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Using Manipulatives to Investigate ESOL Students' Achievement and Dispositions in Algebra

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**USING MANIPULATIVES TO INVESTIGATE
ESOL STUDENTS' ACHIEVEMENT AND DISPOSITIONS IN ALGEBRA**
by

Donna Lynette Marsh

A Dissertation

Presented in Partial Fulfillment of Requirements for the

Degree of

Doctor of Education

in

Secondary Mathematics Education

in the

Bagwell College of Education

Kennesaw State University

Kennesaw, GA

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June 2016

DEDICATION

To my daughter Diana Marsh, and brother, Nathan Edward Marsh (deceased 12/25/2015). You both were the sunshine in my life and the inspiration for pursuing this advance degree. Without you this dream would not have become a reality. I love you both and I thank GOD for blessing me with you.

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ABSTRACT

USING MANIPULATIVES TO INVESTIGATE ESOL STUDENTS' ACHIEVEMENT AND DISPOSITIONS IN ALGEBRA

by

Donna Lynette Marsh

The purpose of this embedded quasi-experimental mixed methods research was to investigate the effectiveness of concrete and virtual manipulatives on the achievement of English Speakers of Other Languages (ESOL) as they employ them to explore linear and exponential functions in high school Sheltered Common Core Coordinate Algebra. Also of interest were the effects concrete and virtual manipulatives have on their disposition towards mathematics and math class. Another goal was to investigate the benefits and disadvantages of using concrete and virtual manipulatives versus traditional instructional practices.

This was a 5-week study. The control group (N=20) was instructed through the use of mathematics textbooks and Power Points (traditional) and compared to the treatment group (N=19), which was instructed using concrete and virtual manipulatives. One ESOL mathematics teacher implemented this study, teaching both groups by utilizing the sheltered instruction observation protocol (SIOP) (2012) model to integrate content and language.

Qualitative research methods, teacher interviews, recorded field notes, students' work samples and artefacts were utilized. Quantitative data analysis techniques were used to analyze departmentalized Linear and Exponential Functions Summative Assessments (pretest and

posttest) to measure mathematics achievement. The one-way ANOVA uncovered no statistically significant difference between the control group and treatment group as they explored linear and exponential functions. The Quantitative Understanding: Amplifying Student Achievement and Reasoning Students Disposition instrument (pre-questionnaire and post- questionnaire) measured dispositions about mathematics and math class. The one-way ANOVA indicated no statistically significant difference between the control and the treatment group's dispositions about mathematics and math class.

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CHAPTER 1 INTRODUCTION

Background of the Study

The United States population of students who are English Language Learners (ELLs) is increasing. The What Works Clearinghouse of the United States Department of Education (2013) defines ELLs as students “with a primary language other than English who have limited range of speaking, reading, writing, and listening skills in English”(p.1). The ELLs are the fastest growing population in United States schools (Teaching English to Speakers of Other Languages International Association, 2014). Between the years of 1980 and 2009, the population of students identified as ELLs increased from 10% to 21% (National Center for Education Statistics, 2012). Students coming to the United States have backgrounds consisting of over 400 different languages. Some ELLs who do not speak English are not even literate in their native language (Goldenberg, 2008); as a result, ELLs may take 7 to 10 years to catch up to their peers (Collier & Thomas, 1997). In the United States, educators are struggling and under tremendous pressure to meet the progressively diverse needs of these students (Goldenberg, 2008).

Meeting the needs of students identified as ELLs and implementing Common Core State Standards (CCSS) has created conversations among educators and other stakeholders in the educational system. However, teachers of English as a second language and their students were not included in policy decisions pertaining to the CCSS

reform movement (Teaching English to Speakers of Other Languages [TESOL] International Association, 2013). This exclusion created a challenging situation for teachers of students identified as English learners because these teachers are responsible for the implementation of CCSS for all their students (Echevarria, Short, & Vogt, 2012; Teaching English to Speakers of Other Languages [TESOL] International Association, 2013).

One element CCSS teachers find problematic is the foundations of literacy are not implemented in grades 6-12, an omission that prevents teachers from meeting the needs of adolescent students who are trying to learn English. ELLs learning to read in English may be comparable to English speakers initially learning to read in English (Goldenberg, 2008). However, in their initial publication, the CCSS did not address the language proficiency of ELLs (Teaching English to Speakers of Other Languages International Association, 2014). The absence of language proficiency strategies in the CCSS hindered teachers from fully meeting the needs of newly arrived immigrant students who lack fluency in English when they enter secondary schools (Goldenberg, 2008). The CCSS for mathematics and English language arts require students to demonstrate comprehension of standards through writing evaluations, analyzing, and developing constructive arguments for both English and mathematics (Common Core State Standards Initiative, 2010). However, the curriculum excludes the teaching of written letters, spelling and constructing sentences, which impacts ELLs' understanding of word choice, syntax, and organizational patterns. Students' ability to comprehend the demanding mathematics curriculum is weakened ELLs' are struggling with reading, writing and comprehension of

mathematical concepts (Fenner, 2013). The CCSS curriculum assumes that students are knowledgeable of the prerequisite skills. However, several ELL students are often two or more years below grade level when entering secondary school. The lack of English ability and academic challenges often result in students in ESOL classes with low self-efficacy in their development of speech, a lack that prevents a smooth consistent transition into an English immersion classroom.

Both the National Council of Teachers of Mathematics (NCTM, 2000), and Berg, Petron, and Greybeck (2012) posit that mathematics teachers often have low expectations for students identified as English learners; however, expectations must be raised because “mathematics can and must be learned by all students” (National Council of Teachers of Mathematics, 2000, p. 13). In their publication, *Teaching Mathematics to English Language Learners* (2013), NCTM indicates that mathematics teachers should address the needs of all students, including students who speak a first language other than English or have cultural differences. NCTM has adopted the philosophy that all students must have access to opportunities to learn mathematics to demonstrate their ability.

NCTM’s previous position on students whose native language was not English stated, “Cultural background and language must not be a barrier to full participation in mathematics programs preparing students for a full range of careers. All students regardless of their language or cultural background must study a core curriculum in mathematics based on the NCTM standards” (National Council of Teachers of Mathematics, 1994, p. 20). The demographic makeup of English Language Learners (ELLs) are students “with a primary language other than English who have a limited

range of speaking, reading, writing, and listening skills in English” (What Works Clearinghouse of the U.S. Department of Education 2013, p. 1). It is essential that teachers of mathematics are aware that students in ESOL classes lack proficiency in English and that these students may not be cognitively limited. It is imperative that teachers remember that students identified as English learners have the dual task of learning a second language and mathematics content standards simultaneously (Kersaint, Thompson, & Petkova, 2013).

According to the National Council of Teachers of Mathematics (2000), one of many teaching strategies and techniques that appears to offer great promise is the use of manipulatives.

Statement of the Problem

The problem exists as a consequence of the increased population of ELLs in the United States, which has awoken a sleepy nation to the alarming problems in our educational systems. Educators are struggling with the dual task of implementing CCSS mathematics and teaching English concurrently. The National Assessment of Educational Progress (NAEP, 2013) indicates the ELLs’ NAEP basic mathematics scores have continuously decreased since 2005; they have decreased by 11 points (127); in 2009, they decreased by 7 points (116), and scores were at 109 in 2013 (a decrease of 7 points). ELLs were successful in answering basic level questions related to reading scatterplots and evaluating functions at a point. However, they were unsuccessful in answering questions at the proficiency level that consisted of determining angle measurement in a three-dimensional figure, evaluating expressions with fractional exponents, and

identifying a formula to solve a problem using a spreadsheet. Additionally, ELLs demonstrated a weakness at the advanced level, which includes answering questions pertaining to completing a proof by mathematical induction, analyzing conjunctions and disjunctions of inequalities, writing a formula to solve a problem using a spreadsheet, and determining the area of three-dimensional figures (NEAP, 2013).

Several following factors may have contributed to these results:

- Some ELLs are limited in their English proficiency, while the NAEP test is written in English (Goldenberg, 2008).
- The increase in ELL student participation in the assessment (J. Brown, personal communication, June 23, 2014).
- The implementation of rigorous Common Core State Standards for Mathematics (2010).
- The decrease in teacher-focused activities while increasing student performance tasks that require higher levels of comprehension of reading and interpreting mathematical concepts (thinking abstractly).

Additionally, the NEAP provides no information regarding whether ELLs scored low on the NEAP because of lagging content knowledge and skills (Goldenberg, 2008). No matter what the cause, the achievement gaps are detrimental to ELL's future educational and vocational options (Goldenberg, 2008). Algebra is the prerequisite skill to learning higher-level mathematics (Haycock, 2003), and the NAEP's 2005 results indicate students who took advanced courses are more likely to attend a four-year college (NAEP, 2005). However, given that the NAEP's 2009 results show that ELLs' scores are continuously

decreasing, there is a decrease in opportunities for ELLs to pursue higherlevel mathematics courses (Goldenberg, 2008). In order to tackle this problem, more extensive research on the use of concrete and virtual manipulatives to teach Algebra in the ELL classroom will be beneficial in increasing their success in higher-level mathematics.

Research Questions

What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about linear functions compared to using traditional instructional practice?

What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about exponential functions compared to using traditional instructional practice?

What difference, if any, exists in student dispositions about mathematics and math class as a result of using concrete and virtual manipulatives as ESOL high school students employ them to learn about linear and exponential functions compared to using traditional instructional practice?

What are the benefits and disadvantages of using concrete and virtual manipulatives versus traditional instructional practice, from a teacher's perspective, in teaching linear and exponential functions?

Statement of the Hypothesis

The use of concrete and virtual manipulatives when teaching linear and exponential functions will improve ESOL students' achievement in high school algebra.

ESOL students' disposition about mathematics and math class will change significantly based on the use of concrete and virtual manipulatives in the mathematics classroom.

Purpose of the Study

The purpose of the current quasi-experimental study was to determine if a specified set of actions (concrete and virtual manipulatives) resulted in a desired outcome, increased scores on mathematics summative assessments and changes in students' dispositions about mathematics and math class. The determination was made by comparing the outcome of a group of students treated by the set of actions with a similar group (the control group) who were not exposed to the intervention to determine whether significant differences existed in outcomes. The design of this experiment involved attempts to isolate the treatment effects from other possible effects.

Significance of the Study

The significance of this study lies in its potential for exploring and validating the effectiveness of a major instructional practice with the use of manipulatives with English language learner's performance in algebra. The study explored ELL dispositions about learning mathematics when using manipulatives. This study will also add to other studies and provide insight to all stakeholders (classroom teachers, administrators, and educational policymakers at the local and state level) who examine techniques, pedagogy and strategies for improving teaching and learning of linear and exponential functions for ELLs; furthermore, globally this study will proved awareness to ELLs' trichotomy of learning tasks categorized into learning English, learning mathematics, and utilizing

manipulatives to enhance mathematical understanding of linear and exponential functions simultaneously.

Common Core Coordinate Algebra Unit 3: Linear and Exponential Functions is the core of the high school mathematics curriculum. Graham, Cuoco and Zimmermann (2009) stress the importance of reasoning with algebraic symbols, building equations, and functions. NCTM and teacher assessments reveal that these skills cause students difficulties in the transition from arithmetic to algebra. In particular, the following areas present challenges:

- “Expressing geometry with algebraic notation, including function notation” (Graham, Cuoco, Zimmermann, 2009, p. 25).
- “Reasoning about slope; graphing line, and finding equations of lines” (Graham et al., 2009, p. 25).
- “Building and using algebraic functions” (Graham, et al., 2009, p. 25).
- “Setting up the appropriate equations to solve word problems” (Graham, et al., 2009, p. 25).

Therefore, instructional programs from prekindergarten through grade 12 should facilitate all students’ understanding of patterns, relations and functions (NCTM, 2000). However, high school algebra students should be encouraged to build and use tabular, symbolic, graphical, and verbal representations and to analyze and comprehend patterns, relations and functions at a more complex level than middle school students (NCTM, 2000).

Existing research has not considered how *affects* (dispositions) in the ELL students’ learning of mathematics and use of manipulatives interact to impact their

algebra achievement. Research in educational psychology indicates students' dispositions play a critical role in impacting cognition and achievement in most any domain (Fatade, Arigbabu, Mogari & Awofala, 2014; Vukovic, Kieffer, Michael, Bailey, Sean, Harari & Rachel, 2013). The power of understanding the *affect* of mathematical learning provides the keys to unlocking students' mathematical power to learn (Debellis & Goldin, 2006). Wlodkowski (1999) suggests that as a student's attitude improves, the student is more receptive to learning, which can lead to higher success in achievement.

The result of this study may assist teachers with the impact manipulatives play in influencing students' dispositions regarding mathematics, thereby increasing both ELLs' and non-ELLs' achievement. Additionally, this study may provide insight into enhancing mathematical teaching strategies and pedagogies that assist ELL students with developing their concrete to abstract understanding of linear and exponential functions and supporting their language development.

Conceptual/Theoretical Framework

The foundation of the theoretical framework which grounded this study was divided into the following major perspectives: the linguist theorist point of view with emphasis on Krashen's (1988) model of second language acquisition; the learning theorist point of view with emphasis on Engerstrom's (1987) activity theory and Vygotsky's (1978) Zone of Proximal Development (ZPD), and Sharma's (1997) *Bridging the Gap* point of view highlighting the six levels of mastering mathematical concepts, including Hedden's (1986) and Underhill's (1977) sequence (concrete level - representational level-abstract level) of using manipulatives.

The theories were selected based upon ELLs' trichotomy of learning tasks: learning English, learning mathematics and utilizing manipulatives to enhance mathematical understanding of linear and exponential functions. Krashen's (1988) model of second language acquisition provides five hypotheses on how we learn a second language. Engerstrom's (1987) activity theory accounts for the classroom environment, roles of the manipulatives, skills of ELL students, standards for coordinate algebra, motivation, activities, and roles of classmates and teachers (Vygotsy's, 1978, ZPD) and proves the interaction of the manipulatives within the learning environment. Sharma's (1997) six levels of mastering mathematical concepts assist with the levels of mathematical comprehension when utilizing concrete and virtual manipulatives to explore linear and exponential functions.

Linguistics theory. The linguist theorist point of view addresses how students learn English as a second language and the concepts applied to the curriculum area of mathematics for this study. Krashen's (1988) model of second language acquisition consists of five hypotheses: (a) the *acquisition-learning hypothesis*, (b) the *natural order hypothesis*, (c) the *monitor hypothesis*, (d) the *input hypothesis*, and (e) the *affective filter hypothesis*.

The *acquisition-learning hypothesis* implies information is stored in the brain through the use of communication; therefore, in this study the ELL mathematics teacher and researcher created situations for ELLs to become engaged in negotiating (speaking English) for meaning of mathematics with their peers, classmates, and teacher (Kersaint, Thompson, & Petkova, 2013). The *natural order hypothesis* process indicates that ELLs

acquire parts of language through natural communication. Krashen (1988) implies learning languages and certain grammatical structures are required early while others are acquired later. This study introduced language concepts which are more accessible for ELLs, and used scaffolding to introduce challenging mathematical concepts of linear and exponential functions using both concrete and virtual manipulatives. The *monitor hypothesis* explains the relationship between acquisition and learning. According to Krashen (1988) monitoring is the result of the learned grammar, and vocabulary; acquisition is the utterances of second language learners. Monitoring sometimes contributes to accuracy, and ELLs' mathematics teachers are challenged to balance acquisition and learning. Depending upon the ELL, monitoring may possibly hinder and force the ELL to slow down and focus more on accuracy as opposed to fluency. The *affective filter hypothesis* suggests that emotional variables, such as anxiety, self-efficacy, motivation and stress, hinder learning. These variables prevent *comprehensible input* from reaching the language acquisition part of the brain (Krashen, 1988). As a result, in this study the ELLs' mathematics teacher provided a learning environment where students were allowed to make mistakes and take risks in learning both English and mathematics through creating a positive classroom environment (Kersaint et al., 2013). The *input hypothesis* is deemed the most significant component of Krashen's theory of second language L₂ acquisition; he has determined that *comprehensible input* (receiving understandable messages) is the fundamental principle in second language acquisition (SLA). The *input hypotheses* component of Krashen's theory of second language acquisition suggests that *i+1 input* should slightly stretch the learner beyond his or her

original stage (being neither too easy nor too difficult). If a learner is at i stage, acquisition takes place when he or she is exposed to *comprehensible input*, which then emerges to the $i+1$ level. Not all students identified as English learners are at the same level of linguistic competence (Krashen, 1988); the five levels include beginning, early intermediate, intermediate, early advanced, and advanced (Goldenberg, 2008). To accommodate the various levels of learners, teachers will need to differentiate by providing a variety of learning strategies. Students in this study were provided with visuals, hand-outs with less complex structures, and paraphrased instructions, and the ESOL teacher spoke slowly and clearly enunciated speech to assist students with making sense of mathematical concepts.

Krashen (1988) specifies that “All factors thought to encourage or cause second language work only when they contribute to comprehensible input and/or a low affective filter” (p. 4). In one of the corollaries of the input hypothesis, Krashen notes that speaking fluently cannot be taught directly; rather, it emerges naturally over time. Intensive listening practice plays a key role in the development of the speaking skills of both first language (L_1) and second language (L_2) ELLs (Krashen, 1988). It takes several years before ELLs are fluent in all four skill areas (listening, speaking, reading, and writing) necessary for academic success. This study took the perspective of using both concrete and virtual manipulatives to provide situations for ELLs to become engaged in negotiating (listening, speaking, reading and writing English), which assisted with building upon their existing English and mathematical skill development. The visual representation of the manipulatives assisted with connecting linear and exponential

functions with mathematical language needed to discuss functions. Students discussed the activity, described patterns and created graphics of linear and exponential functions observed with manipulatives.

Krashen's (1988) model of second language acquisition hypotheses were used to promote comprehensible mathematical thinking and discussions. Students were grouped in small cooperative learning groups, which provided opportunities to use mathematical terminology to communicate their ideas and solutions in English. The ELL mathematics teacher differentiated the small cooperative learning groups with respect to tasks, flexible grouping and teacher observations (assessment). In addition, the ELLs' mathematics teacher differentiated content, process and product according to ELL readiness, interest and *Can Do* descriptors using the Sheltered Instruction Observational Protocol (SIOP) (Detailed in Chapter 2: Integrating Language and Mathematics Content). See Appendix D for additional ESOL mathematics learning strategies.

Learning theorist. The learning theorist point of view for this study was applicable to how ELL students learn mathematics through the social approach learning theory (Vygotsky, 1978; Leontiev, 1981; Engerstrom, 1987: activity theory) indicating that ELLs will learn from their social environment (Schunk, 2012). Theories of Vygotsky (1978), Leontiev (1981), and Engerstrom (1987) guide these approaches. Vygotsky is known for the first generation activity theory (triangle design), which is the notion of mediation between subjects (ELLs), cultural artefacts (manipulatives), and objects (tasks, assignments) (Hardman, 2008). Engerstrom (1987) devised the second generation activity theory, which is an extension of Vygotsky's first generation activity theory, adding the

components of rules, community, and division of labor. Figure 1.1 illustrates Vygotsky's first generation components, combined with Engerstrom's (1987) concepts deriving the second generation with modifications that apply to this study. Vygotsky believes students interact with objects (manipulatives) in the world to learn. He indicates the assistance provided to students (ELLs) should *bridge the gap* between subject and objects, a concept within the Vygotsky's Zone of Proximal Development (ZPD). Vygotsky (1978) defines ZPD as "the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through the problem solving under adult guidance or in collaboration with a more capable peer" (p.86). The ZPD allowed the ELL mathematics teacher or advanced classmate to assist in the next level of mathematical learning.

Leontiev's activity theory includes a three- tiered explanation of social endeavors (motives, emotions and creativity); this study included a survey instrument to measure students' dispositions towards mathematics (see Appendix C) (Triantafillou & Potari, 2010). The activity theorist point of view in this study accounts for the classroom environment, roles of the manipulatives, skills of ELL students, standards for coordinate algebra, motivation, activities, and roles of classmates and ELL teachers. These components interacted, providing opportunities to enhance ELL students' mathematical thinking to solve linear and exponential function problems with real-life applications (Cobb, Wood, & Yackel, 1990). In short, the second generation activity theory components interacted with each other to achieve the outcome (successful learning).

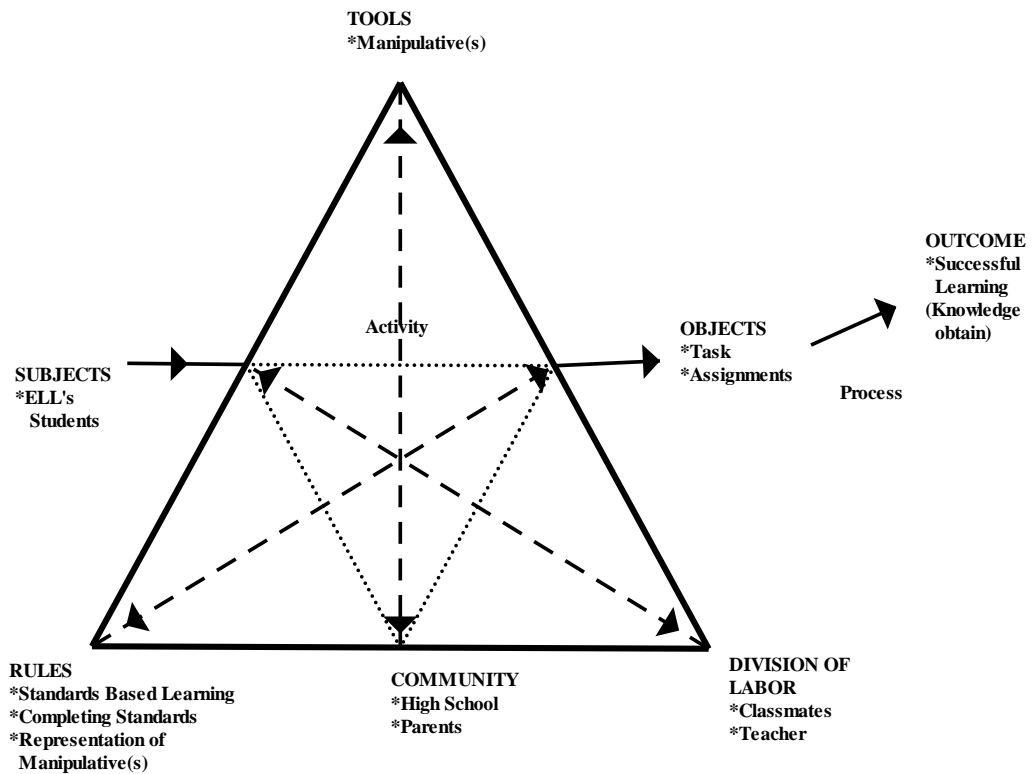


Figure 1.1. Second Generation Activity Theory

Shama's bridging the gap point of view. The *Bridging the Gap* point of view is the sequence involving concrete level to representation level to abstract level (Heddens, 1986; Howell & Barnhart, 1992). This instructional technique used in this study assisted students to formulate the concrete to make connections with the abstract when using math manipulatives (Underhill, 1977; Heddens, 1986; Sharma, 1997; Witzel, Mercer & Miller, 2005). The sequence of *Bridging the Gap* consists of a continuum of learning from concrete to abstract. Figure 1.2 shows a modification of Hedden's (1986) interpretation of the sequence of *Bridging the Gap*. Sharma (1997) argues that there are six levels of mastery of mathematical concepts: intuitive, concrete, representation (pictorial), abstract, applications and communications, whereas Hedden (1986) suggests four levels of

mastery of mathematical concepts (concrete, semi-concrete, semi-abstract, and abstract level). Sharma's (1997) levels of intuitive, applications and communications are important levels for ELLs to experience when utilizing manipulatives. Sharma's communication level (writing and speaking) is the key to making the leap for ELLs to the abstract level of understanding mathematical concepts (Moyer, 2001) used in this study. The Mathematical Association of America (MMA) (2004) emphasizes communication skills through "development of reading, writing, speaking, and listening skills [which] [r]equire students to explain mathematical concepts and logical arguments in words [and r]equire them to explain the meaning –the hows and –whys of their results" (p.4). In addition, the NCTM's (2000) Process Standards for Problem Solving, Reasoning and Proof, Communications, Connections and Representation validate the importance of Sharma's (1997) levels of application and communication, whereas, Heddens's (1986) sequence indicates that students achieve the abstract level and do not fully experience solving applications and communicating what they have learned mathematically.

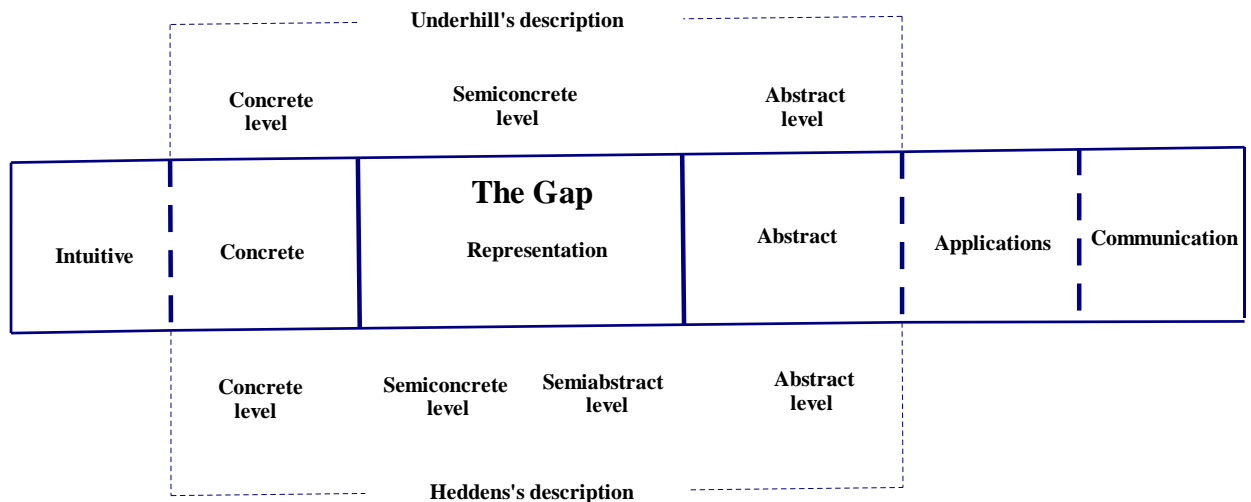


Figure 1.2. Heddens's description

Studies have shown that *Bridging the Gap* helps one formulate the concrete to make connections with the abstract when using manipulatives. Heddens (1986); Boulton-Lewis (1998); Kilpatrick, Swafford, and Findel (2001); Burch (2006); and Reneau (2012) share the philosophy that many students have difficulty understanding mathematics because they are unable to make the connection between the physical world and the abstract. In defining the gap, Heddens (1986) creates two stages: the semi-concrete and the semi-abstract. The semi-concrete level is a representation of a real situation; pictures of the real items are used rather than the items themselves. The semi-abstract level involves a symbolic representation of concrete items, but the symbols or pictures do not look like the objects for which they stand. The gap between concrete and abstract functioning should be considered as a continuum. Assisting students with bridging this gap is crucial because many children cannot cross it without the teacher's assistance. Heddens (1986) claims learners must internalize new knowledge at the concrete level and systematically progress along the continuum to arrive at the abstract representation of knowledge.

Baroody (1989) asserts that strategies for bridging the gap between concrete and abstract ideas involve using pictures. George Bright (1986) continues this assertion by stating that manipulatives hold the promise for helping many students understand mathematics. He further argues that the symbols and the manipulatives used in teaching mathematics must always reflect the same concept. Therefore, manipulatives become tools for thinking and allowing students to correct their own errors (Thompson, 1994; Clements & McMillen, 1996; Boulton-Lewis, 1998; Kilpatrick, Swafford, & Findel,

2001). Furthermore, the contact “touch” with the manipulatives gives students a visual to help with their memory and recall the concept (Boulton-Lewis, 1998; Suh & Moyer, 2007). This visualization of a mathematical concept lessens students’ confusion and allows deeper (mathematical intimacy) understanding to occur (Steen, Brook & Lyons, 2006). The effectiveness of *Bridging the Gap* (concrete level-representational level-abstract level) has been researched in many studies (Allsopp, 1999; Jordan, Miller, & Mercer, 1998; Paulsen & the IRS Center, 2006; Harris, Miller, & Mecer, 1995, Westbook, 2011; Reneau, 2012).

In this study students were encouraged to use scaffolding, and they were provided time to use their English and manipulatives productively while learning about linear and exponential functions as needed. Sharma (1987) quotes, “Visualization is the natural way one begins to think, before words, or images emerge” (p.9). Sharma’s (1997) six levels of mastery of mathematical concepts assisted the researcher with the tools for creating, developing and selecting an appropriate series of mathematical learning activities and tasks which met the requirements of the Common Core State Standard Initiative while supporting ELLs with the dual task of learning a second language and developing an understanding of linear and exponential functions. Each activity moved the ELLs through the six levels of mastery of mathematical concepts. The intuitive level assisted the ELL with connecting the manipulatives to prior experiences not necessary to linear and exponential functions. The concrete level allowed the ELL to use the manipulative to model linear and exponential functions. In the representation level (pictorial) the ELL drew a symbolic picture or representation to illustrate the linear and exponential function.

The abstract level (symbolic) enhanced the ELL's mathematical thinking to translate the linear and exponential function algorithm into mathematical notation. The application level allowed the ELL to apply linear and exponential functions and equations derived to solve real world situations and problems. The communication level created opportunities for ELLs to practice speaking English and writing to express mathematical concepts to classmates and teachers. The Common Core State Standards Initiative (2010) (CCSSI) describes this process as *justification*; students were able to share their reasoning and explain the how's and why's. Once the students demonstrated an understanding of the Common Core Georgia Performance Standard(s) (CCGPS) for linear and exponential functions (see Appendix A) and had no further need for utilization of manipulatives, they were asked to demonstrate the standard without the use of the manipulatives.

Review of Relevant Terms

- algebra achievement- As a measurement of algebra achievement, Unit 3A: Linear Functions and Unit 3B: Exponential Functions (Departmentalized Assessment) was used to compute the gain scores for each participant. The improvement (gain) from pretest to posttest was computed for each ELL by subtracting each student's pretest score from their post-test score.
- attitude toward mathematics-“The general attitude of the class towards mathematics related to the quality of the teaching and to the social-psychological climate of the class” (Hannaula, 2000, p. 3).
- concrete manipulatives (structured, unstructured)- These include objects or items that the pupil is “able to feel, touch, handle, and move. They may be

real objects which have social application in our everyday affairs, or they may be objects which are used to represent an idea” (Grossnickle, Junge, and Metzner, 1951, p. 162).

- disposition – Student dispositions are indicated by University of Pittsburg’s Quantitative Understanding: Amplifying the Student Achievement and Reasoning (QUASAR) Student Disposition Instrument (QSDI), which were administrated at the beginning of Unit 3A: Linear Functions and conclusion of Unit 3B: Exponential Functions to determine students’ dispositions about mathematics and math class (QSDI, 1992-93). The results from students’ responses to questions 7, 8, 10, 12, 15, 16, 19, 28, 29 and 32 difference scores from pre-questionnaires/post-questionnaires were compared.
- English language learner (s)(ELLs) - What Works Clearinghouse of the U.S. Department of Education states ELLs are students “with a primary language other than English who have a limited range of speaking, reading, writing, and listening skills in English” (2013, January, p. 1).
- mathematics manipulatives (structured)- These include “objects that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered” (Swan and Marshall, 2010, p. 14).

- sheltered immersion (SI)- “Instructional approach that promotes English Language development while providing compressible grade-level content” (Kersaint et al., 2013, p. 182).
- virtual manipulatives (VM)- These include “a web-based representation of a dynamic object that allows the students to understand a mathematical concept by manipulating it interactively using the mouse to control physical actions” (Hannan, 2012, p. 2).

Summary

The United States population of students who are English language learners (ELLs) is increasing. Some ELLs do not speak English and are not literate in their native language (Goldenberg, 2008). Teachers of ELLs and, in fact, their students were not included in policy decisions pertaining to the recent CCSS reform movement (Teaching English to Speakers of Other Languages [TESOL] International Association, 2013). It is imperative that mathematics teachers remember that students identified as English learners have the dual task of learning a second language and algebra content standards simultaneously. Language is an important vehicle for thinking (Vykotsky, 1978). Algebra is a necessity in solving problems in today’s technological global economy, and well-developed speech skills are necessary to nurture thinking (Bruner, 1983; Dewey, 1933; Piaget, 1973; Vygotsky 1978). Using manipulatives in Algebra as an instructional strategy nurtures thinking; therefore, it offers an effective strategy to improve students’ mathematics achievement (Gurbuz, 2010; Sherman & Bisanz, 2009).

The foundation of the theoretical framework for this study was divided into three major categories, which include linguist, learning theories and levels of mastery of mathematical concepts. Each of these theories consists of several frameworks:

- Linguist theory, which incorporates:
 - Model of Second Language Acquisition (Krashen, 1988)
 - Zone of Proximal Development (Vygotsky's, 1978)
- Learning theorist point of view, which incorporates:
 - The social approaches school (Vygotsky,1978; Leontiev, 1981; Engerstrom, 1987: second generation activity theory)
- Sharma's (1997) *Bridging the Gap* point of view, which incorporates:
 - Concrete level to representation to abstract (Heddens, 1986; Underhill, 1977: six levels of mastery of mathematical concepts)

Figure 1.3 illustrates the interactions between the model of second language acquisition (Krashen, 1988), the activity theory (Engerstrom, 1987) and ZPD (1978), and the sequence of *bridging the gap* (Heddens, 1986; Sharma, 1997; Underhill, 1977) when ELLs utilize manipulatives. The linkages between the theories and manipulatives are the foundations for the theoretical framework for this study. The use of manipulatives may assist the ELL with mastering mathematical concepts (linear and exponential functions). Sharma (1997) argues that there are six levels of mastering mathematical concepts: intuitive, concrete, representation (pictorial), abstract, application and communication. Therefore, it is imperative that ELL mathematics teachers create engaging activities based on the level of proficiency which require students to listen, speak (negotiate), read

and write as they advance through each of the six levels of mastery (August & Shannahan, 2006; Kersaint, Thompson, & Petkova, 2013). Communication (listening, speaking, reading, and writing) at each level is one of the essential keys for ELLs to make the leap to the abstract level of understanding linear and exponential functions (Moyer, 2001).

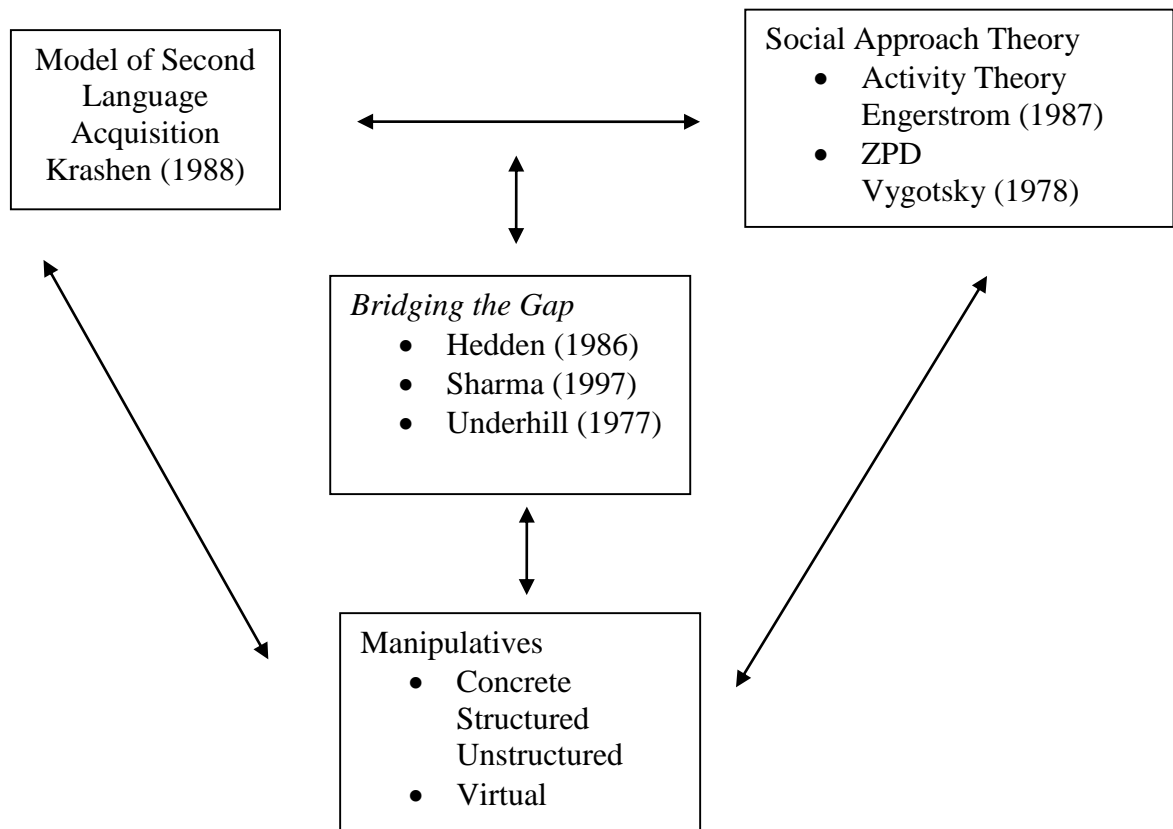


Figure 1.3. Framework Model

The social approach learning theory, which includes activity theory (1987) and ZPD (Vygotsky, 1978), posits that children whose mathematical learning is firmly grounded in hands-on manipulative experiences will be more likely to bridge the gap

between the world in which they live and the abstract world of mathematics (Kennedy, 1986), thereby increasing their chances for success.

CHAPTER 2

LITERATURE REVIEW

Numerous research studies have been conducted in the general area of using manipulatives to teach mathematics in the elementary classroom (Nishida, 2007; Graham, 2013), using virtual manipulatives in the high school classroom (Hollebrands, 2007; Hannan, 2012), using computer software for the bi-lingual student (Kirk, 2011), and using Geogebra software at the high school level (Zulnaidi & Zakari, 2012). However, little research has been conducted on using concrete manipulatives to teach mathematics at the secondary level (Aburime, 2007) and with ELLs. The purpose of this review is to discuss theories and research on math instruction and learning with the use of manipulatives. The review is organized thusly:

- Teaching mathematics to English Language Learners
- Building Understanding of Linear and Exponential Functions
- Discussion of Math manipulatives (concrete and virtual)
- Review of research on manipulatives
- Students' dispositions towards mathematics

Teaching Mathematics to English Language Learners

Limited research models (programs) are available that offer effective strategies and methodologies for teachers of students identified as English learners to use to facilitate the learning of mathematics. Teachers are obligated to make learning comprehensible for their students by integrating the mathematics instructional strategies adapted to the rigor demanded by

the CCSS. Kersaint, Thompson, Petkova (2013) and Ariza, Morales-Jones, Yahy, and Zainuddin (2012) share several ESOL mathematics learning strategies (see Appendix D).

The ESOL mathematics learning strategies (see Appendix D) are supported by various researchers: Robinson, 2006; Zemelman, Daniels, and Hyde, 2005; Kersaint, Thompson, & Petkova, 2013. Krashen (1988) recommends teachers include the use of strategies, tasks, and activities; Shoebottom (2014) claims that this process will “*Make it comprehensible!*” (p. 1). Kersaint, Thompson, and Petkova (2013) insist that ELLs engage in activities that require practicing literacy skills (speaking, reading, and writing). These strategies are not restricted to students in ESOL classes or the teaching and learning of mathematics; all students benefit from these strategies (Ariza et al., 2012).

ELLs have difficulty communicating their mathematical understanding in order to link information to prior knowledge when explaining their thoughts to others (Kersaint et al., 2013). Some are reluctant to speak aloud in front of classmates; therefore, teachers are to provide various language resources and techniques to improve ELLs’ participation in classroom mathematical discussions. Moschkovich (1999) suggests utilizing objects to engage mathematical discussions. For example, the teacher may take a piece of yarn and have ELLs illustrate and discuss linear and exponential function characteristics (Lyster, 2007). Lyster and Mori (2006) and Lyster and Ranta (1997) created the following six feedback moves to assist mathematics teachers with encouraging ELLs to notice their errors and correct their English while participating in mathematical discussions:

- Teacher restates the student’s explanation using correct English and mathematics language
- Teacher requests clarification

- Teacher recasts the ELL's error and provides corrections
- Teacher provides a metalinguistic clue
- Teacher provides elicitation questions
- Teacher repeats ELL's statements and adjusts intonation

Teachers should develop ELLs' mathematical understanding and English skills by helping them make sense of the language of mathematics (i.e. vocabulary, symbols, and syntax). This approach will assist with solving mathematical problems using visuals, manipulatives, and graphic organizers to communicate mathematically.

Teachers are required to implement delivery models (programs) of instruction that facilitate learning for ELLs. The models of facilitation vary from state to state (Kersaint, Thompson, & Petkova, 2013). Georgia provides six approved delivery models (pull-out, push-in, cluster center, resource center and laboratory, a schedule class period, and innovative delivery model). In the pull-out model, students are taken out of a non-academic class. The push-in model provides ELLs with instruction from both content and English Speakers of other Language (ESOL) teachers during an academic block (classes of 60 or 90 minutes). The cluster center model provides ELLs instruction by transporting the students to a central location for intensive English instruction with students from other schools. The resource center and laboratory model provides ELLs group assistance with supplemental materials. The schedule class model provides ELLs language assistance and content instruction during a class period. Any individualized, alternative method must be approved by the Georgia State Department of Education (2013).

Goldenberg (2008) provides insight into needed research to determine whether oral English development can be accelerated. The idea that ELLs will become fluent in English if immersed in all English instruction is a contradiction. For instance, the state language policies in

California and Arizona require mainstreaming ELLs after a year of schooling. However, the National Literacy Panel research indicates that learning to read in the first language promotes reading achievement in the second language (Goldenberg, 2008).

Integrating language and mathematics content. In the state of Georgia, students who are identified as ELLs are taught mathematics through the integration of both language and mathematics content instruction (dual task). Thomas and Collier (2002) define this integration of language and content instruction: “Where teachers use strategies such as speaking slowly and clearly (but using natural language), using visual aids and manipulatives, and building prior knowledge” (p. 10). The state of Georgia implements the Sheltered Instruction Observational Protocol (SIOP) model for ESOL instruction. The SIOP model consists of eight interrelated components: lesson preparation, building background, comprehensible input, strategies, interaction, practice and application, lesson delivery, and review and assessment (see Appendix N). These components have been established as ongoing research-based strategies since 1996. The Georgia Department of Education mandates the SIOP model for ESOL instruction for grades 9 through 12; these students, identified as English learners, may receive a maximum of five day segments. The SIOP was chosen because it provides insight into addressing the dual task simultaneously (content standards and language standards), which allows teachers to facilitate the learning of English through the content areas (Hanse-Thomas, 2008; Flynn & Hill, 2006; Met, 1991; Stoller, 2004; Kersaint, Thompson, & Petkova, 2013). Echevarria et al. (2012) developed the sheltered instruction observation protocol (SIOP) model used in both sheltered instruction (SI) classrooms and in mixed classes of English learners and English-speaking students to integrate content and language through the development of lesson plans and a

delivery approach. Guarino et al. (2001) confirm the SIOP model as valid and reliable measures of sheltered instruction. Conversely, the U.S. Department of Education (2013) indicates, “No studies of Sheltered Instruction Observation Protocol that fall within the scope of English Language Learners review protocol meet What Works Clearinghouse evidence standards” (p. 1). Their negative evaluation is based on the fact that Echevarria’s (2012) study does not use a comparison group design or a single-case design.

The ESOL language standards were established by the World-Class Instructional Design and Assessment (WIDA) consortium (2014). These standards (objectives) are compiled as *Can Do* descriptors and performance definitions that assist teachers with identifying the type of language tasks students should be able to perform within each domain. These domains include listening, speaking, reading, and writing (August & Shannahan, 2006; Genesee, 2006). The WIDA standards include five differing levels of English proficiency: entering, beginning, developing, expanding, bridging, and reaching. The standards were designed for different grade-level clusters, including Pre-K-K, grades 1- 2, grades 3-5, grades 6-8, and grades 9-12. Scores from the WIDA-ACCESS Placement Test (W-APT), which is given to incoming students, and the overall score on the English Language Proficiency (ELP) Assessment, which includes levels 1-4 on the ACCESS for ELLs™ test administered each year, assist teachers with planning differentiated lessons or unit plans (WIDA, 2014).

Teachers of English to Speakers of Other Languages (TESOL) International Association (2013) recommend ESOL teachers focus on depth and rigor and not rush through the materials. Mathematics teachers of students identified as English learners must identify each student’s stage of secondary language acquisition and understand his or her academic background. Previous

schooling experiences in the native language greatly influence learning in the second language. Once a mathematics teacher has an understanding of a student's academic background, he or she should be able to target and differentiate mathematics instruction by implementing effective mathematics strategies (Kersaint et al., 2013). Teaching ELLs with the use of manipulatives is one effective mathematics strategy.

Building Understanding of Linear and Exponential Functions

The idea of building students' understanding of functions is essential to mathematical learning for all grade levels (Dubinsky & Harel, 1992). NCTM (2000, 2009) and teacher assessments of students' understanding functions revealed these skills cause students difficulties in the transitioning from arithmetic to algebra. The term understanding is a dynamic state which allows students to make a connection with pieces of prior knowledge to other related pieces of new knowledge learned (Carpenter & Lehrer, 1999). This study used concrete and virtual manipulatives to assist students with *Bridging the Gap* between the concrete-representational-abstract sequence in using prior knowledge and new knowledge in building an understanding of linear and exponential functions.

Dandola-Depaolo's (2011) research revealed that building students' understanding of functions is a spiraling concept embedded within the historical development that emerged based upon mathematical needs. Researchers suggest that historical information assist teachers comprehending the stages of learning (Barbin, 2000). The comprehension of functions began in 2000 B.C. with Babylonian mathematicians creating numerical tables of values for calculations (Youschkervitch, 1976), moving into the 16th Century when Greek mathematicians became familiar with correspondence, dependence, mapping and binary relations (Bochner, 1970).

Ptolemy (c.a. 150 A.D.) used two column tables, discovering independent and dependent variables (functions of chords) to determine the position of the sun, moon and planets (Pedersen, 1974). Nicole Oresme (1323-1382), mathematician and scientist, is credited with developing early forms of graphing and creating the geometric theory of latitude forms (longitude and latitude) (Bochner, 1970). Longitude and latitude are considered types of coordinates (Youshevitch, 1976). As the history of mathematics continued to unravel, Francois Vieta (1540-1603) established the use of letters (variables) to write algebraic expressions and unknown quantities (Youshevitch, 1976).

Scaffolding students' understanding of linear and exponential functions aligns with history unfolding the development of functions and how the researcher created, developed and selected activities and tasks using concrete and virtual manipulatives for this study. Parallel to the sequence, Akkoc and Tall (2002) suggested six forms of function representation to facilitate student learning; however, this research study employs only four forms: numerical table of values, ordered pair (tables), geometrical graphing, and symbolic formulas or algebraic equations. The other two forms are prerequisites explored prior to this study (mapping diagrams and function machines, which both illustrate input and output relationships). Friel and Bright (1995) suggest students communicating mathematically to determine graphical meaning of the representations should increase. Communication assisting students (ELLs) with making the connection of the order pairs (tables) of graphs, graphical representations, and formulas (equations) using models (manipulatives) helps solidify their understanding (Friel & Bright, 1995; Baron, 2015). Day (2015) posits, "Allowing students to work from the model to the

equation and from the equation to the model encourages a depth of understanding variables” (p. 514) of both linear and exponential functions.

Discussion of Mathematics Manipulatives

This discussion on math manipulatives is divided into the following two categories:

- Teaching with Manipulatives (concrete and virtual)
- The Teacher’s Role in Using Manipulatives

Teaching with manipulatives. Activities involving pictures and objects, which may include textbook illustration models on the active board and drawings, as well as demonstrations by teachers and peers, can smooth the transition between concrete and abstract functioning (Hedden, 1986). Dienes (1960), Dewey (1938), Montessori (1967) and Kersaint et al. (2013) agree that students should be actively engaged with mathematics (doing mathematics), that as a result of communication and touching the material, they learn images, which builds upon the next concept. From these images the student can translate concrete facts into symbolic representation (Antosz, 1987), which generates connections for a deeper level of mathematical understanding (Kersaint et al., 2013).

Pioneers of research on the use of manipulatives, Grossnickle, Junge, and Metzner (1951), provide a concrete definition of manipulatives: “They include offered objects or items that the pupil is able to feel, touch, handle, and move. They may be real objects which have social application in our everyday affairs, or they may be objects which are used to represent an idea” (p. 162). Swan and Marshall (2010) revisited the definition and the use of manipulatives as a result of virtual manipulatives, computers, and interactive white boards in the mathematics classroom. Therefore, an abstract definition was devised: “A mathematics manipulative material

is an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered” (p. 14). Kennedy (1986), Williams (1986), and Moyer (2001) all support the indications of manipulative materials. Marshall and Swan (2005) indicate two types of manipulatives that can be used in the classroom (concrete, either structured or unstructured, and virtual manipulatives). Olkuan (2003) suggests the difference between concrete and virtual manipulatives is their physical nature since one touches concrete manipulatives.

In the 21st century classroom, manipulatives are used as a tool to bridge the gap between the concrete and the abstract. The use of manipulatives with ELLs reinforces opportunities for discovery and leads to actively engaged communication, discussion, and explanations of the students’ ways of solving problems (Caswell, 2007; Kersaint et al., 2013). Due to the increasing development of technology, students are using technology to make “the connections between mathematics and areas outside mathematics such as social studies, science, art, and physical education” (NCTM, 2000, p.44). As computers and calculators become more advanced, comprehension and the ability to perform algorithms have become a priority.

Teaching with virtual manipulatives. The National Council of Supervisors of Mathematics (NCSM) (2014) and researchers Boggan, Harper, & Whitmire (2010), Caglayan & Olive (2010) and Sherman & Bisanz (2009) all recommend teachers integrate both concrete and virtual manipulatives into the mathematics classroom at all grade levels to enhance students’ mathematical thinking. The Common Core State Standard (2010) for *Mathematical Practice 5: Use of Appropriate Tools Strategically* emphasizes students’ utilization of concrete models (manipulatives) and technology. Therefore, virtual manipulatives (VM) are applets, or computer

software (Bouck & Flanagan, 2010), and “a web-based representation of a dynamic object that allows the students to understand a mathematical concept by manipulating it interactively using the mouse to control physical actions” (Hannan, 2012, p. 2).

VM presents a version of the physical manipulative; they are on the computer screen rather than on the student’s desk. Students have the ability to connect the movement and actions on the manipulative to the symbolic notation simultaneously (Moyer, Bolyard, & Spikell, 2002; Suh & Moyer, 2007). This simultaneous action allows students to see and use multiple presentations of the mathematical concept (Dorward, 2002; Suh & Moyer, 2007). Because of this simultaneous action, students are given immediate feedback and a guide to the algorithm being learned (Johnson, Campet, Gaber & Zuidema, 2012). Table 2.1 provides the VM web site and web address that were used in this study. Cannon, Heal and Wellman (2000) provide insight into the advantages of virtual manipulatives to include recording and storing students’ movement; providing web-based accessibility for students, parents, and teachers; providing free availability on the Web; and providing students with access to VM at home without sending home concrete manipulatives that may never be returned to school (p. 1083). Moyer, Bolyard and Spikell (2002) and Johnson et. al (2012) provide questions for evaluating and selecting the appropriate virtual manipulative web site and tools (see Appendix E).

Table 2.1

Virtual Manipulatives and Web sites

<u>Virtual Manipulative Web sites</u>	<u>Web Addresses</u>
National Library of Virtual Manipulatives	http://nlvm.usu.edu/
eNLVM	http://enlvm.usu.edu/
Shodor	http://shodor.org/interactivate/activities/
Desmos	http://desmos.com/

Computer software is an essential component of instruction in the 21st century mathematics classroom that enhances the teaching and learning of mathematics (Heid & Blume, 2008). The Common State Standards Initiative (CCSSI, 2010) suggests the use of appropriate apparatuses (manipulatives) might include “a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software” (p. 7). Ralston (2004) posits the concepts of using mathematics software tools in teaching mathematics concepts are under development. Some mathematicians believe computer software hinders mathematical thinking, while others advocate it enhances mathematical thinking and learning (Quinlan, 2007). The CCSSI encourages providing students opportunities “to use technological tools to explore and deepen their understanding of concepts” (p.7). Unfortunately, some schools disregard updating interactive computer software applications when revamping their hardware to include software

that stimulates students to investigate and discover mathematical concepts (Flores, 2000) due to the rapid development of computer software (financial) and limited teacher effective utilization (Jackson, 2011). Sivin-Kachala and Bialo (2000) indicate the use of computer software as a teaching tool increases student confidence and improves motivation to learn mathematics. There exists numerous software applications used in teaching linear and exponential functions (Cabri Geometry™, GeoGebra, Computer Algebra System (CAS), Derive and Mathematica®); however, only two were utilized in this study: Geometer's Sketchpad (Jackiw, 2001) and Texas Instrument-84 plus Easy Data application.

Swan and Marshall (2010) suggest delaying students' use of VM until they have had the opportunity to experience the *real thing* (physical objects in the hand), for example two-dimensional and three-dimensional representation of objects. In three-dimensional figures the "dimension of the representation is strictly less than that of the figure" (Parzysz, 1988, p.80). Bushell and Fueyo (1998) and Bako (2003) claim a strong need exists for both concrete and virtual manipulatives. Examples of structured manipulatives (cubes and graphic calculators); unstructured manipulatives (balls, paper plates, straws, pipe cleaners, and spaghetti); and virtual manipulatives (Geometer's Sketchpad) were used in this study.

Some researchers question the use of manipulatives and believe they provide no guarantee of mathematical success (Baroody, 1989; Amaya, Uttal, O'Doherty, Liu, & DeLoache, 2007; Jarvin, McNeil, Sternberg, 2006; McNeal, Uttal, Jarvin, Sternberg, 2007; Sowell, 1989). Manipulatives may lead students to focus on having fun at the expense of developing mathematical understanding (McNeil & Jarvin, 2007). In addition, manipulatives may hinder abstract mathematical thinking due to the multiple representations they may provide. Students

may focus on the salient concrete properties of the symbol as an object instead of what the symbol represents and therefore miss learning the underlying concept (Baroody, 1989; Uttal, Scudder, & DeLoache, 1997). While these are valid objections, Gurbuz (2010), Sherman and Bisanz (2009) argue that the impact manipulatives make on students' mathematical learning outweighs these concerns. Heddens (2005) suggests that using manipulatives will assist students with the following:

- Relating world conditions to mathematics symbolism
- Working together in cooperative groups to solve problems
- Exchanging mathematical ideas and concepts
- Expressing their mathematical thinking verbally
- Making presentations in front of large audiences
- Understanding that there are various ways to solve problems
- Comprehending that mathematics problems can be represented in several ways
- Deciphering mathematics problems without teacher assistance

Teachers may utilize various manipulatives as an instructional strategy in teaching a wide variety of topics in math (Cabahug, 2012; Gurbuz, 2010; Sherman & Bisanz, 2009). Noted researchers have similar beliefs on categorizing manipulatives into 11 general categories (Reys & Post, 1973; Jackson & Phillips, 1973) (see Appendix F). The manipulatives utilized in this study are categories as follows:

- Colored Rods, Blocks, Beads and Discs (Also includes pattern blocks and attribute blocks)

- Cards and Charts (Includes flash cards, activity cards, mobiles, manipulative charts, bulletin material, etc.)
- Math Games and Puzzles
- Calculating and computational devices (Includes hand calculators and computer software)
- Videos

Computers, including virtual manipulatives, interactive white boards, and computer tablets, have been more recently available providers of manipulatives.

Manipulatives may also be categorized into grade levels (pre-school, elementary, middle and high school). Manipulatives currently available are multi-purpose devices that can be used to objectify many mathematical concepts. Several of these manipulatives are utilized for a particular concept (Jackson & Phillips, 1973). Swan and Marshall (2010) argue that some of these manipulatives may be clearly identified as teaching tools based on their definition of mathematics manipulatives.

The teacher's role in using manipulatives. Teachers play a significant role in establishing mathematical environments that provide students multiple representations to increase their mathematical thinking while using manipulatives (Moyer, 2001; Uribe-Florez & Wilkins, 2010). Teachers become facilitators of learning when they share their control of learning with their students (National Council of Teachers of Mathematics, 2000; Moyer & Jones, 2004). Teachers who relinquish control allow their students to take responsibility for their own learning, which encourages and deepens their mathematical thinking (Moyer & Jones, 2004; Goracke, 2009, Wiggins, 1990). Mathematical thinking is a fundamental process for students,

and manipulative materials are tools teachers are able to utilize in enhancing students' understanding through the process (Uribe-Florez & Wilkins, 2010). When utilized properly, "A good manipulative bridges the gap between informal math and formal math. To accomplish this objective, the manipulative must fit the development level of the child" (Smith, 2009, p.20).

Reys (1971) and Roberts (2007) provide insight into using manipulative materials at the right time and in the right way if they are to be effective and not hinder mathematical thinking. Failure to select appropriate manipulative material and failure to use them properly can destroy their effectiveness. The task of selecting manipulative materials for classroom instruction is a crucial one, whether the decision involves textbooks, software, or other teaching aids (National Council of Teachers of Mathematics, 1982). The selection process is only the first step in helping students understand mathematics and is therefore an important responsibility of the teacher. The following suggested questions were utilized in this study to select the appropriate manipulative materials for the ELLs:

- Is the manipulative or model a clear and accurate representation of the concept?
- Does the manipulative clearly lead to student discovery in a timely fashion?
- Is the student able to record, reconstruct and generalize the concepts learned using the manipulatives? (Robert, 2007)

In earlier years, Reys (1971) developed a specific set of dos and don'ts for teachers using manipulatives (see Appendix G). Swan and Marshall (2010) provide advantages for teachers using manipulatives (see Appendix H).

Some teachers have difficulty incorporating manipulatives into their lessons (Puchner, Taylor, O'Donnell, & Fick, 2008). Some teachers see them as a diversion and do not believe they

are necessary for understanding (Green, Flowers, & Piel, 2008), and many teachers may lack the training on how to use them (Moyer, P. & Jones, G., 2004). Moyer (2001) and Puchner, Taylor, O'Donnell, and Fick (2008) indicate that teachers' beliefs about how students learn mathematics may influence how and why they use manipulatives; therefore, exploring ELLs' dispositions toward mathematics is an integral part of the research for this study.

Review of Research on Manipulatives

Weiss (1994) reports that the use of manipulatives in the mathematics classroom increased from the mid-1980's to 1993; however, the frequency with which teachers use manipulatives was found to differ by grade level. Elementary school teachers were found to use manipulatives more often than middle school teachers; high school teachers were found to use manipulatives the least (Uribe-Florez, Wilkins (2010). For example, Howard, Berry, and Tracey (1997), comparing elementary and secondary mathematics teachers' use of manipulatives, found that just 4% of the secondary teachers reported using manipulatives in every lesson, while 55% of their colleagues at the elementary level reported manipulative usage in every lesson. Little research has been conducted using concrete manipulatives to teach mathematics at the secondary level; therefore, reviewing the research at all levels provides a holistic prospective of teaching with manipulatives.

Elementary school level. Garcia (2004) investigated using math manipulatives and visual cues with explicit vocabulary with lower achievers in third- and fourth-grade bilingual class rooms for a 5- week study. The pre-test composed of 10 of the 13 Texas Assessment of Academic Skill objectives was administered to 64 third- and fourth-grade students. Students were divided into three groups (manipulatives-based instruction, visual (drawings) cued

instruction and no additional mathematics instruction). Results indicate minimal improvement in the treatment groups. Gradual improvement was made but was not linear. In an analysis of math retention based on this study, Cabonneau, Scott, and Selig (2013) indicate third- and fourth-graders taught with manipulatives performed the same as those taught without manipulatives.

Allen (2007) used an action research project approach to investigate the use of math manipulatives in a fifth-grade self-contained math class (22 students) over a three day period in a program entitled Everyday Math. The students used pattern blocks to understand the relationship of interior angles in polygons. The students were required to take a pretest and posttest, and results indicated that students' mathematics achievement increased, their understanding of mathematics increased, and their dispositions toward mathematics improved using manipulatives.

Nishida (2007) investigated children's (134 six to-seven-year olds) addition and subtraction of fractions. Children were randomly assigned to three groups (self-manipulative, other-manipulate, and comparison conditions). In Experiment 1, students used concrete manipulatives (fraction circles) to solve basic problems. As a result, there was no difference between actively using manipulatives, watching an experimenter use manipulatives, and looking at pictures. Parents also reported that 90% of the children had used manipulatives previously in school. The remaining 5% to 10% had not used manipulatives in previous lessons. Experiment 2 consisted of higher achieving math students, who also used concrete manipulatives (fraction circles). Students who used the manipulatives scored higher than those who watched manipulatives being used and looked at pictures of fractions. All students were excited and interested and enjoyed working with math manipulatives. In an analysis of math retention based

on this study, Cabonneau, Scott, and Selig (2013) indicate retention was the same for both groups.

Battle (2007) used a quantitative research study to determine if manipulatives would increase math grades for 16 low-achieving students in self-contained classes during a one week study. One class was a control group (8 students), and the other was a treatment group (8 students). Both groups were learning addition and subtraction. The treatment group used counter blocks for counting and subtracting numbers from 1 through 20. Each student was given a pretest and a posttest. Results indicate that students taught addition and subtraction with counters performed better than those taught without manipulatives. However, in an analysis of math retention based on this study, Cabonneau, Scott, and Selig (2013) indicate the students scored lower on a measure of retention than those taught without manipulatives.

Smith and Montani (2008) investigated the benefits of multisensory instruction for teaching mathematics to students in resource rooms. Twelve third- and fourth-grade students participated in this study using base-ten block manipulatives to solve word problems. Prepost results indicate that student performance increased through the use of base-ten blocks. In an analysis of math retention based on this study, Cabonneau, Scott, and Selig (2013) indicate third-graders taught with manipulatives answered more questions correctly on a post-measure of retention.

Ogg (2010) investigated the impact of math manipulatives on 12 fifth-grade students using calculators, protractors, rulers, money, counting, base-ten blocks and tangrams, candy, cereal, straws, and computers for math games and geometric transformations. The students were required to take pretests and posttests with and without the use of manipulatives. In addition, the

students completed a survey to determine their perceptions of the manipulatives. The results of 20 teacher surveys indicate that 9 of the 12 students increased their scores using manipulatives to solve math problems. All surveyed teachers indicated that they used rulers, protractors, calculators, counters, and coins.

In a study relating to probability, Gurbuz (2010) used quasi-experimental investigation on the effects of activity-based instruction and traditional based instruction on fifth-grade students (50 students, 25 treatment and 25 control). Open-ended questions were administered before and after learning about probability. The results indicated that activity-based instruction was more effective than traditional in students' learning about probability.

Reneau (2012) used a single-case multiple-baseline across participants to investigate the use of the concrete-to- representation-to-abstract sequence, applying virtual manipulatives to solve equations and word problems with fractions. He investigated five fifth-grade students receiving special education services who had been diagnosed with a specific learning disability. The results indicate that all students gained in performing mathematically when using the concrete-to-representation-to-abstract sequence. Results of this study may be applicable to ELLs and their use of manipulatives when using the concrete to- representation – to-abstract sequence. Results of this study may be applicable to ELLs and their use of manipulatives.

Graham (2013) investigated the use of manipulatives in upper elementary classrooms, exploring third-, fourth- and fifth-grade teacher perceptions. This case study assisted leaders in understanding the association between teachers' perceptions and the problems associated with concrete math manipulatives' disuse. Observations, interviews, and documents from three teachers were analyzed and coded. The results indicate concrete math manipulatives enhance

student learning. However, teachers need training (professional development) to use concrete math manipulatives as components of the state standards.

Morris (2014) investigated the impact of virtual manipulatives on 12 fourth-grade students' mathematics performance in adding and subtracting three- to six-digit whole numbers. One treatment group used virtual manipulatives, and of the two control groups, one used pencil, paper, and worksheets, and the other used concrete manipulatives. The results indicate that the three groups showed improvement between the pre-test and post-test. However, significant improvement exists for those students who participated in the virtual manipulative group.

Dahl (2011) studied the impact manipulatives have in elementary and middle school mathematics classrooms, in addition to the impact manipulatives have on students' understanding and enjoyment for learning mathematics. The research also identified struggles and concerns and the needed increase in professional development for teachers in using math manipulatives.

Middle school level. Goracke (2009) used an action research project approach to investigate the use of manipulatives within an 8th-grade pre-algebra class (19 students over a 5-week period), and its impact on student dispositions and comprehension of mathematics. Students graphed using pegboard, solved integer problems using chips (adding, subtracting, multiplying, and dividing polynomials), and solved equations using algebra tiles. Students also used protractors and compasses to solve geometry problems. Student surveys, interviews (6 students), and math journals (13) were used in determining students' attitudes and dispositions of learning mathematics using manipulatives. The results indicate that student exam scores (4 tests given every 2 weeks) increased, and attitudes, dispositions, and self-efficacy improved. In

addition, math journals revealed that students' understanding of mathematics increases when they draw pictures with the use of math manipulatives.

Yuan, Lee, and Wang (2010) developed virtual manipulatives (polynomials) for junior high school students. This quasi-experimental study compared using physical techniques with using manipulatives and virtual manipulatives in finding the number of polynomials. With 68 participants in the study, students in the treatment group used virtual manipulatives, and students in the control group used physical manipulatives. The results indicate that learning in the treatment group was as effective as that in the control group. In an analysis of math retention based on this study, Cabonneau, Scott, and Selig (2013) indicate eighth-grade students taught geometry with manipulatives answered more questions correctly on a post-assessment of problem solving.

White (2012) used a quasi-experimental non-equivalent control-group to examine 145 seventh-grade general education students using hands-on learning and manipulatives. The results indicate that no significant difference were found between post-test scores of the two sub groups (low-achieving control versus low-achieving experimental, high-achieving control versus high-achieving experimental).

Magruder (2012) used an embedded quasi-experimental mixed methods research to investigate solving simple linear equations comparing concrete and virtual manipulatives. Also, Magruder (2012) investigated unique benefits and drawbacks associated with each manipulative to teach middle school students (60 students: 20 in the control group, 20 in the virtual group, and 20 in the concrete group). The results indicate a statistically significant difference in favor of the

control group because it takes more time to learn how to operate the manipulative and to learn mathematics content.

Doias (2013) used a mixed methods approach to investigate the effects of manipulatives (concrete and virtual) on teaching addition and subtraction of fractions with a seventh-grade math class (44 students: 22 in the experimental group, 22 in the control group) over a two-week, eight-day period. The students were required to take a pretest and posttest, and the researcher's observations and student questionnaires were used to triangulate the data. The results indicate that the combination of concrete manipulatives with virtual manipulatives promotes a measureable change in the students' tested mathematical ability.

High school level. Goins (2001) studied the effects of using algebra tiles with students (30 students) learning polynomial multiplication. Three methods of instruction were used (non-visual and non-manipulative teaching, visual teaching, and teaching with manipulatives). The use of manipulatives had a positive effect on students learning the algorithm of multiplying binomials. The students were better able to explain the process in a written paragraph. There was no statistically significant difference between the non-visual and non-manipulative and the visual methods. In an analysis of math retention based on this study, Cabonneau, Scott, and Selig (2013) indicate that ninth-grade students who were taught algebra with algebra tiles performed better on a post-assessment than students who did not have access to the tiles.

Aburime's (2007) study took place in Nigeria, where 287 high school students participated in learning geometry with math manipulatives in a 10-week study, and stratified random sampling was used to create the 12 groups. Aburime used 6 experimental groups (manipulatives) and 6 control groups (no manipulatives). The Mathematics Achievement pre and

posttests were administered to both groups. Eighteen geometric manipulatives made from cardboard (square, rhombus, rectangle, parallelogram, trapezium, pentagon, hexagon, circle, semi-circle, cube, cuboid, triangular, prism and cylinder) were used in this study. Results indicate a significant difference in students using mathematics manipulatives. In an analysis of math retention based on this study, Cabonneau, Scott, and Selig (2013) indicate both groups were the same.

College level. Maynard (1983) investigated the use of concrete manipulatives on college age remedial students. Four remedial math classes (133 students) also participated with lecture-discussion as the primary method of presenting information. Students were required to participate in a teacher-directed math lab using manipulatives, videos, and study guides to support lectured instruction. Results indicate that the use of mastery testing with the use of manipulatives produces a significant gain on unit tests. In addition, 87 of the 133 students successfully completed the course.

Dyer (1996) investigated the use of algebraic manipulatives with 90 community college students. In an analysis of math retention based on this study, Cabonneau, Scott, and Selig (2013) indicate students taught algebra with algebra tiles performed the same on the measure of retention than students taught without manipulatives.

McGee, Moore-Russo, Ebersol, Lomen, and Quintero (2012) developed a set of manipulatives to help students of science and engineering visualize concepts relating to points, surfaces, curves, contours, and vectors in three dimensions. Three methods (common exam questions, interviews, and questionnaires) were used to assess the effectiveness of the 3D kit. The final examination was taken by 47 control group students and 55 treatment group students.

There was significant improvement in students' attitudes towards the effectiveness of the 3D kit. Students who did not benefit from the manipulative kit revealed weak backgrounds in geometry.

Kindergarten through college level. Carbonneau, Marley and Selig (2013) used a meta-analysis study of the use of manipulatives to teach mathematics. This analysis identified 55 studies for which it compared instruction with manipulatives to instruction without. The sample included 7,237 students from kindergarten to college. The results indicate large effects on retention ($k = 53$, $N = 7,140$) and small effects on problem solving ($k = 9$, $N = 477$) and favors the use of manipulatives over abstract math symbols.

Sowell (1989) used meta-analysis results of 60 studies combined to determine the effectiveness of mathematics instruction with manipulative materials. Studies ranged from kindergarteners to college-age adults. Results indicate that mathematics achievement is increased though the long-term use of concrete instructional materials and those students' attitudes toward mathematics are improved when they have instructions with concrete materials provided by teachers knowledgeable about their use.

Student Dispositions towards Mathematics

The research of students' dispositions (beliefs, attitudes, and emotions (affects)) towards mathematics learning has declined during the last decade (Niss, 2007). This decline may be attributed to "how well-defined and well-delineated the basic notions are, and how clearly they can be disentangled from cognition in mathematics education" (Niss, 2007, p. 1303). The interaction between affect and cognition (Hannula, Evans, Philippou, & Zan, 2004) is also a contributing factor. On the other hand, Harrell and Abrahamson (2010) specify that mathematics education research involving *affect* has risen over the past two decades. Both The National

Council of Teachers of Mathematics (1989) and the National Research Council (1989) recommend that researchers attend to affective, cognitive factors related to mathematics teaching and learning.

Ryes (1984) and McLeod (1992) suggest three categories (variables) of the affective experience related to mathematics learning pertaining to dispositions: beliefs, attitudes, and emotions. Later, DeBellis and Goldin (1997, 2006) added a fourth category, values, which creates a tetrahedral model (Hannula, et al., 2004; DeBellis, et al. 2006). Knowing student beliefs, attitudes, emotions, and values toward mathematics will assist teachers in reducing the mathematics anxiety students experience; also teachers will be able to encourage more students to continue their study of mathematics beyond the minimal requirements in high school by reducing anxiety (Brush, 1981; Ma, 2001). Reys (1984) defines *affective* variables as experience regarding “students’ feelings about mathematics, aspects of the classroom, or about themselves as the learner of mathematics” (Reys, 1994, p. 5). McLeod (1992) explains, “The affective domain refers to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition” (p. 576). For instance, emotions change as students experience solving a mathematical problem (DeBellis, 2006). The theoretical foundations that undergird the affective variables are not quite coherent, and researchers are unable to agree with the theories, terminologies and definitions of *attitudes* (Di Martino & Zan, 2001; Hannula, 2002a), *beliefs* (Furinghetti & Pehkone, 2002), *emotions* (Goldin, 2000; Lazarus, 1991; Mandler, 1989; Power & Dalglish, 1997, Pekrun, Elliot, & Maier, 2009) and *values* (DeBellis & Goldin (1997, 2006; Bishop, 2001). Beliefs, attitudes, emotions, and values do not cover the entire field of affective variables; they might include motivation, feeling, mood, conception, interest, and anxiety

(Hannula et al. 2004). Table 2.2 provides a brief outline of a combination of the theories of McLeod (1992), DeBellis and Goldin (1997, 2006) and their categories of affective experiences.

Table 2.2

The Affective Domain in Mathematics Education

<u>Category</u>	<u>Examples</u>
Beliefs	
About mathematics	Mathematics is based on rules
About self	I am able to solve problems
About mathematics teaching	Teaching is telling
About the social context	Learning is competitive
Attitudes	
	Dislike of graphing functions
	Enjoyment of problem solving
	Preference for hands-on learning
Emotions	
	Joy (or frustration) in solving nonroutine problems
	Aesthetic responses to mathematics
Values	
	Students value correctness in their day-to-day work

Beliefs. Beliefs may or may not be *truth*, but the student finds them comfortable.

Validity, on the other hand, is highly stable, highly cognitive, and highly structured, but it may be uncomfortable. Truths may be pleasant or painful, but they will contribute to a student's stabilization (Hannula et al. 2004; DeBellis et al., 2006). Students' beliefs about mathematics and themselves are essential in the development of their affective responses to mathematical situations (McLeod, 1992). Students experience both positive emotions (relief, pride, and hope)

and negative emotional dispositions (shame, hopelessness, anxiety, boredom, and anger) as they learn mathematics (McLeod, 1992; Hannula et al. 2004; Zan & Di Martino, 2008). These emotional dispositions impact students' behavior and their achievement in mathematics, which influences their willingness to learn advanced mathematics (Eshun, 2004). Fatade, Arigbabu, Mogari, Awofala (2014) indicate that exposing students to problem based learning promotes meaningful learning and enhances beliefs about further mathematical learning. One's beliefs about mathematics can determine how one chooses to solve a problem (Schoenfeld, 1983).

Attitudes. Attitudes are moderately stable orientations or predispositions toward having certain sets of feelings (positive or negative) in particular contexts (how one feels in class); they involve a balance of interacting affect and cognition (Hannula et al., 2004). Mohamed and Waheed (2011) identify the following three factors that influence student attitudes towards learning mathematics:

- Factors associated with students' mathematical achievement, which include anxiety, self-efficacy and self-concept, motivation, and experiences at school.
- Factors associated with the school, teacher, and teaching, such as teaching materials, classroom management, teacher knowledge, attitudes towards math, guidance, and beliefs.
- Factors from the home environment and society, such as educational background and parental expectations.

Hannula (2012) suggests students' attitudes do not really help teachers and some teachers use it as an excuse to surrender when they are unable to help a student (Di Martino and Zan, 2010).

McLeod (1992) suggests that focusing on various types of attitudes, such as feeling anxiously

afraid of failure, being utterly bored, or absolutely hating mathematics, will impact student behavior. Sowell (1989) indicates that students' attitudes toward mathematics improved when they had instructions with concrete materials provided by teachers knowledgeable about their use. Goracke (2009) indicates students' attitudes, dispositions and self-efficacy improved with the use of math manipulatives.

Emotions. Emotions include “feelings; the rapidly changing states of feeling experienced during mathematical (or other) activity” (Hannula et al., 2004, p. 30). Pekrun, Elliot, and Maier (2009) indicate that emotions (enjoyment, boredom, anger, hope, pride, anxiety, hopelessness, and shame) are physiological and involve relations between achievement goals and performance attainment. Emotions during mathematical thinking affect students' cognitive problem solving ability and support their creativity and flexibility in ways to problem solve (Frenzel, Pekrun & Goetz, 2007; Pekrun, 2006, Pekrun, Goetz, Titz, & Perry, 2002; Pekrun & Stephens, 2009). Experts better control emotions than novices (students) (Allen & Carifio, 2007; Schoenfeld, 1985). Emotions are also a dimension of Vygotsky's Zone of Proximal Development (Nelmes, 2003).

Mathematics anxiety. Beliefs, attitudes, and emotions “vary in the level of intensity; from *cold* beliefs about mathematics, *cool* attitudes related to liking or disliking mathematics, to *hot* emotional reactions, to frustration of solving nonroutine problems” (McLeod, 1992, p. 578). Berebisky (1985), Gatuso and Lacases (1987), Hembree (1990) and DeBellis (2006) all agree that beliefs, attitudes, and emotions are involved in the development of mathematics anxiety. Vukovic, Kieffer, Bailey, and Harari (2013) suggest that mathematics anxiety may affect how some students use working memory resources to learn mathematical applications. In addition,

Zakaria, Zain, Ahmad, and Erlan (2012) indicate that math anxiety is one factor that affects student achievement; therefore, teachers should strive to understand mathematics anxiety and implement teaching and learning strategies to reduce students' math anxiety. Marsh and Tapia (2002) indicate that students with low levels of math anxiety feel more excited, more confident and highly motivated to learn mathematics when compared to students who have higher anxiety levels. Stramel (2010) indicates students' negative changes in attitudes toward mathematics and mathematics self-efficacy beliefs are strongly related to the amount of homework and lack of hands-on activities. The ability to understand the *affect* of mathematical learning provides the key to unlocking students' mathematical power to learn (DeBellis & Goldin, 2006). In finding a solution and unlocking students' mathematical power to learn, *Best Practices: New Standards for Teaching and Learning in American Schools* provides a list of best practices for teachers in the mathematics classroom (Zemelman, Daniels & Hyde, 2005) (see Appendix I).

Researchers have found that “as students build strategic competence in solving non-routine problems, their attitudes and beliefs about themselves as mathematics learners become more positive” (Kilpatrick, Swafford, & Findell, 2001, p. 131). Teachers who encourage students to use diverse problem solving approaches further develop confidence in their students' abilities to succeed (Burns, 2006, Kilpatrick, Swafford, & Findell, 2001). Steen, Brooks, and Lyons (2006) advocate that when students form ownership of their learning through the use of manipulatives, the fear is removed from learning mathematical concepts. Furthermore, as the teacher uses concrete and virtual manipulatives (technology), positive student attitudes toward mathematics increase (Brown, 2007; Steen, Brooks, & Lyons, 2006). Burns (2006) claims as

students advance through school, the struggles and consequential dislike for mathematics begin to emerge.

Summary

The purpose of this review is to discuss theories and ELLs learning with the use of manipulatives (concrete and virtual) to build their understanding of linear and exponential functions. This review of literature has been conducted in the general areas of using manipulatives (Aburime, 2007), virtual manipulatives (Hollerbands, 2007 & Hannan, 2012) and computer software (Kirk, 2011 & Zunairdi, Zakari, 2012) in the high school classroom for ELLs. The sequence of *Bridging the Gap* between intuition, and communication assists ELLs with the dual task of learning a second language and mathematic concepts simultaneously. The utilization of manipulatives is beneficial for assisting ELLs with formulating the concrete to make connections with the abstract (Underhill, 1977; Heddens, 1986; Howell & Barnhart, 1992; Sharma, 1997; Witzel, 2005). Sharma's (1997) sequences of six levels of mastery of mathematical concepts (intuitive, concrete, representation (pictorial), abstract, applications and communication) assist the ELLs in making the leap to the abstract level of understanding linear and exponential functions (Moyer, 2001). Building students' understanding of linear and exponential functions is a spiral concept embedded within the historical development that emerged due to mathematical needs of society (Dandola-Depaolo, 2011).

Sharma's (1997) *Bridging the Gap* assists the ELLs with visualization and sense of touch when using manipulatives to represent mathematical concepts to lessen students' confusion and allow for deeper (mathematical intimacy) understanding to occur (Steen, Brook & Lyons, 2006). Teachers are learning to provide opportunities for ELLs to utilize manipulatives that allow for

discovery and lead to actively engaged communication, discussion, and explanation of the students' ways of solving problems (Caswell, 2007; Kersaint et al., 2013). Relinquishing control allows students to take responsibility for their own learning, which encourages and deepens their mathematical thinking (Moyer & Jones, 2004; Goracke, 2009, Wiggins, 1990).

The NCTM has been supporting the use of manipulatives in every decade since 1940, and the National Council of Supervisors of Mathematics (NCSM) (2014) recommends the use of the virtual manipulatives. Computer software is a component of virtual manipulatives. The use of computer software as a teaching tool increases student confidence and improves motivation and self-efficacy to learn mathematics (Sivin-Kachala & Bialo, 2000).

The National Council of Teachers of Mathematics (1989) and the National Research Council (1989) recommend that researchers attend to affective, cognitive factors related to mathematics teaching and learning. Both DeBellis and Goldin (1997, 2006) suggest four categories (variables) of the affective experience related to mathematics learning (beliefs, attitudes, emotions, and values) all affect one's self-efficacy in learning mathematics. Little research has been conducted using concrete manipulatives to teach mathematics at the secondary level; therefore, reviewing the research at all levels provides a holistic perspective of teaching with manipulatives.

NCTM has been supporting the use of manipulatives in every decade since 1940; additionally NCTM encourages the use of manipulatives at all grade levels. NCTM declares that the study of mathematics should include opportunities for students to model situations using oral, concrete, pictorial, graphical, and algebraic methods (National Council of Teachers of Mathematics, 1989). Learning with math manipulatives reduces math anxiety, and students

benefit from the change from lectures and textbooks to hands-on learning (Plaisance, 2009; Woodard, 2004). Math manipulatives help students use concrete objects to make connections with the abstract. The contact provided through kinesthetic engagement with the manipulatives assists students with transference and mental retention (Boulton-Lewis, 1998; Suh & Moyer, 2007). Therefore, the study of mathematics should include opportunities for students to model situations using oral, concrete, pictorial, graphical, and algebraic methods (NCTM, 1989). This use of concrete and virtual manipulatives would better allow students who struggle with achievement in mathematics.

However, the use of manipulatives in the classroom has declined within the past 10 years partially due to lack of teacher knowledge of how to manage and use them (Marshall, L. P. (2005). *The Principles of the NCTM* (2000) state:

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. The use of manipulatives also provides equity in the classroom. Not all students benefit from the same type of instruction. Many students profit from this hands-on collaborative learning that manipulatives afford. (p.20)

CHAPTER 3 METHODOLOGY

This chapter describes the methodology of this sequential embedded quasi-experimental mixed methods research design used to investigate the effects of mathematics manipulatives on ELLs student achievement in high school coordinate algebra. Figure 3.1 illustrates the sequential embedded quasi-experimental mixed methods framework design used in this study. Creswell (2012) suggests using a mixed method study when researching both quantitative and qualitative data. The combination of the data types assists with understanding the research problem and strengthens the study. While quantitative data will prove a statistical difference between treatment groups, qualitative data will provide a picture of the differences of the two groups. The value of qualitative data will be found in the story that it tells. Qualitative data describes teacher and student experiences and opinions and explains student learning. Qualitative research is also beneficial for showing *how* things work, and *how* processes occur over time (Creswell, 2012). Quantitative analysis was used to compare the pretest and posttest results between two groups (concrete, virtual manipulatives and traditional instruction) and their mathematical achievement. Additionally, the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) Students Disposition was used to measure growth in students' dispositions toward mathematics and math class (pre- and post-questionnaire). Posttest data compared the differences between the two groups' conceptual understanding of linear and exponential functions. Qualitative data (teacher interviews, recorded field notes, student work samples and artefacts) was used to reveal the benefits and advantages of using concrete and virtual manipulatives.

The purpose of this sequential embedded quasi-experimental mixed methods research was to explore linear and exponential functions to analyze the effectiveness of manipulatives (concrete and virtual) with ELLs as compared to a control group of ELLs using traditional instruction. In addition it explored ELLs' dispositions about mathematics and the math class. The quantitative data were collected and analyzed, pre and post assessment among two groups (control, and manipulative) was used to measure students' mathematical achievement, and the QUASAR Student Dispositions Instrument was utilized to measure growth with respect to students' dispositions about mathematics and the math class. The strengths of the quantitative data and qualitative data complement each other and offset any weaknesses with equal priority placed on both methods (Fraenkel & Wallen, 2009). The data were analyzed separately, triangulated, and the divergence of the results was discussed. Triangulation assisted with determining overlapping themes, development, and the relations between research questions and data sources. Creswell (2012) posits using triangulation (multiple sources of data) to analyze the data from multiple perspectives neutralizes any bias which may occur in the data source, methodology, and by the researcher, therefore strengthening the validity of the data results. Creswell (2012) indicates triangulation is the process of corroborating evidence from different individuals, types of data, or methods of data collection. . . . This ensures that the study will be accurate because the information is not drawn from a single source, individual, or process of data collection. In this way, it encourages the researcher to develop a report that is both accurate and credible (p. 280). Both quantitative and qualitative analysis of the data provided an understanding of the research problem in multiple views.

The mixed methods limited biased and unbiased, as well as subjective and objective views (Creswell, 2012). The quantitative data demonstrated the differences in performances between the groups (control and manipulative) while qualitative data described these differences and provided specific examples utilized by the ESOL teacher. A mixed method researcher should strategically utilize quantitative and qualitative elements to strengthen data collection. This is the fundamental principle of mixed methods research (Johnson & Onwuegbuzie, 2007).

Mixed Methods Framework: Embedded Quasi-Experimental Model

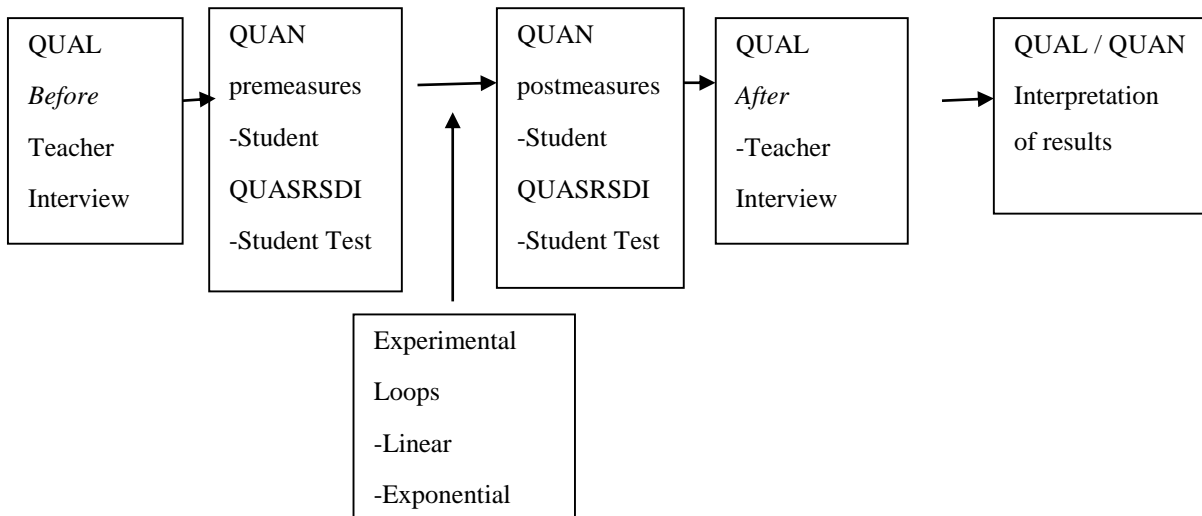


Figure 3.1. Mixed methods design for study (Adapted from Creswell and Clark, 2007).

Steffe, Thompson, and von Glaserfeld (2000) claim that an experimental study allows the researcher a direct and immediate opportunity to observe students engaged in reasoning and learning. The Comparative Experimental Approach Method was used to investigate the effects of mathematics manipulatives on student achievement in high school algebra. According to the *Handbook of Research on Mathematics Teaching and Learning* (1992), this method of

investigation helps determine whether or not a specified set of actions (manipulatives) produces a desired outcome. The outcome of the treatment group was compared with a similar control group to determine if predictable differences in outcomes may exist.

A large suburban high school's Sheltered ESOL Common Core Coordinate Algebra class participated in an experimental study on the effects of manipulatives on student achievement. One ESOL mathematics teacher implemented this study, teaching both groups while utilizing the sheltered instruction observation protocol (2012) method to integrate language and content. The research study included a pre- and post-interview of the ESOL mathematics teacher. The pre-interview took place at the beginning of the research study before the treatment group and control group were administered the prequestionnaire (QUASAR Student Dispositions Instrument). The post-interview took place after the post-questionnaire (QUASAR Student Dispositions Instrument) was administered to the treatment and control groups. The ESOL mathematics teacher and researcher used the small *chunk* instructional strategy for Unit 3: Linear and Exponential Functions. The unit was separated into two units, Unit 3A: Linear Functions and Unit 3B: Exponential Functions. Miller (1956), a psychologist, defines *chunking* as breaking down information into smaller, manageable pieces for the brain to conceptualize new information. The Unit 3A: Linear Function Summative Assessment pretest was administered to the treatment and control groups after the QUASAR Student Dispositions Instrument pre-questionnaire; the Unit 3A: Linear Function Summative Assessment posttest was administered before the Unit 3B: Exponential Function Summative Assessment pretest. The Unit 3B: Exponential Function Summative Assessment posttest was administered to the treatment and

control groups before the QUASAR Student Dispositions Instrument post-questionnaire I. Table 3.1 illustrates the timeline used:

Table 3.1

Timeline

<u>Event</u>	<u>Date</u>
Pre ESOL Teacher Interview	Monday, February 25, 2015
Pre Questionnaire (QUASAR)	Monday, March 2, 2015
Pre Linear Summative Assessment	Tuesday, March 3, 2015
Post Linear Summative Assessment	Monday, March 23, 2015
Pre Exponential Summative Assessment	Monday, March 23, 2015
Post Exponential Summative Assessment	Thursday, April 2, 2015
Post Questionnaire (QUASAR)	Thursday, April 2, 2015
Post ESOL Teacher Interview	Friday, April 3, 2015

Research Questions

This sequential embedded quasi-experimental mixed methods research study investigated and compared ELLs student achievement and growth with respect to their dispositions about mathematics and the math class as a result of using manipulatives (concrete and virtual) in an high school ESOL Algebra course within a large suburban school system. The following research questions were addressed in this study. Table 3.2 illustrates research questions and data alignment:

1. What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about linear functions compared to a control group using traditional instructional practice?
2. What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about exponential functions compared to a control group using traditional instructional practice?
3. What difference, if any, exists in student dispositions about mathematics and math class as a result of using concrete and virtual manipulatives as ESOL high school students employ them to learn about linear and exponential functions compared to a control group using traditional instructional practice?
4. What are the benefits and disadvantages of using concrete and virtual manipulatives versus traditional instructional practice, from a teacher's perspective, in teaching linear and exponential functions?

Table 3.2

Research Questions and Data Alignment

<u>Research Question (RQ)</u>	<u>Data</u>
RQ 1	Linear Assessment pre-test pre-post
RQ 2	Exponential Assessment pre-test pre-post
RQ 3	QUASAR pre-questionnaire post-questionnaire
RQ 4	Teacher Interview pre-interview post-interview

Participants

The participants in this study were high school students who were designated to receive ESOL services based upon demonstrating Level 3 (developing) or higher competency level on

the World-Class Instructional Design and Assessment (WIDA) Placement Test (WAPT) for Assessing Comprehension and Communication in English State to State (ACCESS). The English Language Learners were enrolled in sheltered instruction ESOL CCGPS Coordinate Algebra classes in a large suburban public school system. The treatment group, one class of approximately 18 ESOL students, used concrete manipulatives, virtual manipulatives, and an online textbook (Holt McDougal's *Coordinate Algebra, Georgia Edition* (2014)) in the instruction of mathematics. The control group of approximately 20 ESOL students participated in traditional instruction and used an online textbook (McDougal's *Coordinate Algebra, Georgia Edition* (2014)) instead of concrete and virtual manipulatives. The students for both the control group and treatment group were selected based on predetermined scheduling. The students were enrolled in ESOL Sheltered Instruction (SI) based upon their scores on the WIDA-ACCESS Placement Test (W-APT) (placement test given to incoming students) and overall English Language Proficiency (ELP) level of 2-4 on the ACCESS for ELLs™. These tests are administered each year to assist teachers with planning differentiated lessons or unit plans (WIDA, 2014).

In the 2013-2014 school year, demographic data of this large suburban high school population consisted of 2,383 students (1,132 male and 1,187 female), in grades 9-12. In the same school year, the students' socioeconomic levels consisted of all socio-economic groups. Forty-one percent of the students were eligible for free lunch, and 7% of the students are eligible for reduced-price lunch. In the 2014-2015 school year, the racial and ethnic composition was 4% Asian, 46% Black, 20% Hispanic, 2% Multiracial, 27% White, and less than 1% American

Indian or Alaskan Native. In the 2014-2015 school year 8% of the students were non-English or limited English proficient.

The population sample for this study was representative of the overall school population. Student participation was voluntary, and students were not penalized if they chose not to participate. Following the guidelines for research with human subjects identified by the institutional Review Board (IRB), a parent or legal guardian of each participant signed an informed consent form in the student's first language. The student participants also signed assent forms and approval permission from the local school district, and the Kennesaw State University IRB was obtained before conducting this study.

Procedure and Materials

The researcher conducted two face-to-face interviews with the ESOL teacher of record prior to and after the intervention (see Appendix M). Yin (2011) indicates that interviews allow the researcher to analyze spoken words and phrases in addition to nonverbal communication (voice tone, pauses, interruptions, and mannerisms). The EOSL mathematics teacher implemented this study with two classes (control and treatment). The teacher administered QUASAR Student Dispositions Instrument questionnaire (pre and post) and Unit 3A: Linear Assessment (pre and post) and Unit 3B: Exponential Assessment (pre and post). The ESOL teacher taught the treatment group using concrete and virtual manipulatives as an instructional strategy (Gurbuz, 2010; Sherman & Bisanz, 2009). The same ESOL teacher taught the control group through traditional instruction. The groups are labeled as *Teacher A Treatment Group* and *Teacher A Control Group*.

Assessing differences in achievement groups. Linear and exponential functions in coordinate algebra present significant challenges for ELLs. National Council of Teachers of Mathematics (2009) and teacher assessments reveal that the following skills cause students difficulties in the transition from arithmetic to algebra:

- “Expressing geometry with algebraic notation, including function notation”
- “Reasoning about slope; graphing line, and finding equations of lines”
- “Building and using algebraic functions”
- “Setting up the appropriate equations to solve word problems” (Graham, Cuoco, Zimmermann, 2009, p. 25).

These skills reflect the importance of reasoning with algebraic symbols, building equations, and functions.

Pretests and posttests were administered to both treatment and control groups to evaluate differences in achievement between the groups. As indicated by Gall, Gall, and Borg (2007) one threat to internal validity in this type of study is that differences shown on the posttest could be a result of pre-existing differences between the groups prior to the study.. In order to reduce the effects of initial group differences on the results produced by the study, a pretest served as a co-variant, which strengthens the experiment by removing any extraneous variables that could have a direct impact on the dependent variable (student achievement) (Ary, Jacobs, Razavich, & Sorensen, 2006). For the pretest the Coordinate Algebra departmental assessment was given at the beginning of the Unit 3A: Linear Functions (see Appendix K) and Unit 3B: Exponential Functions (see Appendix L). The posttest, the Coordinate Algebra departmental assessment, was given at the end of the Unit 3A: Linear Functions Summative Assessment (see Appendix K) and

Unit 3B: Exponential Functions Summative Assessment (see Appendix L). To assess reliability, the assessments were scored by two teachers with an intra-class correlation coefficient of .99. The Unit 3A: Linear Functions Summative Assessment and Unit 3B: Exponential Summative Assessment were aligned to the Linear and Exponential Function Common Core Georgia Performance Standards for Mathematics and Standards for Mathematical Practice (see Appendix A). The 41 assessment items range from level 1 to level 3 (Web's Depth of Knowledge Level). The students demonstrated mathematical proficiency in 12 multiple choice questions and 29 free response questions. The mathematics department in this setting used a standardized assessment to measure student ability to comprehend mathematical standards. This study sought to determine if a significant correlation exists between student achievement of linear and exponential functions and instruction using concrete and virtual manipulatives. The following lessons were implemented in Table 3.3:

Table 3.3

Lessons

<u>Linear</u>	<u>Exponential</u>
Graphing Relationships (GR)	Graphing Relationship (GR)
Relations and Functions (RF)	Relations of Functions (RF)
Vertical Line Test (VT)	Vertical Line Test (VT)
Model Variable Relationships (MR)	Model Variable Relationships (MR)
Writing Functions (WR)	Writing Functions (WR)
Arithmetic Sequences (AS)	Geometric Sequences (GS)
Graphing Linear Functions (GF)	Graphing Exponential Functions
Transformations of Linear Functions (TF)	Transformations Exponential Functions
Characteristics of Linear Functions (CF)	Characteristics of Exponential Functions
Functions Operations (FO)	Functions Operations
Average Rate of Change (ARC)	Average Rate of Change
Real world applications with (RWA)	Real world applications with
Linear Functions	Exponential Functions
Compare Linear and Exponential functions (CF)	

Disposition. The QUASAR Student Dispositions Instrument (1992), which was developed by the Learning Research and Development Center at the University of Pittsburgh, was used to measure program outcomes and student growth with respect to student dispositions

about mathematics and math class. This assessment has a 20-minute administration time, and it is a 36 question, 6-point Likert scale with .97 reliability and content validity. QUASAR Student Dispositions Instrument was administered at both the beginning and the end of the Unit 3: Linear and Exponential Functions Summative Assessment to measure changes in student dispositions towards mathematics and math class. The QUASAR Student Dispositions Instrument was used to determine if a significant correlation exists between student dispositions toward mathematics instruction following the use of mathematics manipulatives.

Instructional Design

Echevarria et al.'s (2012) sheltered instruction observation protocol (SIOP) model was used to integrate content and language through the development of lesson plans and delivery approach. The SIOP model consists of eight interrelated components: (a) lesson preparation, (b) building background, (c) comprehensible input, (d) strategies, (e) interaction, (f) practice and application, (g) lesson delivery, and (h) review and assessment. These components have been established as an ongoing research-based strategy, and Guarino et al. (2001) confirm the SIOP model to be a valid and reliable measure of sheltered instruction (Echevarria et al., 2012).

Treatment

The treatment group and the control group were administered pretests and posttests for the Unit 3A: Linear Summative Assessment and Unit 3B: Exponential Summative Assessment. In addition, the QUASAR Student Dispositions Instrument was given before the Unit 3A: Linear Summative Assessment pretest and after Unit 3B: Exponential Summative Assessment posttest. The treatment group used an online edition of Holt M^cDougal's *Coordinate Algebra, Georgia Edition* (2014) and the following activities using manipulatives in Table 3.4:

Table 3.4

Lesson Activities with Manipulatives

<u>Lesson</u>	<u>Activity</u>	<u>Manipulative(s)</u>
MR & WF	Model Variables Relationships	Cubes
VT	Vertical Line Test	Spaghetti
GR	Stations Graphing	TI-84 Plus Calculator, Motion Detector, Temperature probe, Ball, Toy Truck, Card Board Ramp, Remote Control Truck, Paper Plate, Hot Water, Ice Water
AS	Stacking Cubes	Cubes, Pipe Cleaners
CF	Interactive Range and Domain Finder	Paper folding with templates
TF	Exploring Transformations	White Board, Pipe Cleaners
GF	Function of a Toy Balloon (Coes, 1994)	Balloons, Tape Measure, Rulers, Stopwatch, Calculator, Spaghetti
EF	M&M Investigation	M&M's, Pipe Cleaners
EF	Bacterial Growth (Cowen, 2014)	Video You Tube, Cups
GS	Stacking Cubes	Cubes, Pipe Cleaners
TF	Linear and Exponential	Geometer's Sketchpad Sliders
Vocabulary	Vocabulary Builder	Magnetic Flash Cards
RWA	Opening Your Own Business	Cubes, Pipe Cleaners, Promethean Board, TI-84 Plus Cal.

It is important to use a variety of materials to teach a concept to support the multiple learning styles, visual (seeing), kinesthetic (moving), and tactile (hands-on) (Avalos et al., 2005).

Data Collection

The treatment group was treated with concrete and virtual manipulatives and online math textbook instruction as assigned by their ESOL teacher of record. The control group was also instructed by the same ESOL teacher of record through the use of online math textbooks. Before the pretests and before the posttests, both groups completed the QUASAR Student Dispositions Instrument pre-questionnaire and post-questionnaire to measure student growth with respect to their dispositions about mathematics. After the pretest (Unit 3A: Linear Functions), the treatment continued through the end of Unit 3B: Exponential Summative Assessment. A posttest of math achievement was then given to all groups. The ESOL teacher of record participated in two face-to-face interviews prior to and following the experiment (see Appendix M). Recorded field notes, student work samples, and artefacts were obtained..

Analysis of Quantitative Data

After data collection, the pretest and posttest scores were compared in the following ways:

- One-Way ANOVA (single dependent variable and a single independent variable) for parametric techniques for analyzing quantitative data were used to test both hypotheses with a .05 level of confidence to test for the statistical significance of the difference between the mean test scores of the two groups. The QUASAR Student Dispositions Instrument was given at the beginning of the Unit 3A: Linear Functions for the pre-questionnaire and again at the end of the Unit 3B: Exponential Functions for the post-

questionnaire. The results of the QUASAR Student Dispositions Instrument pre-questionnaire and post-questionnaire were compared to determine student dispositions towards mathematics and math class.

- One-Way ANOVA (single dependent variable and a single independent variable) for parametric techniques was used to test both hypotheses with a .05 level of confidence to test for the statistical significance of the difference between the mean scores using questions 7, 8, 10, 12,15,16,19, 28, 29, and 32 (student's belief in math as a problem solving, reasoning, and collaborative activity) of the two groups. The QUASAR was given at the beginning of the Unit 3A: Linear Functions for the pre-questionnaire and again at the end of the Unit 3B: Exponential Functions for the post-questionnaire. The results of the QUASAR Student Dispositions Instrument (question 37) pre-questionnaire and post-questionnaire were compared to determine if ELL students' use of manipulatives increased their understanding of mathematics. In addition, the QUASAR Student Dispositions Instrument (question 38) pre-questionnaire and post-questionnaire were compared to determine if ELL students' use of manipulatives did not increase their understanding of mathematics.
- One-Way ANOVA (single dependent variable and a single independent variable) for parametric technique for analyzing quantitative data were used to test both hypotheses with a .05 level of confidence to test for the statistical significance of the difference between the mean test scores of the two groups. The Unit 3A: Linear and Unit 3B: Exponential Summative Assessments were given at the beginning of the unit for the pretest and again at the end of the unit for the posttest.

- The scores were analyzed using Statistical Package for the Social Sciences (SPSS) software to determine the effects of manipulatives (concrete and virtual) on ESOL student achievement.

Analysis of Qualitative Data

A triangulation design of mixed methods was used to compare both quantitative and qualitative data. The data were collected, and the results of those findings validated each other (Fraenkel & Wallen, 2009). The qualitative data analysis occurred in five iterations. The iterations are methods that Anfara, Brown, and Mangione, (2002) and Harry, Sturges, and Kilinger (2005) have identified as the bottom to top approach. There are three iterations for conducting data analysis through code mapping. The first iteration was the initial process of listening to the audio recorded interviews and recorded field notes and then transcribing the interviews and field notes. The second iteration was reading the transcripts to make meaning of the large set of data. The third iteration was open coding, reading and coding the transcript using ATLAS.ti qualitative research software to organize data. During this initial process, codes emerged. Reading the audio transcripts and applying open coding (labeling the key points) several times provided a holistic understanding of what the data were saying. This process assisted with breaking the data apart, which lead to the fourth iteration where the researcher examined the codes, looked for redundancy, and checked to see if there was any miscoding of the transcript. In the fourth iteration the researcher collapsed (axial coding) codes into groups based on common concepts and characteristics, creating categories for axial coding . The fourth iteration organized the findings into main categories and sub-categories. The researcher returned

to the data (iteration four) for a more theoretical look during this observation. The fifth iteration presented the themes for this study.

Delimitations

- This study was limited to ESOL high school students attending the same large suburban school system.
- This study was limited to one teacher of ESOL mathematics.
- The treatment group was limited to one ESOL Coordinate Algebra class using concrete manipulatives, virtual manipulatives, and an online textbook.
- The control group was limited to one ESOL Coordinate Algebra class that did not use concrete manipulatives and virtual manipulatives.
- The students were confined to predetermined classes.
- The study was limited to 25 instructional days divided into two sections: linear and exponential functions..
- The large class sizes consisted of 20 students in the control group and 19 students in the treatment group.

Limitations

- The number of ESOL students changed during the research study.
- The study did not attempt to identify factors that might affect students' performance on the achievement test other than the use of concrete and virtual manipulatives.
- The Coordinate Algebra course is standardized (made uniform) utilizing Common Core State Standards. The ESOL teacher is required to cover CCSS.

- The short-term use of concrete and virtual manipulatives (5-weeks) limited the effectiveness of the instruction.

Summary

The purpose of this sequential embedded quasi-experimental mixed methods research study was to compare ESOL student achievement results using manipulatives (concrete and virtual) as compared to a control group without using manipulatives to explore linear and exponential functions. Also, this research study compared ESOL students' dispositions towards mathematics and the math class as a result of using manipulatives (concrete and virtual) as compared to a control group without manipulatives to explore linear and exponential functions. Supplemental data (ESOL teacher interview) was collected before and after the dominant data (pre and post assessment, pre- and post-questionnaire). This study revealed some unique benefits and disadvantages of using concrete and virtual manipulatives.

Quantitative (pre and post assessments, pre- and post-questionnaire) and qualitative (before and after ESOL teacher interview) data were collected, analyzed, and triangulated in order to analyze the data and provided an understating of the research problem (Creswell, 2012).

CHAPTER 4

FINDINGS

Introduction

The purpose of this sequential embedded quasi-experimental mixed methods research study was to determine if there were significant performance differences in learning linear and exponential functions between ELLs using concrete and virtual manipulatives (experimental group) and ELLs using traditional instructional learning practices without manipulatives (control group). Additionally, the researcher wanted to investigate if there were significant differences in ELLs' dispositions (which include variables, such as beliefs, attitudes and values) towards learning mathematics between those using concrete and virtual manipulatives and those using traditional learning practices without manipulatives. Quantitative methods compared results from Unit 3A: Linear Functions Summative Assessment (pretest and posttest) and Unit 3B: Exponential Functions Summative Assessment (pretest and posttest) between the groups (control and experimental) to inform research question 1 and question 2. Also, the quantitative methods compared results from Quantitative Understanding: Amplifying Student Achievement and Reasoning Students Disposition Instrument (QUASARQSDI) (pretest and posttest) between the groups (control and experimental) to inform research question 3.

Qualitative methods such as the ELLs' teacher interviews and student work sample artefacts were employed to inform research question 1, question 2, question 3, and question 4. Research question 4 revealed the unique benefits and disadvantages of using concrete and virtual manipulatives. The four research questions and associated hypotheses are as follows:

Research Question 1: What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about linear functions compared to a control group using traditional instructional practice?

Research Question 2: What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about exponential functions compared to a control group using traditional instructional practice?

Research Question 3: What difference, if any, exists in students' dispositions about mathematics and math class as a result of using concrete and virtual manipulatives as ESOL high school students employ them to learn about linear and exponential functions compared to a control group using traditional instructional practices?

Research Question 4: What are the benefits and disadvantages of using concrete and virtual manipulatives versus traditional instructional practice, from a teacher's perspective, in teaching linear and exponential functions?

This chapter provides a presentation of quantitative data in graphic and tabular formats mixed with qualitative results for the three research questions. This chapter contains the results of the quantitative and qualitative analyses, including a reporting of the sample descriptive statistics. Next will be an analysis of the four research questions. One-way ANOVAs were conducted to answer each of the three quantitative research questions because the one-way ANOVAs were preceded by tests that evaluate if the assumptions of the one-way ANOVA have been met. These include an examination of normality and homogeneity of variance (Field, 2013).

Sample

A total of 39 secondary 9th grade students were included in this study. One ESOL mathematics teacher implemented this study, teaching both groups utilizing the sheltered instruction observation protocol (2012) method to integrate language and content. There were 20 (51.3%) students in the control group and 19 (48.7%) in the treatment group. There were 19

males and 20 females in the study, and the average age of all respondents was 15.4 years ($SD = .95$). Mean, median, and standard deviation values were posted for pretest and posttest values of the linear, exponential, and disposition scores for the control and treatment groups. See tables 4.1, 4.2 and 4.3 for descriptive statistics.

Table 4.1

Descriptive Statistics for Age by Treatment and Control Groups

Group	<i>N</i>	<i>M</i>	<i>SD</i>	Minimum	Maximum
Control	20	15.35	1.14	14.00	18.00
Treatment	19	15.42	.77	14.00	17.00

Table 4.2

Descriptive Statistics: Gender by Treatment and Control Groups

	Control		Treatment		Total	
	N	%	N	%	N	%
Female	9	45.0%	11	57.9%	20	51.3%
Male	11	55.0%	8	42.1%	19	48.7%

Table 4.3

Descriptive Statistics: Pretest/Posttest Linear, Exponential, and Student Dispositional Scores by Treatment and Control Groups

Group		<i>N</i>	<i>M</i>	<i>SD</i>	<i>Mdn</i>
Control	CPreLinear	20	10.50	10.57	6.00
	CPostLinear	20	29.75	19.89	20.50
	CPreExponential	20	7.60	8.41	6.50
	CPostExponential	20	22.85	16.10	17.50
	Student_B_A_pre	20	4.01	.75	4.15
	Student_B_A_post	20	3.85	1.03	3.95
Treatment	CPreLinear	19	12.42	9.74	13.00
	CPostLinear	19	27.37	15.80	23.00
	CPreExponential	19	8.16	7.27	8.00
	CPostExponential	19	35.16	24.58	25.00
	Student_B_A_pre	18	4.09	.78	4.00
	Student_B_A_post	18	3.93	.85	3.80

Note: There was no significant difference in pretest linear, exponential, or student B_A scores between the treatment and control groups based on the results of the independent sample t-test. See Table 4.4

Table 4.4

Pretest Differences between the Control and Treatment Groups on Linear Functions, Exponential Functions, and Student Dispositional Scores

	Control		Treatment		df	t	p
	M	SD	M	SD			
CPreLinear	10.50	10.57	12.42	9.74	37	-.590	.559
CPreExponential	7.60	8.41	8.16	7.27	37	-.221	.826
Student_B_A_pre	3.86	1.03	4.09	.78	37	-.324	.748

Analytic Strategy

After the data were entered in to SPSS, the data were examined for missing values and data errors. There were no missing values or data errors. So, no cases were removed because of missing values. Next, difference scores were computed for the linear function and exponential function assessments using pretest and posttest scores. Linear function pretest scores were subtracted from linear function posttest scores to calculate the linear function difference scores for each respondent. Also, exponential function pretest scores were subtracted from exponential function posttest scores to calculate the exponential function difference score for each respondent.

According to Tabachnick and Fidell, (2012), given the stringent limitations of the ANCOVA and potential ambiguity in interpreting results, differences between the posttest and pretest measures can be computed for each respondent and used as the dependent variable in ANOVA as a way of controlling for pretest scores. Pretest and posttest student dispositional scores were also computed for each student by computing a composite mean score using questions 7, 8, 10, 12, 15, 16, 19, 28, 29, and 32 of the QUASAR Student Disposition Instrument. A student dispositional difference score was then calculated by subtracting the mean pretest scores from the mean posttest scores. A one-way ANOVA was later conducted to determine if there were significant differences between the control and treatment groups with regards to student dispositional difference scores. A one-way analysis of variance was later conducted to determine if there were significant differences between the control and treatment groups with regards to student dispositional difference scores. The one-way ANOVA was chosen for two reasons. First and primarily, the one-way ANOVA and the independent samples t-test produce the same results when there are two groups of the independent variable, as the p values are the same and the ANOVA F value is equivalent to the squared t value of the independent samples t-test. (Field, 2013; Hair et. al, 2012; and Tabachnick & Fidell, 2012). Secondly, SPSS only produces effect size and post-hoc power analysis calculations for the ANOVA, not

the t-tests (Field, 2013; Pallant, 2013). So, this was an added benefit of running the one-way ANOVA over the independent samples t-test.

Preliminary analyses were then conducted to evaluate the parametric assumptions of the one-way ANOVA. A Shapiro-Wilk test was conducted to evaluate the assumption of normality, where a p value of less than .05 indicates a violation in normality. The second test that was conducted was Levene's test of homogeneity of variance. Again, p values of less than .05 indicate a violation in the assumption of homogeneity of variance. Given that the samples sizes in each group were at least 15, and the one-way ANOVA is a robust test, violations in either the assumption of normality or homogeneity of variance will still allow for the use of the one-way ANOVA. There are two reasons for this. First, the central limit theorem states that we can assume that the distribution of the sample means is normal if the sample size is 30 or greater (or at least 15 in each group) (Green & Salkind, 2014; Field, 2013, Tabachnick and Fidell, 2012). Second, a statistical test is considered robust if a p value remains between $\pm .02$ of the original p value after an extreme simulated distortion of the sample is generated to produce violations in normality and/or homogeneity of variance (Boneau, 1960; Tabachnick and Fidell, 2012). Through Monte Carlo sample simulation tests, both Posten (1984) and Schmider et al. (2010) have shown that the t-test and ANOVA are robust under very extreme normality (i.e. skewness =2 and kurtosis =9) and homogeneity violations, where the difference in variance is up to 3.5 times in size. So, the ANOVA is very robust when there are violations of normality and homogeneity of variance.

Once the preliminary analyses were completed, the primary analyses were conducted to evaluate the null hypotheses of the three research questions. In addition to statistical significance, the eta squared effect size measure was also computed for each one-way ANOVA. Eta squared tells us the amount of variance or change in the dependent variable that is explained by the independent variable. Essentially, it reveals the size of the effect that the independent variable has on the dependent variable. According to Cohen's (1988) guidelines, an eta squared value of

.01 or 1% is small, .06 or 6% is a medium sized effect, and .16 or 16% or higher is considered a large effect. Table 4.5 contains a summary of the eta squared effect sizes. In the subsequent analyses, I will refer to small, medium, and large effect size.

Table 4.5

Eta Squared Effect Size Guidelines

Size of Effect	Eta Squared
Small	.01 or 1%
Medium	.06 or 6%
Large	.16 or 16%

Results

Research question 1. What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about linear functions compared to a control group using traditional instructional practice?

A one-way ANOVA was conducted to determine if there was a significant difference in student achievement on linear functions between the treatment group using concrete and virtual manipulatives and the control group using traditional instructional practice. The independent variable was grouped, where 1 was the control group and 2 was the treatment group. The dependent variable was linear function difference scores. Again, this variable was calculated by subtracting linear functions pretest scores from linear function posttest scores.

Before the one-way ANOVA was conducted, tests of normality and homogeneity of variance were conducted. Results of the preliminary analyses revealed that there was no violation in the assumption of normality for the control group, $SW(20) = .976, p = .867$, or the treatment group, $SW(19) = .929, p = .169$, as the p values were greater than .05. Additionally, there was no violation in the assumption of homogeneity of variance, $F(1, 37) = 2.82, p = .102$, as the p value

was greater than .05. Figure 4.1 displays the box plots for both the control and treatment group difference scores for linear functions. Despite there being an extreme outlier in the treatment group, there were no statistically significant violations in normality or homogeneity of variance,

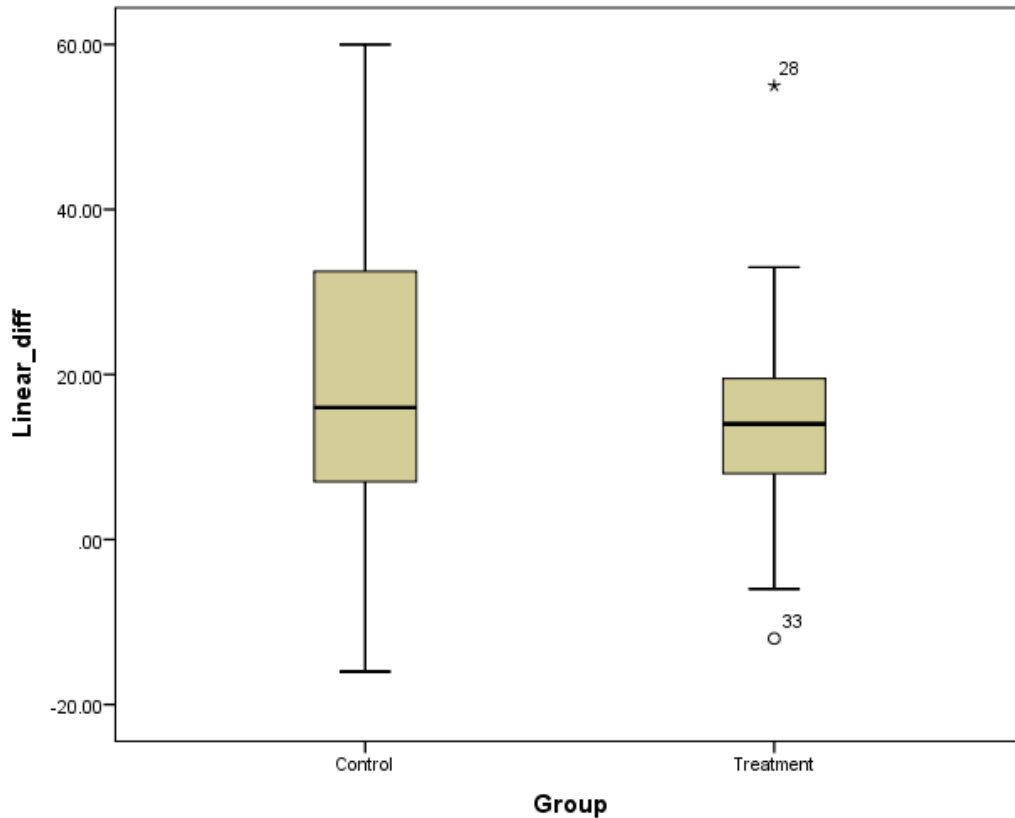


Figure 4.1: Box plots of difference scores for linear functions for the control and treatment groups.

Results of the one-way ANOVA indicated that there was no statistically significant difference in the change in linear function scores from pretest to posttest between the control group ($M = 19.25$, $SD = 19.45$) and the treatment group ($M = 14.95$, $SD = 14.53$) on linear functions, $F(1, 37) = .607$, $p = .441$. This means that the change in linear function scores from the pretest to the posttest was essentially the same for both the control and treatment groups. The eta squared effect size measure indicated that the effect of the independent variable on linear function performance scores was small, $\eta^2 = .016$, accounting for only 1.6% of the variation in

linear function performance difference scores. Based on the results of the one-way ANOVA, the null hypothesis was not rejected. See Tables 4.6, Table 4.7 and Figure 4.2.

Table 4.6

Descriptive Statistics: One-way ANOVA for Linear Function Difference Scores

Group	N	M	SD	Minimum	Maximum
Control	20	19.25	19.45	-16.00	60.00
Treatment	19	14.95	14.53	-12.00	55.00
Total	39	17.15	17.14	-16.00	60.00

Table 4.7

ANOVA Table for Linear Function Difference Scores

Source	SS	df	MS	F	P	Eta Squared	Power
Corrected Model	180.380	1	180.380	.607	.441	.016	.118
Intercept	11394.739	1	11394.739	38.374	.000	.509	1.000
Group	180.380	1	180.380	.607	.441	.016	.118
Error	10986.697	37	296.938				
Total	22643.000	39					
Corrected Total	11167.077	38					

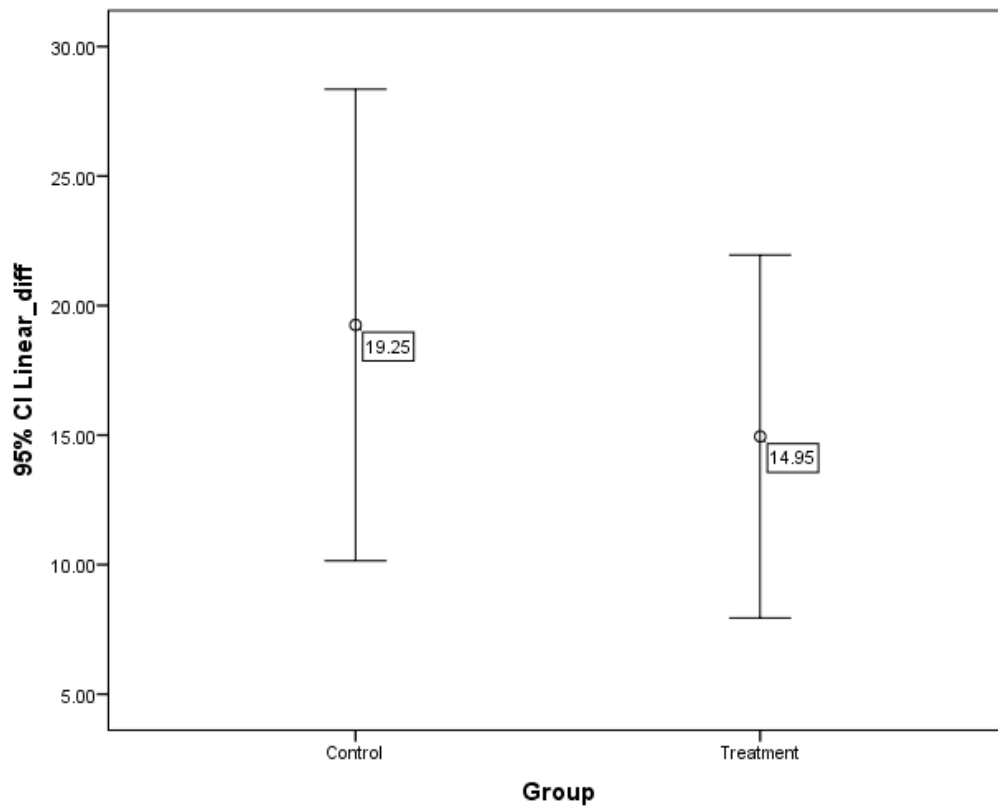


Figure 4.2: Ninety-five percent confidence intervals for the difference in linear function scores between the control and treatment groups indicate that there is overlap. Therefore, there is no significant difference between the difference scores in the groups.

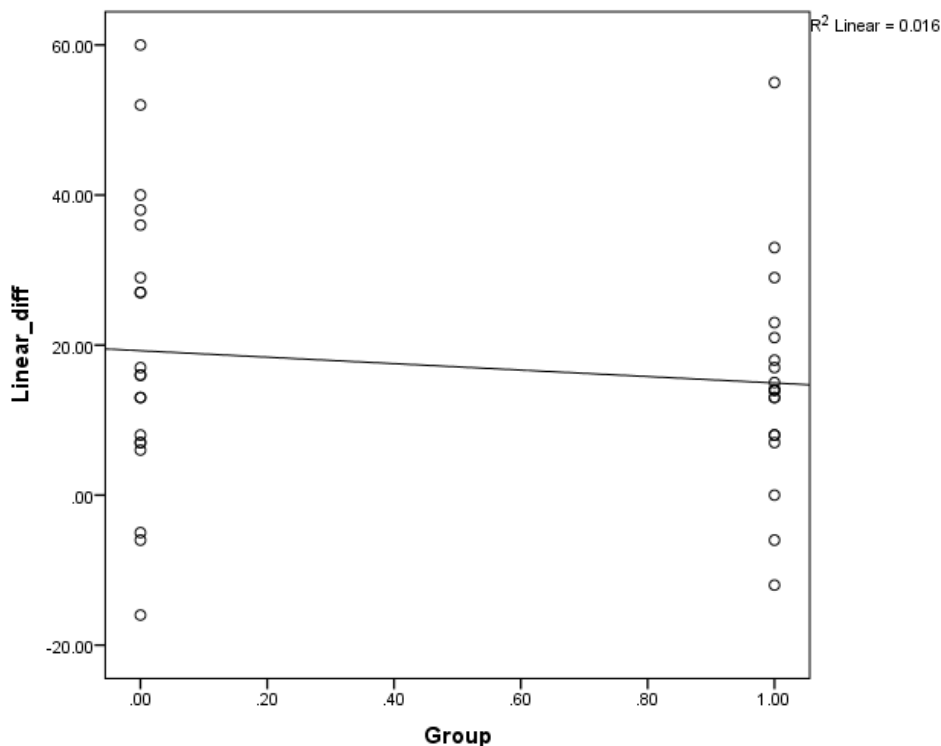


Figure 4.3: Scatterplot of the actual values and the line of prediction using linear differences as the dependent variable and group (0 and 1) as the independent variable.

A post hoc power analysis revealed that the statistical power for this analysis was .12, indicating that given the sample size and the size of the effect, there was only a 12% chance of detecting a significant effect if one actually existed in the real world. The standard for power in the social sciences is .80 or an 80% likelihood of detecting a significant effect (Field, 2013; Tabachnick and Fidell, 2012).

Research question 2. What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about exponential functions compared to a control group using traditional instructional practice?

A one-way ANOVA was conducted to determine if there was a significant difference in student achievement in exponential functions between the treatment group using concrete and

virtual manipulatives and the control group using traditional instructional practice. The independent variable was grouped, where 1 was the control group and 2 was the treatment group. The dependent variable was exponential function difference scores. The dependent variable was calculated by subtracting exponential functions pretest scores from exponential function posttest scores.

Preliminary results of the Shapiro-Wilk test indicated that there was no violation in normality for either the control, $SW(20) = .912, p = .071$, or the treatment group, $SW(19) = .939, p = .270$. However, there was a violation in homogeneity of variance, $F(1, 37) = 9.68, p = .004$, as the p value was less than .05. Given that the ANOVA is robust to extreme violations of homogeneity of variance (up to 3.5 times the size difference in variances), the one-way ANOVA was conducted (Posten, 1984; Schmider et. al., 2010; Tabachnick and Fidell, 2012). A review of the box plots for exponential difference scores reveals that the variance in the control scores was less than 3.5 times the variance of the treatment scores (see Figure 4.3).

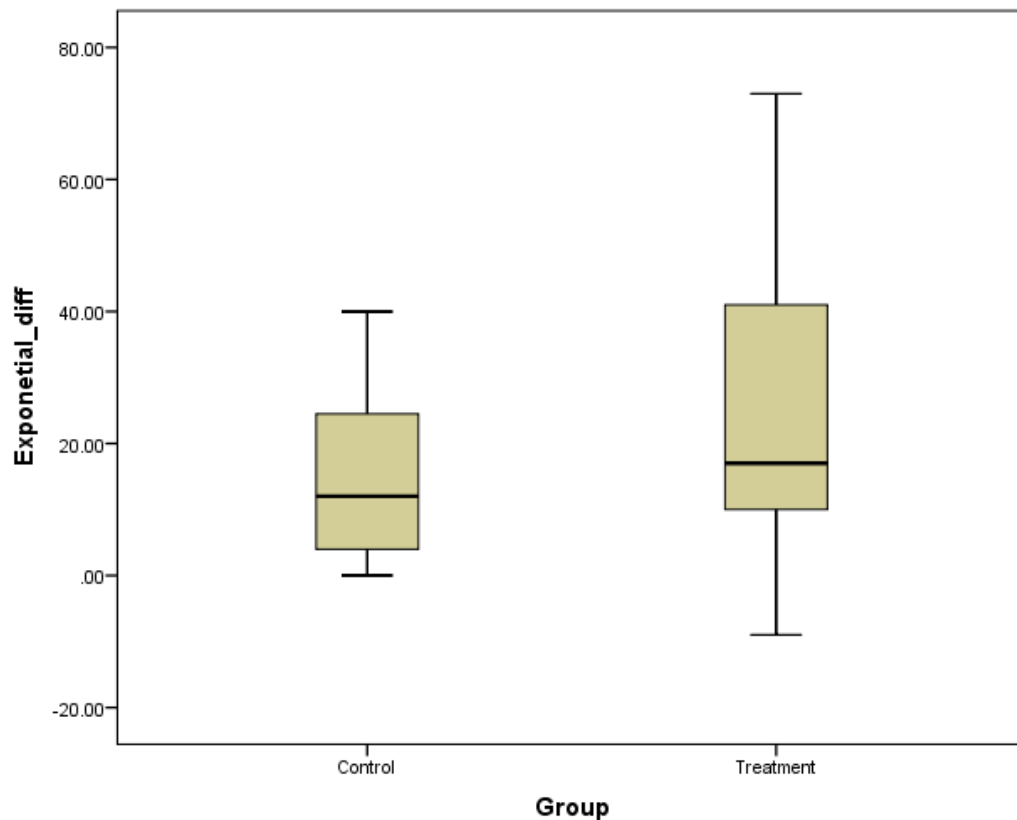


Figure 4.4: Box plots of difference scores for exponential functions for the control and treatment groups.

Results of the one-way ANOVA indicated that there was no statistically significant difference in the change in exponential function performance difference scores from pretest to posttest between the control group ($M = 15.25$, $SD = 12.77$) and the treatment group ($M = 27.00$, $SD = 24.16$), $F(1, 37) = 3.658$, $p = .064$. This means that the change in exponential function scores from the pretest to the posttest was essentially the same for both the control and treatment groups. The eta squared value indicated that the independent variable, group, had a medium sized effect on the change in exponential function scores from pretest to posttest, $\eta^2 = .090$, accounting for 9.0% of the variability in exponential function difference scores. Based the results of the ANOVA, the null hypothesis was not rejected. See Tables 4.8 and 4.9, and Figure 4.4.

Table 4.8

Descriptive Statistics: One-way ANOVA for Exponential Function Difference Scores

Group	N	Mean	SD	Minimum	Maximum
Control	20	15.25	12.77	00.00	40.00
Treatment	19	27.00	24.16	-9.00	73.00
Total	38	19.92	18.96	-9.00	73.00

Table 4.9

ANOVA Table for Exponential Function Difference Scores

Source	SS	df	MS	F	P	Eta Squared	Power
Corrected Model	1345.224	1	1345.224	3.658	.064	.090	.461
Intercept	17392.917	1	17392.917	47.299	.000	.561	1.000
Group	1345.224	1	1345.224	3.658	.064	.090	.461
Error	13605.750	37	367.723				
Total	32108.000	39					
Corrected Total	14950.974	38					

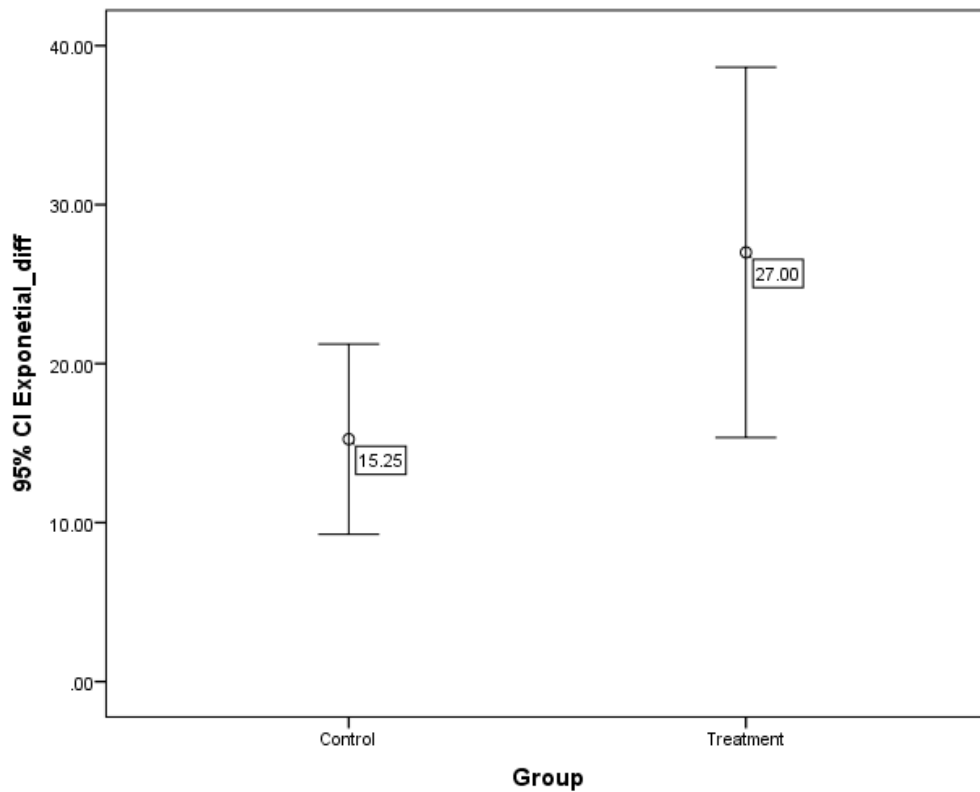


Figure 4.5: Ninety-five percent confidence intervals for the control and treatment groups on the difference in exponential functions scores indicate that there is overlap. Therefore, there is no significant difference between the difference scores in the groups.

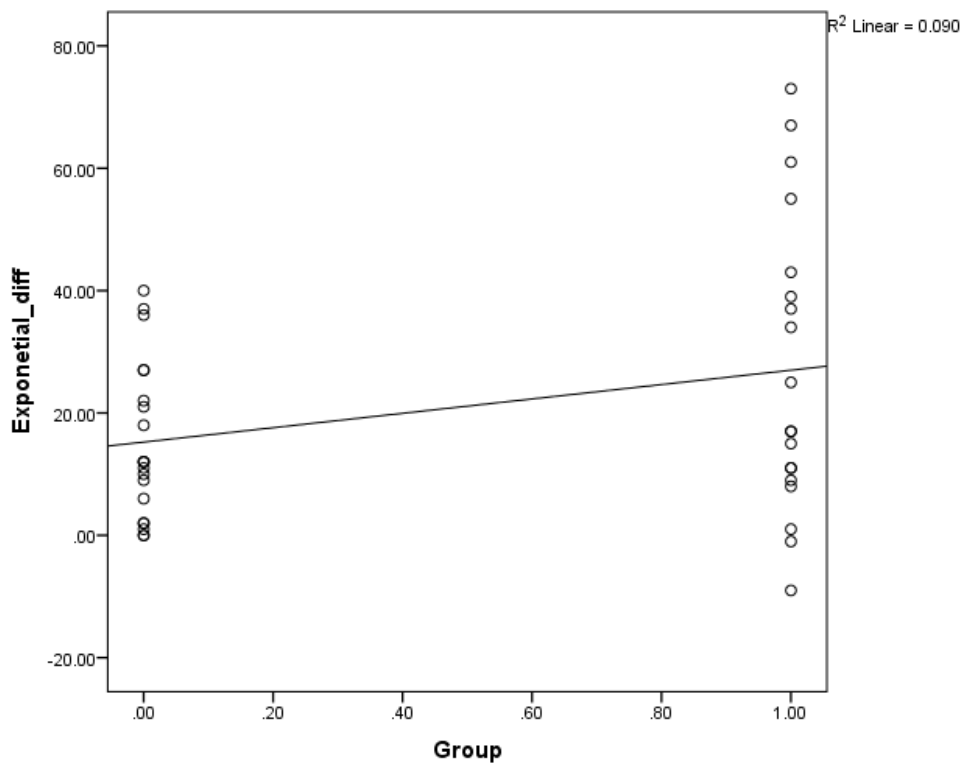


Figure 4.6: Scatterplot of the actual values and the line of prediction using exponential differences as the dependent variable and group (0 and 1) as the independent variable.

A post hoc power analysis revealed that given the size of the effect and the sample size, the power was .461 or 46.1%. This means that there was only a 46.1% chance of detecting a significant effect if one actually existed in the real world. Again, the standard in social scientific research is .80 or an 80% chance of detecting a significant effect.

Research question 3. What difference, if any, exists in student dispositions about mathematics and math class as a result of using concrete and virtual manipulatives as ESOL high school students employ them to learn about linear and exponential functions compared to a control group using traditional instructional practice?

A one-way ANOVA was conducted to determine if there was a significant difference in student dispositions between the treatment group using concrete and virtual manipulatives and

the control group using traditional instructional practice. The independent variable was grouped, where 1 was the control group and 2 was the treatment group. The dependent variable was exponential function difference scores. The dependent variable was calculated by subtracting QUASAR Student Disposition Instrument pretest scores from QUASAR Student Disposition Instrument posttest scores. Negative scores indicated that the pre-questionnaire scores were higher than the post-questionnaire scores, and positive scores indicated that post-questionnaire scores were higher than pre-questionnaire scores.

Preliminary assessments of the parametric assumptions were conducted. Results of the Shapiro-Wilk test indicated that there was no violation in normality for either the control, $SW(20) = .941, p = .249$, or the treatment group, $SW(19) = .952, p = .558$. There was also no violation in homogeneity of variance, $F(1, 37) = .058, p = .811$, as the p value was greater than .05. See Figure 4.7.

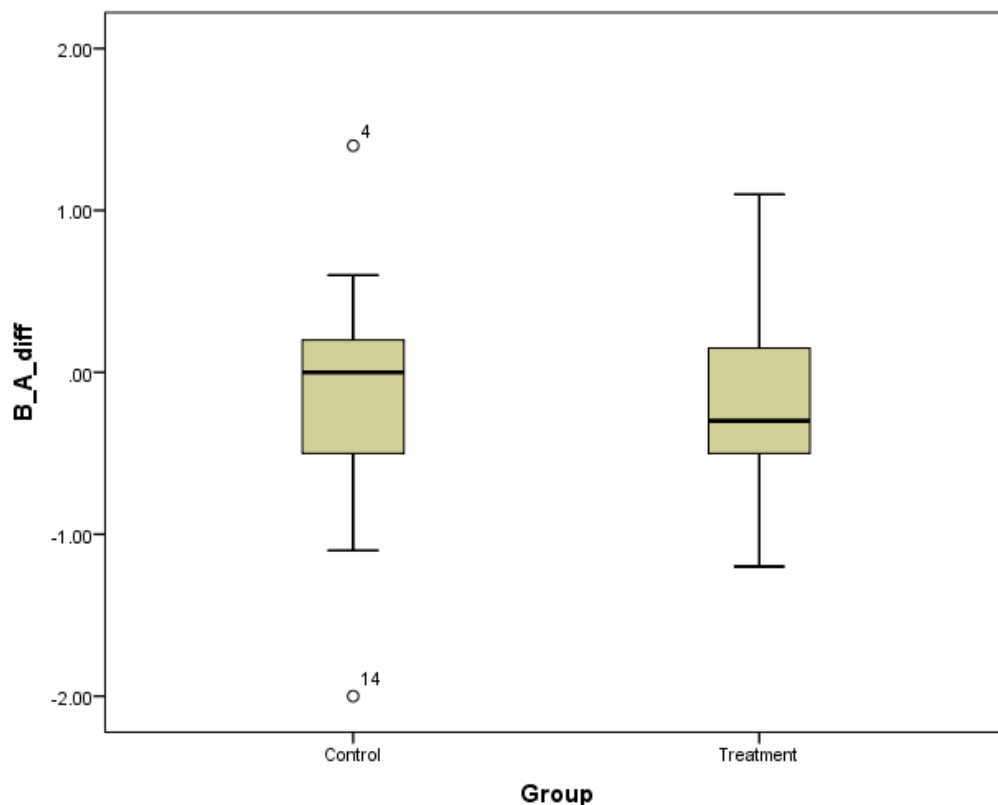


Figure 4.7: Box plots of difference scores of dispositions for the control and treatment groups.

Results of the one-way ANOVA indicated that there was no statistically significant difference in the change in QUASAR Student Disposition Instrument difference scores from pretest to posttest between the control group ($M = -.16, SD = .70$) and the treatment group ($M = -.16, SD = .59$), $F(1, 367) = .002, p = .969$. This means that the change in QUASAR scores from the pretest to the posttest was essentially the same for both the control and treatment groups. The eta squared value indicated that the independent variable, group, had a medium sized effect on the change in exponential function scores from pretest to posttest, $\eta^2 = .002$, accounting for 0.2% of the variability in QUASAR Student Disposition Instrument difference scores. Based on the results of the ANOVA, the null hypothesis was not rejected. See Tables 4.10 and 4.11, and Figure 4.9.

Table 4.10

Descriptive Statistics: One-way ANOVA for QUASAR Student Disposition Instrument

Difference Scores

Group	<i>N</i>	<i>Mean</i>	<i>SD</i>	Minimum	Maximum
Control	20	-.16	.70148	-2.00	1.40
Treatment	19	-.16	.59181	-1.20	1.10
Total	39	-.16	.63184	-2.00	1.40

Table 4.11

ANOVA Table for QUASAR Student Disposition Instrument Difference Scores

Source	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P</i>	Eta Squared	Power
Corrected Model	.001	1	.001	.002	.969	.000	.050
Intercept	.986	1	.986	2.331	.135	.059	.319
Group	.001	1	.001	.002	.969	.000	.050
Error	15.654	37	.423				
Total	16.640	39					
Corrected Total	15.654	38					

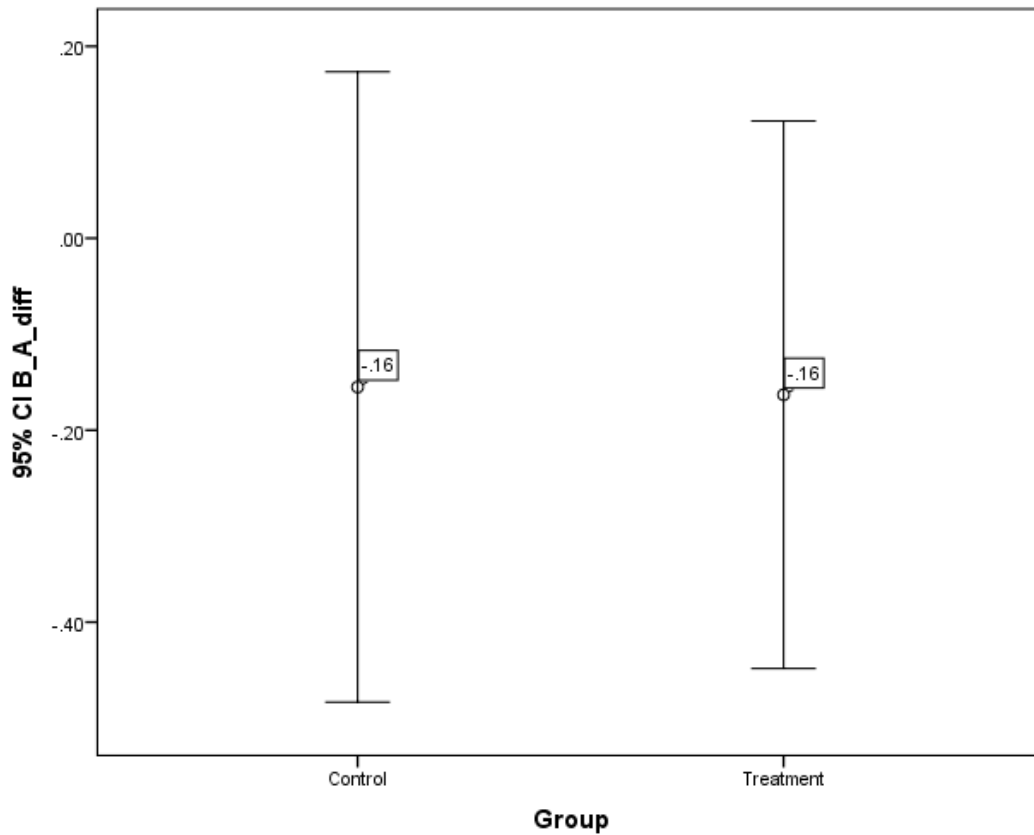


Figure 4.8: Ninety-five percent confidence intervals for the control and treatment groups on the difference in QUASAR Student Disposition Instrument scores indicate that there is overlap.

Therefore, there is no significant difference between the difference scores in the groups.

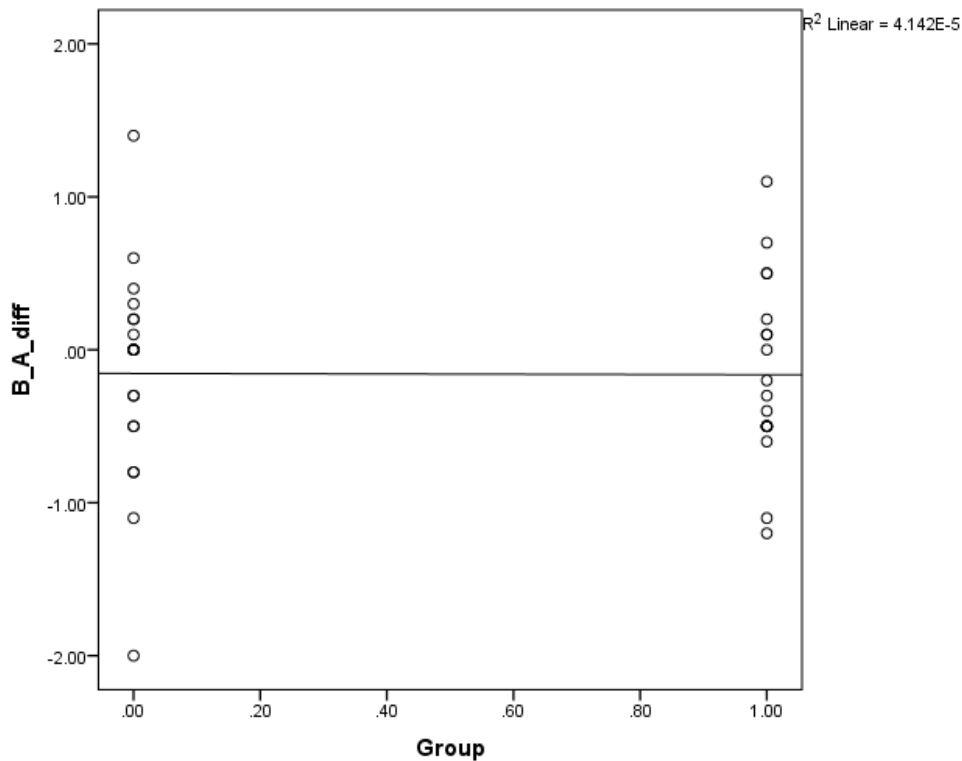


Figure 4.9: Scatterplot of the actual values and the line of prediction using dispositional differences as the dependent variable and group (0 and 1) as the independent variable

A post hoc power analysis revealed that given the size of the effect and the sample size, the power was .050 or 5.0%. This means that there was only a 5.0% chance of detecting a significant effect if one actually existed in the real world. Again, the standard in social scientific research is .80 or an 80% chance of detecting a significant effect.

Additional Analyses

To determine if there were significant changes in scores from the pretest and posttest on linear functions, exponential functions, and student dispositions within the control and treatment groups, a dependent samples t-test was used.

Results of the one-way ANOVA for the treatment group indicated that there was a statistically significant improvement in scores on linear functions from the pretest ($M = 12.42$,

SD = 9.74) to the posttest (M = 27.37, SD = 15.80), $t(18) = 4.48, p < .001$. Additionally, there was a statistically significant improvement in scores on exponential functions from pretest (M = 8.16, SD = 7.27) to posttest (M = 35.16, SD = 24.58), $t(18) = 4.87, p < .001$. Finally, results indicated that there was no significant change in pretest (4.09, SD = .78) and posttest (M = 3.93, SD = .85) scores on the student disposition test, $t(18) = -1.20, p = .245$, as the p value was greater than .05. See Table 4.12 and Figure 4.10

Table 4.12

Pretest Differences on Linear Functions, Exponential Functions, and Student Dispositional Scores for the Treatment Group

	Pretest		Posttest		df	t	p
	M	SD	M	SD			
Linear	12.42	9.74	27.37	15.80	18	4.48	<.000
Exponential	8.16	7.27	35.16	24.58	18	4.87	<.000
Student_B_A_pre	4.09	.78	3.93	.85	18	-1.20	.245

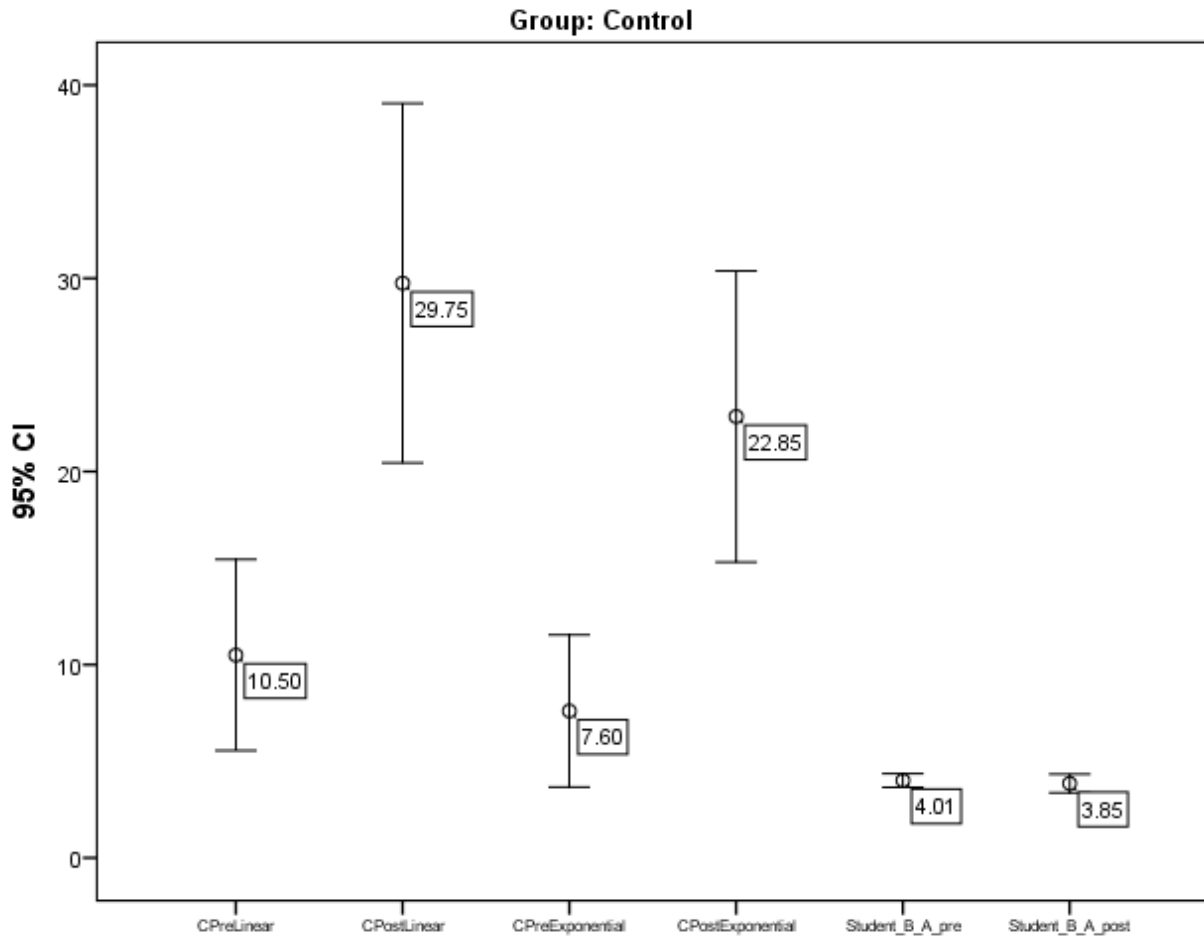


Figure 4.10: Ninety-five percent confidence intervals for the treatment group indicated that there was significant improvement in scores from pretest and posttest for the linear functions and exponential functions, but no difference for student disposition scores.

Result of the one-way ANOVA for the treatment group indicated that there was a statistically significant improvement in scores for linear functions from the pretest ($M = 10.50$, $SD = 10.57$) to the posttest ($M = 29.75$, $SD = 19.89$), $t(19) = 4.43$, $p < .001$. Additionally, there was a statistically significant improvement in scores for exponential functions from pretest ($M = 7.60$, $SD = 8.41$) to posttest ($M = 22.85$, $SD = 16.10$), $t(18) = 5.34$, $p < .001$. Finally, results indicated that there was no significant change in pretest ($M = 4.01$, $SD = .75$) and posttest ($M = 3.85$,

SD = 1.03) scores on the Student Disposition test, $t(18) = -.99$, $p = .335$, as the p value was greater than .05. See Table 4.13 and Figure 4.11

Table 4.13

Pretest Differences on Linear Functions, Exponential Functions, and Student Dispositional Scores for the Control Group

	Pretest		Posttest		df	T	p
	M	SD	M	SD			
Linear	10.50	10.57	29.75	19.89	19	4.43	<.000
Exponential	7.60	8.41	22.85	16.10	19	5.34	<.000
Student_B_A_pre	4.01	.75	3.85	1.03	19	-.99	.335

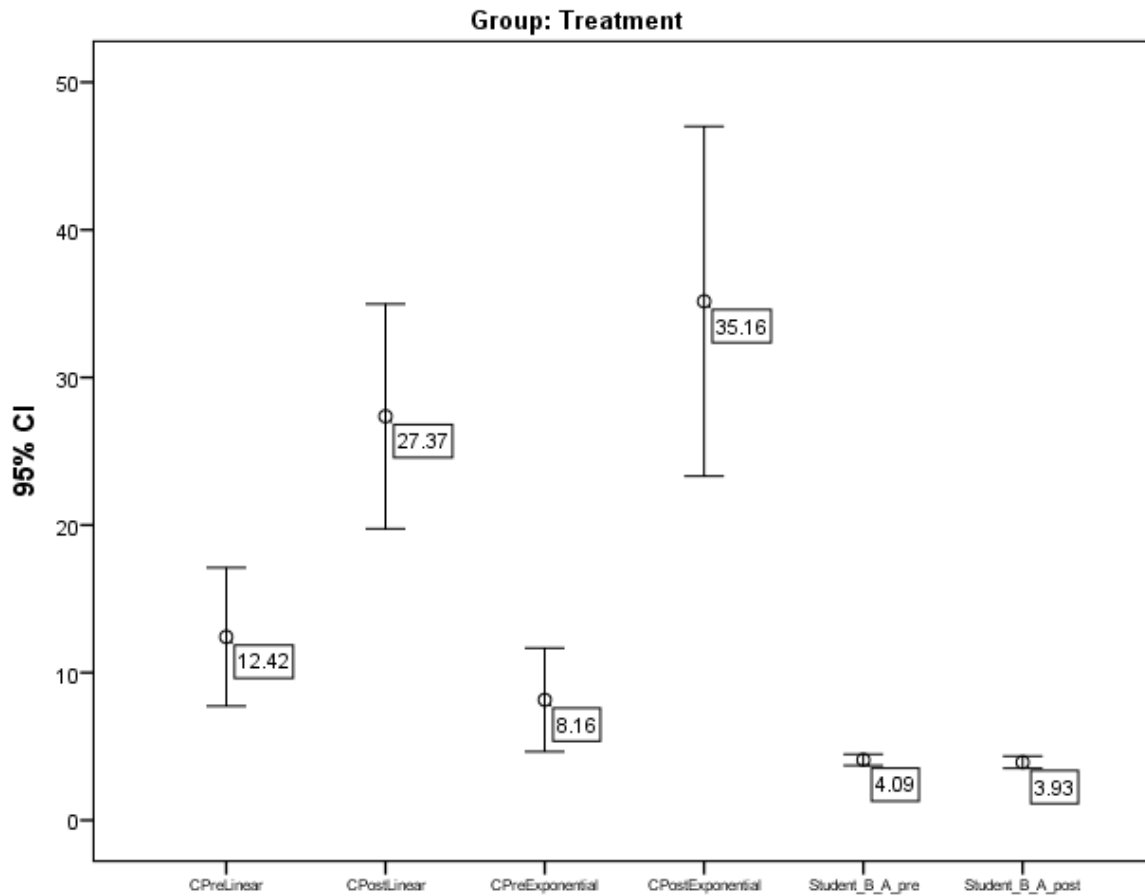


Figure 4.11: Ninety-five percent confidence intervals for the control group indicated that there was significant improvement in scores from pretest and posttest on the linear functions and exponential functions, but no difference in student disposition scores.

Research question 4. What are the benefits and disadvantages of using concrete and virtual manipulatives versus traditional instructional practice, from a teacher’s perspective, in teaching linear and exponential functions?

To answer research question 4, a qualitative analysis was conducted using the bottom to top approach developed by Anfara, Brown, and Mangione, (2002) and Harry, Sturges, and Kilinger (2005). Unlike quantitative analysis, there are no independent and dependent variables in this analysis. The qualitative data (teacher interviews, recorded field notes, students’ work samples and

artefacts) revealed the benefits and advantages of using concrete and virtual manipulatives. The following six themes emerged from the qualitative data analysis. These themes included 1) ELLs were able to make a connection and build upon their prior math knowledge, 2) ELLs were actively engaged during mathematical problem solving, 3) manipulatives created an interference (free play) with ELLs' exploration of linear and exponential functions, 4) large class size created classroom management issues, 5) the teacher found time management was an issue for implementation; 6) the teacher found a lack of availability of concrete and virtual manipulatives. Of the six themes, one and two were considered advantages, theme three was both an advantage and disadvantage, and themes four through six were disadvantages. Each of the themes and their impact will be discussed below.

Theme One: ELLs were able to make a connection and build upon their prior math knowledge. Based on the interviews, recorded field notes and students' work samples from the ESOL teacher, Theme One found that math retention could be increased by having ELLs make a connection with the math manipulatives using Sherman's (1997) six levels of mastery of mathematical concepts, which include intuitive, concrete, representation, abstract, application, and communication. Additionally, building upon prior math knowledge also increases retention and makes a connection (bridges the gap) between the concrete level and the abstract level. For example, when the teacher in the intervention group had the students stack cubes on top of each other, she was able to convey the principles of both arithmetic and geometric sequence as they relate to observing the patterns in stacking the cubes. The students observed that the arithmetic sequence formula ($A_n = A_1 + (N-1) D$) was a linear function with a common difference between two consecutive terms that was constant, and that the geometric sequence formula ($A_n = A_1 * R^{(N-1)}$) was an exponential function with a common ratio between two consecutive terms that was constant. The students were able to write a function algebraically from a given graph, a given table and a given pattern observed from stacking the cubes. By visually and kinetically demonstrating this concept through the stacking of cubes, students were able to take a concept they were familiar with (i.e. stacking blocks) and associate it with new concepts of the arithmetic sequence formula and the geometric sequence formula. This linkage is an example of building upon prior math knowledge to improve retention of a new math concept. Also, the ESOL mathematics teacher expounded on how she built math retention by

making the connection (*bridging the gap*) between the concrete level and the abstract level using Sherman's (1997) six levels of mastery of mathematical concepts:

Like I said, I'm going to make the connection to the equation from the pattern ... and see what if they can ... if they understand that. They are like, "We've got to have like words." I said, "Yes, this is writing an equation ...I mean write in front you," and today, like I said, we did arithmetic sequences and I explained to them that the reason why it's in this section is because we are doing linear functions ... (ESOL Teacher, recorded field notes, March 12, 2015)

1. Draw a picture (histogram) of the cube sackings. Use the grid on the right. The first one has been done for you.

2. What pattern did you observe in stacking the cubes and in the table (output)?
Each one of the cubes grows
by 5 cubes

3. This number is called the common difference
 (d) = 5

4. Use the common difference to write a rule (formula) to find the n th term.
 This is the arithmetic sequence formula.
 $a_n = a_1 + (n-1)d$

5. Use the formula to determine how many blocks would appear in the 10th position.

$a_n = a_1 + (n-1)d$
 $a_{10} = 6 + (10-1)5$
 $a_{10} = 6 + 45$
 $a_{10} = 51$

Place answer here: 51

Sacking Number (x)	Number of Blocks (y)
1	6
2	11
3	16
4	21

Figure 4.12 Treatment group page 2, response to Stacking Cubes arithmetic.

6. Plot the data from the table, using (X, Y) coordinates on the graph to the right. Connect the points and label graph. What function does the graph make?

a linear function

Bonus: Write an equation for the graph?

$y = 5x + 1$

7. What did you discover about placing the pipe cleaner (s) on top of the stack cubes? Explain using complete sentences below:
- I discover placing the pipe cleaner on top of the stack cubes will increase. It will in a straight line.



Figure 4.13 Treatment group page 3, response to Stacking Cubes arithmetic.

1. Draw a picture (histogram) of the cube sackings. Use the grid on the right. The first one has been done for you.

2. What pattern did you observe in stacking the cubes and in the table (output)?

The domain increases by 1

$2^0=1, 2^1=2, 2^2=4, 2^3=8, 2^4=16$

$2^5=32, 2^6=64$

3. This number is called the common ratio

$(r) = \underline{2}$

4. Use the common ratio to write a rule (formula) to find the n th term.

This is the geometric sequence formula.

$n^{\text{th}} \text{ term} = 2^{n-1}$

5. Use the formula to determine how many blocks would appear in the 10th position.

Place answer here: 512

Position (n)	Number of Blocks
1	1
2	2
3	4
4	8
5	16
6	32

Figure 4.14. Treatment group page 2 response to Stacking Cubes geometric.

6. Plot the data from the table, using (X, Y) coordinates on the graph to the right. Connect the points and label graph. What function does the graph make?

a curve line

Bonus: Write an equation for the graph?

$y = 1.2^x$
 $y = 2^x$

7. What did you discover about placing the pipe cleaner (s) on top of the stack cubes? Explain using complete sentences below:

When I place the pipe cleaner on top of the stack cubes, it makes a curve.

X	Y
1	2
2	4
3	8
4	16
5	32

Figure 4.15 Treatment group, page 3 response to Stacking Cubes geometric.

In another example, the ESOL mathematics teacher iterated the differences between common ratio and common difference in stating, “Okay, you remember how it kept on going at a constant difference of five and how the cubes increased each time by adding the same number?”

(ESOL Teacher, recorded field notes, March 13, 2015). Figure 4.12, Figure 4.13, and Figure 4.16 illustrate students' cube stacks to determine the common difference; Figure 4.14, 4.15 and 4.17 illustrate students' cube stacks to discover the common ratio. The activity provided the students with a solid concrete example they visualized, touched and manipulated to make a meaningful connection in building on prior knowledge. The ESOL mathematics teacher additionally indicated, "I could relate it to something. You could go back and build on concrete. . . Like, 'Okay, remember when you did this?' And you could build onto something when you're explaining the rules and things like that" (ESOL Teacher, ESOL teacher interview, April 3, 2015).



Figure 4.16. Common difference.



Figure 4.17. Common ratio.

Theme Two: ELLs were actively engaged during mathematical problem solving. Based on the interviews, recorded field notes and students' work samples from the ESOL teacher, Theme Two found that ELLs were actively engaged during mathematical problem solving when utilizing manipulatives. Theorists and researchers Dienes (1960), Dewey (1938), Montessori (1967), Moschkovich (1999), and Kersaint et al. (2013) agree that students should be actively engaged with mathematics (doing mathematics); and as a result of communication (listening, speaking, reading and writing English) and touching the material, they learn images, which builds upon the next mathematical concept. From these images, the student can translate concrete facts into symbolic representation (Antosz, 1987), which generates connections for a deeper level of mathematical understanding and problem solving while engaging with their peers, classmates, and teacher (Kersaint, Thompson, & Petkova, 2013).

For example, during the *Functions of a Toy Balloon* (Coes, 1994) activity, the students were actively involved in collecting, examining and graphing the effects of filling a toy balloon with various amounts of air in exploring the relationship of circumference versus diameter, diameter versus breadth, flight time versus breadth and flight time versus diameter. The ESOL mathematics teacher stated,

The idea that they all have a job, that's a good thing. Okay? And oh yeah. They were on task, they complained about the measuring, you're not holding it right, and you're not... but they were on task for the most part. Everybody was productive at different levels.
(ESOL Teacher, recorded field notes, March 13, 2015)

The students also verbally communicated, in writing, the accuracy or inaccuracy of the data they collected.

Another example illustrates how the concrete manipulatives (stacking cubes) caught the attention and engaged one special education ELL student. The ESOL teacher expressed how

captivated she was with the special education ELL student's engagement during the *Opening Your Own Business* task; she indicated,

"*Student X*" who usually doesn't do anything, actually did his, drew the pictures and went back and did the line. So he did that and I was impressed, I thought he was playing around. (ESOL Teacher, ESOL teacher interview, April 2, 2015)

The ESOL mathematics teacher and the researcher (an active participant) observed that ELLs were more actively engaged when employing concrete manipulatives (hands-on) to explore linear and exponential functions than when utilizing virtual manipulatives. The ESOL mathematics teacher stated, "The blocks, the stacks were an asset to ELLs learning about algebraic concepts because they could relate to arithmetic and geometric sequences." (ESOL Teacher, ESOL teacher interview, April 3, 2015).

An additional example of engagement took place while the ELLs were practicing literacy skills. The researcher provided the treatment group with a word bank (reading strategy) with the TI-84 Plus Stations Activities: Graphing Relations to serve as a reference for students' interaction while practicing speaking, reading and writing mathematically. The ESOL teacher solidified the activity, explaining Figure 4.18 and Figure 4.19 and having the students write a story:

Then they could tell the story. I told them make sure they write what was happening ... what they were doing. Yeah. I told them these are the key words to use. Some of them, did the bubble just like you did. One little girl wrote a whole paragraph up here (wall).
'When we rolled down ... down the fire truck first it was increased sharply on a constant speed then it dropped and rose variable at an uneven pace then it decreased sharply and rose variable. Finally it decreased ...' We have run-on sentences here. . . Here she used, *'Increased sharply, dramatically, quickly, rapidly...'* (Figure 4.18). This one was the ...

falling ball. Falling ... Rolling ball. Yeah, rolling the ball. Here, she put 'stay the same', 'increase', and 'decrease' (Figure 4.19). Okay and 'rose steady', 'constant', 'increase and started to get colder' ... that's the water. 'Stayed the same. (ESOL Teacher, recorded field notes, March 17, 2015)

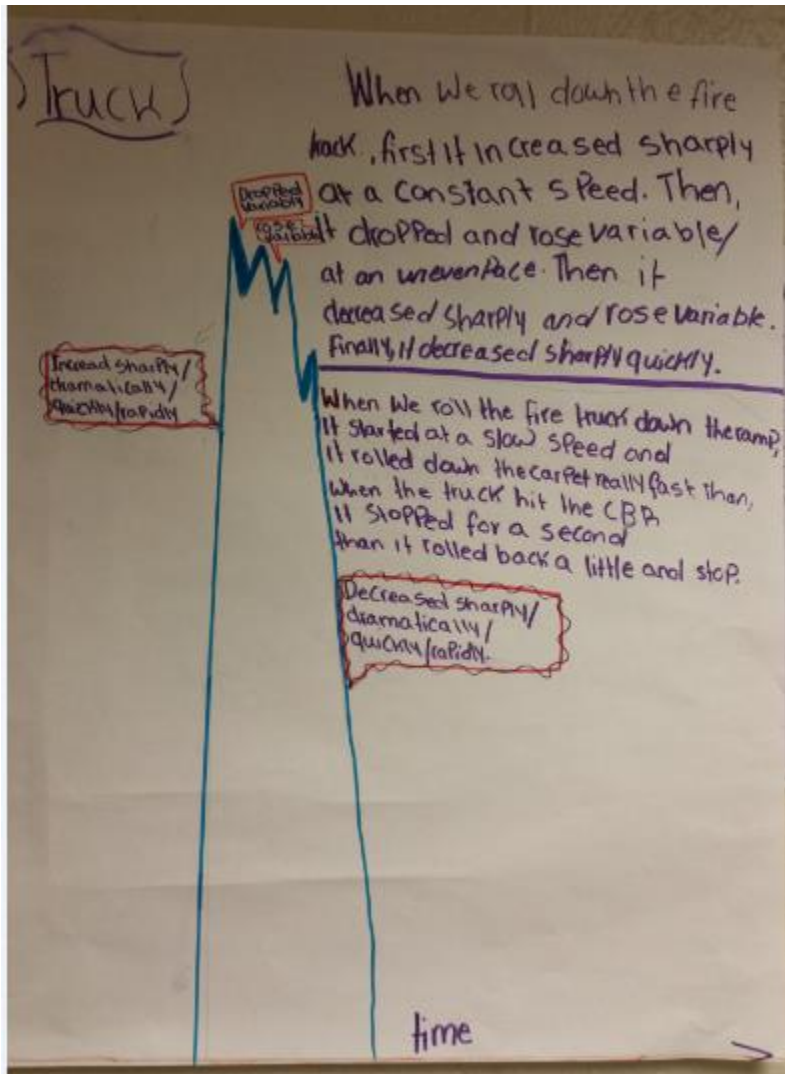


Figure 4.18 Treatment group responses to rolling a fire truck.

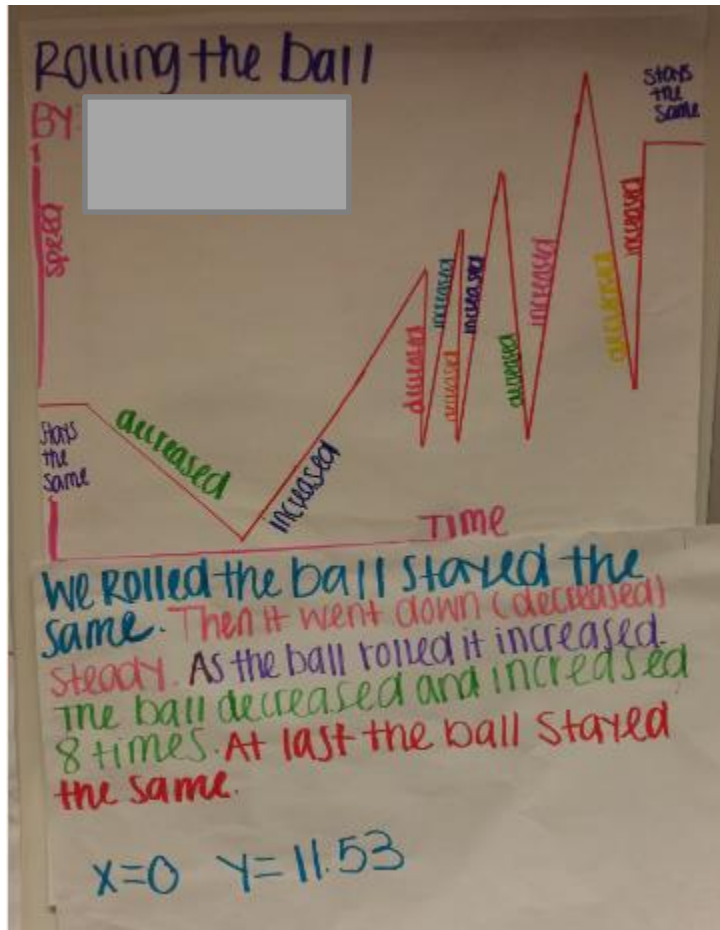


Figure 4.19. Treatment group responses to rolling a ball.

By telling the story, the students were better able to understand the problem which employed practicing literacy skills (speaking, reading and writing) aligned with Krashen's (1988) five models of second language acquisition hypothesis (*acquisition-learning, natural order, monitor, input, and the affective filter*) and Vygotsky's (1978) Zone of Proximal Development. The continuous opportunities for engagement while utilizing manipulatives were advantageous during mathematical problem solving and supported the ELLs with the dual task of learning both English and mathematics concepts simultaneously (see Appendix O).

Theme Three: manipulatives created an interference (free play) with ELLs' exploration of linear and exponential functions. Based on the ESOL teacher interviews, recorded field notes and students' artefacts, Theme Three found that manipulatives created an interference. *Free play* with the

manipulatives interrupted the ELLs' exploration of linear and exponential functions. Dienes (1971) describes *free play* as one of six stages of learning mathematics, which is vital in formulating the first understanding of a new concept. Rabardel's (2003) *from artefact to instrument* theoretical framework stems from Vygotsky's (1978) activity theory, which provided understanding of ELLs' first desired impression (knowledge) to play, explore, and create figures with the stacking blocks (manipulatives as artefacts) prior to learning the actual meaning and usage of the mathematical tool (instrument). Figures 4.20 through 4.22 illustrate some figures students created. For example, the ESOL mathematics teacher was disappointed when the ELLs' first desired impression was to play with the manipulatives, unaware of the *from artefact to instrument* theoretical framework. In a disappointed tone she stated,

Some of the kids decided to make cars and buildings. After I passed out everything I had to go around and say, "No, this is ...," and show them, give them ... this is what you are doing. Then after I had some kids that wanted to mix the colors. I'm like, "No, do the colors like the way they have it." (ESOL Teacher, recorded field notes, March 13, 2015)

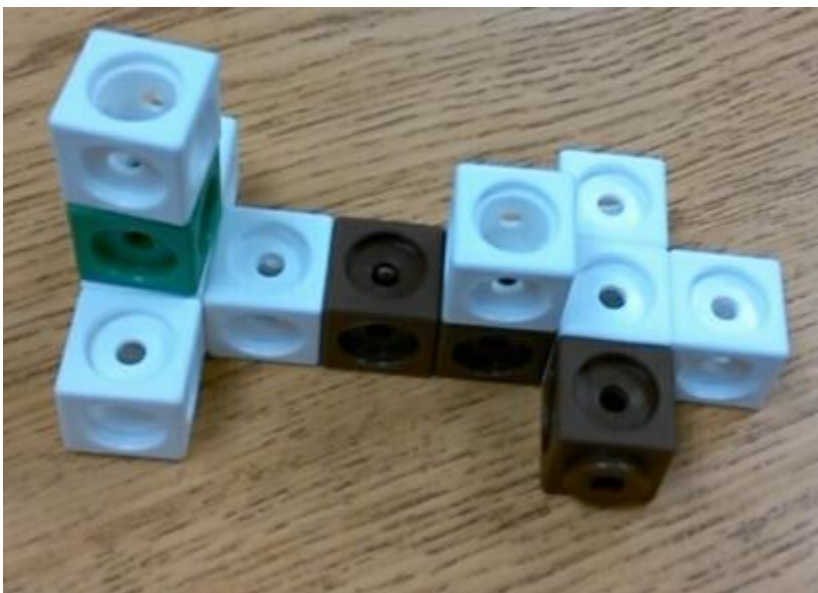


Figure 4.20. A car.

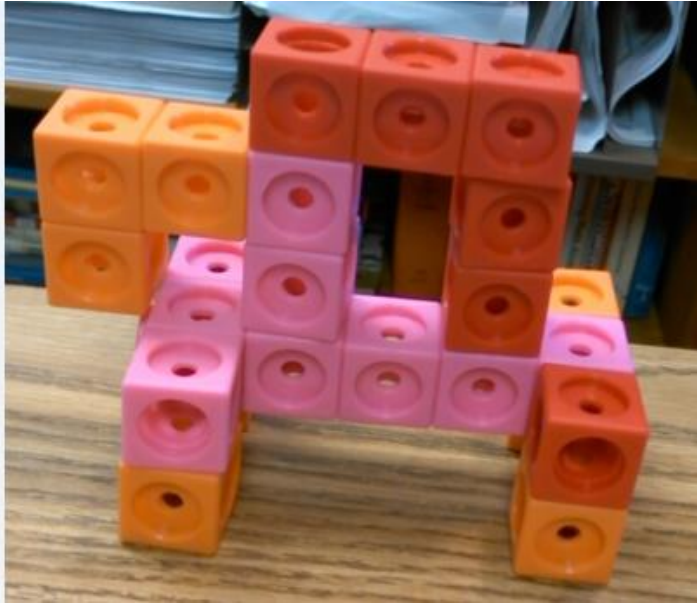


Figure 4.21. An elephant.

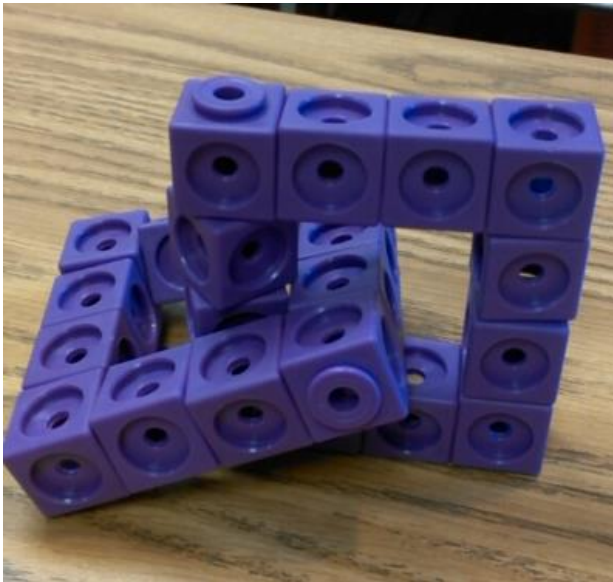


Figure 4.22. Interlocking cubes.

As the research study progressed, one advantage of utilizing manipulatives was that the ESOL teacher developed flexibility and understanding as to how through *free play* the students developed a deeper comprehension of mathematical concepts. In the course of *The Functions of a Toy Balloon* activity (Coes,1994) the students conducted the lab, collected data, graphed

functions, used technology and made inferences pertaining to the data collected. The ESOL teacher specified the following:

Like I said, they're active. There were a couple of the boys of course, I had to try to split them up as best as I could but the three that managed to stay together, they did get the work done. That was good. There were a couple of times I had to tell them about playing, but they did accomplish the goal like the other kids. (ESOL Teacher, recorded field notes, March 13, 2015)

Free play provided opportunities for ELLs to encounter stages (levels) of the instrumental genesis during mathematical problem solving based upon developing usage schema for the manipulatives (Rabardel, 2003). Additionally, *free play* using manipulatives reduced mathematics anxiety (Plaisance, 2009; Woodard, 2004) and encouraged students to continue their study of mathematics beyond the minimal requirements in high school (Brush, 1981; Ma, 2001).

Theme Four: large class size created classroom management issues. Based on the ESOL teacher interviews and recorded field notes, Theme Four found that large class sizes often created classroom management issues. The local school district reduced ESOL teacher allowance, which in turn increased class sizes, and this contributed to a number of classroom issues. Salaudeen's (2013) research on large class size and Gann's (2013) research on meeting the needs of ELLs in the secondary mathematics classroom interplays with the challenges the ESOL mathematics teacher experienced utilizing manipulatives in teaching linear and exponential functions. For example, ESOL teacher indicated:

My biggest problem this year is because my classes are so large, and I know large relatively to your class, no, but for an ESOL class where I have Special Ed (education) and IEL (Integrated English Literacy) kids, you know, who can barely speak English and then regular kids who can speak English very well but

probably not write as well, you have behavioral issues. (ESOL Teacher, ESOL teacher interview, April 3, 2015)

Additionally, the ESOL mathematics teacher found herself frustrated with accountability for the concrete manipulatives, students cleaning up and properly storing the concrete manipulatives. Another disadvantage occurred when students used virtual manipulatives like Geometer's Sketchpad and Desmos. The students had autonomy when utilizing the computers to explore transformation parameters of linear and exponential functions. The ESOL teacher observed some students were off-task surfing the internet, listening to music, watching videos and playing on-line games as she offered assistance with the exploration. The ESOL teacher stated with disappointment, "I didn't have the class management in order to keep them on task" (ESOL Teacher, ESOL teacher interview, March 2, 2015). A large ESOL class size often created classroom management issues and was a disadvantage in that the ESOL teacher found herself concentrating on classroom management while using manipulatives to achieve the goal of the lesson. The large class-size hindered the quality of time needed to work one-on-one with ELLs using the manipulatives.

Theme Five: the teacher found time management was an issue for implementation. Based on the ESOL teacher interviews and recorded field notes, Theme Five found time management for implementation of manipulatives was very limited. There was not enough time for planning and classroom implementation of manipulatives. Garcia (2004) indicated that long term use of manipulatives may be necessary to increase achievement and understanding of mathematics. For example, the continuous changes within the transitioning to Common Core State Standard mathematics curriculum and the rapid pace of covering the Common Core State Standard for Coordinate Algebra hindered the implementation of utilizing manipulatives. Furthermore, the ESOL mathematics teacher stated, "Every time I start to get used to a curriculum where I'm comfortable enough to plan ahead, to get these manipulatives and know what is going to come

up, the curriculum changes and I'm lost again" (ESOL Teacher, ESOL, teacher interview, February 25, 2015).

Another contributing factor in managing time was the need for students to learn how to utilize the manipulatives to solve mathematical problems. Although the treatment group had an additional 18 minutes added within each block daily due to the lunch scheduling, (unlike the control group) the ESOL mathematics teacher stated:

If we had started using manipulatives earlier in the semester, the adjustment would not have been as difficult. Students are just now getting into a routine. I think one of the behavior issues was, it was something they weren't used to. Within that two weeks, they got used to it. They we're doing it more. They were ready. (ESOL Teacher, recorded field notes, April 2, 2015)

Additionally, the ESOL teacher stated:

After a while, they got used to it, because even the time when you asked them to do the graph, they're so used to drawing the blocks, they went and added their blocks. Even today I said, "That's really cool, but you got the right thing, just make a line. It's right there." They're getting used to that. I think if there was more time, if we were doing it from the beginning of the semester, we would have less problems with them. We could space them out just a little more. This I could relate it to something. You could go back and build on concrete ... Like, "Okay, remember when you did this?" and you could build onto something when you're explaining the rules and things like that. (ESOL Teacher, ESOL teacher interview, April 3, 2015)

Time management for implementing using manipulatives was a disadvantage; it hindered establishing students' routines, norms and expectations of using manipulatives to build their mathematical understanding of linear and exponential functions. Routines, norms and expectations for ELLs are critical in establishing appropriate classroom behavior (Kersaint et al., 2013).

Theme Six: the teacher found a lack of availability of concrete and virtual manipulatives.

Based on the ESOL teacher interviews and recorded field notes, Theme Six found the limited availability of concrete and virtual manipulatives hindered ELLs' explorations of mathematical concepts. The concrete manipulatives utilized in this research study were provided by the researcher and Texas Instrument calculator loan program. The virtual manipulatives were provided using the schools' computer lab and software. The treatment group's limited access to the computer lab, time to create concrete manipulatives, and funding to purchase materials all hindered availability. For example, the researcher scheduled the treatment group for the computer during the course of this study but was limited to forty-five minutes intervals and scheduling around other school-wide classes' usage of the computer lab. The ESOL teacher expressed her overall opinion of teaching mathematics using manipulatives during the post interview by stating,

I think it's good in moderation, evenly spaced out with manipulatives and the lecture and stuff. Together, I think it's a good thing. I think it's valid. Money would be an issue. I think it's valid. I think that it's needed. It's another way of looking at things. Some kids are those kinds of learners, of course. Whether I will use it all the time, I don't know. Because, like I said, time and money. Yeah. Everything is a time crunch. I looked the other day, and I was like "Okay, when we come back we have three weeks before the EOC test. (ESOL Teacher, ESOL teacher interview, April 3, 2015)

The accessibility of concrete and virtual manipulatives is a disadvantage which hinders students' ability to gain a deeper understanding of mathematical ideas while transitioning through Sherman's (1997) six levels of mastery of mathematical concepts to *bridge the gap* from concrete to abstract understanding.

Summary

A research study was conducted to explore differences in ELLs' learning about linear and exponential functions. A total of 38 students participated in this study. The students were divided into two groups, control and treatment. One group of students used concrete and virtual manipulatives while the control group used traditional instructional learning practices without manipulatives.

In this study four research questions were examined. The first research question asked, what difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about linear functions compared to a control group using traditional instructional practice? The results indicated that there was no statistically significant difference in performance related to linear functions between the groups. Therefore, the null hypothesis was not rejected. The second research question asked, what difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about exponential functions compared to a control group using traditional instructional practice? The results indicated that there was no statistically significant difference in performance on exponential functions between the groups. Therefore, the null hypothesis was not rejected. The third research question asked, what difference, if any, exists in student dispositions about mathematics and math class as a result of using concrete and virtual manipulatives as ESOL high school students employ them to learn about linear and exponential functions compared to a control group using traditional instructional practice? The results indicated that there was no statistically significant difference in students' disposition about mathematics and math class between the control and treatment groups. As a result, the null hypothesis was not rejected.

The final research question was qualitative and asked, what are the unique benefits and disadvantages of using concrete and virtual manipulatives versus traditional instructional practice? The results of the qualitative analysis revealed six themes that addressed this research question. Two themes revealed advantages of the intervention, three themes revealed disadvantages, and one theme revealed both a disadvantage and an advantage. The two advantages of using the intervention were that math retention could be increased by building upon students' prior math knowledge, and that the students were actively engaged in learning. The three disadvantages were first, the ESOL class sizes, due to county budget cuts, are too large, which makes it difficult to use manipulatives with a large number of students. Second, time management was an issue, as there was not enough time for planning and classroom implementation of manipulatives. Third, there was limited availability of the virtual or computer-based manipulatives. The theme that was both an advantage and a disadvantage was that the students were distracted by the manipulatives. ELLs' saw the manipulatives as toys and wanted to play. This was the disadvantage. However, the teacher later used the students' *free play* with the manipulatives in the learning process.

In chapter 5, the results of the study will be reviewed in the context of the theoretical framework and the significance of findings compared to the theoretical framework. Additionally, the limitations of the study will be discussed, along with implications and suggestions for future research. Chapter 5 will end with a personal reflection and conclusion section.

CHAPTER 5

DISCUSSION, CONCLUSIONS, & IMPLICATIONS

Introduction

The purpose of this embedded quasi-experimental mixed methods research was to determine if English Language Learners' (ELLs') student achievement is affected by using a specified set of actions (concrete and virtual manipulatives) versus traditional instructional practices in learning about linear and exponential functions. In addition, the researcher wanted to explore ELLs' dispositions in learning about linear and exponential functions when using manipulatives (concrete and virtual manipulatives) versus traditional instructional practices. Finally, another goal was to explore the benefits and disadvantages of using concrete and virtual manipulatives. Qualitative methods such as teacher interviews, recorded field notes, student work samples and artefacts were utilized to inform research question 4. This chapter contains the discussion of findings, significance of findings compared to theoretical framework, implications, future research, summary, personal reflections and conclusions.

Discussion of Findings

This section of the chapter is organized based on the four research questions. The results from research question 1 and question 2 revealed no statistically significant difference in the change in linear function and exponential function scores from pretest to posttest between the control group and the experimental group. Although, one study on the high school level conducted by Aburime (2007) indicates a significant difference in student achievement using mathematics manipulatives. On the other hand, Goins's (2001) study on the high school level indicated no statistically significant difference in using manipulatives. Also, Magruder's (2012)

study of using concrete and virtual manipulatives on the middle level indicates no statistically significant difference in using manipulatives. My research results are more aligned with her findings. She attributes her results to students needing the time to learn how to utilize the manipulatives and to learn the mathematical concepts. The ELLs in my research study not only have the task of learning how to implement the manipulatives, but need additional time to acquire the mathematical concepts for both linear and exponential functions, while learning English. Garcia (2004) indicates using math manipulatives made a minimal improvement in the bilingual treatment groups' classroom. However, gradual improvement was made but was not linear, indicating that long term use of manipulatives has a larger increase in students' achievement and understanding of mathematics.

The results from research question 3 revealed mixed findings in regards to student dispositions about mathematics and math class. Conversely, a study on the college level by McGee, Moore-Russo, Ebersol, Lomen, and Quintero (2012) reveals significant improvement in students' attitudes toward the effectiveness of manipulatives. Additionally, the implications of the results of research question 3 are difficult to pinpoint due to the multiple meanings of dispositions. This is aligned with McLeod's (1992) explanation; "The affective domain refers to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 576). Therefore, the theoretical foundations that undergird the affective variables are not quite coherent, because researchers are unable to agree with the theories, terminologies and definitions of *attitudes* (Di Martino & Zan, 2001; Hannula, 2002a), *beliefs* (Furinghetti & Pehkone, 2002), *emotions* (Goldin, 2000; Lazarus, 1991; Mandler, 1989; Power &

Dalgleish, 1997, Pekrun, Elliot, & Maier, 2009) and *values* (DeBellis & Goldin (1997). ELLs' dispositions (affective domains) are an *umbrella* which includes all of the above.

The results from research question 4 were aligned with Magruder's (2012) six themes to investigate the benefits and drawbacks associated with using concrete and virtual manipulatives. She indicated "time considerations (time on-task, time lost); student perseverance and initiative; play/distraction caused by manipulatives; active and passive learning; and cost and availability of resources" (p. 65) as themes. My research study specified making a connection and building upon the ELLs' prior math knowledge to increase retention (a *reference point*) as an additional advantage using manipulatives. Boulton-Lewis (1998) and Suh and Moyer (2007) share the philosophy that contact provided through kinesthetic engagement with the manipulatives assists students with transference and mental retention. ELLs have difficulty communicating their mathematical understanding in order to link information to prior knowledge when explaining their thoughts to others (Kersaint et al., 2013). Having a *reference point* assisted with ELLs' memory retention and building their levels of mathematical understanding while bridging from the concrete to abstract (Sharma's, 1997).

Research question 1. What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about linear functions compared to using traditional instructional practice?

The Unit3 A: Linear Functions Assessment results of the one-way ANOVA indicated that there was no statistically significant difference in the change in linear function scores from pretest to posttest between the control group ($M = 19.25$, $SD = 19.45$) and the treatment group ($M = 14.95$, $SD = 14.53$) concerning linear functions, $F(1, 37) = .607$, $p = .441$. This means that

the change in linear function scores from the pretest to the posttest was essentially the same for both the control and treatment groups. The eta squared effect size measure indicated that the effect of the independent variable on linear function performance scores was small, $\eta^2 = .016$, accounting for only 1.6% of the variation in linear function performance difference scores. Based on the results of the one-way ANOVA, the null hypothesis was not rejected. However, there were significant changes in scores between the pretest and posttest Linear Functions Assessment within both the control and treatment groups. The result of the one-way ANOVA for the treatment group indicated that there was a statistically significant improvement in scores on Linear Functions from the pretest ($M = 12.42$, $SD = 9.74$) to the posttest ($M = 27.37$, $SD = 15.80$), $t(18) = 4.48$, $p < .001$.

The research study noted several factors may have contributed to these results; one factor is vocabulary. The ESOL mathematics teacher and the researcher accounted for ELLs' learning of mathematical vocabulary associated with linear and exponential functions by having the students participate in magnetic interactive vocabulary walls. However, learning vocabulary for ELLs is an ongoing process, and all students are not at the same level of linguistic competency. This may have contributed to a lack of a statistically significant difference between the control group and the treatment group. The Kessler, Quinn, and Hayes (1985) share the philosophy that vocabulary is the utmost essential tool of second language competence when learning academic content, while learning mathematics vocabulary is challenging for ELLs (Kersaint et al., 2013). The challenges ELLs experience in solving word problems are dependent upon their understanding of the linguistics and mathematical meaning to create a solution to solving the word problem (Kessler et al., 1985).

Another factor is semantics, the process of making meaning from language, including mathematical language (Dales & Cuevas, 1992). In this study ELL students were having mathematical misconceptions for identifying and labeling the x- and y-axis correctly in relationship to the independent and dependent variables while making connections among rates involving time and rates involving other variables. These findings are aligned with Acuna's (2007) study, which indicated most students are knowledgeable of identifying the slope and y-intercept of a linear function from a given graph and the y-intercept form ($y = mx + b$); however, students are unable to make the mathematical connections when asked to make predictions, or explain and interpret the graph of the linear functions. Researchers Herbert and Pierce (2008) suggest students' mathematical difficulties with making the connections in conceptualizing rate of change are due to teachers introducing rate of change while applying the slope formula (calculations) without making sure students understand the results. Other factors include ELLs' misunderstanding and usage of the terms slope, rate of change, and steepness interchangeably.

The final contributing factor to the lack of a statistically significant difference between the control group and the treatment group is that ELL students were having mathematical misconceptions with generating a table of values, plotting ordered pairs to construct a graph, and then deriving an algebraic equation. These results are associated with Birgin's (2012) research, which specified students have difficulties explaining and transitioning among the interrelations between the tabular, graphical, and algebraic representations of equations.

Research question 2. What difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about exponential functions compared to using traditional instructional practice?

The Unit3 B: Exponential Functions Assessment results using the one-way ANOVA indicated that there was no statistically significant difference in the change in exponential function performance difference scores from pretest to posttest between the control group ($M = 15.25$, $SD = 12.77$) and the treatment group ($M = 27.00$, $SD = 24.16$), $F(1, 37) = 3.658$, $p = .064$. This means that the change in exponential function scores from the pretest to the posttest was essentially the same for both the control and treatment groups. The eta squared value indicated that the independent variable, group, had a medium sized effect on the change in exponential function scores from pretest to posttest, $\eta^2 = .090$, accounting for 9.0% of the variability in exponential function difference scores. Based on the results of the ANOVA, the null hypothesis was not rejected. However, there were significant changes in scores from the pretest to the posttest on exponential functions within both the control and treatment groups. The result of the one-way ANOVA for the treatment group indicated that there was a statistically significant improvement in scores on exponential functions from pretest ($M = 8.16$, $SD = 7.27$) to posttest ($M = 35.16$, $SD = 24.58$), $t(18) = 4.87$, $p < .001$.

The research study noted several factors may have contributed to these results, such as vocabulary and word problems. Dale and Cuevas (1992) suggest linguistic difficulties are associated with ELLs' mathematical discourse in understanding oral and written language. The mathematical discourse presents a challenge as ELLs have the dual tasking of learning English and the mathematical content (Kersaint et al., 2013). ELLs are applying their understanding of the English language and combining their mathematical experiences and cultural background to

plan and derive solutions to solving word problems (Kersaint et al., 2013). Often word problems contain both social (Basic Interpersonal Communications Skills) and academic (Cognitive/Academic Language Proficiency) language which enhances the mathematical misconceptions when the English language has several words with multiple meanings (homonyms, homophones, and homographs) (Kersaint et al., 2013). For instance, polysemous words, words with the same spelling and pronunciation, caused misconceptions for ELLs. The word “geometric” in geometric sequence presented difficulty; some students initially perceived the term in affiliation to geometric shapes (squares, rectangles).

The reasons ELLs had difficulties with exponential functions are comparable to the reasons for ELLs’ misconceptions with linear functions. Norman (1993) and Lo et. al (2012) posit that students occasionally have difficulties generating appropriate relations between tabular, graphical and algebraic situations for exponential functions. Also, Lo and Kratky (2012) suggest that student misconceptions of interpreting graphs are attributed to being provided formulas and not having a deep understanding of the rate of change. In this study ELLs were faced with the challenge of identifying mathematical notations and comprehending the rules and meaning for using the mathematical notations (Ker saint et al., 2013). Rubenstein and Thomas (2001) suggests the challenge appears when the student is required to use several words to articulate the meaning of a mathematical notation. Also, if the mathematical notations utilized in America are different from the ELLs’ native country, they present a challenge (Kersaint et al., 2013). Metcalf (2007) suggests students should comprehend the mathematical connections between representing functions algebraically and representing the functions in a given graph, table and pattern. Additionally, Markovits (1986) suggests students have challenges interpreting

and creating graphs of functions to satisfy given constraints when provided with characteristics of a given function.

Research question 3. What difference, if any, exists in student dispositions about mathematics and math class as a result of using concrete and virtual manipulatives as ESOL high school students employ them to learn about linear and exponential functions compared to using traditional instructional practice?

From the perspective of student dispositions, the results of the one-way ANOVA indicated that there was no statistically significant difference in the change in QUASAR Student Disposition Instrument (questions 7, 8, 10, 12, 15, 16, 19, 28, 29, and 32) difference scores from pretest to posttest between the control group ($M = -.16$, $SD = .70$) and the treatment group ($M = -.16$, $SD = .59$), $F(1, 367) = .002$, $p = .969$. The eta squared value indicated that the independent variable, group, had a very small effect on the change in exponential function scores from pretest to posttest, $\eta^2 = .002$, accounting for 0.2% of the variability in QUASAR Student Disposition Instrument difference scores. Based on the results of the ANOVA, the null hypothesis was not rejected. Additionally, there were no significant changes in scores from the pre-questionnaire and post-questionnaire on the Student dispositions. The result of the one-way ANOVA for the treatment group indicated that there was a statistically Finally, results indicated that there was no significant change in pretest (4.09, $SD = .78$) and posttest ($M = 3.93$, $SD = .85$) scores on the Student Disposition test, $t(18) = -1.20$, $p = .245$, as the p value was greater than .05.

The major factor which contributed to the negative results of students dispositions about mathematics and math class, are associated with ELLs limited English proficiency due to the QUASAR Student Dispositions Instruments written in English. Also, Krashen's (1988) *affective filter hypothesis* which indicates emotional variables, such as anxiety, self-efficacy, motivation

and stress, hinder learning. These variables prevent *comprehensible input* (receiving understandable messages) from reaching the language acquisition part of the brain; in addition, accents possibly hinder (*monitoring hypothesis*) ELLs comprehension of mathematical concepts. Therefore the ESOL mathematics teacher provided a safe interactive learning environment where students were allowed to make errors and take risks in learning both English and mathematics utilizing manipulatives with hands-on activities. Some students were hesitant to speak and read due to their lack of proficiency in English which initiated the ESOL mathematics teacher to pair English learners with strong English speakers to assist with translation thereby minimizing student frustration. In short, the participants in this research study had a wide range of reading and speaking abilities which may have affected their ability to interrupt the QUASAR Student Disposition instrument questionnaire.

Research question 4. What are the benefits and disadvantages of using concrete and virtual manipulatives versus traditional instructional practice, from a teacher's perspective, in teaching linear and exponential functions?

From the view point of benefits and disadvantages when using concrete and virtual manipulatives, the following six themes emerged from the bottom to top approach developed by Anfara, Brown, and Mangione, (2002) and Harry, Sturges, and Kilinger (2005).

Theme one: ELLs were able to make a connection and build upon their prior math knowledge (advantage). Based on the interviews, recorded field notes and students work samples from the ESOL teacher Theme One found that math retention could be increased by having ELLs make a connection with the math manipulatives using Sherman's (1997) six levels of mastery of mathematical concepts which include intuitive, concrete, representation, abstract,

application, and communication. Additionally, building upon prior math knowledge also increases retention and making a connection (bridge the gap) between concrete level to abstract level of mathematical understanding. This linkage is an example of building upon prior math knowledge to improve retention of a new math concept.

Theme two: ELLs were actively engaged during mathematical problem solving (advantage). Based on the interviews, recorded field notes and student's work samples from the ESOL teacher Theme Two found that ELL's were actively engaged during mathematical problem solving when utilizing manipulatives. Theorist and researchers Dienes (1960), Dewey (1938), Montessori (1967), Moschkovich (1999), and Kersaint et al. (2013) agree that students should be actively engaged with mathematics (doing mathematics); and as a result of communication (listening, speaking , reading and writing English) and touching the material, they learn images, which builds upon the next mathematical concept. From these images the student can translate concrete facts into symbolic representation (Antosz, 1987), which generates connections for a deeper level of mathematical understanding and problem solving while engaging with their peers, classmates, and teacher (Kersaint, Thompson, & Petkova, 2013). Shoebottom (2014) claims that this process will "*Make it comprehensible!*" (p. 1).

Theme three: manipulatives created an interference (free play) with ELLs' exploration of linear and exponential functions (advantage/ disadvantage). Based on the ESOL teacher interviews, recorded field notes and student's artefacts Theme Three found that manipulatives created an interference (free play) with ELL's exploration of linear and exponential functions. Dienes (1971) describes *free play* as one of six stages of learning mathematics which is vital in formulating the first understanding of a new concept. Rabardel's (2003) *from artefact to*

instrument theoretical framework stems from Vygotsky's (1978) activity theory which provided understanding of ELLs first desired impression (knowledge) to play, explore, and create figures with the stacking blocks (manipulatives as artefacts) prior to learning the actual meaning and usage of the mathematical tool (instrument). As the research study progressed the advantage of utilizing manipulative revealed the ESOL teacher developed flexibility and understanding as to how through *free play* the students accomplished goals.

Theme four: large class size created classroom management issues (disadvantage).

Based on the ESOL teacher interviews and recorded field notes Theme Four found that large class size often created classroom management issues. The local school district reduced ESOL teacher allowance which in turn increased class sizes, and this contributed to a number of classroom issues. Salaudeen (2013) research on large class-size and Gann (2013) research on meeting the needs of ELLs in the secondary mathematics classroom interplays with the challenges the ESOL mathematics teacher experienced utilizing manipulatives in teaching linear and exponential functions. A large ESOL class size often created classroom management issues was a disadvantage in that ESOL teacher found herself concentrating on classroom management while using manipulatives to achieve the goal (objective, standard) of the lesson. The large class-size hindered the quality of time needed to work one-on-one with ELLs using the manipulatives.

Theme five: the teacher found time management was an issue for implementation (disadvantage). Based on the ESOL teacher interviews and recorded field notes Theme Five found time management for implementation for utilizing manipulatives was very limited. There was not enough time for planning and classroom implementation of manipulatives. Garcia (2004) indicated that long term use of manipulatives may be necessary to increase achievement

and understanding of mathematics. Another contributing factor in managing time was the need for students to learn how to utilize the manipulatives to solve mathematical problems. Time management for implementing using manipulatives was a disadvantage; it hindered establishing students' routines, norms and expectations of using manipulatives to build their mathematical understanding of linear and exponential functions. Routines, norms and expectations for ELLs is critical in establishing appropriate classroom behavior (Kersaint et al., 2013).

Theme six: the teacher found a lack of availability of concrete and virtual manipulatives (disadvantage). Based on the ESOL teacher interviews and recorded field notes Theme Six found the limited availability of concrete and virtual manipulatives hindered ELLs explorations of mathematical concepts. The concrete manipulatives utilized in this research study were provided by the researcher and Texas Instrument calculator loan program. The virtual manipulatives were provided using the schools computer lab and software. Accessibility of the treatment group utilizing the computer lab, time to create concrete manipulatives, and funding to purchase materials hinders productivity. The accessibility of concrete virtual manipulatives is a disadvantage which hinders students' ability to explore in gaining a deeper understanding of mathematical ideas while transitioning through Sherman's (1997) six levels of mastery of mathematical concepts to *bridge the gap* from concrete to abstract understanding.

Significance of Findings Compared to Theoretical Framework

The foundation of the theoretical framework which grounded this study was divided into the following major perspectives: the linguist theorist point of view with emphasis on Krashen's (1988) model of second language acquisition and the learning theorist point of view with emphasis on Engerstrom's (1987) activity theory and Vygotsky's (1978) Zone of Proximal

Development (ZPD). During this research study Rabardel's (2003) theory from *artefact to instrument* emerged, which stems from Vygotsky's (1978) activity theory. The last theoretical framework is Sharma's (1997) *Bridging the Gap* point of view highlighting the six levels of mastering mathematical concepts, including Hedden's (1986) and Underhill's (1977) sequence (concrete level-to-representational level-to-abstract level) of using manipulatives.

Linguistics theorist. The linguistics theoretical framework addressed how students learn English as a second language. These theories conceptualize Krashen's (1988) model of second language acquisition, which consists of five hypotheses: (a) the *acquisition-learning hypothesis*, (b) the *natural order hypothesis*, (c) the *monitor hypothesis*, (d) the *input hypothesis*, and (e) the *affective filter hypothesis*; and Vygotsky's (1987) Zone of Proximal Development incorporates learning to speak a second language while learning the concepts applied to the curriculum area of mathematics (linear and exponential functions). The ESOL mathematics teacher employed Echevarria et al.'s (2012) sheltered instruction observation protocol model to integrate both language and mathematics content for ELLs.

With this framework it is understood that ELLs store information in the brain through the use of communications (*acquisition-learning hypothesis*); therefore, the ELL mathematics teacher and researcher created situations for ELLs to become engaged in negotiating (speaking English) the meaning of mathematics with their peers, classmates, and teacher (Kersaint, Thompson, & Petkova, 2013). Also, ELLs acquire parts of language through natural communication (*natural order hypothesis*) and were introduced to language concepts that were more accessible. The ELL mathematics teacher employed scaffolding to introduce challenging mathematical concepts of linear and exponential functions. Additionally, with this framework the

ELL mathematics teacher stressed high-frequency vocabulary terms and used fewer idioms while carefully monitoring (*monitor hypothesis*) ELLs' learned grammar, vocabulary, and the speaking of English. Also, the ELLs' mathematics teacher was challenged with balancing acquisition and learning with carefully monitoring ELLs' speech, focusing on fluency rather than accuracy. The ELL mathematics teacher focused on their positive dispositions (*affective filter hypothesis*) in order to facilitate the learning (*comprehensible input*) of English from the language acquisition part of the brain (Krashen, 1988). ELLs' mathematics teacher focused on positive dispositions to create a positive learning environment where students were allowed to make mistakes and take risks in learning both English and mathematics (Kersaint et al., 2013).

For ELLs, Krashen's theory of second language L₂ acquisition is deemed the most significant component, and *comprehensible input* (receiving understandable messages) is the fundamental principle in second language acquisition (SLA)(*input hypotheses*). The ELL mathematics teacher slightly stretched the learner beyond his or her original stage of *i+1 input*, being neither too easy nor too difficult; and keep in mind that not all of the students are at the same level of linguistic competence (five levels include beginning, early intermediate, intermediate, early advanced, and advanced (Goldenberg, 2008))(Krashen, 1988). The ELL mathematics teacher differentiated the instruction to accommodate the various levels of learners by providing a variety of learning strategies (see Appendix D). The ESOL mathematics teacher also provided the students with visuals, hand-outs with less complex structures, and paraphrased instructions. Additionally, the ESOL teacher spoke slowly and clearly while enunciating words to assist students with making sense of mathematical concepts of linear and exponential functions.

Another framework of the linguistics theory that was used in this research study is Vygotsky's Zone of Proximal Development (ZPD). The ZPD allowed the ELL mathematics teacher or advanced classmates to collaboratively assist struggling ELLs in their next levels of both mathematical learning and speaking of English (Vygotsky, 1978). These theoretical frameworks provide an explanation of how the ways ELLs learn a language (listen, speak, read, and write) are affected by their social environment (Vygotsky, 1978; Leontiev, 1981; and Engerstrom, 1987) activity theory). Current study findings are analyzed and interpreted in the context of this theoretical framework.

The linguistics theories disconfirmed the results of research question 1 and question 2, indicating no statistically significant difference in the change in linear and exponential function performance difference scores from pretest to posttest between the control group and the treatment group. This means that the change in both linear function and exponential function understanding from the pretest to the posttest was essentially the same for both the control and treatment groups. However, the linguistic theories confirmed the results of statistically significant improvement in scores from pretest to posttest on both the Linear Functions and Exponential Functions assessment within each group. This means ELLs' progress indicated gain in mathematics achievement in learning about linear and exponential functions for both groups. The progression was observed by the classroom teacher and researcher as ELLs enhanced their listening, reading, writing and speaking skills in English. The progression was demonstrated within their work samples, artefacts, and their articulations about using manipulatives to learn about linear and exponential functions.

Additionally, research question 3 (quantitative data) was unable to reveal any confirmation of the linguistic theory as it relates to ELLs' dispositions about mathematics and math class. Because of ELLs' limited English proficiency skills, the QUASAR Student Dispositions Instrument was written in English and not all ELLs are on the same levels of linguistic competency (Goldenberg, 2008). As a result of comparing any statistically significant difference between the groups (control and treatment) and determining statistically significant changes in ELLs' dispositions about mathematics and math class within the groups, were aligned indicating none existing changes.

The results from the qualitative data obtained in research question 4, from teacher interviews, recorded field notes, student work samples and artefacts, and revealed the benefits and advantages of using concrete and virtual manipulatives. Six themes emerged that included the following: 1) ELLs were able to make a connection and build upon their prior math knowledge (advantage), 2) ELL's were actively engaged during mathematical problem solving (advantage), 3) manipulatives created an interference (free play) with ELLs' exploration of linear and exponential functions (advantage and disadvantage), 4) large class size created classroom management issues (advantage), 5) the teacher found time management was an issue for implementation (advantage); and 6) the teacher found a lack of availability of concrete and virtual manipulatives (disadvantage). The linguistic theories confirmed the themes and provided new information.

Learning theorist. The learning theoretical framework addresses how ELL students learn mathematics through the social approach learning theory (Vygotsky, 1978; Leontiev, 1981; Engerstrom, 1987: activity theory and Rabardel's, 2003: from *artefact to instrument*), indicating that ELLs learn from their social environment (Schunk, 2012). With these frameworks, it is understood that ELLs' cultural artefacts (manipulatives), objects (tasks, assignments),

components of rules, community, and division of labor are modifications that interact within the social environment (Hardman, 2008). ELLs' interactions with objects (manipulatives) in the classroom environment assists with the learning and exploring of linear and exponential functions (Vygotsky, 1978). Within the social environment, ELLs first desired impression (knowledge) is to play, explore, and create figures with the stacking blocks; drive the remote control trucks; roll and bounce balls; and blow air into balloons (manipulatives as artefacts) prior to learning the actual meaning and usage of the mathematical tool (instrument) (Rabardel's, 2003). Leontiev's activity theory explains that ELLs' social endeavors (student dispositions: motives, emotions and creativity) interact with the roles of the manipulatives, skills of ELL students, standards for coordinate algebra, motivation of the students, activities and roles of classmates, and ELL teacher components to achieve the outcome (successful learning). Theorists Vygotsky (1978), Leontiev, (1981), and Engerstrom's (1987) activity theory and Rabardel's (2003) from *artefact to instrument* were used as frameworks for this study because these theories are based upon ELLs' learning of mathematics within their social environment using manipulatives. Current study findings are analyzed and interpreted in the context of these theoretical frameworks.

The activity theory disconfirmed the results of research question 1 and question 2; findings indicate no statistically significant difference in the change in linear and exponential function performance difference scores from pretest to posttest between the control group and the treatment group. This means that the change in both linear function and exponential function from the pretest to the posttest was essentially the same for both the control and treatment groups. However, the activity theory confirms the results of statistically significant improvement

in scores from pretest and posttest on both the Linear Functions and Exponential Functions Assessment within each group. ELLs' progress indicated gain in mathematics achievement in learning about linear and exponential functions for both groups. The progression was observed by the classroom teacher and researcher in student work samples, artefacts, and articulations about the use of manipulatives to learn about linear and exponential functions.

Additionally, research question 3 (quantitative data) was unable reveal any confirmation of the activity theory as it relates to ELLs' dispositions about mathematics and math class; because of ELLs' limited English proficiency skills and the QUASAR Student Dispositions Instrument was written in English. However, results of both comparing any statistically significant difference between the groups (control and treatment) and determining changes in ELLs' dispositions about mathematics and math class from within each group were aligned.

The results from the qualitative data obtained in research question 4, findings from teacher interviews, recorded field notes, student work samples and artefacts revealed the benefits and advantages of using concrete and virtual manipulatives. The identical six themes emerged, which addressed the advantages and disadvantages of virtual manipulatives versus traditional instructional practice, from a teacher's perspective, confirmed the linguistics theories also, confirmed the activity theory and Rabardel's (2003) theory from *artefact to instrument*. These themes and theories interacted together with the components of the social environment for the activity theory. The activity theory confirmed the themes and provided new information.

Sharma's bridging the gap theory. Shama's (1997), Hedden's (1986) and Underhill's (1977) *Bridging the Gap* theory addressed how ELLs' formulate the concrete to make the connections with the abstract when using manipulatives. ELLs transition though Sharma's six

levels of mastery of mathematical concepts (intuitive, concrete, representation (pictorial), abstract, applications and communications) when using manipulatives to explore and learn about linear and exponential functions in making the connections with the abstract world. With these frameworks, it is understood that ELLs were able to make the leap to the abstract level of understanding mathematical concepts as they internalized new knowledge at the concrete level and systematically progressed along the continuum to arrive at the abstract representation of knowledge (Heddens, 1986). This theoretical framework provides an explanation of the sequence ELLs transition through when utilizing manipulatives. Figure 5.1, Figure 5.2, Figure 5.3, Figure 5.4, and Figure 5.5 illustrate how the ESOL mathematics teacher and the researcher scaffold experimental students' understanding of exponential functions using a table, pattern, algebra and graph to guide students through Sharma's (1997) six levels of mastering mathematical concepts.

These are as follows:

1. Intuitive: Building upon prior knowledge (stacking)
2. Concrete: Students utilized the manipulatives (stacking cubes) to construct a model of the geometric sequence $2, 4, 8, 16, 32, \dots$
3. Representation (pictorial): Students drew a picture (histogram) of the cube stacking and placed pipe cleaners on top of the stacked cubes (visualizing the exponential growth function). Additionally plotting the (x, y) coordinates to create an exponential function graph.
4. Abstract (symbolic): Students identified the pattern in stacking cubes (Collaborative Group B indicated, "*it Double each time multiplied by 2*") and the common ratio

- ("2"), and the ESOL teacher assisted with deriving the geometric formula. Also, the students wrote an equation for the graph.
5. Application: Students applied the geometric sequence formula to determine how many blocks appeared in the 10th position (project).
 6. Communication: Some ELL students were able to discuss and write about their discovery, placing the pipe cleaner on top of the stack cubes using complete sentences. Collaborative Group B indicated, "*When I Place the pipe cleaner on top of the stack cube, it makes a curve.*"

The ESOL mathematics teacher expounded on students' conceptualizing the exponent in $f(x) = ab^x$ (exponential function) and students deriving the geometric formula during the *Geometric Activity: Stacking Cubes*:

I will show them the formula and everything. I already had the formula up on the white board, but they didn't notice that. They came up with the 512, but they didn't come up with the exponent. This is the stacks. When they were doing that, they would ask me, "Do I multiply?"

They couldn't understand that it was an exponent. I'm like, "How do you get bigger?" They're like, "Do I multiply?" Like I said, they came up with the 512, but they couldn't grasp that it's an exponent.

I did the powers of two thing. I'm like, "How much is it going up every time? Two." I did two to the first power, two to the zero power, two to the first power.

Then, that's how we came up to Y equals two to the X . We did that. The reason why I came up with that is because in my other class there's an opener, where the grandmother is saving money for college. Every year, she decides to double the previous year.

The next step is you fill in the table. Then, the next step is you find the R , you find the ratio, and then change it all to powers of two, base two. Then, from there, predict the formula and then predict the rule. Then, use the rule to come up with your ten, which is very similar to what you're doing [with stacking the cubes]. (A. Horton, recorded field notes, March 24, 2015)

The ELLs sequence of instruction (scaffolding) was aligned with Sharma (1997) six levels of mastering mathematical concepts.

1. Draw a picture (histogram) of the cube sackings. Use the grid on the right. The first one has been done for you.

2. What pattern did you observe in stacking the cubes and in the table (output)?
The domain increases by 1
 $2^0=1, 2^1=2, 2^2=4, 2^3=8, 2^4=16$
 $2^5=32, 2^6=64$

3. This number is called the common ratio
 $(r) = 2$

4. Use the common ratio to write a rule (formula) to find the n th term.
 This is the geometric sequence formula.
 $n^{\text{th}} \text{ term} = 2^{n-1}$

5. Use the formula to determine how many blocks would appear in the 10th position.

Place answer here: 512

Figure 5.1. Experimental collaborative group A, page 1 response to Stacking Cubes.

1. Draw a picture (histogram) of the cube sackings. Use the grid on the right. The first one has been done for you.

2. What pattern did you observe in stacking the cubes and in the table (output)?

It doubles each time
multiplied by 2.
 $2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16$
 $2^5 = 32, 2^6 = 64$

3. This number is called the common ratio

$(r) = 2$

4. Use the common ratio to write a rule (formula) to find the n th term.
 This is the geometric sequence formula.

$a^n - r + m = 2^n - 1$

5. Use the formula to determine how many blocks would appear in the 10th position.

$2^{10} - 1$

Place answer here: 511

Figure 5.2. Experimental collaborative group B, page 1 response to Stacking Cubes.

1. Draw a picture (histogram) of the cube sackings. Use the grid on the right. The first one has been done for you.

2. What pattern did you observe in stacking the cubes and in the table (output)?

$2^0=1, 2^1=2, 2^2=4, 2^3=8, 2^4=16$

$2^5=32, 2^6=64$

3. This number is called the common ratio

$(r) = 2$

4. Use the common ratio to write a rule (formula) to find the n th term. This is the geometric sequence formula.

$a_n = a_1 r^{n-1}$

$n^{\text{th}} \text{ term} = 2^{n-1}$

5. Use the formula to determine how many blocks would appear in the 10th position.

$2^{(10-1)}$
 2^{9}
 512

Place answer here: 512

Position (n)	Number of Blocks
1	1
2	2
3	4
4	8
5	16
6	32
7	64

Figure 5.3. Experimental collaborative group C, page 1 response to Stacking Cubes.

6. Plot the data from the table, using (X, Y) coordinates on the graph to the right. Connect the points and label graph. What function does the graph make?

Curve exponential
Function

Bonus: Write an equation for the graph?

$y = 1.2^x$ ← variable
 $y = 2^x$

7. What did you discover about placing the pipe cleaner (s) on top of the stack cubes? Explain using complete sentences below: *it makes a curve.*

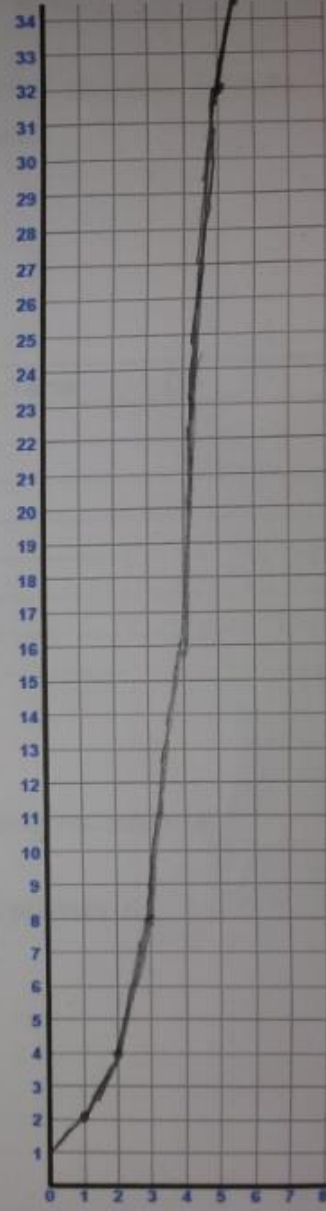


Figure 5.4. Experimental collaborative group A, page 1 response to Stacking Cubes.

6. Plot the data from the table, using (X, Y) coordinates on the graph to the right. Connect the points and label graph. What function does the graph make?

a curve line

Bonus: Write an equation for the graph?

$y = 1.2^x$
 $y = 2^x$

7. What did you discover about placing the pipe cleaner (s) on top of the stack cubes? Explain using complete sentences below:

When I place the pipe cleaner on top of the stack cubes, it makes a curve.

X	Y
1	2
2	4
3	8
4	16
5	32

Figure 5.5. Experimental collaborative group B, page 1 response to Stacking Cubes.

The ELL students in the treatment group established reference points, which increased their memory and retention. The ELLs in the treatment group were more dexterous in writing functions algebraically from a given graph, a table, a pattern and from applications as a results of

utilizing the manipulatives than the control group. The findings confirm Sharma's *bridging the gap* theory, and provided new information.

Implications

The findings of this study are significant to classroom teachers, administrators, and educational policymakers at both local and state levels. This study validates the use of manipulatives to enhance ELLs' performance in algebra. This study also adds to other studies that examine techniques, strategies, and pedagogies for improving ELLs' comprehension of linear and exponential functions. Therefore, the researcher recommends the following implications:

Classroom teacher. The results from teacher interviews, recorded field notes, student work samples, and artefacts indicated that when utilizing manipulatives teachers should be aware of how to reduce mathematics anxiety for ELLs. In this research study ELLs were excited and actively engaged while using manipulatives to explore and learn about linear and exponential functions. Research in educational psychology indicates that students' beliefs, attitudes, and dispositions provide the keys to unlocking students' mathematical power to learn (Debellis & Goldin, 2006). Wlodkowski (1999) suggests that as a student's attitude improves, the student is more receptive to learning, which can lead to higher success in achievement and pursuing higher-level courses. Additional researchers, Grouwns (1992) and Vinson, Haynes, Brasher, Sloan, and Gresham (1997) revealed a positive connection between the use of manipulatives and a decrease in students' mathematics anxiety. During this research study, ELLs' dispositions about mathematics and math class changed once they realized that *doing mathematics* is fun. Teachers should participate in professional development opportunities to learn how to utilize both concrete and virtual manipulatives on an ongoing basis. In this research study, the ELL mathematics

teacher became aware of how to create learning activities employing Sharma's (1997) six levels of mastery of mathematical concepts. Additionally, the ELL mathematics teacher indicated that she never thought to explore linear functions (arithmetic sequence) or exponential functions (geometric sequence) by using stacking cubes to create a histogram and then laying pipe cleaners on top of the cubes to visualize the shape of linear and exponential functions. These ELLs were able to *Bridge the Gap* between the abstract and the concrete by using manipulatives. The collaboration between the researcher and ELL mathematics teacher provided opportunities to enhance student learning during the research study (Dove & Honigsfeld, 2010). Vinson et al. (1997) suggests professional development enhances mathematics teachers' learning of how and when to teach with manipulatives. Additionally, teachers become facilitators of learning when they share their control of learning with their students (National Council of Teachers of Mathematics, 2000; Moyer & Jones, 2004). When ELLs' used the manipulatives to explore and learn about linear and exponential functions, they took control of their mathematical comprehension.

School administrators. The results from both qualitative and quantitative data revealed school administrators should be aware of how large ESOL classes affect student learning. On several occasions during this research study the ELL mathematics teacher indicated that a large ESOL class often created classroom management issues while using manipulatives to achieve the goal of the lesson. Additionally, the large class size hinders the quality of time needed to work one-on-one with ELLs. Salaudeen's (2013) research on large class size and Gann's (2013) research on meeting the needs of ELLs in the secondary mathematics classroom interplays with the challenges the ESOL mathematics teacher experiences with large ESOL classes.

Additionally, school administrators should be mindful of the preparation time classroom teachers need for curriculum planning. The ESOL teacher expressed her concern for not having enough preparation time to create learning tasks using manipulatives due to excessive meetings, teaching other colleagues' classes, and the continuous changes within the state's Common Core Curriculum. According to Fink (2005) teachers need adequate time for developing learning activities and developing strategies to assess students' mathematical understanding. School administrators may consider returning to a long-standing strategy of scheduling common planning periods during designated times to assist teachers with the needed preparation for curriculum planning (Abdal-Haqq, 1996).

Policymakers both local and state. Policymakers at both local and state levels should be aware that the ELL population is steadily increasing and some ESOL students are not literate in their native language (Teaching English to Speakers of Other Languages International Association, 2014; Goldenberg, 2008). Therefore, an allocating of more funding is needed to impact the resources and reduce class sizes, which in turn decreases the ratio of students to teacher. Currently, "there is no federal mandate to provide specialized services to ELL students as there is for special education students" (Education Commission of the States, 2013, p. 4). Therefore some states include the ELL population with Special Education or low-income students in order to allocate instructional funding (Jimenez-Castellanos & Topper, 2012). In this study the ESOL teacher expressed concern about not being able to implement manipulatives next semester due to large class sizes and unavailability of funding. The concrete manipulatives for this study were provided by the researcher, and the virtual manipulatives were supplied by Texas

Instrument's teacher loan program. There was limited availability in using the school-wide computer labs.

Additionally, the school board and state policymakers should support the 2001 Development, Relief and Education of Alien Minors (DREAM) Act, to assist with providing illegal immigrants' conditional residency and then later providing permanent United States residency. The need for both local school boards and state government support was revealed during an interview with the ESOL mathematics teacher, who identified the hopelessness a student felt about pursuing a higher-level mathematics course. Due to non-documentation of citizenship, the student questioned the purpose of comprehending math, which led to low self-efficacy and negatively affected the student's disposition towards mathematics and math class:

Because, it's like, "*I've been here, all my life. I can't get it, I'm not documented. What's the sense in pursuing higher academics?*" I saw another kid, and asked him, "Hey, are you taking Math Four?" He was a good student in my class. I teased him. It was so fun. He said, "*What was the sense of taking it? Can't do anything with it since that amnesty thing.*" Later, I ran into him one day after school, and said he did apply. He was getting ready to get his driver's license that day. He was going to school. I was so excited for him. (ESOL teacher interview, February 25, 2015)

State policymakers. State policymaker should make modifications within the Common Core State Standards Curriculum to include the foundations of literacy implemented in grades 6-12 (teaching of written letters, spelling and constructing sentences) because some ELLs are not even literate in their native language (Collier & Thomas, 2008). Also, not all students identified as

English learners are at the same level of linguistic competence (Krashen, 1988). This is indicated and demonstrated within this research study by the various levels and limited range of speaking, reading, writing, and listening skills of ELLs. Also, the National Association of Educational Progress Test specifies that limited English proficiency was a factor in students' low performance as the test is written in English (Goldenberg, 2008). Furthermore, an additional factor supporting the need to include the foundations of literacy is indicated in this research study. The QUASAR Student Disposition Instrument results indicated no statistically significant differences between student disposition scores from the pre-questionnaire to post-questionnaire for both the control and treatment groups. Additionally, there were no indications of change in students' disposition scores within each group. The QUASAR instrument was written in English, and ELLs were unable to comprehend the questionnaire. This is an indicator that the foundations of literacy must be included in grades 6 – 12.

Future Research

This mixed methods study contributes to research regarding using concrete and virtual manipulatives with high school English Language Learners. The researcher recommends a quantitative study investigating the use of one type of manipulative (either concrete or virtual) on one particular algebraic concept, as well as a quantitative study investigating teacher perceptions of utilizing manipulatives. These recommendations emerged from the limitations of this study. ESOL mathematics teacher indicated one particular factor was the short-term use (5-weeks) of a large quantity of concrete and virtual manipulatives. If the experiment had been conducted over a longer term and with a limited number of activities using various manipulatives, there may have been an increase in students' mathematics achievement in learning about linear and exponential

functions. Basically, utilizing the various types of manipulatives consumed a large amount of instructional time. The ELLs were not accustomed to exploring mathematics with manipulatives and wanted to play (*free play*) (McNeil & Jarvin, 2007). Therefore, we were unable to see the effects of the manipulatives because the researcher was implementing too many at once. McNeil and Jarvin (2007) suggest limiting *cognitive* resources which may be overwhelming as the students utilize manipulatives. The utilization of one manipulative (concrete or virtual) supports the need of a quantitative study to visualize students' improvements. According to Garcia (2004) the improvement is a gradual process, but not linear, indicating that long term use of manipulatives has a larger increase in students' achievement and understanding of mathematics. Also, the change in both linear and exponential function scores from the pretest to the posttest was essentially the same for both the control and treatment groups. The low statistical power of eta squared indicates if this study is repeated we need to increase the sample size.

A quantitative study to investigate teachers' perceptions of utilizing concrete and virtual manipulatives will assist teachers with pedagogy and strategies for classroom implementation (Sowell, 1989). For example, the ESOL mathematics teacher indicated manipulatives contributed to classroom management issues pertaining to students' off task behavior. The teacher later learned to develop flexibility in understanding how through *free play* the students developed a deeper comprehension of mathematical concepts. The concept of *free play* is Rabardel's theory (2003) *from artefact to instrument*, which provided opportunities for ELLs to encounter stages of the instrumental genesis during mathematical problem solving based upon developing usage schema for the manipulatives. The ESOL mathematics teacher's initial perception of manipulatives as a distraction may have hindered students' ability to progress through

Sharma's(1997) six levels of mathematical mastery if the researcher had not shared Rabardel's (2003) theory *from artefact to instrument*. In this research the ESOL mathematics teacher enhanced students' ability to make the connection between the physical world and abstract in how mathematical knowledge is constructed though the use of manipulatives (Cobb & Steffe, 1983).

Summary

The ELLs are the fastest growing population in the United States schools (Teaching English to Speakers of Other Languages International Association, 2014). In the United States, educators are struggling and under tremendous pressure to meet the progressively diverse needs of these students (Goldenberg, 2008). The National Assessment of Educational Progress (NAEP, 2013) indicates the ELL's NAEP basic mathematics scores have continuously decreased since 2005 by 11 points (127); in 2009, they decreased by 7 points (116), and scores were at 109 in 2013 (a decrease of 7 points). Due to the ELLs poor performance in mathematics, we must identify strategies and methods for teaching mathematics curriculum that will assist in excelling our students' math achievement. Using manipulatives in Algebra as an instructional strategy nurtures ELLs thinking; therefore, it offers an effective strategy to improve their mathematics achievement (Gurbuz, 2010; Sherman & Bisanz, 2009). It is imperative that mathematics teachers remember that students identified as English learners have the dual task of learning a second language and algebra content standards simultaneously.

Therefore the theoretical framework for this study was divided into three major categories, which include linguist, learning theories and levels of mastery of mathematical concepts. Each of these theories consists of several frameworks:

- Linguist theory which incorporates:
 - Model of Second Language Acquisition (Krashen, 1988)
 - Zone of Proximal Development (Vygotsky's, 1978)
- Learning theorist point of view which incorporates:
 - The social approaches school (Vygotsky, 1978; Leontiev, 1981; Engerstrom, 1987: second generation activity theory; Rabardel, 2003: *from artefact to instrument* theory)
- Sharma's (1997) *Bridging the Gap* point of view which incorporates:
 - Concrete level to representation to abstract (Heddens, 1986; Underhill, 1977: six levels of mastery of mathematical concepts)

Swan and Marshall (2010) define manipulatives as “A mathematics manipulative material is an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered” (p. 14). Research reveals using manipulatives with ELLs reinforces opportunities for discovery and leads to actively engaged communication, discussion, and explanations of the students' ways of solving problems (Caswell, 2007; Kersaint et al., 2013). The NCTM has been supporting the use of manipulatives in every decade since 1940, and the National Council of Supervisors of Mathematics (NCSM) (2014) recommends the use of the virtual manipulatives. Computer software is a component of virtual manipulatives. The use of computer software as a teaching tool increases student confidence and improves motivation, and self-efficacy to learn mathematics (Sivin-Kachala & Bialo, 2000). Manipulatives need not be expensive. Items such as centimeter grid paper, pipe cleaners, balloons, paper plates and free online graphing software (Desmos).

The review discussed theories, and ELLs learning with the use of manipulatives (concrete and virtual) to build their understanding of linear and exponential functions. This review of literature has been conducted in the general area of using manipulatives (Aburime, 2007), virtual manipulatives (Hollerbands, 2007 & Hannan, 2012) and computer software (Kirk, 2011 & Zunairdi, Zakari, 2012) in the high school classroom for ELLs. The utilization of manipulatives is beneficial for assisting English Language Learners with formulating the concrete to make connections with the abstract (Underhill, 1977; Heddens, 1986; Howell & Barnhart, 1992; Sharma, 1997; Witzel, 2005). Building students' understanding of linear and exponential functions is a spiral concept embedded within the historical development that emerged due to mathematical needs of society (Dandola-Depaolo, 2011).

The National Council of Teachers of Mathematics (1989) and the National Research Council (1989) recommend that researchers attend to affective and cognitive factors related to mathematics teaching and learning. Both DeBellis and Goldin (1997, 2006) suggest the four categories (variables) of the affective experience related to mathematics learning (beliefs, attitudes, emotions, and values) all affect one's self-efficacy in learning mathematics. Little research has been conducted using concrete manipulatives to teach mathematics at the secondary level; therefore, reviewing the research at all levels provides a holistic perspective of teaching with manipulatives.

The purpose of this sequential embedded quasi-experimental mixed methods research was to explore differences in learning about linear and exponential functions for investigating the effectiveness of concrete and virtual manipulatives with ELLs as compared to a control group of ELLs using traditional instructional learning practices without manipulatives. Additionally, the

researcher wanted to investigate ELLs' beliefs, attitudes, and dispositions (variables) about learning mathematics and math class. Lastly, the unique benefits and disadvantages of using concrete and virtual manipulatives were discussed. The control group (N= 20), was instructed through the use of a math textbook and Power points (traditional instruction); the treatment group (N=19), was instructed using concrete and virtual manipulatives. One ESOL mathematics teacher implemented this study teaching both groups utilizing the sheltered instruction observation protocol (2012) method to integrate language and content.

Quantitative methods compared results from Unit 3A: Linear Functions Summative Assessment (pretest and posttest) and Unit 3B: Exponential Functions Summative Assessment (pretest and posttest) between the groups (control and experimental) to inform research questions 1 and question 2. Also, the quantitative methods compared results from Quantitative Understanding: Amplifying Student Achievement and Reasoning Students' Disposition (QUASARQSDI) (pretest and posttest) between the groups (control and experimental) to inform research question 3. Qualitative methods such as ELLs' teacher interviews and student work sample artefacts were employed to inform research question 1, question 2, question 3, and question 4.

The first research question asked what difference, if any, exists in student achievement as a result of using concrete and virtual manipulatives as ESOL high school students use them to learn about linear functions compared to a control group using traditional instructional practice. The results indicated that there were no statistically significant differences in performance related to linear functions between the groups. Therefore, the null hypothesis was not rejected. The second research question asked what difference, if any, exists in student achievement as a

result of using concrete and virtual manipulatives as ESOL high school students use them to learn about exponential functions compared to a control group using traditional instructional practice. Results indicated that there was no statistically significant difference in performance on exponential functions between the groups. Therefore, the null hypothesis was not rejected. The third research question asked what difference, if any, exists in student attitudes, beliefs, and dispositions about mathematics and math class as a result of using concrete and virtual manipulatives as ESOL high school students employ them to learn about linear and exponential functions compared to a control group using traditional instructional practice? Results indicated that there were no statistically significant differences in students' attitudes, beliefs, and disposition about mathematics and math class between the control and treatment groups. As a result, the null hypothesis was not rejected.

The final research question was qualitative and asked what are the unique benefits and disadvantages of using concrete and virtual manipulatives versus traditional instructional practice? The results of the qualitative analysis revealed 6 themes that addressed this research question. Two themes revealed advantages of the intervention, three themes revealed disadvantages, and one theme revealed both a disadvantage and an advantage. The two advantages of using the intervention were that math retention could be increased by building upon students' prior math knowledge, and that the students were actively engaged in learning. The three disadvantages was first, the ESOL class sizes, due to county budget cuts, are too large, which makes it difficult to use manipulatives with a large number of students. Second, time management was an issue, as there was not enough time for planning and classroom implementation of manipulatives. Third, there was limited availability of the virtual or computer

based manipulatives. Finally, the theme that was both an advantage and a disadvantage was that the students were distracted by the manipulatives as they saw them as toys and wanted to play with them. This was the disadvantage. However, the teacher later used the students' free play with the manipulatives in the learning process.

Personal Reflections

I have been teaching school for 28 years and have always engaged my students with utilizing manipulatives and hands on activities to enhance their mathematical thinking and understanding. There is a Chinese Proverb, "I hear, and I forget. I see and I remember. I do, and I understand." This has guided my journey for teaching mathematics to middle school, high school and college students. The Chinese Proverb is also aligned with Engerstrom's (1987) Activity Theory. Although my research indicated no statistically significant difference between the control group and treatment group as they explored linear functions and then exponential functions, there was statistically significant achievement gain within each group. The teacher interviews, recorded field notes, student work samples and artefacts revealed ELLs in the treatment group had reference points, increase in memory and retention, and were more dexterous in writing functions algebraically from a given graph, table, pattern and application as opposed to the ELLs in the control group. The ESOL mathematics teacher stated:

I think it gives them a reference point because their memory is really bad. I call it the blank slate syndrome. One day to a next, you start off with a blank slate because they don't remember what you taught them the day before. Here you have a reference point; they can refer to something. It's like, "Oh yeah, when we did that." You know, manipulatives, they can refer to something rather than to just a sheet of paper and notes,

you know what I'm saying? It's in addition to; it's an additional tool they can use for their memory for reinforcement. Reference point. You know? That's what I think ideally. They should be able to go from the concrete manipulative to the abstract. You know; they should be able to wean off of that concrete to the abstract ideally. But at the same time, when they forget, they have something concrete to refer back to. (ESOL teacher interview, February 25, 2015)

I strongly believe if the ESOL students had utilized manipulatives prior to my research study or the treatment had been conducted over a long term, the use of manipulative materials would have indicated statistically significant differences between the control group and treatment group test scores as opposed to not using manipulatives. This may also positively improve their student dispositions about mathematics and math class.

Although the quantitative data analysis revealed no statistically significant difference in ELLs' student attitudes, beliefs and dispositions towards mathematics and math class, the qualitative data analysis indicated the manipulatives created opportunities where ELL's were actively engaged, excited and having fun during mathematical problem solving. Once the ELLs saw that you have to experiment with mathematics, the fun in learning about linear and exponential functions began. Students were able to conceptualize abstract ideas. Also, Garrity (1998) suggests that manipulatives foster students' motivation (disposition) to learn mathematical concepts.

The NCTM (2000) recommends students experience a repertoire of functions for mathematical modeling. The experiences provided within this research study enhanced students'

mathematical thinking and ability to solve real-world applications, as well as assisted ELLs' with organizing, describing, explaining, studying, comprehending, and making predictions about using linear and exponential functions in the real world (Kersaint et al., 2013; McNeil & Jarvin, 2007). Additionally, these applications of linear and exponential functions created interactions between subjects (ELL's), cultural artefacts (manipulatives), objects (tasks, assignments), and components of rules, community, and division of labor (Engerstrom's Activity Theory, 1987), while providing opportunities for ELLs to collaborate with peers in solving real-life applications (Kersaint et al., 2013). Reading and comprehending word problems presented challenges for ELLs, therefore I made the following modifications to *Opening Your Own Business* task to assist students with *comprehensible input* (Krashen, 1988):

- Changed the document's font
- Bolded key terminology to assist with mathematical operations
- Provided space within document for students to show work
- Inserted first quadrant coordinate plane for graphing
- Separated the activity into two parts (Plan A and Plan B), which limited the number of assigned problems (Kersaint et al., 2013)

Conclusion

Due to the increase in the ESOL student population in United States Public Schools and the implementation of Common Core State Standards educators are struggling and under tremendous pressures to meet the progressively diverse needs of these students (Goldenberg, 2008). The National Council of Teachers of Mathematics (2000) proposes one of many teaching strategies and techniques, which appears to offer great promise in the use of manipulatives.

Additionally, NCTM declares that the study of mathematics should increase opportunities for students to model situations using oral, concrete, pictorial, graphical, and algebraic methods (National Council of Teachers of Mathematics, 1989). Kersaint, Thompson, and Petkova (2013) insist that ELLs are engaged in activities that require practicing literacy skills (speaking, reading and writing). Manipulatives assist students with using concrete objects to make connections with abstract ideas; manipulatives also improve students' dispositions about mathematics and math class. If a student's disposition improves, the student is more receptive to learning, which can lead to higher success in algebra achievement (Wlodkowski, 1999).

Reference List

- Abdal-Haqq, I. (1996). Making Time for Teacher Professional Development. ERIC Digest.
- Aburime, F. E. (2007). How Manipulatives Affect the Mathematics Achievement of Students in Nigerian Schools. *Educational Research Quarterly*, 31(1), 3-15.
- Acuna, C. (2007). Use of slope and y-intercept in prediction and description, as seen from students' perspective. In: Conference of the European Society for Research in Mathematics Education, 3rd, 2007, Bellaria. Proceedings... Bellaria, Italy.
- Akkoç, H., & Tall, D. (2002). The simplicity, complexity and complication of the function concept. Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education. 2, (pp. 25-32). Norwich, England.
- Alkan, V. (2013). Reducing Mathematics Anxiety: The Ways Implemented by Teachers at Primary School. *International J. Soc. Sci & Education*, 3(3), 795 – 807.
- Allen, C. (2007). *An action based researched study on how using manipulatives will increase students' achievement in Mathematics*. Marygove College. Retrieved from <http://files.eric.ed.gov/fulltext/ED499956.pdf>
- Allen, B.D., and J. Carifio. (2007). Mathematical sophistication and differentiated emotions during mathematical problem solving. *Journal of Mathematics and Statistics* 3(4), 163-7.
- Allsopp, D. H. (1999). Using modeling, manipulatives, and mnemonic with eight-grade math students. *Teaching Exceptional Children*, 32(2), 74-81.

- Amaya, M. M., Uttal, D. H., O'Doherty, K.D., Liu, L.L. & DeLoache, J.S. (2007). *Two-digit subtraction: Examining the link between concrete and abstract representations of knowledge*.
- American College Test. (2007). *EPAS State of the Nation Report 2007: Mathematics*. Retrieved from www.act.org/research/policymakers/pdf/math_report.pdf
- Anfara, V. A., Brown, K. M. & Mangione, T.L. (2002). Qualitative Analysis on Stage: Making the Research Process More Public, *American Educational Research Association*, 31, 28 - 38.
- Antosz, E. (1987). Teaching Math with Manipulatives. Proceeding of the honors teacher workshop of Middle Grades Mathematics. Michigan State University 67-78. ERIC ED 295m792 SE 049 106.
- Arem, C. (2003). *Conquering Math Anxiety 2nd: A Self-Help Workbook*. Pacific Grove, CA
- Ariza, E. N. (2009). *Not for ESOL teachers: What every classroom teacher needs to know about the linguistically, culturally, and ethnically diverse students* (2nd ed.). New York, NY: Pearson Education.
- Ariza, E. N., Morales-Jones, C., Yahy, N., & Zainuddin, H. (2012). *Why TESOL? Theories and issues in teaching English to speakers of other languages in K-12 classrooms*. Dubuque, IA: Kendall Hunt.
- Ary, D., Jacobs, L., Razavieh, A., & Sorensen, C. (2006), *The imperfect induction, Introduction to Research in Education* (7th ed.) New: Wadsworth Group.

- August, D. L., & Shanahan, T. (2006). Synthesis: Instruction and professional development. In D. L. August, & T. Shanahan (Eds.), *Developing literacy in a second language: Report of the National Literacy Panel*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Battle, T. S. (2007). *Infusing math manipulatives: The key to an increase in academic achievement in the mathematics classroom*. (ERIC Document Reproduction Service No. ED498579).
- Bako, M. 2003. Different projecting method in teaching spatial geometry. In Proceeding of the Third Conference of the European Society for Research in Mathematics Education, Bellaria, Italy.
- Barbin, E. (2000). Integrating history: Research perspectives. Introduction. In J. Fauvel & J. van Maanen (Eds.), *History in mathematics education, The ICMI Study* (pp. 63-66). Boston: Kluwer Academic.
- Baron, M. (2015). An Authentic Task that Models Quadratics. *Mathematics Teaching In the Middle School*, 20 (6), 335-340.
- Baroody, J. 1989. Manipulatives Don't Come with Guarantees. *Arithmetic Teacher*, 37 October, pp. 4-5.
- Berebisky, R. D. (1985). An annotated bibliography of the literature dealing with mathematics anxiety. *Dialog*, ERIC, ED 257 684.
- Berg, H., Petron, M., & Greybeck, B. (2012). Setting the foundation for working with English. *American of Secondary Education*, 40(3), 33-44.
- Berger, K. S. (1983). The developing person through the lifespan. Language learners in the secondary classroom. *American Secondary Education Journal*, 40(3), 34-44.

- Birgin, O. (2012). Investigation of eighth-grade students' understanding of the slope of the linear function. *Bolema: Boletim de Educação Matemática*, 26(42A), 139-162.
- Bishop, A. J. (2001). What values do you teach when you teach mathematics? In P. Gates (Ed.), *Issues in Mathematics Teaching*, 93-104. London: Routledge Falmer.
- Bright, G. (1986). One point of View: Using Manipulatives. *Arithmetic Teacher*, 33(6), 4.
- Bochner, S. (1970). The rise of functions. *Rice University Studies*, 56, 3-21.
- Boggan, M., Harper, S., & Whitmire, A. (2010). Using manipulatives to teach elementary mathematics. *Journal of Instructional Pedagogies*, 3, 1-6.
- Boneau, C. A. (1960). The effects of violations of assumptions underlying the t test. *Psychological Bulletin*, 57(1), 49-64.
- Bouck, E.C., & Flanagan, S. M (2010). Virtual manipulatives. *Intervention in school and Clinic*, 45(3), 186-191. Doi:<http://dx.doi.org/10.1177/1053451209349530>
- Boulton-Lewis, G. W. (1998). Children's strategy use and interpretation of mathematical representation. *Journal of Mathematical Behavior*, 17(2), 219-237.
- Brown, S. (2007). Counting block or keyboards? A comparative analysis of concrete versus virtual manipulative in elementary school mathematics. Detroit: Marygrove College.
- Bruner, J. S. (1961). *The Process of Education*. Cambridge: Harvard University Press.
- Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge, MA: Belknap Press.
- Bruner, J. S. (1983). *Child's Talk: Learning to Use Language*. New York: Norton.
- Brush, L. (1981). Some thoughts for teachers on mathematics anxiety. *The Arithmetic*, 29, 37-39.

- Burch, T. (2006). The affects of manipulatives in mathematics classrooms. *Action Research Project*, Marygrove College. 1-29.
- Burns, M. (2007). *About teaching mathematics: A K-8 Resource (3rd ed)*. Sausalito, CA: Math Solutions Publications.
- Bushell, Jr.,D., & Fueyo, V. (1998). Using number line procedures and peer tutoring to improve the mathematics computation of low performing first graders. *Journal of Applied Behavior Analysis*, 31(3) 417-30.
- Cabonneau, K., Scott, M. and Selig, P. (2013). A Meta-Analysis of the Efficacy of Teaching Mathematics with Concrete Manipulatives. *Journal of Education Psychology*, 105(2), 380-400.
- Cabahug, J.A. (2012). The Use of Bruner's Modes of Representation in Teaching Factoring Second-Degree Polynomials. *International Journal of Education*, 1, 85-103.
- Caglayan, G., & Olive, J. (2010). Eighth grade students' representations of linear equations on a cups and tiles model. *Educational Studies in Mathematics*, 74, 143-162.
- Cannon, L., Heal, R. & Wellman, R (2000). Serendipity in interactive mathematics: Virtual (electronic) manipulatives for learning elementary mathematics. In C. Crawford et al. (Eds), *Proceedings of the Society for Information Technology and Teacher Education International Conference (2000)* (pp. 1083-1088). Retrieved 20 February form http://www.editlib.org/index.cfm?fuseaction=Reader.ViewAbstract&paper_id=15782
- Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics Classrooms that Promote Teaching for Understanding* (pp. 19 - 32). Mahwah, NJ: Lawerance Erlbaum Associates.

- Caswell, R. (2007). Fraction from Concrete to Abstract Using “Playdough Mathematics”.
Australian Primary Mathematics Classroom, 12(2), 14-17.
- Clements, D. H. & McMillian, S. (1996). Rethinking “concrete” manipulatives. *Teaching Children Mathematics*, 2, 270-280.
- Steffe, L. P., Richards, J., & Cobb, P. (1983). Children’s counting types: Philosophy, theory, and application.
- Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 125-146). Reston, VA: National Council of Teacher of Mathematics.
- Cohen, J. (1988), *Statistical Power Analysis for the Behavioral Sciences*, 2nd Edition. Hillsdale, N.J.: Lawrence Erlbaum.
- Creswell, J. W. (2012). *Educational research, planning, conducting, and evaluating quantitative and qualitative research*. Boston, MA: Addison-Wesley.
- Creswell, J.W., & Plano-Clark, V. L. (2007). *Mixed methods research*. Thousand Oaks, CA: Sage.
- Collier, V., & Thomas, W. (1997). School effectiveness for language minority students. Washington, DC: *National Clearinghouse for Bilingual Education*. Retrieved from http://www.thomasandcollier.com/1997_Thomas-Collier97-1.pdf
- Coes, L. (1994). The Functions of a Toy Balloon. *Mathematics Teacher*, 87 (8), 619-629.
- Common Core Coach. (2014). *Georgia common coach coordinate algebra assessments*. New York, NY: Triumph Learning.

- Common Core State Standards Initiative (2010). *Common Core State Standards for Mathematics*. Washington, DC: Retrieved from http://corestandards.org/assets/CCSSI_Math%20Standards.pdf
- Common Core State Standards for Mathematics (2010). *Common Core State Standards Initiative Preparing America's Students For College & Career*. Retrieved from <http://www.corestandards.org/Math/>
- The Common Core State Standards for Mathematics (2010). *Common Core State Standards Initiative Standards of Mathematical Practices*. Retrieved from <http://www.corestandards.org/Math/Practice/>
- Cowen, K. (2014, May 21). *Exponential Bacterial Growth* [Video file]. Retrieved from <https://www.youtube.com/watch?v=NSK4Wh-7orw>
- Dahl, L. (2011). The impact of manipulatives on learning in the elementary and middle school mathematics classroom. (M.A.) Bemidji State University. Bemidji, Minnesota.
- Dale, T., & Cuevas, G. (1992). Integrating mathematics and language learning. In P. Richard-Amato, & M. Snow (Eds.), *The multicultural classroom* (pp. 271-284). White Plains, NY: Longman.
- Day, L. (2015). Mathematically Rich, Investigative Tasks for Teaching Algebra. *Mathematics Teacher*, 108 (7), 512-518.
- DeBellis, V., Goldin, G. (1997). *The affective domain in mathematical problems solving*. In: E. Pekhone (Eds), *Proceeding of the 21st International Conference for Psychology of Mathematics Education 2*, 209-216. University Helsinki: Lahti, Finland.

- Debellis, V., & Goldin, G. (2006). Affect and Meta-Affect in Mathematical Problem Solving: A Representational Perspective. *Educational Studies in Mathematics*, 63 (2), 131-147.
- Dewey, J. (1938). *Logic: The theory of inquiry*. In J. A. Boydston (Ed.), *John Dewey: The later works*, 12, 1925-1953, Carbondale: Southern University Press
- Dewey, J. (1902). *The Child and the Curriculum*. Chicago: University Chicago Press.
- Dewey, J. (1915). *The School and Society*. Chicago: University Chicago Press.
- Dewey, J. (1933). *How We Think*. New York: Heath.
- Dienes, Z. P. (1960). A theory of mathematics-learning. In *Building up mathematics* (pp. 19-36). London, England: Hutchinson Educational.
- Dienes, Z. P. (1971). An example of the passage from the concrete to the manipulation of formal systems. *Educational Studies in Mathematics*, 3, 337-352.
- Di Martino, P., and Zan, R. (2001) Attitudes towards mathematics: some theoretical issues. In M. van den Heuvel-Panhuizen (Ed.) *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education*, 3, 209-216, Utrecht, the Netherlands: Freudenthal Institute, University of Utrecht.
- Di Martino, P., and Zan, R. (2010). 'Me and maths': Towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teachers Education* 13(1), 27-48.
- Doias, E. D. (2013). The effect of manipulatives on achievement scores in the middle school mathematics class. (Ed.D., Lindenwood University). *ProQuest Dissertations and Theses*.
- Dove, M., & Honigfeld, A. (2010). ESL coteaching and collaboration: Opportunities to develop teacher leadership and enhance student learning. *TESOL journal*, 1(1), 3-22.

- Dubinsky, E., & Harel, G. (1992). The nature of the process conception of function. In E. Dubinsky & G. Harel (Eds.), *The concept of function: Aspects of epistemology and pedagogy* (pp. 85-106). Washington, D.C: Mathematical Association of America
- Dyer, L. (1996). *An investigation of the use of algebraic manipulatives with community college students* (Unpublished doctoral dissertation). Central Missouri State University, Columbia, MO
- Echevarria, J., Short, D., & Vogt, M. (2008). Making content comprehensible for non-native speakers of English: The SIOP model. *International Journal of Learning, 14*(11), 41-49.
- Echevarria, J., Short, D., & Vogt, M. (2012). *Making content comprehensible for English learners: The SIOP model*. Boston, MA: Pearson Education.
- Education Commission of the States (2013). *English Language Learners: A growing-yet underserved-student population*. Retrieved from <http://www.ecs.org/clearinghouse/01/10/20/11020.pdf>
- Eshun, B. (2004). Sex-differences in attitudes of students towards mathematics in secondary schools. *Mathematics Connection, 4*, 1-13.
- Fatade, A., Arigbabu, A., Mogari, D., & Awofala, A. (2014). Investigating Senior Secondary School Students' Beliefs about Further Mathematics in a Problem Based Learning Context. *Bulgarian Journal of Science Policy, 8*(1), 5-46.
- Fazey, M. (1999). *Guidelines to help instructors help their learners get the most out of video lessons*. Unpublished manuscript. (Available from Kentucky Educational Television, Lexington, KY).

- Fenner, D. (2013, May 21). Culture and writing for ELLs in the CCSS. Color in Colorado. [Blog]. Retrieved from [http://blog. colorincolorado.org/2013/05/21/culture-and-writing-for-ells-in-the-ccss/](http://blog.colorincolorado.org/2013/05/21/culture-and-writing-for-ells-in-the-ccss/)
- Field, A. P. (2013), *Discovering statistics using SPSS*. London, England: SAGE.
- Fink, D. L. (2005). *Integrated course design*. Manhattan, KS: The IDEA Center. Retrieved from http://ideaedu.org/wp-content/uploads/2014/11/Idea_Paper_42.pdf
- Flynn, K., & Hill, J. (2006). *Classroom instruction that works with English language learners*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Flores, A. (2000). Learning and teaching mathematics with technology. *Teaching Children Mathematics*. 308-310
- Fraenkel, J. R., Wallen, N. E. (2009). *How to Design and Evaluate Research in Education 7th Edition*. New York, NY: McGraw-Hill Higher Education.
- Friel, S. N., & Bright, G. W. (1995). Graph knowledge: understanding how students interpret data using graphs. Paper presented at the 17th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Columbus, OH.
- Furinghetti, F., and E. Pehkonen. (2002). Rethinking characterizations of beliefs. In *Beliefs: A hidden variable in mathematics education?* ed. G.C. Leder, Pehkonen and G. Torner, 39-58. Dordrecht, Netherlands: Kluwer.
- Gall, M., Gall, J. P., Borg, W.R. (2007). *Educational Research: An Introduction (8th ed.)*. Boston, MA: Pearson.

- Gann, L. (2013). Secondary mathematics teachers' beliefs, attitudes, knowledge base, practices in meeting the needs of English language learners. (Ed.D. University of Texas). *ProQuest Dissertations and Theses*.
- Garcia, E. P. (2004). Using manipulatives and visual cues with explicit vocabulary enhancement for mathematics instruction with grade three and four low achievers in bilingual classrooms (Unpublished doctoral dissertation). Texas A&M University, College Station, TX.
- Genesee, F., Lindholm-Leary, K., Saunders, W.M., & Christian, D. (2006). *Educating English language learners: A synthesis of research evidence*. New York: Cambridge.
- Georgia Common Core Coach (2014). *Coordinate Algebra Assessments*. Triumph Learning.
- Georgia State Department of Education. (2013). *ESOL/Title III resource guide*. Retrieved from <http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Documents/2012-2013%20ESOL%20Title%20III%20Resource%20Guide.pdf>
- Green, S. B., & Salkind, N. J. (2013). *Using SPSS for Windows and Macintosh: Analyzing and understanding data* (7th ed.). Upper Saddle River, N.J.: Pearson/Prentice Hall.
- Goldenberg, C. (2008). Teaching English Language Learners What the Researches Does- and Does Not-Say. *American Educator*, 8-44.
- Goldenberg, E. P., Mark, J., & Cuoco, A.. (2010). An algebraic-habits-of-mind perspective on elementary school. *Teaching Children Mathematics*, 16(9), 548–556. Retrieved from <http://www.jstor.org.proxy.kennesaw.edu/stable/41199563>
- Goldin, G. A. (2000). Affective pathways and representation in mathematical problem solving. *Mathematical Thinking and Learning*, 17, 209-219.

- Goins, K. B. (2001). Comparing the effects of visual and algebra tile manipulative methods on student skill and understanding of polynomial multiplication. *Dissertation Abstracts International: Section A. Humanities and Social Sciences*, 62, 12.
- Goracke, M. A. (2009). *The Role of Manipulatives in the Eighth Grade Mathematics Classroom*.
- Graham, J. M. (2013). *Concrete Math Manipulatives in Upper Elementary Classrooms*. (Ph.D., Walden University). *ProQuest Dissertations and Theses*, 170.
- Graham, K., Cuoco, A., Zimmermann, G. (2010). *Focus in High School Mathematics: Reasoning and Sense Making Algebra*. Reston, VA: National Council of Teacher of mathematics (NCTM).
- Green, M. Flowers, C., & Piel, J. (2008). Reversing education majors' arithmetic misconception with short-term instruction using manipulatives. *The Journal of Educational Research* (Washington, D.C.), 101(4), 234 -42.
- Grossnickle, F., Junge, C., and Metzener, W. (1951). *Instructional Material for Teaching Arithmetic. In the Teaching of Arithmetic. Fiftieth yearbook of the National Society for the Study of Education*, pt 2. Chicago: University of Chicago Press.
- Grouwns, D. A. (1992). In Grouwns DA. *Handbook on research on mathematics teaching and learning*.
- Guarino, A. J., Echevarria, J., Short, D., Schick, J. E., Forbes, S. & Rueda, R. (2001). The sheltered instruction observation: Reliability and validity assessment. *Journal of Research Education*, 11(1), 138-140.

- Gürbüz, R. (2010). The effect of activity-based instruction on conceptual development of seventh grade students in probability. *International Journal of Mathematical Education in Science and Technology*, 41, 743–767. doi:10.1080/00207391003675158.
- Hannan, S.R. (2012). The role of virtual manipulatives in the high school teaching mathematics. (M.A., Kean University). ProQuest Dissertation and Theses.
- Hannula, M. S. (2000). *Mathematics emotions and attitude- two case studies*: Norway: Trondheim, 75-92.
- Hannula, M. S. (2002). Attitudes towards mathematics: emotions, expectation and values. *Educational Studies in Mathematics Education*, 4, 73-80.
- Hannula, M., Evans, J., Philippou, G., & Zan, R. (2004). Affect in mathematics education- exploring theoretical frameworks. Proceeding of the 28th Conference of International Group for *Psychology of Mathematics Education*, 1, 107-136.
- Hannula, M., Evans, J., Philippou, G., & Zan, R. (2004). Affect in mathematics education- exploring theoretical frameworks. Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 1, 107-136.
- Hansen-Thomas, H. (2008). Sheltered Instruction: Best Practices for ELLs in the Mainstream. *Kappa Delta Pi*, 44(4), 165-169.
- Hardman, J. (2008). Researching Pedagogy: An Activity Theory Approach. *Journal of Education*, 45, 63-93.
- Harrell, V., & Abrahamson, D. (2010) Second Life Unplugged: A Design for Foster At-risk Students' STEM Agency, *Journal of Virtual World Research*, 2(5), 1-23.
- Harris, C. A., Miller, S.P., & Mercer, C.D. (1995). Teaching initial multiplication skills to

- students with disabilities in general education classrooms. *Learning Disabilities Research & Practices*, 10, 180-195.
- Harry, B. H., Sturges, K.M. & Kilinger, J. K. (2005). Mapping the Process: An Exemplar of Process and Challenge in Grounded Theory Analysis. *Educational Researcher*, 34 (2), 3-13.
- Haycock, K. (2003). Foreword to Carnevale and Desrochers, *Standards for What?: Economic Roots of K-12 Reform*. Princeton, N. J.: Educational Testing Services.
- Heddens, J. (1986). Bridging the gap between the concrete and the abstract. *Arithmetic Teacher*, 33, 14-17.
- Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, 21, 33-46.
- Henningsen, M., & Stein, M.K. (1997). Mathematical tasks and student cognition: Classroom-base factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28, 524-549.
- Heid, M. K., & Blume, G.W. (2008). *Research on technology and the teaching and learning of mathematics: Syntheses, cases, and perspectives. Vol. 1: Research syntheses*. Charlotte, NC: Information Age.
- Herbert, S., & Pierce, R. (2008). An 'emergent model' for rate of change. *International Journal of Computers for Mathematical Learning*, 13(3), 231-249.
- Hollerands, K. (2007). The Role of a Dynamic Software Program for Geometry in the Strategies High School Mathematics Students Employ. *Journal for Research in Mathematics Education*, 28 (2), 164.

- Howard, P., Berry, B., & Tracey, D. (1997). Mathematics and manipulatives: Comparing primary and secondary mathematics teacher's views. Paper presented at Australian Association for Research in Education, Brisbane, Australia.
- Howell, S. C., & Barnhart, R. S. (1992). Teaching word problem solving at the primary level. *Teaching Exceptional Children*, 1992 (winter), 44-46.
- Jackiw, N. (2001). The Geometer's Sketchpad (Version 4.0) [Computer software]. Emeryville, CA: KCP Technologies.
- Jackson, C. (2011). *Has today's technology changed or affected the way you teach? It should!* Paper presented at the Internal Conference on Technology in Collegiate Mathematics, Denver, CO.
- Jackson, R. L., & Phillips, G. (1973). Manipulative Devices In Elementary School Matheamtics. *Instructional Aids in Mathematics National Council Teachers of Matheamtics Yearbook Thirty-fourth*. NCTM. 299-339.
- Jarvin, L., McNeil, N., & Sternberg (2006). *Understanding students' mathematical competencies: An exploration of the impact of contextualizing math problems*. Paper presented at the Institute of Educational Sciences Research Conference, Washington, DC.
- Jimenez-Castellanos, O., & Topper, A. M. (2012). The Cost of Providing an Adequate Education to English Language Learners A Review of the Literature. *Review of Educational Research*, 82(2), 179-232.
- Johnson, P., Campet, M., Gaber, K. & Zuidema, E. (2008). Virtual manipulatives to assess understanding. *Teaching Children Mathematics*, 19(3), 202-206.

- Johnson, R. B., Onwueguzie, A. J., & Turner, L.A. (2007). Toward a definition of mixed methods research. *Journal of Mixed Methods Research*, 1, 112 -133.
- Johnston, J. (1999). *Enhancing adult literacy instruction with video*. Unpublished manuscript, University of Michigan-Ann Arbor.
- Jordan, L., Miller, M.D., & Mercer, C.D. (1998). The effects of concrete to semi concrete to abstract instruction in the acquisition and retention of fraction concepts and skill. *Learning Disabilities*, 9, 115-122.
- Kennedy, L. M., (1986). A rationale. *The Arithmetic Teacher*, 33(6), 6-7, 32.
- Kersaint, G. (2007) The learning environment: Its influence on what is learned. In W.G. Martin and M.E. Strutchen (EDs.), *The learning of mathematics*, 83-86. Reston, VA: National Council of Teachers of Mathematics
- Kersaint, G., Thompson, D., Petkova, M. (2013). *Teaching Mathematics to English Language Learners*. New York, NY: Routledge.
- Kessler, C., Quinn, M., & Hayes, C. (1985). Processing mathematics in a second language: Problems for LEP children. In A. Labarca, and L. Bailey (Eds.), *Issues in L2: Theory as Practice, Practice as Theory*. Delaware Symposium on Language Studies, 151-163.
- Kilpatrick, J., Swafford, J., & Findel, G. (Eds.). (2001). Teaching for mathematical proficiency. In *Adding it up: Helping children learn mathematics* (pp. 313-368). Retrieved from National Academies Press database.
- Kirk, R. (2011). *The impact of bilingual treatment on the math skills of Hispanic high school algebra students*. (Order No. 3490442, Gardner-Webb University). *ProQuest Dissertation and Theses*, 168.

- Krashen, S. D. (1988). *Second language acquisitions and second learning*. Upper Saddle River, NY: Prentice-Hall.
- Krashen, S., & Terrell, T. (1983). *The natural approach - Language acquisition in the classroom*. Oxford, United Kingdom: Pergammon.
- Lazarus, R.S. (1991). *Emotions and Adaptation*. NY: Oxford University Press.
- Leontiev, A.N. (1981). The problem of activity in psychology. In Wretch, J.V. (ed.). *The concept of activity in Soviet psychology*. Armonk, N.Y.: M.E. Sharpe, 120-142.
- Lo, J., & Kratky, J. (2012). Looking for Connections between Linear and Exponential Functions. *The Mathematics Teacher*, 106(4), 295–301.
<http://doi.org.proxy.kennesaw.edu/10.5951/mathteacher.106.4.0295>
- Lyster, R. (2007). *Learning and teaching languages through content: A counterbalanced approach*. Amsterdam: John Benjamins.
- Lyster, R., & Mori, H. (2006). Interactional feedback and instructional counterbalance. *Studies in Second Languages Acquisition*, 28:321-341.
- Lyster, R., & Ranta, L. (1997). Corrective feedback and learner uptake: Negotiation of form in communicative classrooms. *Studies in Second Language Acquisition*, 19:37-66.
- Ma, Xin. (2001). Participation in Advanced Mathematics. Do Expectation and Influences of Students, Peers, Teacher, and Parents Matter? *Contemporary educational psychology*, 26 (1).
- Maccini, P., & Gagnon, J. (2001). Preparing Students with Disabilities for Algebra. *Council for Exceptional Children*, 34(1), 8-15.

- Magruder, R. L. (2012). Solving linear equations: A comparison of concrete and virtual manipulatives in middle school mathematics.
- Mandler, G. (1989). *Affect and learning: Causes and consequences of emotional interaction*. New York: Springer-Verlag, 3-19.
- Metcalf, R. (2007). *The Nature of Students' Understanding of Quadratic Functions*. (Doctoral dissertation). Retrieved from ProQuest Dissertations and Theses Database. (3277750)
- Markovits, Z., Eylon, B., & Bruckheimer, M. (1986). Functions today and yesterday. *For the Learning of Mathematics*, 6(2), 18-28.
- Marsh, G., Tapia, M. (2002) Feeling good about mathematics: Are there sex difference? Proceeding of the Annual Meeting of the Mid-South Educational Research Association, Nov 6-8, Chattanooga, TN, 1-12
- Marshall, L., & Swan, P. (2005). Developing Mathematical Thinking with the Assistance of Manipulatives. Paper presented at The Mathematics Education into the 21st Century Project, Johor Bahry, Malaysia, November 25 – December 2005.
- Mathematical Association of America. (2004). *A Collective Vision: Voices of the Partner Disciplines*. In *The Curriculum Foundations Project: Voices of the Partner Disciplines*, edit by Ganter and Barker, pp. 1-13. Washington D.C.
- Mathematics from English Language Learners (MELL) Project. (2005). *Teaching guide to teaching mathematics for English language learners*. Round Rock, TX: Author.
- Maynard, L. E. (1983). *A study of the effects of required mastery strategies and the use of concrete manipulatives on college-age remedial students* (Doctoral dissertation. University of Carolina at Greensboro).
- McGee, D., Jr, Moore-Russo, D., Ebersol, D., Lomen, D. O., & Quintero, M.M. (2012).

- McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grows (Ed.). *Handbook of Research on Mathematics Teaching and Learning*. A project of the National Council of Teachers of Mathematics. Information Age Publishing Inc. 575-596.
- McNeil, N. & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory In to Practice*, 46 (4), 309-316.
- McNeil, N. M., Uttal, D. H., Jarvin, L., & Sternberg, R.J. (2007). *Should you show me the play money? Concrete objects both hurt and help performance on math problems*.
- Met, M. (1991). Learning language through content: Learning content through language. *Foreign Language Annals*, 24, 281–295.
- Met, M. (1999). *Content-based instruction: Defining terms, making decisions*. Washington, DC: Author.
- Miller, G. (1956). The Magical Number Seven, Plus or Minus Two Some Limits on Our Capacity for Processing Information. *American Psychological Association*, 101(2), 343-353.
- Mohamed L., & Waheed, H. (2011). Secondary students' attitudes towards mathematics in a selected school of Maldives. *International Journal of Humanities and Social Science*, 1(15), 277-281.
- Moore, C. G. (1994). Research in Native American mathematics education. *For the Learning of Mathematics*, 14(2), 9-14.

- Morris, J. (2014). The use of virtual manipulatives in fourth grade to improve mathematics performance. (M.A., State University of New York. Fredonia, New York. Retrieved from https://dspace.sunyconnect.suny.edu/bitstream/handle/1951/63485/jaimie_morris_masters_project_december2013.pdf?sequence=1
- Moschkovich, J. (1999). Supporting the participation of English language learners in mathematics discussions. *For the Learning of Mathematics*, 19(1), 11-19.
- Moyer, P., Bolyard, J., Spikell, M. (2002). What Are Virtual Manipulatives. *Teaching Children Mathematics*, 8(6), 372-377.
- The National Assessment of Educational Progress (2013). Are the nation's twelfth-graders making progress in mathematics and reading? The Nation's Report Card. Retrieved from http://nationsreportcard.gov/reading_math_g12_2013/#/
- The National Council of Supervisors of Mathematics (2013). Improving Student Achievement in Mathematics by Using Manipulatives with Classroom Instruction. Retrieved from <http://www.mathedleadership.org/resources/position.html>
- Moyer, P. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, 47, 175-197. Retrieved August 22, 2012, from Wilson Web data base.
- Moyer, P., Bolyard, J. & Spikell (2002). What are virtual manipulatives? *Teaching Children Mathematics*, 8(6), 372-377.
- Moyer, P. & Jones, G. (2004). Controlling Choice: Teachers, Students, and Manipulatives in Mathematics Classroom. *School Science and Mathematics*. 104 (1), 16- 31.

National Center of Educational Statistics. (2012). *Fast facts. English language learners*. Institute of Educational Sciences. Retrieved from <http://nces.ed.gov/fastfacts/display.asp?id=96>

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards For School Mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1994). *News bulletin*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (2008). *Algebra: What, When and for Whom. A position of the National Council of Teachers of Mathematics*. Retrieved from http://www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/Algebra%20final%2092908.pdf

National Council of Teacher of Mathematics. (2013). *Teaching mathematics to English learners: A position of the National Council of Teachers of Mathematics*. Reston, VA: Author.

The National Assessment of Educational Progress (2013). Are the nation's twelfth-graders making progress in mathematics and reading? The Nation's Report Card. Retrieved from http://nationsreportcard.gov/reading_math_g12_2013/#/

The National Foreign Language Center. (2013). *Content-based instruction: Defining terms, making decisions*. Retrieved from: <http://www.carla.umn.edu/cobaltt/modules/principles/decisions.html>

- Nelmes, P. (2003). Developing a conceptual framework for the role of the emotions in the languages of teaching and learning. Forthcoming in Proceedings of CRÈME 3; February 28-March 3, 2003, Bellaria, Italy.
- Nishida, T. K. (2007). The use of manipulatives to support children's acquisition of abstract math concepts. (Ph.D., University of Virginia). *ProQuest Dissertations and Theses*.
- Niss, M. (2007). Reflection on the state of and trends in research in mathematics teaching and learning from here to Utopia. *Second Handbook of Research on Mathematics Teaching and Learning*. A project of the National Council of Teachers of Mathematics. Information Age Publishing Inc. 31, 1293-1311.
- Norman, D. A. (1993). Things that make us smart: Defending human attributes in the age of the machine. Reading, MA: Addison-Wesley
- Olkun, S. (2003). Comparing computer verses concrete manipulatives in learning 2D geometry. *The Journal of Computers in Mathematics and Science Teaching*, 22(1) 43-56.
- Ogg, B. (2010). What is the Impact of Math Manipulatives on Student Learning. (MA., Ohio University).
- Parzysz, B. (1988). Knowing versus seeing: Problems of the plane representation of space geometry figures. *Educational Studies in Mathematics*. 19(1), 79-92.
- Paulsen, K., & the IRS Center (2006). *Algebra 2 (Part 2): Applying learning strategies to intermediate algebra*. Retrieved from <http://iris.peabody.vanderbilt.edu/wp-content/uploads/2013/07/ICS-010.pdf>
- Pedersen, O. (1974). Logistics and the theory of functions. *Archives Internationales D'Histoire Des Sciences*, 24, 29-50.

- Pekrun, P. (2006). The control-value theory of achievement emotions: Assumptions, 53 corollaries, and implication for educational research and practice. *Educational Psychology Review*. 18(4), 315-341.
- Pekrun, R., Elliot, A.J., & Maier, M.A. (2009). Achievement goals and achievement emotions: Testing a model of their joint relations with academic performance. *Journal of Educational Psychology*, 101, 115-135.
- Pekrun, P., R., Goetz, T., Tiz, W., & Perry, R. P. (2002). Academic emotions in students' self regulated learning and achievement: A program of qualitative and quantitative research. *Educational Psychologist*, 37 (2), 91-105.
- Pekrun, P., & Stephens, E. J. (2009). Goals, emotions, and emotion regulation: Perspectives of the control-value theory. *Human Development*, 52, 357-365.
- Perry, B., & Howard, P., (1997). Manipulatives in primary mathematics: Implications for teaching and learning. *Australian Primary Mathematics Classroom*, 2(2), 25-30.
- Piaget, J. (1952). *The origins of intelligence in children*. New York, NY: International University Press.
- Piaget, J. (1955). *Language and Thought of the Child*. New York: New American Library.
- Piaget, J. (1970). *Science of education and the psychology of the child*. (D. Coleman, Trans.). New York, NY: Orion Press.
- Piaget, J. (1973). *Child and Reality*. New York: Grossman.
- Plaisance, D.V. (2009). A Teacher's Quick Guide to Understanding Mathematics Anxiety. *Louisiana Association of Teachers of Mathematics Journal*, 6(1). Retrieved from http://www.lamath.org/journal/vol6no1/anxiety_guide.pdf

- Posten, H.O. (1984). Robustness of the two-sample t-test. In D. Rasch and M. L. Tiku (Eds.), *Robustness of statistical methods and nonparametric statistics* (pp. 92-99). Dordrecht, Germany, Reidel.
- Power, M. & Dalgleish, T. (1997). *Cognition and Emotion; From order to disorder*. UK: Psychology Press.
- Project GLAD. (n.d.). Retrieved from <http://www.projectglad.com/>
- Puchner, L., Taylor, A. O'Donnell, B. and Fick, K (2008). Teaching Learning and Mathematics Manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics*, 108(7), 313-325.
- QUASAR Student Disposition Instrument. (1992). *QUASAR Student Disposition Instrument*. University of Pittsburgh. Retrieved from <http://files.eric.ed.gov/fulltext/ED343776.pdf>
- Quinlan, J.E. (2007). *Profiles of software utilization by university faculty*. Retrieved from Dissertation & Theses. (AAT 3262107)
- Rabardel, P. (2003). From artefact to instrument. *Interacting with computers*, 15(5), 641-645.
- Ralston, A. (2004). Research mathematics and mathematics education: A critique. *Notices of the American Mathematical Society*, 51, 502-514
- Reneau, J. L. (2012). Using the concrete-representational-abstract sequence to connect manipulatives, problem solving schemas, and equations in word problems with fractions. (Ed.D., West Virginia University). *ProQuest Dissertations and Theses*.
- Reys, R.E. 1971. Considerations for Teaching Using Manipulatives Materials. *Arithmetic Teacher*, 181 December, 551-558.

- Reys, R. E., & Post, T.R. (1973). *The mathematics laboratory: Theory to practice*. Boston, MA: Prindle, Weber, and Schmidt.
- Roberts, S. (2007). On My Mind: Not All Manipulatives and Models Are Created Equal. *Mathematics Teaching In the Middle School*, 13 (1), 6-9.
- Robinson, S. (2006). Teaching math to English learners-Myths and methods. In E.N. Whelan Ariza (ed.). *Not for ESOL teachers: What every classroom teacher needs to know about the linguistically, culturally, and ethnically diverse student*. Boston: Person.
- Roessingh, H. (2006). BICS-CALP: An Introduction for Some, A Review for Others. *TESL Canada Journal*, 91-96.
- Rubenstein, R.N. & Thompson, D.R., (2001). Learning Mathematical Symbolism: Challenges and Instructional Strategies. *Mathematics Teacher*, 94(4):265-271.
- Salaudeen, W. A. (2011). Effects of Class-Size on the Teaching and Learning of English Language. Kwara State College of Education, Ilorin, Nigeria
- Schmider, E., Ziegler, M., Danay, E., Beyer, L., & Buhner, M (2010). Is it really robust? Reinvestigatin the rubustness of ANOVA against violations of the normal distribution assumption. *Methodology: Erurpean Journal of Research Methods for the Behavioral and Social Sciences*, 6, 147-151.
- Schunk, D. H. (2012). *Learning Theories: An Educational Perspective (6th ed.)*. Upper Saddle River, NJ: Pearson.
- Schoenfeld, A. H. (1983). Beyond the purely cognitive: Belief systems, social cogntition, and metacognition as driving forces in intellectual performance *Cognitive Science*, 7, 329-363.

- Sharma, M. (1997, July). Improving mathematics instruction for all. Fourth Lecture in Colloquium "Improving Schools from Within: Your Role. 2-12.
- Sharma, M. (1987). How to take a child from concrete to abstract Math Note book, 5, 8-10.
- Sherman, J., & Bisanz, J. (2009). Equivalence in symbolic and nonsymbolic contexts: Benefits of solving problems with manipulatives. *Journal of Educational Psychology*, 101(1), 88-100.
- Shoebottom. (2014). *The most important. A guide to learning*. Oberursel, Germany: Author. Retrieved from <http://esl.fis.edu/teachers/support/sum.htm>
- Shulman, L. S. (1998). Theory, practice, and the education of professionals. *The Elementary School Journal*, 98, 511–526.
- Sivin-Kachala, J., & Bialo, E. R. (2000). *Research Report on the Effectiveness of Technology in Schools* (7th ed.). Washington, D.C.: Software Information Industry Association.
- Smith, L. F. & Montani, T. O. (2008). The effects of instructional consistency: Using manipulatives and teaching strategies to support resource room mathematics instructions. *Learning Disabilities*, 15(2), 71-76.
- Smith, S.S. (2009). *Early Childhood Mathematics* (4th ed.) Boston: Pearson Education Using manipulatives (2009). Retrieved July, 1, 2014, from: <http://www.teachervision.fen.com/pro-dev/teaching-methods/48934.html>
- Sowell, E. J. (1989). Effects of manipulative material in mathematics instruction. *Journal for Research in Mathematics Education*, 20(5), 498-505.

- Steen, K., Brooks, D., Lyon T. (2006). The impact of Virtual Manipulatives on First Grade Geometry Instruction and Learning. *Journal of Computers in Mathematics and Science*, 25(4), 373-391.
- Steffe, L.P., Thomopson, P.W., & vonGlaserfeld, E. (2000). Teaching experiment methodology: Underlying principles and essential elements. In Kelly, A. & R. Lesh (Eds.) *Handbook of research design in mathematics and science education*, p 267-306. Mahwah, NJ: Erlbaum.
- Stoller, F. L. (2004). Content-based instruction: Perspectives on curriculum planning. *Annual Review of Applied Linguistics*, 24, 261–283.
- Stoller, F. L. (2008). Content-based instruction. In N. Van Deusen-Scholl, & N. H. Hornberger (Eds.), *Encyclopedia of language and education: Volume 4: Second and foreign language education* (2nd ed. [pp. 59–70]). New York, NY: Springer Science/ Business Media.
- Stoller, F. L., & Grabe, W. (1997). A six-T's approach to content-based instruction. In M.A. Snow, & D. M. Brinton (Eds.), *The content-based classroom: Perspectives on integrating language and content* (pp. 78-94). White Plains, NY: Longman.
- Stramel, J.K. (2010). A naturalistic inquiry into the attitudes towards mathematics and mathematics self-efficacy beliefs of middle school students. Proquest Dissertations and Theses
- Suh, J. & Moyer, P.S. (2007). Developing students' representational fluency using virtual and physical algebra balances. *The Journal of Computers in Mathematics and Science Teaching*, 26(2), 155-173.

Swan, P., and Marshall, L. 2010. Australian Primary Mathematics Classroom. 15(2), June, pp. 13-19.

Tabachnick, B.G., & Fidell, L.S. (2012). Using multivariate statistics (6th edn). Boston: Pearson Education.

Teaching English to Speakers of Other Languages (TESOL) International Association. (2013). *Implementing the common core state standards for English learners: The changing role of the ESL teacher*. Alexandria, VA: Author.

Teaching English to Speakers of Other Languages (TESOL) International Association. (2014). *Advancing Excellence in English Language Teaching: Overview of the Common Core State Standards Initiatives for ELLs* . Alexandria, VA: Author.

Teachers of English to Speakers of Other Languages. (2006). *Pre K–12 English language proficiency standards*. Alexandria, VA: Author.

Tessema, T. G. (2010). Classroom Instruction and Students' Attitudes Towards Mathematics. (Ed.D., Arizona State University). *ProQuest Dissertations and Theses*.

Thomas, W. P., & Collier, V. P. (2002). *A national study of school effectiveness for language minority students' long-term academic achievement*. Santa Cruz, CA: Center for Research on Education, Diversity, and Excellence.

Thompson, P.W. (1994). Notations, conventions, and constraints: Contributions to effective uses of concrete materials in elementary mathematics. *Journal of Research in Mathematics Educator*, 23, 123-147.

Triantafillou, C., & Potari, D. (2010). *Mathematical practices in a technological work place: the role of tools*. Educational Studies in Mathematics, 74 (3), 275-294.

- Underhill, R.G. (1977). *Teaching Elementary School Mathematics*. 2d ed. Columbus, Ohio: Charles E. Merrill Publishing Co.
- Uribe-Florez, L., & Wilkins, J. (2010). Elementary teachers' manipulative use. *School Science and Mathematics*, 110(7), 363-71.
- U.S. Department of Education. (2013). *English language learners intervention report: Sheltered instruction observation protocol® (SIOP®)*. Institute of Education Sciences, What Works Clearinghouse. Retrieved from <http://whatworks.ed.gov>
- Uttal, D. H., Scudder, K. V., & DeLoaches, J.S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Development Psychology*, 18, 37-54.
- Vinson, B. M., Haynes, J., Brasher, J., Sloan, T., & Gresham, R. (1997). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives (Research Presentation. *Athens, AL: Athens State College*.
- Vukovic, R., Kieffer, M., Bailey, S., Harari, R. (2013). Mathematics anxiety in young children: concurrent and longitudinal association with mathematical performance. *Contemporary Educational Psychology*, 38(1), 1-10.
- Vygotsky, L. S. (1962). *Thought and language*. Cambridge, MA: MIT Press
- Vygotsky, L. S. (1978). *Mind in society: The Development of Higher Psychological Process*. Cambridge, MA: Harvard University Press.
- Weiss, I. (1994). *A profile of science and mathematics education in the United States, 1993*. Retrieved April 13, 2006, from <http://www.horizon-research.com/reports/1994/profile.php>

- Westbrook, T. R. (2011). Evaluating the effectiveness of experiential learning with concrete-representational-abstract instructional technique in a college statistics and algebra course. (Ph.D., Texas State University - San Marcos). *ProQuest Dissertations and Theses*.
- White, K. M. (2012). The effect of an instructional model utilizing hands-on learning and manipulatives on math achievement of middle school students in Georgia (Doctoral dissertation, Liberty University).
- Wiggins, G., & McTige, J. (2005). *Understanding by design* (2nd ed.). Upper Saddle River, NJ: Prentice Hall.
- Williams, D. (1986). Activities for Algebra. *Arithmetic Teacher*, 33, 42-27.
- Witzel, B., Mercer, C.D., & Miller, D. M. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research and Practice*, 1,121-31.
- Wlodkowask, R. J. (1999). *Enhancing adult motivation to learn* (2nd edition). San Francisco, CA: Jossey-Bass.
- World-Class Instructional Design. (2014). *Can do descriptors*. Madison, WI: Author. Retrieved from http://www.wida.us/standards/CAN_DOs/
- Woodard, T. (2004). The Effects of Math Anxiety on Post-Secondary Development Students as Related to Achievement, Gender, and Age. *Inquiry*, 9(1). ERIC Document Reproduction Service No. EJ876845
- Yin, R. K. (2011). *Qualitative Research from Start to Finish*. New York, NY: The Guilford Press.
- Yuan, Y., Lee, C. Y. & Wang. (2010). A comparison study of polyominoes exploration in a

- physical and virtual environment. *Journal of Computer Assisted Learning*, 26 (4). 307-316.
- Youschkevitch, A. P. (1976). The concept of function up to the middle of the 19th century. *Archive for History of Exact Sciences*, 16, 37-85.
- Zan, R., Mortino, P. (2008). Attitudes towards mathematics overcoming the positive/ negative dichotomy in Beliefs and Mathematics, B. Sriraman, Ed. *The Montana Mathematics Enthusiast: Monograph Series in Mathematics Education*, 197-214.
- Zakaria, E., Zain, N. M., Ahmad, N. A., Erlina, A. (2012). Mathematics anxiety and achievement among secondary school students. *American Journal of Applied Science*, 9 (11), 1828-1832.
- Zemelman, S., Daniels, H., Hyde, A. (2005). *Best Practice Today's Standards for Teaching and Learning in America's Schools (3rd ed)*. Portsmouth, NH: Heinemann.
- Zoltan, P. D. (1960). Mathematical thinking and learning. *An International Journal*, 9(1), 59-75.
- Zulnaldi, H., & Zakaria, E. (2012). The effect of using GeoGebra on conceptual and procedural knowledge of high school mathematics students. *Asian Social Science*, 8(11), 102- 106.

Appendix A
Coordinate Algebra Common

Core Georgia Performance Standard(s) (CCGPS) for
Linear and Exponential Functions

MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

MCC9-12.F.IF.2 Use function notation; evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

MCC9-12.A.REI.11 Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make where $f(x)$ and/or $g(x)$ are linear and exponential functions.

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

MCC9-12.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MCC9-12.F.IF.7a Graph linear functions and show intercepts, maxima, and minima.

MCC9-12.F.IF.7e Graph exponential functions, showing intercepts and end behavior.

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

MCC9-12.F.BF.1 Write a function that describes a relationship.

MCC9-12.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations.

MCC9-12.F.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations and translate between the two forms.

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $k f(x)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs.

Experiment with cases and illustrate an explanation for the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that quantity increasing exponentially eventually exceeds a quantity increasing linearly.

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Appendix B
Quantitative Understanding: Amplifying Student Achievement and Reasoning Students
Disposition Instrument (Control)
University of Pittsburg

Please answer all of the following questions. Check only one answer.

1. How far do you want to go in school?
 - 1 Not finish
 - 2 Graduate from high school
 - 3 Vocational school after high school
 - 4 College after high school

2. Compared to other classes, how much do you like math class?
 - 1 Like it much less
 - 2 Like it-less
 - 3 Like it a little less
 - 4 Like it a little more
 - 5 Like it more

3. Compared to other classes, how good are you in math class?
 - 1 Much worse in math
 - 2 Worse in math
 - 3 A little worse in math
 - 4 A little better in math
 - 5 Better in math
 - 6 Much better in math

4. Compared to other classes, how much thinking and reasoning is done in math class?
 - 1 Much less in math
 - 2 Less in math
 - 3 A little less in math
 - 4 A little more in math
 - 5 More in math
 - 6 Much more in math

5. Compared to other classes, how hard is the work in math class?
 - 1 Much less in math
 - 2 Easier in math
 - 3 A little easier in math
 - 4 A little harder in math
 - 5 Harder in math
 - 6 Much harder in math

6. I am good at math
 - 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

7. I like to work on math problem that make me think hard
 - 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

8. When doing math problems, I like to work with other students.
 - 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

9. I think it is important to do well in math.
 - 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

10. In math class I like to think of my own ways to solve problems instead of following the teacher's way.

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

11. I like math

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

12. After I get an answer to a math problem I usually check my work

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

13. I understand most of what goes on in math class

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

14. I would be good in a job that requires math.

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

15. I usually keep working on hard problems until I solve them.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
16. I think about how to solve a math problem before I start to solve it
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
17. For most problems, I would rather watch the teacher solve the problem than solve it by myself.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
18. When doing math problems, I like to work by myself.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
19. I like to work on math problem that make me think hard
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

20. My friends think it is important to get good grades in math.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
21. My friends think that people who like math are weird.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
22. Someone at home thinks I can do well in math.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
23. Someone at home usually makes sure that I do my math school work.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
24. Someone at home usually asks me how I am doing in math.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

25. Someone at home thinks math is important.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
26. Math is useful for solving problems every day.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
27. Learning math is mostly memorizing facts.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
28. Some math problems have more than one correct answer.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
29. Some math problems can be solved in more than one way.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

30. Almost all people use math in their jobs
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
31. Math is more for boys than for girls.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
32. Explaining why an answer is correct is just as important as getting the correct answer.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
33. Knowing math is useful for learning other subjects in school.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
34. Girls are just as good at math as boys.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

35. How old are you?

- 1 Thirteen
- 2 Fourteen
- 3 Fifteen
- 4 Sixteen
- 5 Seventeen
- 6 Eighteen

36. Gender

- 1 Female
- 2 Male

Appendix C
Quantitative Understanding: Amplifying Student Achievement and Reasoning Students
Disposition Instrument (Treatment)
University of Pittsburg

Please answer all of the following questions. Check only one answer.

1. How far do you want to go in school?
 - 1 Not finish
 - 2 Graduate from high school
 - 3 Vocational school after high school
 - 4 College after high school

2. Compared to other classes, how much do you like math class?
 - 1 Like it much less
 - 2 Like it-less
 - 3 Like it a little less
 - 4 Like it a little more
 - 5 Like it more

3. Compared to other classes, how good are you in math class?
 - 1 Much worse in math
 - 2 Worse in math
 - 3 A little worse in math
 - 4 A little better in math
 - 5 Better in math
 - 6 Much better in math

4. Compared to other classes, how much thinking and reasoning is done in math class?
 - 1 Much less in math
 - 2 Less in math
 - 3 A little less in math
 - 4 A little more in math
 - 5 More in math
 - 6 Much more in math

5. Compared to other classes, how hard is the work in math class?
 - 1 Much less in math
 - 2 Easier in math
 - 3 A little easier in math
 - 4 A little harder in math
 - 5 Harder in math
 - 6 Much harder in math

6. I am good at math
 - 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

7. I like to work on math problem that make me think hard
 - 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

8. When doing math problems, I like to work with other students.
 - 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

9. I think it is important to do well in math.
 - 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

10. In math class I like to think of my own ways to solve problems instead of following the teacher's way.

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

11. I like math

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

12. After I get an answer to a math problem I usually check my work

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

13. I understand most of what goes on in math class

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

14. I would be good in a job that requires math.

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

15. I usually keep working on hard problems until I solve them.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
16. I think about how to solve a math problem before I start to solve it
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
17. For most problems, I would rather watch the teacher solve the problem than solve it by myself.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
18. When doing math problems, I like to work by myself.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
19. I like to work on math problem that make me think hard
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

20. My friends think it is important to get good grades in math.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
21. My friends think that people who like math are weird.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
22. Someone at home thinks I can do well in math.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
23. Someone at home usually makes sure that I do my math school work.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
24. Someone at home usually asks me how I am doing in math.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

25. Someone at home thinks math is important.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
26. Math is useful for solving problems every day.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
27. Learning math is mostly memorizing facts.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
28. Some math problems have more than one correct answer.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
29. Some math problems can be solved in more than one way.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

30. Almost all people use math in their jobs
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
31. Math is more for boys than for girls.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
32. Explaining why an answer is correct is just as important as getting the correct answer.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
33. Knowing math is useful for learning other subjects in school.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree
34. Girls are just as good at math as boys.
- 1 Strongly Disagree
 - 2 Disagree
 - 3 Somewhat Disagree
 - 4 Somewhat Agree
 - 5 Agree
 - 6 Strongly Agree

35. How old are you?

- 1 Thirteen
- 2 Fourteen
- 3 Fifteen
- 4 Sixteen
- 5 Seventeen
- 6 Eighteen

36. Gender

- 1 Female
- 2 Male

37. The use of manipulatives increases my understanding of mathematics.

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

38. The use of manipulatives does not increase my understanding of mathematics.

- 1 Strongly Disagree
- 2 Disagree
- 3 Somewhat Disagree
- 4 Somewhat Agree
- 5 Agree
- 6 Strongly Agree

Appendix D

ESOL Mathematics Language Strategies

- Teaching vocabulary by modeling real-life applications
- Relating math problems and vocabulary to prior knowledge
- Applying math problems using manipulatives as a means of bridging abstract and concrete ideas
- Giving students sketches to assist with deciphering word problems
- Providing adequate wait time
- Inspiring students to follow the *four-step* problem-solving process
- Rephrasing word problems in simple terms
- Inspiring children to give oral justifications for their solutions when solving word problems
- Clarifying directions and repeating key terms
- Recognizing that all math notations are not universal
- Generating and displaying word bank charts
- Pairing ELL and non-ELL students
- Grouping students heterogeneously
- Making cultural connections when teaching
- Taking internet field trips to assist with conceptualizing mathematics concepts
- Using children's literature to teach mathematics

Appendix E
Evaluating Virtual Manipulative Sites

- What is the mathematical objective for the students use?
- Does objective correlate with mathematics state standards?
- Is the site to easy access for students?
- Is the site user friendly for students?
- Are the images stimulating for the students?
- Are students able to connect between the concrete or virtual pictorial and the symbolic?
- Are students able to make the connection between the concrete and abstract?
- What is the flexibility of the virtual manipulative site?
- Are teachers able to generate their own problems?
- Does the site offer helpful prompts for student use?

Appendix F
Categorizing Mathematics Manipulatives

- Colored Rods, Blocks, Beads and Discs (Also, includes pattern blocks and attribute block)
- Manipulatives devices for teaching counting and sorting
- Manipulatives devices for teaching place value
- Manipulatives devices for teaching operation and fraction (Also, includes devices for working with percent and decimals)
- Number boards and Demonstration boards
- Cards and Charts (Includes flash cards, activity cards, mobiles, manipulative charts, bulletin material, etc.).
- Measurement Devices
- Model of Geometric Relationship
(Includes plane figures, solids figures, conic section, polyhedral, trig models, problems dealing with geometric relationships)
- Math Games and Puzzles
- Calculating and computational devices (include slide rulers, trig devices, tables, hand calculators, and computers).
- Videos

Computers, including virtual manipulatives, interactive white boards, and computer tablets have been more recently available providers of manipulatives.

Appendix G

Teacher Do's and Don'ts for Using Manipulatives

Do's for teachers:

- Do consider pedagogical and physical criteria in selecting manipulative material.
- Do construct activities that provide multiple embodiment of the concept.
- Do prepare the classroom.
- Do encourage pupils to think for themselves.
- Do ask pupils questions.
- Do allow students to make errors.
- Do provide follow-up activities.
- Do evaluate the effectiveness of materials after using them.
- Do exchange ideas with colleagues.

Don'ts for teachers:

- Don't use manipulative materials indiscriminately.
- Don't make excessive use of manipulatives materials.
- Don't hurry the activity.
- Don't rush from the concrete to the abstract level.
- Don't provide all the answers.

Appendix H

Advantages for using Manipulatives

- They engage student interest
- They provide concrete visuals
- They provide hands-on learning
- They build understanding
- They assist and reinforce mathematical understanding
- They are appropriate for all learning styles (Caswell, 2007; Brown, 2007)
- They provide an introduction to mathematical concepts
- They provide assessment of students' mathematical thinking
- They encourage oral language

Appendix I

Best Teaching Practices

- Provide manipulatives
- Provide cooperative work
- Provide opportunities for discussion when teaching
- Provide opportunities for questioning and making conjectures
- Provide opportunities for justification
- Provide students with the use of problem solving approach
- Provide students with integration of other contents
- Provide students with calculators and computers
- Provide students the opportunities to facilitate their learning
- Provide assessment of learning
- Provide opportunities for students to write about mathematics

Appendix J
Reducing Mathematics Anxiety

- Keep calm, even when feeling anxious or intimidated. Breathe slowly before working the math problems.
- Stop negative self-talk.
- Visualize success with solving math problems.
- Know and understand your learning style.
- Review your math lessons.
- Break down challenging math problems (Brush, 1981, Arem, 2003, Alkan, 2013).

Appendix K
Pre and Post Assessment: Linear Functions

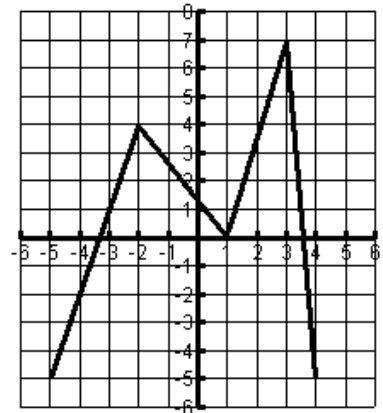
Multiple Choice: Choose the best answer for each question.

3 pts each

Use the following graph for problems 1.

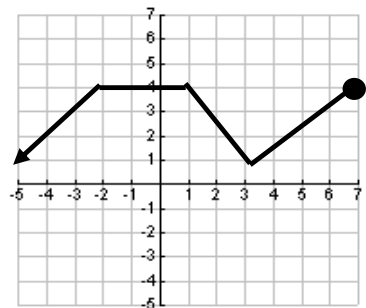
_____ 1. Use the graph above to find the rate of change from $x = -2$ to $x = 1$?

- A. $-\frac{4}{3}$ B. 3
C. -3 D. undefined



_____ 2. On which interval is the graphed portion of the function **decreasing**?

- A. $1 < x < 3$
B. $-\infty < x < -2$
C. $3 < x < 7$
D. $-\infty < x < \infty$ (all reals)

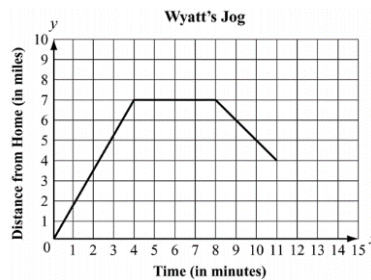


3. Find the 40th term of the sequence 6, 15, 24, 33, ...

- A. 348
B. 357

- C. 366
D. 339
-

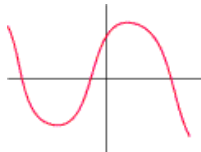
4. Which statement best describes what is being modeled by the graph?



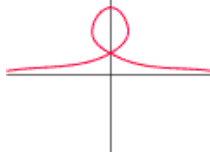
- A. Wyatt started from a standstill, gradually picked up speed, jogged at a constant rate for 4 minutes, gradually slowed down and stopped.
B. Wyatt began jogging at a constant rate and increased his pace steadily until coming to a complete stop after jogging 11 minutes.
C. Wyatt jogged at a steady pace for 4 minutes, took a 4 minute break, walked at a steady pace for 3 minutes, and stopped.
D. Wyatt jogged uphill for 4 minutes, jogged on a flat surface for 4 minutes, jogged downhill for 3 minutes, and then stopped.
-

5. Which of the following graphs is a function?

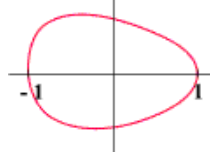
A.



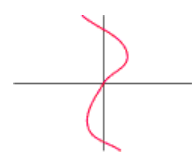
B.



C.



D.



Free Response Questions: Show all work!

4 pts each

6. Given the functions: $g(x) = 9 - \frac{3}{5}x$, evaluate $g(12)$.
7. For the given function $f(x) = -\frac{5}{3}x + 5$ identify the following:

Slope (m): _____ X intercepts: _____ Y intercepts: _____

2 pts each

Bryce is selling coupon books for their club fundraiser. Bryce has a goal of selling 4 per day.

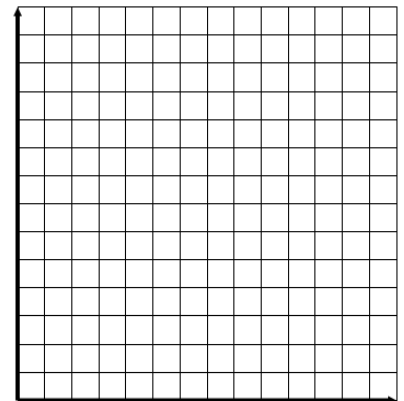
8. Write a function that represents the number of coupon books sold in terms of number of day (x).

a. Bryce: $b(x) =$

b. Graph and label the function.

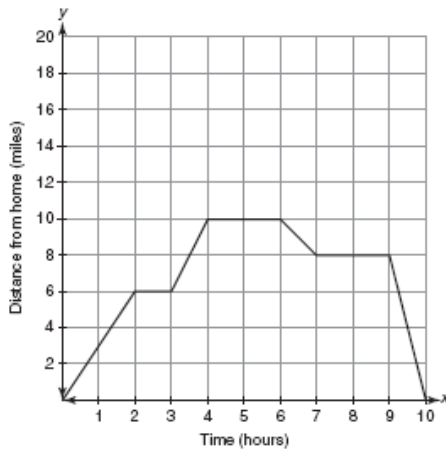
c. What is the rate in which Bryce sells the books?

d. How many books would Bryce have sold on the 12th day?



3 pts each

9. The graph below represents distance that a bird is from his nest during a 10-hour period.



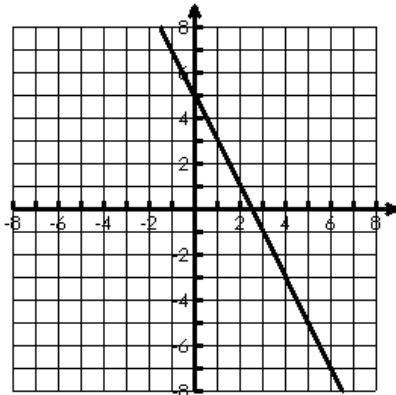
a. What is his average rate of change from hours 2 to 6?

b. How many stops did the bird make?

Analyzing Functions

1 pt each

10. Given the following functions, describe the characteristics.



a. Domain: _____

b. Range: _____

c. x-int/y-int: _____

d. Increasing/Decreasing:

e. Rate of Change from $x = -1$ to $x = 4$

Transforming Functions**3 pts each**

11. Write a function that is shifted 2 units up from the function $f(x) = x + 3$

Sequences**3 pts each**

Write an explicit formula for the following sequence.

12. 8, 12, 16, 20, ...

Appendix L
Pre and Post Assessment: Exponential Functions

Multiple Choice: Choose the best answer for each question. 3 pts each

_____ 1. State the **domain** for the function to the right.

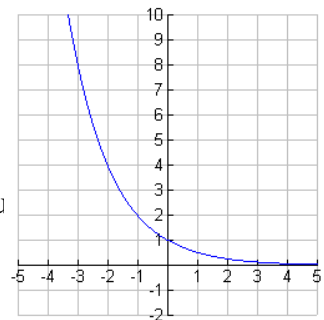
A. $0 \leq y < \infty$

B. $-3.5 < x < 4$

C. $0 \leq x < \infty$

D. All Real Nu

E.



_____ 2. The explicit formula for a geometric sequence is $a_n = 3(-2)^{n-1}$. What is the fifth term of the sequence?

A. -96

B. 48

C. 19

D. -48

_____ 3. What is the **y-intercept** of $f(x) = -4\left(\frac{3}{2}\right)^x$?

A. (0, -6)

B. (0, 1.5)

C. (0, 0)

D. (0, -4)

_____ 4. How would you transform the graph of $y = 1.4^x$ to produce $f(x) = -1.4^x$?

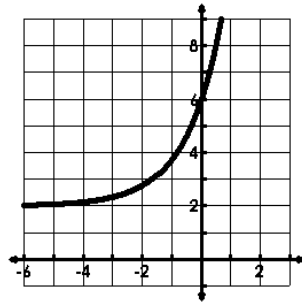
- A. Reflect over the line $y = x$ C. Reflect over the x – axis
B. Reflect over the line $x = 0$ D. Reflect over the y – axis
-

_____ 1. The value (in millions of dollars) of a large company is modeled by: $y = 241(1.04)^x$.
What is the projected annual percent of growth and what is the initial value?

- A. 10.4%; \$241 million C. 241%; \$4 million
B. 2.41%; \$104 million D. 4%; \$241 million
-

_____ 2. Which function is shown by the graph?

- A. $y = 2(2.3)^x - 2$
B. $y = 4(2.3)^x + 2$
C. $y = 4(2.3)^x$
D. $y = 5(2.3)^x - 3$



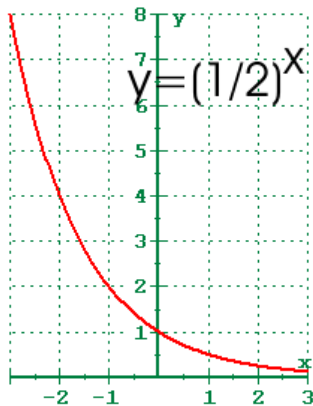
_____ 3. Which models show are exponential decay models?

- i. $y = (0.032)^x$ ii. $y = (1.01)^{x+3}$ iii. $y = (3.22)^x$ iv. $y = (1-0.12)^x$

- A. I and II
B. I and IV
C. II and III
D. III and IV

Free Response Questions: Show all work!

4 pts eachGiven the functions: $f(x) = 3^x$ and $g(x) = 2^{x+1}$ 8. Find $f(2)$ _____9. Find $g(3)$ _____

10. Use the graph and table to answer the following questions.**2 pts each**

X	$g(x)$
-2	6
-1	4
0	2
1	0
2	-2

a. $f(-2) =$ _____b. $g(2) =$ _____c. $x =$ _____, if $f(x) = 2$ d. $x =$ _____, if $g(x) = 0$

e. Would the two functions ever intersect? _____ If yes,

Where? _____

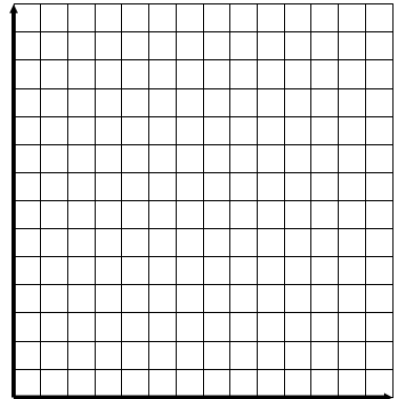
2 pts each

Bryce and Amelia are having a contest to see who can sell the most number of coupon books for their club fundraiser. Bryce has a goal of selling 4 per day. Amelia plans to sell 2 the first day, 4 the 2nd day, 8 the 3rd day, and so on.

11. Write a function that represents the number of coupon books sold in terms of number of day (x).

i. Bryce: $b(x) =$

ii. Amelia: $a(x) =$



c. Graph each function labeling the two functions.

d. Where do they intersect?

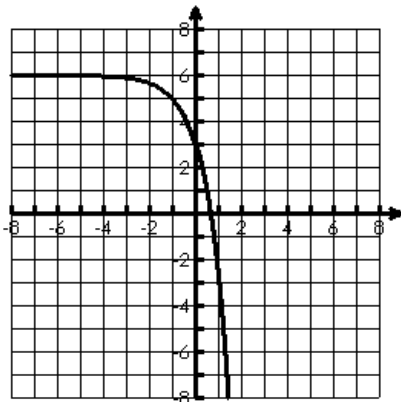
e. What does the intersection mean?

f. When will Bryce have sold more books? When will Amelia have sold more books?

Analyzing Functions

1 pt each

12. Given the following functions, describe the characteristics.



a. Domain: _____

b. Range: _____

c. x-int/y-int: _____

d. Increasing/Decreasing:

e. Rate of Change from $x = -1$ to $x = 1$

Transforming Functions**3 pts each**

Given the following functions, describe at least three transformations for each.

13. $y = \frac{1}{3}(4)^{x+6} + 8$ _____, _____, _____

14. $y = -7\left(\frac{2}{9}\right)^{x-5}$ _____, _____, _____

Sequences**3 pts each**

Write an explicit formula for each of the following sequences.

15. 4, 12, 36, 108, ...

4 pts each

16. The student population in a high school increases by 3% a year. When it opened, the school had 1440 students.

a. Write a formula that models this situation.

b. How many students will there be in 5 years?

17. A new car has a value of \$35,000 and depreciates by 12% a year.

a. Write a formula that models this situation.

b. What will be its value?

Appendix M

Teacher Interview Protocol

Pre Instruction

1. Describe your experience as teacher; how long have you been in the classroom, what courses have you taught, etc.?
2. What is your experience with teaching mathematics to ELLs?
3. What training have you had using manipulatives to teach mathematics?
4. Describe your prior experiences using manipulatives of any kind to teach mathematics?
5. Describe your prior experiences using virtual manipulatives to teach mathematics?
6. Describe your prior experiences using concrete manipulatives to teach mathematics?
7. What are some barriers you might see with teaching mathematics with concrete manipulatives?
8. What are some barriers you might see with teaching mathematics with virtual manipulatives?
9. How have you taught ELLs linear and exponential functions in the past?
10. What difficulties do ELLs have as they explore linear and exponential functions?
11. What is your opinion of the overall effect of teaching mathematics with manipulatives?
12. What have you found to be rewarding in teaching mathematics to ELLs?
13. What have you found to be the most frustrating in teaching mathematics to ELLs?
14. What do you anticipate the outcome will be of using manipulatives in the classroom?
15. What do you sense are ELL students' attitudes toward mathematics?
16. On a typical teaching day, what might I see happening in your classroom?
17. Is there anything else you would like to add?

Post Instruction

1. Tell me how you integrate technology in your mathematics class on a typical day.
2. What aspect did you find the most beneficial in teaching mathematics using concrete manipulatives?
3. What aspect did you find the most frustrating teaching mathematics using concrete manipulatives?
4. Which manipulatives did you find most beneficial?
5. What benefits do you think ELLs obtained by completing this unit using manipulatives?
6. What benefits do you think ELLs obtained by completing this unit with concrete manipulatives?

7. Did you have any problems refraining from using the manipulatives with the control group?
8. What aspect did you find the most frustrating teaching mathematics using manipulatives?
9. What are some attitudes ELL students' experienced while using manipulatives?
10. What is your opinion of the overall effect of teaching mathematics with manipulatives?
11. What type of manipulative, virtual or concrete impact ELLs mathematical thinking the most?
12. Which manipulative was most help for the ELL to learn algebraic concepts?
13. In our original interview, you discussed _____ as a perceived barrier to using concrete and virtual manipulatives. How has your opinion changed?
14. How have concrete and virtual manipulatives helped students improve conceptual and abstract understanding of linear and exponential functions?
15. Next semester, if you are teaching linear and exponential functions what method would you choose?
16. Is there anything else you would like to add?

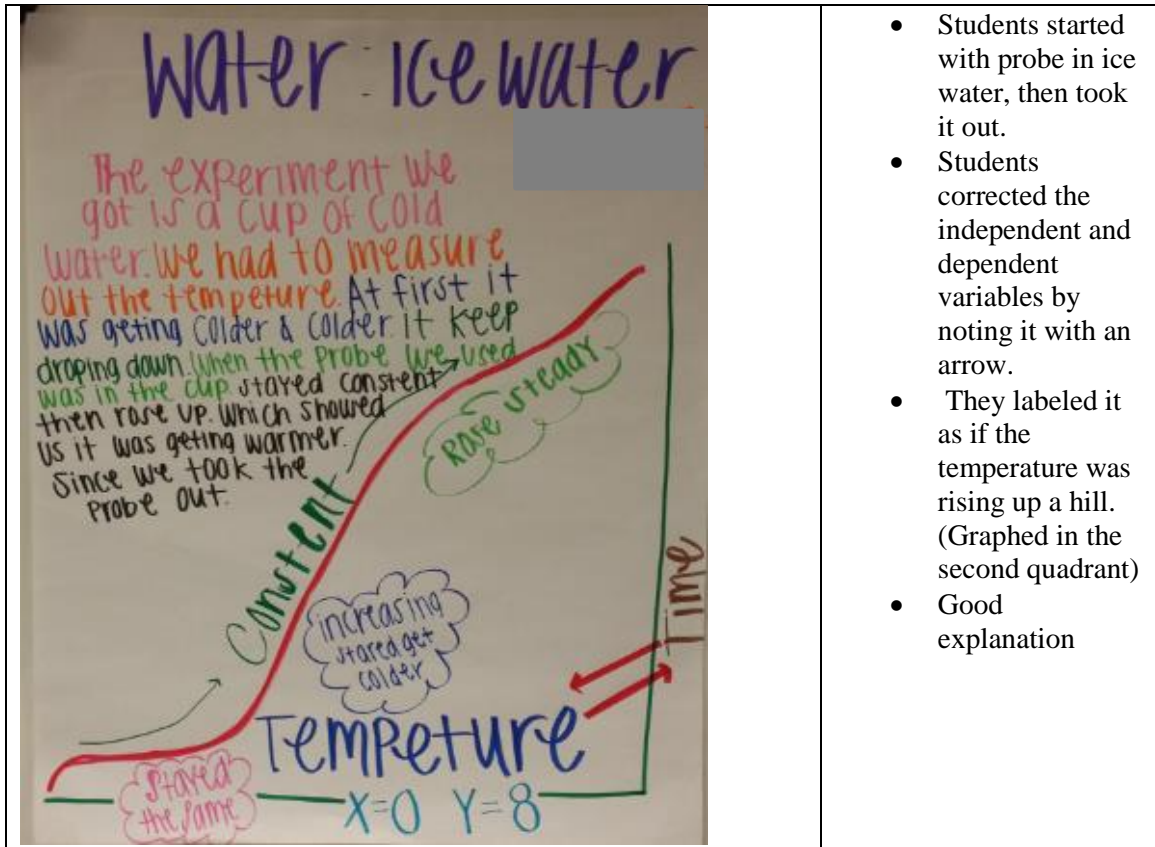
Appendix N SIOP Components

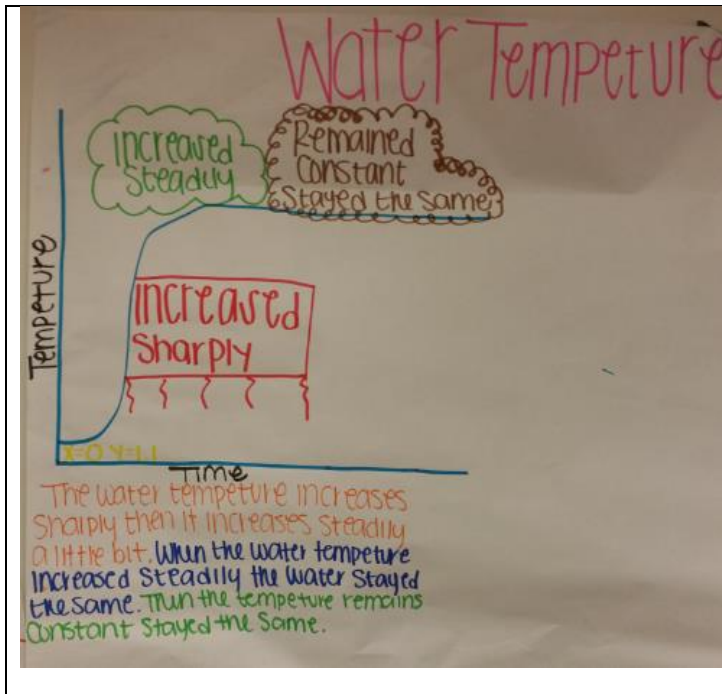
Table N1

SIOP Components (Echevarria et al., 2008)

<u>Component</u>	<u>Focus</u>
Lesson Preparation	Demonstrates planning and organization, together with the appropriate language and subject objectives, and implementing supplemental materials and activities that are impactful to the student.
Building Background	Use the students' prior knowledge to teach them new material in way that they best understand. Link information across disciplines so that it retainable and is relatable to each student.
Comprehensible Input	The teacher must be willing to alter the way the delivery information to the students. This could include: The manner of speech and willingness to slow, repeat, or change examples.
Strategies	Shines a light on the strategies that teachers use to teach their students, this should be done in way that allows the students to develop their critical thinking skills.
Interaction	Encourage the students to build vocabulary so that are able to do express their ideas and demonstrate their comprehension of the studied material. Place the students in groups so they can communicate amongst each other improving language and content development.
Practice and Application	Provide activities to reinforce the students' knowledge.
Lesson Delivery	Ensures that the lesson moves at an appropriate pace for the students while meeting all of the criteria and that the teacher delivers a quality lesson to the students.
Review and Assessment	At the end of each lesson the teacher should assess the students on the language and key concepts presented on material. After the assessment the teacher should provide the student with feedback to make sure that they have a full understanding of the criteria.

Appendix O
Activities of Graphing Relations Stories

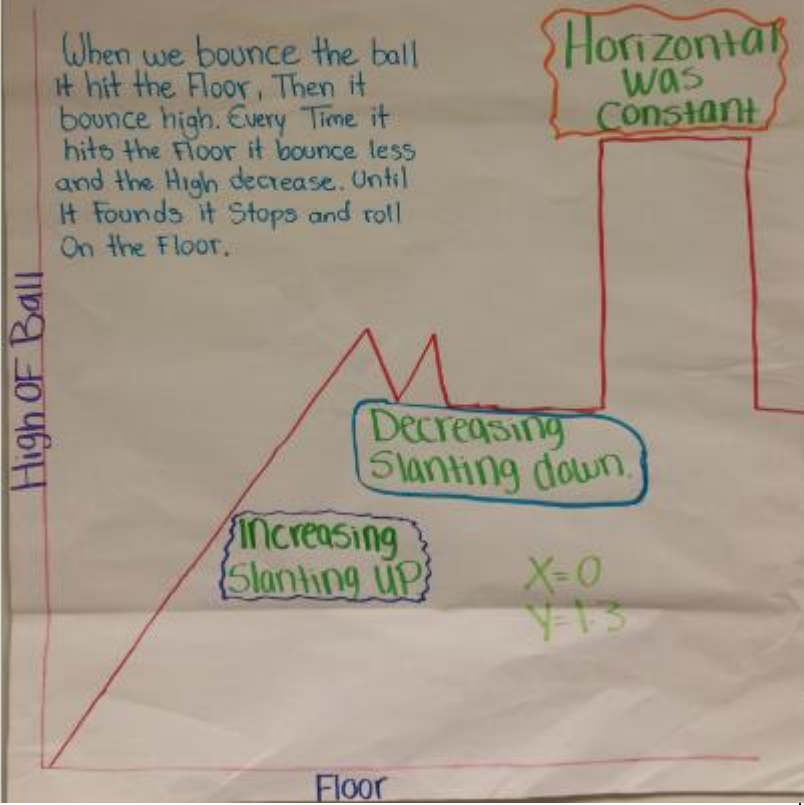




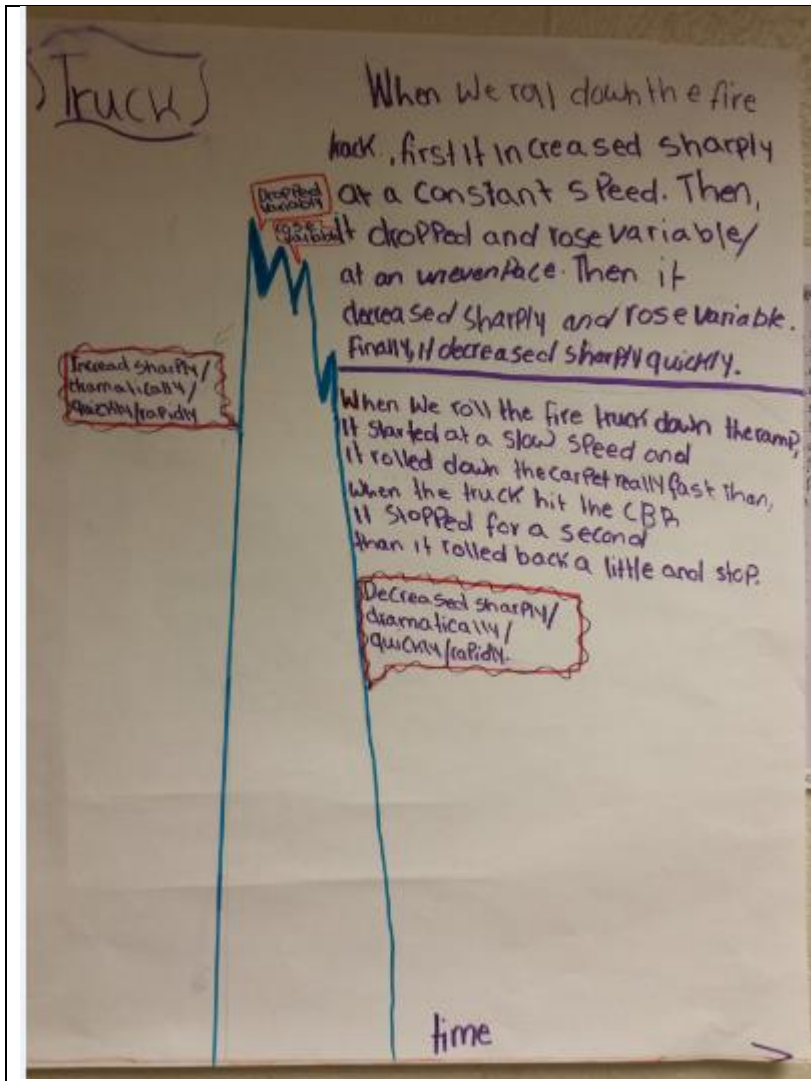
- Inserted probe into ice water.
- Took probe out.
- Labeled the axis, x (time) and y (temperature). Missing the units.
- Vocabulary slope, increased, steadily, and constant

Ball Bounce

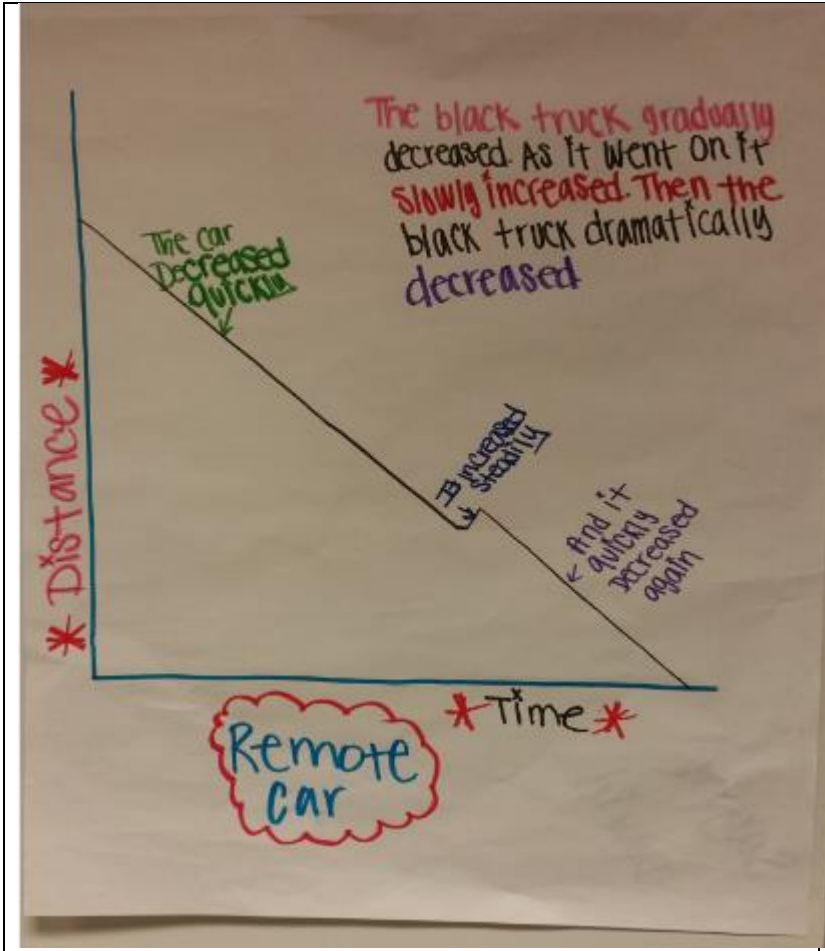
When we bounce the ball it hit the Floor, Then it bounce high. Every Time it hits the Floor it bounce less and the High decrease. Until It Finds it Stops and roll On the Floor.



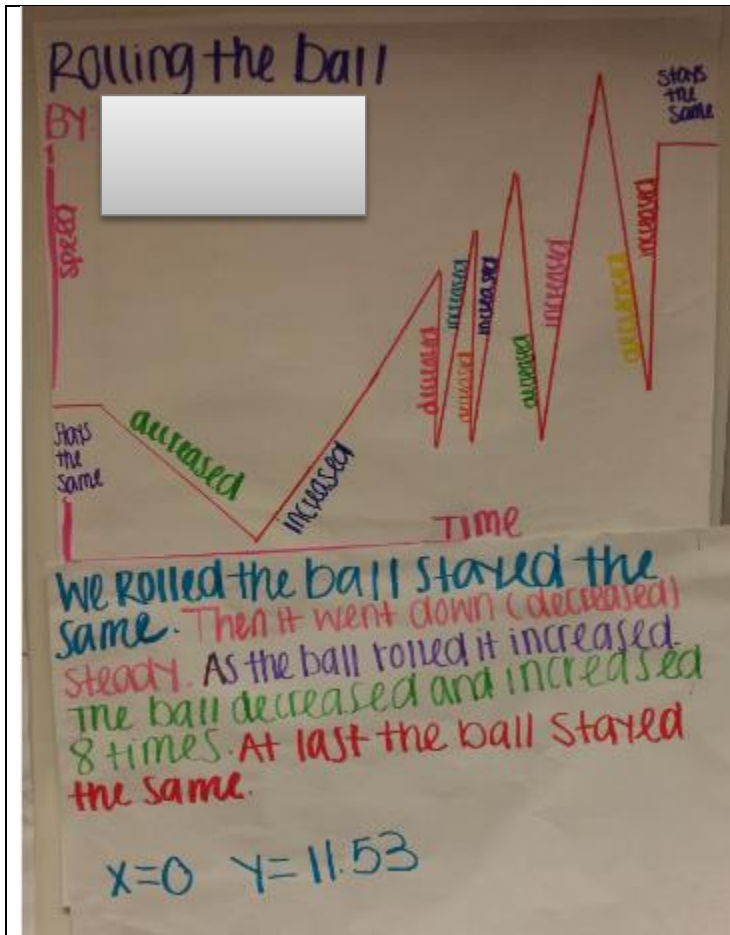
- Incorrect label of x axis time (floor).
- The y axis is correctly labeled for height.
- Initial high not recorded.



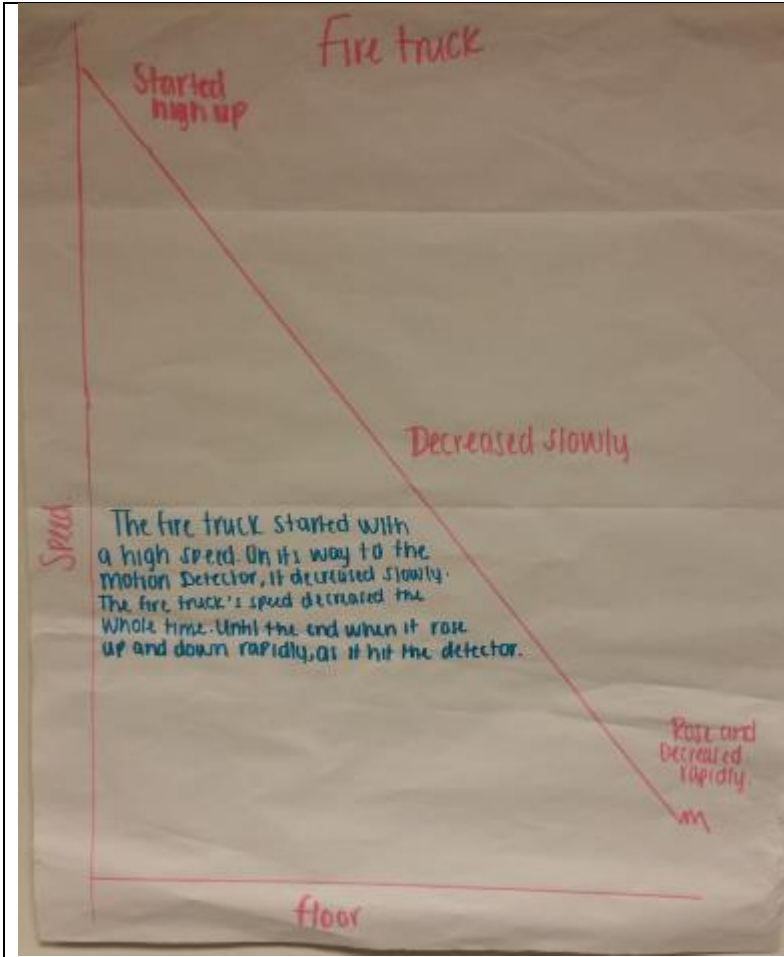
- Rolled fire truck down the ramp.
- Did not label the y axis (distance)



- Drove the truck toward motion detector
- Label axis correctly



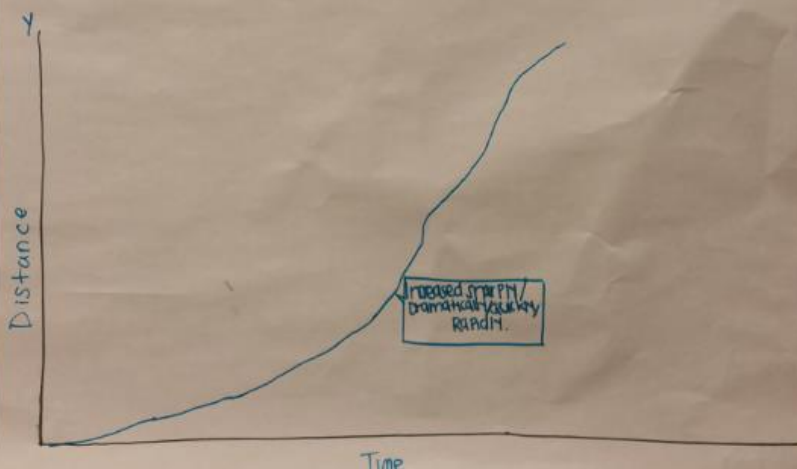
- Rolled the ball away from the motion detector.
- Labeled y axis incorrectly (speed) distance.
- Confused speed (rate of change)
- Initially holding the ball. Pushed the ball



- Distance away from the motion detector versus the time it took to travel the distance. The student pushed the fire truck at the start of the run.
- Incorrectly labeled x (time) and y (distance)
- Increasing constant rate of change.
- Student started the truck out with push, in which case her synopsis is correct.

The Walking

when the boy started walking the numbers started to increase rapidly. The more he walk the most it increased. Then when he stopped the numbers either decrease or stopped. If the boy would have walked faster the numbers would have increase more rapidly. But, if he walked slowly the numbers would have slowly increase but not a lot.



- Walking away from the motion detector
- Distance (Y)
- Time (X).
- The relationship between distance and time. Students discovered the rate of change (slope) is velocity.
- Numbers increase (Distance) The more he walked the most the distance increased.
- When he stopped the distance remained constant.
- If he walked faster his distance would have increased faster.
- If he walked slower the distance would have increased but at a slower rate.