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The Relationship Between Middle School Students' Mathematical Vocabulary and Their Achievements in Mathematics: A Mixed Method Study

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THE RELATIONSHIP BETWEEN MIDDLE SCHOOL STUDENTS' MATHEMATICAL VOCABULARY AND THEIR ACHIEVEMENT IN MATHEMATICS

A MIXED METHOD STUDY

by

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DEDICATION

I would I would like to thank my Lord and Savior Jesus Christ for all my praises, honor and glory shall go to you. Thank you Father God.

I dedicate this dissertation to my family. To my mother and father (Arlene & Joseph), who have loved, supported, and believed in me regardless of my mistakes. To my beloved husband Velton, who helped me begin this journey and has held my hand throughout the course of it, I am eternally grateful to you. To my beautiful daughter, Niya, you are my inspiration and my joy, I am so proud to be your mother. I love all of you and I am so blessed to call you my family.

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ABSTRACT

THE RELATIONSHIP BETWEEN MIDDLE SCHOOL STUDENTS' MATHEMATICAL VOCABULARY AND THEIR ACHIEVEMENT IN MATHEMATICS

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In 2000, National Council of Teachers of Mathematics (NCTM) included communication in the standards encouraging students to develop their mathematical language to sufficiently and accurately explain their ideas through discourse. For years, there has been a growing movement for students to attain abilities to articulate problem-solving methods utilizing mathematics vocabulary (Pierce & Fontaine, 2009).

In this study, a mixed method design was utilized to examine the relationship between middle school students' understanding of mathematics vocabulary and their success in mathematics. The quantitative study was conducted to determine if there is a correlation between eighth grade mathematics vocabulary acquisition and students' achievement on Georgia's Criterion Referenced Competency Test (CRCT). Using Ericsson and Simon's (1980) think-aloud protocol, the qualitative study was conducted to examine whether conceptual understanding of mathematics vocabulary impacts students' ability to problem-solve. The results from both studies indicated an association between students' acquisition of mathematics vocabulary, student achievement, and their problem-solving abilities.

Keywords: Procedural/Conceptual Understanding, Discourse, Problem-Solving

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CHAPTER ONE

INTRODUCTION

Nearly fourteen years ago, I decided to leave my employment in the private business sector and return to college to pursue my life's ambition of becoming an educator. My goal was to change students' lives and the direction of education. I began my new career as a Pre-kindergarten teacher, later deciding to teach mathematics to middle school students. The shift from teaching prekindergarten students to students at the middle school level was challenging. Yet, I was determined to make mathematics exciting to the least engaged student and change his or her view of the unpopular subject.

However, within a few days of school starting, reality settled in. I welcomed my first class of middle school students who came from a variety of different backgrounds and held a range of philosophies about education. Once the bell rang, I was instantly the center of attention. I felt as though every student was carefully scrutinizing me and determining my ultimate purpose as their new mathematics teacher. Needless to say, much of what I learned in school went out of my mind. I was standing in front of a classroom full of impressionable young people who were counting on me to provide them with security, comfort, and a quality education.

Consequently, my focus shifted because I knew that my students required more from me than just the ability to teach mathematics. Hence, my role as a teacher broadened. I was now a counselor, role model, mediator, facilitator, and collaborator. Embracing my new roles, I worked tirelessly to get my students to excel in mathematics.

I practiced the then state-mandated Quality Core Curriculum in hopes I could curb the downward trend of poor mathematics performance based on the previous annual scores produced by the students.

Interestingly, with every implementation and application of the state's mandated curriculum, my students' understanding of mathematics did not appear to grow. Each passing year, I would receive a new population of students who did not show any more interest in learning mathematics than students from the previous years. The excitement for learning mathematics was the same and the expectation on how to learn mathematics was troubling. Students' expectation regarding their work was minimal while they expected me to do all of the work as they watched and attempted to mimic or replicate my efforts.

Thankfully, the performance standards changed the classroom. Students were now expected to do the work while the teacher facilitates the learning. It took years for the students' dispositions and educators' philosophies to change from a teacher-led to a student-centered approach. I expected that once the new curriculum was fully implemented, students' mathematical comprehension would increase making mathematics class more informative and exciting.

Upon implementation of the performance standards, the excitement of learning mathematics did increase. Students became more interested in participating in mathematical discussions including students who were less than enthusiastic about the subject. Yet, the combination of typical classroom discourse and increased excitement still did not reveal one of the possible underlying issues of their marginal success in mathematics. Students' learning remained static, possibly due to their inability to make connections from one lesson to the next. I would have to constantly review material and use valuable time explaining content to the students that I had presumed they already understood.

This mystery continued for several months until one of my students (who was completing his part of a group assignment) asked a simple yet important question. “What does *mean* mean?” Thinking that he was joking, I paid little attention to this very valid question or the basis for it. I returned to the student moments later wondering why he had not completed his portion of the assignment for his group. He stated while everyone was listening “I’ve asked my group what does *mean* mean and no one knows the answer.” Obviously frustrated, I asked a series of questions that lead them to answering their own question but then I posed a question to the group “why don’t you know the definition?” With a blank expression, the original student replied, “I don’t know the meaning of many of the words you say in class. I think that is why I get so confused and why I don’t get math sometimes.” Intently listening, the group nodded in unison. This single incident profoundly changed my entire perspective on teaching mathematics. Needless to say, it was that moment when I knew vocabulary was a necessity for students to conceptually understand mathematics.

Purpose of the Study

The purpose of this study is to determine if there is a relationship between mathematics vocabulary and students’ understanding of mathematics. Lager (2006) notes mathematics is no longer viewed as a universal language that is based primarily on mathematical concepts. Contrarily, mathematics is a language dependent subject because it can serve dual roles as a means of representation and as a means of communication (Lager, 2006). The study will be guided by the following research questions:

- Is there a correlation between the acquisition of mathematics vocabulary and students' achievement in mathematics?
- How does conceptual understanding of mathematics vocabulary impact students' ability to problem solve?

Significance of the Study

Mathematics is a language with its own unique characteristics. This language must be taught and frequently used for students to understand it (Usiskin, 1996). The information from this study may increase the importance of vocabulary instruction in mathematics class and hence effect the way mathematics is taught. Thus, as students begin to develop mathematical reasoning they need to also develop a balance of conceptual and procedural understanding of mathematics vocabulary terms. This involves students learning new vocabulary terms they may not ordinarily use in their daily discourse (Capraro and Joffrion, 2006).

The significance of this study is the insight it can provide on how mathematics vocabulary and students' discourse may increase students' ability to interpret and solve mathematical problems while developing their conceptual understanding of mathematics.

Limitations

The following is a list of possible limitations:

1) The study will be limited to one NW middle school in Georgia.
2) The study will be limited to 8^{th} grade participants only.
3) The entire population of 8^{th} grade students may not participate in the study.
4) The timing of the administration of the vocabulary assessment and the limited time frame permitted for conducting the interviews were not optimal.

Definition of Terms

Affect – A biological response that manifests itself physically in response to various stimuli (Holinger, 2009).

Commognition Theory – A theory (and coined term) created by Anna Sfard who states that the origins of thinking are mired within the realm of other human capacities (Sfard, 2007).

Conceptual understanding – A mental link or association between procedures, ideas or concepts and mathematical facts (Brownell, 1935; Davis, 1984; Hiebert & Carpenter, 1992; Hiebert & Grouws, 2007).

Mathematical Discourse – The ability to communicate mathematically with others as well as with oneself (Sfard, 2007).

Procedural understanding – Algorithms and mnemonic strategies used to complete a series of sequential actions (Byrnes, 1992).

Sociocultural Theory – A theory largely inspired by the work of Lev Vygotsky (and other notable psychologists) who focused on the process of how people develop forms of reasoning while participating in traditional cultural practices (Cobb, 2007).

CHAPTER TWO

THEORETICAL FRAMEWORK & REVIEW OF LITERATURE

Pierce and Fontaine (2009) state that language is increasingly playing a vital role in learning mathematics. Consequently, there is a growing movement for students to attain abilities to articulate problem-solving methods utilizing mathematics vocabulary to coherently express their ideas. It is gradually becoming more evident that mathematics vocabulary is vitally relevant and important in influencing students' level of success (Pierce & Fontaine, 2009).

Thompson and Rubenstein (2000) state that in order for students to master the unique language of mathematics, students would have to read, recognize, understand and verbally participate in mathematics discussions. Those students who readily misuse vocabulary terms and ignore the discernable differences of word meanings may often contribute to the many reasons they have problems with mathematics (Thompson & Rubenstein, 2000). Consequently, it is the ambiguous nature of mathematics terminology that confuses students from comprehensively understanding and learning the subject (Pierce & Fontaine, 2009). Monroe and Orme (2002) state that vocabulary acquisition is an important component in learning the language of mathematics because of the vital connection of unfamiliar words with mathematical literacy. Likewise, Lee and Herner-Patnode (2007) found that students' conceptual understanding of mathematics is closely aligned with their vocabulary comprehension.

Theoretical Framework

Theoretical frameworks provide a variety of perspectives for researchers to examine and possibly utilize in shaping their own research (Izsak, 2005). The objective for educational researchers is to properly identify who or what they will be researching (i.e. individuals, lessons, district) and associate the appropriate theoretical perspective with their topic (Simon, 2009). For this study, the researcher associated sociocultural, commognition and affect as the three theoretical theories or frameworks to shape the research. Sociocultural theory was the framework utilized to examine conceptual understanding; commognition theory was the framework for discourse and/or vocabulary acquisition, and affect theory was the framework utilized for problem solving.

Sociocultural Theory

Sociocultural theory focuses on the process of how people develop forms of reasoning while participating in traditional cultural practices (Cobb, 2007). Cobb (2007) states that the work of Lev S. Vygotsky and the works of other psychologists, notably Soviet psychologist Alexei Leont’ev who continued Vygotsky’s work after his untimely death, largely inspired sociocultural theory. Vygotsky was a Russian psychologist whose work was strongly influenced by the writings of Karl Marx. Vygotsky believed that language, writing, and counting techniques are not necessarily newly acquired attributes with each passing generation; contrarily, it is intelligence that’s the inherited trait and is culturally passed from generation to generation (Cobb, 2007). The empirical research conducted by Lave (1988), Saxe (1991), Carraher, Carraher, and Schliemann (1985), and Scribner (1984) support Vygotsky’s perspective demonstrating that students are indeed highly influenced by their cultural practices particularly when engaging in individual arithmetical activities (Cobb, 1994).

According to Steele (2001), Vygotsky believed that the cultural development of child's word meaning is first processed on a social level and subsequently on an individual level (Steele, 2001; Vygotsky, 1962). Steele (2001) states that Lev Vygotsky understood that students possess some mathematical language; however, students' mathematical understanding is created when they are better able to connect new mathematical terminology with previous learning experiences. This belief moreover suggested that word meanings cannot be directly taught from one student to another because neither students nor teachers can successfully provide their own constructed understanding to another person (Sierpinska, 1998; Steele, 2001). Furthermore, Steele (2001) states that Vygotsky believed that the more students utilize and internalize new mathematical words in the presence of a more knowledgeable person, the more likely they will find themselves in the zone of proximal development (ZPD) for learning. Zone of proximal development is a place where learning occurs between a students' present understanding and their potential understanding. Once a student enters ZPD, they become more willing participants in their learning or co-participants one day and independent learners the next day. Moreover, the teacher is substantially able to increase the student's learning ability by amplifying on their current knowledge base when students are in ZPD. This permits the student to acquire and develop additional culturally established concepts while fostering their own inherent conceptual understanding (Steele, 2001; Vygotsky, 1978).

Similar to Vygotsky's vision, philosopher Lugwig Wittgenstein also suggested that culture is embedded in the study of mathematics (Knott, 2010). Knott (2010) states that Wittgenstein noted that mathematics was a different language that was systematically interwoven into our daily language usage. He described language and cultural skills as central components in learning or teaching mathematics. Wittgenstein believed in the importance of discourse in the mathematics classroom. Knott asserts that current mathematical research has illustrated that teachers who facilitate discussions regarding thought-provoking problems increases students' reasoning abilities (Knott, 2010).

Like Knott, published author Anna Sfard (2008) respects Vygotsky's and Wittgenstein's work. Her allegiance is briefly captured in the following passage she wrote in one of her books:

> .….Lev Vygotsky and Ludwig Wittgenstein, two giants whose shoulders proved wide enough to accommodate legions of followers and a wide variety of interpreters. Although libraries have already been filled with exegetic treatises, the Byelorussian psychologist and the Austrian-born philosopher continue to inspire new ideas even as I am writing these lines. (Sfard, 2008, p. 435).

Sfard continues her positive assertion by stating that Vygotsky and Wittgenstein had a significant impact on her thinking (Sfard, 2008). However, Sfard (2007) also believes that there were additional factors for human development and the subsequent transition from a participatory individual to an acquisitionist of mathematical knowledge (Sfard, 2007).

Commognition Theory

Anna Sfard (2001) suggests that traditional research practices regarding mathematical thinking (i.e. students' misconceptions) has prompted a reevaluation of teaching and learning mathematics. Sfard (2007) believes that the evolution of a student's discourse is a critical component and an underlying principle for learning mathematics. Sfard defines mathematical discourse as the ability to communicate mathematically with others as well as with oneself. Noted in her previous studies, Sfard asserts that effective mathematics communication occurs when all of the participants feel confident in their mathematical discourse using the same vocabulary terms when referencing the same topic (Sfard, 2007).

In 2000, the National Council of Teachers of Mathematics (NCTM, 2000) included communication in the standards encouraging students to develop a language to sufficiently and accurately explain their mathematical ideas through discourse (NCTM, 2000). Pierce and Fontaine (2009) stated that the language requirement, established by NCTM, is due in large part to the high-stakes benchmarks, which are administered in many states throughout the country. Students are expected to possess a sound conceptual understanding of complex word problems irrespective that many of the words used in the benchmark assessments are uniquely specific to mathematics. Consequently, students' success in mathematics is profoundly based on their foundation of mathematics vocabulary and the depth of their previous mathematics vocabulary experience (Pierce & Fontaine, 2009).

Sfard (2001) suggests that communication in the mathematics classroom is the missing component for students' comprehension. She states that when teachers engage students using mathematical discourse, then students' learning will increase possibly changing the educators' teaching practice (Sfard, 2001). Sfard (2007) regards thinking as a method of communication because as individuals we ask ourselves questions, make arguments, update ourselves of new information to ultimately seek our own answers. Sfard states that communication is not a residual benefit to thinking but rather a critical element to thinking itself. Based on this principle, Sfard decided to merge the words cognitive and communication together to coin a new term *commognition*. Sfard's commognition theory assumes that origins of thinking are mired within the realm of other human capacities. Thus, thinking emerges as an inherently individual activity, which can be regarded as a form of interpersonal communication. Sfard utilizes the commognitive theoretical framework to explain the importance and application of discourse in the mathematics classroom (Sfard, 2007).

In a review of Sfard's work, Felton and Nathan (2008) state that Sfard's commognition paradigm addressed topics that have been central to mathematics education and cognition such as the development of numerical thinking, concerns about transfer and the process of separating from arithmetic to algebra. Sfard examines how students can learn counting routines in one situation successfully but are unable to transfer the same knowledge to another but similar situation. She concludes that children are unable to change or associate their numerical learning from an independent learning experience to a social interaction. Sfard's commognition framework offers the potential of uniting dissimilar concepts (thinking and communication) by edifying mathematics education with a new and promising agenda (Felton & Nathan, 2008).

Affect Theory

Silvan Tomkins was an American psychologist who was renowned for his work in affect particularly noted in his books *Affect Imagery Consciousness* (Holinger, 2009). Tomkins was regarded as the "founder of modern affect theory" (Basch, 1991, p. 296); he defined affect as a biological response that manifests itself physically in response to various stimuli (Holinger, 2009). Shmurak (2006) notes that Tomkins believed that emotion was an important component of life and he wanted the science of psychology to further examine the study of emotions. His opportunity to study emotions was realized when his son was born. Tomkins observed his infant son's emotions for hours, which lead him to conclude there was a need to further examine and reassess emotions. During his observation, he noted that his son displayed a range of innate patterns (six negative, two positive and one neutral), which he later described as affect (Shmurak, 2006).

Similarly, OP't Eynde, DeCorte, & Verschaffel (2006) note that there are several scholars who have cultivated a collection of theoretical perspectives about affect and its role in learning mathematics (Evans, 2000; Goldin, 2002; Hannula, 2002; Malmivuori, 2001; OP't Eynde, DeCorte, & Verschaffel, 2006). According to OP't Eynde et al. (2006), the interaction between students' behavior in the classroom and their identity or personal relationship with learning is associated with their mathematical understanding. Based on this perspective, students' emotional and affective processes are vital elements for understanding how some students' problem solve and learn mathematics (OP't Eynde et al., 2006). McLeod (1988) note that students are emotional when solving mathematical problems; however, inexperienced problem solvers emotions are more intense. The gamut of feelings from frustration, joy and even panic are typical emotions students possess when they are performing problem-solving tasks (Buxton, 1981; Confrey, 1984; McLeod, 1988). OP't Eynde et al. (2006) state that frustration is an emotion that might signal that a student is motivated in achieving some level of success. However, in many cases, students are unaware of their feelings and McLeod (1988) suggests that educators be prepared to assist students with coping strategies to help with them deal with their feelings. Thus, it is noteworthy and critically important to state that students may experience either positive or negative emotions when engaged in problem solving activities (Mason, Burton, & Stacey, 1982; McLeod, 1988).

Philipp (2007) states in a NCTM journal that while the 1990's phenomena of the sociocultural and participatory frameworks were being established, theoreticians were already advancing research on psychological theories to explain the complexities that are associated with learning. Describing the relationship of affect and achievement as an important aspect in mathematics education, Philipp believes that complex conceptual and procedural problem solving tasks can somewhat predict students' achievement and attitudes towards mathematics. Concluding that there should be more research conducted on affect and how it should be measured (Philipp, 2007).

Review of Literature

Conceptual Understanding

Stylianides and Stylianides (2007) collectively state that learning with understanding is increasingly receiving critical attention from educators, school administrators, and psychologists, which has substantially elevated it as one of the important goals for all subjects. The realization of this goal has been perceived as somewhat problematic, especially in the domain of mathematics. While the vision of students learning mathematics with acquired understanding has often appeared in curriculum guidelines, the implementation of the vision has been mediocre (Stylianides & Stylianides, 2007). In the NCTM (1989) the following was written about conceptual understanding:

> A conceptual approach enables children to acquire clear and stable concepts by constructing meaning in the context of physical situations and allows mathematical abstractions to emerge from empirical experience. A strong conceptual framework also provides anchoring for full skill acquisition. Skills can be acquired in ways that make sense to children and in ways that result in more effective learning. (NCTM, 1989, pg. 17).

The goal of improving mathematical skills and providing explanations has been the focus for the NCTM for several years (NCTM, 1989).

In the NCTM (2007), Hiebert and Grouws define conceptual understanding as making more enriched mental link or association between procedures, ideas, and mathematical facts (Brownell, 1935; Davis, 1984; Hiebert & Carpenter, 1992). Byrnes (1992) describes conceptual understanding as a mental connection that has a relational link to multiple entities and procedural knowledge as algorithms and mnemonic strategies used to complete a series of sequential actions. Hiebert and Lefevre (1986) characterize conceptual understanding as knowledge that has many links to previous networks whereas procedural knowledge is the implementation of rules or the completion of a task.

In 2014, an article published in the NCTM suggests that conceptual understanding is a necessary component for procedural fluency. Students should acquire conceptual understanding prior to and concurrently with procedural knowledge. This is largely due to students losing interest for the reasoning behind the mathematics once they have practiced and memorized the procedures (Hiebert, 1999, NCTM, 2014). Therefore, students should possess some level reasoning skills to enable them to analyze and select the best procedural method to solve a mathematical problem (NRC, 2001, 2005, 2012; NCTM, 2014; Star, 2005).

Ghazali and Zakaria (2011) conducted a study to investigate students' procedural and conceptual understanding of mathematics specifically in the algebra domain. The study consisted of 132 participants (62% female and 38% male) from secondary schools in Malaysia. The researcher administered an algebra test, which consisted of twenty-two items measuring procedural knowledge, and six items measuring conceptual knowledge.

The results from Ghazali and Zakaria research indicated that students scored better when they were asked superficial questions and scored lower when the questions required reasoning. This encouraged the researchers to determine that procedural and conceptual understandings are highly correlated. The students with higher conceptual knowledge were able to manipulate and solve problems they had yet to learn. Ghazali and Zakaria determined that educators should focus on teaching for conceptual meaning because this approach will help students develop their ability to solve unfamiliar problems reducing their need to memorize algorithms (Ghazali & Zakaria, 2011).

In 1998, Canobi, Reeve, and Pattison conducted a qualitative study of first through third grade students. The focus of their study was to examine the strategies students utilize to solve addition problems. Students were administered multiple tasks that involved problem solving and judgment tasks. The problem solving tasks assessed speed, accuracy, and problem-solving strategies while the judgment tasks assessed conceptual understanding. Canobi, Reeve and Pattison (1998) results supported the theory that students who understand addition conceptually solved the problems quicker, more accurately, and demonstrated more flexibility in utilizing multiple strategies than students who only possessed procedural knowledge. Specifically, students who possess conceptually understanding of addition problems could justify and explain their procedures or strategies unlike the students who only understood step-by-step procedures. The researchers were able to establish that students' conceptual understanding was linked to their retrieval cues. Theorizing that students who understand and focus on the relationships of problems and solutions were better equipped to retrieve addition combinations from memory. The researchers concluded that additional research of conceptual understanding is necessary to determine if conceptual development does indeed foster better problem solving skills (Canobi et al., 1998).

In a study conducted in 1976, authors M. K. Otterburn and A. R. Nicholson, discovered that many students possess a rather inept conceptual understanding of mathematical terms utilized daily in mathematics classrooms. Using the data from 300 students from several different schools, Otterburn and Nicholson compiled a list of thirty-six commonly used vocabulary terms. Students were instructed to indicate if they understood a term either by actually writing a verbal description, an example, a pictorial/symbolic representation, or a combination of any of the aforementioned techniques. The compiled findings indicated that students inexplicably failed to fully explain or they did not completely comprehend many of the terms readily spoken by their mathematics teacher. This profound evidence demonstrated that teachers should be aware that their students' do not necessarily understand the language of mathematics often communicated in class (Otterburn & Nicholson, 1976).

In the NCTM (2007), Judith Sowder describes how effective educators should be able to analyze how well their students understand mathematics and effectively ascertain some of their misunderstandings. In 1995, Sowder and her colleagues conducted a study (NCTM, 2007), which investigated middle school teachers' mathematical understanding. In the study, teachers were provided an opportunity to assess their own teaching practices. The researchers and teachers met weekly for the first year and every four to eight weeks for the subsequent two years. The conducted meetings focused on the content of the curriculum particularly examining proportional reasoning, rational numbers and quantity. Additionally, the teachers were observed multiple times by the researchers and voluntarily participated in follow-up interviews. During the first year the teachers were toiling over the complexities of the mathematics content because the teachers themselves did not conceptually understand the mathematics content, which they were teaching to their students. This was indicative that the content was more complex than their level of teaching. However, after conducting several strategic conversations, the selected teachers actually changed their perspective of the content presented. Over time the teachers developed a more comprehensive understanding of mathematics, which helped them feel more comfortable with teaching the subject. Subsequently, they expanded their roles in the classroom setting and began to probe their students for more depth of understanding of the mathematics content. Fundamentally, the study placed greater emphasis on how teachers' personal understanding of mathematics can affect their own teaching styles and practices (NCTM, 2007).

Franke, Kazemi and Battey (2007) believe that a classroom should be a place where students and teachers both are engaged in the learning process. Teachers should be afforded the opportunity to refine their teaching skills while students equally have an opportunity to develop their individual mathematics skills and understanding. Thus, there should be an accepted practice between the teacher and students to create and establish a constructive learning environment for which both parties can productively participate in an engaging discourse (Franke et al., 2007).

Discourse (Vocabulary Acquisition)

Lager (2006) states that many students associate learning mathematics tantamount to learning a second language. Students invariably assume that mathematics content is initially difficult to understand at the outset, resulting in students feeling frustrated and believing that the content is beyond their capabilities. Lager indicates that vocabulary and linguistic abilities are a growing acknowledged essential in making mathematical connections. The misconception that mathematics is not a language-dependent subject is slowly eroding. The interdependence relationship of language and mathematics is becoming a necessity for students to learn algebra and higher-level concepts (Lager, 2006).

Adams, Thangata and King (2005) state that the language of mathematics is a specialized language, which requires students to have both proficient and complete understanding of the mathematics vocabulary. Monroe and Orme (2002) indicate that language of mathematics is similar to reading comprehension; students must know the vocabulary to comprehend what they are reading. Several obstacles can adversely interfere with vocabulary acquisition. One obstacle is the rarity in which mathematics is spoken. Seldom is mathematics vocabulary spoken in everyday life; consequently, students frequently miss the opportunities to speak mathematically outside of the classroom (Monroe and Orme, 2002). Monroe and Orme (2002) point out that many mathematics teachers mistakenly disregard teaching vocabulary in the classroom further limiting students ability to learn, build and broaden their individual vocabularies. Additionally, many mathematical vocabulary terms have different meanings within the realm of mathematics rather than those found in everyday life. This can further hinder students' ability to utilize their background knowledge to construct logical inferences about unfamiliar concepts (Monroe and Orme, 2002).

Morgan (2005) asserts that a key feature of mathematics language is embodied in the vocabulary connection. The process of acquiring mathematical discourse requires that the student extend beyond writing and simply memorizing definitions. The acquisition of mathematical vocabulary requires students to develop and explain their meaning of vocabulary terms by sorting through any misconceptions or ambiguities. Morgan states that it is essential for the teacher and the students to participate in meaningful dialogue in mathematics class so both participants are able to successfully extract the fundamental meaning of the mathematical terms (Morgan, 2005). Walshaw and Anthony (2008) argued that students who do not engage in conversations may put themselves at a disadvantage and might not fully develop conceptual understanding of mathematics. Therefore, when teachers and students engage in classroom discourse, there is a shift in the students' attention from simply implementing procedural rules to making meaning of their mathematical experiences (Walshaw & Anthony, 2008).

Pirie states in the NCTM (1996) that mathematical discourse can naturally take place in an environment where communication is valued. When students are afforded the opportunity to use mathematical discourse in the classroom, it allows the students to verbalize, write, and interpret their current mathematical knowledge. Additionally, the teacher has the opportunity to provide formative feedback to the student correcting any misconceptions. These discussions permit students to listen to the context in which the words are being used to extract meaning. Thus, requiring the teacher to be acutely aware of students' discourse in the classroom. The misuse of mathematics terminology should be the teacher's priority to intently listen and identify when students use words out of context. Using the example from Pirie (NCTM, 1996), if a student repeats the word twice to represent the word squared, the student may have misinterpreted the problem or it may be a common misinterpretation of the word's meaning throughout the class. Thus, it is fundamentally important for the teacher, as well as the students, to listen to the discourse being communicated throughout the classroom to correct the misuse of any vocabulary terms (NCTM, 1996).

Sfard (2001) spoke of her observation of two classrooms at the elementary level. The students were learning a new mathematic topic and Sfard transcribed the discourse that took place between the teacher and her students. Based on the teachers' expectations, the discourse among the students was unproductive. Nevertheless, she continued to facilitate the students' learning. The teacher specifically asked leading questions while encouraging the students to come up with their own creative solutions. Upon conclusion of her observation, Sfard highlighted how discourse in the mathematics classroom, regardless of any misunderstandings, can be beneficial as long as all students are active participants in the discussions. While it is important to correct misconceptions at some point, it is equally important not to stress accuracy during students' discourse. It is counterproductive to identify inaccuracies during discussions because it will discourage some students from participating in the conversations. Fundamentally, it is important for students to communicate their knowledge, regardless if it is correct. If all students do not provide their insight in a group discussion on a mathematics topic, then the teacher won't be able to assess their mathematics comprehension, which is the ultimate goal (Sfard, 2001).

In the NCTM (1996) Rubenstein suggests that students creatively invent their own words to promote personal meaning with vocabulary terms in mathematics. When students are able to invent their own mathematical words, they are able to crystalize their understanding while making new mathematical connections. In the NCTM, a class of students made up a term to represent bisecting an angle. Instead of the students using the correct term, angle bisector, they chose the term midray. The benefit of allowing students to make up their own words for various mathematic terms outweighs the possibility that they won't remember the conventional term later. However, it is important to occasionally translate the students' invented words to the conventional mathematical terms to ensure that the two dichotomies of word usage are not lost in translation (NCTM, 1996).

Additionally, Rubenstein and Thompson (2002) suggest in an effort to get children to connect unfamiliar terms with new terms, teachers should have students perform an assortment of activities to build on the concepts. Once the students have completed the activities, they should informally communicate their understanding of each vocabulary term. When the students can correctly present their interpretation of every mathematical term, it should be written, illustrated and formally communicated. The formal discussion will ensure that the students understand the exact definition of all of the newly acquired terms. Other strategies for vocabulary acquisition can be generalized to writing stories, poetry, drawing cartoons or keeping journal entries (Rubenstein and Thompson, 2002).

Strategically teachers should be encouraged to examine relevant and current literature to assist in vocabulary acquisition (Rubenstein & Thompson, 2002). Rubenstein and Thompson (2002) state that there are numerous literature books available to assist and substantially reduce students' negative viewpoint of mathematics and its concepts. Additionally, the books can help resolve students' misconceptions of a mathematical topic. Rubenstein and Thompson suggest that students should write questions about the story or draw pictorial representations of the story to build on to the literature. Both authors acknowledged that the mathematics classroom is one of the few places mathematics is spoken. Consequently, teachers need to generate new ideas to teach vocabulary to students to build onto their conceptual understanding (Rubenstein & Thompson, 2002).

Gay (2008) also acknowledges that teachers should readily use mathematical vocabulary in the classroom. While observing new and experienced teachers in their classroom, Gay heard teachers using ambiguous vocabulary terms when providing instruction to their students. Teachers were asking students to *evaluate a set of exponents* and *graph the following expressions*. The vagueness of their directions, initiated Gay to address vocabulary issues with the pre-service teachers and provide them with strategies on how to teach mathematics vocabulary for the upcoming semester. One strategy Gay suggested for teaching mathematics vocabulary was for the students to utilize graphic organizers. Mathematics organizers are particularly useful in geometry in identifying polygons and categorizing figures. The second strategy Gay suggested was the concept circle, which is more helpful with defining, identifying and categorizing new vocabulary terms. The concept circle is typically divided into four sections and the students have to write a phrase or word in each section later filling in the sections with attributes or word association to enhance their understanding of the written terms (Gay, 2008). Efforts by Gay (2008) raised the pre-service teachers' awareness for mathematics vocabulary. The teachers were consistently able to see how vital it was for them to use the correct terminology when describing a mathematical object or when providing instructions to their students. Upon the conclusion of the semester, one teacher wrote that she understands she must be precise and clear when addressing the students. Furthermore, she understands that many of the concepts she articulates to the students will presumably be new content (Gay, 2008).

Authors, Blanton, Berenson, and Norwood (2001) examined classroom discourse and its role in the development of a student teacher. The current reforms in mathematics are consistently emphasizing the importance of discourse in mathematics education. The authors' purpose of this interpretive study was to examine the linkage between learning to teach mathematics and classroom discourse. Blanton et al. understood that discourse not only structure how students' think about mathematics but how teachers' think about teaching mathematics (Blanton, Berenson, & Norwood, 2001). Discourse, which promotes a mathematical inquiry, is regarded as meaningful form of communication (Curio, Schwartz, & Brown, 1996). Similar to Blanton et al., authors, Curio, Schwartz, and Brown (1996) believe that learning mathematics genuinely entails students being able to make connections and construct new meaning by using their prior knowledge and new experiences. Based on this viewpoint discourse between teachers and students is an effective instructional tool utilized in the mathematics classroom (Curio et al., 1996).

In 2005, authors Cook and Buchholz wrote an article about a kindergarten teacher who promoted mathematics vocabulary with her students. Melissa (pseudonym) was teaching at a school located in a predominantly African American neighborhood with a class of 20 students that contained nine girls and eleven boys. In an effort to promote the language of mathematics, Melissa utilized six informal strategies. The first strategy Melissa utilized with her students was the opportunity for them to talk with her and with each other about mathematics. Adding manipulatives like pattern blocks to their mathematics activities the students were encouraged to express themselves in informal conversations (Cook & Buchholz, 2005). Cook and Buchholz (2005) indicated that the second strategy utilized was for Melissa to serve as a facilitator in the classroom. Melissa chose to listen to the children's discourse to facilitate their use of mathematical language and help foster their ideas. The third strategy was to promote and provide students the opportunity to connect previous knowledge with newly acquired knowledge. A day before the students were instructed about measurement, Melissa brought out a balancing scale and asked the students if they have seen anything similar to the scale on the playground. Many of the students responded excitedly about the seesaw. This provided Melissa the opportunity to guide her students to connect their old knowledge, the seesaw, with their new knowledge of the balancing scale. The fourth, fifth, and sixth strategies collectively incorporated some type of pre-determined activities to engage the students to actively participate in mathematical discussions. Upon the conclusion of the 3-month observational period, Melissa's kindergarten class spoke with confidence regarding mathematical concepts providing Melissa with the necessary evidence of their understanding and learning (Cook & Buchholz, 2005).

Authors, Hardcastle and Orton (1993) conducted a study to investigate whether or not eighth grade students properly use mathematics terminology when problem solving. While being recorded, a couple of students were left alone, with a few manipulatives to openly discuss the necessary steps to solve an assigned task. Upon successful completion of the task, it was noted that the students failed to use specialized mathematics vocabulary. The researchers felt this was widely due to the students pointing to objects stating a more natural response of "this" or "two of these" throughout the problem-solving task. Consequently, the researchers decided to blindfold the teacher and place her into the room as a tactic to reduce students utilizing visual aids to generically describe various objects. As a result, the students started using more specialized mathematics vocabulary to describe objects to their teacher and to keep her abreast of their progress as they completed the assigned task. Using this strategy forced the students to utilize mathematics vocabulary and provided the researchers an evaluative tool to measure the students' understanding of appropriate usage of mathematical terminology (Hardcastle & Orton, 1993).

Hardcastle and Orton (1993) conducted another study, similar to the 1976 study conducted by Otterburn and Nicholson, with 12-year-old pupils requesting them to classify 12 familiar mathematical terms (e.g. area, digit, edge, face, vertical). The students' responses were categorized as correct, blank, or confused (answers were categorized as confused because they were unclear or ambiguous). Upon conclusion of the test, the unsettling results revealed that only thirty-nine percent of the students' responses were accepted as correct. If these results were used as a broad brush regarding 12-year-old students understanding, it would suggest students comprehend approximately forty percent of the information the teacher is communicating in mathematics class. Accordingly, Hardcastle and Orton recognized that their method to measure and quantify students understanding of mathematical terminology had its imperfections. However, the quick assessment easily showed that the language, which is often spoken in mathematics classrooms, might be a barrier for students' comprehension of mathematical concepts. This suggests that mathematics terminology is an important aspect of teaching mathematics (Hardcastle & Orton, 1993).

Rangecroft (2002) ascertains that learning to communicate mathematically extends beyond the traditional formal educational institution. The author exclaimed that career positions, like statisticians, require that they are fluent in the language of mathematics. Rangecroft referenced that it is a long recognized issue that language is an important component of teaching and learning mathematics. Thus, learning mathematical vocabulary might be an essential for employment further verifying the need to teach mathematics terminology in the classroom (Rangecroft, 2002).

In a recorded audiotaped study conducted by Christine Renne (2004), she requested her students to discuss both the similarities and differences of a square and a rectangle. After providing them with concrete examples of both shapes and thirty minutes to discuss their findings, she called the class back to order for a full class discussion. Students raised their hands ready to offer their interpretation of the similarities and differences they concluded from their groups' discussions. Calling on one group of students to freely communicate their thoughts, Renne asked them about the word congruent. Earlier she had overheard the word congruent being used by one of the members of the group to describe a square. The group fell silent until one student spoke up and inaccurately defined congruent but instead defined parallel lines (Renne, 2004).

This important discovery Renne (2004) made through listening to the students' discourse assisted her in understanding why her students were often confused about geometric figures. Renne made an effort to redirect her students' thinking, but confusion was already established among the students. Within minutes of moving on to another group, congruency was correctly defined and the first group was able to learn the correct meaning. During the class discussion, one student started to challenge another student's rationale for identifying a square from a rectangle. The discourse became more intense and the excitement of learning categorically reached a new level (Renne, 2004).

Upon the conclusion of class, Renne (2004) listened to the recorded class discussions. She concluded that she missed opportunities to clarify some misconceptions. However, Renne noted that she did not believe that the misconceptions would have been revealed if she had held a teacher led lesson. Renne concluded that discourse in the mathematics classroom is essential for students to conceptually understand mathematical vocabulary while providing the teacher the opportunity to expound on teachable moments (Renne, 2004).

Problem Solving

Although, life is filled problem-solving events, the idea of solving word problems is often viewed as a dreadful experience for many students and some adults (Monroe & Panchyshyn, 2005). Monroe and Panchyshyn (2005) state that there is a genuine effort in the education community to bridge the connection of problem solving in the mathematics classroom with real-world mathematics to elevate students' success. Monroe and Panchyshyn offer several strategies teachers could incorporate to help alleviate some of the emotional ties associated with problem solving (i.e. frustration). One strategy devised to reduce students' frustration levels would be to embed mathematics vocabulary instruction into the actual lesson. Yet another strategy would be to encourage students to rewrite the problem as reflected by their own interests. The goal here is to redirect and refocus students' energies from ultimately looking for a pre-packaged algorithm to solve mathematics problems utilizing their own mathematical reasoning to successfully find a solution (Monroe and Panchyshyn, 2005). Ali, Hukamdad, Akhter and Khan (2010) indicate that all engaged learners learn at their own pace suggesting that learning is a personal process. Individuals are faced with multi-dimensional problems in their daily lives and they attempt to solve these problems by using their previously acquired experiences and knowledge. Expectantly, students will have to gain the necessary skills to find appropriate solutions when they are faced with real life challenges (Ali, Hukamdad, Akhter and Khan, 2010).

Muir, Beswick and Williamson (2008) defined problem solving as a sequence of actions used to find the solution to a problem not instantly known. Muir et al. believed that using a variety of heuristics approaches to solve problems could assist students in becoming successful problem solvers. Heuristics in this instance is defined as a strategy problem solvers use to construct appropriate knowledge to effectively solve problems. George Polya, who is known for his renowned work in problem solving, developed a four-step heuristic plan. The four steps include understanding the problem, devising a plan, carrying out the plan and looking back. Muir et al. conceded that Polya's heuristics plan, along with a few other strategies, like guess and check, and working backwards are most likely the most widely used strategies in problem-solving (Muir et al., 2008).

Passmore (2007) asserts that Polya proposed that students should feel comfortable with guessing the result of a problem. He believed that intuition and judgment were two indispensable tools for fine-tuning students' ability to guess an answer leading to the possibility of constructing a proof. Oftentimes, Polya would address how he believed that mathematics teachers should improve their perspective on mathematics and choose more meaningful and interesting problems. This act, Polya believed, would increase students' confidence in solving a range of problems and encourage students to formulate and explore more viable solutions (Passmore, 2007).

Author Frank K. Lester, Jr. (2013) has written about problem solving for over 40 years, stated that many mathematics educators commonly agree that the development of students' problem solving abilities is the primary objective for mathematics instruction. However, the teacher's consideration to accomplish this goal involves a broad range of factors and decisions. Lester suggests that problem solving indifference compared to other mathematical areas uniquely requires several different proficiencies that teachers must attain, in order to be successful in promoting students to become better problem solvers. Lester suggests that teachers should have an instructional plan to effectively teach students problem solving strategies. Lester and a colleague wrote a book illustrating an optional instructional plan (Charles & Lester, 1982). Charles and Lester's (1982) book described the three phases of instruction as part of ten *teaching actions*. The before, during, and after phases identifies and specifies the teacher's role and the environment of the classroom, which is unparalleled to conventional lesson plans (Charles & Lester, 1982). Lester (2013) suggests that educators must be able to identify any possible challenges that may occur when teaching problem solving tasks. Moreover, teachers must be skilled at selecting and creating activities, ensuring that the students' activities are challenging, remaining attentive to students' discourse and strategies, and intervening at the appropriate time (Cai, 2010; DiMatteo & Lester, 2010; Hiebert, 2003; Lester, 2013).

Like Lester, Schoenfeld (1992) states that one can assume that the primary goal of mathematics instruction is to have students become successful problem solvers.

However, the goal itself may sometimes be too vague regarding how this can be accomplished consistently. Schoenfeld examined a concept of what he describes as 'thinking mathematically' or the way people view mathematics. Schoenfeld believes that mathematics education should be perceived more as a social process than an instructional one. This process involves transferring the mathematics problems to an activity that makes sense to the student. This would allow the student to find a solution based on their current knowledge and/or with the assistance of their peer. Schoenfeld's theory of mathematics education extends beyond procedures and facts, which may be the turning point for mathematics education (Schoenfeld, 1992).

Silver and Smith (NCTM, 1996) assert that structured small group activities whereas students are engaged in questioning, providing explanations and elaborating on their ideas provides a positive productive learning environment. To foster positive interdependence among the group members, group structure is an important component in getting students to feel comfortable in creating appropriate discussions. The group should comprise of members who can cooperatively (individually or collectively) reason through mathematical problem solving tasks while positively contributing to the mathematical discourse. These actions of cooperative group work, activities, and class discussions will later aid an individual student when he or she is working alone and have to communicate with oneself on a problem-solving task (NCTM, 1996).

Muir et al. (2008) conducted a study consisting of 20 six-grade students who had to identify which strategies they would independently utilize to solve non-routine mathematic problems. In the study, the researchers examined the effectiveness of students' strategies and their abilities to write and verbally communicate their thinking.

Typical problem solving behaviors were observed along a continuum ranging from naïve (routine) strategies to sophisticated (expert-like) strategies. Although the researchers did not expect for all twenty participants' behaviors to neatly fit into one category, it was noted that two students were categorized as having no problem solving behaviors. Students, whose problem-solving behaviors were classified as naïve, tended to utilize one or two strategies in order to solve a problem and were more likely to adopt a more specific procedural strategy. Routine or average problem solvers persisted in adopting new strategies. However, sophisticated problem solvers uniquely explored and generated their own strategies to solve the problems. Muir et al. believe that many of the students might have been taught a limited number of problem solving strategies, which may have lessen their ability to creatively solve complex mathematical problems, thus decreasing their mathematical thinking. Similar to vocabulary acquisition, teaching practices may best be served when teachers encourage their students to discover their own problem solving strategies. This approach may produce more sophisticated problem solvers and encourage metacognitive thinking (Muir et al., 2008).

This study amplifies the NCTM (2014) article regarding procedural fluency. In the article it explains about the importance of procedural fluency and its impact on learning mathematics. The critical element of mathematical comprehension can no longer be restricted to conceptual understanding. Therefore, it may be beneficial for students to learn mathematics conceptually prior to learning the procedural skills associated with the lesson providing students options regarding which strategy or method would best be applicable to a given situation (NCTM, 2014). Thus, students who possess both procedural and conceptual knowledge have an extensive understanding of mathematics (Ghazali & Zakaria, 2011).

In 2012, Abdullah, Zakaria, and Halim designed a quasi-experimental study to determine how visual aids could affect thinking strategies to solve mathematics problems. In the study of 193 primary grade students, the treatment groups were provided thinking strategies with visual aids to assist them in solving mathematics word problems. One of the visual strategies utilized with the treatment group was Polya's heuristics plan that displayed an in-depth analysis of each step. The treatment lasted for four hours a week for ten weeks. The participants in the control group attended normal classes with no additional instruction using conventional teaching practices involving drills, memorizing facts and/or formulas. The two instruments utilized in this study were an achievement test and a conceptual understanding instrument. The aim of the conceptual understanding instrument was to assess the students' knowledge of mathematical concepts required to solve mathematical word problems (Abdullah, Zakaria, and Halim, 2012).

Abdullah, Zakaria, and Halim (2012) findings revealed that visual aids are important in solving mathematical word problems. Students who are able to conceptually visualize the word problems are more likely to perform appropriate mathematical procedures. When students are subjected to a learning environment where the teacher promotes drill and grill practice, the students' ability to creatively problem solve decreased along with their ability to apply thinking strategies. Thus, students who experience a more innovative teaching approach have a better understanding of mathematics. When students were taught to think analytically they were able to make connections to additional mathematics concepts (Abdullah, Zakaria, and Halim, 2012).

Acknowledging thinking strategies and the language component in word problems, Abdullah, Zakaria, and Halim (2012) wrote the following:

> The problems need to be analyzed and interpreted as the basis for selection and decision making. To achieve this goal, students need to be guided and exposed to strategic thinking and representation skills so that mathematical problem-solving skills can be achieved effectively. It is necessary to build a relationship between knowledge of language and knowledge to manipulate, in addition to the development of thinking processes and representation skills in building a relationship between all of the important parts in a problem (Abdullah, Zakaria, & Halim, 2012, pg. 30).

Convincingly, students may have to acknowledge that word problems may require some dismantling or manipulation for better interpretation and analysis (Johnson, 2010; Lager, 2006; Abdullah, Zakaria, and Halim, 2012). Accordingly, one of the reasons students may have challenges in solving mathematical word problems may be the words themselves (Abdullah, Zakaria, and Halim, 2012).

Summary

There exists a compelling amount of literature, which asserts that students' comprehension of mathematics vocabulary is profoundly related to their aptitude in mathematics. Using the theoretical frameworks as a compass, the researcher attempted to find corroborating literature available which supports this hypothesis. Previous researchers and education authorities like Vygotsky, Thompson, Rubenstein, Sfard, Lester, Schoenfeld and Polya have paved the way to establish the importance of cultivating a culturally responsible classroom where students are encouraged to have ongoing and engaging discourse utilizing acquired vocabulary.

In summary, this chapter was designed to develop a detailed exploration of the conceptual framework for a study based on mathematics vocabulary and student achievement. Ultimately, the study may help in determining the degree and extent to which mathematics vocabulary can enhance, and possibly increase a student's chance to acquire a correct answer to a mathematics problem. Although, it is only speculation by which the absence of vocabulary knowledge may preclude students from making pertinent and relevant connections; mathematics vocabulary procurement is regarded as an important component for students to conceptually understand the subject. Thus, mathematics vocabulary may be a missing component, yet an underlying tool, necessary for students to learn, communicate and comprehend all facets of current and future mathematic concepts.

CHAPTER THREE

METHODOLOGY

In an article written by Smith and Angotti (2012), a high school teacher was quoted as saying "….it is a challenge to trim it down to just essential vocabulary in lessons, because often students need to know all of the vocabulary terms" (Smith & Angotti, 2012, p. 49). The challenge of teaching mathematics vocabulary is often characterized as a daunting task because many students are trying to understand the concept while learning the discipline-specific vocabulary often in a limited time span (Smith & Angotti, 2012).

Usiskin (1996) characterized mathematics as having its own unique language, which must be taught and utilized frequently in the classroom, in order for students to understand it. Rangecroft (2002) suggests that teachers speak mathematically in the classroom encouraging students to learn how to read, write and speak the language of mathematics. Furthermore, Rangecroft (2002) asserts that the language of mathematics is an essential tool for teaching and learning mathematics. However, mathematics is often referred to as a unique and an isolated language and students' appropriate usage of mathematical terminology is usually limited exclusively to the classroom environment (Adams, Thangata and King, 2005; NCTM, 1996; Monroe and Orme, 2002; Morgan, 2005).

Matteson (2006) states in the *Journal of Reading Psychology* that educators needed to understand the importance of teaching mathematics literacy. She recognized the complexities of mathematics and indicated that comprehensive understanding could be compromised if students do not have in-depth knowledge of theorems, formulas, and technical terminology. Among other factors, Mattson (2006) describes students' discourse as a critical component in developing mathematical literacy. She states that a classroom void of communication prohibits students from conceptually understanding mathematics. Thus, researchers should further examine possible connections between mathematical literacy and student achievement on assessments (Matteson, 2006).

Research Questions

This study was designed to determine if there is a connection between mathematics vocabulary comprehension and student achievement. In this chapter, the researcher provides a systematic approach detailing how the study was conducted and how the compiled data were collected. Using a mixed method design, the researcher's rationale for conducting this study was to answer the following research questions:

- Is there a correlation between the acquisition of mathematics vocabulary and students' achievement in mathematics?
- How does conceptual understanding of mathematics vocabulary impact students' ability to problem solve?

Description of the Setting

The study was performed in a northwest rural county located in the state of Georgia during the 2013-2014 school year. The county had a population of approximately 146,900 people with a median income of $63,190 (http://www.census.gov/en.html). In the last few years, the county experienced a recent business growth, which may be attributed to the increased population expanding the school district to fourteen elementary, nine middle and five high schools.

The study took place in a middle school with a student population of 847 students and approximately 70 teachers and staff members. The student body comprised 271 sixth graders, 286 seventh graders and 290 eighth graders. Despite having over 35% economically disadvantaged families enrolled, the NW school impressively met adequate yearly progress or AYP every year in all categories since the 2005-2006 school year. Table 1 displays the racial/ethnicity breakdown of the student population for the 2013-2014 school year.

Table 1

Student population by race/ethnicity by grade level

Grade	White	Black/African American	Hispanic/ Latino	Asian	American Indian or Alaska Native	Native Hawaiian or Pacific Islander	Two or more races
6	132/16%	94/11%	27/3%	2/0.2%	1/0.1%	0/0%	15/1.8%
7	156/18%	80/9%	30/4%	4/0.5%	1/0.1%	0/0%	15/1.8%
8	158/19%	91/11%	21/2%	0/0%	2/0.2%	2/0.2%	16/01.9%
All Grades	**446/53%**	**265/31%**	**78/9%**	**6/0.7%**	**4/0.5%**	**2/0.2%**	**46/5.5%**

Based on the schools' demographics the Principal would often examine researched based methods in an effort to raise student achievement. One morning the Principal called a faculty meeting to express his mission to place academic vocabulary in the forefront of the school's curriculum for every content and grade level. He requested that all teachers read a book written by R. Marzano and D. Pickering called *Building*

Academic Vocabulary in an effort to raise the importance of teaching and developing students' comprehension of grade-level vocabularies. The book provides a six-step comprehensive approach on best practices of teaching students academic vocabulary. All teachers were compelled to immediately implement Marzano's and Pickering's strategies utilizing a list of content-based vocabulary words created by the teachers. Interestingly, administrators did not require teachers to administer a pre/post test to assess students' vocabulary comprehension, instead, they urged teachers to administer an end-of-the-year assessment to determine how many vocabulary terms students acquired for the current academic school year.

The new agenda was aligned with the study the researcher was conducting at the school. Fully apprised of the study, the Principal requested that the researcher present and share quarterly updates with the staff to further the school's initiative. Additionally, the Principal was extremely supportive of the creation of the vocabulary assessment utilized in this study for the eighth grade mathematics department, and indicated which school week he wanted the test to be administered to produce optimal results.

Description of the Student Population

There were 290 eighth grade students during the time the study was being conducted, which consisted of 46% females and 54% males, who were assigned to one of three eighth grade teams. For practicality purposes, the researcher identified each team by a letter (A, B, and C respectively). Team A, which was the most diverse team, had a student population of 107 students. Team A's population consisted of 22 special education students, 29 gifted and 56 general education students who were taught by four academic and three special education teachers. Team B's student population consisted of two gifted and 115 general education students who were taught by four academic teachers. Team C was the smallest team with a student population of 62 regular education students. However, it was unlike the other teams because there were only two academic teachers who taught two different contents. One academic teacher (the researcher) taught mathematics and social studies while the second academic teacher taught English language arts and science. The researcher used the convenience method and only solicited data from the students on team A, B and C. There was one small self-contained class of eighth grade students; the four students' data were not collected for this study. Thus, a total of 286 eighth grade students were invited to participate in this study. Table 2 displays the student population for the three eighth grade teams.

Table 2

Student population (Team A, B, and C)

Student population	**Team A**	**Team B**	**Team C**
Gifted	29/10%	2/0.7%	0/0%
Special Education	22/7%	0/0%	0/0%
General Education	56/20%	115/40%	62/22%
Total	**107/37%**	**117/41%**	**62/22%**

Instruments

The Standards for Educational and Psychological Testing (1999) describes an instrument as a way to measure an individual's interests, skills, and knowledge in an organized format according to a specified plan. While developing an instrument, there must be consideration of the context, format, and content (American Educational

Research Association, 1999). Likewise, it is equally important that the selected instrument is reliable and valid (Creswell, 2012; Merriam, 2009).

There were two instruments needed to effectively conduct this study. The first instrument was the state mandated test known as the Georgia Criterion Referenced Competency Test (CRCT). The CRCT assessment is designed to measure the students' overall academic performance in a specific content area. The second instrument was an eighth grade vocabulary assessment, which was designed to measure students' comprehensive understanding of middle grades mathematics vocabulary terms. Unlike the state test, there wasn't a valid and reliable vocabulary test to utilize, compelling the researcher to create an assessment. The researcher, with the assistance of eight mathematics teachers, created and developed an instrument to measure students' comprehension of eighth grade mathematics vocabulary.

Vocabulary Assessment

With the invaluable support of eight mathematics teachers (two sixth grade, three seventh grade and three eighth grade), the researcher commenced the arduous task of creating a comprehensive eighth grade vocabulary assessment. Individually the teachers were provided with a list of 89 content-based words obtained from the Georgia Common Core Curriculum (Appendix A). Each teacher was requested to circle 45 terms they believed were conceptually relevant or important for students to understand in order to perform well on the mathematics portion of the CRCT. To ensure that every teacher understood conceptual relevancy, the researcher discussed and explained in great detail that the 45 terms they selected should be vocabularies (he or she believed) the students should know meticulously well by the end of eighth grade. Each of the selected terms were tallied and organized in a hierarchical format based on the number of votes the term received (Appendix B). Subsequently, it was collectively decided by the committee that some terms should be merged together to produce a paired phrase (i.e. base number/exponent). Basically, the committee agreed that one question could effectively ascertain if a student was knowledgeable of each term although two terms were combined to produce one term. Once the list was finalized, the terms were organized into one of four categories corresponding to the content weights predicted for the CRCT (Appendix C). Below is a table of the eighth grade content weights for the 2013 – 2014 academic school year.

Table 3

CRCT content weights for eighth grade 2013-2014

Mathematics (CCGPS)	**Approximate Percentage for Content Weight**
Numbers & Operations	20%
Geometry	27%
Algebra	41%
Data Analysis & Probability	12%

http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Documents/CRCT_Content_Weights_%202013-14_Final.pdf

Collectively, there were 37 terms that were listed into four categories. There were six terms listed under the numbers and operations category, twelve terms for geometry, fourteen terms for algebra, and five terms for the data analysis and probability category. Based on the 37 terms, the researcher drafted a comprehensive multiple-choice vocabulary assessment, which totaled 41 problems. Some of the ideas for the assessment were developed from old tests, homework assignments, and images of graphs retrieved from the Internet. Upon drafting the assessment, the committee members were provided a copy of the vocabulary test along with a teacher's rating scale (Appendix D). The rating scale was a document the researcher created for the purpose of eliciting commentary regarding the complexity level of each question and to note any grammatical errors or poorly written problems. Accordingly, all of the teachers completed the vocabulary test and returned their copy of the rating scale along with constructive comments and suggested revisions. One committee member noted that the term dilation was inadvertently omitted from the original list of 89 content-based words. After consulting with the other members, it was unanimously agreed that dilation should have been included on the original list and was subsequently added to the vocabulary assessment (Appendix E). The addition of the term dilation to the vocabulary test increased the total number of questions on the assessment to 42.

In an effort to establish validity, the researcher secured the assistance of two ninth grade students (who completed and signed their assent/consent forms) to provide their feedback of the vocabulary test. The expectations were explained to the students and they completed both the vocabulary assessment and a student's edition of the rating scale (Appendix F). The student's scale requested information similar to the teacher's edition; however, there was additional information requested from the students. The students were asked about the fairness of the assessment and if eighth grade students should know the terms listed in the assessment. Both students indicated that the test was objective and fair and eighth grade students should know most of the terms by the end of their eighth grade school year. Table 4 displays the results of the teachers and students' rating scale and the difficulty level for each question on the assessment.

Table 4

Classification of the vocabulary test based on the level of difficulty per question

	Easy	**Medium**	**Difficult**
Teacher (6th grade)	1, 3, 4, 5, 11, 15, 20, 21, 22, 23, 25, 27, 28, 32, 34, 36, 37, 38	2, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 24, 26, 29, 30, 31, 33, 35, 39, 40, 41, 42	
Teacher (6th grade)	1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 20, 21, 22, 23, 28, 31, 37, 38, 40	2, 14, 16, 19, 24, 26, 27, 29, 30, 32, 33, 34, 35, 39, 41, 42	25, 36
Teacher (7th grade)	1, 3, 5, 6, 7, 8, 10, 11, 15, 17, 18, 20, 21, 22, 23, 26, 28, 32, 33, 34, 37, 41	2, 4, 9, 12, 13, 14, 16, 19, 24, 25, 27, 29, 35, 36, 38, 39, 40, 42	30, 31
Teacher (7th grade)	1, 3, 7, 8, 10, 11, 13, 20, 21, 22, 26, 27, 28, 32, 34, 37	4, 5, 6, 9, 12, 14, 15, 16, 17, 18, 19, 23, 24, 25, 29, 30, 31, 33, 35, 36, 38, 39, 41, 42	2, 40
Teacher (7th grade)	1, 2, 3, 5, 6, 7, 10, 11, 14, 15, 20, 23, 28, 37, 38, 39, 41	4, 8, 9, 12, 13, 16, 17, 18, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 40, 42	19, 26, 33, 35, 36
Teacher (8th grade)	1, 3, 4, 5, 6, 7, 10, 13, 20, 21, 22, 23, 26, 27, 28, 32, 34, 36, 37, 38, 40, 41, 42	2, 8, 9, 11, 12, 14, 15, 16, 17, 18, 24, 25, 29, 31, 35, 39	19, 30, 33
Teacher (8th grade)	1, 3, 4, 12, 15, 16, 34, 37, 40, 42	2, 5, 6, 7, 8, 9, 10, 11, 13, 14, 17, 18, 20, 21, 22, 23, 24, 25, 26, 31, 32, 33, 35, 36, 38, 39, 41	19, 27, 28, 29, 30,
Teacher (8th grade)	1, 3, 6, 7, 10, 11, 13, 17, 20, 21, 22, 23, 24, 27, 28, 30, 34, 36, 37, 38, 40, 41, 42	4, 5, 8, 12, 14, 16, 18, 19, 25, 29, 31, 32, 33, 39	2, 9, 15, 26, 35
Teacher (8th grade)	1, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 15, 17, 18, 20, 21, 22, 23, 24, 26, 28, 29, 30, 31, 32, 34, 36, 37, 39, 40, 41, 42	2, 9, 14, 16, 19, 25, 27, 33, 35, 38	
Student (Female)	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42	7	25
Student (Male)	1, 2, 3, 4, 5, 6, 37, 38, 39, 40, 41, 42	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36	

The table indicates both teachers and students believed that the test questions largely range from easy to medium. The first student in the table marked the majority of the test questions as easy and earned a test score of 85%. The second student marked many of the questions in the medium range and he earned a 74% on the test. Neither of the student's data were calculated with the eighth grade participants' data because their percentile scores were utilized to examine their knowledge and perception of the vocabulary assessment.

Reliability

Creswell (2012) defines reliability as a set of scores that are consistent and stable regardless of when or how often the same instrument is administered. In the American Educational Research Association or AERA (1999), the *Standards for Educational and Psychological Testing* expounds on Creswell's definition of reliability expressing that participants may exhibit some level of stability in their behavior but it is rare for any individual to consistently exhibit the same behavior in every situation. Given the likeliness of variability, AERA (1999) asserts that there is an expectation of some amount of measurement error but this kind of variability is not necessarily related to the instrument.

The researcher utilized the item response theory (IRT), which can effectively compare the probability of success on an item to the person's overall ability (Camilli & Shepard, 1994; Guler, Uyanik & Teker, 2013). Unlike Classical Test Theory (CTT), which focuses on the test, the IRT model focuses more so on test items (Fan, 1998; Guler, Uyanik & Teker, 2013). Guler, Uyanik, and Teker (2013) state that an IRT analysis gauges the complexity of the test items and provides valuable insight regarding an individual's ability. Each of the measures has a standard error and fit statistic that are considered population independent. Thus, reliability is assessed in terms of the amount of error and fit for each of the items (Guler, Uyanik & Teker, 2013).

The WINSTEPS application, which is an IRT measurement system, conveniently converts the IRT scored responses into useful data. The WINSTEPS application is equipped to individually identify participants' ability (or their vocabulary knowledge) when the instrument is established as reliable (Bond & Fox, 2007). The more reliable the test items, the less chance for standard error and the better chance the participants' traits (low to high vocabulary knowledge) will be identified (Bond & Fox, 2007).

The results of the scored vocabulary assessment were presented on a continuous scale individually displaying the position of each student. With the WINSTEPS program, there is an expectation for students with low vocabulary knowledge to get many of the easy vocabulary terms correct but struggle with intermediate to difficult mathematic terms. However, students with high conceptual understanding of mathematics vocabulary should get easy, intermediate and most of the difficult problems correct. It was expected for some students with comprehensive vocabulary knowledge to get an easy problem incorrect and students with low vocabulary comprehension to get a difficult problem correct; however, those students were easily identifiable because of the logit (log odd units) scale. The logit scale is a unit of measurement that commonly ranges from negative three to positive three. Thus, students who are positioned on the continuum in the negative three range would be categorized as having the lowest ability; respectively, students positioned in the positive three range would be categorized as having the highest ability. (Bond & Fox, 2007).

Validity

Creswell (2012) defines validity as the development of a sound instrument that measures its intended purpose. The purpose of the test was to assess students' comprehensive knowledge of eighth grade mathematics vocabulary. AERA (1999) states that the validation process requires a sound fundamental argument supporting the instrument's intended purpose. This suggests that a diverse set of interested parties with similar content offer their expert advice on whether the test is adequately measuring its proposed construct (AERA, 1999). To follow the suggestions made by AERA (1999), the researcher actively recruited the support of several mathematics teachers from different grade levels to analyze and revise the mathematics vocabulary instrument. The teachers met several times to objectively discuss and evaluate the assessment. The periodic meetings were collectively regarded as valuable benchmarks and greatly assisted in establishing validity. Based on the four phases of development (planning, constructing, evaluating, and checking), the researcher sought to properly evaluate the assessment for validity (Benson & Clark, 1983; Creswell, 2012).

Creswell (2012) states that instruments are evaluated for construct validity when participants are interviewed upon completing the instrument. If the participants' responses are determined to fit what the instrument was intended to measure, then the instrument is more likely to be classified as a valid instrument (Creswell, 2012). Subsequently, the researcher decided to run a test of the vocabulary assessment. With the assistance of two ninth grade students, the vocabulary test was administered, along with the student's rating scale, to help establish construct validity. Upon completion of the assessment and the student's rating form, the researcher examined the documents for commentary regarding any ambiguities the students found in the overall assessment. Specifically, the researcher was seeking to determine if the students observed any potential problems with the language or semantics used in the assessment. Additionally, the researcher sought to examine whether or not the students had any questions or concerns they encountered while taking the actual vocabulary test. The information provided by the students was compiled, organized, and thoroughly reviewed to assist with the specific purpose of establishing construct validity. A copy of the vocabulary assessment is located in the appendix (Appendix G).

Georgia's Criterion Referenced Competency Test (CRCT)

Implemented in the spring of 2000, the CRCT is a yearly high stakes standardized test that is designed to measure Georgia's content standards in English language arts, mathematics, science, reading, and social studies. Due to budget constraints in the spring of 2013, the assessment was administered to third to eighth grade students only. The scores from the test were utilized to effectively analyze students' strengths and weaknesses in each content area to help gauge the quality of education and instruction being provided to the students for the academic school year. Additionally, the scores provide information regarding students' academic achievement and/or their overall performance in their content classes.

The CRCT is administered to all general education students regardless of gender, race/ethnicity, and any other subcategories including students with limited proficiency in English and students with disabilities. For this study, the researcher used participants' scores from the newly created mathematics vocabulary assessment and compared them to the results from the mathematics portion of the 2013 – 2014 CRCT.

Research Design

Johnson & Christensen (2004) assert that the combination of both qualitative and quantitative methods provides a better understanding of the research than either quantitative or qualitative data alone. The primary goal for each paradigm is different. In qualitative research the objective is to describe and explore whereas in quantitative research the objective is to predict and explain (Johnson & Christensen, 2004). A combination of methods or a mixed method design is regarded as time consuming, but provides a better understanding of the research questions (Creswell, 2012). In this study, the researcher implemented a mixed research method, indicating that both qualitative and quantitative data were collected.

The quantitative component of the research was collected at a single point in time demonstrating that the data were cross sectional. The scores from both assessments were analyzed using descriptive statistics and the Rasch model (a specific model within the item-response theory or IRT). The results from the IRT model were disaggregated into multiple representations, including a person-item map, two statistical summary maps, an item map, and a student map. Furthermore, descriptive statistics were computed then analyzed and later organized in a tabular format.

In the qualitative study, the researcher sought to gain better insight regarding the association between conceptual understanding of mathematics vocabulary and students' performance on problem solving tasks. The researcher used the convenience method and selected only three participants, based on their vocabulary scores, from the quantitative study. The three participants completed the tasks during the last week of school in an eighth grade classroom. The participants were asked to verbalize their thinking using the think-aloud technique (Johnson and Christensen, 2004). The interviews were audiotaped and later transcribed along with a narrative summary of the researcher's reflection of each participant based on what the researcher observed.

Data Collection

Assessment, questionnaires, interviews and observations are a few methods to collect data for educational research studies (Johnson & Christensen, 2004). Merriam (2009) states researchers are continuously collecting data; although, they do not know what will be discovered or what the final analysis will uncover. This requires the data to be systematically analyzed while it is being collected for interpretation and relevance.

In this study, 286 eighth grade students received a copy of both the student assent and parent consent forms requesting permission for the researcher to use their student's data. A total of 136 or approximately 48% of consent/assent forms were returned giving the researcher permission to use their student's data. However, only 135 students' data were utilized for the vocabulary assessment and 131 students' data were utilized for the comparative analysis of the vocabulary assessment and the mathematics portion of the CRCT. The discrepancy was in part due to four students who did not attempt to complete the mathematics portion of the CRCT test and one student who did not complete the vocabulary assessment.

Quantitative Design

In the quantitative study, the entire population of eighth grade students (with the exception of the self-contained special education students) was administered the mathematics vocabulary assessment the second full week of May 2014. The mathematics teachers administered the assessment to their students in a quiet classroom, absent of visual aids, to assist with establishing the validity of the assessment. Students who required extra time for testing (according to their Individualized Education Program or IEP) were provided extended time as needed. However, it is noteworthy to point out that the majority of the students completed the test within the allotted class period of 60 minutes. The vocabulary assessments were completed on a scantron form and graded utilizing a scantron machine. This device is capable of numerically grading and tabulating multiple tests within a short period of time. The scores from the vocabulary assessment were compared to the mathematics CRCT results, which were administered in April 2014, several weeks prior to the administration of the mathematics vocabulary assessment. It was the researcher's intention to examine if the results from the two assessments show a correlation between mathematics vocabulary knowledge and student achievement.

Qualitative Design

For the qualitative study, the researcher utilized an Internet based random generator (www.random.org) to select three participants from the eighth grade population. Charters (2003) states that qualitative researchers believe that any interested participant can offer a unique perspective or something valuable to a study (Charters, 2003). Thus, all of the students who returned their signed assent/consent forms names were alphabetically listed in an excel spreadsheet with their corresponding vocabulary scores. There were 136 students' names on the list (prior to discovering five students' data would be eliminated); therefore numbers 1 and 136 were inputted into the random generator as the minimum and maximum values accordingly. Dozens of numbers were generated until one participant from each category (below average, average, above average) was selected per their vocabulary scores, which ranged from 24 to 100. The mathematics vocabulary assessment scores were divided as follows: 24 – 49 (below average), 50 – 75 (average), and 76 – 100 (above average). The researcher selected participants from each category to examine their knowledge of mathematics vocabulary as it relates to their problem-solving skills using the think-aloud technique. A copy of the assent and consent forms are in the appendix (Appendix H and Appendix I).

Think Aloud Technique

Johnson and Christensen (2004) state that the think-aloud technique requires the participants to verbalize their perceptions and thoughts while completing a task. This process is helpful in determining whether the participants are properly interpreting an assessment (Johnson & Christensen, 2004). Charters (2003) describes the think-aloud technique as a form of information processing. It was Vygotsky's work that developed the relationship between thought and verbalized inner speech. Vygotsky believed that it was difficult to truly access what participants were thinking when solving problems. Yet, he believed that their spoken utterances are closely aligned with their inner speech thus providing insight into their individual thinking (Charters, 2003). Ideally, Charters (2003) states that the task should be cognitively demanding but not too overwhelming because it may interfere with the verbal utterances from the working memory. However, the task should not be too easy requiring a mere automatic response. For these reasons, the cognitive task should be constructed based on an intermediate level for the participants to employ verbal utterances that provide a natural and correct think-aloud response (Akyel & Kamisli, 1996; Charters, 2003; Ericsson & Simon, 1980; Pressley & Afflerbach, 1995). These utterances are later organized and processed beginning the initial phase in data analysis. In the analysis stage, the data were segmented producing descriptions or themes, a process known as coding (Creswell, 2012). Coding the data was a multi-step inductive process that the researcher utilized to find emerging themes to describe the qualitative data as a result from the think-aloud technique (Creswell, 2012). The entire coding process is completely described in chapter four (findings).

Think Aloud Protocol

During the last week of school each of the three selected students were individually administered the two problem solving tasks in a classroom with only the researcher present. Prior to giving the students the tasks, the students were instructed of their expectations (i.e. no questions or time limits) and were asked to try to relax during the recorded session. Following Ericsson and Simon's (1980) protocol, the researcher did not suggestively coach nor ask the students leading questions to substantially reduce perceived biases; rather the students were advised to keep talking if they initiated long pauses. Upon the conclusion of the recorded session, the researcher played the audiotape back to each participant requesting each of them to recall their thoughts as they were completing the task. This additional step, known as reflective questioning, added depth and expanded on the think-aloud results. Additionally, Charter (2003) notes that the think-aloud technique is a valuable method utilized in qualitative research because it assists in exploring participants' thought processes while aiding the researcher to develop a generalization about the conducted study (Charter, 2003). A copy of the directions is located in the appendix (Appendix J).

Problem-Solving Tasks

Muir, Beswick and Williamson (2008) define problem solving as a sequence of steps used to find a solution to a problem that is not readily known. Stein, Smith, Henningsen and Silver (2000) state that high-level cognitive (problem-solving) tasks place demands on students to use reasoning skills based on their prior knowledge. In order for students to use their reasoning skills, they must have sufficient time to complete complex tasks and asked thought provoking questions by the teacher (or a capable peer) to sustain pressure for a realistic answer (Stein, Smith, Henningsen & Silver, 2000).

Although the students did not sustain any pressure, the researcher remained determined in examining how students' conceptual understanding of mathematics vocabulary impacted their ability to problem-solve. The two tasks the participants were required to complete for the study were based on Georgia's Common Core Performance Standards. The first task was centered around unit 5 (linear equations), primarily focused on the concept of slope or rise/run. Students were asked to create two points on a coordinate plane and find the slope of their line. Additionally, the students were required to explain the slope of their line if x represented hours worked and y represented money earned. The expectation for the students was to plot two random points, connect them and explain their line.

The second task originated from the transformations unit. The transformations unit is arguably one of the densest vocabulary units taught in the eighth grade Common Core curriculum. Due to its high volume of mathematic terms, students are required to have fundamental knowledge of the vocabulary situated in the unit to effectively complete many of the pre-designed tasks suggested by the state. One of the lessons students are required to learn in the transformations unit involve transversals of parallel lines. Thus, in the second task, students were expected to find the missing angle measures from a diagram displaying a set of parallel lines and two transversals. Although there were multiple algorithms the participants could have utilized in order to solve the second task, the students had to possess some fundamental knowledge of congruency, parallel lines and transversals to adequately complete the task. Consistent with Stein, Smith, Henningsen & Silver (2000) recommendation, the interrelated problem-solving task involving vocabulary and mathematics compelled the participants to utilize their prior knowledge to sufficiently improve their reasoning skills. Following Johnson and Christensen (2004) suggestion, the researcher instructed the participants to utilize the think-aloud technique for both tasks in an effort to determine how the participants were utilizing their reasoning skills and vocabulary understanding to problem solve. Similar to the quantitative results, the results for the qualitative study have been documented, interpreted, and summarized in chapter four. A copy of both problem-solving task are in the appendix (Appendix K).

Ethical Consideration

Trustworthiness/Credibility

Merriam (2009) states that it is the researcher’s responsibility to ensure trustworthiness and credibility to carry out a study in an ethical manner (Merriam, 2009). During this process the researcher opened herself to a journey in which she was uncertain of the eventual outcome. She continuously took precautionary measures and requested guidance from more knowledgeable peers and from committee members to ensure that she did not make adverse decisions, which could later be considered as questionable or objectionable.

Johnson and Christensen (2004) stated that it is the researcher's responsibility to ensure that the interview has been accurately transcribed verbatim between the participants and the researcher (Johnson & Christensen, 2004). The researcher transcribed and hand coded each of the participants' words verbatim, integrating, and fully disclosing observations and journal notes in the transcripts. All participants had proper consent documentation (consent/assent forms) prior to the researcher utilizing their data. The researcher did not bargain, wager, coerce, intimidate or influence any of the participants into committing to the study. All testing materials and results were secured in a locked safe for no other person to access. During the study, no information was purposely omitted or changed in any regard, which could compromise the study or be construed as unethical behavior.

Institutional Review Board

In compliance with Kennesaw State University guidelines, the researcher ensured that the data utilized in this study was from students who signed and returned the appropriate documentation. The researcher understood that there were human participants utilized in this study. As a standard, the researcher always practiced academic honesty, integrity, and followed protocol when working with the participants, handling their work, and/or analyzing their data. All documentation has been secured in a locked safe at the middle school facility for a total of three years upon completion of this study. After the storage time expires the information gathered will be destroyed.

Limitations

As with any research study there are limitations. Creswell (2012) defines limitations as potential weaknesses including but not limited to small sample sizes, losing participants, errors in measurement and insufficient measures of variables (Creswell, 2012). The researcher distributed assent and consent forms to the eighth grade population of students to create an ample sample size. However, only 136 forms were returned and of which only 131 participants' data were utilized making a less than desirable sample population. Although there were a host of explanations for lack of participation, the researcher believed that some of the reasons were as follows: lack of trust in the study or of the researcher (What is she really going to do with my child's data?), low interest in participating, and simple disregard or memory lapse for returning the requested form. Furthermore, the vocabulary assessment was administered in the last week of school when many students are less than enthusiastic about their educational setting and considerably more focused on the summer break. Thus, the low scores on the vocabulary assessment may somewhat be attributed to students' motivation rather than their actual knowledge.

Last, The CRCT assessment is an instrument utilized to measure students' overall academic understanding of each content area. The CRCT test was not regulated or examined by the researcher to determine the quantity or depth of vocabulary terminology required to successively pass the standardized test. Therefore, the researcher had to presume that the mathematics vocabulary listed on the CRCT assessment is comparable with the terminology specified in the mathematics vocabulary assessment.

CHAPTER FOUR

FINDINGS

The purpose of this study is to determine if there is a relationship between vocabulary acquisition and students' achievement in mathematics. Johnson and Christensen (2004) stated that for every mixed research study, a collection of qualitative and quantitative data are appropriately collected, analyzed, and interpreted (Johnson & Christensen, 2004). In this chapter, the results for both quantitative and qualitative data have been collected and analyzed in order to carefully answer both research questions. For the quantitative analysis the researcher sought to answer the following research question:

- Is there a correlation between the acquisition of mathematics vocabulary and students' achievement in mathematics?

Analysis of Quantitative Data

While there were 136 students and parents who gave the researcher permission to use their student's data, the results for the vocabulary assessment are primarily based on a sample population of 135 eighth grade students because one of the students did not complete the vocabulary assessment. Unfortunately, four additional students did not complete the mathematics portion of the CRCT thereby reducing the data size to 131 for the comparative analysis of the vocabulary assessment and the mathematics portion of the CRCT.

Vocabulary Assessment

The vocabulary assessment was actually created over a period of weeks with the assistance of eight middle school mathematics teachers. It was designed to measure eighth grade students' comprehensive understanding of key middle grades mathematics vocabulary terms. Originally, the vocabulary assessment was based on 42 questions, however, it was discovered (after the test was administered to the population of students) that question number seven could yield two possible answers. Consequently, the results for the vocabulary assessment specified in this chapter is based on a 41-question assessment due to the omission of question number seven. Described in this chapter are results from the quantitative and qualitative tests and reasonable evidence that the mathematics vocabulary assessment created and utilized by the researcher is a valid and reliable instrument.

Reliability and Validity

To establish reliability and construct validity, the Rasch measurement model was utilized to critically analyze the students' data on the vocabulary assessment. The Rasch model, unlike conventional statistics, utilizes interval measures to estimate difficulty and ability levels for statistical analysis (Bond & Fox, 2007). Figure one is a person-item map representing the Rasch analysis of the vocabulary assessment completed by 135 students and was constructed using the WINSTEPS software program. The data distributed on the person-item map is displayed on a logit (log odd units) scale with measures ranging from negative three to positive five. On the left-hand side near the top of the logit scale are the students with the highest ability and students with the lowest ability are placed near the bottom of the scale. In compliance with confidentiality,

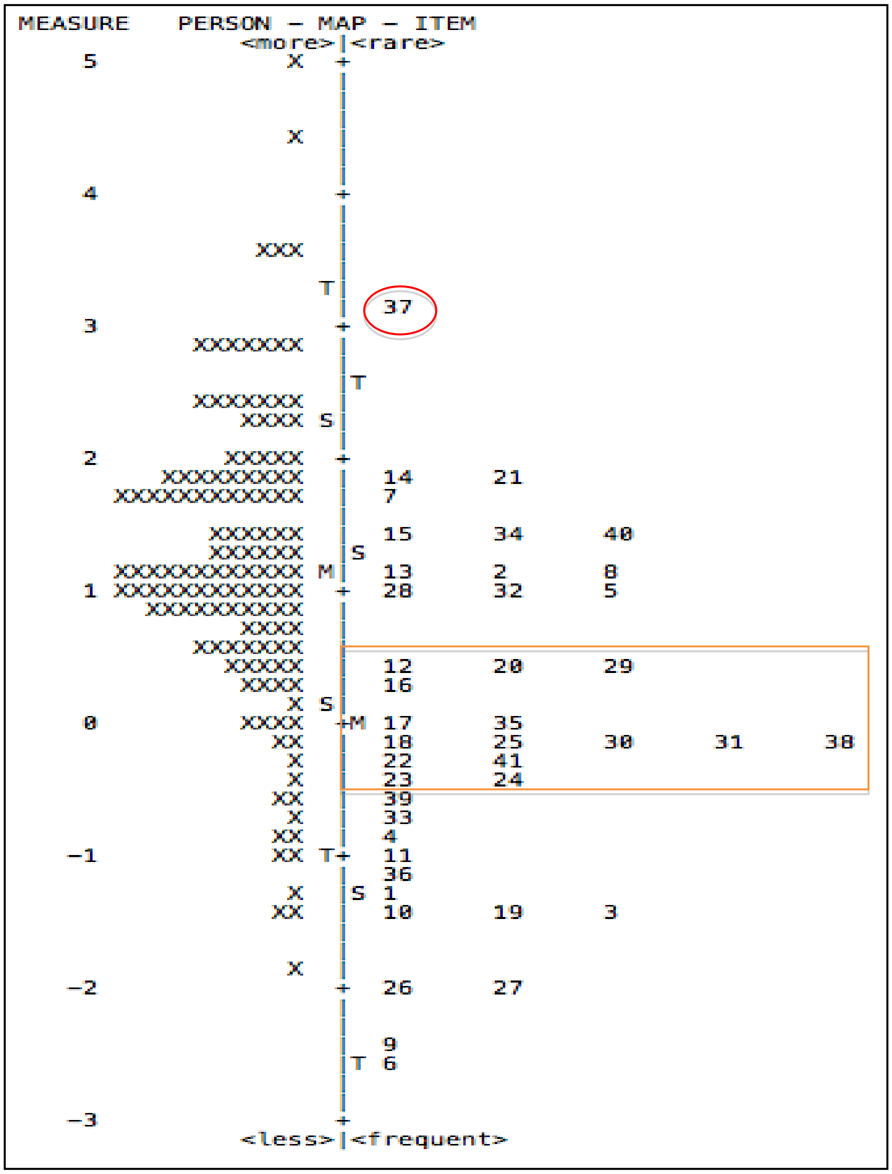


Figure 1. Person-item map for the vocabulary assessment.

each of the 135 students is represented with an x on the logit scale. Accordingly, items placed near the top on the right-hand side of the logit scale are the items from the vocabulary assessment that were identified as difficult questions. The items placed near the bottom of the scale have been identified as the easiest questions. Based on the person-item map, item number thirty-seven is the most difficult problem and item number six is the easiest problem. Ideally, the researcher would have preferred that the person-item map be created without any gaps in the data. However, there are some noticeable gaps in the data displayed in figure one. The largest gap occurs between measures 1.89 and 3.13 (specifically enumerated in table five), which may indicate that there is a possible problem with the construct validity with item number 37. Additionally, the item statistic map (table five) shows that item number 37 has an outfit mean square (mnsq) value of 1.73, which is considerably outside the reasonable range of 0.8 – 1.2. Bond and Fox (2007) state that the infit and outfit statistics are reported in chi-squared statistical analysis as mean squares. With an expected value of +1, the mean square statistics is used to check the variation between observed and the predicted response patterns to determine if the data and the "model-predicted response patterns" are compatible (Bond & Fox, 2007). This means that item number 37 does not fit what it was intended to measure. However, this did not have any effect on the general analysis regarding the reliability and validity of the vocabulary assessment.

Table 5

Item statistic map (measure order)

Entry Number	Total Score	Total Count	Measure	Model S.E.	Infit MNSQ	Infit ZSTD	Outfit MNSQ	Outfit ZSTD	PTMEASURE-A CORR.	PTMEASURE-A EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%
37	23	135	3.13	.25	.98	-.1	1.73	2.2	.34	.37	83.6	84.6
14	49	135	1.89	.20	.94	-.7	1.07	.5	.45	.42	72.4	70.9
21	49	135	1.89	.20	1.09	1.1	1.20	1.3	.34	.42	69.4	70.9
7	55	135	1.66	.19	1.24	3.1	1.41	2.9	.21	.42	59.7	68.9
34	60	135	1.47	.19	1.14	2.0	1.24	1.9	.30	.42	64.9	67.9
15	61	135	1.44	.19	1.01	.2	1.13	1.1	.39	.42	70.1	67.8
40	63	135	1.36	.19	1.07	1.0	1.25	2.0	.35	.42	67.2	67.6
2	69	135	1.15	.19	1.08	1.2	1.17	1.5	.35	.42	65.7	67.0
8	70	135	1.11	.19	1.15	2.2	1.20	1.7	.30	.42	56.0	67.1
13	70	135	1.11	.19	1.06	.9	1.08	.8	.37	.42	64.9	67.1
5	72	135	1.04	.19	1.10	1.4	1.23	1.9	.34	.42	61.9	67.2
28	72	135	1.04	.19	1.00	.0	1.06	.5	.41	.42	70.9	67.2
32	74	135	.96	.19	.99	-.1	1.00	.0	.43	.42	66.4	67.4
29	87	135	.47	.20	1.09	1.1	1.05	.4	.35	.41	67.9	71.4
12	89	135	.39	.20	.97	-.3	.90	-.7	.44	.41	70.1	72.3
20	89	135	.39	.20	1.00	.0	.95	-.3	.42	.41	71.6	72.3
16	90	135	.35	.20	1.02	.2	1.00	.1	.40	.41	73.9	72.8
17	99	135	-.04	.21	1.11	1.0	1.31	1.6	.28	.40	76.1	77.3
35	99	135	-0.4	.21	.89	-1.0	.76	-1.4	.49	.40	79.1	77.3
25	100	135	-0.8	.22	.99	-.1	.94	-.2	.40	.39	80.6	77.8
18	101	135	-.13	.22	.94	-.5	.88	-.6	.45	.39	76.9	78.3
30	101	135	-.13	.22	.92	-.7	.78	-1.2	.47	.39	78.4	78.3
38	101	135	-.13	.22	1.02	.2	.90	-.5	.39	.39	76.9	78.3
31	102	135	-.18	.22	.95	-.4	.88	-.5	.43	.39	80.6	78.8
22	103	135	-.23	.22	.96	-.3	.90	-.4	.42	.39	81.3	79.4
41	104	135	-.28	.22	.94	-.5	.89	-.4	.43	.38	82.1	79.9
23	107	135	-.44	.23	1.13	1.0	1.12	.6	.28	.38	79.1	81.5
24	107	135	-.44	.23	.93	-.5	.91	-.3	.43	.38	83.6	81.5
39	109	135	-.55	.24	.84	-1.1	.73	-1.1	.49	.37	85.8	82.6
33	112	135	-.73	.25	.89	-.7	.69	-1.2	.46	.36	85.1	84.2
4	113	135	-.79	.25	.97	-.1	.76	-.8	.41	.35	83.6	84.8
11	116	135	-.99	.27	94	-.3	.85	-.4	.39	.34	85.8	86.6
36	118	135	-1.14	.28	.72	-1.6	.63	-1.1	.53	.33	90.3	87.8
1	119	135	-1.22	.29	1.11	.6	1.21	.7	.22	.32	88.1	88.5
19	121	135	-1.40	.30	.85	-.7	.59	-1.1	.44	.31	90.3	89.7
3	122	135	-1.49	.31	.92	-.3	.69	-.7	.38	.30	91.0	90.4
10	122	135	-1.49	.31	.89	-.5	.70	-.6	.39	.30	91.0	90.4
26	126	135	-1.94	.36	.71	-1.1	.26	-1.9	.51	.26	93.3	93.3
27	126	135	-1.94	.36	.81	-.6	.48	-1.1	.41	.26	93.3	93.3
9	129	135	-2.42	.43	.96	.0	.69	-.3	.27	.22	95.5	95.5
6	130	135	-2.62	.47	1.00	.1	.67	-.3	.23	.21	96.3	96.3
Mean	93.4	135.0	.00	.24	.98	.1	.95	.1			78.1	78.6
S.D.	25.8	.0	1.27	.07	.11	1.0	.27	1.1			10.4	9.0

To examine if the teachers and students' perception of the vocabulary assessment align with the intermediate-level items listed on the logit scale, the researcher elected to compare the participants' perceived level of difficulty with the Rasch analysis. Item numbers 12, 16, 17, 18, 20, 22, 23, 24, 25, 29, 30, 31, 35, 38, 41 are highlighted in the rectangular box in figure one and are identified as the intermediate-level problems ranging from -0.5 to 0.5 on the logit scale. The same items are bolded in table six to visually enhance where the numbers are displayed and categorized based on the participants' perception. Based on the bolded numbers, the teachers and students' perceived the majority of intermediate-level items as either easy or medium. This indicates that the theoretical data reasonably coincides with the experimental data, by which the participants' perceived level of difficulty was a realistic representation of the actual exam administered. Interestingly, the Rasch analysis identified item number 37 as the most difficult problem; however, none of the participants marked item number 37 as neither difficult nor as a medium-level question. Intriguingly, item number 37 was marked as easy by all of the participants; this irregularity will be further examined in chapter five.

Table 6

Question/item #37 (level of difficulty)

	Easy	Medium	Difficult
Teacher (6th grade)	1, 3, 4, 5, 11, 15, **20**, 21, **22**, **23**, **25**, 27, 28, 32, 34, 36, 37, **38**	2, 6, 7, 8, 9, 10, **12**, 13, 14, **16**, **17**, **18**, 19, **24**, 26, **29**, **30**, **31**, 33, **35**, 39, 40, **41**, 42	
Teacher (6th grade)	1, 3, 4, 5, 6, 7, 8, 9, 10, 11, **12**, 13, 15, **17**, **18**, **20**, 21, **22**, **23**, 28, **31**, 37, **38**, 40	2, 14, **16**, 19, **24**, 26, 27, **29**, **30**, 32, 33, 34, **35**, 39, **41**, 42	**25**, 36
Teacher (7th grade)	1, 3, 5, 6, 7, 8, 10, 11, 15, **17**, **18**, **20**, 21, **22**, **23**, 26, 28, 32, 33, 34, 37, **41**	2, 4, 9, **12**, 13, 14, **16**, 19, **24**, **25**, 27, **29**, **35**, 36, **38**, 39, 40, 42	**30**, **31**
Teacher (7th grade)	1, 3, 7, 8, 10, 11, 13, **20**, 21, **22**, 26, 27, 28, 32, 34, 37	4, 5, 6, 9, **12**, 14, 15, **16**, **17**, **18**, 19, **23**, **24**, **25**, **29**, **30**, **31**, 33, **35**, 36, **38**, 39, **41**, 42	2, 40
Teacher (7th grade)	1, 2, 3, 5, 6, 7, 10, 11, 14, 15, **20**, **23**, 28, 37, **38**, 39, **41**	4, 8, 9, **12**, 13, **16**, **17**, **18**, 21, **22**, **24**, **25**, 27, **29**, **30**, **31**, 32, 34, 40, 42	19, 26, 33, **35**, 36
Teacher (8th grade)	1, 3, 4, 5, 6, 7, 10, 13, **20**, 21, **22**, **23**, 26, 27, 28, 32, 34, 36, 37, **38**, 40, **41**, 42	2, 8, 9, 11, **12**, 14, 15, **16**, **17**, **18**, **24**, **25**, **29**, **31**, **35**, 39	19, **30**, 33
Teacher (8th grade)	1, 3, 4, **12**, 15, **16**, 34, 37, 40, 42	2, 5, 6, 7, 8, 9, 10, 11, 13, 14, **17**, **18**, **20**, 21, **22**, **23**, **24**, **25**, 26, **31**, 32, 33, **35**, 36, **38**, 39, **41**	19, 27, 28, **29**, **30**,
Teacher (8th grade)	1, 3, 6, 7, 10, 11, 13, **17**, **20**, 21, **22**, **23**, **24**, 27, 28, **30**, 34, 36, 37, **38**, 40, **41**, 42	4, 5, 8, **12**, 14, **16**, **18**, 19, **25**, **29**, **31**, 32, 33, 39	2, 9, 15, 26, **35**
Teacher (8th grade)	1, 3, 4, 5, 6, 7, 8, 10, 11, **12**, 13, 15, **17**, **18**, **20**, 21, **22**, **23**, **24**, 26, 28, **29**, **30**, **31**, 32, 34, 36, 37, 39, 40, **41**, 42	2, 9, 14, **16**, 19, **25**, 27, 33, **35**, **38**	
Student (Female)	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, **12**, 13, 14, 15, **16**, **17**, **18**, 19, **20**, 21, **22**, **23**, **24**, 26, 27, 28, **29**, **30**, **31**, 32, 33, 34, **35**, 36, 37, **38**, 39, 40, **41**, 42	7	**25**
Student (Male)	1, 2, 3, 4, 5, 6, 37, **38**, 39, 40, **41**, 42	7, 8, 9, 10, 11, **12**, 13, 14, 15, **16**, **17**, **18**, 19, **20**, 21, **22**, **23**, **24**, **25**, 26, 27, 28, **29**, **30**, **31**, 32, 33, 34, **35**, 36	

Table 7 is a WINSTEPS table that provides the summary statistics of the population of students studied. In table 7, the student achievement levels ranged from 8 to 41 with a mean score of 28.40 and the standard deviation measure of 6.7. The maximum raw score of 41 indicates that one student earned a perfect score on the mathematics vocabulary assessment. The person reliability index of .83/.84 with a standard error mean of .10 indicates that if the same sample population of students were administered another test with the same type of questions measuring the same construct; the students would achieve a comparable score (Bond & Fox, 2007). The data displayed in table 7 establishes (person) reliability for the vocabulary assessment. Additionally, both person to raw score-to-measure correlation and the Cronbach alpha person raw score test reliability is .97/.86, which is close to the expected value of 1.0, further establishing (person) reliability.

Table 7

Summary statistics (person) map for the vocabulary assessment

	Total Score	Count	Measure	Model Error	Infit MNSQ	Infit ZSTD	Outfit MNSQ	Outfit ZSTD
Mean	28.4	41.0	1.20	.43				
S.D.	6.7	.0	1.14	.15				
Maximum	41.0	41.0	5.67	1.85				
Minimum	8.0	41.0	-1.83	.36				
REAL RMSE	.47	TRUE SD	1.04	SEPARATION	2.20	PERSON RELIABILITY		.83
MODEL RMSE	.46	TRUE SD	1.05	SEPARATION	2.27	PERSON RELIABILITY		.84
S.E. OF PERSON MEAN = .10								
PERSON RAW SCORE-TO-MEASURE CORRELATION = .97								
CRONBACH ALPHA (KR-20) PERSON RAW SCORE “TEST” RELIABILITY = .86								

In table 8, the summary for the item analysis displays the mean infit of .98, a mean outfit of .95 and a standard deviation of .11/.27 respectively. With both infit and outfit values close to the expected value of 1.0, the researcher concluded that the test items fit its intended construct. The -.98 on the item raw score-to-measure correlation is near the expected value of -1.0, indicating that there is a high probability of success on the test items. The high person reliability of .96 indicates that the test has strong reliability and low variability establishing that the researcher has met the requirements of proving that the test is (item) reliable.

Table 8

Summary statistics (item) map for the vocabulary assessment

	Total Score	Count	Measure	Model Error	Infit MNSQ	Infit ZSTD	Outfit MNSQ	Outfit ZSTD
Mean	93.4	135.0	.00	.24	.98	.1	.95	.1
S.D.	25.8	.0	1.27	.07	.11	1.0	.27	1.1
Maximum	130.0	135.0	3.13	.47	1.24	3.1	1.73	2.9
Minimum	23.0	135.0	-2.62	.19	.71	-1.6	.26	-1.9
REAL RMSE	.25	TRUE SD	1.24	SEPARATION	4.88	PERSON RELIABILITY		.96
MODEL RMSE	.25	TRUE SD	1.24	SEPARATION	4.94	PERSON RELIABILITY		.96

S.E. OF ITEM MEAN = .20

ITEM RAW SCORE-TO-MEASURE CORRELATION = -.98

5494 DATA POINTS. LOG-LIKELIHOOD CHI-SQUARE: 4953.68 with 5320 d.f. p=.9999

Global Root-Mean-Square Residual (excluding extreme scores): .3846

Capped Binomial Deviance = .1944 for 5535.0 dichotomous observations

UMEAN = .0000 USCALE = 1.0000

Georgia Criterion Reference Competency Test (CRCT)

The second section of the quantitative analysis compares the scores of the mathematics vocabulary assessment with the scores on the mathematics portion of the yearly Georgia CRCT assessment. The students' data were formatted in an Excel spreadsheet in two columns for analysis. The vocabulary assessment scores were based on the number of correct responses out of the total number of problems. Accordingly, the CRCT scores were collectively accessed, evaluated, and gathered for this study. In the

Georgia Department of Education Interpretation Guide (2014), there was a genuine effort made to effectively communicate the process of how students' scale scores are transformed from a raw score to an achievable performance level. Based on the interpretation guide, a minimum performance level of 800 will yield a meets level on the Georgia CRCT assessment, while an 850 or better results will yield an exceed performance level.

Statistical Analysis of the Assessments

The two scores from the CRCT and the vocabulary assessment were analyzed using descriptive and inferential statistics. Table nine displays a descriptive analysis of both assessments; the independent variable (*x*) was the mathematics vocabulary

Table 9

Descriptive statistics for vocabulary scores (x) and CRCT scores (y)

	Min Score	Max Score	Mean	Standard Deviation
Vocabulary Scores (x)	24%	100%	69.82	15.77
Students' CRCT Scores (y)	765	990	846.20	38.80

assessment, while the dependent variable (*y*) was the CRCT scores. The data show that the vocabulary assessment has a mean score of approximately 70 (passing) and a mean score for the CRCT as 846 (meets), with standard deviation of 15.77/38.80 respectively.

The Pearson correlation (r) was utilized to examine if there was a correlation between the two variables using the formula below:

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{X_i - \bar{X}}{s_X}\right)\left(\frac{Y_i - \bar{Y}}{s_Y}\right)$$

Moore, McCabe and Craig (2012) described how the correlation r measures the strength and direction associated between two quantifiable variables. Correlation r determines the strength of the variables $-1 \leq r \leq 1$, whereas a score close to -1 is an inverse correlation and a score close to +1 is a direct correlation (Moore, McCabe & Craig, 2012). The correlation *r* for the x and y variables was .67 establishing that there is a positive association between the variables (fig. two). The correlation between the two variables and the p-value was calculated at p=0.0001, which is less than the significance level of p=0.05 convincingly indicating that the data were statistically significant.

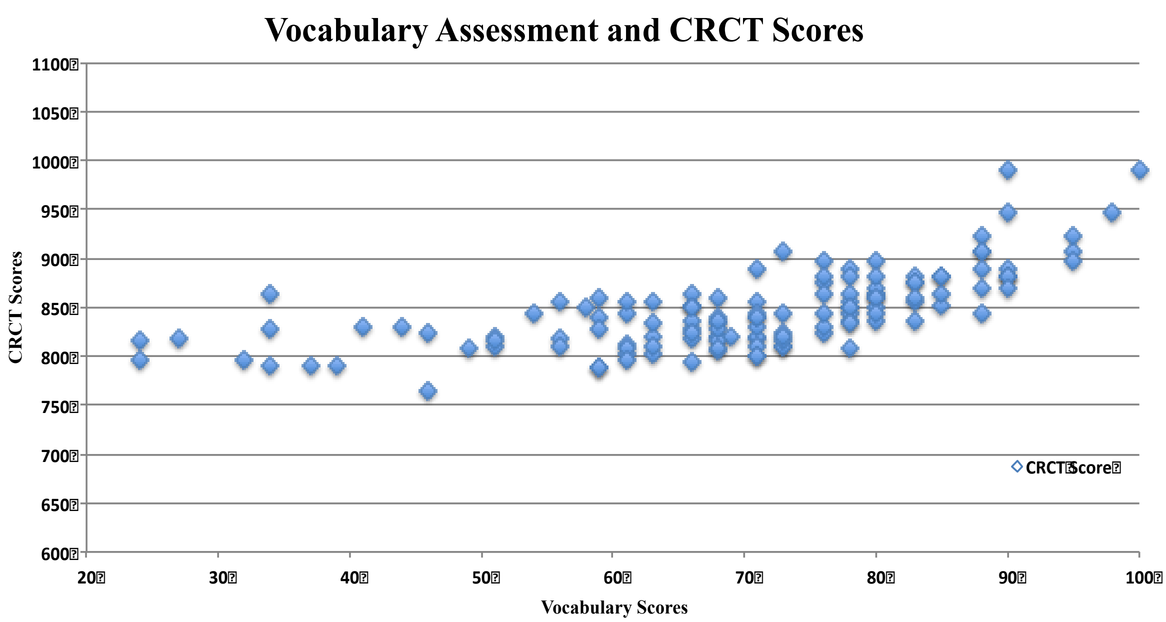


Figure 2. Scatter plot results for the Pearson Coefficient Correlation r (131 students).

Analysis of Qualitative Data

In the qualitative research study, three participants from a sample population of 135 students were randomly selected to participate in the qualitative study based on their individual vocabulary assessment scores. The random selection process involved the researcher listing all of the approved students names and vocabulary scores in an excel spreadsheet. With the assistance of a random generator, three students were selected and asked to participate in the qualitative study. Two of the students selected were males and one student was a female. Using the pseudonyms Lawrence, Phillip, and Sara, the researcher made sure that each of the students fell into one of the following three categories: 24 – 49 (below average), 50 – 75 (average), and 76 – 100 (above average). Using participants from each category provided the researcher with the opportunity to examine a range of students' knowledge of mathematics vocabulary as it relates to their mathematical skills. The data aided in answering the research question for the qualitative component of the study.

- How does conceptual understanding of mathematics vocabulary impact students' ability to problem solve?

Participants

Lawrence

Lawrence was an active and energetic student who had a difficult time staying focused in mathematics class. His choice of entertaining the class hindered his ability to perform well in mathematics. Although it took the teacher several attempts to get Lawrence refocused, he was always respectful and would instantly correct his behavior. However, due to his behavior Lawrence missed many learning opportunities resulting in poor participation in group activities and classroom discussions, which affected his grades and further exacerbated his limited understanding of mathematics. Nevertheless, Lawrence's teachers regarded him as a smart student making inappropriate choices. Understandably, Lawrence would occasionally exhibit frustration and sadness in mathematics class when he was not the focus of attention. Lawrence earned a 24 on the vocabulary assessment and a 798 (does not meet) on the mathematics portion of the CRCT.

Sara

Sara was an enthusiastic student who was learning deficient in reading comprehension. Sara would read and re-read word problems to ensure that she understood the problem before attempting to solve it. She was very involved in class discussions and would often raise her hand to answer questions or to offer an alternative algorithm to solve a problem. Sara was comfortable in her mathematics abilities and incorrect answers did not necessarily deter her from trying harder to get the subsequent problem correct. Sara required the teacher, in some instances, to illustrate and frequently explain solutions in detail. Her inquisitive nature wouldn't allow her to accept an algorithm as an acceptable form of learning; she needed to analyze the material before she could process it in order to achieve more thorough understanding. Sara earned a 63 on the vocabulary assessment and an 833 (meets) on the mathematics portion of the CRCT.

Phillip

Phillip was an advanced mathematics student who earned an 80 on the vocabulary assessment and an 864 (exceeds) on the mathematics portion of the CRCT. Phillip was an extremely respectful and an attentive student in mathematics class. He would often offer his assistance to struggling students and performed well on many of the assessments administered throughout the school year. He regularly used mathematics terminology and the think-aloud protocol to explain his algorithm for solving mathematic problems. Phillip loved the challenge of word problems and would seek alternate algorithms to solve an assortment of mathematics problems.

Interview Process

The students were individually audiotaped in a quiet classroom with only the researcher present during the last week of school. Prior to audiotaping the students, the researcher briefly explained the purpose of the study to ensure that the students felt comfortable participating in the study and was aware of their expectations. Using the think-aloud technique, the students completed two problem-solving tasks, which was later coded for analysis. Audiotaped interviews for each participant are in the appendix (Appendix L).

Coding

The researcher hand coded all of the data for each of the interviews. Creswell (2012) describes coding as a process of dissecting written material to form general themes within the data. Creswell (2012) regards the coding process as inductive because it involves condensing broad themes to fewer or more specific themes. Creswell (2012) notes that there are no specific guidelines in coding data because it is a gradual process used to make sense out of the data. Merriam (2009) states that it is important to code data as it is being collected to ensure that the emerging themes are relevant to the study

Chart 1

Sara's interview for the first problem-solving task

Interview	**Initial Code (Level 1)**	**Category Code (Level 2)**	**Theoretical Concept (Level 3)**
Do you want me to read the question out loud?			
If x equals zero, then you…would….like, put in the origin and then you would…..	Thinking about the problem and correct vocabulary terms	Has some background knowledge of x and y axes Using internal discourse to confirm her thinking	Problem Solving Commognition
You would….uh, uh….you would gain money….but….x would equal zero.	Devising a plan (Polya) through internal discourse (Sfard)	Attempting to retrieve her background knowledge	Problem Solving Commognition
The…the equation of my line would be y = 0, wait no… y….yeah y = 0 cause the, oh I forgot what it is called, no the, the y-intercept would be 2!?	Asking questions and answering herself (Sfard) Trying to recall the vocabulary Thinking/Strategizing or devising a plan (Polya)	Attempting to retrieve her knowledge of linear equations Internal discourse	Commognition Problem Solving Procedural Understanding
Yeah…if x equals 0, then y would have to equal the umm…..it…..it would be undefined because the umm, x would equal 0….ok.	Student is more certain about her answers Answering herself (Sfard) Has been exposed to linear equations	Concluded the first problem – appears to feel good about her answer Internal discourse	Affect Theory (Positive) Commognition Procedural Understanding

because the purpose of data collection is to specifically process and analyze it to ultimately answer the research questions (Merriam, 2009).

Chart one displays a portion of Sara's interview as she completed the first problem-solving task. It is an example of the coding process the researcher utilized to code the participants' interviews from the initial coding level to the construction of the theoretical concepts.

Initial Coding

Yin (2011) states that initial coding or open codes are the first level of coding. Initial coding involves consolidating, interpreting, and analyzing data based on what the researcher has seen and participants have communicated (Yin, 2011). Merriam (2009) describes open coding as bits of data such as notes, comments, questions, and observations that are jotted in the margins. Open coding is the beginning stages of analyzing the data because it is during this process the researcher is open to any emerging information. Assigning codes to bits of data is how the researcher begins to construct categories. The first set of categories might be lengthy initially but many of the categories will be revised, renamed or combined with other categories as the researcher continuously reviews the data (Merriam, 2009). Per Merriam's recommendation, the researcher continuously reviewed and revised the data for this study. Figure three is an example of how the researcher constructed an initial code from Sara's interview as she completed the first task. In the excerpt, Sara is discussing how she would find the equation of her line based on her two points.

Interview

The…the equation of my line would be y = 0, wait no… y….yeah y = 0 cause the, oh I forgot what it is called, no the, the y-intercept would be 2!?

Initial Code

Asking questions and answering herself (Sfard)

Trying to recall the vocabulary

Thinking/Strategizing and/or devising a plan (Polya)

Figure 3. Excerpt of Sara's interview.

Categories

Based on Creswell's (2012) theory for constructing codes, the researcher began with an assortment of codes and slowly reduced it to create categories (Creswell, 2012). The categories were analyzed and reconstructed accordingly. This permitted the researcher to begin the process of drawing inferences based on the data's relevance. Merriam (2009) states that category construction should be exhaustive, conceptually congruent and sensitive to the data. The emerging themes theoretically move the data from concrete to an abstract analysis allowing the researcher to begin theorizing about the data. The conceptual link is how the researcher was able to analyze the data to sufficiently answer the research questions (Merriam, 2009). Figure four is an example of how the researcher progressed from the initial code to category coding (a continuation of the same link from figure three).

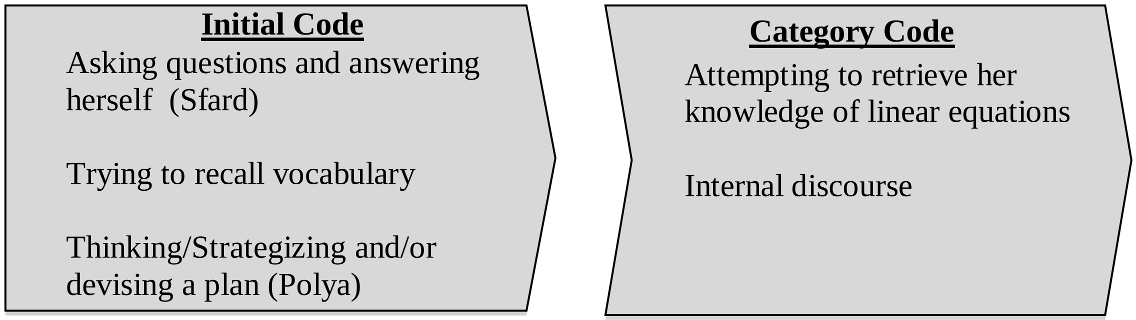


Figure 4. Initial coding to category coding.

Theoretical Concept

Merriam (2009) describes the next step of coding as theorizing the data. However, Merriam warns that theorizing the data can restrict the researcher's thinking rather than expand it (Merriam, 2009). Thus, category scheme may not necessarily provide a comprehensive picture of the guiding principles the researcher requires to further analyze the data to develop a connection with the findings. Figure five diagrams the last step the researcher utilized to code the data (a continuation from figure four).

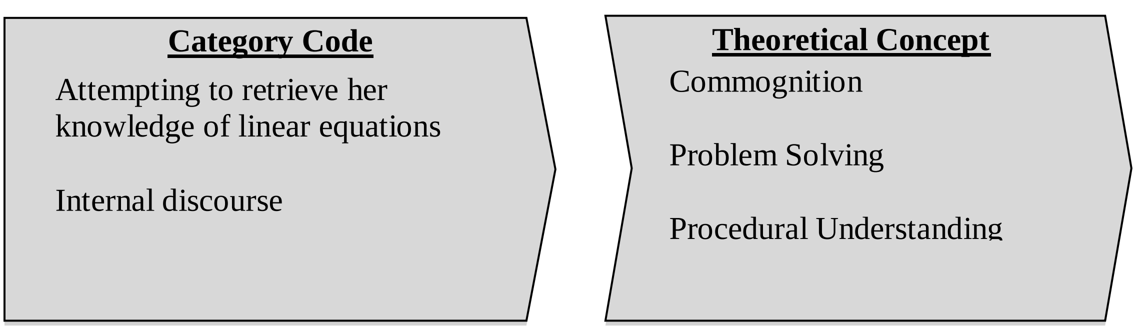


Figure 5. Category coding to theoretical concepts.

Themes

During the coding process, four hierarchal themes (theoretical concepts) emerged (problem solving, commognition, affect, and conceptual understanding) with mathematics vocabulary embedded into all of the themes. The researcher examined the frequency of the codes for both tasks illustrated in table ten.

Table 10

Four emerging themes from the three interviews

Categories	**Lawrence**	**Sara**	**Phillip**
Problem Solving	3	8	7
Commognition	8	7	4
Affect (Positive/Negative)	Positive – 0 Negative – 2	Positive – 3 Negative – 0	Positive – 2 Negative – 0
Understanding (Procedural/Conceptual)	Procedural – 0 Conceptual – 0	Procedural – 4 Conceptual – 0	Procedural – 3 Conceptual – 2

Problem Solving

George Polya, who is highly regarded as the father of problem solving for modern mathematics (Passmore, 2007), believed that students should fully understand a problem prior to attempting to solve it (Poly, 1945). Consequently, Polya developed the four principles of problem solving to assist students in obtaining a solution to a mathematics problem. The first phase was to understand the problem; Polya noted that was important to know what was required of the participant in order to solve the problem. Secondly, there must be a plan in connecting the unknown or undetermined with some information (or previous data) to obtain a solution. The third phase is carrying out the plan, and reviewing the solution is the concluding or fourth phase (Polya, 1945). The two problem solving tasks designed for the participants were selected because they required basic to intermediate vocabulary knowledge. Based on the research question, the researcher sought to examine if conceptual understanding of mathematics vocabulary impact students ability to problem solve. Thus, the two problem-solving tasks were fundamentally created and designed to examine the relationship between the mathematics vocabulary knowledge and problem solving abilities.

The first task requested the students to explain how they would find the slope of a line utilizing two points. The students were instructed to select two points (of their choosing) and identify the equation of the line based on those two points. Although each student read the problem, it was observed that none of the students actually completed it correctly. As a result, the researcher determined, among many possibilities that the instructions for the first task might have been too ambiguous for the students to follow. However, it is noteworthy to state that all of the students interpreted the problem in similar fashion and reached similar conclusions. The amount of hours worked will yield the amount of money earned, thus if you work zero hours you will not earn any money. Nevertheless, the data from both problem-solving tasks provided helpful information for the results for the qualitative portion of the study.

In the second problem-solving task, the students were asked to find the measures of all of the angles in the figure provided and identify each angle (e.g. complementary angle). The second task revealed that there were some noticeable differences between the students. Phillip appeared to be more knowledgeable about solving the task than the other two students. The researcher observed that he calculated his numbers with relative ease and fluently described some of the mathematical vocabulary associated with angle congruence. However, Phillip did not solve for the angles correctly because he made an incorrect assumption regarding angles two, four, and six. Phillip incorrectly stated that angles two, four, and six were adjacent angles. Additionally, he misspoke of angle seven because there was no angle seven. Although the angles shared a common vertex, all three angles did not share a common side.

Excerpt of Phillip's interview:

> I know angle four, two, six and seven are 180 degrees so Imma need to subtract 180, because they are adjacent angles, Imma subtract 180 minus 46 (46 is the value of angle four – Phillip begins computations).

In the excerpt, Phillip utilized his understanding of adjacent angles and straight angles and embarked on a series of incorrect answers failing to use the necessary knowledge to complete the task. Unlike Phillip, Lawrence did not possess any comprehension of the vocabulary associated with the problem. He was unable to accurately identify or calculate any angle as indicated in the following excerpt.

Excerpt of Lawrence's interview:

> Ummm….six and three are comple….., six and three are….complementary and one and four are…complementary (mispronouncing complementary). One and two are complementary. Ummm…four and six are congru…congr…umm congruent, I guess. Let's say congruent…that's all I know.

In the excerpt, Lawrence repeatedly misused and stumbled over the term complementary and incorrectly identified angles four and six as congruent. Due to Lawrence's lack of vocabulary comprehension, he was unable to make any inferences to logically attempt the second problem-solving task.

Commognition

Several years ago, Sfard (2008) coined the term commognition in an effort to stress that individualized thinking and inter-personal communication are interrelated to one another (Sfard, 2008). Thus, if thinking is a form of communication then, it was the researcher's objective to get the students to verbalize their thinking. Using Ericsson and Simon's (1980) protocol the researcher requested that the students continue talking throughout the problem-solving process to gain insight into their thinking. The interview that would best represent a participant communicating all of their inner speech or utterances was the interview the researcher experienced with Phillip. The researcher never had to tell Phillip to talk while he was completing either of the two tasks. He consistently uttered his thoughts except when he was performing basic mathematics operations.

Sara's interview was the lengthiest because she actually verbalized her computations in great detail. The researcher observed that she was meticulously careful about computing her work accurately. Sara spoke clearly while completing her tasks, she would often initiate long pauses when she was unsure of her next step or was searching for the correct terminology she wanted to use to describe her work. She would often tap her pencil in a rapid pattern as if it was a retrieval cue for the correct mathematical term she was trying to recall. Below is an excerpt of Sara's interview exhibiting some of the characteristics the researcher observed.

Excerpt of Sara's interview:

> You would subtract…six, no, you would (tapping)…oh, they are complem e n t a r y angles (Sara slowly pronounces complementary, dragging out the word) and complementary angles equal 180… (under her breath, she whispers) supplementary…supplementary, ok (Speaking normally she begins again.).

In the excerpt, Sara repeated the term complementary angles, almost to herself, to confirm that she was using the correct term. Sara's verbal repetition was clearly one of the basic tenets of Sfard's (2001) research. Sara was thinking or communicating with herself, which Sfard (2001) explains is a private form of communication where we inform, argue, ask ourselves questions, and wait for our own answer (Sfard, 2001).

Lawrence, however, inaudibly spoke and clearly fumbled over his words as he slowly wrote a few notes as he completed the first task in the following excerpt.

Excerpt of Lawrence's interview:

> So, if x is zero and you didn't work anything so….and then find the slope of the…using two points…using two point (repeats)….ummm….x equals zero, so…(pause)….x equals (in a whisper)…. ummm, I don't know how to figure out this problem.

The researcher had to remind Lawrence to talk more than the other two students. It was later determined that Lawrence could not do either problem because he informed the researcher that he did not know how to figure out the first problem and he abruptly stopped working on the second problem. Although his silence was not substantial enough to conclude that he was completely unaware of the material; it was, however, indicative that he lacked the necessary skills to initiate any form of self-communication with himself.

Affect (Positive/Negative)

DeBellis and Goldin (2006) state that affect is a fluid state of emotional feelings in which an individual may or may not be consciously aware of when he or she is problem solving. Thus, affect can empower students motivating them to seek better understanding or it can disempower them, which can lead to frustration (DeBellis & Goldin, 2006). McLeod (1988) describes frustration and panic as one of the more intense emotions students can experience when problem solving; especially if they are inexperienced problem solvers and have worked on the problem for an extended period of time. Additionally, students who are successful problem solvers tend to express satisfaction and even joy. McLeod describes the range of emotions students experience (both negative and positive) are essential factors for problem solving performance (McLeod, 1988).

Based on table ten, the data indicate that Lawrence was the only student who exhibited negative affect during his interview. Lawrence's non-verbal body language indicated that he was uncomfortable or possibly embarrassed by his lack of knowledge. He would often look at the researcher as though he was seeking feedback regarding his performance. Periodically, looking down at his paper, Lawrence appeared like he was gazing at the problems not necessarily processing what he needed to do next to complete the task. Upon the conclusion of the interview, the researcher asked Lawrence if knowing the vocabulary would have helped him perform better on the tasks. Lawrence simply replied, "yes" later stating that he felt that he could have done better. The researcher reassured him that he did well and thanked him for his participation.

Sara was positive during her interview and appeared self-assured with her work. As she worked on her task, she would shift in her seat in an anticipatory state. Similar to tapping her pencil, her movements indicated that she was on the cusp of retrieving the correct answer. Her movements did not appear nervous but motivated to prove that she was knowledgeable on the topic she was requested to complete. Although, she stumbled with some of the correct mathematic terms, it did not deter her from using them in context based on her level of understanding. Sara's positive demeanor revealed that Sara was reasonably comfortable with her mathematics ability and satisfied with her personal understanding of mathematics and the derived vocabulary associated with it.

Unlike Sara or Lawrence, Phillip's emotion was difficult to read because he appeared casual and nonchalant throughout the entire interview. Phillip did not exhibit nor verbally express any spectrum of emotion until he completed the tasks. When he completed a task, he would nod in approval of his work, which could be contrived as exhibiting a prideful performance. He was goal oriented and was fully focused on the assignment as though he was completing a mission. His answers were more automatic and his lack of emotion or affect reflected in his overconfident demeanor. During the interview, Phillip would show some satisfaction with his mathematics skills with verbal remarks like "yea" and nodding in approval with his answer. Similar to McLeod's (1988) views, the researcher categorized Phillip's brief expression of satisfaction as positive affect. In short, Phillip's conviction in the quality of his own work may have superseded any noticeable emotional response associated with his mathematics ability.

Procedural/Conceptual Understanding

Ghazali and Zakaria (2011) state that procedural knowledge is a form of understanding, which focuses on procedures and skills without a clear reference to mathematical ideas. Simple procedural knowledge fails to provide the necessary schemes to solve mathematics problems. However, conceptual knowledge involves a thorough understanding of fundamental and the core concepts related to the algorithms executed in mathematics. Students who possess conceptual understanding of mathematics are able to apply their understanding recreating proofs and formulas related to the mathematics concept (Ghazali & Zakaria, 2011).

In a study conducted by Hallet, Nunes and Bryant (2010) regarding the differences between conceptual and procedural knowledge, they identified which characteristics would be listed as procedural versus conceptual understanding. Hallet et al. studied 318 grade four and five students measuring their conceptual and procedural knowledge of fractions. Students with knowledge of equivalent fractions and the ability to compare two quantities were coded as having conceptual knowledge of fractions. However, students who could only solve fractions using simple rules were coded as having procedural knowledge of fractions (Hallet, Nunes, & Bryant, 2010). Based on Hallet et al. (2010) study, the researcher used the same coding model. In this study, students were coded with conceptual knowledge if they are able find the measures of the angle utilizing their understanding of angle relationships. Students were coded as procedural knowledge when they solely used recall to solve the problem.

The only student who did not get coded with either procedural or conceptual knowledge was Lawrence. Unfortunately, he was unable to accurately calculate any of the angle measures nor was he able to identify the angles. Instead he repeated incorrect responses to himself in the same manner Sfard (2007) describes as interpersonal communication; however, it personified his lack of mathematics vocabulary. His word association did not correspond with the correct definition indicating that Lawrence was randomly recalling material associated with angle congruence. Based on his internal discourse, negative affect, and inability to properly solve either of the two problems with some accuracy, the researcher concluded that Lawrence did not possess either procedural or conceptual knowledge with either task.

Contrary, Sara was more familiar with the vocabulary but predominantly on a superficial level. Her reference to various terms identified that Sara was familiar with some mathematics terminology (or possessed procedural understanding); however, she did not have conceptual understanding of the terms. In the following statement Sara is attempting to find the slope of the line in the first problem.

Excerpt of Sara's interview:

> If x equals 0, then y would have to equal the umm…..it…..it would be undefined because the umm, x would equal 0.

In her statement, there is evidence that Sara has some information regarding linear equations including the importance of the variables x and y. Based on her internal discourse, she revealed that y was undefined because x was equal to zero. Although her recall regarding the association of x and y was misconstrued, her knowledge of the linear equations (y=mx+b) was evident, yet vague. Consequently, Sara lacked fundamental knowledge to accurately complete either problem and she failed to recall and accurately identify many of the necessary vocabulary terminology associated with the either of the two tasks.

Phillip was a bit more knowledgeable than either of the two students. Although his conceptual knowledge was limited, his procedural knowledge surpassed either of the other students' understanding. Beginning with angle two, four, and six, Phillip used his procedural understanding of supplementary angles (Phillip may have assumed that the three angles totaled 180 degrees, forgetting that supplementary angles are two angles whose total measures are 180 degrees.) to find the measures of angle two and six. As stated earlier, he incorrectly identified angle two, four, and six as adjacent angles because he may have recalled that adjacent angles share a common vertex but forgot that they must also share a common side concluding that adjacent angles are two angles and not three angles. He correctly identified angles two and five as vertical angles and labeled both angles with the same measure. He did not notice or did not know that angles four and one were alternate interior angles, which would have quickly helped him calculate the entire problem correctly based on the procedural knowledge he already possessed.

Summary

The purpose of this study was to examine the relationship between students' understanding of mathematics vocabulary and their success in mathematics. The study was completed using a mixed method design representing both qualitative and quantitative data for analysis. The instrument utilized to measure vocabulary has been established as valid and reliable. The data for both methods have shown that there is a correlation between the acquisition of mathematics vocabulary and student achievement. Likewise, there is a possible link between conceptual understanding of mathematics vocabulary and students' ability to problem solve. In chapter five, the researcher provides a more comprehensive discussion of the findings, the relationship of the findings to previous literature, implications for future research, and the limitations of the findings.

CHAPTER FIVE

DISCUSSIONS, IMPLICATIONS AND LIMITATIONS

In this chapter the researcher discusses the findings (using previous literature to make connections), the implications of the findings, the implications for future research and practice, and the limitations of the findings. This study was designed to examine if there is an association between students' understanding of mathematics vocabulary and their achievement in mathematics.

Discussion of Findings

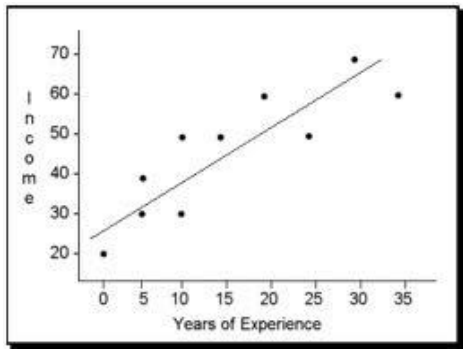
For the quantitative study the researcher sought to examine if there is a correlation between the acquisition of mathematics vocabulary and students' achievement in mathematics. In chapters three and four, the researcher outlined how she created and administered a valid and reliable mathematics vocabulary instrument to eighth grade students and compared their corresponding scores to the Georgia mathematics CRCT. The scores were entered in an Excel spreadsheet and the Pearson correlation was calculated yielding a correlation of .67 and a p-value of $p=0.0001$. The results indicated that there is a positive correlation (or statistical association) between the acquisition of mathematics vocabulary and students' achievement in mathematics. Thus, the researcher has concluded that there may exist a possible link between mathematics vocabulary acquisition and student achievement.

Sfard (2001) believes that it is the communication piece that is missing for students to understand mathematics. Monroe and Orme (2002) acknowledge the obstacles of learning mathematics could be the rarity it is spoken in everyday life limiting the discourse to the classroom. In the NCTM (1996), it states that students should hear and use mathematical terminology so they can extract meaning from the vocabulary and provide the teacher with the opportunity to correct students who use terms out of context. Curio, Schwartz and Brown (1996) believe that discourse promotes mathematical inquiry and is an effective instructional tool to be utilized in the classroom (Curio et al., 1996). These and other researchers have communicated the importance of students using acquired mathematics vocabulary for a better comprehensive understanding of mathematics. The researcher believes that the results derived from this study support their theories. The results indicated that many of students who passed the vocabulary assessment also passed the mathematics portion of the CRCT. Additionally, there was some indication that the better the students scored on the vocabulary assessment the higher they scored on the CRCT. The study conducted by Hardcastle and Orton (1993) utilizing 12-year-old students provided an interesting correlation with this study. Their study revealed that students were only able to correctly illustrate or define approximately 40% of the commonly spoken vocabulary terms utilized in the classroom. Furthermore, 40% of the time, the students thought they knew what teachers were communicating but they were unclear on the vocabulary. The result derived from this study identifies vocabulary as a possible weakness in students' performance on the yearly-standardized assessment. Furthermore, it implies that students could be more proficient in mathematics if they conceptually understood the vocabulary readily spoken in class.

The results of this and previous studies suggest the need for educators to frequently incorporate vocabulary lessons into daily class assignments. Teachers can utilize graphic organizers as suggested by Gay (2008) or place the students into group discussions enabling the teacher to hear the discourse and informally evaluate their students' understanding as noted in the literature from NCTM (1996), Renne (2004), Sfard (2001), Walshaw and Anthony (2008).

However, unlike the previous literature, the researcher believes that there is at least one additional reason as to why students have a difficult time learning mathematics and the vocabulary associated with it. It is the inconsistencies in how the vocabulary is taught from one grade level to the next that causes confusion among the students. For example in earlier grades, consider the variables *m* and *b* in the linear equation y = mx+ b whereas *m* is the coefficient of x and *y* is the starting point. Although this is true, the confusion is recognizable when the students reach the higher grades and they are asked to adjust their understanding and refer to the variables as slope and y-intercept. If the students were informed that the variables represent multiple meanings including slope and y-intercept when the concept is initially introduced, then there may be less confusion later. Furthermore, if the term *variable* were taught in context (particularly in pre-algebra) with previous lessons the students would not enter into the latter grades referring to them as letters. It is for these reasons the researcher believes that the change in rules, directions, and content may be another culprit for low mathematics achievement.

Closely examining the association of student achievement and content, the researcher critically examined why problem number 37 was identified as the most difficult problem on the logit scale and problem numbers six and nine were listed as the easiest problems. In chapter four, table six displays the teachers' and students' perception of each question on the vocabulary assessment. Oddly, question number 37 was unanimously listed as an easy question and the participants marked question numbers six and nine, which were identified as the two easiest problems, in all three categories. The researcher found these discrepancies oddly interesting and further examined each of the questions, beginning with question number 37 (figure six).



37.) The graph above is how bivariate data is usually displayed. What type of graph is displayed above?

a.) Line plot
b.) Scatter plot
c.) Bar graph
d.) Histogram

Figure 6. Question #37 from the vocabulary assessment.

Upon reviewing the question, the researcher was somewhat confused as to why many of the students incorrectly answered this problem. The general assumption is that most eighth grade students can easily identify scatter plots. The aha moment came after the researcher reread the question which states, “The graph above is how bivariate data is usually displayed. What type of graph is displayed above?” Bivariate! The mathematical term bivariate was what the students did not understand. The researcher felt that the majority of eighth grade students could identify a scatter plot but how many of them knew what bivariate meant? This revelation underscored the importance of this study.

However, the excitement was short lived once the researcher reviewed the students' answer choices, and saw that line plot was the most selected answer at 73%. Scatter plot was only chosen as the correct answer by 19% of the participants. As a result, there was a second conclusion the researcher could draw regarding problem number 37. The researcher concluded that the students incorrectly identified the *line of best fit* on the scatter plot as a graph displaying a linear relationship. The two conclusions reached by the researcher could explain why problem number 37 was listed on the logit scale as the more difficult item but was perceived as an easy problem by the participants. This could further explain the gap in the measures and the misfit of the mean squares, which was listed as 1.73. Therefore, item #37 did not meet the requirement for construct validity.

After examining problem number six (figure seven), the researcher could not speculate as to why it was identified as the easiest problem positioned along the bottom of the logit scale. Item number six was a recall question, however, there were several recall questions (e.g. items one, three, and five) that did not have 96% of the participants choose the correct answer. Similarly, item number nine was also a recall question regarding independent and dependent variables. However, the researcher believes that

6.) The distance between the sun and the earth is approximately 93,000,000 miles. A quicker or shorter method to write this number is known as ________________ _________________.

a.) Exponent Notation
b.) Scientific Notation
c.) Base Notation
d.) Number Notation

Figure 7. Question #6 from the vocabulary assessment.

many of the participants may have arrived to the correct answer on this item partly due to graphing being taught in sixth, seventh, and eighth grade. The repetition and regularity of learning the terms independent and dependent variable in the same context (graphing), might have helped the students' conceptually understand the concept.

As a noteworthy comment, the researcher wants to restate that these reasons are only conjectures as to why these items were identified as easy or difficult because none of the participants were interviewed upon the conclusion of the vocabulary assessment regarding the test items. Nonetheless, the researcher wanted to examine why those items (thirty-seven, six, and nine) were perceived so differently than actual test results.

For the qualitative research, the researcher sought to examine how conceptual understanding of mathematics vocabulary impacts students' ability to problem solve. With the assistance of three randomly selected students from a pool of 131 eighth grade participants, the researcher requested the students to complete two problem-solving tasks using the think-aloud protocol. The three selected participants were Lawrence (scored a 24 on the vocabulary test and 798 on the CRCT), Sara (scored a 63 on the vocabulary test and an 833 on the CRCT) and Phillip (scored an 80 on the vocabulary assessment and an 864 on the CRCT).

Based on the results, the researcher was able to determine that Phillip's answers displayed more vocabulary knowledge than the other two participants. For the first task, all of the students arrived at an incorrect answer. This prompted the researcher to conclude that the either the directions were unclear or the participants were nervous as they read and completed the first problem. However, as the students were completing the second problem they were noticeably less rushed and were able to verbalize their thinking better. Upon the conclusion of the interviews and the coding process, the results indicated that there is an association between affect, mathematics vocabulary, and problem solving abilities.

Table 10

Four emerging themes from the three interviews

Categories	**Lawrence**	**Sara**	**Phillip**
Problem Solving	3	8	7
Commognition	8	7	4
Affect (Positive/Negative)	Positive – 0 Negative – 2	Positive – 3 Negative – 0	Positive – 2 Negative – 0
Understanding (Procedural/Conceptual)	Procedural – 0 Conceptual – 0	Procedural – 4 Conceptual – 0	Procedural – 3 Conceptual – 2

As described in George Polya's (1945) writings *How to solve it*, each of the students performed at least one of the four steps in solving the tasks. George Polya devised a four-step heuristics plan that the researcher observed the students were utilizing as they were completing the two tasks. The researcher noted that two of the students limited their problem solving abilities to the first two heuristics (understanding the problem and devising a plan). Phillip reached the third step (carrying out the plan) while Lawrence utilized the fourth and final step (looking back). During the interviews, the students' dialogue would often indicate that they were not speaking to the researcher directly regarding the task but utilizing an internal discourse to retrieve correct terminology. Their repeated internal dialogue represented Anna Sfard's (2008) description of communicating with oneself or commognition.

The next two categories (affect and understanding) displayed a difference between the participants. Both categories displayed Lawrence as the only participant with negative affect and seemingly no procedural or conceptual understanding of either task. The literature for problem solving suggested that teachers incorporate several strategies to assist in alleviating the frustration commonly associated with solving problems (Monroe & Panchyshyn, 2005). In chapter four, the researcher described Lawrence as an energetic student, however, during the interview Lawrence was much more calm and timid. Unlike the other two interviews, once Lawrence's interview concluded, he was quiet and appeared disappointed. His display of emotion was more intense than the other students, which would be an example of McLeod's (1988) description of an inexperienced problem solver.

In contrast, Sara's interview was positive. She displayed optimism and confidence as she answered the questions. She would often repeatedly whisper to herself and continuously tap her pencil in rhythmic pattern. Sara's understanding appeared to be somewhat superficial; she displayed familiarity with some of the mathematic terms but did not know how to apply them correctly. She knew that y-intercept and slope were interrelated, however, she could not effectively communicate the association between the terms. Although there might have been a multitude of explanations for her memory lapse, the researcher believed that her lack of recall could be attributed to her lack of conceptual understanding of the vocabulary. Muir et al. (2008) believe that students with limited number of problem solving strategies may lessen their ability to find creative ways to solve problem thus reducing their mathematical thinking (Muir et al., 2008). This was exhibited more in Sara's interview where she made a concerted effort to recall methods to solve the tasks but failed to do so because of her limited knowledge of the mathematics content.

Similar to Sara, Phillip did not exhibit that he understood all of the vocabulary but he did have a better understanding of the task than the other two students. Phillip was more comfortable with his mathematics skills and asserted more confidence indicating that he felt like he was solving the problems correctly (positive affect). The researcher believed that Phillip was more knowledgeable about the tasks but he failed to review his work and ultimately did not subsequently identify all of the angles. Upon the conclusion of his interview, the researcher asked Phillip to review his work for accuracy. It was during this time; Phillip noticed his errors and stated that he had to "get use to it (the problems) because I have not done this in a while." Nevertheless, Phillip was the only student who appeared to possess conceptual understanding of the two tasks. He was able to find the measures of a couple of angles based on his recall and understanding of vertical and adjacent angles. Although his recall was somewhat limited, he attributed his memory lapse to the fact that it had been a long time since he had studied angle congruence. One could debate that conceptual knowledge is constant or fixed; however, it is commonplace for even knowledgeable mathematicians to experience memory fog.

In previous literature Canobi, Reeve and Pattison (1998) conducted a study regarding students' understanding of addition and found that students with conceptual understanding of addition were able to solve the problems quicker and more accurately than those with procedural understanding. Based on their findings, Canobi et al.

concluded that additional research was needed to determine if conceptual understanding does increase problem-solving skills. The results from this study support and add to Canobi, Reeve and Pattison's research because it helps to establish that conceptual understanding of mathematics does assist with students' problem-solving skills.

Implications of the Findings

Students who conceptually understand mathematics vocabulary appear to perform better on mathematic achievement tests and display more positive affect during problem solving. The study also provided some credible evidence and insight regarding the importance of vocabulary acquisition and its association with mathematics achievement. The results did reveal that new and challenging methods might be necessary in mathematics education in order to achieve fluency with mathematics vocabulary and measurable student success.

In the figure eight, the researcher diagramed how mathematics vocabulary can result in students developing conceptual understanding. The top of the concept map is mathematics vocabulary because vocabulary is presented, not necessarily taught, in many mathematics classrooms. The deviation manifests when students do not read, write, speak, and problem solve using mathematics vocabulary. When the vocabulary is simply spoken by the teacher with little regard for students' knowledge, then the students' problem solving skills suffer because they do not understand the language communicated in the classroom. As a result, negative affect towards mathematics increases and knowledge is more procedural. Consequently, students who are able speak, write, and perform real-life application in the classroom, tend to make more meaningful connections from one mathematical topic to the next. Students' affect is more positive and conceptual understanding may increase.

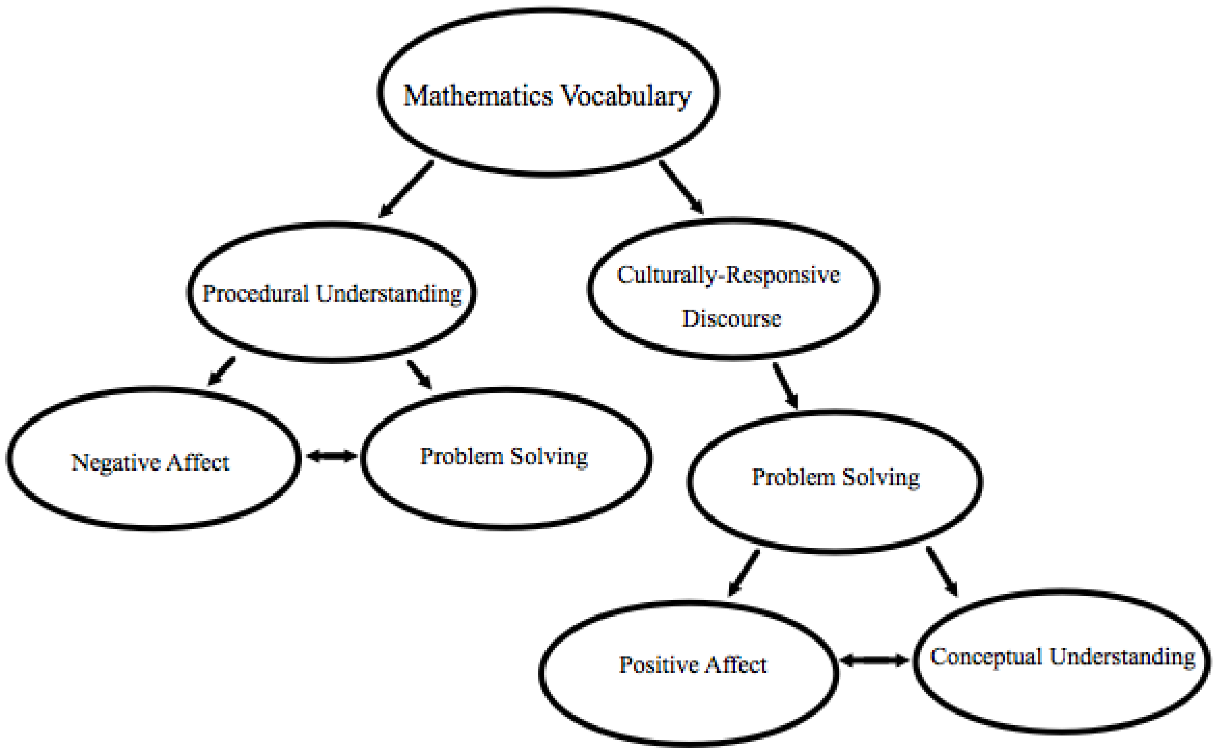


Figure 8. Concept map – The impact of mathematics vocabulary on understanding, problem solving, and affect.

While the researcher conducted this particular study it became abundantly clear that more comprehensive empirical studies of mathematics vocabulary should be conducted. Thus, the researcher hopes that mathematics vocabulary is continuously examined as a realistic and viable option for student comprehension putting it in the forefront of educational research.

Implications of Future Research and Practice

This study was limited to one eighth-grade population of students. The researcher believes that future studies could be conducted in earlier grades to examine when students' vocabulary understanding begins to decline or simply identify when students begin to misunderstand critical vocabulary terminology. Additionally, the researcher believes that it would be interesting to examine if vocabulary acquisition is equally important in other content areas. However, the researcher believes that another form of measurement would make for a stronger study. The use of the multiple-choice assessment is limited and may not fully expose or justify categorizing students as possessing (or not possessing) conceptual understanding. It is the researcher's judgment that open-ended questions would provide more substance than a multiple-choice assessment.

Limitations of Findings

One of the limitations the researcher noted was late administration of the vocabulary assessment. The students took the assessment in May when they were already beginning to mentally withdraw from school. The researcher was requested (by the school's administration) and approved to administer the assessment within a small time frame, which also limited the amount of participants she could interview. The researcher believes that if she had additional time to interview more students, then the qualitative study would have yielded better and stronger results. Thus, the timing of the test may have been a factor to the results of the study.

Another revelation the researcher could not foretell until after the conclusion of the study deals with the qualitative research. During the think-aloud interview, Ericsson and Simon's (1980) protocol was utilized. Thus, the researcher did not suggestively coach nor ask the students any leading questions. Unfortunately, it is the researcher's belief that she should have been able to request that the students reread the problems to ensure that the participants understood the questions that were being asked. As previously stated, none of the students properly completed the first problem-solving task. The students may have been somewhat anxious during this time and wanted to quickly begin solving the problem without first assessing what was being asked in the problem. The researcher's believes that if she was able to verbally redirect the students; they would have attempted the problem with a different perspective, thereby reaching a different result. Furthermore, the researcher believes that there should have been an additional sheet attached requesting that the students' review their answers after completing the reflective questioning portion of the interview. Yet, again this would ensure that the students did not hastily complete the tasks, without comprehensively thinking through their answer.

Researcher's Comments

Some of the literature the researcher read raised questions in her mind regarding education and mathematics. The researcher pondered whether or not educators should practice Renne (2004) behaviors and audiotape their classes to monopolize on teachable moments. Likewise, could the fault lie in educators' laps because they don't fully understand all the complexities of mathematics being taught in the classroom (NCTM, 2007). Ultimately, it is important to understand that achievements are made when all facets of learning and teaching have been exhaustively explored and applied correctly to achieve desired results.

Summary

The research conducted was predicated on the premise that students overall performance in mathematics could improve based on their ability to mathematically communicate. While the researcher only examined a relatively small portion of a student sample population a convincing conclusion was achieved. Based on the literature and the evidence supported by the researcher's findings, there is a correlation between mathematics vocabulary and student achievement. While mathematics vocabulary research appears to be somewhat in its infancy stages, it is clearly a topic that warrants additional in-depth examination and further exploration.

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Appendix A
Original Vocabulary List

Thank you for helping me with my research. The purpose of the research is to determine if there is a correlation between mathematics vocabulary and student achievement. Please circle 45 terms you believe are relevant or important for the students to understand to perform well on the 8th grade Criterion Referenced Competency Test (CRCT) in mathematics. Thank you!

Addition Property of Equality	Exponential Notation	Rate of Change
Additive Inverses	Functions	Rational Number
Adjacent Angles	Geometric Solid	Reflection
Algebraic Expression	Graph of a Function	Reflection Line
Alternate Exterior Angles	Hypotenuse	Right Triangle
Alternate Interior Angles	Imperfect Square	Rotation
Altitude of a Triangle	Independent Variable (x-axis)	Same-Side Exterior Angles
Angle of Rotation	Initial Value	Same-Side Interior Angles
Base (of a Polygon)	Intersecting Lines	Scale Factor
Base Number	Inverse Operation	Scatter Plot
Bivariate Data	Irrational	Scientific Notation
Clustering	Leg of a Triangle	Significant Digits
Complementary Angles	Like Terms	Similar Figures
Cone	Line of Best Fit	Slope
Congruent Figures	Linear	Solution
Converse of Pythagorean Th.	Linear Equations	Solve
Coordinate Plane	Linear Pair	Sphere
Coordinate Point of a Plane	Multiplication Property of Equality	Square Root
Corresponding Angles	Multiplicative Inverses	Standard Form
Corresponding Sides	Non-linear	Supplementary Angles
Cube Root	Origin	System of Linear Equations
Cylinder	Outlier	Transformation
Decimal Expansion	Perfect Cubes	Translation
Dependent Variable (y-axis)	Perfect Square	Transversal
Diameter	Proportional Relationships	Trend Line
Distance Formula	Pythagorean Theorem	Unit Rate
Distributive Property	Pythagorean Triples	Variable
Domain	Radical	Vertical Angles
Equation	Radius	Volume
Exponent	Range of a Function	

Please print, sign and date this page. Thank you again!

Print: ______________________ Sign: ________________________ Date: ______

Appendix B
Vote Count for Each Term

9 votes	8 votes	7 votes	6 votes	5 votes
Functions	Congruent Figures	Alternate Exterior Angles	Adjacent Angles	Distributive Property
Perfect Square	Dependent Variable (y-axis)	Alternate Interior Angles	Algebraic Expressions	Equation
Pythagorean Theorem	Hypotenuse	Base Number	Corresponding Angles	Non-Linear
Slope	Independent Variable (x-axis)	Complementary Angles	Cube Root	Radical
Square Root	Like Terms	Exponent	Graph of a Function	Reflection
Variable	Rate of Change	Linear Equations	Irrational Number	
	Similar Figures	Scale Factor	Line of Best Fit	
		Scientific Notation	Linear	
		Systems of Linear Equations	Perfect Cubes	
		Volume	Rational Number	
			Rotation	
			Standard Form	
			Supplementary Angles	
			Transformations	
			Translations	
			Transversal	
			Vertical Angles	
6	**7**	**10**	**17**	**5**

4 votes	3 votes	2 votes	1 vote	0 votes
Angle of Rotation	Altitude of a Triangle	Additive Inverses	Bivariate Data	Addition Property of Equality
Coordinate Plane	Intersecting Lines	Base (of a Polygon)	Clustering	Decimal Expansion
Coresponding Sides	Inverse Operations	Cone	Coordinate Point of a Plane	Distance Formula
Domain	Pythagorean Triples	Converse of Pythagorean Th.	Exponential Notation	
Leg of a Triangle	Range of a Function	Cylinder	Geometric Solid	
Multiplicative Inverse	Right Triangle	Diameter	Imperfect Square	
Proportional Relationshi	Solve	Initial Value	Multiplication Property of Equaltiy	
Radius		Linear Pair	Origin	
Reflection Line		Same-side Exterior Angles	Outlier	
		Same-side Interior Angles	Significant Digits	
		Scatterplot	Trend Line	
		Solution		
		Sphere		
		Unit Rate		
9	**7**	**14**	**11**	**3**

Appendix C
Terms Listed by Standards

Below is a comprehensive list, 9 teachers selected, as the most relevant terms 8th grade students need to understand to perform well on the CRCT. Based on the listing below, do you agree or disagree that this list is a reflective and valid representation of 8th grade terms for each of the following standards?

Numbers & Operations	Algebra	Geometry	Data & Probability
Base Number/Exponents	Algebraic Expressions	Adjacent/Corresponding/Supp./Vertical Angles	Bivariate Data
Irrational/Rational Numbers	Dependent/Independent Variable	Alt. Interior/Alt. Ext. Angles	Clustering
Perfect Cubes/Cube Roots	Distributive Property	Complementary Angles	Line of Best Fit/Trend Line
Perfect Squares/Square Roots	Equation	Congruent/Similar Figures	Outlier
Radical	Like Terms	Function	Scatterplot
Scientific Notation	Linear/Non-Linear	Graph of a Function	
	Linear Equations	Hypotenuse	
	Multiplicative Inverse	Pythagorean Theorem	
	Non-Linear	Scale Factor/Dilation	
	Proportional Relationships	Transformations/Trans./Rotation/Reflection	
	Slope/Rate of Change	Transversal	
	Standard Form	Volume	
	Systems of Linear Equations		
	Variable		

Yes, I agree that the vocabulary list is a valid representation of 8th grade terms for each standard.

Print Name: ______________________________ Signature: ______________________________ Date: _______________

No, I do not agree that the vocabulary list is a valid representation of 8th grade terms for each standard. Please indicate your changes below.

____

____

Print Name: ______________________________ Signature: ______________________________ Date: _______________

Appendix D
Teacher's Rating Scale

Fellow Mathematics Teachers:

I would like to express my sincere gratitude for helping me with my research.

Your assistance has helped me develop the attached mathematic vocabulary assessment. This is last time I will request your assistance ☺.

Please evaluate the attached vocabulary assessment I have developed using the bank of terms compiled by a team of mathematics teachers. Please read each problem carefully deciding the appropriate difficulty level. Using the following criteria, circle one of the three number choices indicating which level you believe best classifies the problem.

1	2	3
easy	medium	difficult

Last, if you see any errors (including grammatical) or if you think it is a "bad" problem, please make note on the comment line. Once again, I want to thank you for being positive, supportive, and gracious through this entire process. I am humbled by your commitment and very grateful!

Alanna Bowie

Appendix D (pg. 2 of 8)

Problem #1 – Numbers & Operations

1 2 3

Comment(s): ____

____

Problem #2 – Numbers & Operations

1 2 3

Comment(s): ____

____

Problem #3 – Numbers & Operations

1 2 3

Comment(s): ____

____

Problem #4 – Numbers & Operations

1 2 3

Comment(s): ____

____

Problem #5 – Numbers & Operations

1 2 3

Comment(s): ____

____

Problem #6 – Numbers & Operations

1 2 3

Comment(s): ____

____

Appendix D (pg. 3 of 8)

Problem #7 – Algebra

1 2 3

Comment(s): ____

____

Problem #8 – Algebra

1 2 3

Comment(s): ____

____

Problem #9 – Algebra

1 2 3

Comment(s): ____

____

Problem #10 – Algebra

1 2 3

Comment(s): ____

____

Problem #11 – Algebra

1 2 3

Comment(s): ____

____

Problem #12 – Algebra

1 2 3

Comment(s): ____

____

Appendix D (pg. 4 of 8)

Problem #13 – Algebra

1 2 3

Comment(s): ____

____

Problem #14 – Algebra

1 2 3

Comment(s): ____

____

Problem #15 – Algebra

1 2 3

Comment(s): ____

____

Problem #16 – Algebra

1 2 3

Comment(s): ____

____

Problem #17 – Algebra

1 2 3

Comment(s): ____

____

Problem #18 – Algebra

1 2 3

Comment(s): ____

____

Appendix D (pg. 5 of 8)

Problem #19 – Algebra

1 2 3

Comment(s): ____

____

Problem #20 – Algebra

1 2 3

Comment(s): ____

____

Problem #21 – Geometry

1 2 3

Comment(s): ____

____

Problem #22 – Geometry

1 2 3

Comment(s): ____

____

Problem #23 – Geometry

1 2 3

Comment(s): ____

____

Problem #24 – Geometry

1 2 3

Comment(s): ____

____

Appendix D (pg. 6 of 8)

Problem #25 – Geometry

1 2 3

Comment(s): ____

____

Problem #26 – Geometry

1 2 3

Comment(s): ____

____

Problem #27 – Geometry

1 2 3

Comment(s): ____

____

Problem #28 – Geometry

1 2 3

Comment(s): ____

____

Problem #29 – Geometry

1 2 3

Comment(s): ____

____

Problem #30 – Geometry

1 2 3

Comment(s): ____

____

Appendix D (pg. 7 of 8)

Problem #31 – Geometry

1 2 3

Comment(s): ____

____

Problem #32 – Geometry

1 2 3

Comment(s): ____

____

Problem #33 – Geometry

1 2 3

Comment(s): ____

____

Problem #34 – Geometry

1 2 3

Comment(s): ____

____

Problem #35 – Geometry

1 2 3

Comment(s): ____

____

Problem #36 – Geometry

1 2 3

Comment(s): ____

____

Appendix D (pg. 8 of 8)

Problem #37 – Data & Probability

1 2 3

Comment(s): ____

____

Problem #38 – Data & Probability

1 2 3

Comment(s): ____

____

Problem #39 – Data & Probability

1 2 3

Comment(s): ____

____

Problem #40 – Data & Probability

1 2 3

Comment(s): ____

____

Problem #41 – Data & Probability

1 2 3

Comment(s): ____

____

Problem #42 – Data & Probability

1 2 3

Comment(s): ____

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Appendix E
Addition of Dilation

In my original list of terms, I neglected to add the term *dilation*. If dilation were on the original list, now completing your list with a total 46 terms (instead of 45), would you have chosen dilation?

Yes	No	Initial _______________
Yes	No	Initial _______________
Yes	No	Initial _______________
Yes	No	Initial _______________
Yes	No	Initial _______________
Yes	No	Initial _______________
Yes	No	Initial _______________
Yes	No	Initial _______________
Yes	No	Initial _______________

Appendix F
Student's Rating Scale

Dear Student:

I would like to express my sincere gratitude for you helping me with my research.

Your assistance will help me develop the attached mathematic vocabulary assessment ☺.

Please review the attached vocabulary assessment I have developed. Please read each problem carefully deciding the appropriate difficulty level. Using the following criteria, circle one of the three number choices indicating which level you believe best classifies the problem.

1	2	3
easy	medium	difficult

If you see any errors (including grammatical) or if you think it is a "bad" problem, please make note on the comment line. Last, after you've completely reviewed the test, please answer the following questions located on the bottom of this page. Once again, I want to thank you for helping me with this assessment.

Mrs. Bowie

Do you think 8th grade students should know **some**, **most** or **all** of the vocabulary terms on this test by the end of their 8th grade school year?

Circle One: Some Most All

Please explain why you choose the answer above.

____

____

____

Do you think this is a "fair" test? Yes or No

Please explain your answer on the lines below.

____

____

Appendix G
Mathematics Vocabulary Assessment

Mathematic Vocabulary Assessment

1.) The expression 5^2 is equal to 25. What does the number two represent?
   a.) Base
   b.) Coefficient
   c.) Function
   d.) Exponent

2.) The number 3.14159265359….. or pi (π) is a popular irrational number. Which of the following reasons **best** describes why pi is an irrational number?
   a.) Irrational numbers cannot be graphed
   b.) Once a number exceeds 10 digits, it is no longer identified as a rational number
   c.) Irrational numbers cannot be written as a common fraction
   d.) Irrational numbers are not part of the real number system

3.) $\sqrt[3]{64} = 4$ is an example of which of the following?
   a.) Irrational number
   b.) Scientific Notation
   c.) Perfect square root
   d.) Perfect cube root

4.) The square root of 36 is 6, what are the next 3 perfect squares?
   a.) 46, 56, 66
   b.) 49, 64, 81
   c.) 42, 56, 64
   d.) 49, 56, 81

5.) The $\sqrt{\ }$ symbol is known as a ______________ symbol.
   a.) Radical
   b.) Radicand
   c.) Root
   d.) Rational

6.) The distance between the sun and the earth is approximately 93,000,000 miles. A quicker or shorter method to write this number is known as _______________ ________________.

a.) Exponent Notation
b.) Scientific Notation
c.) Base Notation
d.) Number Notation

7.) How many terms are there below?

$2x^2 + 3x^2 - 4x^2y - 5y$

a.) 2
b.) 3
c.) 4
d.) 5

8.) Niya says that an **algebraic expression** is a mathematical phrase that include numbers, variables and constants, while Blake says that an **algebraic expression** is a mathematical sentence that has all of those items including an equal sign. Who is correct?

a.) Niya
b.) Blake
c.) Both
d.) Neither

9.) Sara noticed that her average in mathematics was increasing as the number of hours she studied increased. Sara decided to graph her data, which variable should she label “number of hours studied”?

a.) Independent variable
b.) Dependent variable
c.) Coefficient variable
d.) Correlation variable

10.) $a\,(b + c) = ab + ac$

Above is an example of which property?

a.) Distributive property
b.) Associative property
c.) Commutative property
d.) Flip property

11.) Which of the following is the best example of an **equation**?

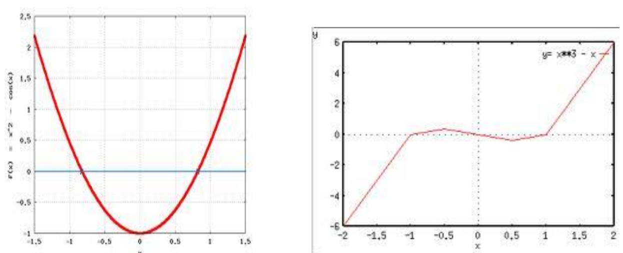
a.) $3x + 2$
b.) $3x + 2x$
c.) $3x + 2x – 10$
d.) $3x + 2x – 10 = 20$

12.) Rachel solved the problem:

$$3x + x – 2 = 10$$
$$4x – 2 = 10$$
$$\underline{+2} \quad \underline{+2}$$
$$4x = 12$$
$$x = 3$$

What step did Rachel complete first?

a.) Add the inverse of -2
b.) Divide by 4
c.) Combine like terms
d.) Subtract 3x

13.) 

Both of the graphs above represent _______________ functions.

a.) curved
b.) linear
c.) non-linear
d.) sloped

14.) 7 x $\square$ = 1

The answer to the equation above is……

a.) the multiplicative inverse of 7
b.) a negative number
c.) the square root of 7
d.) an irrational number

15.) $\frac{7 * (2)}{8 * (2)} = \frac{14}{16}$

Above is an example of two equivalent ratios also known as…..

a.) fraction relationship
b.) a correlation
c.) linear system
d.) proportional relationship

16.) In the following table, which number represents the slope?

X	0	2	4	6
Y	5	9	13	17

a.) 0
b.) 2
c.) 4
d.) 5

17.) The equation 4x + 8y = 16 is a linear equation currently written in:

a.) Point-slope form
b.) Standard form
c.) Y-intercept form
d.) Slope intercept form

18.) Arlene just discovered that the point (1, 3) is the solution for a set of linear equations. This is the first time Arlene has correctly solved which type of equation?

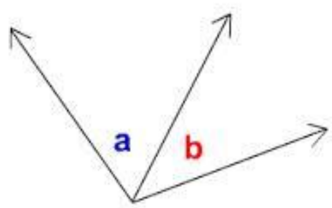
a.) Systems of equations
b.) Quadratic equations
c.) Equivalent equations
d.) Point equations

19.) Joe's equation for renting a car is y = 24x + 75. Which of the following situations best fits his equation?

a.) Each day Joe rents a car he pays $75.00 and a flat fee of $24.00.
b.) Joe is renting the car for 24 days and pays a total of $75.00.
c.) Joe can only rent the car for 24 hours because he only has $75.00.
d.) Joe will pay $24.00 for each day he rents the car and a $75.00 service fee.

20.) A variable is…
- a.) a number in front of a letter
- b.) the origin on a coordinate plane
- c.) an ordered pair
- d.) a letter which represent an unknown number

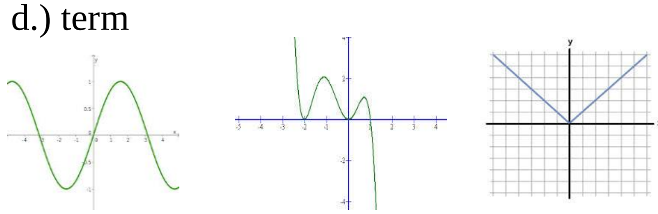
21.) Angle A and angle B are what type of angles?



- a.) Vertical angles
- b.) Corresponding angles
- c.) Adjacent angles
- d.) Straight angles

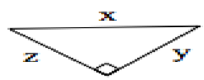
22.) Two angles are ________________ if the sum of both angles equals 90 degrees.
- a.) Supplementary
- b.) Corresponding
- c.) Interior
- d.) Complementary

23.) A relationship between elements where one input has exactly one unique output is defined as a ________________.
- a.) relation
- b.) equation
- c.) function
- d.) term



24.) Each of the above graphs represent which of the following?
- a.) Reflection
- b.) Function
- c.) Rotation
- d.) Translation

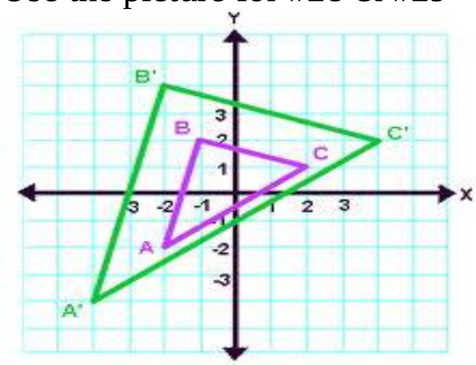
25.) Which of the graphs displayed in problem #24 are non-linear?
a.) 1st and 2nd graph
b.) 1st and 3rd graph
c.) 2nd and 3rd graph
d.) 3rd graph only



26.) Kali was struggling in finding the hypotenuse in the right triangle above. Leighton told Kali one piece of information to help her always locate the hypotenuse. What was the information?
a.) The hypotenuse is next to the right angle.
b.) There is no hypotenuse in right triangle.
c.) The hypotenuse is always across from the right angle.
d.) The hypotenuse is next to the shortest leg of the triangle.

27.) The square of the hypotenuse is equivalent to the sum of squares of the other two sides (or legs) of a right triangle. This is known as what theorem?
a.) Euclidean Theorem
b.) Galileo's Theorem
c.) Fermat's Theorem
d.) Pythagorean Theorem

Use the picture for #28 & #29



28.) Which type of transformation does above the picture represent?
a.) Rotation
b.) Reflection
c.) Dilation
d.) Translation

29.) What is the scale factor for the picture above?

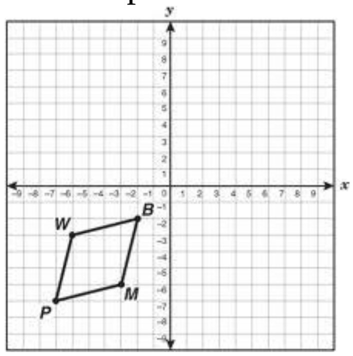
a.) 2

b.) ½

c.) -2

d.) – ½

Use the picture below for #30 – #31



30.) After completing the following rule (x + 15, y +12), which quadrant would the copy be placed?

a.) Quadrant 1

b.) Quadrant 2

c.) Quadrant 3

d.) Quadrant 4

31.) Which transformation was completed in problem #30?

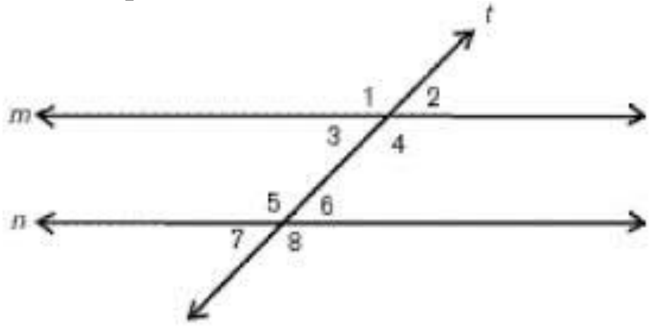
a.) Rotation

b.) Reflection

c.) Dilation

d.) Translation

Use the picture below for #32 – #34



32.) Angle 2 and angle 6 are known as __________________ angles.

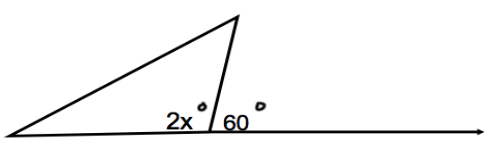
a.) vertical
b.) complementary
c.) corresponding
d.) adjacent

33.) Which of the following angles listed below are all congruent to one another.

a.) Angles: 1, 2, 5 and 6
b.) Angles: 1, 3, 6 and 8
c.) Angles: 1, 4, 5 and 8
d.) Angles: 3, 4, 5 and 6

34.) Line ____ is the transversal.

a.) m
b.) n
c.) t
d.) There is no transversal.

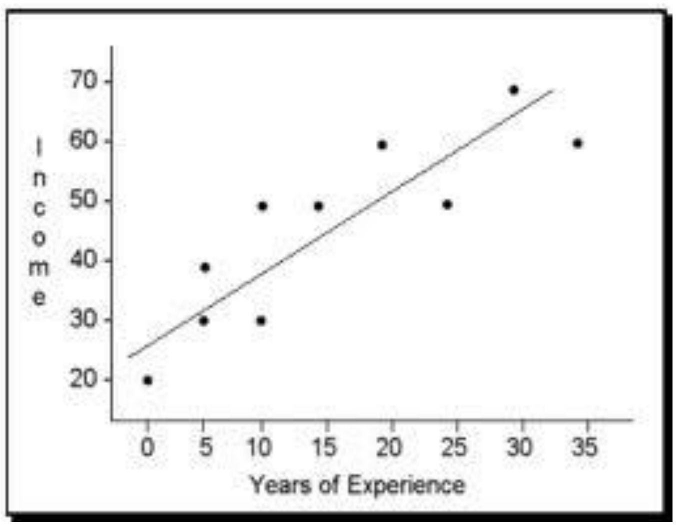


35.) To find the value of x, Luis decided to add $2x^o$ and 60^o to equal 180^o. What did Luis know about the angles to correctly complete this problem?

a.) The angles were complementary angles.
b.) The angles were adjacent angles.
c.) The angles were vertical angles.
d.) The angles were supplementary angles.

36.) Victor has a small rectangular prism that he wants to fill with sand to use as a paperweight. How would Victor find the amount of sand needed to fill the prism?
a.) Victor could calculate the volume of the prism.
b.) Victor could calculate the area of the prism.
c.) Victor could calculate the surface area of the prism.
d.) Victor could calculate the thickness of the prism.

Use the picture below for #37 – #42



37.) The graph above is how bivariate data is usually displayed. What type of graph is displayed above?
a.) Line plot
b.) Scatter plot
c.) Bar graph
d.) Histogram

38.) Based on the line, if an individual has 25 years of experience, then **approximately** how much income will he/she will make? *(Income is in $1,000's.)*
a.) $40,000
b.) $45,000
c.) $50,000
d.) $55,000

39.) Most of the clustering occurs around which set of years of experience?
  a.) 0 – 5 years
  b.) 5 – 10 years
  c.) 20 – 25 years
  d.) 30 – 35 years

40.) If one of the points on the graph were located at (50, 20), then the coordinate would be identified as which of the following?
  a.) Outlier
  b.) Distant
  c.) Closer
  d.) Relevant

41.) “Income” is identified as the ____________________.
  a.) title
  b.) trend
  c.) independent variable
  d.) dependent variable

42.) Which term best describes the line extending down the center of the graph?
  a.) Line of best fit
  b.) Linear trend
  c.) Function line
  d.) Line of correlation

Appendix H
Assent Form

Background information
The researcher, Alanna Bowie, is an 8th grade teacher working at your school. Mrs. Bowie has been a student at Kennesaw State University since January 2011. She is currently seeking to complete her graduate studies and this research study is the final step in her graduate studies. The research study requires using your data only! No identifiable information will ever link you to this research. All data is strictly confidential.

Purpose of the study
You are invited to participate in a research study. The purpose of this study is to examine the relationship between your background knowledge and mathematics vocabulary. Teachers who have a better understanding of how students acquire knowledge can design instruction to most effectively support student learning. Additionally, the researcher would like to conduct an audiotape interview with 6 students on problem-solving skills. If you would not like to participate in the interview, please check the box below. Giving the researcher permission to use your data does not mean you are also giving permission to the interview. You can check the box below to say no to the interview. Please remember that any and all of your information will be confidential.

Participants
You are being asked to participate in the study because you are an eighth grade student.

Procedures
The researcher is asking for your consent to use your data or scores in a research study. One set of scores will come from the yearly criterion referenced competency test administered in April. The other set of scores will come from a vocabulary assessment, which will be administered in May. The vocabulary assessment will be administered in mathematics class and should take approximately 25 to 30 minutes to complete.

Benefits of Participation
The benefit of participating is the intrinsic knowledge that your data was influential in improving research in math education.

Risks of Participation
There are risks in all research studies. However, this study has minimal risks. You may feel tired or bored when completing either of the two tests.

Contact Information
If you have any questions or concerns about the study, you may contact Alanna Bowie at 770-443-4875. This research is to fulfill a requirement for a class at Kennesaw State University. All research involving human participants is carried out under the oversight of an Institutional Review Board. Questions or problems regarding these activities should be addressed to the Institutional Review Board, Kennesaw State University, 1000 Chastain Road, #0112, Kennesaw, GA 30144-5591, (678) 797 – 2268.

Voluntary Participation
Your participation in this study is voluntary. You may refuse for your data to be used in any part of this study. You may withdraw the use of your data any time without prejudice.

Confidentiality
All information gathered for this study will be kept completely confidential. **No reference will ever be made in written or oral materials that will link you to this study**. All records will be stored in a locked safe at the middle school facility for one year after completion of this study. After the storage time the information gathered will be destroyed.

Participant Consent
I have read the above information and I agree to participate in this study.

__________________________________ __________________
Name of the Participant Date

☐ Please check the box to decline an interview.

____
Signature of the Participant

All research involving human participants is carried out under the oversight of an Institutional Review Board. Questions or problems regarding these activities should be addressed to the Institutional Review Board, Kennesaw State University, 1000 Chastain Road, #0112, Kennesaw, GA. 30144-5591, (678) 797 – 2268.

Appendix I
Consent Form

Background information
The researcher, Alanna Bowie, is an 8th grade teacher working at your child's school. Mrs. Bowie has been a student at Kennesaw State University since January 2011. She is currently seeking to complete her graduate studies and this research study is the final step in her graduate studies. The research study requires using your child's data only! No identifiable information will ever link your child to this research. All data is strictly confidential.

Purpose of the study
Your child has been invited to participate in a research study. The purpose of this study is to examine the relationship between your child's background knowledge and mathematics vocabulary. Teachers who have a better understanding of how students acquire knowledge can design instruction to most effectively support student learning. Additionally, the researcher would like to conduct an audiotape interview with 6 students on problem-solving skills. If you would not like for your child to participate in the interview, please check the box below. Giving the researcher permission to use your child's data does not mean you are also giving permission to the interview. You can check the box below to say no to the interview. Please remember that any and all of your information will be confidential.

Participants
The participants will only encompass eighth grade mathematics students.

Procedures
The researcher is asking for your consent to use your child 's data in a research study. Also, the researcher is seeking your child's permission to use his/her data by signing and dating the assent form, located on the other side of this form. One set of scores will come from the yearly criterion referenced competency test administered in April. The other set of scores will come from a vocabulary assessment, which will be administered in May. The vocabulary assessment will be administered in mathematics class and should take approximately 25 to 30 minutes to complete.

Benefits of Participation
The benefit of participating is the intrinsic knowledge that your child's data was influential in improving research in mathematics education.

Risks of Participation
There are risks in all research studies. However, this study has minimal risks. Your child may feel tired or bored when completing either of the two assessments.

Contact Information
If you have any questions or concerns about the study, you may contact Alanna Bowie at 770-443-4875. This research is to fulfill a requirement for a class at Kennesaw State University. All research involving human participants is carried out under the oversight of an Institutional Review Board. Questions or problems regarding these activities should be addressed to the Institutional Review Board, Kennesaw State University, 1000 Chastain Road, #0112, Kennesaw, GA 30144-5591, (678) 797 – 2268.

Voluntary Participation
Your child's participation in this study is voluntary. You may refuse for your child's data to be used in any part of this study. Your child may withdraw at any time without prejudice.

Confidentiality
All information gathered in this study will be kept completely confidential. **No reference will ever be made in written or oral materials that could link your child to this study**. All records will be stored in a locked safe at the middle school facility for one year after completion of this study. After the storage time the information gathered will be destroyed.

Parent Consent
I have read the above information and agree for my child's data to be used in this study.

PLEASE SIGN AND RETURN, I WILL MAKE A COPY FOR YOU WITH ALL SIGNATURES.

____ ____________________
Name of the Participant Date

☐ Please check the box to decline the interview.

____
Signature of the Parent/Guardian

All research involving human participants is carried out under the oversight of an Institutional Review Board. Questions or problems regarding these activities should be addressed to the Institutional Review Board, Kennesaw State University, 1000 Chastain Road, #0112, Kennesaw, GA. 30144-5591, (678) 797 – 2268.

Appendix J
Directions for the Think-Aloud Protocol

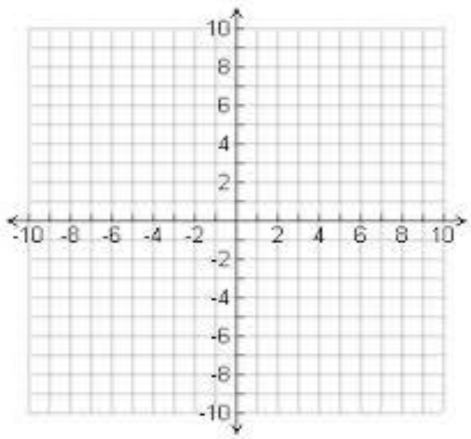
Directions

Thank you for participating in my study ☺. The purpose of this study is for the researcher to observe you while you solve two problems. For the researcher to obtain an accurate understanding of your work, you will be using a technique called "think-aloud". This technique requires for you to **verbalize** everything that you are thinking or wanting to write as you solve each of the problems. Again, you must talk through everything as you solve the problems. Please don't keep any thoughts or information to yourself because the researcher won't be able to get an accurate understanding of how you problem solve. Last, this interview will be audiotaped for the researcher to later write in a paper about students' problem solving methods. Thank you again, please relax, and do your best!

Appendix K
Problem-Solving Tasks

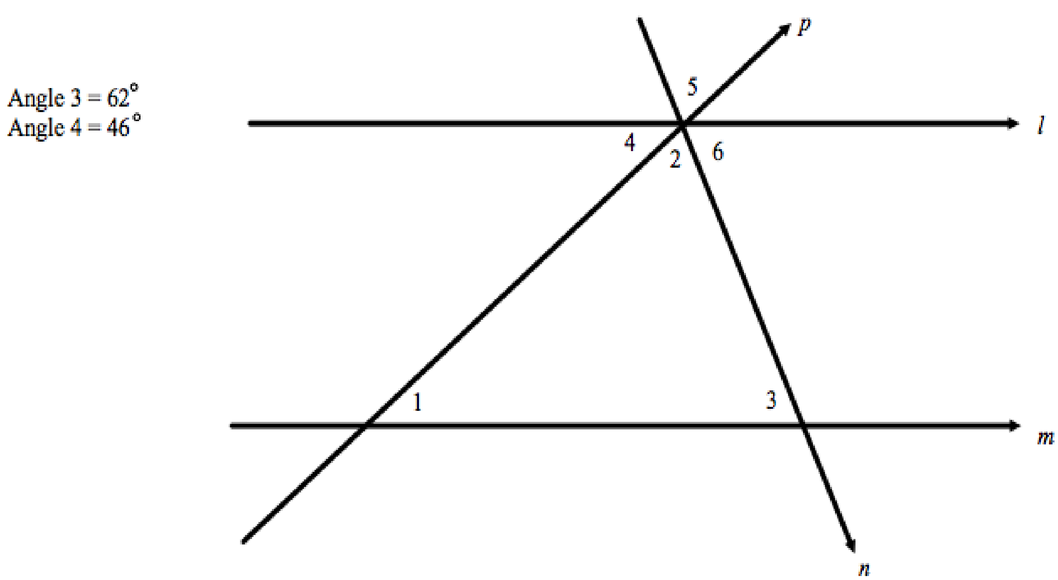
<u>Problem-solving Task #1</u>

Using the graph below, show and verbally explain how you would find the slope of a line using two points. Based on your points, what is the equation of your line?



For your line, if x represents the number of hours worked and y represents money earned. What could you conclude if x = 0?

<u>Problem-solving Task #2</u>



Find the **<u>measures</u>** of all of the angles and **<u>identify</u>** each of them (e.g. complementary angle).

Appendix L
Audiotaped Interviews

Color Key – {Blue – Information}, {Green – Researcher}, {Black – Participant}, {Italics – Notes}, {Orange [underlined] – Key Vocabulary Terms}

Lawrence – 5/20/14

PROBLEM #1

Researcher reads directions……upon concluding reading the directions, Lawrence responds….

No ma'am.

Ok, thank you Lawrence…..here you go (researcher handed student the problem solving task).

Using the problem...using the graph below show the variable, explain how you would find the slope of the line using two points. Based on your points what is the equation of your line….of your line? For your line if x represents the number of hours worked and y represents money you've earned, what would you conclude if x equals 0?

You work zero hours (*student starts writing*)…(pause)…

Please talk

You work zero hours and….so if you work zero hours you work, umm, and you wouldn't earn anything because you haven't work anything yet. So, zero hours would be zero dollars…

Student asks if he can write on the sheet and he was given permission

So, if x is zero and you didn't work anything so….and then find the slope of the…using two points…using two point (repeats)….ummm….x equals zero, so…(pause)….x equals (in a whisper)….

Please talk

Ummm, I don't know how to figure out this problem.

PROBLEM #2

Problem 2, angle 3 equals 62 degrees, angle 4 equals 46 degrees, find the measures of all of the tri.., all the angles. Identify each of them…eg. contenutent (mispronounced complementary) angles. Ummm. Angle 1, so angle 3 is…so angle 3 is (repeats), angle 3 is (repeats again) 6 (inaudible), so angle 6 is….46 degrees (starts writing), angle 2 is….wait no angle 6 is 62 degrees and ang..(erases), angle 2 is….(pause)

Please talk

Angle 2 is 62 degrees (writing). Angle 5 is 46 degrees (writing), angle 1 would be….46 degrees (writing)……(pause)…..

Is that all? That's all the angles.

Please read the directions.

Find the measures, of the measures, of the angles (inaudible), and identify each of them (inaudible…humbling). Is that when I use the vocabulary words? Use the vocabulary words?

Researcher does not respond.

Please talk.

Ummm…(clears throat) six and three are comple…(trails off), six and three are….complementary (repeats)…(starts writing) and one and four are….(writing)…complementary (mispronouncing complementary). One and two are complementary (student is writing and is barely speaking above a whisper).

Please talk.

Ummm…(clears throat) four and six are congru…congr…umm (fumbles with word) congruent, I guess. Let's say congruent (starts writing)……..and that's all I know.

Thank you.

Session Ended.

Phillip – 5/22/14

PROBLEM #1

Researcher reads directions……upon concluding reading the directions, Phillip responds….

No.

Ok, here you go Phillip (researcher handed student the problem solving task).

Ok, when I first start this problem I'm going to…..(reads the directions but mumbles), if x equals zero that means that you would like, umm…uhhh…x right here, x one…can I write on this?

Yes.

Umm, during this I would like, if x equals zero that means to me that ttthhheee (stretches out the word "the") the hours that he worked would be zero, so the money that he earned would be zero and that means the slope of this graph would be…..will be…zer…uh.. (trails off). (Begins to mutter to himself) undefined is vertical (pause), based on the points what's the equation of the line, oh the equation of the line (repeats), uhh, the equation of the line would be y equals….0x + 0y and the…and the slope would be like a horizontal line because there…because vertical is unidentified and zero is like the zero, if its horizontal (tapping). Yea, I'm done.

Ok, please talk about the second problem.

PROBLEM #2

The second problem, I'm going to plug in angle 3 as 62 degrees, angle 4 at 46 degrees, and when I first start this, the easiest to me would be, umm, I know angle 4, 2, 6 and 7 are 180 degrees so Imma need to subtract 180, because they are adjacent angles, Imma subtract 180 minus 46, ummm, that's 7 (Kevin is talking while he subtracts), one, four, three so that's 134 and…ummm…so that's 134 (mumbling, inaudible – student is working out his division) and eight, umm, 34 divided by 2 is 6 and 12 and drop 4, 2 times...is 7.

So each of these is 67 degrees, umm, and since this triangle right here, angle 2, 1, and 3, that is a triangle and all triangles equal 180 degrees, Imma add 67 and 62 and that's 9 and 129, so Imma subtract 180 minus 129, and 7 and 1 and that's 5, so angle 1 is 51 degrees. Angle 4, the measures, identify each of them and since angle 2 is a vertical angle to angle 5 that becomes 67 degrees, to angle 5…..

Phillip nonverbally requests a tissue by pointing to his noise. After Phillip blows his nose he remarks...

I'm finish.

You're finish?

Yes, ma'am.

Ok, thank you.

You're welcome.

Session Ended.

Sara – 5/23/14

PROBLEM #1

Researcher reads directions……upon concluding reading the directions, Sara responds….

No.

Ok. Do you want me to read the question out loud?

Yes ma'am, please.

Sara reads the directions verbatim.

Umm…(tapping)….(pause)….

Please talk

If x equals zero, then you…would….like, put in the origin and then you would…..

Sara starts rereading the problem.

You can read it out loud.

Alright, find the line, if x equals...if x represents the number of hours worked and y represents money earned…you would….uh, uh….you would gain money….but….x would equal zero. Ummm, you first….(pause)….

Please talk

Long pause again (12 seconds)

Please talk

The….(sigh), the equation of the line would be y = 0, wait no (erasing her work), y….yeah y = 0 cause the, oh I forgot what it is called, no the, the y-intercept would be (erasing) 2!? (Stating the answer 2 as though she was certain of the answer yet questioning it at the same time.) Yeah (erasing)…if x equals 0, then y would have to equal the umm…..it…..it would be undefined because the umm, x would equal 0 (writing)….ok.

PROBLEM #2

Find the measures of all angles, of all of the angles (repeat) and identify each of them. (Tapping) Angle 3 equals 62 degrees and angle 4 equals 46 degrees, you would…you would (repeat), you….

Please talk

You would subtract…six, no, you would (tapping)…oh, they are complem e n t a r y angles (Sara slowly pronounces complementary, drags out the word) and complementary angles equal 180. (Under her breath, she whispers) supplementary…supplementary, ok. (Speaking normally she begins again) So you would subtract 180 from the, uh, from angle 3 and 4, 62, you would subtract 62 and 46 to get the…the um…difference you subtract from 180. (Sara is working out the subtraction problems) 6, 12 minus 6 equals 6, 5 minus 4 equals 1, 10 minus 6 is 4, 7 minus 1 is 6 and 1 minus 0 is 1. So….wait, I did that wrong, umm, you would…subtract 90 and 46, 10 and 6 is 4, 8 and 4 is 4, so angle…2….I believe…is 44 degrees. Umm, angle 6….you would subtract 180 and 46, 10 and 6 is 4, 7 and 4 is 3, 1 and 0 is 1. So, angle 6 would be one thirty.…134 degrees. Umm, angle 5 would be…(pause, but she is computing her math) ….forty…no…it would be 44 degrees as well as angle 2 because it…is… hang on…yea, it would be 44 degrees. Angle one would be….(writing) 62 and 180, 10 and 2 is 8, 7 and 6 is 1, and then1 and 0 is 1, so angle 1 would be 118 degrees.

Sara pauses like she is done.

Finish?

Yes, ma’am

Session Ended.