you feel that way?
How long have you had this belief?
Do you feel this way in other classes or just mathematics? Explain
How is that belief different from when you are solving a word problem?
Are there any items you do not understand? Please explain.
Can you tell me more?

Appendix E*:* Survey Instrument – Teacher Protocol

Please read this information to students before passing out the surveys

Thank you for helping us with this survey. It has been developed so you can tell us your thoughts and beliefs towards mathematical problem-solving. The information you give will be used to develop better mathematics education for young people like yourself. Please DO NOT put your name on the survey. You do not have to answer any questions you do not want to answer and can stop participating at any time. Make sure to read every question. If you have questions about any of the survey items, you may raise your hand and ask the administrator. If he/she cannot answer your question, you can make the best possible choice or leave the answer blank. You may also write any thoughts you would like to share on the scale next to those items. No names will ever be reported. There are no right or wrong answers and your answers will be completely anonymous because your name will not be on the survey. Once you have completed the questionnaire, put your pencil down and place the questionnaire in the folder on the front table.

Appendix F: Preliminary MPSDB Scale

Thank you for participating in a research project by completing this questionnaire .Through this questionnaire, I would like to understand your feelings and beliefs about mathematical problem-solving. There is **NO right or wrong answer** to each item on this questionnaire. It is very important that you answer each item with **HONEST feelings about yourself and your beliefs.** There are eight sections in the questionnaire, and it may take you 30 minutes to complete the whole survey. To ensure the integrity of the data collected from you, we ask you to take time to

1. **read each item carefully,**
2. **respond to each item individually,**
3. **and complete the whole survey.**

In order to ensure the responses are anonymous on the survey, please don't put down your name on the survey.

If you see something you do not understand or are not clear about the instructions or items on the survey, you may ask questions to your advisement teacher. After you complete the survey, please place it in the envelope located on the front table.

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	By trying hard, I can become better at math	1 2 3 4 5 6
2	Hard work can increase my ability in math	1 2 3 4 5 6
3	The more mathematics I learn, the more my math ability grows	1 2 3 4 5 6
4	The harder I try, the better I can be at math	1 2 3 4 5 6
5	I learn from making mistakes in math , which pushes me to work harder next time	1 2 3 4 5 6
6	I will never be good at math	1 2 3 4 5 6
7	I get better in math because I learn more every year	1 2 3 4 5 6
8	If I can't seem to solve a math problem, I work harder and try new strategies	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	If I have difficulty problem-solving, I keep working and do my own research to figure a solution out	1	2	3	4	5	6
2	When problem-solving, I stay committed until I can develop a solution to the problem	1	2	3	4	5	6
3	If I cannot develop a solution for a math task in a few minutes, I usually stop trying	1	2	3	4	5	6
4	I am unwilling to spend more than five minutes finding solutions to math tasks	1	2	3	4	5	6
5	Problem-solving takes too long to complete	1	2	3	4	5	6
6	I am willing to work as long as it takes when problem-solving	1	2	3	4	5	6
7	If I become frustrated while problem-solving , I usually stop trying	1	2	3	4	5	6
8	If I can't find a solution to a problem-solving task in a single setting, I stop looking for a solution	1	2	3	4	5	6
9	I am willing to try several times before I find solutions to math tasks	1	2	3	4	5	6
10	I give up after my first few attempts to find solutions to math tasks	1	2	3	4	5	6
11	After being assigned a challenging math task, I keep working to find solutions	1	2	3	4	5	6
12	Even if I don't know how to solve the problem or feel like I don't have enough information, I stick with it to develop a solution	1	2	3	4	5	6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	When completing a problem-solving task, I evaluate and refine my solutions	1 2 3 4 5 6
2	I reflect on the appropriateness of my solutions	1 2 3 4 5 6
3	It is important to find alternative solutions when problem-solving	1 2 3 4 5 6
4	When creating solutions to problem-solving task, I think about whether or not my solution could be used in a similar situation	1 2 3 4 5 6
5	If my solution is not working, I am willing to revise my thinking	1 2 3 4 5 6
6	I find value in testing out my solution	1 2 3 4 5 6
7	Once I have solved a problem, I evaluate how it is working	1 2 3 4 5 6
8	When problem-solving, revising my solutions creates a better model that applies to the real world	1 2 3 4 5 6
9	When solving real life problems, I improve my solutions as I gain additional knowledge, even if I have already found an answer	1 2 3 4 5 6
10	When problem-solving, understanding how I developed a solution is more important than the fact that actually have a solution	1 2 3 4 5 6
11	In addition to creating a solution, it is important to know why the solution works	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	When problem-solving, I value other people's input when creating solutions	1 2 3 4 5 6
2	When problem-solving, I find my peers' input to be helpful	1 2 3 4 5 6
3	I do not like to depend on my peers to help create solutions to problem-solving tasks	1 2 3 4 5 6
4	When comparing solutions, I compare each possible solution with my peers' solutions to find the best one	1 2 3 4 5 6
5	It's better to work with a team of people than alone	1 2 3 4 5 6
6	I like working on problem-solving tasks alone	1 2 3 4 5 6
7	When working on a problem-solving task, it is important to describe my thinking to others	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	I'll need to know problem-solving skills for my future job	1 2 3 4 5 6
2	When I am older, I don't plan on having a job that requires mathematical problem-solving	1 2 3 4 5 6
3	Working on problem-solving tasks in math class will help me in the future	1 2 3 4 5 6
4	Math is a worthwhile subject to learn because it teaches me problem-solving	1 2 3 4 5 6
5	I will use mathematical problem-solving as an adult	1 2 3 4 5 6
6	My job one day will not involve problem-solving	1 2 3 4 5 6
7	I will never use mathematical problem-solving after I graduate high school	1 2 3 4 5 6
8	Problem-solving will not be important for my life	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	An important part of problem-solving is developing my own steps to find answers	1	2	3	4	5	6
2	I develop my own procedures when problem-solving	1	2	3	4	5	6
3	When problem-solving, I often create a formula for myself	1	2	3	4	5	6
4	When given a problem-solving task, I first identify what the goal is	1	2	3	4	5	6
5	Mathematical problem-solving is a process without specific procedures	1	2	3	4	5	6
6	Being creative is important when problem-solving	1	2	3	4	5	6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	**2**	**3**	**4**	**5**	**6**
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	I study mathematics because I know how useful it is	1 2 3 4 5 6
2	As an adult I will use mathematics	1 2 3 4 5 6
3	Taking mathematics is a waste of time	1 2 3 4 5 6
4	In terms of my adult life it is not important for me to do well in mathematics in high school	1 2 3 4 5 6
5	I will use mathematics later in life	1 2 3 4 5 6

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	I believe I am the type of person who can do mathematics	1 2 3 4 5 6
2	I get nervous when I have to do mathematics outside of school	1 2 3 4 5 6
3	I feel confident when using mathematics outside of school	1 2 3 4 5 6
4	I feel confident to ask questions in my mathematics class	1 2 3 4 5 6
5	I get nervous when asking questions in my mathematics class	1 2 3 4 5 6
6	I believe I can complete all of the assignments in a mathematics course	1 2 3 4 5 6
7	I believe I am the kind of person who is good at mathematics	1 2 3 4 5 6

Directions: Circle your selection(s) below.

Gender (circle one): 1- Male or 2-Female

Ethnicity (circle one):
1-White 2-Hispanic/Latino 3-African American

4-Native American/ American Indian 5- Asian 6-Pacific Islander 7-Other

What teacher do you have for math class (circle one)?
1-Mr. Diaz 2-Ms. Barrett 3-Mr. Aubrey 4-Ms. Tieles

5-Ms. Hires/Sullivan 6-Mr. Sarris 7- Ms. Howell 8. Ms. Hayes

9- Ms. Hatchett 10- Ms. Merritt 11-Ms. Ferenczy 12- Ms. Isabell 13- Mrs. King

What was your grade in <u>math class</u> on your report card from last quarter (circle one)?

A+ (100-95)
A (94-90)
B+ (89-85)
B (84-80)
C+ (79-75)
C (74-70)
F (69-below)

What was your <u>overall GPA</u> last quarter (circle one)?
A (4.0+) B (3.0-3.9) C (2.0-2.9) F (0-1.9)

Appendix G: Final Version of the MPSDB

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	By trying hard, I can become better at math	1	2	3	4	5	6
2	Hard work can increase my ability in math	1	2	3	4	5	6
3	The more mathematics I learn, the more my math ability grows	1	2	3	4	5	6
4	The harder I try, the better I can be at math	1	2	3	4	5	6
5	I learn from making mistakes in math , which pushes me to work harder next time	1	2	3	4	5	6
6	I will never be good at math	1	2	3	4	5	6
7	I get better in math because I learn more every year	1	2	3	4	5	6
8	If I can't seem to solve a math problem, I work harder and try new strategies	1	2	3	4	5	6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	If I have difficulty problem-solving, I keep working and do my own research to figure a solution out	1 2 3 4 5 6
2	When problem-solving, I stay committed until I can develop a solution to the problem	1 2 3 4 5 6
3	I am willing to work as long as it takes when problem-solving	1 2 3 4 5 6
4	I am willing to try several times before I find solutions to math tasks	1 2 3 4 5 6
5	After being assigned a challenging math task, I keep working to find solutions	1 2 3 4 5 6
6	Even if I don't know how to solve the problem or feel like I don't have enough information, I stick with it to develop a solution	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	When completing a problem-solving task, I evaluate and refine my solutions	1	2	3	4	5	6
2	I reflect on the appropriateness of my solutions	1	2	3	4	5	6
3	It is important to find alternative solutions when problem-solving	1	2	3	4	5	6
4	When creating solutions to problem-solving task, I think about whether or not my solution could be used in a similar situation	1	2	3	4	5	6
5	If my solution is not working, I am willing to revise my thinking	1	2	3	4	5	6
6	I find value in testing out my solution	1	2	3	4	5	6
7	Once I have solved a problem, I evaluate how it is working	1	2	3	4	5	6
8	When problem-solving, revising my solutions creates a better model that applies to the real world	1	2	3	4	5	6
9	When solving real life problems, I improve my solutions as I gain additional knowledge, even if I have already found an answer	1	2	3	4	5	6
10	When problem-solving, understanding how I developed a solution is more important than the fact that actually have a solution	1	2	3	4	5	6
11	In addition to creating a solution, it is important to know why the solution works	1	2	3	4	5	6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	When problem-solving, I value other people's input when creating solutions	1 2 3 4 5 6
2	When problem-solving, I find my peers' input to be helpful	1 2 3 4 5 6
3	When comparing solutions, I compare each possible solution with my peers' solutions to find the best one	1 2 3 4 5 6
4	It's better to work with a team of people than alone	1 2 3 4 5 6
5	When working on a problem-solving task, it is important to describe my thinking to others	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	I'll need to know problem-solving skills for my future job	1 2 3 4 5 6
2	When I am older, I don't plan on having a job that requires mathematical problem-solving	1 2 3 4 5 6
3	Working on problem-solving tasks in math class will help me in the future	1 2 3 4 5 6
4	Math is a worthwhile subject to learn because it teaches me problem-solving	1 2 3 4 5 6
5	I will use mathematical problem-solving as an adult	1 2 3 4 5 6

There are NO right or wrong answers to these statements. Please use the following rating scale to indicate how much you agree or disagree with each statement about ***your true feelings and beliefs.***

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	An important part of problem-solving is developing my own steps to find answers	1	2	3	4	5	6
2	I develop my own procedures when problem-solving	1	2	3	4	5	6
3	When problem-solving, I often create a formula for myself	1	2	3	4	5	6
4	When given a problem-solving task, I first identify what the goal is	1	2	3	4	5	6
5	Being creative is important when problem-solving	1	2	3	4	5	6

Appendix H: Teacher Questionnaire on Model-Eliciting Activities and Teaching Practice

Thank you for participating in a research project by completing this questionnaire .Through this questionnaire, we would like to understand how often you use model-eliciting activities or how often your instruction includes principles involved in modeling eliciting activities. There is **NO right or wrong answer** to each item on this questionnaire. It is very important that you answer each item with **HONEST feelings about your current use of these activities**. It may take you 15 to 20 minutes to complete the whole survey. To ensure the integrity of the data collected from you, we ask you to take time to

1. **read each item carefully,**
2. **respond to each item individually,**
3. **and complete the whole survey.**

In order to keep the **confidentiality** of your responses on the survey, please don't put down your name on the survey. **Please provide your id number in the box below:**

You may ask questions to the researcher if you are not clear about the instructions or items on the survey. After you complete the survey, please place it in the envelope located on the front table.

In this section, we are interested in understanding how often your mathematics instruction uses principles of model-eliciting activities. There are NO right or wrong answers to these statements. Please use the following rating scale to indicate your feelings about your instructional practices and tasks in your classroom

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1	Students must work as teams to produce and explain their solutions	1	2	3	4	5	6
2	Tasks I assign have correct answers	1	2	3	4	5	6
3	When assigning tasks, I keep in mind the main goal is for students to develop a model to use in solving a problem	1	2	3	4	5	6
4	My role as a teacher is to lead students to desired solutions	1	2	3	4	5	6
5	Social learning is important in a mathematics class	1	2	3	4	5	6
6	The problems I assign do not have correct solutions	1	2	3	4	5	6
7	Students have to make predictions and apply their models to a new problem	1	2	3	4	5	6
8	Students must learn to identify patterns and rules governing relationships	1	2	3	4	5	6
9	Reading and writing are not a component of my activities	1	2	3	4	5	6

In this section, we are interested in understanding how often your class engages in model-eliciting activities. There are NO right or wrong answers to these statements. Please use the following rating scale to indicate your feelings about your instructional practices and task in your classroom

.

1	2	3	4	5	6
Never	**Rarely**	**Occasionally**	**Sometimes**	**Often**	**All the time**

1	Tasks I use have students identify an audience who will be served by the solutions	1	2	3	4	5	6
2	Tasks I give encourage students to compare solutions and select promising ones	1	2	3	4	5	6
3	Tasks I use encourage students to extend and refine solutions	1	2	3	4	5	6
4	Tasks I give require students to plan, monitor, and assess their progress	1	2	3	4	5	6
5	Tasks I use state why the audience needs a solution	1	2	3	4	5	6
6	Solutions students create can be used on other problems in my class	1	2	3	4	5	6
7	Tasks I give promote classroom discourse	1	2	3	4	5	6
8	Tasks I use encourage students to detect deficiencies	1	2	3	4	5	6
9	Tasks I use require student self-assessment and need for improvement	1	2	3	4	5	6
10	The tasks I give are open ended	1	2	3	4	5	6
11	The tasks I give may be completed by students working alone	1	2	3	4	5	6
12	The tasks I give can only be completed by a team of students	1	2	3	4	5	6

Faculty Advisor Routing Sheet

(ONLY submit this page with student research applications)

All student research at KSU must be supervised by a faculty advisor. In order to ensure that the advisor has reviewed the IRB application materials and agrees to supervise a student's proposed human subject research project **this routing sheet, along with the application materials, must be submitted by the faculty advisor from their KSU email address** to irb@kennesaw.edu.

By checking the boxes below, the faculty advisor for this project attests the following:

☐ I have personally reviewed each of my student's IRB application documents (approval request, exemption request, informed consent documents, child assent documents, survey instruments, etc.) for completeness, and all documents pertaining to the conduct of this study are enclosed (consents, assents, questionnaires, surveys, assessments, etc.)

☐ I verify that the proposed methodology is appropriate to address the purpose of the research.

☐ I have completed a CITI training course in the ethics of human subject research within the past three years as have all researchers named within this application.

☐ I approve of this research and agree to supervise the student(s) as the study is conducted.

Faculty Advisor Name:

Date:

Kennesaw State University
Institutional Review Board

Approval Request for Research with Human Participants

To ensure a more timely review of your study:

- Go to http://www.kennesaw.edu/irb/application_instructions.html and review the instructions for submitting an IRB Application.
- **Answer each question on this form.**
- **Check spelling and grammar.** This is a protected form. You must cut and paste your answers into the question blocks or unprotect the form to run the spell check feature in Word. To unprotect the form, select the Developer tab, select the "Restrict Editing" tool, select the "Stop Protection" button, run spell check. When you have finished checking spelling and grammar, select the "Yes, Start Enforcing Protection" button, and save your document. The form is not password protected, so there is no need to enter a password when prompted.
- **Ensure consent documents contain all of the required elements of informed consent** (see http://www.kennesaw.edu/irb/forms.html for examples of consent forms, cover letters, assent for minors, and online consent documents). If required elements are missing, your documents will be returned for revision.
- **Reference all materials cited** (you may do so within the body of this form or in a separate document).
- **Submit the following documents to** irb@kennesaw.edu.
    - IRB Approval Request
    - Consent documents

- Survey instruments
- IRB training certificate for all researchers (unless CITI course is completed at KSU)

Refer all questions to the IRB at (470) 578-2268 or irb@kennesaw.edu.

Status of Researcher: ☐ Faculty ☐ Staff ☒ Student ☐ Other (explain):

Title of Research:

Mathematical Problem-solving Dispositions and Beliefs

Proposed Research Start Date: 12/1/15 * **Proposed Ending Date:** 7/30/16

***The official start date for research is the date the IRB approval letter is issued. Studies should be submitted well in advance of the proposed start date to allow for processing, review, and approval. Research activities may not begin prior to final IRB approval.**

NOTE: It is each researcher's responsibility to ensure that their CITI Certificate does not expire during the course of the approved study. Failure to maintain a current certificate will invalidate your approval.

Research is Funded: ☐ Yes* ☒ No

***Name of Funding Agency**

By submitting this form, you agree that you have read KSU's "Assurance of Compliance" (http://www.kennesaw.edu/irb/policies/assurance.doc) and agree to provide for the protection of the rights and welfare of your research participants as outlined in the Assurance. You also agree to submit any significant changes in the procedures of your project to the IRB for prior approval and agree to report to the IRB any unanticipated problems involving risks to subjects or others.

Primary Investigator

Name:

Laura Barrett

Department:

Secondary and Middle Grades Education

Telephone:	**Email:**
770-833-7209	**lleduc123@gmail.com**

Co-Investigator(s) who are faculty, staff, or students at KSU:

Name: Email:	☐ Faculty ☐ Staff ☐ Student
Name: Email:	☐ Faculty ☐ Staff ☐ Student
Name: Email:	☐ Faculty ☐ Staff ☐ Student
Additional Names (include status and email):	

Co-Investigator(s) who are NOT employees or students at KSU:

Name: Email: Home Institution:
Name: Email: Home Institution:
Additional Names (include email and home institution):

FOR RESEARCH CONDUCTED BY STUDENTS OR NON-FACULTY STAFF. This study, if approved, will be under the direct supervision of the following faculty advisor who is a member of the KSU faculty:

Faculty Advisor

Name:

Dr. Mei-Lin Chang

Department:

Secondary and Middle Grades Education

Telephone:	Email:
470-578-7795	mchang6@kennesaw.edu

1. Prior Research

Have you submitted research on this topic to the IRB previously? ☐ Yes* ☒ No

*If yes, list the date, title, name of investigator, and study number, if known:

See http://www.kennesaw.edu/irb/application_instructions.html for detailed explanations of questions 2-8.

2. Description of Research

a. Purpose of research:

The purpose of this research is to measure students' mathematical dispositions and beliefs towards problem-solving from a models-and-modeling perspective. This research is being conducted for my dissertation and I plan to use a survey to measure students' mathematical problem-solving dispositions and beliefs.

b. Nature of data to be collected:

The data will involve two surveys, one given to students and one given to teachers.

The student survey will include items associated to students' mathematical problem-solving dispositions and beliefs. The survey will contain responses to Likert-scaled statements. The items will include statements related to different beliefs and dispositions important in a models–and-modeling perspective on problem-solving. Items will measure students' mathematical mindset, students' perseverance in problem-solving, students' beliefs about the mathematical revision process, students' beliefs on communities of practice in mathematical problem-solving, students' belief about the utility of mathematical problem-solving, and students' value in understanding mathematics, as emphasized in a models-and-modeling perspective where solutions to mathematical problem are complex artifacts.

The teacher survey will include items involving items referencing how much instructional time is spent involving students in modeling eliciting activities through mathematical problem-solving. The survey will include Likert-scaled statements.

c. Data collection procedures:

Participants will be asked to complete a paper and pencil survey.

d. Survey instruments to be used (pre-/post-tests, interview and focus group questionnaires, online surveys, etc.):

A survey will be used to collect the data and this survey will be taken as part of a normal classroom activity.

e. Method of selection/recruitment of participants:

All 7th and 8th grade students will be invited to participate in this survey. The students and teachers invited to participate in the survey attend/teach at the same school located in a suburban school district in Georgia. A group of mathematics teachers will administer the survey to students. A training session will be provided to the mathematics teachers to ensure the same protocol is followed when the surveys are administered to students. No instructor will administer the survey to his or her own class.

f. Participant age: **Students- <u>13-15 years old</u>** Number: Goal of **<u>300</u>** Sex: ☐ Males ☐ Females ☒ Both

<u>Teachers – 25-60 years old</u>

g. Incentives, follow-ups, compensation to be used:

Participation is entirely voluntary and there will be no penalties or consequences for withdrawing from the study at any time.

3. Risks

Describe in detail any psychological, social, legal, economic or physical risk that might occur to participants. *Note that all research may entail some level of risk, though perhaps minimal.*

☒ No known risks (if selected, must be reflected within consent documents)

☐ Anticipated risks include (if selected, must be reflected within consent documents):

4. Benefits

University policy requires that risks from participation be outweighed by potential benefits to participants and/or humankind in general.

a. Identify benefits to participants resulting from this research (reflect within consent documents):

Individuals might benefit indirectly from participation in this study from sharing information about their mathematic dispositions and beliefs through the survey, or gaining personal satisfaction from participating in the study.

b. Identify benefits to humankind in general resulting from this research (reflect within consent documents):

This research has benefits to society as the knowledge gained in this study will impact the local community as well as local institutions. Institutions benefit, for example, by receiving information to improve a preservice mathematics education program as a result of this study. The local community can benefit through improved public educational programs arising based on data about students' dispositions and beliefs involved in mathematics problem-solving.

5. Informed Consent

All studies must include informed consent (see IRB approved templates). Consent may require signature or may simply require that participants be informed. If deception is necessary, please justify and describe, and submit debriefing procedures. What is the consent process to be followed in this study?

This study will require a parental consent from with child assent as well as an adult consent form for teachers.

Online Surveys

Will you use an online survey to obtain data from human participants in this study?

☒ No. If no, skip to Question 6 below.

☐ Yes, I will use an online survey to obtain data in this study. If yes:

a. How will **online data** be collected and handled? Select one and add the chosen statement to your consent document.

☐ Data collected online will be handled in an anonymous manner and Internet Protocol addresses **WILL NOT** be collected by the survey program.

☐ Data collected online will be handled in a confidential manner (identifiers will be used) but Internet Protocol addresses **WILL NOT** be collected by the survey program.

☐ Data collected online will be handled in a confidential manner and Internet Protocol addresses **WILL** be collected by the survey program.

b. Include an "I agree to participate" **and** an "I do not agree to participate" answer at the bottom of your consent document. Program the "I do not agree to participate" statement to exclude the participant from answering the remainder of the survey questions (this is accomplished through "question logic" in Survey Monkey).

Ensure that the online consent document is the first page the participant sees after clicking on the link to your online survey.

Although you may construct your own consent document, see the IRB approved Online Survey Cover Letter template (http://www.kennesaw.edu/irb/forms.html#consentdocs), which contains all of the required elements of informed consent that must be addressed within any online consent document.

6. Vulnerable Participants

Will minors or other vulnerable participants be included in this research?

☒ Yes. Outline procedures to be used in obtaining the agreement (assent) of vulnerable participants. Describe plans for obtaining consent of the parent, guardian, or authorized representative of these participants. For research conducted within the researcher's own classroom, describe plans for having someone other than the researcher obtain assent so as to reduce the perception of coercion.

The parental consent form will be sent home for parents', guardians', and authorized representatives' signatures. In addition, the consent from will include the child assent statement. Before beginning the survey, the first page will include an additional assent statement and note of voluntary participation. The students' instructor will not be the one administering the survey, therefore reducing any perception of coercion.

☐ No. All studies excluding minors as participants should include language within the consent document stating that only participants aged 18 and over may participate in the study.

7. Future Risks

How are participants protected from the potentially harmful future use of the data collected in this research?

a. Describe measures planned to ensure anonymity or confidentiality.

The survey for students will not ask for any identifiable information. Signed informed consents will not be used as the identifying link to the research data and will NOT contain participant ID numbers nor be filed with other research data files. The survey for teachers will ask for a given ID number that will be linked to that teacher. The signed consent forms as well as the surveys will be held in a locked file cabinet, in a locked room, in a locked building. These ID numbers will not be given out at any time to ensure confidentiality.

b. Describe methods for storing data while study is underway.

Data will be stored on a computer device and will be encrypted to prevent unintentional breaches of security. Digital files will be password protected. Sensitive data will also be encrypted, stored, and securely erased when appropriate. Signed parental consent forms will be stored in a locked cabinet in a locked office in a locked building.

c. List dates and plans for storing and/or destroying data and media once study is completed. Please note that all final records relating to conducted research, including signed consent documents, must be retained for at least three years following completion of the research and must be accessible for inspection by authorized representatives as needed.

At the appropriate time (3 years after study), paper records (signed consent forms) will be shredded and destroyed, and physical electronic files used to store data will be erased and the drives will be scrubbed after the files are deleted.

d. If audio, videotape, or other electronic data are to be used, when will they be erased?

My hard drive has a built-in, secure self-erase feature that can be activated with the proper software from Windows and will be utilized when data from this study can be erased. Secure destruction of a computer disk renders the disk completely unusable by degaussing the unit and by punching a hole through the disk platter; I plan to use this process to ensure that data is not available for future use.

8. Illegal Activities

Will collected data relate to any illegal activities? ☐ Yes* ☒ No

*If yes, please explain.

Is my Study Ready for Review?

Every research protocol, consent document, and survey instrument approved by the IRB is designated as an official institutional document; therefore, study documents must be as complete as possible. Research proposals containing spelling or grammatical errors, missing required elements of informed consent (within consent or assent documents), not addressing all questions within this form, or missing required documents will be classified as incomplete.

All studies classified as incomplete may be administratively rejected and returned to the researcher and/or faculty advisor without further processing.

Appendix J: Kennesaw State University Parental Consent With Student Assent Form

PARENTAL CONSENT FORM WITH CHILD ASSENT STATEMENT

Title of Research Study: Mathematical Problem-solving Dispositions and Beliefs

My name is Laura Barrett, and I am a teacher at Crabapple Middle School and a doctoral student at Kennesaw State University. I am conducting a study on students' mathematical problem-solving dispositions and beliefs. Should you wish to contact me or my university advisor, our contact information is below:

Ms. Laura Barrett	Dr. Mei-Lin Chang
770-552-4520	470-578-7795
BarrettLN@fultonschools.org	mchang6@kennesaw.edu

Your child is being invited to take part in a research study conducted by Laura Barrett of Kennesaw State University. Before you decide to allow your child to participate in this study, you should read this form and ask questions if you do not understand.

Description of Project
The purpose of the study is to develop a valid and reliable scale that measures students' mathematical dispositions and beliefs towards problem-solving.

Explanation of Procedures
Your child will be asked to fill out a survey that contains items relating to their beliefs about mathematical problem-solving, such as valuing the interaction of peers when solving problems, or valuing time spent in the problem-solving process. The survey will be given as part of a normal classroom activity.

Time Required
The survey is expected to take your child 20 minutes to complete.

Risks or Discomforts
There are no known risks or anticipated discomforts in this study. Participation is entirely voluntary and there will be no penalties or consequences for withdrawing from the study at any time.

Benefits
Individuals might benefit indirectly from participation in this study from sharing information about their mathematical problem-solving dispositions and beliefs through the survey, or gaining personal satisfaction from participating in the study.

This research has benefits to society as the knowledge gained in this study will impact the local community as well as local institutions. Institutions benefit, for example, by receiving information to improve a pre-service mathematics education program as a result of this study. The local community can benefit through improved public educational programs arising based on data about students' dispositions and beliefs involved in mathematics problem-solving.

Confidentiality
The results of this participation will be anonymous. This study does not collect identifying information of individual subjects (e.g., name, address, email address, etc.). The survey cannot link individual responses with participants' identities.

Inclusion Criteria for Participation
The age of intended participants ranges from 11-14 years.

Parental Consent to Participate

I give my consent for my child, ____ (please print), to participate in the research project described above. I understand that this participation is voluntary and that I may withdraw my consent at any time without penalty. I also understand that my child may withdraw his/her assent at any time without penalty.

____
Signature of Parent or Authorized Representative, Date

____
Signature of Investigator, Date

Research at Kennesaw State University that involves human participants is carried out under the oversight of an Institutional Review Board. Address questions or problems regarding these activities to the Institutional Review Board, Kennesaw State University, 585 Cobb Avenue, KH3403, Kennesaw, GA, 30144-5591, (470) 578-2268.

Child Assent to Participate

My name is Laura Barrett. I am inviting you to be in a research study about students' mathematical beliefs about problem-solving. Your parent has given permission for you to be in this study, but you get to make the final choice.

If you decide to be in the study, I will ask you to take a short survey about your mathematical problem-solving beliefs. By taking part in this survey you will be helping not only me gain understanding into students' beliefs about mathematical problem-solving, but you will also be helping our society, as they too will gain insight into students' beliefs. Everything you say and do will be private, your teachers will not be told what you say or do while you are taking part in the study. When I tell other people what I learned in the study, I will not tell them your name or the name of anyone else who took part in the research study.

If anything in the study worries you or makes you uncomfortable, let me know and you can stop. No one will be upset with you if you change your mind and decide not to participate. You are free to ask questions at any time. Please print your name on the line below if you are willing to participate and check the box:

☐ I want to be part of this study

Child's Name	Signature	Date

Check which of the following applies *(completed by person administering the assent)*

☐ Child is capable of reading and understanding the assent form and has signed above as documentation of assent to take part in this study.

☐ Child is not capable of reading the assent form, but the information was verbally explained to him/her. The child signed above as documentation of assent to take part in this study.

____

Signature of Person Obtaining Assent, Date

Appendix K: Kennesaw State University Teacher Consent Form

CONSENT FORM

Title of Research Study: Mathematical Problem-solving Dispositions and Beliefs

My name is Laura Barrett, and I am a teacher at Crabapple Middle School and a doctoral student at Kennesaw State University. I am conducting a study on students' mathematical problem-solving dispositions and beliefs. Should you wish to contact me or my university advisor, our contact information is below:

Ms. Laura Barrett	Dr. Mei-Lin Chang
770-833-7209	470-578-7795
lleduc@students.kennesaw.edu	mchang6@kennesaw.edu

You are being invited to take part in a research study conducted by Laura Barrett of Kennesaw State University. Before you decide to participate in this study, you should read this form and ask questions if you do not understand.

Description of Project
The purpose of the study is to develop a valid and reliable scale that measures students' mathematical dispositions and beliefs towards problem-solving.

Explanation of Procedures
You will be asked to fill out a survey that contains items relating to model-eliciting activities you use in your class. These items are related to mathematical problem-solving, such as valuing the interaction of peers when solving problems, or valuing time spent in the problem-solving process

Time Required
The survey is expected to take you 20 minutes to complete.

Risks or Discomforts
There are no known risks or anticipated discomforts in this study. Participation is entirely voluntary and there will be no penalties or consequences for withdrawing from the study at any time.

Benefits
Individuals might benefit indirectly from participation in this study from sharing information about their mathematical dispositions and beliefs through the survey, or gaining personal satisfaction from participating in the study.

This research has benefits to society as the knowledge gained in this study will impact the local community as well as local institutions. Institutions benefit, for example, by receiving information to improve a pre-service mathematics education program as a result of this study. The local community can benefit through improved public educational programs arising based on data about students' dispositions and beliefs involved in mathematics problem-solving.

Confidentiality

The results of this participation will be confidential. This study does not collect identifying information of individual subjects (e.g., name, address, email address, etc.). The survey cannot link individual responses with participants' identities.

Consent to Participate

I, _________________________, understand that this participation is voluntary and that I may withdraw my consent at any time without penalty.

____

Signature of Participant , Date

____

Signature of Investigator, Date

____

Research at Kennesaw State University that involves human participants is carried out under the oversight of an Institutional Review Board. Address questions or problems regarding these activities to the Institutional Review Board, Kennesaw State University, 585 Cobb Avenue, KH3403, Kennesaw, GA, 30144-5591, (470) 578-2268.

____

Appendix L: Fulton County IRB Research Application

Department of Research & Program Evaluation
Office of Accountability
Fulton County School District
Phone: 470-254-4906
Email: fcsresearch@fultonschools.org

Dear Research Study Applicant:

Thank you for your interest in conducting research in the Fulton County School District (FCS). It is the goal of FCS and the Department of Research & Program Evaluation (DRPE) to participate in research efforts that will substantially benefit FCS, its students, and/or staff.

Each year, FCS receives a number of requests to participate in research investigations. While we are eager to participate in research that will substantially benefit our system, students, and/or staff, it is not feasible or desirable for FCS to participate in every proposed research project. Thus, researchers are required to provide a Research Study Application for proposed research projects that fall within the guidelines and policies regarding research adopted by FCS. The Research Study Application is designed to provide the review committee with sufficient information in order to reach a decision about the appropriateness of FCS participation in the research project. Your application will be evaluated based on the following criteria:

- Alignment of proposed research with FCS strategic priorities
- Technical soundness of the research methodology, measures, and proposed analysis
- Feasibility of study design in terms of time requirements from staff and students
- Confidentiality of data and privacy of participants
- Compliance with human consent procedures
- Appropriateness of the research topic for support in the public school setting
- Clarity of purpose and thoroughness of research plan

Please carefully review the *Research Application Resource Guide* available on the *DRPE website* prior to completing this application. The Resource Guide provides detailed information about the application submission process including the timeline for when proposals will be reviewed and when notifications of committee decision will be sent out. Questions about the application process and/or application materials should be directed to 470-254-4906 or fcsresearch@fultonschools.org. Again, thank you for your interest. We look forward to receiving your research application.

Department of Research and Program Evaluation

Application for Conducting Research

SECTION A: APPLICATION INFORMATION			
Project Title: Mathematical Problem-solving Dispositions and Beliefs			
Researcher's Full Name(s): Laura Barrett		Title/Position: Teacher	
University/Institution/Organization: Kennesaw State University			
Mailing Address: 550 Saddle Creek Circle Roswell, GA 30076			
Email Address: BarrettLN@fultonschools.org		Daytime Phone: 770-833-7209	
Date Submitted: 10/26/15	Is this the final version of the proposal?		Yes ☒ No ☐
Projected Start Date: December 2015	Projected Completion Date: July 2016		
I have reviewed and understand:			
FCS Policy ICC – Educational Research			Yes ☒ No ☐
Research Application Resource Guide			Yes ☒ No ☐
The research is related to a: ☒ Doctoral Study ☐ Masters Study ☐ Class Project ☐ Professional Project (Non-profit organization) ☐ Professional Project (For-profit organization)			
Have you included a copy of your Institutional Review Board (IRB) approval form with your application?		Yes ☒ No ☐ *(no institutional research will be approved without prior IRB approval)*	
Has any FCS staff member already agreed to assist you upon approval of this study? (Please note that this agreement is not binding on the District)		Yes ☐ No ☒ *(if yes, please provide documentation of the agreement)*	

Graduate students only: Please fill out the information below *Fulton County Schools reserves the right to contact university faculty associated with a proposed research project. Advisor contact information is required for ALL graduate student projects.*			
Advisor's Name:	Dr. Mei-Lin Chang	Title/Position: Professor/Advisor/Dissertation Chair	
E-mail Address:	mchang6@kennesaw.edu	Daytime Phone: 470-578-7795	
Mailing Address:	Bagwell College of Education Kennesaw State University 580 Parliament Garden Way NW, MD # 0122 Kennesaw, GA 30144		
Have all advisory/regulatory committee members formally approved this research?			Yes ☒ No ☐

Fulton County Employees Only: Please fill out the information below *School principals may approve research studies in which only one school is involved and in which the research will be conducted entirely by FCS employees who work at that school.*	
School Leader's Name: Dr. Rako Morrissey	Title/Position: Principal
Email Address: morrisseyr1@fultonschools.org	Daytime Phone: 770-552-4520
Mailing Address:10700 Crabapple Road	

Roswell GA 30075	
Has the principal/designee formally approved this research?	Yes☒ No☐ (*if yes, please provide documentation of the agreement)*

SECTION B: STUDY SPECIFICATIONS
Study supports the following FCS Strategic Plan Strategic Initiative Focus Area(s): ***Check all that apply***
Instruction-Ensure that all students are engaged in learning that enables them to reach their full potential for college and career readiness. ☐ Continuous Achievement ☒ Effective Assessment of Learning and Feedback ☐ Tailored Supports ☒ Challenging and Innovative Instruction ☒ Application of Learning
People-Ensure FCS attracts and retains the most talented and effective employees in K-12 education ☒ Supportive Culture ☐ Accountability ☐ Support and Development ☒ Top Talent ☐ Effective Employees
Technology-Ensure teachers, parents and students have the tools and information necessary to accelerate learning. ☐ Student Access ☐ Data-Driven Decision Making ☐ Stakeholder Skills
Effective Schools-Ensure effective schools through collaborative leadership that balances innovation with accountability for achievement of all students. ☐ School Governance ☐ Strong School Support
Resources-Ensure student needs are supported with efficient and effective allocation of staff, instructional materials, and equipment. ☐ Resource Flexibility
Relevance of Research to FCS- Describe why the research is specifically relevant to FCS and its Strategic Plan and how it would substantially benefit FCS, its students, and/or staff. (Response should not exceed 200 words.)

Fulton county's mission is to educate every student to be a responsible, productive citizen. The models-and-modeling perspective on mathematical problem-solving promotes students to use their mathematical knowledge to benefit their local community and design artifacts that can be used by society. Model-eliciting activities encourage communities of practice within a mathematics class to achieve this mission. This study's purpose is to development and validate a scale that would measure students' mathematical problem-solving disputations and beliefs. Measuring this particular construct allows schools to recognize productive depositions and beliefs that do exist and foster development in those that display low levels of productive dispositions and beliefs by focus on preparing all students with the needed skills to be productive and responsible in a world relying increasingly on technology, collaboration and communication, through mathematical problem-solving as defined in a models-and-modeling perspective. This study supports the strategic theme of instruction as it promotes challenging and innovate instruction, application of learning, and effective assessment of learning and effective feedback.

Study includes participants at: ***Check all that apply.***	☐ Elementary School ☐ Start-Up Charter School ☐ Other (*please explain):*	☒ Middle School ☐ Alternative School	☐ High School ☐ Administrative Office
Area(s) of Study: ***Check all that apply.***	☐ Language Arts ☐ Social Studies ☐ Foreign Language ☐ Talented & Gifted ☐ Students with Disabilities	☒ Mathematics ☐ Art ☐ Career/Technical Education ☐ ELL/ESOL/LEP ☐ Other(*please explain*):	☐ Science ☐ Health/Physical Education ☐ Technology ☐ Economically Disadvantaged
Type of Study	☒ Quantitative	☐ Qualitative	☐ Mixed-Methods

Does study employ:	***Check all that apply.***
Non-school personnel surveys (e.g., surveys of District level staff, like superintendents) All surveys must be attached.	Yes☐ No☒
School administrator surveys. All surveys must be attached.	Yes☐ No☒
Teacher surveys. All surveys must be attached.	Yes☒ No☐
Student surveys. All surveys must be attached.	Yes☒ No☐
Parent surveys. All surveys must be attached.	Yes☐ No☒
Non-school personnel interviews	Yes☐ No☒
School administrator interviews	Yes☐ No☒
Teacher interviews	Yes☐ No☒
Student interviews	Yes☒ No☐

Parent interviews	Yes☐ No☒
Classroom observations	Yes☐ No☒
Audio recording of FCS staff	Yes☐ No☒
Audio recording of FCS students	Yes☐ No☒
Video recording of FCS staff	Yes☐ No☒
Video recording of FCS students	Yes☐ No☒

Does study access, require, or record:	***Check all that apply.***
De-identified student-level information	Yes☐ No☒
De-identified staff-level information	Yes☐ No☒
Aggregated student-level information	Yes☐ No☒
Aggregated staff-level information	Yes☐ No☒
School-level information	Yes☐ No☒
Other (*please explain):*	

Section C. Proposal Summary

Instructions: Please answer each question below within this document.

Relevant documents (e.g., questionnaires, consent forms, IRB approvals, etc.) should be sent electronically with this application.

Mathematical Problem-solving Dispositions and Beliefs

2. **Purpose and research questions:** Specify purpose of research study and the primary research questions to be addressed. (Response should not exceed 200 words)

The purpose of this research is to measure students' mathematical dispositions and beliefs towards problem-solving from a models- and- modeling approach. This study will involve developing a valid and reliable scale to measure students' mathematical dispositions and beliefs towards problem-solving.

3. **Rationale for Research-** Provide a brief description of the theoretical background for the study, including references, where appropriate. (Response should not exceed 500 words.)

A models-and-modeling perspective is associated with philosophies of constructivism and the sociocultural view. Lesh and Zawojewski (2007) claim "the development of problem-solving ability are highly interdependent and far more socially constructed and contextually situated than traditional theories have supposed" (p. 779). Thus, Lesh and Doerr (2003) propose a models-and-modeling perspective for conducting research and interpreting results.

Through a models-and-modeling perspective students are able to experience a process of revision and analysis as they create products for the real world.

Models-and-modeling perspectives adopt more sophisticated conceptions of development based on the observation that when students (or groups) go through a series of modeling cycles in which they integrate, differentiate, revise, and refine their existing relevant ways of thinking development seldom occurs along a single, one-dimensional, ladder-like sequence. Thus, problem-solving is a process in which students develop tools for use in the everyday context of the mathematical world. Teams of students must learn to perform research that informs their decisions while making meaning of the mathematics. An important characteristic of a models-and-modeling perspective is that student research is planned around the construction of tools that are then tested in the classroom (Lesh & Zawojewski, 2007).

A models-and-modeling perspective is based in the creation of tangible tools and artifacts for school-based use by employing the principles portrayed by a suggested conceptual model of learning and teaching (Lesh & Zawojewski, 2007).Solutions to mathematical problem-solving tasks become reusable not only to the students but also to other people. These solutions require refinement, revision, and testing.

Additionally, the models-and-modeling perspective has characteristics of situated cognition and communities of practice. As stated, “recent themes of research – such as those related to situated cognition, communities of practice, and representational fluency – seem to be converging to a models-and-modeling perspective on mathematical learning and problem-solving” (Lesh & Zawojewski, 2007, p. 793). The reasons for this shift is that knowledge is socially situated and through the models-and-modeling approach students engage with others as they develop conceptual tools and mathematical concepts for a problem-solving context.

One thing to remember is that “neither concept development nor the development of problem-solving abilities proceeded in the absence of beliefs, feelings, dispositions, values and other concepts of a complete problem-solving persona” (Lesh & Zawojewski, 2007, p. 779). Beliefs play a role in problem-solving as they impact interpretation of situations. The effect of beliefs cannot be ignored. The models-and-modeling perspective has the capacity to “develop productive beliefs and affect” (Lesh & Zawojewski, 2007, p. 793). Thus, students as well as teachers must be aware of beliefs that impact one’s problem-solving identity. The models-and-modeling perspective creates an environment where the class is a safe place to develop these beliefs about mathematical problem-solving.

Students need to learn their beliefs are complex and flexible. Beliefs impact one’s success in a mathematics classroom and therefore students need to not only be aware of these beliefs, but understand that their beliefs can change. McLeod (1989) cautioned teachers as he recognized the emotional state of students is expected to range from positive to negative during problem-solving, however; consistently viewing problem-solving negatively could affect one’s permanent view.

4. **Methods-** This section should include your procedures for what will you do, with whom, and when you will conduct the different activities in sufficient detail for a review to fully understand the implementation plan. Also include a description of the sample and proposed analyses. (Note: The technical soundness of your research methodology, measures, and analysis will be considered in the review process) Please attach all surveys and assessments you propose to use during the research.

The design of the MPSDB scale development procedures began with DeVellis’ suggestion from Scale Development: Theory and Applications (2012). The following steps will be carried out in this study

Step 1: Item Generation → Literature Review

Step 2: Expert Review of Measures→ Content Validity

Step 3: Collect Data → Initial Survey

Step 4: Purify Measures→ Factor Analysis

Step 5: Collect Data→ Factor Analysis

Step 6: Assess Reliability→ Coefficient Alpha/Split Half Reliability

Step 7: Assess Validity → Criterion (GPA), Construct (Fennema-Sherman Correlation & May Correlation), Content (Expert Panel & Ratings)

Step 8: Conclusion of Statistics → Summarize Distribution of Scores

Initial items were generated through a review of mathematics education literature on problem-solving. An initial list of items was generated based on the themes that ran through the literature and an examination of other scale. The following sections provides an overview of these themes.

Six of 13 invited experts received the initial pool of items presented in MPSDB scale. At least two individuals participated in the panel from each area of expertise including Measurement/Scale Development (n= 3); Secondary Education (n=2); Mathematics Teaching (n=3); and Mathematics Problem-solving (n=3). A rating from designed to evaluate potential MPSDB items was sent to each expert via email. This from included four sections that asked experts to judge the following: 1) relevancy of each item to the conceptual definition of Mathematical Problem-solving as approached in a models-and-modeling perspective(high relevance = 1, moderate relevance = 2, low relevance= 3); 2)realistic beliefs related to mathematical problem-solving (very realistic = 1, realistic = 2, not realistic = 3); 3) word choice with respect to its appropriateness to the target audience(very appropriate= 1, appropriate= 2, not appropriate = 3); and 4) response format with respect to its relevance to the items(very relevant= 1, relevant= 2, not relevant= 3).

This study will be conducted with 330 middle school students and 8 middle school teachers from Fulton County Public Schools. The students will be enrolled in either 7th or 8th grade mathematics. The teachers will teach either 7th or 8th grade mathematics. All 330 students will be invited to take a paper version of the MPSDB survey. All teachers will be invited a paper version of the models-and-modeling questionnaire. During the recruitment phase, an email invitation along with a written letter will be provided to all 7th and 8th grade teachers and parents with a brief overview of the study, guidelines of data collection procedures, and letter of consent. Students will also be asked to read and sign an assent form directly before data is collected. To maximize the validity of self-reports, the confidently and anonymity of responses will be emphasized to participants. The researcher gained access to the site as she taught 8th grade mathematics at this school and out of convenience selected the 7th and 8th grade students and teachers to participate. The principal of the school acted as the gatekeeper between the school district and the researcher giving permission to the researcher to conduct the study at this public school pending county approval. According to Costello & Osborne (2005), subject to item ratios of 10:1 are acceptable. This “early and still-prevalent rule-of-thumb” is still suggested by researchers for determining *a priori* sample size. Thus the study will need to include around 300 participants.

A focus group will need to be conducted with 8th grade students (n=14), females (n=7) and males (n=7). Focus groups involve carefully planned and documented discussions among homogenous individuals around specified topics of interest (Kitzinger, 1995). Focus groups will be employed to get feedback on initial scale items. Focus groups discussions will delve into perceptions and interpretations of the scale items as well as other beliefs and dispositions students might have about mathematical problem-solving .The focus groups will be led by the researcher of this study.

The preliminary scale will be prepared for administration following the expert panel review, and analysis of focus group data. The preliminary scale will be administered to a group of 7th grade students (n=50) in two Fulton county public school mathematics classes. The lead researcher will administer the anonymous paper/pencil self -report instrument during class time. She will follow the protocol she developed by advising students to choose the best answer for each question, and if the respondent is unsure or unclear about a question, they will be asked to leave it blank or write in what their own thoughts on that particular topic.

After expert panel review and focus group analysis, the final scale will then be administered to all 7th and 8th grade students. To run validity measures students will be also asked to report both their overall and mathematics grade point average (GPA), in addition to their mathematics teacher. In addition the MPSDB scale will include items from the Fennema-Sherman Mathematics -Usefulness of Mathematics Scale (1976) and items from May’s Mathematics Self-Efficacy and Anxiety Questionnaire (2009) to establish construct validity.

Exploratory factor analysis will be used to determine which factors account for the most variance. Factor analysis involves the last recommended procedure presented by DeVellis (2012) as he encouraged the optimization of scale length. Factor analysis involves Churchill's (1979) emphasis for the researcher to provide guidance on the interpretation of the results, as this statistical procedure provides a frame of reference to describe relations among the variables by defining the number of variables and allowing for interpretation.

Using The Statistical Package for the Social Sciences (SPSS), exploratory factor analysis (EFA) will be performed to indicate the factor structure of the MPSDB scale. This statistical analysis is convenient as it was suspected that a measure designed to assess mathematical problem-solving dispositions and beliefs among secondary mathematics students contains a dimensional structure and that measuring the separate dimensions would lead to a better understanding of the construct. According to Fabrigar, Wegener, MacCallum, & Strahan (1999), "the primary purpose of EFA is to arrive at a more parsimonious conceptual understanding of a set of measured variables by determining the number and nature of common factors needed to account for the pattern of correlations among the measured variables"(pg.275).

After extraction the researcher will decide how many factors to retain for rotation. Cattell's (1996) scree test will be used to determine how many factors to retain. Costello & Osborne (2005) claim that the scree test is the "best choice for researchers" as it is contained in most statistical software packages and commonly used. In the scree test the eigenvalues are given in decreasing order and linked with a line. The researcher will examine the eigenvalues of the graph created in SPSS to determine the point at which the last significant drop or break took place. The researcher will create a scree plot that plots the eigenvalues against the corresponding factor numbers as this graph gives insight into the number of factors to extract as one can examine when the rate of decline tends to become almost horizontal, indicating that each successive factor is accounting for smaller and smaller amounts of variance. According to Ledesma & Valero-Mora (2007), this "point divides the important or major factors from the minor or trivial factors." The scree plot suggest a maximum of 6 factors in this study.

Rotation will be performed to simplify and clarify the data structure. According Costello& Osborne (2005), educational fields generally anticipate some correlation among factors, since human feelings and beliefs are rarely segregated into boxed units that function independently of one another. For this reason oblique rotation will be used. After performing oblique rotation in SPSS the researcher will examine the pattern matrix for item loadings and the factor correlation matrix, which reveal correlation between the factors. The factor matrix will examined to determine the communalities. Generally communalities are considered high if their value is greater than or equal to .8. However, common magnitudes in the social sciences tend to be more moderate with values of .40 to .70(Costello & Osborne, 2005). If the item has a magnitude of less than .4 that item will be dropped. Also note that factors with fewer than three items is generally weak; 5 or more strongly loading items (.50 or better) are desirable and indicate a solid factor (Costello & Osborne). A factor with fewer than three items is generally weak and unstable; 5 or more strongly loading items (.50 or better) are desirable and indicate a solid factor. Thus factors in this study will have 5or more items.

The reliability coefficient is defined as "the proportion or percent of test score variance due to true score differences" (Friedenberg, 1995, p. 182). The formula used calculate the reliability coefficient in this study can be seen below, where r_{xy} is the reliability coefficient, ${\sigma_t}^2$ is the true score variance, and ${\sigma_x}^2$ is the observed score variance:

$$r_{xy} = \frac{{\sigma_t}^2}{{\sigma_x}^2}$$

The ratio should be close to 1 if there is little error, and hence, a high reliability. Conversely, if the ratio is close to 0, it implies no correlation, and no reliability. A ratio between .7 to .9 is adequate to establish reliability (Nunnally, et al., 1967). The ratio was calculated using reliability analysis in SPSS.

Alternate form, which DeVellis (2012) refers to as parallel form, is used to assess the degree to which two different forms measure the same characteristic. As time will be a limiting factor when administering a survey in school, alternate form will be utilized. For instance, the even and odd items can count as the two different forms that can be correlated to establish reliability. Cronbach's (1951) alpha is typically used to examine reliability. According to Churchill (1979) and DeVellis (2012) the recommended measure of internal consistency is provided by coefficient alpha. Nunnally & Bernstein (1978) also recognize that coefficient alpha provides a worthy estimate of reliability. "Alpha is defined as the proportion of a scale's total variance that is attributable to a common source, presumably the true score of a latent variable underling the items" (DeVellis, 2012, p. 37). The formula used in this study to calculate alpha is below where α is the coefficient alpha, k is the number of items, and $\frac{\sum \sigma_i^2}{\sigma_{yi}^2}$ is the total proportion of total variance:

$$\alpha = \frac{k}{k-1}\left(1 - \frac{\sum \sigma_i^2}{\sigma_{yi}^2}\right)$$

The alternate form of alpha can be calculated by using Spearman-Brown (1910) formula below:

$$\alpha = \frac{kr}{1 + r(k-1)}$$

Again, where α is the coefficient alpha, k is the number of items, and r is the average inter item correlation. Alpha will be calculated using reliability analysis in SPSS.

Test length and test score variability are two other considerations when developing a scale in terms of the reliability. Longer tests usually provide a more representative sample of reliability. The Spearman-Brown (1910) formula can be used to determine if increasing or decreasing the number of items on the MPSDB scale results in more reliable results. In fact the following equation was used to estimate the number of items needed to obtain highly reliable results where k is the number of items the test would have to be lengthened to, r_{kk} is the desired reliability, and r_{11} is the reliability of the existing test:

$$k = \frac{r_{kk}(1 - r_{11})}{r_{11}(1 - r_{kk})}$$

In addition, the scale needs to truly test the characteristic it intends. "The most reliable tests are those that include a representative sample from this set of possible test items" (Friedenberg, 1995, p. 185). Purifying items using reliability formulas as well as performing factor analysis procedures to determine grouping and clusters of variables occurred at this stage to ensure reliability with optimal test length.

According to Friedenberg (1995), criterion validity "is the ability of a test to predict performance on another measure" (p. 94). This type of validity is important when making decisions about future performance. In the case of this study, if the MPSDB scale designed has criterion validity, the scale would predict a relevant criterion measure, such as grade point average (GPA). This type of validity is sometimes referred to as concurrent validity (DeVellis, 2012). This name is given based on the approach used to obtain the criterion validity. The criterion validity coefficient can be calculated, which represents the relationship between scores: the predictor and the criterion. This statistic is known as r_{xy}. r_{xy} indicates the relationship between predictor and criteria. In this concurrent validity study, the researcher determined the correlation between test scores and a current criterion measure using SPSS. Friedenberg (1995) states that theoretically, the proportion of interest in criterion validity is $\frac{\sigma_R^2}{\sigma_X^2}$ (p. 227). The square of the coefficient, which would be $r_{xy}{}^2$ is the coefficient of determination. This statistic indicates the proportion of variance in criterion scores predicted by test scores. SPSS will be used to correlate the scale scores with scores of overall GPA and mathematics' GPA, as those students who have productive dispositions and beliefs on problem-solving potentially have correlations with these measures. Correlation will be run as this particular scale and correlation has not been explored before.

Content validity consists of detailed domains of items included on the scale. According to DeVellis (2012), "a scale has content validity when its items are a randomly chosen subset of the universe of appropriate items:" (p. 60). An expert in the field is required to judge whether the subset reflects the specific domain. This is why this study will employ an expert panel when constructing items to determine if the items on the MPSDB scale are appropriate to mathematical problem-solving dispositions and beliefs. For this study the expert panel will consist of mathematics education professors as well as experienced mathematics teachers. It is essential that differences in test scores, σ_X^2, reflect differences in domain relevant characteristics, σ_R^2. Although content validity seems to involve qualitative measures, Brown (1983) and Cronbach & Thorndike (1971) suggest using statistical measures to support any conclusions made by the expert judges. This study will use SPSS to analyze the content validity. Brown's (1983) suggestion involves creating a scale that judges would use to rate a particular scale. The ratings will then be analyzed and compared using SPSS. A higher degree of interrelated reliability implies consensus and thus establishes content validity.

Finally, performing measures of construct validity will determine whether the MPSDB scale accurately measured mathematical problem-solving dispositions and beliefs. Congruent validity measures will be used to demonstrate whether the MPSDB scale measured what it intended to measure. According to Friedenberg (1995), a common procedure is "to correlate scores on the test with scores on another established test measuring the same construct" (p.254). The correlation coefficient should be positive, establishing congruent validity. This study will use MPSDB scale scores and correlate them with items from the Mathematics Self-Efficacy and Anxiety Questionnaire (MSEAQ) (May, 2009), as mathematics self-efficacy has been associated with achievement and persistence to problem solve. In addition, items from the Fennema-Sherman Mathematics Attitudes Scale-Usefulness of Mathematics Scale (1976) will be used to correlate scores as "perceived usefulness of mathematics is an important component of motivation" and problem-solving (Kloosterman & Stage, 1992).

5. **Timeline:** Describe your timeline for the research study, include when you plan to analyze and report on the data.

Pending approval with the county, data will be collected in late December 2015 to early January 2016, after consent and assent forms have been signed and returned, as this will be when the scales are administered. The data will be analyzed in the spring of 2016 and the report of the data will be presented to dissertation committee members in the summer of 2016.

6. **How much time will each of the various sets of participants be required to commit to this study?**

Participants are invited to spend around 20 minutes to complete the surveys.

7. **Describe any existing data that you will need from the school system (e.g., demographics, test scores) and how this will help you answer your research questions.**

No data will be collected from the school as this is a self-reported survey on feelings and beliefs toward mathematical problem-solving.

8. **Identify any ethical or privacy issues that may be of concern to FCS or parents, and explain how you have addressed them.**

All research methods and necessary consent forms were approved and stamped by Kennesaw State University Institutional Review Board (IRB). IRB applications including student assent, parent consent forms, and teacher consent forms are attached. In addition all Fulton County approval forms including county application, parent guardian permission form, and principal support form are attached.

There are no known risk in this study. Individuals might benefit indirectly from participation in this study from sharing information about their mathematic dispositions and beliefs through the survey, or gaining personal satisfaction from participating in the study. This research has benefits to society as the knowledge gained in this study will impact the local community as well as local institutions. Institutions benefit, for example, by receiving information to improve a preservice mathematics education program as a result of this study. The local community can benefit through improved public educational programs arising based on data about students' dispositions and beliefs involved in mathematics problem-solving.

The parental consent form will be sent home for parents', guardians', and authorized representatives' signatures. In addition, the consent from will include the child assent statement. Before beginning the survey, the first page will include an additional assent statement and note of voluntary participation. Signed parental consent forms will be stored in a locked cabinet in a locked office in a locked building. The students' instructor will not be the one administering the survey, therefore reducing any perception of coercion. The survey for students will not ask for any identifiable information. Signed informed consents will not be used as the identifying link to the research data and will NOT contain participant ID numbers nor will they be filed with other research data files. The survey for teachers will ask for a given ID number that is linked to that teacher. The signed consent forms as well as the surveys will be held in a locked file cabinet, in a locked room, in a locked building. These ID numbers will not be given out at any time to ensure confidentiality.
Data will be stored on a computer device and will be encrypted to prevent unintentional breaches of security. Digital files will password protected. Sensitive data will was also encrypted, stored, and securely erased at the appropriate time.

9. **What is your plan to share and disseminate results? -Describe with whom and how you will share your results.**

The data will be shared with my committee members during a public defense held at Kennesaw State University. The results will be shared in the written dissertation as well as orally through a power point presentation.

10. **Participant consent:** Researchers must obtain written permission from the student participant's parent/guardians using the Parent/Guardian permission form (template available on department website). Please attach a copy of the consent forms you will us to obtain permission from all study participants, including parents if students are involved.

[Note]Forms are attached

11. **Confidentiality:** Researchers will only be provided with de-identified student data. Please explain how you will maintain the confidentiality of your study participants. Specifically, who will have access to this data? For what purposes will research data be shared? What will you do with the data after the analyses are complete? What security measures will you take with the data?

The parental consent form will be sent home for parents', guardians', and authorized representatives' signatures. In addition, the consent from will include the child assent statement. Before beginning the survey, the first page will include an additional assent statement and note of voluntary participation. Signed parental

consent forms will be stored in a locked cabinet in a locked office in a locked building. The students' instructor will not be the one administering the survey, therefore reducing any perception of coercion. The survey for students will not ask for any identifiable information. Signed informed consents will not be used as the identifying link to the research data and will NOT contain participant ID numbers nor will they be filed with other research data files. The survey for teachers will ask for a given ID number that is linked to that teacher. The signed consent forms as well as the surveys will be held in a locked file cabinet, in a locked room, in a locked building. These ID numbers will not be given out at any time to ensure confidentiality.
Data will be stored on a computer device and will be encrypted to prevent unintentional breaches of security. Digital files will password protected. Sensitive data will was also encrypted, stored, and securely erased at the appropriate time.

12. **Human Subjects:** How do you plan to protect human subjects during this research?

All research methods and necessary consent forms were approved and stamped by Kennesaw State University Institutional Review Board (IRB). IRB applications including student assent, parent consent forms, and teacher consent forms are attached. In addition all Fulton County approval forms including county application, parent guardian permission form, and principal support form are attached.

There are no known risk in this study. Individuals might benefit indirectly from participation in this study from sharing information about their mathematic dispositions and beliefs through the survey, or gaining personal satisfaction from participating in the study. This research has benefits to society as the knowledge gained in this study will impact the local community as well as local institutions. Institutions benefit, for example, by receiving information to improve a preservice mathematics education program as a result of this study. The local community can benefit through improved public educational programs arising based on data about students' dispositions and beliefs involved in mathematics problem-solving.

The parental consent form will be sent home for parents', guardians', and authorized representatives' signatures. In addition, the consent from will include the child assent statement. Before beginning the survey, the first page will include an additional assent statement and note of voluntary participation. Signed parental consent forms will be stored in a locked cabinet in a locked office in a locked building. The students' instructor will not be the one administering the survey, therefore reducing any perception of coercion. The survey for students will not ask for any identifiable information. Signed informed consents will not be used as the identifying link to the research data and will NOT contain participant ID numbers nor will they be filed with other research data files. The survey for teachers will ask for a given ID number that is linked to that teacher. The signed consent forms as well as the surveys will be held in a locked file cabinet, in a locked room, in a locked building. These ID numbers will not be given out at any time to ensure confidentiality.
Data will be stored on a computer device and will be encrypted to prevent unintentional breaches of security. Digital files will password protected. Sensitive data will was also encrypted, stored, and securely erased at the appropriate time.

13. **Compliance:** Are you prepared to comply with all the terms in the Resource Manual?

YES, I will comply with all terms in the resource manual and look forward to hearing back from Fulton County's department research about the status of my application.

Appendix M: Principal Support Form

Fulton County Schools
Where Students Come First

Principal Support to Conduct Research in Schools

Dear Principal:

The researcher identified below has submitted a proposal to the Department of Research and Program Evaluation (DRPE) and requested that your school serve as a site for his/her research. While the DRPE evaluates the proposal in terms of research design, methodology, and compliance with federal regulations and also obtains district-level feedback, the researcher must secure your support and permission to conduct the study in your school.

Researchers should clearly describe their project and provide you with a detailed description of the research activities that will take place in the school. **Please complete this form and return it to the researcher so that he/she can submit it to the *Department of Research and Program Evaluation*.** Forms must be on file prior to the initiation of the study.

Researcher/Principal Investigator: Laura Barrett

Title of Study: Mathematical Problem Solving Dispositions and Beliefs

Research will involve: Scale development and validation to measure mathematical problem solving dispositions and beliefs among secondary mathematics students based on a models-and-modeling perspective.

Cooperating School: Crabapple Middle School Grade(s): 8

#Classes: 20 #Students: 330 # Staff: 8

Data Collection Start Date: 12/1/15 Data Collection End Date: 6/1/16

This study has been explained to my satisfaction: ☒ Yes ☐ No

This study may be conducted in my school: ☒ Yes ☐ No

Principal Name(print): Rako Morrissey

Signature: Rako Morrissey Date: 11-11-15

Please contact the Department of Research and Program Evaluation (fcsresearch@fultonschools.org) with any questions.

Department of Research and Program Evaluation - Office of Accountability – Fulton County Schools

Appendix N: Fulton County Parent Guardian Permission Form

Parent/Guardian Permission Form

My signature below indicates that I have read the information provided and have decided to allow my child to participate in the study titled "**Mathematical Problem-solving Dispositions and Beliefs**" to be conducted at my child's school between the dates of **January 2016 to July 2016**.

I understand the purpose of the research project will be to develop a valid and reliable scale measuring mathematical problem-solving dispositions and beliefs and that my child will participate in the following manner:
1. Take a 20 minute survey during regular school hours

I understand that the following data pertaining to my child will be requested/collected:

1. Self-reported feelings and dispositions in relation to mathematical problem-solving
2. Self-reported beliefs about mathematical problem-solving
3. Self-reported grade point average (GPA)

This research has benefits to society as the knowledge gained in this study will impact the local community as well as local institutions.
1. Institutions benefit, for example, by receiving information to improve a preservice mathematics education program as a result of this study.
2. The local community can benefit through improved public educational programs arising based on data about students' dispositions and beliefs involved in mathematics problem-solving.

I agree to the following conditions with the understanding that I can withdraw my child from the study at any time should I choose to discontinue participation.

- The identity of participants will be protected. The survey is anonymous. The survey will not ask for any identifiable information.
- Information gathered during the course of the project will become part of the data analysis and may contribute to published research reports and presentations.
- Participation in the study is voluntary and will not affect either student grades or placement decisions (or if staff is involved, will not affect employment status or annual evaluations.) If I decide to withdraw permission after the study begins, I will notify the school of my decision in writing.

If further information is needed regarding the research study, I can contact:
Laura Barrett
BarrettLN@fultonschools.org
(770)-552-4520
10700 Crabapple Rd, Roswell, GA 30075

If I wish to review any instrument or instructional material used in connection with any protected information or marketing survey, I may submit a request to the school principal. The school principal will notify me of the time and place where I may review these materials. I have the right to review a survey and/or instructional materials before the survey is administered to my student.
This also serves as assurance that the Fulton County School District complies with requirements of the Family Educational Rights and Privacy Act (FERPA) and the Protection of Pupil Rights Amendment (PPRA) and will ensure that these requirements are followed in the conduct of this research. The District provides parents/guardians information regarding rights under FERPA and PPRA annually in the Code of Conduct & Discipline Handbook. Additional information regarding compliance of research studies with FERPA and PPRA may be found in District Policy/Procedure ICC – Educational Research. By signing this letter you are disclosing you are aware of those rights.

Student Name
____

____________________________________ _______________
Parent/Guardian Signature
Date

Appendix O: Pilot Study Factor Loadings

Table 1

Factor Loadings and Communalities Based on Exploratory Factor Analysis with Direct Oblimin Rotation for 11 items from the Mathematical Mindset Scale (N=150)

Item	Factor 1 Mathematical Mindset	Factor 2 Mathematical Mindset
By trying hard, I can become better at math	.821	
Hard work can increase my ability in math	.878	
The more mathematics I learn, the more my math ability grows	.726	
The harder I try, the better I can be at math	.850	
I am afraid to give an incorrect solution to a math problem because I feel judged		.470
I learn from making mistakes in math , which pushes me to work harder next time	.534	
There is nothing I can do to increase my math ability		.495
I will never be good at math	.403	
I get better in math because I learn more every year	.702	
If I can’t seem to solve a math problem, I work harder and try new strategies	.585	

Note: Factor loadings <.4 are suppressed

Table 2
Factor Loadings and Communalities Based on Exploratory Factor Analysis for 12 Items from the Mathematical Problem-solving Perseverance Scale (N=150)

Item	Factor 1 Mathematical Problem-solving Perseverance
If I have difficulty problem-solving, I keep working and do my own research to figure a solution out	.665
When problem-solving, I stay committed until I can develop a solution to the problem	.725
If I can't develop a solution to a math tasks in a few minutes I usually stop trying	.648
I am unwilling to spend more than five minutes finding solutions to math tasks	.541
Problem-solving takes too long to complete	.634
I am willing to work as long as it takes when problem-solving	.632
If I become frustrated while problem-solving, I usually stop trying	.814
If I can't find a solution to a problem-solving task in a single setting I stop looking for a solution	.768
I am willing to try several times before I find solutions to math tasks	.659
I give up after my first few attempts to find solutions to math tasks	.671
After being assigned a challenging math task, I keep working to find solutions	.788
Even if I don't know how to solve the problem or feel like I don't have enough information, I stick with it to develop a solution	.562

Note: Factor loadings <.4 are suppressed

Table 3

Factor Loadings and Communalities Based on Exploratory Factor Analysis with Direct Oblimin Rotation for 16 Items from the Mathematical Revision Scale (N=150)

Item	Factor 1 Mathematical Revision	Factor 2 Mathematical Revision
When completing a problem-solving task, I evaluate and refine my solutions	.744	
I reflect on the appropriateness of my solutions	.563	
It is important to find alternative solutions when problem-solving	.524	
When creating solutions to problem-solving tasks, I think about whether or not my solution can be used in a similar situation	.524	
If my solution is not working, I am willing to revise my thinking	.708	
I find value in testing out my solution	.770	
After I find a solution that works, I never look back to refine my solution		.892
Once I have solved a problem, I evaluate how it is working	.756	
When problem-solving, revising my solutions creates a better model that applies to the real world	.667	
Revising solutions, when problem-solving, takes too much time		.480
When solving real life problems , I improve my solutions as I gain additional knowledge, even if I have already found an answer	.656	
When problem-solving , understanding how I developed a solution is more important than the fact that I actually have a solution	.518	.
In addition to creating a solution, it is important to know why the solution works	.680	

Note: factor loadings <.4 are suppressed

Table 4

Factor Loadings and Communalities Based on Exploratory Factor Analysis for 12 Items from the Mathematical Communities of Practice Scale (N=150)

Item	Factor 1 Mathematical Communities of Practice
When problem-solving, I value other people’s input when creating solutions	.688
When problem-solving, I find my peers’ input to be helpful	.822
I do not like to depend on my peers to help create solutions to problem-solving tasks	.500
When comparing solutions, I compare each possible solution with my peers’ solutions to find the best one	.632
It’s better to work with a team of people than alone	.537

I like working on problem-solving tasks alone	.448
When working on a problem-solving task, it is important to describe my thinking to others	.497

Note: Factor loadings <.4 are suppressed

Table 5

Factor Loadings and Communalities Based on Exploratory Factor Analysis with Direct Oblimin Rotation for 11 Items from the Mathematical Problem-solving Utility (N=150)

Item	Factor 1 Mathematical Problem-solving Utility	Factor 2 Mathematical Problem-solving Utility
I'll need to know problem-solving skills for my future job	.447	
When I am older, I don't plan on having a job that requires mathematical problem-solving	.514	
Working on problem-solving tasks in math class will help me in the future	.662	
Math is a worthwhile subject to learn because it teaches me problem-solving	.558	
I will use mathematical problem-solving as an adult	.688	
My job one day will not involve problem-solving	.722	
I will never use mathematical problem-solving after I graduate high school	.754	
Problem-solving will not be important for my life	.752	
Once I create solutions to a problem, I think about how others can use my solutions in solving future problems		.760
When developing solutions to a math task, I ensure they can be used by others in solving future problems		.821

Note: Factor loadings <.4 are suppressed

Table 6

Factor Loadings and Communalities Based on Exploratory Factor Analysis with Direct Oblimin Rotation for 11 Items from the Mathematical Problem-solving Process (N=150)

Item	Factor 1 Mathematical Problem-solving Process	Factor 2 Mathematical Problem-solving Process	Factor 3 Mathematical Problem-solving Process
An important part of problem-solving is developing my own steps to find answers	.722		
I develop my own procedures when problem-solving	.681		
In order to problem solve, a list of steps needs to be given to me		.640	
When problem-solving, I often create a formula for myself	.590		
When given a problem-solving task, I first identify what the goal is	.628		
Mathematical problem-solving is a process without specific procedures	.563		
Mathematical problem-solving is done by following the steps the teacher gives me			.574
Memorizing steps is one of the best strategies to use when problem-solving			.468
Being creative is important when problem-solving	.567		
Memorizing specific procedures is not helpful when problem-solving		.528	

Note: factor loadings <.4 are suppressed

Appendix P: Adopted Items From Established Scales

Fennema-Sherman (1976) Attitude Scale- Usefulness of Mathematics Scale:

- I study mathematics because I know how useful it is
- As an adult I will use mathematics
- Taking mathematics is a waste of time
- In terms of my adult life it is not important for me to do well in mathematics in high school

Mathematics Self-Efficacy and Anxiety Questionnaire (MSEAQ) (May, 2009):

- I believe I am the type of person who can do mathematics
- I get nervous when I have to do mathematics outside of school
- I feel confident when using mathematics outside of school
- I feel confident to ask questions in my mathematics class
- I get nervous when asking questions in my mathematics class
- I believe I can complete all of the assignments in a mathematics course
- I worry that I will not be able to complete all the assignments in a mathematics course.