ABSTRACT

Title of dissertation: EQUILIBRIUM MODELS WITH DYNAMIC DEMAND AND DYNAMIC SUPPLY

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Doctor of Philosophy, 2019

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This dissertation comprises two studies of equilibrium models with both dynamic demand and dynamic supply sides. The first is an empirical study of the US video games industry, and the second is a theoretical study.

Chapters 1 and 2 develop a model for quantifying the role of social learning in consumers' dynamic demand and finding optimal intertemporal prices for profit maximizing firms in a market populated by forward-looking social learners. Optimal prices are a result of a Markov perfect equilibrium played between the firm and the consumers. Nested in the market equilibrium is a demand equilibrium played among consumers who make the "right" purchase/wait decisions given endogenously produced product information. The empirical exercises are conducted in two steps. The first step estimates demand parameters, including those associated with social learning. Endogeneity of prices is remedied with a pseudo pricing policy function of relevant state variables. In the second step, optimal prices are found by the Mathematical Programming with Equilibrium Constraints (MPEC) approach. The

model is applied to the US video games industry with sales data of PlayStation 3 games. The results reveal that (1) compared to static social learning, forward-looking social learning reduces equilibrium profits of games in the sample by \$5.2M (28.4%) on average; (2) an incorrect belief of consumers' forward-looking behavior reduces a firm's profits by a maximum of 29.92%. These results indicate great value for researches on consumers' forward-looking social learning behavior.

In chapter 3 we study the effect of adding strategic buyers to the computational model of dynamic price competition when sellers experience learning-by-doing and organizational forgetting developed by Besanko et al. (2010) (BDKS). The addition is motivated by the presence of repeat buyers in many industries where learning-by-doing has been documented, and the role that the assumption of a monopsony strategic buyer has played in the theoretical literature. We characterize the degree of strategic buyer behavior using a single parameter, and show that even quite limited strategic behavior changes the equilibrium correspondance by almost entirely eliminating the multiplicity of equilibria emphasized by BDKS, and ensuring that no seller is likely to dominate the industry in the long-run. We examine how the welfare of both buyers and sellers varies with the degree of strategic buyer behavior.

EQUILIBRIUM MODELS WITH DYNAMIC DEMAND AND DYNAMIC SUPPLY

by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2019

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Dedication

To my parents.

Acknowledgements

The PhD experience is a humbling and life-changing one. I owe my thanks to everyone that saw this big project through to its fruition and made this thesis possible.

First and foremost, I'd like to thank my advisors, Prof. Andrew Sweeting, Prof. Ginger Jin, Prof. Daniel Vincent, Prof. Emel Filiz-Ozbay and Prof. Sergio Urzua for their guidance and support. I am deeply indebted to Prof. Sweeting for his teaching and mentoring. Andrew introduced me to empirical Industrial Organization and computational economics. His expertise, inspiration and continuous encouragement have been invaluable to have. I also have the privilege of working closely with him on our joint project, which has become the third chapter of this thesis. This experience has been immensely educational. I am grateful to Prof. Jin and Prof. Vincent, who are both my first teachers of Industrial Organization and my professional role models. Ginger and Daniel are forever generous in providing insightful and much-needed feedback throughout my entire research process over the years. I would also like to thank Prof. Filiz-Ozbay and Prof. Urzua for graciously agreeing to serve on the committees of both my proposal defense and my dissertation defense. Emel and Sergio are renowned experts in their fields. They provide me fresh perspectives on my paper and teach me to speak to a broader audience. I also thank Prof. Liye Ma for agreeing to serve on my dissertation committee as the Dean's Representative.

I thank Prof. Guido Kuersteiner, Prof. Nolan Pope, Prof. John Shea and Ms. Emily Molleur for their help on the job market. One could not ask for more professional and efficient placement service. I also thank the participants of my presentations at the Economics department and other events for useful comments.

My colleagues at the Economics department deserve a special mention for all the valuable discussions, encouragement and support throughout my doctoral studies, as well as for enriching my graduate life in many ways.

I would like to also acknowledge the Department of Economics at the University of Maryland for the financial support in obtaining the NPD data. I am grateful to Prof. John Shea and Ms. Vickie Fletcher for this and the many other favors they granted me during my time here.

My PhD journey started with three Professors at Vanderbilt University who saw a Doctor in me: Prof. Emmanuele DiBenedetto, Prof. Myrna Wooders and Prof. Kamal Saggi. I thank them for trusting me when I knew little economics and had but passion for the discipline.

Lastly, I owe my deepest thanks to my parents, whose endless support I try not to let down. I am blessed to have Diyue, my wife and classmate at the Economics department, by my side. There are things to be said for a marriage to your co-worker, and empathy and camaraderie are not the least of them.

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1.1 Introduction

"Single digits (of online reviews) didn't seem to move the needle at all. It wasn't enough to get people comfortable with making that purchase decision."

— John McAteer, Google Retail and Tech

The modern era has greatly transformed retailing. On the one hand, many products have become increasingly sophisticated and harder to appraise before purchase. On the other hand, the abundance of product review information in the public domain has greatly eased consumers' anxiety of "making the right choice". Such information is especially important for durable goods, including consumer electronics like smart phones and personal computers and entertainment products like video games and movies, for which each consumer typically has only a unit demand. Barring opportunities to try out the product at a store and the like, consumers of durable goods largely need to rely on so-called social learning to resolve their uncertainty, which means learning the opinions of others either in the real world or on the Internet. Indeed, according to a recent poll by the Ipsos company, 78% of Americans read online reviews before they make purchases (Bassig 2013). Perhaps inevitably, the desire to become more informed leads to purchase delay for the sake of better information, as is evidenced by the above quote by John McAteer, who runs shopping.google.com. While strategic informational delay poses new challenges for retail practitioners, it also opens up new channels through which consumers' purchase decisions can be influenced by a firm. In this paper, I focus on the pricing policies for sellers of durable experience goods. Set in the US video games industry, I attempt to answer the three following questions. 1) How important a role does social learning play in consumers' purchase decisions? 2) How should a profit-maximizing firm set optimal intertemporal prices to a population of forward-looking social learners? 3) What is the implication of consumers' forward-looking social learning behavior for the firm's profits?

My answer to the first question is that relative to their prior beliefs, consumers on average assign a weight of 65.5% to social learning in the first month that a product in the sample is released to the market. To answer the second question, I use the method of Mathematical Programming with Equilibrium Constraints (MPEC) and develop an algorithm to find optimal intertemporal prices as functions of relevant state variables, which include consumers' belief variables. Armed with optimal prices, I find the answer to the third question by making two counterfactual comparisons: 1) Compared to myopic social learners, a market populated by forward-looking social learners reduces a firm's maximum profits by 28.4%; 2) Compared to correctly assessing their patience level, wrong belief of consumers' ability to look forward costs a firm up to 29.92% of its optimal profits.

To answer these questions, I develop an empirical model with which to analyze the optimal pricing policy of a firm selling a durable good with initially uncertain quality to a population of homogeneous consumers. At the beginning, consumers hold a common prior belief of a product's quality. This belief then evolves in a Bayesian manner as more consumers buy and report their experiences to the public opinion pool. Such mechanism of endogenous information accumulation creates an interdependence of decisions among consumers: a consumer's value of waiting depends on how many other consumers make purchases in the current period. The demand equilibrium concept then naturally arises: demand is in equilibrium when the marginal consumer finds it equally beneficial to purchase and to wait.[1] Put differently, when more consumers than the equilibrium level make purchases, too much information is generated that some of them prefer to wait. Conversely, when too few consumers buy, some delayers shall deem the informational gain too small to justify the wait. This demand equilibrium is nested in the market equilibrium, which is a Markov Perfect equilibrium (MPE) where both the firm and the consumers play best responses to the other party's action in every state. That is, consumers make the correct purchase/wait decisions according to firm's optimal prices; and the firm sets prices to maximize its present discounted value (PDV) of future profits, fully accounting for consumers' decisions and the resulting evolution of state variables.

The empirical exercise is conducted in two steps. In the first step, I use the demand model to find demand estimates. In the second step, these estimates are input

[1]The model is actually of a continuum of consumers. I use the discrete language because it's more intuitive.

to the equilibrium-finding algorithm, which solves for optimal prices. In demand estimation, an endogeneity problem arises because price potentially depends on both the unobserved product characteristic and the (also unobserved) belief variables. I control for this problem by estimating the demand side together with a pseudo pricing policy function, which is a polynomial function of the relevant state variables. This function also serves as a proxy for the firm's true pricing function, which is used when consumers calculate value of waiting. Simulating evolution of unobserved belief variables, I use Maximum Simulated Likelihood approach to find demand parameters. Finding optimal prices requires the firm to maximize its value function in every state given consumers' optimal responses and the resulting state evolutions. I solve this problem by the Mathematical Programming with Equilibrium Constraints (MPEC) approach, which is shown by Su and Judd (2012) to be more efficient than the nested fixed point (NFXP) approach. In this approach, the firm's problem is written in such a way that, in addition to the pricing function, consumers' value function of waiting is also regarded as a control variable, and the two functions satisfy the constraint that consumers play the demand equilibrium given the firm's pricing function in every state.

I apply this framework to the US video games industry. The data I use are monthly sales prices and quantities obtained from NPD Group of video games released on the SONY PlayStation 3 between January and December 2009. I observe the complete history of sales prices and quantities since the introduction of every product. Demand estimates are consistent with the observed downward-sloping patterns of prices. They also reveal that social learning is an important determinant in consumers' purchase decisions. To be precise, for the games in the sample, in the first month after their introduction, an average weight of 65.5% is assigned to the social learning signal when consumers make their purchase decision. This weight gradually decreases over time and most learning happens within the first three to four months, however, it's clear that social learning plays a significant role in consumers' purchase decisions.

Demand estimates are then used to find equilibrium prices and profits for the firm. These results allow me to do two counterfactual exercises. In the first exercise, I compare firm profits in two different worlds: one where consumers are static decision makers who learn; one where consumers are forward-looking social learners. The result shows that the forward-looking behavior reduces firm's profits by $5.2M (28.4%) on average. This result is regardless of whether the product is introduced with a quality belief higher or lower than its true quality. The intuition is that forward-looking social learners are more rational than their myopic counterparts in that they recognize the value of waiting for better information. This additional rationality limits the firm's ability to extract profits. In the second exercise, I calculate the firm's profit loss when it incorrectly assumes consumers' discount parameter. With the true patience level set at 0.975, the firm loses 4.81%, 8.21% and 17.58% of its optimal profits when it assumes consumers' patience level is 0.9, 0.75 and 0.5 respectively. In the extreme case where patient consumers are assumed to be myopic, firm suffers a loss of 29.92% of its PDV of profits. This exercise quantifies the monetary value to the firm of the information about the extent to which consumers look forward. It indicates that researches regarding consumers' forward-looking learning behavior is immensely valuable to the firm.

The rest of the first two chapters is organized as follows. Section 1.2 reviews the relevant literature. Section 1.3 describes the model. Section 1.4 then discusses the empirical strategy and derives the simulated likelihood function for estimation of the dynamic model of consumer demand. Section 1.5 introduces the video game data and discusses the demand estimates. The firm's decisions are discussed in chapter 2. Section 2.1 develops the firm model. Section 2.2 discusses the pricing implications corresponding to these estimates of demand. Section 2.3 concludes.

1.2 Related literature

The social learning literature and the dynamic pricing literature are both expansive.[2] In this section, I review only researches most relevant to my study.

Of the theoretical literature on social learning, Frick and Ishii (2016) is the only paper that studies the innovation adoption problem of forward-looking social learners. They provide a careful analysis of consumers' informational incentives and their dependence on quantitative and qualitative features of the news environment through which social learning occurs. They show that qualitatively different news environments give rise to observable differences in aggregate adoption dynamics, which implies a purely informational explanation for two of the most commonly observed adoption patterns (S-shaped vs. concave curves). Quantitatively, they identify news environments where beyond a certain level, increased opportunities for

[2]See Mobius and Rosenblat (2014) for a review of the theoretical and empirical literature of social learning. Gönsch et al. (2013) provides a review of the newer theoretical literature of dynamic pricing. Chan et al. (2009) reviews structural studies of the empirical pricing problem.

social learning can slow down adoption and learning and do not increase consumer welfare. These results depend non-trivially on the ease and nature of information transmission of the news environment.

On the empirical side, many researchers have used reduced form studies to show that social learning exists in many industries, and often has sizable impact on consumers' adoption decisions and firms' profits. In the movie industry, Moretti (2011) uses the number of screens dedicated to a movie in its opening weekend to identify ex ante demand expectations. He then compares the sales trajectories of movies with positive surprise and negative surprise in the opening weekend, i.e. movies with higher than expected demand and lower than expected demand. Based on a few pieces of evidence, he concludes that social learning is an important determinant of sales in the movie industry, accounting for 32% of sales for the typical movie with positive surprise. In the book industry, Chevalier and Mayzlin (2006) studies the valence of online book reviews. They use public data from Amazon.com and Barnesandnoble.com. By comparing the sales of the same book on the two platforms with a "differences-in-differences" approach, they find that the addition of new, favorable reviews at one site results in an increase in the sales of a book at that site relative to the other site. Furthermore, analysis of the length of reviews suggest that consumers actually read and respond to written reviews, not merely the summary statistics provided by the websites. In agriculture, there is also evidence for forward-looking social learning: Bandiera and Rasul (2006) analyze the decision of farmers in Mozambique to adopt a new crop, sunflower. They find that if a farmer's network of friends and family contains many adopters of the new crop, knowing one more adopter may make him less likely to initially adopt it himself. Munshi and Myaux (2006) compares farmers' willingness to experiment with new high-yield varieties (HYV) across rice and wheat growing areas in India. Farmers in rice growing regions, which compared with wheat growing regions display greater heterogeneity in growing conditions that make learning from others' experiences less feasible, are found to be more likely to experiment with HYV than farmers in wheat growing areas. Ching (2010) is one of the few structural studies of social learning. He investigates the prescription drug market after patent expiration during the 80s, when the diffusion of generic drugs is fairly slow. He estimates consumer demand for both branded and generic drugs in a static setting, and find evidence that brand-name firms might set their prices lower than what they would do if they were myopic, in order to slow down the learning process for generic qualities. Compared to the previous works, mine is the first to incorporate a structural demand side with forward-looking social learners.

There has been a long series of theoretical work studying the pricing of experience goods in economics. Early works include Shapiro (1983) , Cremer (1984), Milgrom and Roberts (1986), Farrell (1986), Tirole (1988). These studies either assume myopic consumers (Shapiro 1983), consider only the commitment price path (Cremer 1984), or have unbiased expectations (Milgrom and Roberts 1986, Farrell 1986, Tirole 1988). More recently, Villas-Boas (2004) considers the equilibrium pricing of experience goods in a duopoly model with differentiation along a location and a taste dimension. The location is known at the outset whereas tastes are learned through experience. It presents a sufficient condition on the skewness of the distribution under which brand loyalty exists in equilibrium. A common feature of these papers is that they consider private learning only, i.e. they study repeat purchase products, rather than durable goods. Bergemann and Välimäki (2006) was the first to study experience goods pricing in a fully dynamic model with a population of heterogeneous buyers with independent private valuations. They concluded that optimal paths of sales and prices can be either decreasing or increasing depending on whether the market is "mass" or "niche". In an extension of their model, the authors integrate elements of social learning. The opportunity to learn from other buyers is transformed into an increase in the discount rate. In consequence, the equilibrium policies of the seller are exactly as if he would face buyers with a larger discount rate. Papanastasiou and Savva (2017)'s setup is closest to this study. They investigated how the presence of social learning affects the strategic interaction between a dynamic-pricing monopolist and a forward-looking consumer population, within a two-period model. They find that with the presence of social learning, firm's responsive pricing could be either increasing or decreasing. Moreover, even though the social learning process exacerbates strategic consumer behavior (i.e., increases strategic purchasing delays), its presence results in an increase in expected firm profit.

In stark contrast to the richness of theoretical models is the sparsity of empirical studies on dynamic pricing, even in the absence of the experience good aspect.[3] Che et al. (2007) study pricing competition when consumer demand is state-

[3] Without specifically studying dynamic pricing, quite a few recent papers incorporate a dynamic demand side. Some of these papers are Gandal et al. (2000), Hendel and Nevo (2006) Esteban and Shum (2007), Melnikov (2012), Song and Chintagunta (2003), Gordon (2009) Goettler and Gordon (2011) and Gowrisankaran and Rysman (2012).

dependent (e.g. switching cost, inertia or variety-seeking in consumer behavior) in the breakfast cereal market. They assume firms are forward-looking but boundedly rational. They find that observed retail prices are consistent with firms having short time horizons when setting prices (i.e., they look ahead by only one period). Nair (2007) builds a dynamic model of demand incorporating forward-looking behavior of heterogeneous consumers and a forward-looking firm that takes this consumer behavior into account in formulating its optimal pricing policy. He presents an empirical application to the video games market and show that consumer forward-looking behavior has a significant effect on optimal pricing of games in the industry. Relative to the existing empirical literature on dynamic pricing, my paper is the first to study the fully dynamic pricing problem of a firm with a dynamic demand side and social learning.

1.3 Model

This section discusses the main model of the paper. The model describes the market dynamics of a monopoly firm selling a product with uncertain quality to a population of forward-looking consumers who resolve the quality uncertainty through social learning. I develop a discrete-time finite-horizon dynamic programming model that has two parts: a demand model of forward-looking social learners, and a supply model of a firm who maximizes present discounted value (PDV) of future profits by dynamically setting prices over time. That is, the firm does not commit to a price path beforehand.

At the beginning of the first period, the firm introduces a product with uncertain quality. Both the firm and the consumers are equally uninformed, and they always share the same belief of the product's quality. The uncertainty is gradually resolved through social learning. The firm can set a different price in each period with the purpose of maximizing future profits. The market is populated by a continuum of ex-ante homogeneous consumers, who choose to buy or wait in every period. Because the product is durable, once a purchase is made, the consumer leaves the market. The sale process ends in finite periods. If consumers miss the last chance to buy, they drop out of the market and get zero utility. The sequence of the events are:

1. At the beginning of the first period, the firm introduces a product. However, the true quality A_j is unknown to both the firm and the consumers and they share a common prior ζ_{j1} of the quality, which is unobservable to the econometrician.

2. At the beginning of every period, period-specific product characteristic ξ_{jt} and belief of quality ζ_{jt} is learnt by both the firm and the consumers. The firm sets a price p_{jt} for the product for the current period.

3. Consumers observe the price and choose between (i) wait and enter the next period, (ii) purchase and leave the market.

4. Buyers experience a utility and may report their experiences of the product to form an aggregate signal. It is then used to update the population's belief.

5. By the end of the last period, if a consumer has not purchased the product, he drops out of the market and gets zero utility.

Below I describe different pieces of the model in detail. Since the model is monopolistic, I suppress product subscript j when it's not confusing.

1.3.1 Demand

The consumption of the product yields the following *ex-post* indirect utility for consumer i:

$$\upsilon_{it} = \alpha A + rA^2 + \omega h_t + \gamma p_t + \xi_t + \varepsilon_{i1t}. \tag{1.1}$$

A is the present discounted lifetime utility, or "true quality" of the product. It is the also the object of learning. r is a measure of consumers' risk attitude. h_t is a time indicator defined as

$$h_t = \begin{cases} t-1 & \text{if } t \leq 12, \\ 12 & \text{if } t > 12. \end{cases} \tag{1.2}$$

This variable reflects the fact that consumers value the "newness" of the product: the longer the product has been on the market, the less its value. However, after the first year, "newness" no longer matters. p_t is the period t price. ξ_t is a period-specific product characteristic, observable to the firm and the consumers, but not to the econometrician. This could be factors such as the screening of a movie that is based on the game, or the good performance of a soccer star that is featured in a soccer game, etc. ε_{i1t} is an individual-specific demand shock for purchase that is only observed by consumer i. Assume $\boldsymbol{\varepsilon_{it}} \equiv (\varepsilon_{i0t}, \varepsilon_{i1t}) \overset{i.i.d.}{\sim}$ Type I extreme value, where ε_{i0t} is the idiosyncratic shock for waiting, which will be introduced below.

Bayesian social learning. The ex-post indirect utility is unknown to consumer i at the time of making a purchase because A is uncertain. I now describe the social learning process through which consumers establish and update their beliefs of this uncertainty.

As is conventional in the learning literature (Ching et al. 2013), I assume normality for beliefs of quality. These are denoted by $\zeta_t \sim N(A_t, \sigma_t^2)$. Similarly, the aggregate signal produced by early buyers is also assumed normal: $\bar{A}_t \sim N(A, \sigma_e^2/Q_t)$. σ_e^2 signifies precision of the aggregate signal, a result of both the credibility of individual consumer's signal and the number of quality reports the public can access. Q_t is the units sold in period t. The assumptions are that the aggregate signal is centered on the true quality, and that its precision (inverse of variance) of the aggregate signal is proportional to the demand in period t, i.e. the more sales are made in a period, the more precise the aggregate signal.

Bayesian updating implies the following transition rule for the mean and variance of the belief:

$$A_{t+1} = (1 - \eta_t)A_t + \eta_t \bar{A}_t, \tag{1.3}$$

$$\sigma_{t+1}^2 = \frac{1}{1/\sigma_t^2 + Q_t/\sigma_e^2}. \tag{1.4}$$

Some discussion is in order regarding η_t, the weight placed on the aggregate signal. The Bayesian formula gives:

$$\eta_t = \frac{\sigma_t^2}{\sigma_t^2 + \sigma_e^2/Q_t}. \tag{1.5}$$

Define $\iota_t \equiv \frac{\sigma_t^2}{\sigma_e^2/Q_t}$, then $\eta_t = \frac{\iota_t}{\iota_t+1}$ is increasing in ι_t. ι_t is the ratio of period t starting quality uncertainty and the uncertainty in the aggregate signal. When $\iota_t \to 0$, social learning is irrelevant. Either the quality of the product is certain ($\sigma_t^2 \to 0$) or consumers' reports are pure noises ($\sigma_e^2 \to \infty$). When $\iota_t \to \infty$, social learning is instantaneous: the population is either completely uninformed about the product ($\sigma_t^2 \to \infty$) or are capable of producing fully revealing signals ($\sigma_e^2 \to 0$). Thus $\iota_1 = \frac{\sigma_1^2}{\sigma_e^2/Q_1}$ can tell us how relevant social learning is when the product is first introduced.

Expected utilities. Armed with quality beliefs, consumer i can form an expectation of the ex-post indirect utility from purchase using the time t belief:

$$u_{i1t} \equiv E_t[\upsilon_{it}] = \underbrace{\alpha A_t + r(A_t^2 + \sigma_t^2) + \omega h_t + \gamma p_t + \xi_t}_{\delta(p_t, \mathbf{x}_t)} + \varepsilon_{i1t}. \tag{1.6}$$

Both ξ_t and ε_{i1t} are known to the consumer and pass through the expectation operation. For simplicity, we can denote the period-specific part of the expected utility by $\delta(p_t, \mathbf{x}_t)$. $\mathbf{x}_t$ denotes the non-price (public) state variables of period t.

The value of consumer i waiting till the next period is defined recursively as

$$u_{i0t} = \beta_c E_t[\max(u_{i1,t+1}, u_{i0,t+1})] + \varepsilon_{i0t}, \tag{1.7}$$

where β_c is consumers' discount factor, and ε_{i0t} is the demand shock associated with waiting. E_t represents the period t conditional expectation of the stochastic state variables: p_{t+1}, A_{t+1} and ξ_{t+1}.

Regarding p_{t+1}, I make the assumption that consumers are rational predictors of the future price. That is, consumers know firm's pricing policy function $p(\mathbf{x}_t)$, and they form correct expectation of p_{t+1} based on how state variables evolve.

The distinguishing feature of consumer i's' option value of waiting in this model, however, is that it depends on other consumers' actions. To see this, refer to equations (1.6), (1.3), (1.4) and (1.5) and notice that the both A_{t+1} and σ^2_{t+1} depend on Q_t, which is determined by all consumers' "purchase/wait" decisions. Therefore, to pinpoint the value of waiting, I introduce the demand equilibrium concept.

Value of waiting in demand equilibrium. I now describe how consumers' value of waiting is determined in the demand equilibrium. Firstly, following Rust (1987), the equilibrium value of waiting can be written in a "choice-specific" value function:

$$u_{i0t} = W(\mathbf{x}_t) + \varepsilon_{i0t}. \tag{1.8}$$

Given the Type I extreme value distribution of $\boldsymbol{\varepsilon}_{i,t+1}$, I can further write

$$W(\mathbf{x}_t) = \int \log\left[\exp(\delta(p(\mathbf{x}_{t+1}), \mathbf{x}_{t+1})) + \exp(W(\mathbf{x}_{t+1}))\right] dF(\mathbf{x}_{t+1}|\mathbf{x}_t). \tag{1.9}$$

In all but the last period, consumers play a game of "buy or wait" among themselves. Each consumer has public state variables $(p_t, \mathbf{x}_t)$ and private state variables $\boldsymbol{\varepsilon}_{it}$ and choose between actions {Wait, Buy}, or $\{0, 1\}$. I look for a symmetric Bayesian Nash equilibrium in which consumers play the same strategy. I use $s'(\mathbf{x}_t, \boldsymbol{\varepsilon}_{it})$ to denote a generic strategy, and use $s'(\mathbf{x}_t)$ to denote the probability of purchase implied by $s'(\mathbf{x}_t, \boldsymbol{\varepsilon}_{it})$:

$$s'(\mathbf{x}_t) = E_{\boldsymbol{\varepsilon}}[s'(\mathbf{x}_t, \boldsymbol{\varepsilon}_{it}) = 1]. \tag{1.10}$$

Let $\tilde{u}_{i0t}$ be consumer i's generic expected payoff from waiting, then

$$\tilde{u}_{i0t}(s'(\mathbf{x}_t, \boldsymbol{\varepsilon}_{it})) = \Psi(\mathbf{x}_t, s'(\mathbf{x}_t)) + \varepsilon_{i0t}, \tag{1.11}$$

where $\Psi(\cdot)$ is the generic value of waiting. That is

$$\Psi(\mathbf{x}_t, s'(\mathbf{x}_t)) = \int \log\left[\exp(\delta(p(\mathbf{x}_{t+1}), \mathbf{x}_{t+1}|s')) + \exp(W(\mathbf{x}_{t+1}|s'))\right] dF(\mathbf{x}_{t+1}|s'). \tag{1.12}$$

Here $\mathbf{x}_{t+1}|s'$ means the distribution of $\mathbf{x}_{t+1}$ conditional on all consumers playing strategy s'. Since we are considering only the stage game of period t, we only allow consumers to deviate from the equilibrium for this period, and force them to comply to the equilibrium strategies for all future periods. Therefore, the right hand side of equation (1.12) has $W(\cdot)$ rather than $\Psi(\cdot)$. In plain words, the generic value of waiting is the option value of waiting until the next period, with state variables' evolution dictated by a generic strategy, and given optimal strategies in future periods.

Consumer i's payoff of purchase is independent of other consumers' actions, so

$$\tilde{u}_{i1t} = u_{i1t} = \delta(p_t, \mathbf{x}_t) + \varepsilon_{i1t}. \tag{1.13}$$

Therefore, consumer i's optimal strategy is simply

$$s'(\mathbf{x}_t, \boldsymbol{\varepsilon_{it}}) = \begin{cases} 1 \text{ if } \tilde{u}_{i1t} \geq \tilde{u}_{i0t}, \\ 0 \text{ if } \tilde{u}_{i1t} < \tilde{u}_{i0t}. \end{cases} \tag{1.14}$$

Definition 1. *A Bayesian Nash equilibrium is a decision rule $s(\mathbf{x}_t, \boldsymbol{\varepsilon}_{it})$ such that equations (1.11), (1.13), and (1.14) hold for (almost) all consumers for (almost) all $\boldsymbol{\varepsilon}_{it}$.*

Because the optimal strategy is of the "cutoff" form, I can simply describe the equilibrium in terms of consumers' choice probability. $s'(\mathbf{x}_t) \in [0, 1]$ constitutes an equilibrium of consumers' game if

$$s'(\mathbf{x}_t) = \frac{\exp(\delta(p_t, \mathbf{x}_t))}{\exp(\delta(p_t, \mathbf{x}_t)) + \exp(\Psi(\mathbf{x}_t, s'(\mathbf{x}_t)))}. \tag{1.15}$$

Denote the equilibrium purchase probabilities with $s(\mathbf{x}_t)$. Then the (equilibrium) value function of waiting is

$$W(\mathbf{x}_t) \equiv \Psi(\mathbf{x}_t, s(\mathbf{x}_t)). \tag{1.16}$$

Consumers' purchase probability is given by

$$s(p_t, \mathbf{x}_t) = \frac{\exp(\delta(p_t, \mathbf{x}_t))}{\exp(\delta(p_t, \mathbf{x}_t)) + \exp(W(\mathbf{x}_t))}. \tag{1.17}$$

Although equation (1.17) is of the familiar logit form, it's important to recognize that $s(p_t, \mathbf{x}_t)$ and $W(\mathbf{x}_t)$ enter the definition of the other, and that both are results of equilibrium consumer decisions.

Then period t demand is

$$D(p_t, \mathbf{x}_t) = Q_t = M_t s(p_t, \mathbf{x}_t), \tag{1.18}$$

where M_t is the period t market size.

Discussion of demand equilibrium. Before turning to the firm's problem, I provide a brief discussion of the demand equilibrium concept to make it more concrete.

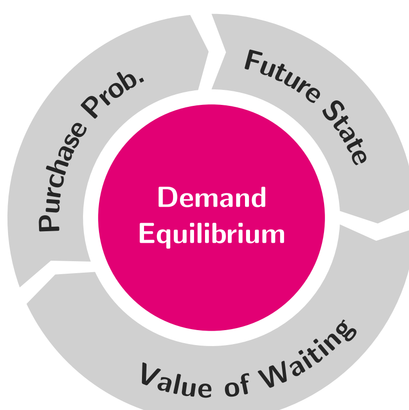

Figure 1.1: A graphical illustration of demand equilibrium

Figure 1.1 provides a graphical illustration of the demand equilibrium concept.

In the demand model, consumers' purchase probability determines the distribution of state variables of the next period, which in turn determines the value of waiting. Consumers' rationality then links their value of waiting back to the purchase probability. The demand is in equilibrium when this feedback loop is at a fixed point. That is, consumers at the margin shall find buying and waiting equally preferable. Put differently, when the purchase level is higher than the equilibrium level, information generated by buyers is so much that some buyers are better off waiting until the next period. On the other hand, if the purchase level is lower than equilibrium, then some of the consumers who delay shall find it unworthwhile to do so, as there's not enough information being generated.

1.3.2 Evolution of state variables

The model's non-price state variables are $\mathbf{x}_t = (h_t, A_t, \sigma_t^2, \xi_t, M_t)$. The transition of (A_t, σ_t^2) has been described in subsection 1.3.1. I now state the assumptions for M_t and ξ_t.

Transition of market size. In the video games industry, with the sales of new hardware (video game consoles such as PlayStation 3) in each period, buyers of these hardware become potential consumers for video game software. For simplicity, I assume a constant inflow of N new consumers per period, and I calculate N as the average of monthly hardware sales. The transition of market size is then:

$$M_{t+1} = M_t(1 - s(p_t, \mathbf{x}_t)) + N. \tag{1.19}$$

Unobserved product characteristic. The unobserved product characteristic ξ_{jt} is assumed to be i.i.d. across periods and products with mean 0, i.e. $\xi_{jt} \sim N(0, \sigma_\xi^2)$.

1.4 Demand estimation and numerical strategy

In this section, I will use the demand model to estimate demand parameters. The assumption is that the observed data are from demand equilibria. However, I do not assume the firms to be profit maximizing as described in section 2.1.1. This is due to two reasons: 1) the purpose of this paper is to find optimal prices for the firm, so I do not assume the prices to be already optimal; 2) estimating demand with a reduced-form supply side, as I will describe in subsection 1.4.1, is computationally easier than doing it with a fully structural supply side. In the following, I first discuss the price endogeneity problem and the pseudo pricing function approach that I adopt to alleviate this problem. I then discuss the details of the maximum simulated likelihood approach for demand estimation.

1.4.1 Pseudo pricing policy function

Because the researcher does not know the firm' true pricing policy functions, he faces two challenges in demand estimation. First, the researcher cannot use firms' true pricing policy functions to proxy consumers' expectations for future prices. Second, he has to address the fact that unobserved product characteristic ξ_t and the public information variable A_t may be correlated with prices. If such correlation is uncontrolled for, an endogeneity problem arises in demand estimation and parameter estimates will be biased.

Nair (2007) employed a "limited information" approach to deal with the endogeneoty problem, which effectively uses lagged price as an instrument for current price. This approach is however not appropriate for my model. This is because unobserved demand shock A_t in my model is serially correlated by design. Therefore, if p_t is endogenous, so is p_{t-1}.

The technique I adopt is known as the pseudo policy function approach (e.g. Ching (2010)). This technique approaches firm's true policy function with polynomials of the relevant state variables. [4] For now, I use the linear policy function:

$$\rho_{jt}(\mathbf{x}_{jt}) = \rho_{j0} + \rho_1 t + \rho_2 A_{jt} + \rho_3 \sigma_{jt}^2 + \rho_4 \xi_{jt} + \rho_5 M_{jt} + \varepsilon_{jt}, \tag{1.20}$$

where $\varepsilon_{jt} \sim N(0, \sigma_\varepsilon^2)$. This function directly builds in correlation between p_{jt} and (A_{jt}, ξ_{jt}), which are represented by the coefficients ρ_2 and ρ_4. By estimating this pseudo pricing policy function together with the demand side, any linear correlation should be picked up by these coefficients. However, it's possible that we could miss correlation of p_{jt} with higher orders of A_{jt} and ξ_{jt}.

[4]The theoretical foundation for this method is the Stone-Weierstrass theorem. One implication of this theorem is that if the firm's true pricing policy function is a continuous function $\rho(\mathbf{x})$ on $\prod_{n=1}^{N}[a_n, b_n] \subset \mathbb{R}^N$, then $\forall \varepsilon$ there exists a polynomial function $p(\mathbf{x})$ such that $|p(\mathbf{x}) - \rho(\mathbf{x})| < \varepsilon$, $\forall \mathbf{x} \in \prod_{n=1}^{N}[a_n, b_n]$. Moreover, the Stone-Weierstrass theorem implies that the tensor product bases of orthogonal polynomials are also dense in $C[R^N]$. The orthogonal polynomials I use are the Chebyshev polynomials. Details of these results can be found in Chapter 6 of Judd (1998). The caveat of this method, however, is that only finite explorations of polynomial approximations can be done. If the polynomials I use are of orders too low or that I miss relevant state variables as covariates, the estimation results will be biased.

1.4.2 Maximum simulated likelihood estimation

The demand is estimated with the Maximum Simulated Likelihood approach (e.g. Cameron and Trivedi 2005). Let $\{Q_{jt}, P_{jt}\}_{(j,t)=(1,1)}^{(J,T)}$ be the monthly units sold and prices data of all products in the sample, and denote all parameters to estimate by Θ, i.e. $\Theta = (\gamma, \omega, \sigma_\varepsilon^2, \sigma_\xi^2, \{A_j\}_{j=1}^J, \{A_{j1}\}_{j=1}^J, \{\sigma_{j1}^2\}_{j=1}^J, \{\rho_{j0}\}_{j=1}^J, \{\rho_k\}_{k=1}^5)$. In demand equilibrium, the randomness in the model has 3 pieces: $\xi_{jt}, \varepsilon_{jt}$ and A_{jt}. Given $\{A_{j1}\}_{j=1}^J, \{\sigma_{j1}^2\}_{j=1}^J$ and $\{Q_{jt}\}_{(j,t)=(1,1)}^{(J,T)}$, I simulate $\{A_{jt}\}_{(j,t)=(1,2)}^{(J,T)}$ according to the Bayesian formulas. Given a simulated A_{jt}, the demand and pricing functions can be inverted to find the unobserved values:

$$\begin{aligned}\hat{\xi}_{jt} =& D^{-1}(Q_{jt}, P_{jt}; A_{jt}) = \log(\frac{Q_{jt}}{M_{jt} - Q_{jt}}) - \alpha A_{jt} - r(A_{jt}^2 + \sigma_{jt}^2) - \omega h_{jt} \\ & - \gamma P_{jt} + W_{jt}(A_{jt})\end{aligned} \tag{1.21}$$

$$\hat{\varepsilon}_{jt} = \rho_{jt}^{-1}(P_{jt}; A_{jt}, \hat{\xi}_{jt}) = P_{jt} - \rho_{j0} - \rho_1 t - \rho_2 A_{jt} - \rho_3 \sigma_{jt}^2 - \rho_4 \hat{\xi}_{jt} - \rho_5 M_{jt} \tag{1.22}$$

In demand equilibrium, the log-likelihood of observing data $\{Q_{jt}, P_{jt}\}_{(j,t)=(1,1)}^{(J,T)}$ is

$$L(\Theta; \{Q_{jt}, P_{jt}\}_{(j,t)=(1,1)}^{(J,T)}) = \sum_{j=1}^{J} \sum_{t=1}^{T} \log \left\{ \int f_{\xi,\varepsilon}(\hat{\xi}_{jt}, \hat{\varepsilon}_{jt}; A_{jt}) \frac{\partial(\hat{\xi}_{jt}, \hat{\varepsilon}_{jt})}{\partial(Q_{jt}, P_{jt})} dF(A_{jt}) \right\}. \tag{1.23}$$

Based on (1.21) and (1.22) and the distribution assumptions for ξ_{jt} and ε_{jt},

$$
\begin{aligned}
f_{\xi,\varepsilon}(\hat{\xi}_{jt}, \hat{\varepsilon}_{jt}; A_{jt}) &= f_{\xi}(\hat{\xi}_{jt}; A_{jt}) f_{\varepsilon}(\hat{\varepsilon}_{jt}; A_{jt}, \hat{\xi}_{jt}) \\
&= \frac{1}{\sigma_{\xi}} \phi(\frac{\hat{\xi}_{jt}(A_{jt})}{\sigma_{\xi}}) \frac{1}{\sigma_{\varepsilon}} \phi(\frac{\hat{\varepsilon}_{jt}(A_{jt}, \hat{\xi}_{jt})}{\sigma_{\varepsilon}}),
\end{aligned}
$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-1/2x^2}$ is the pdf of the standard normal distribution. And the Jacobian is given by

$$
\begin{aligned}
\frac{\partial(\hat{\xi}_{jt}, \hat{\varepsilon}_{jt})}{\partial(Q_{jt}, P_{jt})} = &= \left| det \begin{pmatrix} \frac{\partial \hat{\xi}_{jt}}{\partial Q_{jt}}, \frac{\partial \hat{\xi}_{jt}}{\partial P_{jt}} \\ \frac{\partial \hat{\varepsilon}_{jt}}{\partial Q_{jt}}, \frac{\partial \hat{\varepsilon}_{jt}}{\partial P_{jt}} \end{pmatrix} \right| \\
&= \left| det \begin{pmatrix} \frac{\partial \hat{\xi}_{jt}}{\partial Q_{jt}}, \gamma \\ -\rho_4 \frac{\partial \hat{\xi}_{jt}}{\partial Q_{jt}}, 1 - \rho_4 \gamma \end{pmatrix} \right| \\
&= \left| (1 - \rho_4 \gamma) \frac{\partial \hat{\xi}_{jt}}{\partial Q_{jt}} + \rho_4 \gamma \frac{\partial \hat{\xi}_{jt}}{\partial Q_{jt}} \right| \\
&= \left| \frac{\partial \hat{\xi}_{jt}}{\partial Q_{jt}} \right| \\
&= \frac{M_{jt}}{Q_{jt}(M_{jt} - Q_{jt})}
\end{aligned}
$$

1.4.2.1 Values of waiting

For the purpose of demand estimation, the assumption is that the observed data are from a demand equilibrium. Therefore, the product-and-period-specific value function needs only to depend on A_{jt}. This is because first, given initial conditions and data, h_{jt}, σ^2_{jt} and M_{jt} are deterministic; second, ξ_{jt} doesn't affect values of waiting as it's assumed to be non-correlated over time. Given the assumption that consumers are rational price predictors, and that we use the pseudo pricing policy function as proxy pricing policy function, value functions of waiting are defined recursively as:

$$W_{jT}(\cdot) = 0,$$

and

$$\begin{aligned} W_{jt}(A_{jt}) =& \beta_c \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \log\left\{\exp\left[x - r(x^2+\sigma_{jt}^2) - \omega h_{j,t+1} - \gamma\rho_{j,t+1} + z\right]\right\} \\ &+ \left\{\exp\left[W_{j,t+1}(x)\right]\right\} \frac{1}{\sigma_t[A_{jt+1}]}\phi(\frac{x-\mu_t[A_{jt+1}]}{\sigma_t[A_{jt+1}]})\frac{1}{\sigma_\varepsilon}\phi(\frac{y}{\sigma_\varepsilon})\frac{1}{\sigma_\xi}\phi(\frac{z}{\sigma_\xi})\, dxdydz, \end{aligned}$$

for $t = 1, \ldots, T-1$, where

$$\rho_{j,t+1} = \rho_{j0} + \rho_1 h_{t+1} + \rho_2 x + \rho_3\sigma_{j,t+1}^2 + \rho_4 z + \rho_5 M_{j,t+1} + y$$

is the pseudo pricing policy function. Also,

$$\mu_t[A_{j,t+1}] = E(A_{j,t+1}|A_{jt}),$$

$$\sigma_t[A_{j,t+1}] = (\sigma_{A_{j,t+1}}^2|A_{jt})^{1/2}$$

are the conditional mean and standard deviation of $A_{j,t+1}$ given A_{jt}.

1.5 Data and estimation results

In this section, I first briefly describe the video games industry and the data I use. I then present estimates of demand and the pseudo pricing policy function.

1.5.1 The video games industry

The computer and video games industry is a major entertainment industry with 67% of American households playing computer or video games. The hardware medium that runs the game software separates these markets: computer games can be run on general computers, whereas video games are only run on game consoles they are specifically designed for. The main players in the so-called 7th generation video game era (since 2005) include Microsoft, Sony and Nitendo. These companies develop and sell consoles (Xbox 360, PlayStation 3 and Wii, respectively), and charge royalty fees to firms producing software. Software firms, mainly independent publishers, develop games for one or more consoles, and pay royalty fees to the hardware manufacturers for every game unit sold. Traditionally, the hardware has been sold at or below cost, subsidizing the sales of the software, which in turn accounts for most of the profits. In 2009, software revenues from video games in the US totaled $9.9 billion, and more than 250 million units of video game software were sold (Williams 2002, Entertainment Software Association 2010).

Critic reviews vs consumer reviews. Many review platforms exist in the video games industry. The more popular ones include *metacritic.com*, *IGN.com*, *GameSpot.com* and *YouTube.com*. There are two types of reviews: critic reviews written by product experts and consumer reviews written by regular buyers. Critic reviews are typically released in the early days of the life cycle of the game, while consumer reviews are gradually accumulated as more sales are made. The two types of reviews often agree, but discrepancies, even sharp differences, frequently occur.

For example, Call of Duty: Ghost, a sequel game of a highly successful video game franchise, Call of Duty, received critic ratings of 8.0 and 8.8 from GameSpot.com and IGN.com, respectively. However, it only received an average consumer rating of 5.7 on GameSpot.com over the first two months post-release (Liu and Ishihara 2017). [5]

1.5.2 Data

The data I use are units sold and prices of all new video games released on the Sony PlayStation 3 in the US market between January and December 2009. The sample includes the complete history of aggregate retail sales and prices of 131 games since their date of introduction. The Point of Sales (POS) data were collected by the NPD Group using scanners linked to over 90% of the consumer electronics retail ACV in the US.[6]

The three main stylized features of the data are as follows:

- Though not monotonic, prices generally trend downwards. The average rate of change over the first 18 months ranges from 0.96% to 15.38%, with the median monthly rate of decline being 2.9%.

- Units sold have a less pronounced time trend. The average monthly decline rate for all games in the sample is 0.8% or 4,416 units.

- There is wide variance in the unit sales of the games. The least successful

[5] As critic reviews are early and influential, it's considered by other researches (e.g. Liu and Ishihara 2017) to form consumers' prior beliefs. In this version, the source of consumer learning is unobserved. In future iterations, I shall make the learning process more concrete.

[6] The data doesn't include digital downloads.

game in the data ("Tornado Outbreak") had 3,942 units sold, while the most successful game ("Call Of Duty: Modern Warfare 2") had a total of 4,789,738 units sold.

- Most sales and revenues are made early in the game's life cycle. By the end of the first 18 months, all games in the sample have made over 92% of their lifetime sales and earned 97% of their lifetime revenue.

Figure 1.2 and table 1.1 provide more detailed exposition of these facts.

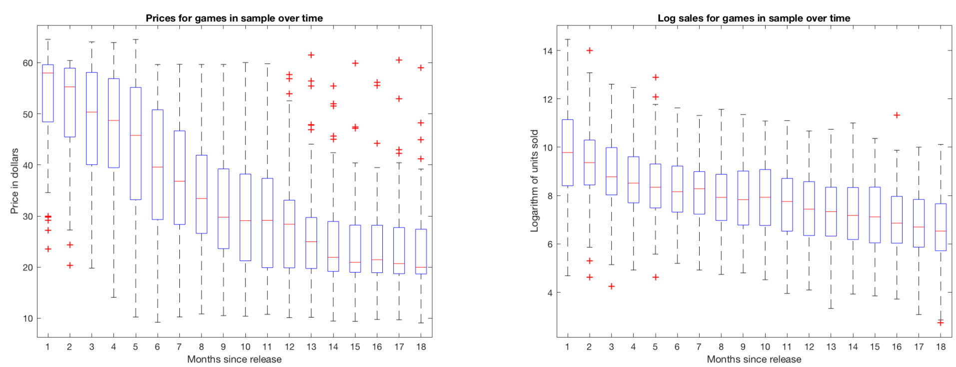

Figure 1.2: Time trends in price and units sold

1.5.3 Estimates of demand and pseudo pricing function parameters

Table 1.2 displays estimates for parameters in the demand function and the pseudo pricing policy function. By reading the demand estimates, it can be seen that all parameters are of the expected signs. Also, of all estimates, only the coefficient for market size in the pseudo pricing policy function is insignificant. All other parameters are significant at least at the 5% level. Some features of consumers' demand are: first, the consumers value high quality products, and are slightly risk averse. In general, belief variance is much less important than belief mean to the

Age of game	Prices		Unit sales	
	Mean	Std. dev.	Mean	Std. dev.
1	52.47	9.62	77,216.56	196,804.66
2	50.78	9.69	43,369.63	121,218.89
3	48.42	11.10	22,622.25	44,331.13
4	46.64	11.25	16,010.11	33,682.85
5	42.78	12.79	14,202.47	40,857.52
6	40.20	13.22	9,072.98	15,814.85
7	37.21	12.66	7,889.43	12,725.45
8	34.53	11.94	7,294.43	14,390.28
9	32.70	11.40	6,350.20	10,579.07
10	31.39	11.48	6,245.35	9,481.39
11	29.69	10.74	5,126.00	8,509.35
12	27.94	9.53	4,226.94	6,272.46
13	26.24	9.14	3,854.08	6,695.44
14	24.64	8.53	4,046.75	7,969.64
15	23.73	7.80	3,736.96	6,115.56
16	23.58	7.57	3,310.58	8,075.67
17	23.34	7.90	2,388.32	3,976.90
18	22.70	7.84	2,138.36	3,780.19

Table 1.1: Summary statistics for price and units sold over time

consumers. Second, the negative value of ω means that consumers value the "newness" of the products. This fact is consistent with the downward-sloping prices we observe in the data. The monetary equivalent of a product being one month newer is \$2.35 on average. Lastly, the mean of $\iota_{j1} = \sigma_{j1}^2/(\sigma_e^2/Q_{j1})$ for the sample products is 1.90, which translates to an average weight parameter of 0.655. That is, consumers on average place a weight of 65.5% on the aggregate signal in the first month that the product is introduced. While this weight decreases over time as belief variance decreases, it's however clear that social learning plays an important role in consumers' purchase decisions.

Variable	Parameter	Std. err.
Demand parameters		
Belief mean (α)	0.9447***	0.1962
Risk attitude (r)	-0.0018***	0.0005
Price (γ)	-0.2016***	0.0309
Newness (ω)	-0.4747***	0.1554
Demand shock variance (σ_ξ^2)	2.1518**	0.9702
Signal noisiness[c] (σ_e^2)	1.9832**	1.0057
Pseudo pricing parameters		
Time (ρ_1)	-0.0360***	0.0095
Belief mean (ρ_2)	6.3809***	2.0186
Belief variance (ρ_3)	-3.3329***	0.7653
Demand shock (ρ_4)	2.5148***	0.8522
Market size in millions (ρ_5)	4.7354	10.4075
Log-likelihood	-760.67	
Number of observations	360	

a. Full set of prior mean, variance and true quality, as well as price intercepts estimated, but not reported.
b. ***: significant at 1%, **: significant at 5%
c. Quantity measured in millions

Table 1.2: Demand and pseudo pricing parameter estimates

1.5.4 An example of the importance of social learning

To make the impact of social learning more concrete, I provide an illustration using an example game named "Borderlands". This game is estimated to have true quality $A = 53.24$, prior mean of quality $A_1 = 22.73$, and prior variance of mean $\sigma_1^2 = 16.58$. To separate the effect of social learning, I fix price at $p_t = \$50$, and unobserved demand shock $\xi_t = 0$.

Figure 1.3 plots the demand dynamics of "Borderlands" with and without social learning. Firstly, as expected, belief variance monotonically decreases over time. Secondly, the dashed horizontal red line plots the true quality, and it can be seen that belief mean gradually approaches this value as time passes. Belief mean, however, does not converge to the true quality, because after a few months, belief variance is so low that new information does not matter much for the consumers and learning effectively stops barring small disturbances. Lastly, the blue lines plot purchase probabilities with and without social learning. Without social learning, the probability monotonically decreases. With social learning, the purchase probability increases at first as a result of belief mean being corrected upwards, and eventually decreases as the "newness" effect dominates after a few months. The total sales quantity over 18 months with social learning is 2,835,502. Compared to the sales of 1,643,736 units without social learning, the total impact of social learning is a 72% increase in sales for this game. However, this result obviously depends on the fact that the prior mean is lower than the true quality. If the relation of the two values is reversed, then social learning shall hurt sales compared to when consumers don't learn.

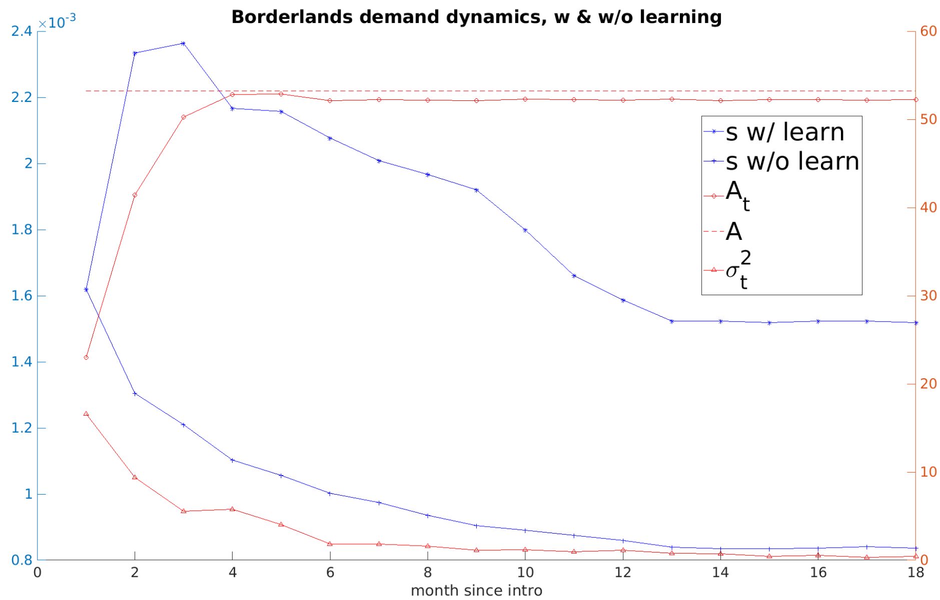

Figure 1.3: An illustration of the impact of social learning

Chapter 2: Dynamic Pricing with Forward-looking Social Learners: The Case of the US Video Games Industry

In this chapter, the demand estimates obtained in chapter 1 are used as inputs to find optimal prices for each game. I first develop the model of the firm's behavior and define the market equilibrium concept. I then present the MPEC algorithm I use to find the optimal prices. Next I describe patterns of firm's optimal prices and compare market outcomes from optimal pricing with the observed data. Lastly, I will conduct two counterfactual exercises to study the effect of forward-looking social learning for the firm's profits.

2.1 Firm Model

2.1.1 Firm decisions

The firm's per-period profit is given by:

$$\pi(p_t, \mathbf{x}_t) = D(p_t, \mathbf{x}_t)(p_t - c), \tag{2.1}$$

where c is the constant marginal cost. Assume the firm to be risk neutral, and maximizes present discounted value of current and future profits. Its value function is:

$$V(\mathbf{x}_t) = \max_{p_t > 0}[\pi(p_t, \mathbf{x}_t) + \delta_f \int V(\mathbf{x}_{t+1}) dF(\mathbf{x}_{t+1}|\mathbf{x}_t)], \tag{2.2}$$

where δ_f is the firm's discount factor.

The firm's pricing policy function is the maximizer of the value function:

$$p^*(\mathbf{x}_t) = \underset{p(\mathbf{x}_t) > 0}{\text{argmax}}[V(\mathbf{x}_t)]. \tag{2.3}$$

2.1.2 Market equilibrium

I now present the equilibrium concept of the entire model.

Definition 2. *A Markov-perfect equilibrium in prices in the above model is defined by a set of value functions of waiting* $W(\mathbf{x}_t), t = 1, 2, \ldots, T-1$*, and pricing functions* $p^*(\mathbf{x}_t)$*, such that equations* (1.17)*,* (1.18)*,* (2.1)*,* (2.2) *and* (2.3) *are satisfied at every state* $\mathbf{x}_t$*.*

In plain words, the market equilibrium requires that in every state, every period, consumers play the equilibrium strategy in their game of information waiting. The firm maximizes its PDV of profits by setting optimal prices, fully accounting for consumers' actions and the resulting evolution of state variables.

It turns out that the market equilibrium is unique. This result is shown by Proposition 5 of Papanastasiou and Savva (2017). The authors use a two-period model and show this result by (1) showing that in the last (second) period, there is a unique equilibrium in the pricing-adoption game between the firm and the consumers; (2) there is a unique optimal purchasing strategy by the consumers in the previous (first) period. It is straightforward to adapt this proof to any finite-horizon game by backward induction and realize that these two conditions are all I need to guarantee uniqueness of equilibrium. Result (1) is obvious. As for result (2), there are two differences between my model and theirs. The first is that consumers in their model is risk neutral with respect to product quality while mine are potentially risk-averse. This difference does not change the result as the fact that information is beneficial is true no matter consumers' attitude toward risk and therefore doesn't change the fact that consumers' optimal purchasing strategy is unique in every state. The second difference is the unobserved product characteristic that I assume. When consumers calculate their value of waiting, however, this value is integrated out as it is not serially correlated. It therefore also has no effect on the uniqueness of consumer strategy in early periods. Therefore, I conclude that given initial conditions $\mathbf{x}_1$ and appropriate parameter values, there is a unique Markov perfect market equilibrium played between the firm and the consumers in the T-period pricing-adoption game.

2.2 Pricing implications

2.2.1 The algorithm to find optimal prices

I solve for the equilibrium prices using the method of Mathematical Programming with Equilibrium Constraints (MPEC), first discussed in Su and Judd (2012). The key to this method is that in addition to the traditional control variables, which are the pricing functions for my case, some auxiliary variables are also considered as control variables. These two types of control variables are required to satisfy some equilibrium constraints, which take the place of solving explicitly for the auxiliary variables for given control variables as done in the Nested Fixed Point (NFXP) approach. Su and Judd (2012) show that MPEC in general is much faster than NFXP. For my model, the auxiliary variables are the value functions of waiting, and the equilibrium constraints are the demand equilibrium conditions.

The steps of the algorithm are as follows:

1. Discretize the state space of $\mathbf{x}_t = (A_t, \sigma_t^2, \xi_t, M_t)$ into G points.

2. Start backwards from period T, at each state, find optimal price by solving

$$V_T(\mathbf{x}_T) = \max_{p>0} M_T \frac{\exp(\delta(p, \mathbf{x}_T))}{\exp(\delta(p, \mathbf{x}_T)) + 1}(p - c).$$

3. Starting from period $T-1$, in every state, solve

$$V_t(\mathbf{x}_t) = \max_{p>0, W_t(\mathbf{x}_t)} M_t s(p, \mathbf{x}_t, W(\mathbf{x}_t))(p - c) + \beta_f E_t V_{t+1}(\mathbf{x}_{t+1})$$

   s.t. (demand equilibrium conditions)

   (a) calculate $s_t(p, \mathbf{x}_t)$ according to equation (1.17), M_{t+1} according to (1.19), Q_t according to equation (1.18).

   (b) calculate belief variance and weight σ_{t+1}^2, η_t according to equations (1.4) and (1.5).

   (c) calculate conditional mean and variance of A_{t+1}: $\mu_t[A_{t+1}] = (1 - \eta_t)A_t +$

$\eta_t A$, $\sigma_t^2[A_{t+1}] = \eta_t^2 \frac{\sigma_e^2}{Q_t}$.

(d) The below updated value function is equal to $W(\mathbf{x}_t)$, with expectation taken with respect to A_{t+1}, ξ_{t+1}:

$$TW_t(\mathbf{x}_t) = \beta_c E_t \log \left\{ \exp \left[\delta(p_{t+1}(\mathbf{x}_{t+1}), \mathbf{x}_{t+1}) \right] + \exp \left[W_{t+1}(\mathbf{x}_{t+1}) \right] \right\}$$

4. Stop after period 1 problem is solved.

The expectation in the above calculation is done numerically with Gauss-Hermite quadrature with 8 nodes.

2.2.2 Equilibrium pricing policy

Figure 2.1 displays equilibrium pricing policy in period 1 and 7 of the game "Borderlands", which has a estimated true quality of 53.24, prior quality mean of 22.73, and prior quality variance 16.58. Several qualitative features of equilibrium pricing are obvious. 1). Equilibrium prices are increasing in the mean of consumer belief and decreasing in its variance. 2). Equilibrium prices are monotonically related to the demand shock. 3). Equilibrium prices trend down over time.

Figure 2.2 presents firm's value function of the game "Borderlands" in periods 1 and 7. The value function decreases with time. It is increasing in consumer belief mean, and is decreasing in its variance, with the former having a more significant effect. Demand shock shifts value function, but its effect is transient and less pronounced than that in equilibrium prices.

For consumers' value of waiting and purchase hazard, similar to Nair (2007),

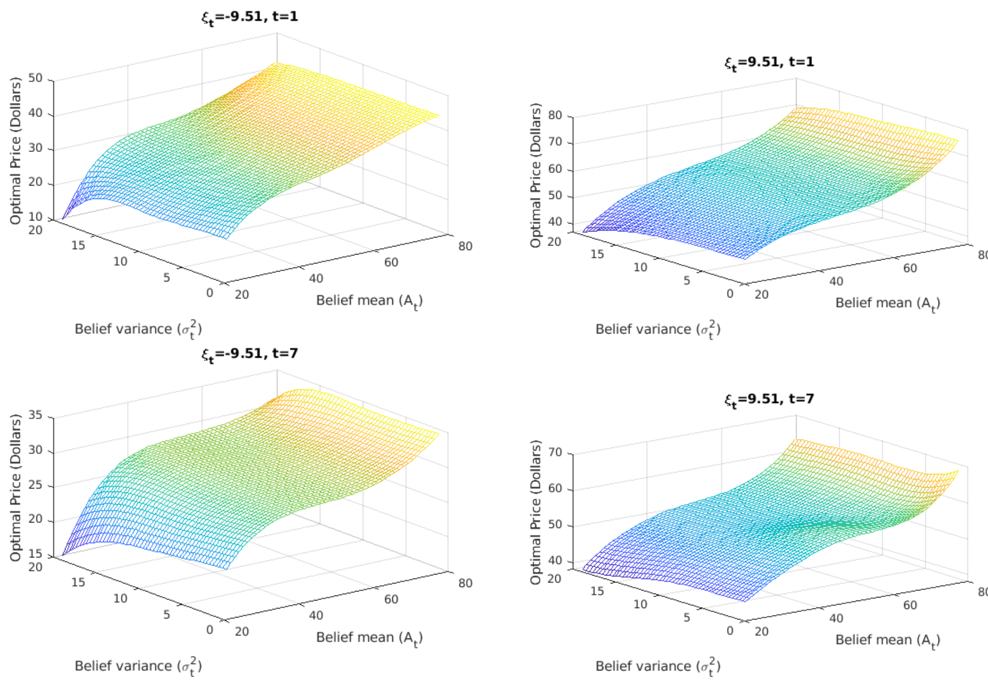

Figure 2.1: Equilibrium pricing policy of "Borderlands" in period 1 and 7

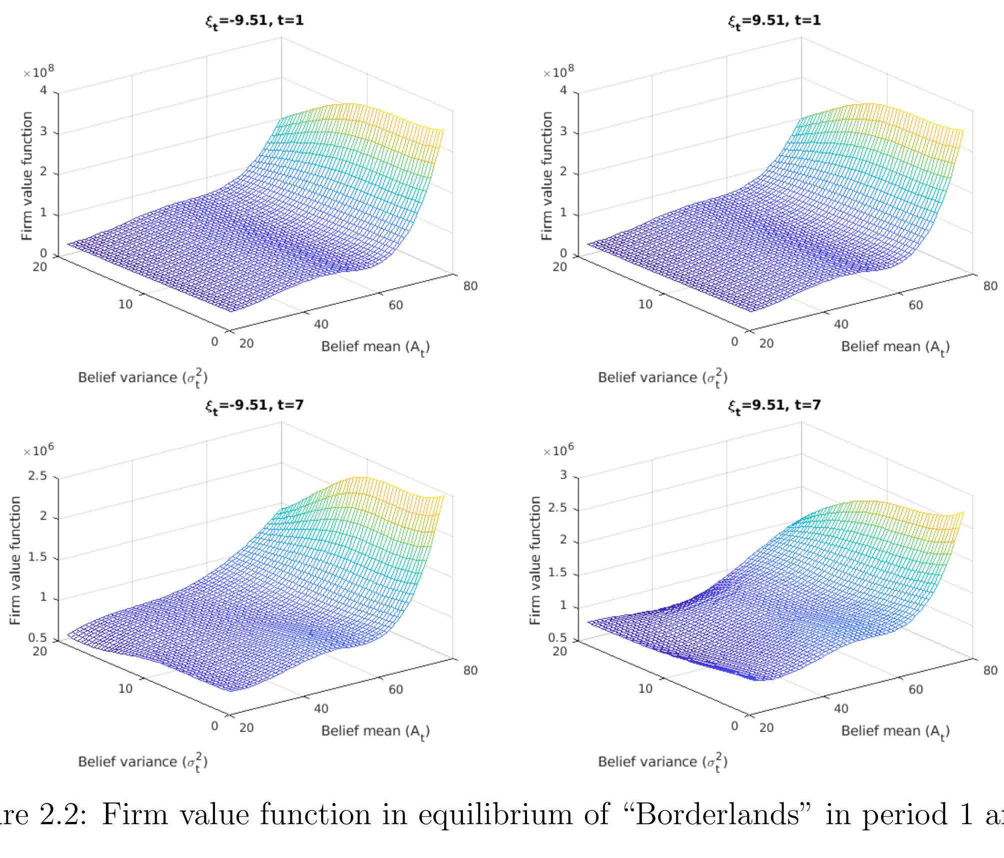

Figure 2.2: Firm value function in equilibrium of "Borderlands" in period 1 and 7

I plot against usual price points. For period 1, I set belief mean and variance at the estimated values, i.e. $A_1 = 22.73$ and $\sigma_1^2 = 16.58$. To find period 7 values, I take 30 demand shock paths $\{(\xi_1, \xi_2, \ldots, \xi_7)\}$, and for each shock path I simulate 30 paths of belief mean $\{A_1, A_2, \ldots, A_7\}$, resulting in a total of 900 demand realizations. I plot the mean value of waiting and purchase hazard of these realizations.

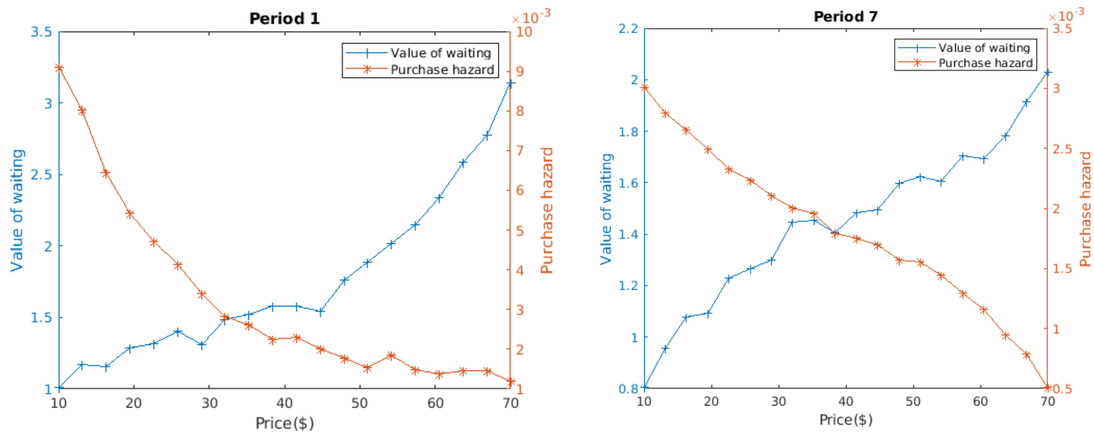

Figure 2.3: Consumer value of waiting and purchase hazard of "Borderlands"

Purchase hazard is inversely related to price, and value of waiting is positively related with price. In Nair (2007), price declining is correlated with time passing, high value consumers leaving and low value consumers taking over the market. Without consumer heterogeneity and for a specific time period, however, the results are driven by consumer learning rather than change of population composition. Because the game "Borderlands" start with a disadvantage (prior mean smaller than true quality), the firm will initially price low. With social learning in motion, belief mean will increase and belief variance decrease, both leading to higher prices. When product uncertainty is largely resolved after some time, the only driving forces of price change will be consumer's valuation of game "newness" and their discount of late consumption. Comparing period 1 with period 7, we see consumer's value of waiting is larger for period 1. This is largely driven by lower initial valuation of the product. However, due to product "newness", purchase hazard for period 1 is still higher. While the relationship between price and purchase probability for a given time period remains negative, the relationship between price and value of waiting is not general. This is because the value of waiting depends on two different forces. On the one hand, high price leads to low sales and low informational gain from waiting. On the other hand, disadvantaged belief means that firm will price low in the next period, which increases the value of waiting. That is, the informational incentive and price incentive of waiting may counteract each other, leading to undetermined relationship between price and value of waiting.

2.2.3 Observed vs. predicted prices

Table 2.1 compares the observed prices and the predicted equilibrium prices. Again, for each game, I simulate 30 paths of $\{\xi_1, \xi_2, \ldots, \xi_{18}\}$, and for each of these paths, I simulate 30 paths of $A_1, A_2, \ldots, A_{18}$. The presented numbers are averages of the simulated ones. In general, predicted prices are lower than the observed ones. However, caution is advised when making conclusions, because firm's prices correspond to only one history of realization of the unobserved variables.

Age of game	Observed prices		Predicted prices	
	Mean	Std. dev.	Mean	Std. dev.
1	52.47	9.62	48.55	15.67
2	50.78	9.69	47.55	12.98
3	48.42	11.10	46.91	10.74
4	46.64	11.25	44.69	8.66
5	42.78	12.79	40.22	7.63
6	40.20	13.22	38.50	6.23
7	37.21	12.66	34.77	6.17
8	34.53	11.94	29.93	5.16
9	32.70	11.40	26.60	5.27
10	31.39	11.48	21.40	5.05
11	29.69	10.74	18.30	5.18
12	27.94	9.53	15.99	6.02
13	26.24	9.14	12.95	5.96
14	24.64	8.53	11.91	5.56
15	23.73	7.80	10.54	6.01
16	23.58	7.57	9.58	4.70
17	23.34	7.90	9.33	4.43
18	22.70	7.84	8.19	3.38

Table 2.1: Observed and predicted prices for games in sample

2.2.4 Impact of consumer rationality

I now return to the main empirical question raised at the beginning of this study: what is the implication of consumers' forward-looking social learning behavior on firms' profits? The next two subsections answer this question from two different perspectives. In this subsection, I consider two different types of consumers and compare firm's optimal prices when faced with each: (1) forward-looking social learners; (2) "myopic social learners": short-lived consumers who exit the market after one period no matter the purchase decisions, who nevertheless report their experiences if they buy and are able to utilize the updated information in the market.

These assumptions represent two levels of consumer rationality: both types of consumers have the ability to correct wrong beliefs, with the latter further recognizing the value of waiting for better information.

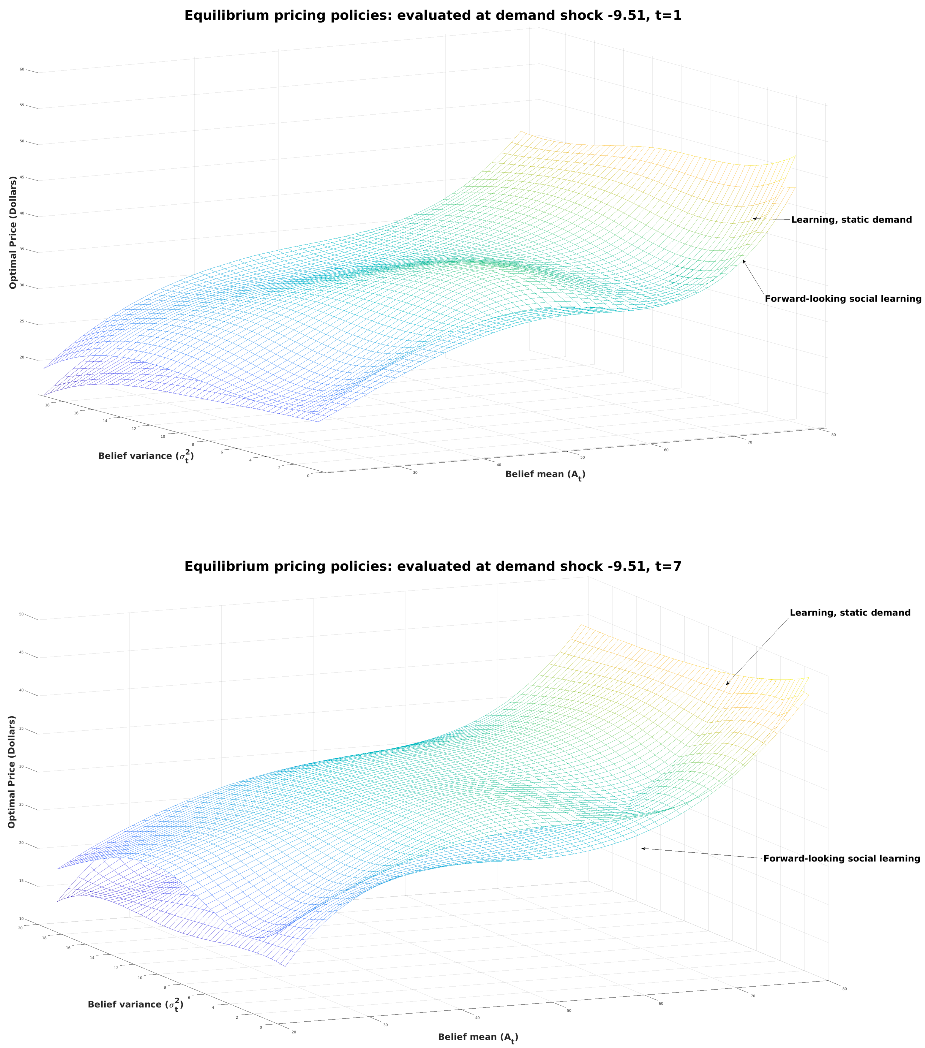

Figure 2.4: Equilibrium pricing policies of "Band Hero" in 2 scenarios

Figure 2.4 plots the equilibrium pricing policies of the game "Band Hero" in these 2 worlds. This game is estimated to have a true quality of 22.73, but sales start with an advantage at prior mean 35.33, prior variance 11.14. The results show that optimal prices with forward-looking consumers is lower than the static ones. This is intuitive because the forward-looking consumers value waiting for information, and is therefore less inclined to buy at the same price than their static counterparts. This is true for both high and low values of belief mean: the rationality of the forward-looking social learners limit the firm's ability to profit even when they overvalue the product.

To investigate the profit impact of consumer rationality, I compare firms' profits with myopic vs. forward-looking social learners. For each game in the sample, I simulate 30 paths of demand shocks $\{\xi_1, \xi_2, \ldots, \xi_{18}\}$. For each path of demand shocks, I simulate 30 belief evolutions $\{A_1, A_2, \ldots, A_{18}\}$ for both myopic and forward-looking consumers. Difference in profits for each game is the difference in the means for both worlds. I then take the average over the entire sample.

Figure 2.5 plots the difference in firm's PDV of profits with different consumers. Firms make an average of \$18.3M with myopic social learners, and \$13.1M with forward-looking social learners. The total difference (period 1 number) is \$5.2M, i.e. forward-looking social learners reduce firms' profits by about 28.4%.

2.2.5 Profit loss from wrong rationality assessment

The above results show that the level of consumer rationality has a big impact on firms' profitability. To further gauge the value of information on consumers' forward-lookingness, I consider the situation where consumers are forward-looking social learners, but firm holds incorrect belief of their discount factor. In this exercise, consumers' value functions of waiting $W_t(A_t, \sigma_t^2, \xi_t, M_t), t = 1, \ldots, T-1$ are

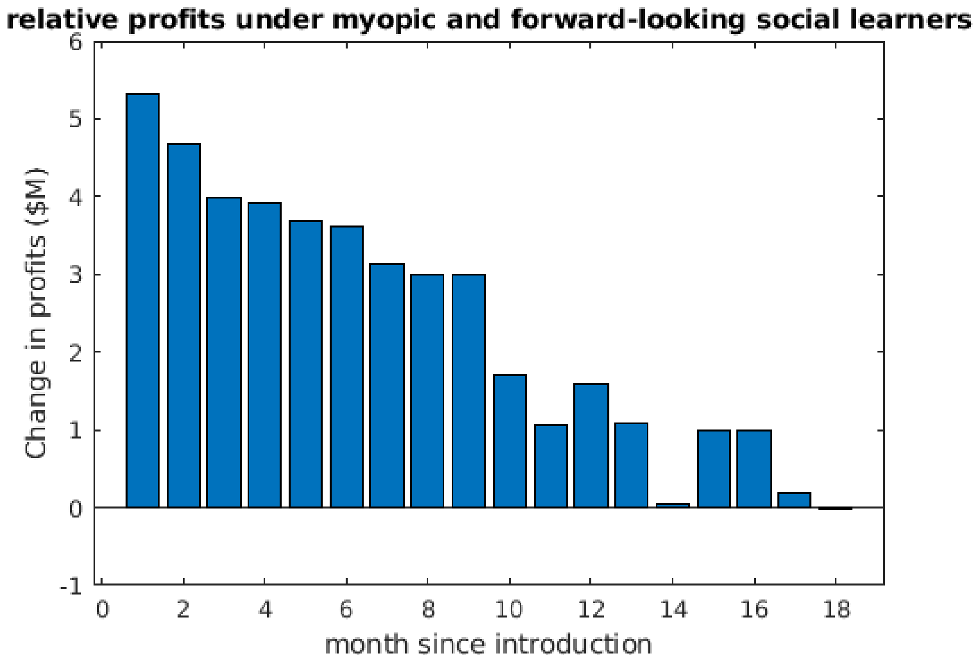

Figure 2.5: Difference in firm PDV under myopic and forward-looking social learners

solved for patience level (β_c) 0.975. But firm's optimal prices are solved at 4 lower levels: 0.9, 0.75, 0.5 and 0, where discount level of zero means that consumers are myopic. I then simulate 30 paths of demand shocks for each game in the sample, and 30 paths of belief evolution for each demand shock path. Demand is calculated with equations (1.17) and (1.18). The results are then averaged across all demand shock and belief evolution simulations.

Figure 2.6 reports the average difference in firm's PDV of profits when the firm uses different levels of incorrect discount factors for the consumers. I find that when the firm uses a discount factor of 0.9, its profits are on average 4.81% lower than the optimal case. When discount factor is set at 0.75, 8.21% profits will be lost. Discount factor of 0.5 results in 17.58% of profits loss. In the extreme case where patient consumers are assumed to be myopic, firm suffers a 29.92% profit loss.

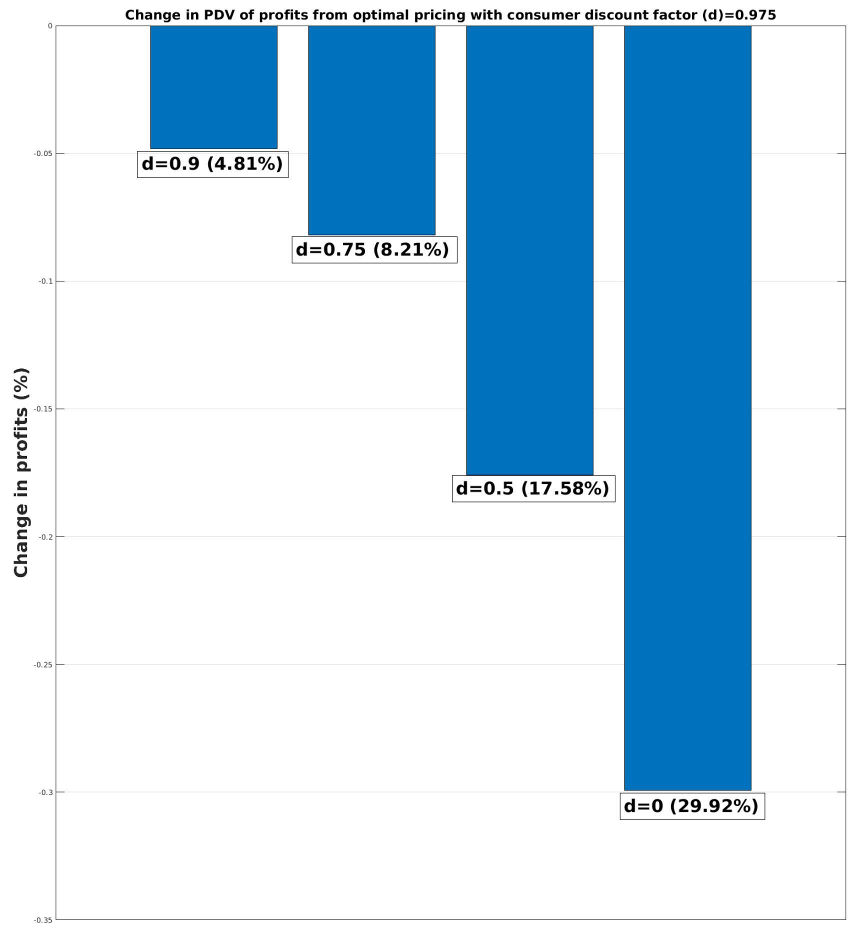

Figure 2.6: Effect of incorrect beliefs of firm about consumer forward-looking behavior on profits

These results place a monetary value on the correct information regarding the extent to which consumers look forward. It's therefore a worthwhile exercise for the firms to conduct researches into this value.

2.3 Conclusions

In this paper, I study the dynamic pricing of durable experience goods with forward-looking social learning consumers. I propose a finite-horizon dynamic programming framework that produces firm's optimal pricing strategy as result of a Markov perfect equilibrium between the firm and the consumers. The distinguishing feature of consumers' forward-looking social learning behavior is that consumers play a coordination game when they strategically wait for better information. The demand equilibrium from this game is nested in the Markov equilibrium between the firm and consumers.

To ease computational burden, I take 2 steps to find optimal prices. In the first step, I estimate demand parameters together with a pseudo pricing policy function of the firm, which is potentially non-optimal. Because of unobserved information evolution, demand is estimated with the Maximum Simulated Likelihood approach. In the second step, I use demand estimates as inputs and find optimal prices using the Mathematical Programming with Equilibrium Constraints (MPEC) approach.

The estimated parameters show that learning plays an important part in demand and are consistent with the downward-sloping observed prices. I use the equilibrium solution to explore the implications of consumers' forward-looking social learning on profits for the video-game firms in the sample. For the firms in the sample, I find that (1) compared to static demand with social learning, forward-looking social learning reduces equilibrium firm profits significantly; (2) firm's incorrect belief of the extent to which forward-looking social learning happens also materially detriments profits.

A limitation of the model is that I don't consider consumer heterogeneity. A heterogeneous model assumes that consumers in the market comprise two or more discrete types, differentiated by some aspects of their preferences (e.g. patience levels, valuations of the product). Such differences lead to different purchase probabilities, therefore endogenously varying type composition over time and richer demand dynamics. While the loss of this richness is partly remedied with the time trend in product value of the homogeneous model (one could consider this time trend to reflect the fact that over time, there are more low-valuation consumers and fewer high-valuation ones), this control nevertheless is simplistic, and heterogeneity remains a key element for the model to be considered comprehensive enough to guide real-world practice. This limitation is mainly due to the significant curse of dimensionality heterogeneity brings as I need at least one more state variable to characterize the population composition. Moreover, a heterogeneous model would also be more difficult to estimate. These difficulties could hopefully be solved by more efficient algorithms or by simply running the programs on more powerful hardware. At any rate, in future iterations of this paper, I shall try to incorporate consumer heterogeneity.

Finally, the learning process in this model is totally unobserved. Given the richness of critics' and consumers' review data. This should be possible to be improved. To make the learning process more concrete, I shall make use of the reviews data to supplement the sales data.

Chapter 3: Dynamic Price Competition and Learning-by-Doing: The Effect of Strategic Buyers on Equilibria

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3.1 Introduction

In a series of papers that provide some of the most well-known and widely-cited applications of computational economics to Industrial Organization, David Besanko and co-authors analyze duopoly models where there is the potential that firms may lower their marginal costs by accumulating production experience, but a firm's rival is also able to lose this know-how. In Besanko et al. (2010) (BDKS) learning-by-doing is combined with the possibility of "organizational forgetting", so that both firms can move both up or down their marginal cost curves, while in Besanko et al. (2014) (BDK) and Besanko et al. (2017) a firm can choose to exit the industry so that it is possible that a firm will face a new rival at the top of its cost curve in the future. The papers show that these features can generate "aggressive" price equilibria where firms set very low prices, including prices that are well below marginal cost, in some states firms not only try to lower their own marginal costs but also to reduce their rival's future efficiency or cause their rival to exit (in BDK the equilibria are thought of as involving "predation"). While customers can obviously benefit from low prices in the states where they are offered, aggressive price equilibria also tend to lead to a single firm dominating the industry in the long-run and, once dominance is achieved, that firm is able to charge high margins, reducing long-run consumer surplus.

BDK and BDKS show that the parameter values that support aggressive price equilibria can also often support equilibria that can be viewed as more "accomodative", where equilibrium pricing behavior tends to sustain the existence of fairly symmetric competitors in the long-run. While accomodative prices tend to be significantly above marginal costs, these equilibria often tend to generate higher long-run welfare (this point is made more explicitly in BDK). From a policy perspective, the papers raise a natural question: how likely are aggressive price equilibria? Within the BDK and BDKS frameworks, this amounts to asking whether the parameters (for example, the progress ratio (measured by a parameter ρ) and the probability of forgetting (δ) in BDKS) that can support aggressive equilibria are plausible for real-world industries. The authors argue that they are (e.g., BDKS p. 462). This leads the authors to suggest that antitrust policy towards predatory pricing potentially faces a delicate balancing act between a real need to try to prevent dominance while avoiding undermining firms' incentives to lower their costs by generating sales (BDK, conclusion).

However, it is also relevant to ask whether the conclusions of BDK and BDKS are robust to changing their model in ways that make sense for the types of industry where sizable learning-by-doing effects can occur. In this paper we consider one simple change to these models, the addition of "strategic buyers", and investigates how this affects the multiplicity of equilibria, the level of equilibrium prices, long-run market structure and welfare. By "strategic buyers" we mean customers who are forward-looking and take into account how their purchases may affect future market structure, and therefore the prices they may have to pay if they want to buy in future periods. Given the computational burden of trying to enumerate as many equilibria as possible in these models, we do so by changing the BDKS model (the same analysis with the BDK model is in progress) in the simplest way possible.

In their models, each buyer is assumed to purchase just once, and then disappears from the market forever. We assume instead that, when she is able to make a purchase, each buyer assumes that she will be selected to be the buyer in any future period with probability, b^p ("buyer-p"). The BDKS framework would correspond to the case when $b^p = 0$. A monopsonist buyer would correspond to $b^p = 1$ and an industry with 20 symmetric buyers, one of whom is called to purchase in each period, would correspond to $b^p = 0.05$. While not allowing a buyer to commit to its future behavior, buyers can act strategically by allowing their interest in the future evolution of the industry to affect their current decision about which company to buy from given the prices that are set. Under this approach the state space remains the same as in the models with short-lived buyers, and the only additional complexity is that we need to keep track of the value functions of a representative buyer as well as the value functions of the sellers.[1]

One could imagine augmenting the BDKS model in many other interesting ways (for example, allowing multiple unit purchases, allowing buyers to tend to prefer to buy from a single supplier or replacing take-it-or-leave-it price competition with some element of bargaining or negotiation). Our decision to focus on strategic buyers is motivated by two considerations. First, many of the industries where substantial learning-by-doing effects have been identified, such as aircraft manufacture (Alchian (1963), Benkard (2000)), shipbuilding (Thompson (2001), Thornton and Thompson (2001)), semiconductors (Irwin and Klenow (1994), Dick (1991))

[1]In particular, a strategic buyer does not care about how many times he has purchased or which firm he has purchased from in the past, and instead she is only concerned about which states, defined by combinations of seller know-how, she may experience in the future.

and chemicals (Lieberman (1984), Lieberman (1987)), have a non-trivial number of large, repeat customers. Even in learning-by-doing markets where one might assume that individual customers reflect only a tiny proportion of the volume, such as hospital procedures (Gaynor et al. (2005), Dafny (2005)), it is plausible that intermediaries that can affect the flow of purchases and are able to extract some customer surplus, such as local physician groups or insurance companies, would act in the type of strategic, forward-looking way considered in our model.

Second, strategic buyers have been a subject of much focus in the analytical theory literature on dynamic models with duopoly sellers, where dynamics arise through learning-by-doing, switching costs or capacity constraints. For example Lewis and Yildirim (2002) consider a model where two sellers, who benefit from learning-by-doing, with no forgetting, compete for sales to a single forward-looking buyer who repeatedly makes purchases in a infinite horizon model. The paper shows that there is a unique equilibrium where the buyer will generally skew his purchasing in a way that, relative to a myopic buyer, slows learning but maintains competition. Subsequent papers by Lewis and Yildirim (2005) and Anton et al. (2014) consider models with a single strategic buyer whose choices are influenced by the desire to maintain competition, even if this type of behavior ends up increasing equilibrium prices. Saini (2012) solves a computational model to show that a strategic monopsonist procurer may purchase in a way to preserve competition. However, it is unclear from this literature how far strategic buyer behavior should affect equilibrium prices and market structure outside of the extreme case of monopsony.[2] To address

[2]One paper that does consider the non-monopsonist case is Clark and Polborn (2011). They

this question, we introduce strategic buyer behavior into the computational framework of BDKS. More broadly one can view our paper as contributing to research on the topical question of "buyer power", by analyzing a setting with dynamics arising from learning-by-doing on the supply side.

We find two main results. First, as soon as we raise the degree of strategic buyer behavior, the area of the (ρ, δ) parameter space that supports multiple equilibria becomes smaller.[3] For intermediate values of b^p above 0.1 we find no multiplicity for parameter values that have any economic relevance. We provide intuition for the existence of single equilibria as b^p increases by examining changes to the implied demand functions of buyers, and the effects that this have on what we label the "dynamic best response functions" of buyers. The basic pattern is that when we consider equilibria where sellers price aggressively to establish dominance and gain significant market power, the demand of strategic buyers tends to shift in a way that makes this type of strategy unprofitable.

Second, we analyze how different values of b^p affects long-run market structure and the welfare of sellers and customers. Not surprisingly, what happens reflects a trade-off between the strategic buyer's incentive to try to lower producer costs (which will tend to favor buying from the lower cost firm, promoting dominance) and constraining market power. For (ρ, δ) parameters which imply that concentration

analyze a model with two periods, duopoly sellers, no learning-by-doing but some possibility that a seller may exit after the first period. They show that N buyers may purchase strategically in order to prevent exit from happening, so that they maintain competition in the second period.

[3]To find multiple equilibria we use the "homotopy path-following" approach used by BDKS and BDK for different values of b^p. The homotopy approach is not guaranteed to find all equilibria in the type of model that we considering, but we have also found that other approaches, for example, by solving the set of equations implied by equilibrium behavior from a large number of different starting values would lead to similar conclusions.

is the only way to offset forgetting, strategic buyers will tend to produce a more concentrated, and more efficient, market structure. However, these outcomes are not necessarily associated with the type of aggressive pricing that characterizes concentrated market structures with no strategic buyers.

Before continuing, we should be clear that we do not view our paper as implying any type of criticism of the BDKS or BDK models, which we view as highlighting how computation can shed light on the economics at play in rich strategic interactions that analytical approaches may not be able to isolate. Instead, we interpret the fact that multiplicity is much less of an issue when we introduce strategic buyers as a positive result, in the sense that it is much easier to use any type of model to guide policy and as the basis for empirical analysis when is not so concerned that multiple equilibria are a pervasive feature of the model.

The rest of the paper proceeds as follows. Section 3.2 explains the model with and without strategic buyers (in the latter case, it corresponds exactly to the BDKS model). Section 3.3 briefly explains how we search and identify multiple equilibria, with the many numerical details presented in the Appendix. Our main results are presented in Sections 3.4 and 3.5, together with our presentation of the intuition for the changes in the nature of equilibria that we identify. Section 3.6 concludes.

3.2 Model

We begin with a short presentation of the BDKS model where buyers are not strategic, which is the limiting case of our model ($b^p = 0$). We then explain what changes when $b^p > 0$.

3.2.1 Learning by Doing and Organizational Forgetting with Non-Strategic Buyers

States and Costs. Consider a discrete time, discrete state infinite horizon game between ex-ante symmetric duopolists, $n = 1, 2$. The common discount factor is $\beta = 0.95$. Each firm's know-how can take on values $e_n = 1, ..., M$. The marginal cost of firm n, $c(e_n)$ is κe_n^η for $1 \leq e_n \leq m$ and κm^η for $m \leq e_n \leq M$. $\eta = \log_2 \rho$, where ρ is the "progress ratio". For BDKS's computations, and our own, $m = 15$ and $M = 30$. The state of the model consists of the marginal costs of each firm $\mathbf{e} = (e_1, e_2) \in ((1, 1), (1, 2)...(M, M - 1), (M, M))$. It is assumed that the state is observed by both firms throughout the game.[4] Figure 3.1 shows the implied marginal cost curves for four values of ρ. As can be seen high values of ρ imply that marginal costs will be low once a relatively low level of know-how has been achieved, but implies that there might be quite intense competition to sell the first few units as it will create a large cost advantage.

Demand. A short-lived buyer who wants to buy a single unit arrives in each period. At the beginning of the period the firms simultaneously set prices, and then the buyer makes her purchase decision. The indirect utility of purchasing from firm n is $v - p_n + \sigma\varepsilon_n$ where p_n is the price set by firm n, ε is a private information Type

[4] Asker et al. (2018), Sweeting et al. (2019a) and Sweeting et al. (2019b) consider dynamic models where a persistent state variable is private information. In these games solving for a single equilibrium imposes a significant computational burden, and the type of search over a wide parameter space that we consider here would likely be completely infeasible.

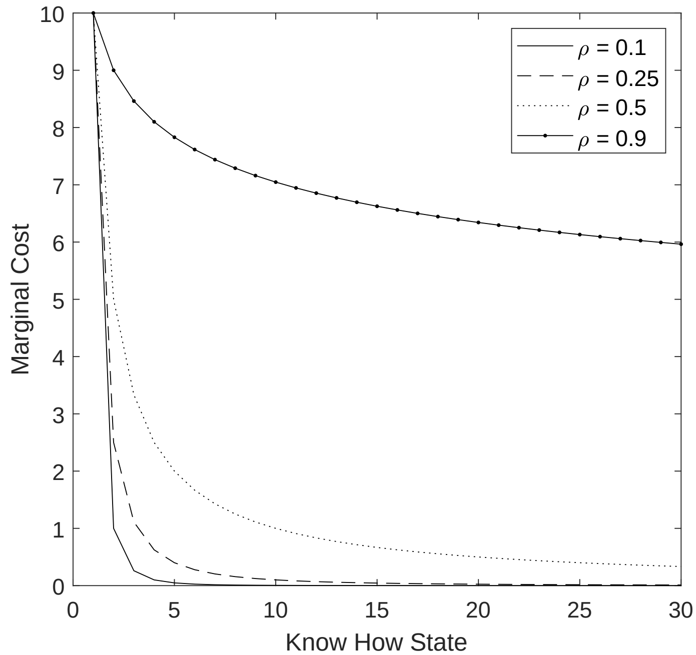


Figure 3.1: Marginal Cost Functions for Different ρs

I extreme value preference shock, and σ is a scaling parameter. We follow BDKS in assuming that there is no "outside good", so that the buyer must buy from one of the firms. The probability that the buyer purchases from firm n given prices p_n and p_{-n} is simply

$$D_n(p_n, p_{-n}) = \frac{1}{1 + \exp\left(\frac{p_n - p_{-n}}{\sigma}\right)} \tag{3.1}$$

so that demand only depends on the scaled difference in prices. We use BDKS's baseline assumption that σ=1.

State Transitions. Dynamics are generated by learning-by-doing and organizational forgetting. Specifically,

$$e_{n,t+1} = e_{n,t} + q_{n,t} - f_{n,t}$$

where $q_{n,t}$ is equal to one if n makes a sale in period t (0 otherwise) and $f_{n,t}$ is equal to one with probability $\Delta(e_n) = 1 - (1 - \delta)^{e_n}$ with $\delta \in [0, 1]$ (0 otherwise), except at the boundaries of the state space.[5] A firm that makes a sale can therefore either have more or the same know-how in the next period, whereas a firm that does not make a sale will have either the same or lower know-how. This structure means that a firm will have an incentive to price aggressively both to lower its own marginal cost (all else equal, this will allow it to earn higher margins and/or make more sales in future periods) and to try to raise its rival's marginal cost (reducing future competition). The assumed functional form implies that the probability of (one unit of) forgetting increases with know-how, but at a decreasing rate, and that the probability of forgetting is quite high for $e_n > 10$ even when δ is small. This is illustrated in Figure 3.2 for three different values of δ. For example, several of BDKS's examples involve $\delta = 0.0275$ and $\delta = 0.08$. For these values, the probability of forgetting when $e_n = 10$ are 0.24 and 0.56 respectively. This implies that if a firm makes sales less frequently than its rival because of a small cost disadvantage, the size of that disadvantage may tend to increase quite rapidly.

Equilibrium. BDKS use the notion of symmetric Markov Perfect Equilibrium (Maskin and Tirole (2001), Ericson and Pakes (1995), Pakes and McGuire (1994)), which restricts strategies to be functions of the current values of the payoff-relevant state variables, which are the know-how states in this model. The symmetry assumption implies that firm 2 in state (e_1, e_2) will use the same price as firm 1 in state (e_2, e_1),

[5]For example, when $e_{n,t} = 1$ and $q_{n,t} = 0$ the firm n cannot forget, and when $e_{n,t} = M$ and $q_{n,t} = 1$ the firm n has to forget.

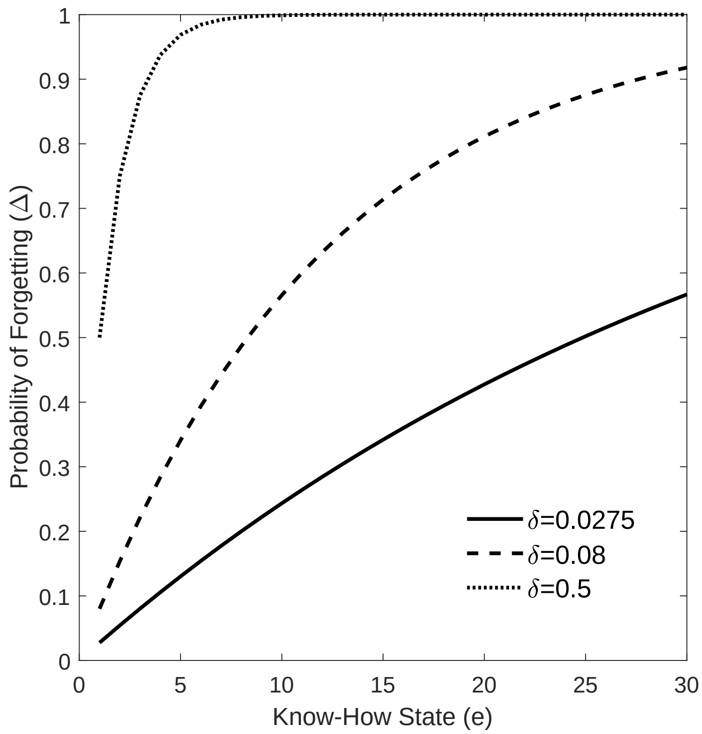


Figure 3.2: Probability of Forgetting for Different Values of δ

so that we can define the equilibrium using the prices and value functions of firm 1 only. The model is stationary in the sense that current and future payoffs only depend on the know-how states, not the time period t.

The seller's value function, $V^S(\mathbf{e})$, is defined at the beginning of a period before the buyer's payoff shocks are realized. To specify the Bellman equation, it is useful to first define the "conditional value functions" that defines the seller's continuation value as a function of which seller the buyer decides to purchase from. For example, in the case where seller 1 makes the sale, the conditional value function is defined as

$$\overline{V}_1^S(\mathbf{e}) = \beta \sum_{e_1'=e_1}^{e_1+1} \sum_{e_2'=e_2-1}^{e_2} V^S(\mathbf{e}') \Pr(e_1'|e_1, 1) \Pr(e_2'|e_2, 0)$$

where $\Pr(e_1'|e_1, 1)$ is the probability that firm 1 transitions to know-how state e_1' in the next period given that firm 1's current know-how state is e and it makes the sale, and the sums are taken over the experience states that can be reached given any possible realization of forgetting for each firm. The value function is then defined from the Bellman equation

$$V^S(\mathbf{e}) = \max_{p_1} D_1(p_1, p_2(\mathbf{e}))(p - c(e_1)) + \sum_{k=1,2} D_k(p_1, p_2(\mathbf{e}))\overline{V}_k^S(\mathbf{e})$$

where $p_2(\mathbf{e})$ is the pricing policy function of firm 2, and D represents the demand function defined in equation 3.1.

The policy function is then defined using a first-order condition for pricing.

$$D_1(p^*, p_2(\mathbf{e})) + \left(p^* - c(e_1) + \beta\left[\overline{V}_1^S - \overline{V}_2^S\right]\right)\frac{\partial D_1(p^*, p_2(\mathbf{e}))}{\partial p} = 0$$

which corresponds to the standard static first-order condition and an additional term: $\beta\left[\overline{V}_1 - \overline{V}_2\right]\frac{\partial D_1(p^*, p_2(\mathbf{e}))}{\partial p}$ which captures the dynamics of the problem. The form of logit demand guarantees that, holding fixed the continuation values, the equilbrium prices are unique.[6] This does not, however, imply that there is a unique equilibrium in the dynamic model, when we allow for prices in other states to affect continuation values.

Stacking together the Bellman equations and first-order conditions for each

[6]This follows from the uniqueness of static price equilibria in logit demand models for any linear marginal costs (Caplin et al. (1991), and the fact that the difference in continuation values enters the first-order condition in the same way as a marginal cost shift.

state, we can express the equilibrium as a set of equations,

$$\begin{bmatrix} F_e^S(\mathbf{V}^*, \mathbf{p}^*) \\ F_e^p(\mathbf{V}^*, \mathbf{p}^*) \end{bmatrix} = \begin{bmatrix} V^S(\mathbf{e}) - D_1(p^*(\mathbf{e}))(p^*(\mathbf{e}) - c_1(\mathbf{e})) - \sum_{k=1,2} D_k(p^*(\mathbf{e}), \mathbf{e})\overline{V}_k^S(\mathbf{e}) \\ D_1(p^*(\mathbf{e}), \mathbf{e}) + \left(p^*(\mathbf{e}) - c_1(\mathbf{e}) + \beta \left[\overline{V}_1^S(\mathbf{e}) - \overline{V}_2^S(\mathbf{e})\right]\right) \frac{\partial D_1(p^*(\mathbf{e}),\mathbf{e})}{\partial p} \end{bmatrix} = 0.$$

3.2.2 Strategic Buyers

We change the BDKS model by introducing strategic buyers. Specifically, we assume that the buyer in any period believes that she will be chosen, by nature, to be the buyer in any future period with probability b^p. One way to rationalize this, if $b^p > 0$ is to imagine that there is a pool of $\frac{1}{b^p}$ potential buyers and that each period one of them is selected randomly and with replacement, to be the customer. However, one might also view b^p as being influenced by the patience of sellers or their probability of going out of business. The buyer's per-period indirect utility function remains the same as before. This implies, for example, that a buyer's preferences do not depend on their past purchases. Therefore, the only reason for deviating from the static purchase probabilities shown in equation 3.1 is because a purchase may influence the future prices that the buyer may face. If $b^p = 0$, the model corresponds exactly to the BDKS model.

The only change to the equilibrium equations outlined above, is that there is now an additional equation representing the value function of buyers, and a change to the form of the demand functions, D_n.

For the representative strategic buyer, the value function, $V^B(\mathbf{e})$, is defined before she knows whether she has been selected to be the buyer this period. It is useful to define the mean continuation utility (in the sense that it excludes the realized ε) for the selected buyer who chooses seller 1,

$$\mu_1^B(\mathbf{e}) = v - p_1 + \beta \sum_{e_1'=e}^{e_1+1} \sum_{e_2'=e_2-1}^{e_2} V^B(\mathbf{e}') \Pr(e_1'|e_1, 1) \Pr(e_2'|e_2, 0).$$

Given the assumed distribution of the ε's and values for $V^B(\mathbf{e})$, the optimal strategy of the selected buyer in a particular state can be characterized by the probability of purchasing from seller 1, under the assumption that $\sigma = 1$,

$$\begin{aligned} D_1(p_1, p_2(\mathbf{e})) &= \frac{\exp(\mu_1^B(\mathbf{e}))}{\exp(\mu_1^B(\mathbf{e})) + \exp(\mu_2^B(\mathbf{e}))} \\ &= \frac{1}{1 + \exp\left(\begin{array}{c} p_1 + \beta \sum_{e_1'=e}^{e_1+1} \sum_{e_2'=e_2-1}^{e_2} V^B(\mathbf{e}') \Pr(e_1'|e_1, 1) \Pr(e_2'|e_2, 0) - \\ \left[p_2 + \beta \sum_{e_1'=e-1}^{e_1} \sum_{e_2'=e_2}^{e_2+1} V^B(\mathbf{e}') \Pr(e_1'|e_1, 0) \Pr(e_2'|e_2, 1) \right] \end{array} \right)} \end{aligned}$$

and, it follows that

$$V^B(\mathbf{e}) = b^p \log \left(\sum_{k=1,2} \exp\left(\mu_k^B(\mathbf{e})\right) \right).$$

This provides a full set of equations defining the equilibrium,

$$\begin{bmatrix} F_e^S(\mathbf{V}^*, \mathbf{p}^*) \\ F_e^B(\mathbf{V}^*, \mathbf{p}^*) \\ F_e^p(\mathbf{V}^*, \mathbf{p}^*) \end{bmatrix} = \begin{bmatrix} V^S(\mathbf{e}) - D_1(p^*(\mathbf{e}))(p^*(\mathbf{e}) - c_1(\mathbf{e})) - \sum_{k=1,2} D_k(p^*(\mathbf{e}), \mathbf{e}) \overline{V}_k^S(\mathbf{e}) \\ V^B(\mathbf{e}) - b^p \log \left(\sum_{k=1,2} \exp \left(\mu_k^B(\mathbf{e}) \right) \right) \\ D_1(p^*(\mathbf{e}), \mathbf{e}) + \left(p^*(\mathbf{e}) - c_1(\mathbf{e}) + \beta \left[\overline{V}_1^S(\mathbf{e}) - \overline{V}_2^S(\mathbf{e}) \right] \right) \frac{\partial D_1(p^*(\mathbf{e}), \mathbf{e})}{\partial p} \end{bmatrix} = 0. \tag{3.2}$$

In the BDKS model, the equilibrium is defined by the Bellman equations and first-order conditions for 900 (M^2) states, giving 1,800 equations in total, with 1,800 unknowns. With $b^p > 0$, we add 465 equations, exploiting the fact that the buyer's value in (e_1, e_2) must be the same as in (e_2, e_1), giving 2,265 equations and 2,265 unknowns in total.

3.3 Computation

To find a single equilibrium in their model, for given parameters, BDKS use the iterative best-response approach proposed by Pakes and McGuire (1994). In practice, we find that in both the BDKS model and our augmented model it is quicker to use the Levenberg-Marquardt algorithm with analytic gradients called by `fsolve` in MATLAB.[7]

However, as emphasized by BDKS, some parameters of interest may support many equilibria with different policies. BDKS propose using a path-following "homotopy" routine, to trace out the equilibrium correspondance. This is implemented

[7]We find that this approach converges to equilibria given reasonable ranges of plausible starting values. This equation solving approach can find equilibria that are not stable to small changes in pricing strategies, unlike the Pakes and McGuire algorithm (BDKS, p. 467-470).

using the HOMPACK90 Fortran algorithm (Watson et al. (1987)). The idea of the algorithm is that, starting at a vector of values and prices that corresponds to an equilibrium, a path can be traced that keeps all of the the equations holding as one of the parameters (for example, the forgetting parameter δ) is changed. Multiple equilibria can be found when a path folds back on itself, or when the same pair of parameters is passed in another direction (for example, when changing the learning-by-doing progress ratio parameter ρ) when following a different path of equilibria. BDKS use this type of criss-crossing of the parameter space in different directions to search in a systematic way for multiple equilibria, although we should be clear that there is no guarantee that all equilibria will be identified. They begin the procedure starting at values of $\rho = 0$.

We use this type of criss-crossing homotopy procedure for a discrete set of b^p values. The procedure is a multi-stage numerical process, and we describe all of the details, including our choices of numerical tolerances and the way in which the output of the homotopies are used to establish the number of equilibria at particular gridpoints in (ρ, δ) space in Appendix A. We also illustrate how sensitive our results for counting equilibria are to different choices. It is the case that using different tolerances can change the number of equilibria that we find for very low values of b^p, including the BDKS model, but our finding that multiplicity is eliminated when we increase b^p seems entirely robust.

We do note, however, one caveat with the current results presented in the following sections. As noted by BDKS, homotopies can stop because the mathematical problem that defines the homotopy is not "regular". As a consequence, we do not have any equilibria for some values of (ρ, δ). However, as will become clear in some of the diagrams, this problem is very largely restricted to values of $\delta > 0.2$, implying a rate of forgetting that is unlikely to be interesting. However, we can find at least one equilibrium for these values using a simpler approach, and we will use this to fill-in the missing values in future iterations.

3.4 Results: Strategic Buyers and Multiplicity of Equilibria

3.4.1 Extent of Multiplicity

BDKS emphasize their finding that there are many economically-plausible values of (ρ, δ) that support multiple equilibria as one of the main results of their analysis. The first part of this section documents how our results compare with BDKS, in particular showing that multiplicity disappears quickly as we increase b^p. The second part of the section develops intuition for why this occurs, and connects to our discussion in the next section where we examine the effects of strategic buyers on equilibrium pricing, market structure and welfare.

Figures 3.3 and 3.4 shows the multiplicity results reported in BDKS, and our equivalent results when $b^p = 0$. The cases where the homotopies do not provide solutions are marked as if there is a unique equilibrium (this appears to be how BDKS have treated these outcomes). The forgetting parameter, δ, is on the x-axis and the progress ratio, ρ is on the y-axis. As our models are the same in these cases, we would expect equivalent results, so the differences should be explained by differences in our numerical implementation (for example, the criteria we use to specify that small numerical differences in prices are sufficient to say that there are multiple equilibria).

We identify multiplicity in broadly similar areas, but there are some differences: for example, for low values of ρ they identify a large contiguous area with three equilibria, whereas we identify some areas within this zone where the equilibrium appears to be unique. However, the small area where they identify as many as nine equilibria is common to both. For high values of ρ (learning is able to reduce marginal costs dramatically) they identify more equilibria for slightly lower values of δ than we do. For the moment we use the similarity between the figures as evidence that we can replicate BDKS's results well enough to get meaningful results.

Figures 3.5 and 3.6 show our results for values of $b^p = 0.01, 0.025, 0.05, 0.1$ and 0.2. For the case of a monopsonist buyer we find no multiplicity for any (ρ, δ) values.

The clear pattern is that as b^p is increased, the multiplicity of equilibria is reduced. For $b^p = 0.01$ the pattern is similar to the $b^p = 0$ case, but for low values of ρ both the number of equilibria for given parameters and the range of parameters that support multiplicity are clearly falling once $b^p \geq 0.025$ (equivalent to 40 symmetric buyers). The pattern is a little bit different for very high values of the progress ratio, as the area that supports multiplicity grows larger as we go from $b^p = 0$ to $b^p = 0.025$ before falling. For $b^p = 0.1$ (ten equally sized buyers) multiplcity persists only in a narrow stretch of the parameter space with low progress ratios and moderately fast forgetting. We will examine below whether the economic differences between the equilibria in this region are also becoming less marked. For

Figure 3.3: Extent of Multiplicity for $b^p = 0$: BDKS Results (their Fig. 2)

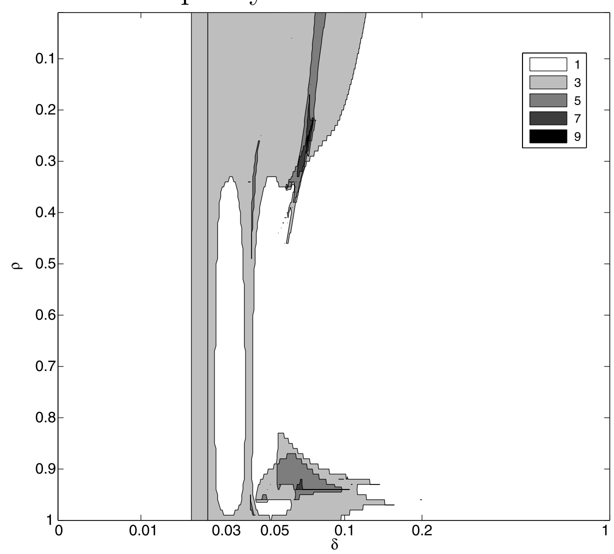
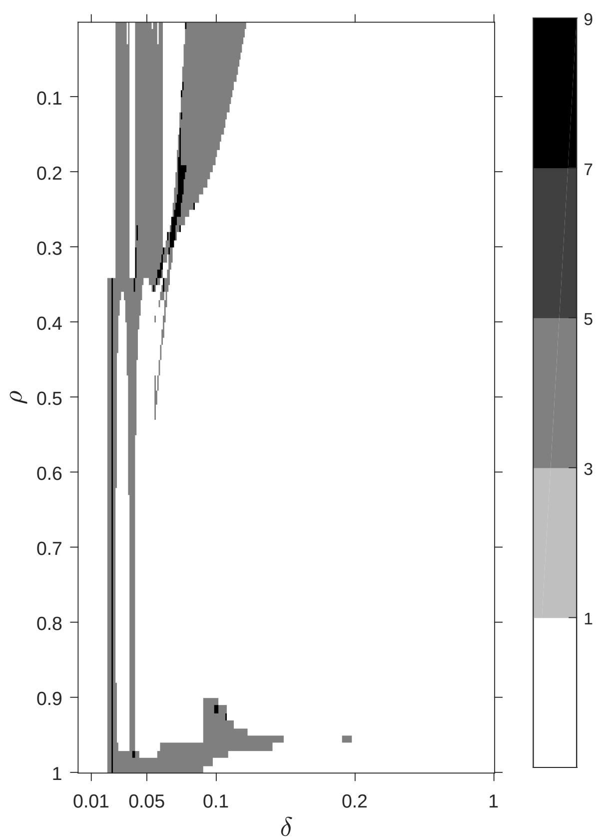


Figure 3.4: Extent of Multiplicity for $b^p = 0$: Our Results



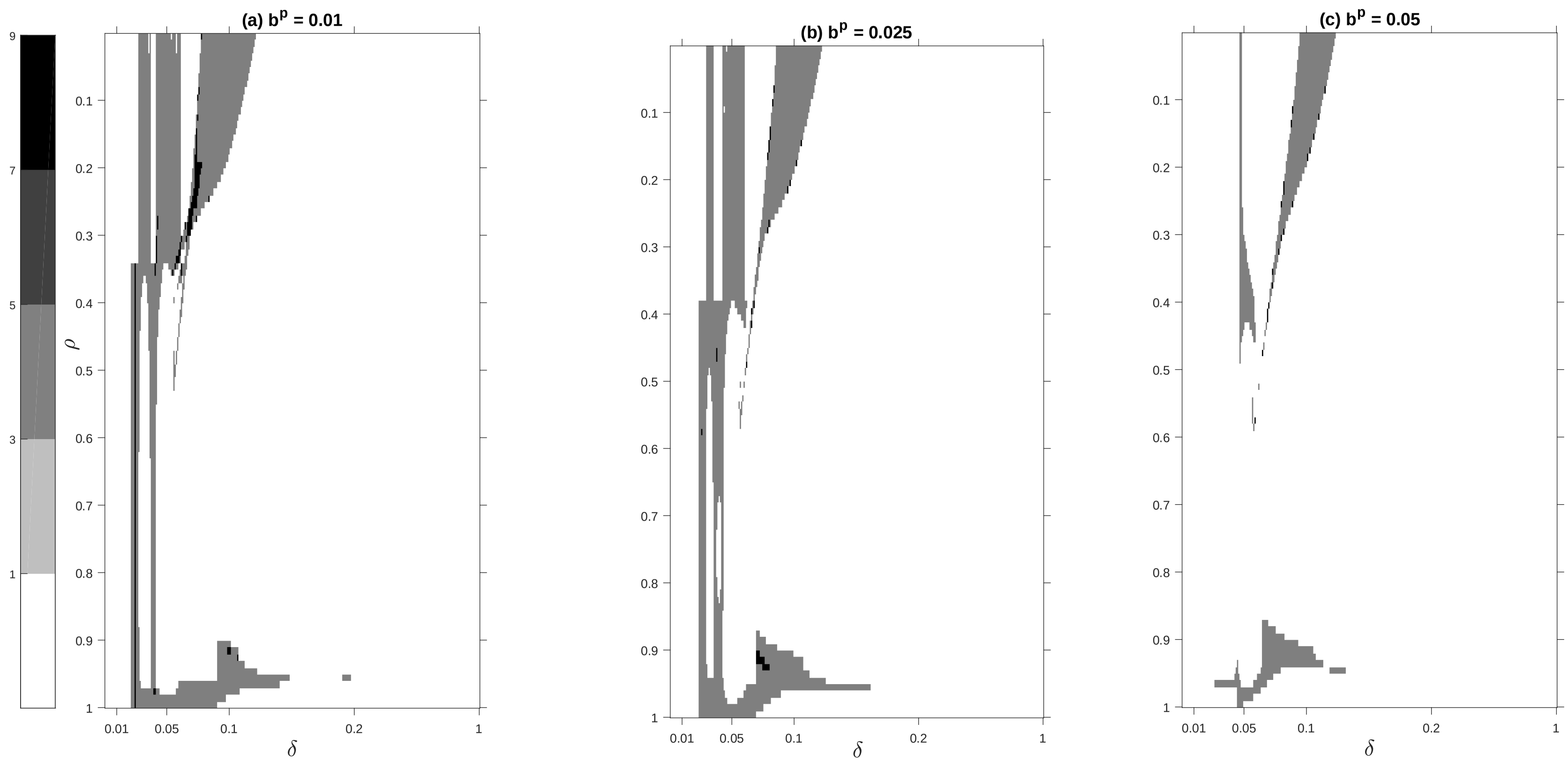


Figure 3.5: Number of Equilibria For Different Values of b^p

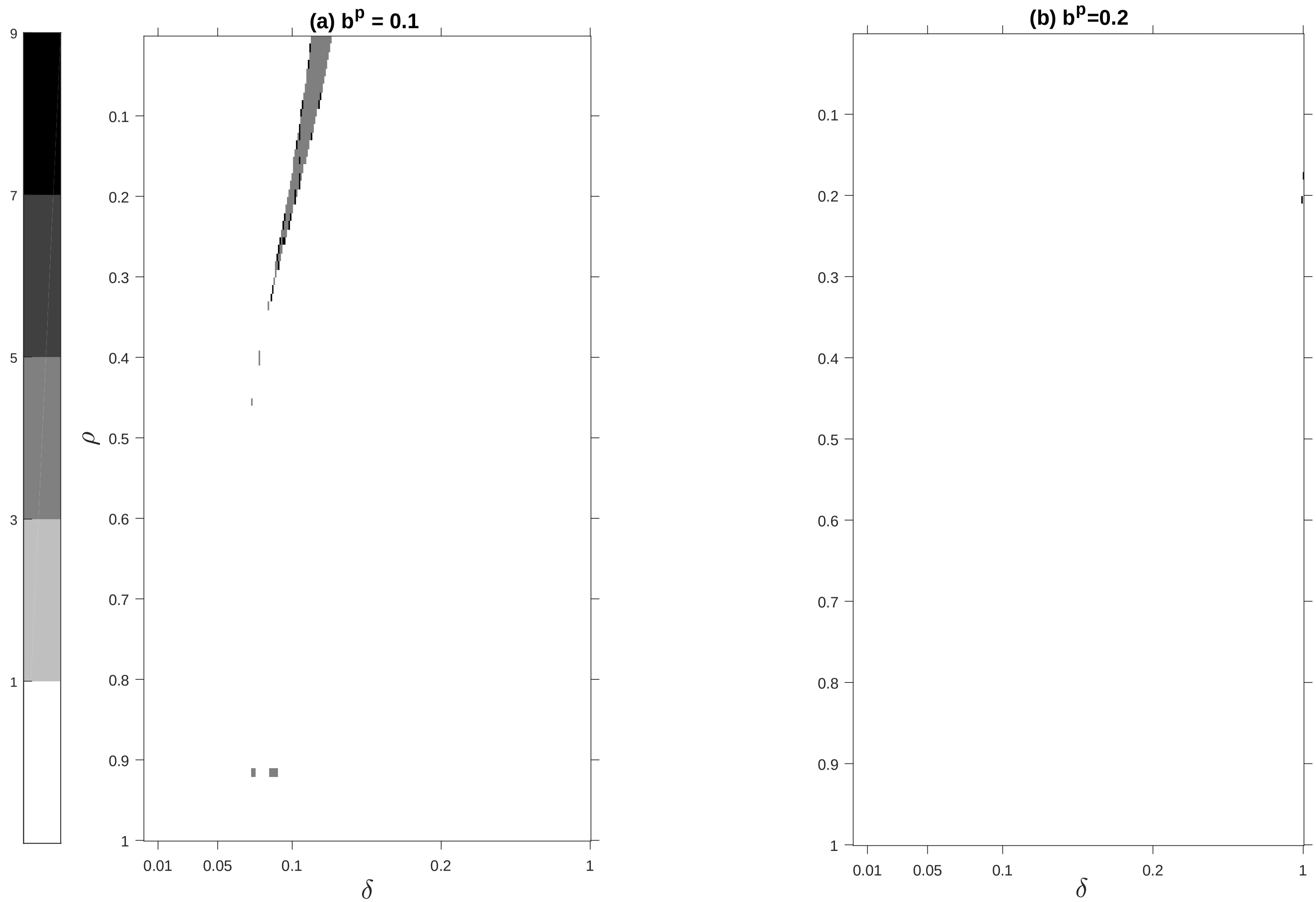


Figure 3.6: Number of Equilibria For Different Values of b^p

$b^p = 0.2$ we find multiple equilibria for a couple of gridpoints with δ close to 1, and low progress ratios. This seems inconsistent with earlier results, and surprising given that the industry is close to being trapped at the lowest level of know-how for both firms, independent of what a strategic buyer does. We are investigating whether this results reflects a numerical issue, or has some economic rationalization.

3.4.2 Explanations for Why Strategic Buyers Reduce Multiplicity

We now try to provide an explanation for why strategic buyers tend to undo the multiplicity of equilibria. The logic behind multiplicity when $b^p = 0$ is entirely rooted in different types of competition amongst the suppliers: if they expect relatively even competition in all future states, then there is little incentive to price aggressively to gain a temporary or permanent advantage, supporting pricing functions that are relatively flat across the states. On the other hand, if suppliers expect that one firm will emerge as a dominant supplier then they will price aggressively to gain an advantage when they are symmetric and a leader will also price agressively whenever there is some chance that the rival will catch up. If both types of logic works given the parameters, we can generate equilibrium pricing functions that are flat and pricing functions that have significant wells and trenches, which may vary in their depth and their precise forms.

Strategic buyers enter the picture because of their incentives to avoid purchasing from firms who will exploit gaining a sale to increase future prices. If this makes a buyer less responsive to a current price decrease, the cost of gaining an advantage may increase so much that the strategy becomes unprofitable. One prediction of this logic is that we should expect to see aggressive price equilibria (i.e., those with deep wells and trenches) eliminated whereas as those with flatter prices are likely to survive.

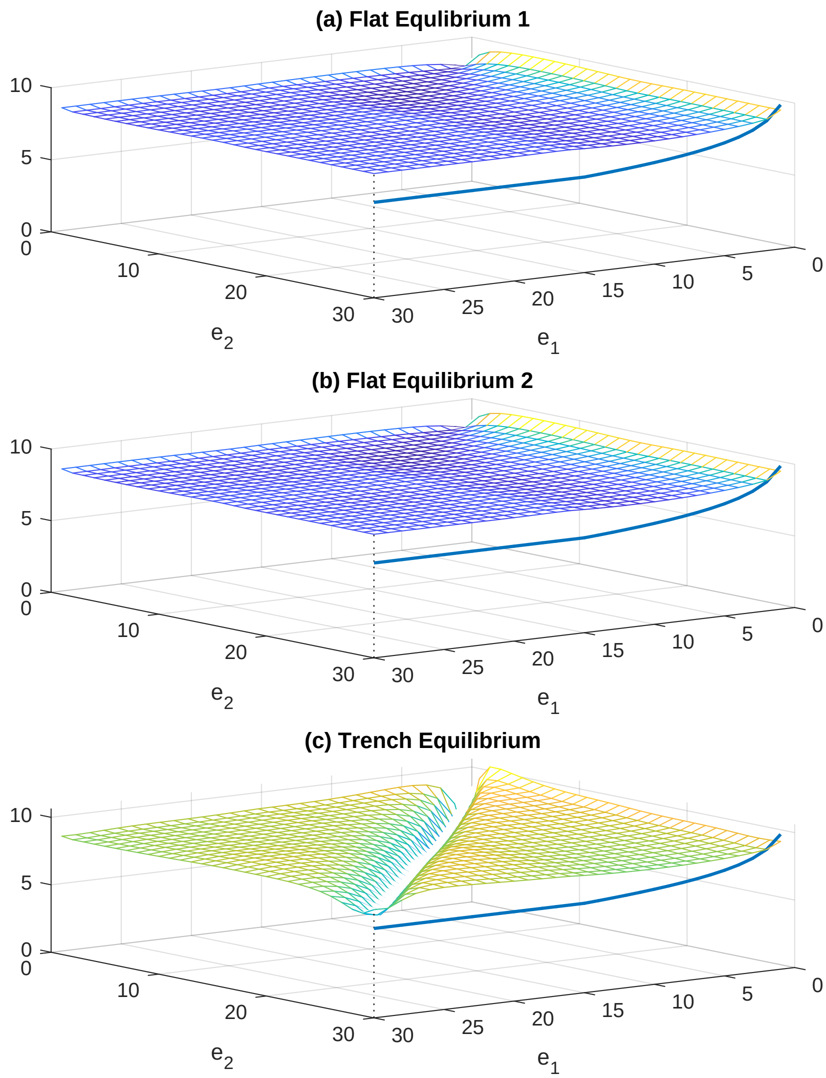


Figure 3.7: Pricing Policy Functions for Firm 1 (surface) and Firm 1 Marginal Cost for $(\rho = 0.9, \delta = 0.028)$

To illustrate consider the parameters $\rho = 0.9$ and $\delta = 0.028$. Figure 3.7 shows the three equilibria that we identify for those parameters when $b^p = 0$. Two of

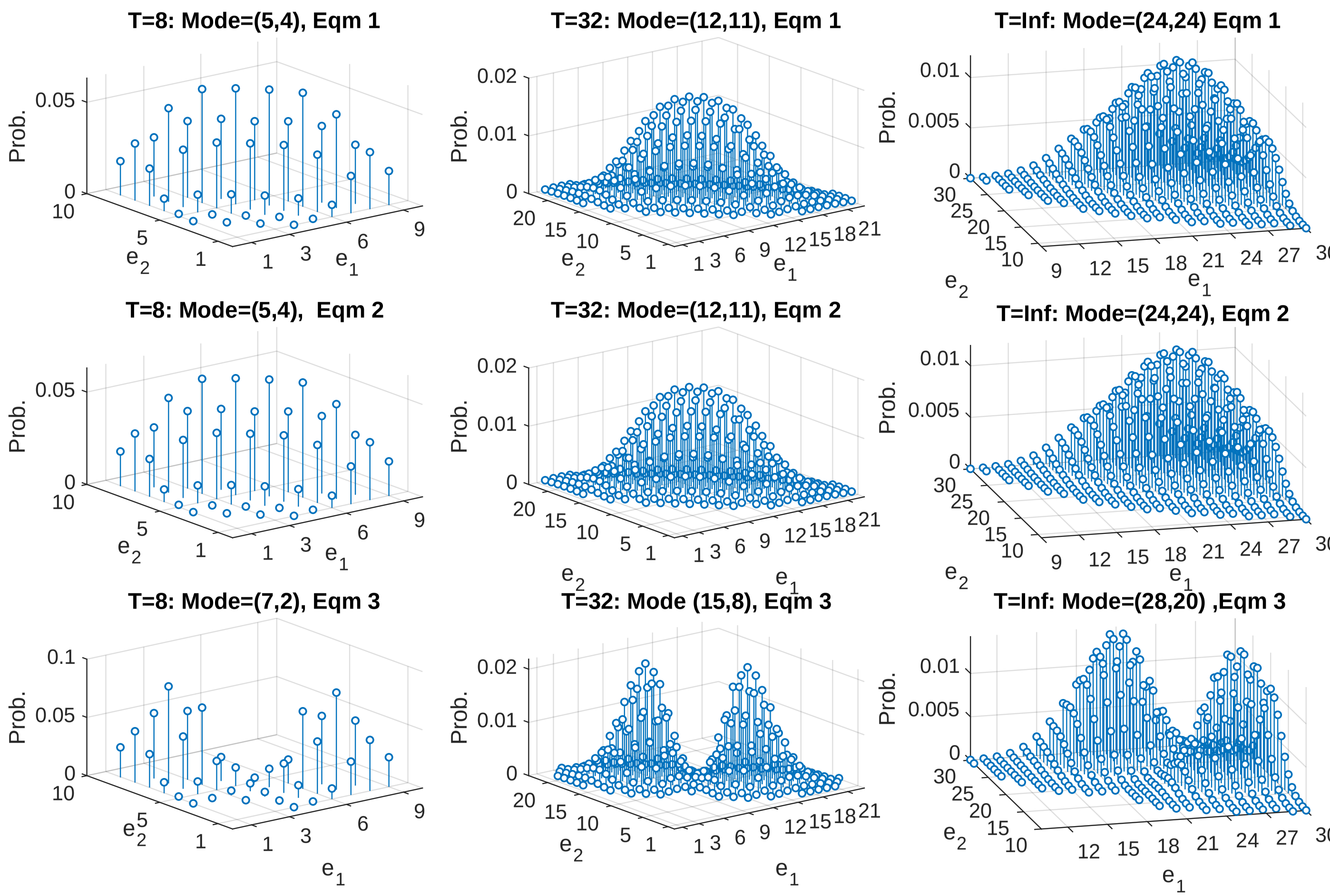


Figure 3.8: Distributions of States for the Three Equilibria Given ($\rho = 0.9, \delta = 0.028$)

them are flat (there are small differences in prices for low levels of know-how), and one of them has a marked trench. Figure 3.10 shows the distribution over states in each equilibrium after 8 and 32 periods (starting at (1,1)) and in the limiting case where the game is played for an infinitely long-time. The two flat equilibria generate almost identical distributions. Both in the short-run and the long-run the aggressive price equilibrium is associated with a more asymmetric structure where one firm has more know-how, although we note that in the modal state the marginal costs of the two firms are actually equal when the states are $(28, 20)$. However, given the low prices in the trench, a buyer would clearly prefer the industry to be in a state of (24,24) rather than (28,20) if firms are playing equilibrium 3.

Figure 3.9 shows how buyer demand would change given forward-looking incentives given these parameters, and the equilibirum pricing behavior is shown in Figure 3.7. To understand the figure consider the middle figure in the top row. This shows how the probability of firm 1 making a sale depends on its price given that firm 2 charges its price in equilibrium 1 for the state, and the buyer takes into account different degrees of future incentives, according to the value of b^p, with the red-line reflecting the actual equilibrium demand and the other lines reflecting how a more strategic buyer would behave when facing $b^p = 0$ prices.

Given that the buyer always buys from one of the sellers, there are no effects on demand when the firms are symmetric (left-hand column). But when the firms are not symmetric, strategic buyers favor the weaker (lower know-how) firm, with the size of the preference depending on the size of the relevant b^p (we do not show the case when $b^p = 1$ as it requires changing the scales on the plot). The effects

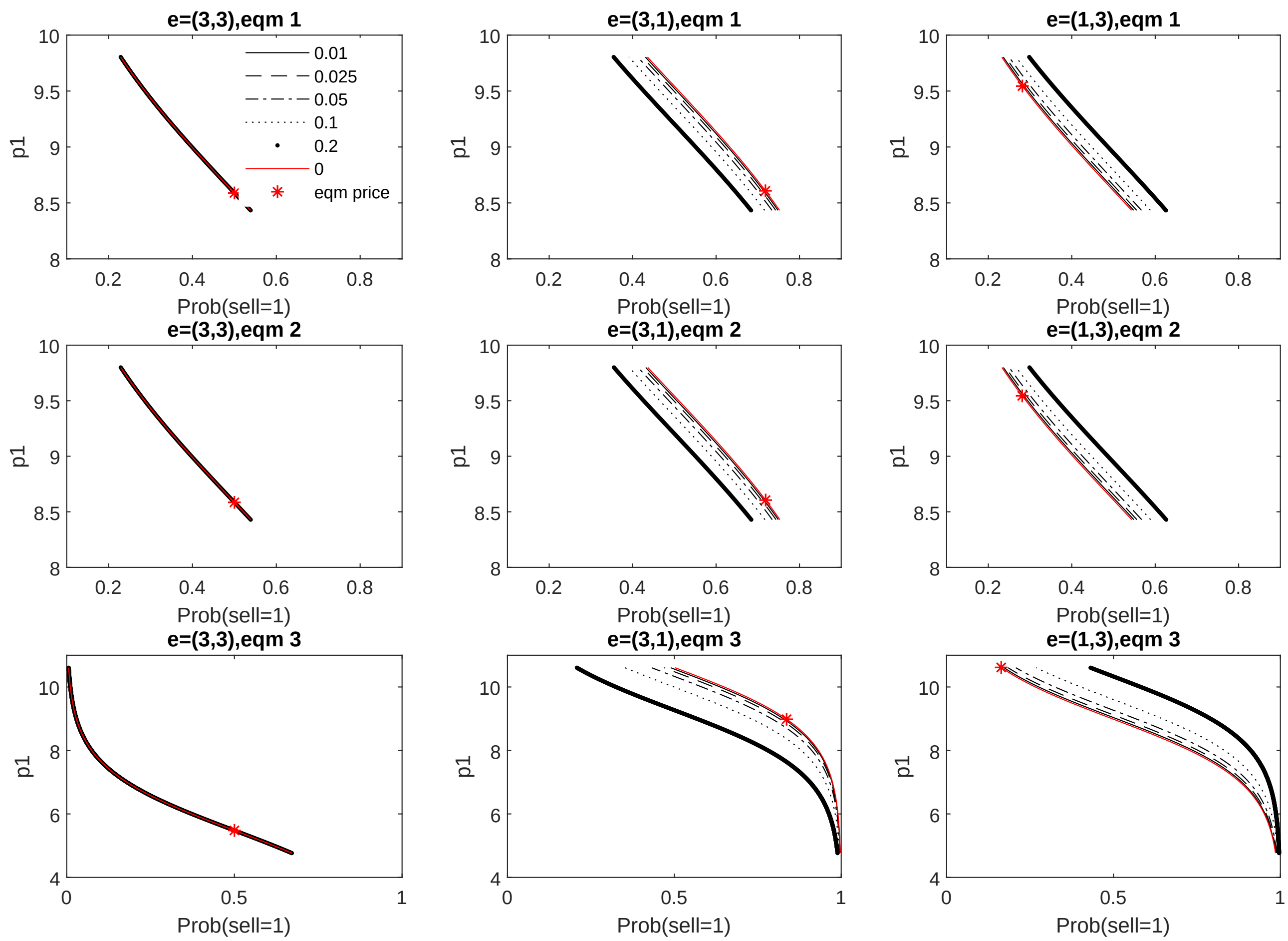


Figure 3.9: Demand Functions for Firm 1 for ($\rho = 0.9, \delta = 0.028$)

are noticeable in asymmetric states for all of the equilibria, including the flat ones, but they are largest in equilibrium 3, consistent with a buyer potentially benefiting from getting the industry back towards the low-price trench.

We now examine how these demand changes affect the incentives of the seller. Recall that the motivation to set low prices in symmetric or near-symmetric states in an aggressive price equilibrium is that it raises the probability that the seller will end up, for a sustained period of time, in states where it is has a significant advantage and can charge large markups. Figure 3.10 show the distribution of states after 8, 32 and a limiting number of periods when buyers use the $b^p = 0.2$ strategies implied by the demand curves (in all states) implied by the analysis above. Comparing to Figure 3.10, the changes for the flat equilibria are relatively small, whereas for the aggressive price equilibrium the expected experience levels of the two firms become clearly more symmetric, reflecting the incentives of buyers to move the industry towards symmetry. For lower levels of b^p strategic buyer behavior has the same directional effect (more symmetric states become more common), but the magnitude of the changes is smaller.

To understand how this affects multiplicity we construct graphical "dynamic reaction functions" where we can interpret multiple crossing as reflecting multiple equilibria. Of course, to summarize the whole problem we would have to show what happens to pricing incentives in 900 different states, which is impossible in a 2-D figure. The reaction functions that we present are constructed as follows. Start with $b^p = 0$. We define the strategy of firm 1 using a single parameter λ_1, which we think of measuring the aggressiveness of firm 1's pricing strategy. Given λ_1 the

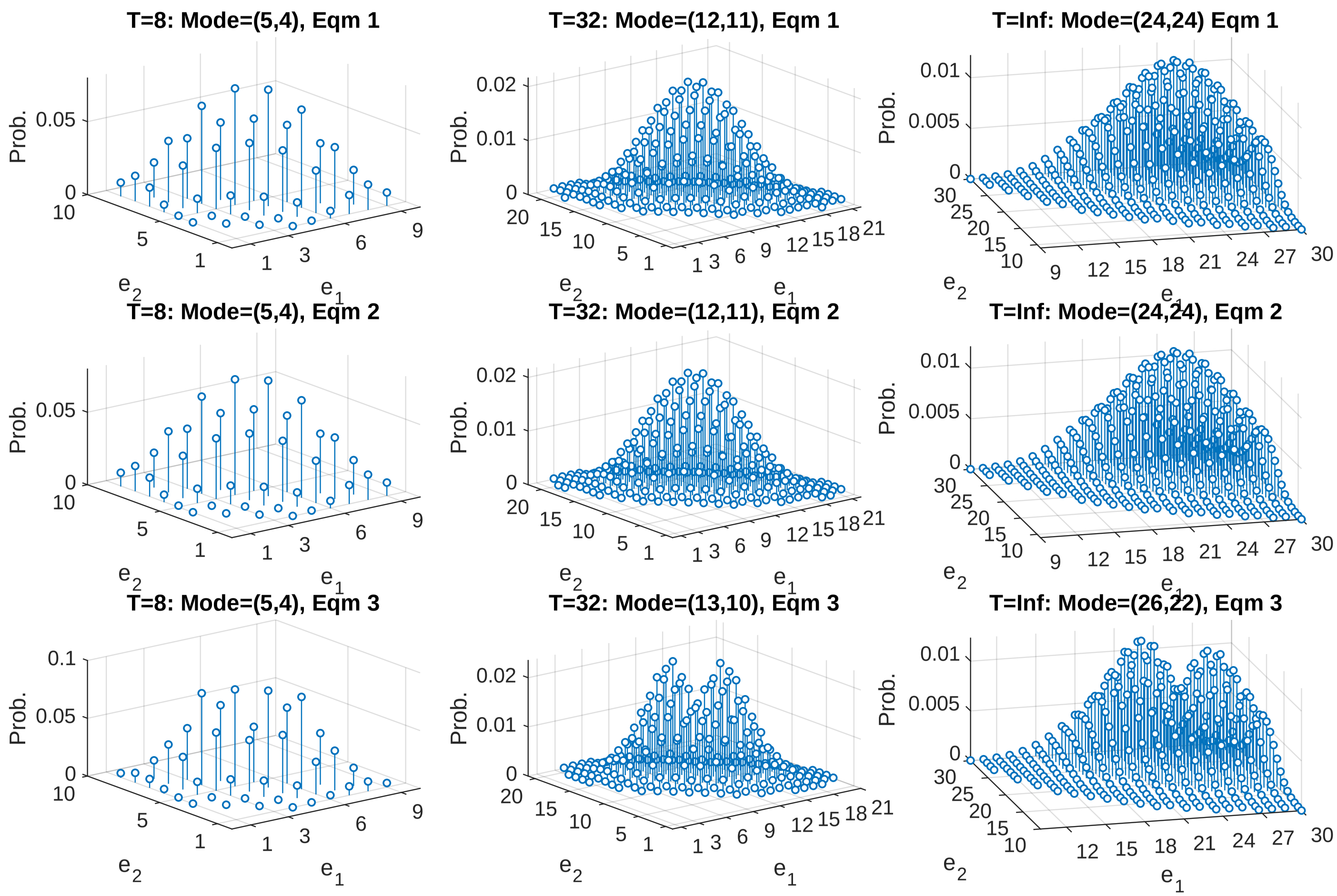


Figure 3.10: Distributions of States for the Three Equilibria Given ($\rho = 0.9, \delta = 0.028$) and Buyer Demands Implied by $b^p = 0.2$

prices charged by firm 1 will be

$$\mathbf{p}(\mathbf{e}, \lambda_1, b^p = 0) = (1 - \lambda_1)\mathbf{p}^{*,1}(\mathbf{e}, b^p = 0) + \lambda_1 \mathbf{p}^{*,3}(\mathbf{e}, b^p = 0)$$

where $\mathbf{p}^{*,\mathbf{1}}$ and $\mathbf{p}^{*,\mathbf{3}}$ are the vector of equilibrium prices in equilibria 1 and 3 respectively so that we are using equilibrium 1 prices in all states when $\lambda_1 = 0$ and equilibrium 3 prices when $\lambda_1 = 1$. We define and use λ_2 in a similar way. For values of λ_1 on a fine grid, we calculate the best response $\lambda_2^*(\lambda_1)$ for firm 2, i.e., the λ_2 that maximizes firm 2's value from playing the game starting in state (1,1).[8] We repeat this process to derive the best response reaction function of firm 1. By construction, there is an MPE, and an intersection of the best response functions, represented by the red and blue solid lines in the Figure 3.11, for $\lambda_1 = \lambda_2 = 0$ and $\lambda_1 = \lambda_2 = 1$. The convex shape of the reaction functions suggests that, with $b^p = 0$, there is an increasing return to aggressive pricing behavior as the rival firm becomes more aggressive. The intuition is that when the rival firm is more aggressive, it becomes even more important to gain an advantage.

The figure also includes reactions functions (with other markers) for other values of b^p. In these cases we may only have one identified equilibrium (this is the case for $b_p \geq 0.05$), so we define pricing strategies as follows

$$\mathbf{p}(\mathbf{e}, \lambda_1, b^p) = (1 - \lambda_1)\mathbf{p}^{FLAT,*}(\mathbf{e}, b^p) + \lambda_1(\mathbf{p}^{*,3}(\mathbf{e}, b^p = 0) - \mathbf{p}^{*,1}(\mathbf{e}, b^p = 0))$$

[8] This means that we can identify the best response by comparing the values of the seller in this state for different values of λ_2. It is possible that if we used values in other states we might find slightly different best responses.

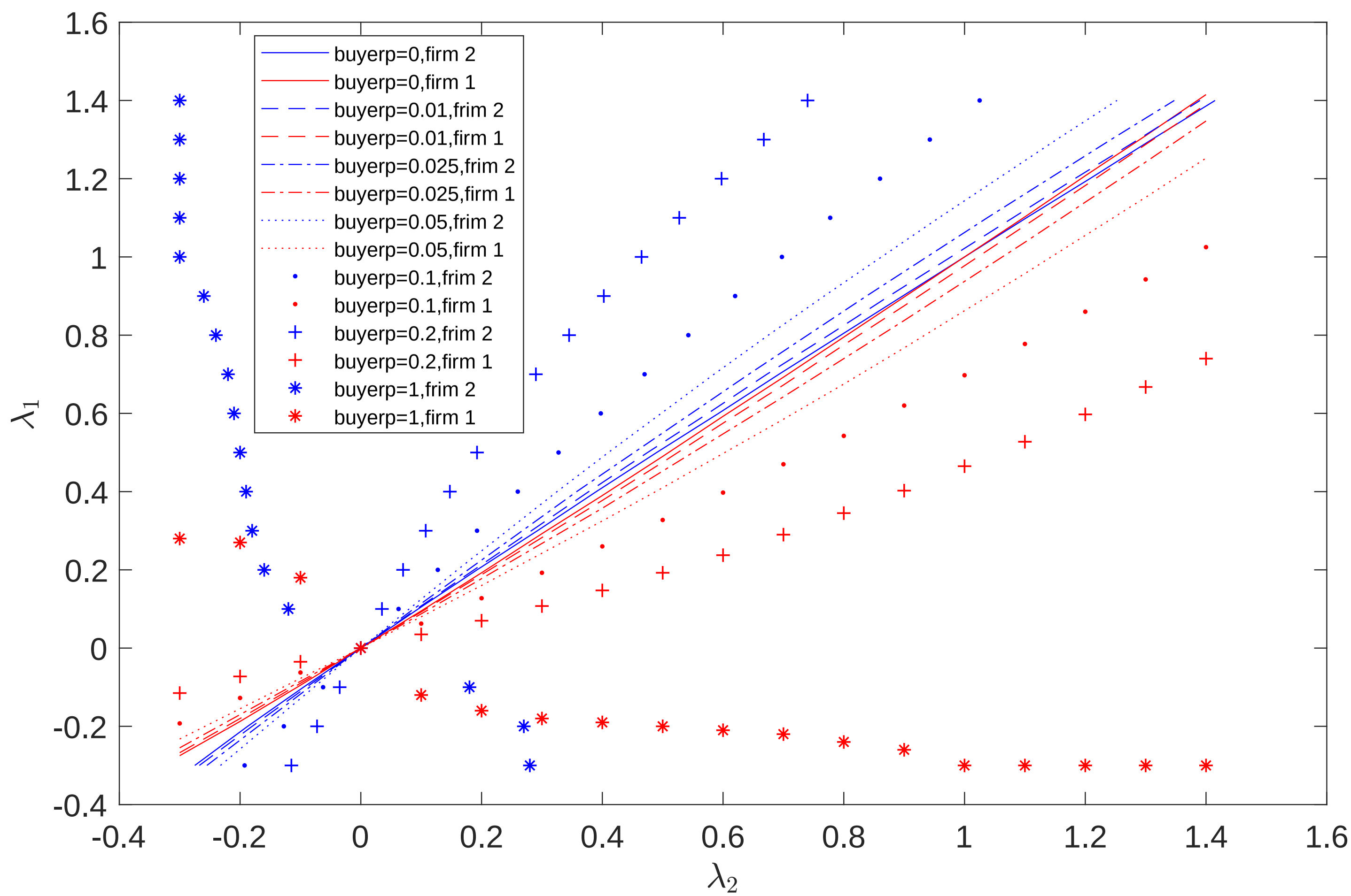


Figure 3.11: Dynamic Best Response Functions for $(\rho = 0.9, \delta = 0.028)$

where $\mathbf{p}^{FLAT,*}$ is the unique equilibrium (when all players account for the value of b^p) or a flat one if there is more than one. For $\lambda_1 = \lambda_2 = 0$ we still have an intersection. While we cannot guarantee that a different way of defining best response functions would produce different results, what happens to these reaction functions when we increase b^p is intuitive. The strategic behavior of buyers, which tends to favor the firm with a disadvantage, makes it less valuable for a seller to increase the aggressiveness of its own pricing behavior when its rival becomes more aggressive. This tends to make the reaction functions both flatter and less convex for $\lambda > 0$, and downward sloping when we consider the extreme case of $b^p = 1$, which tends to work against multiplicity.[9]

3.5 Results: Strategic Buyers, Equilibrium Pricing Strategies, Market Concentration and Welfare

We now examine how strategic buyers affect welfare. To connect to the previous discussion, we begin by considering ($\rho = 0.9, \delta = 0.028$). Figure 3.12 shows how several outcomes change as a function b^p. When there are multiple equilibria the outcomes from different equilibria are shown separately, although the outcomes from the flat equilibria are so close to being identical that there is no visible difference between these outcomes. We observe that it is the high concentration, high expected price aggressive equilibrium that is eliminated. On the other hand the flat

[9] For low values of b^p extrapolation beyond the right-hand edge of the figure would imply that the reaction functions would cross consistent with the fact that we continue to find multiplicity for these values.

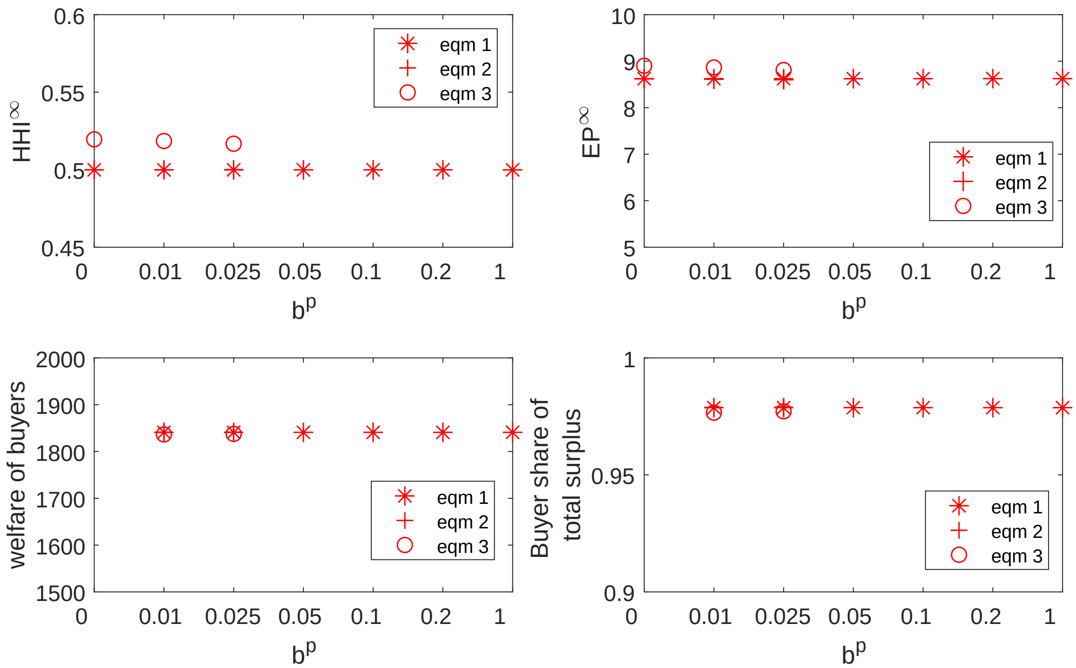


Figure 3.12: Outcomes for ($\rho = 0.9, \delta = 0.028$). Moving clockwise the outcomes are the long-run HHI, the long-run expected price of a purchase, consumer surplus as a fraction of total surplus, and the total welfare of buyers.

equilibrium which persists gives very similar outcomes across the values of b^p.

In this example, the market outcomes are fairly similar between the flat and aggressive equilibria. There are examples where the differences are larger. Figure 3.13 shows an example using ($\rho = 0.19, \delta = 0.099$) where there is a faster rate of forgetting. In this case, some of the equilibria with low b^p values involve long-run dominance (HHI close to monopoly) and others involve relatively even competitors. Even though the dominant firm will make most of the sales and have low costs, expected transaction prices are much higher in the dominant equilibrium and the welfare of buyers is much lower. When we arrive at a single equilibrium ($b^p > 0.1$) it is the low concentration/low price equilibriuim that survives.

These illustrations speak to which equilibria are robust to increases in strategic

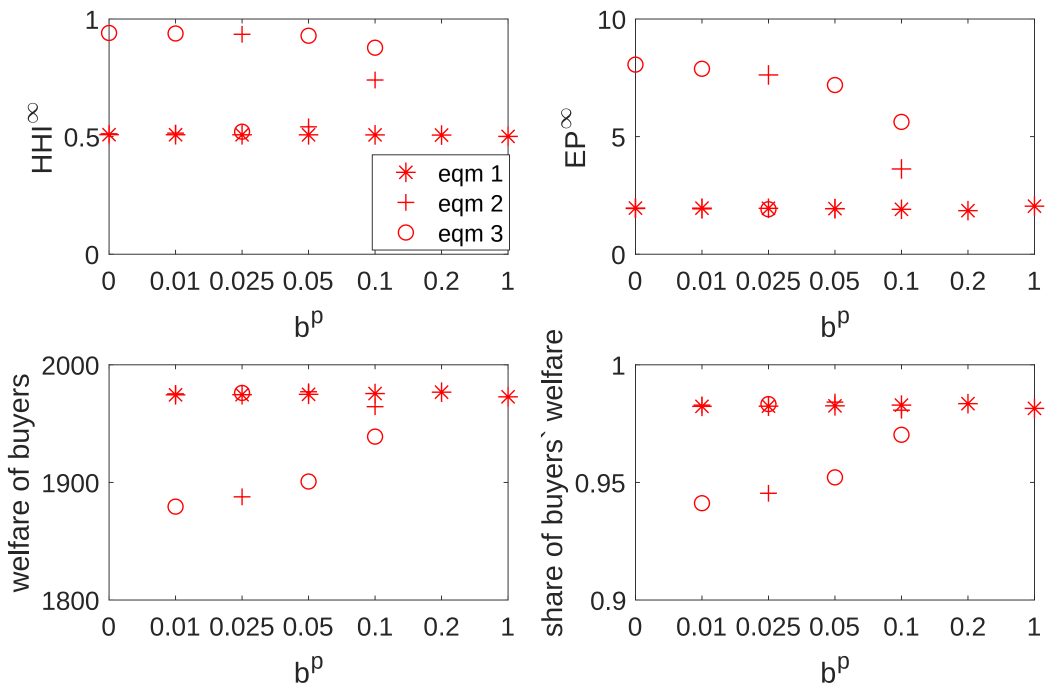


Figure 3.13: Outcomes for $(\rho = 0.19, \delta = 0.099)$

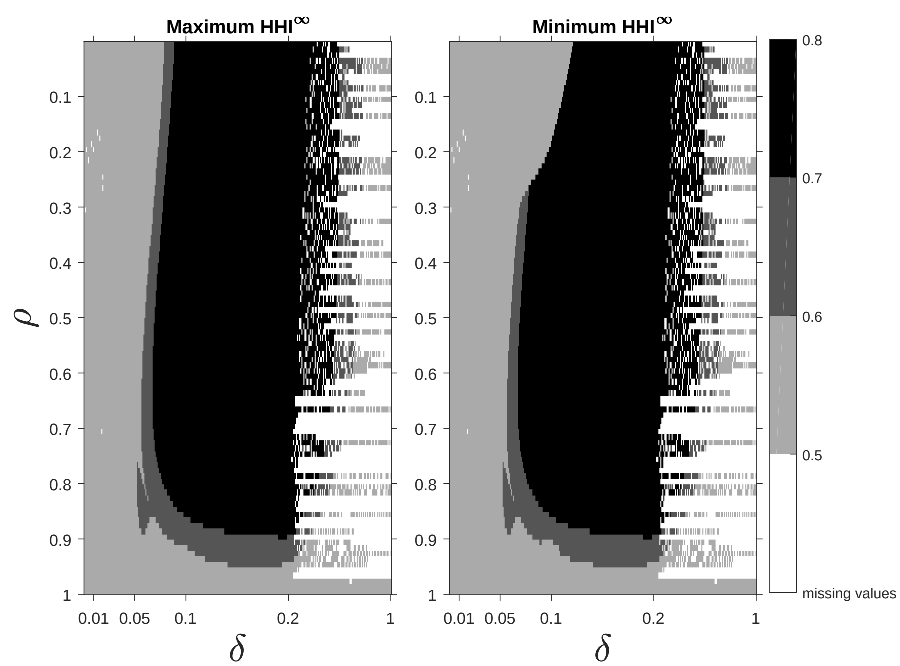


Figure 3.14: Minimum and Maximum Long Run Expected HHIs for $b^p = 0$

buyer behavior, they do not necessarily clarify what happens to choices across the parameter space. Strategic buyers are interested in both limiting future seller market power, but also reducing the marginal costs of the supplier that they prefer to buy from. When forgetting is very likely, but costs would be reduced by know-how, the only way to realize the potentially large benefits of learning may be to coordinate purchases on a single seller.

To illustrate this point, Figures 3.14- 3.16 show the minimum and maximum HHI (taken across equilibria) for each gridpoint for $b^p = 0, 0.05$ and 0.2.

In these figures the speckled area to the right of $\delta = 0.2$ reflects the fact that our homotopies often stop in this area of fast forgetting, so that we do not have solutions. As emphasized previously, this area is not necessarily of primary interest, although we are working to fill in this plot with at least one equilibrium (not using a homotopy approach). What we observe is that for low ρ but δs in the 0.1-0.2 range that, as b^p increases so does the long-run expected HHI, reflecting an increase in concentration.

3.6 Conclusion

In this paper we augment the learning-and-forgetting model of BDKS by adding strategic buyers, and we examine what happens as the degree of strategic buyer behavior is increased. Our motivation comes from trying to understand whether multiplicity is robust to adding a feature that seems to describe many industries where learning-by-doing effects have been documented, and the emphasis that BDKS and BDK give to aggressive price equilibrium that can result in markets with ex-ante symmetric firms coming to be dominated by a single seller.

Our results document at least two types of changes that are directly related to these motivations. First, while multiplicity remains a feature of the model for low values of our strategic buyer parameter, the area of the parameter space that supports multiplicity becomes progressively smaller, and by the time we consider a model with five symmetric buyers multiplicity is essentially eliminated. Second, we find that it is the aggressive price equilibria (i.e., those associated with more concentration, higher long-run transaction prices and lower customer surplus) that are eliminated and that, in general, more strategic buyer behavior increases the welfare of buyers, at least until we reach the point where there are around 5 buyers, when the effect of chilling competition can cause customer surplus to fall. The logic of these effects is in line with what one might have expected from the analytical theory literature, but the contribution of our framework is to examine this in a more quantitative way in a model that combines features (learning-by-doing, forgetting and multiple levels of know-how for each firm) that are analytically intractable.

There are clearly areas that we can build on from this draft. We would like to understand in more detail why our results are not quite the same as those of BDKS when there are no strategic buyers, and we would like to develop our intuition for why aggressive price equilibria disappear in a more systematic way, quantifying the changes to the benefits and costs of building dominance for different values of the parameters. We would also like to incorporate some data from industries where learning-by-doing is apparent, such as aircraft manufacture, to try to quantify what the degree of seller concentration looks like, and to use it as a starting point for looking at the effects of buyers some of whom may be more strategic than others.

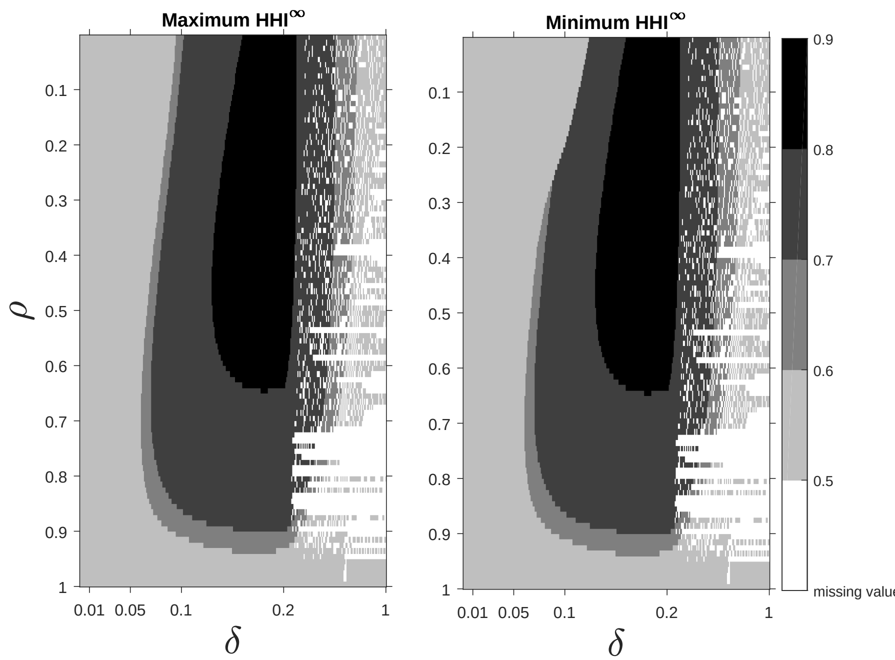


Figure 3.15: Minimum and Maximum Long Run Expected HHIs for $b^p = 0.05$

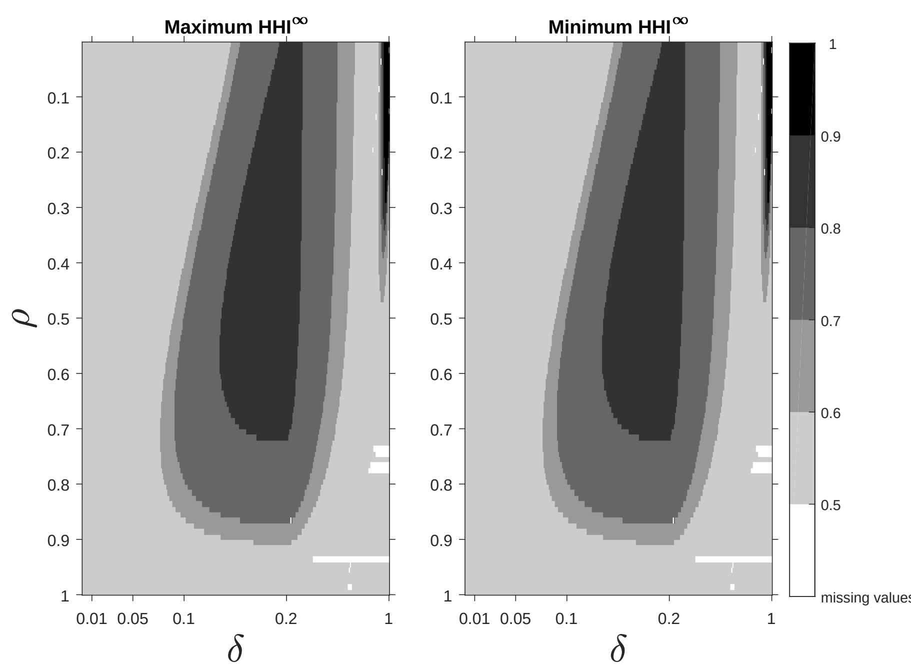


Figure 3.16: Minimum and Maximum Long Run Expected HHIs for $b^p = 0.2$

Appendix A: Computational Methods of Dynamic Price Competition and Learning-by-Doing: The Effect of Strategic Buyers on Equilibria

This appendix details the methods used to solve the model, with and without strategic buyers, and to enumerate the number of equilibria for different combinations of the progress ratio and forgetting parameters (ρ, δ). We briefly explain how we have investigated the effects of a several changes to the method. We try to follow the method outlined by Besanko et al. (2010) closely. However, because they do not explain in detail some of the choices that need to be made when interpreting the FORTRAN output, there are some differences in the results, especially with regard to the number of equilibria, even when we set our strategic buyer parameter, b^p, equal to zero.

A.1 Preliminaries

We identify equilibria at particular gridpoints in (ρ, δ) space. We specify a 1000-point evenly-spaced grid for the forgetting rate $\delta \in [0, 1]$ and a 100-point evenly-spaced grid for the learning progress ratio $\rho \in [0, 1]$. We perform our procedure for six different values of b^p are $\{0 \text{ (BDKS model)}, 0.01, 0.025, 0.05, 0.1, 0.2, 1\}$. As described in the text, the state space of the game is defined by an (30 x 30) grid of values of the know-how of each firm.

A.2 System of Equations Defining Equilibrium

As explained in the text, a Markov Perfect Equilibrium will be a combination of value functions for the buyer and the sellers, and a set of prices, that satisfy a system of 2,265 equations.

$$\begin{bmatrix} F_e^S(\mathbf{V}^*, \mathbf{p}^*) \\ F_e^B(\mathbf{V}^*, \mathbf{p}^*) \\ F_e^p(\mathbf{V}^*, \mathbf{p}^*) \end{bmatrix} = \begin{bmatrix} V^S(\mathbf{e}) - D_1(p^*(\mathbf{e}))(p^*(\mathbf{e}) - c_1(\mathbf{e})) - \sum_{k=1,2} D_k(\mu^B(\mathbf{e}, V^B(\mathbf{e})))\overline{V}_k^S(\mathbf{e}) \\ V^B(\mathbf{e}) - b^p \log\left(\sum_{k=1,2} \exp\left(\mu_k^B(\mathbf{e}, V^B(\mathbf{e}))\right)\right) \\ D_1(p^*(\mathbf{e}), \mathbf{e}) + \left(p^*(\mathbf{e}) - c_1(\mathbf{e}) + \beta\left[\overline{V}_1^S(\mathbf{e}) - \overline{V}_2^S(\mathbf{e})\right]\right)\frac{\partial D_1(p^*(\mathbf{e}), \mathbf{e})}{\partial p} \end{bmatrix} = 0.$$

where μ_k^B is the selected buyer's mean continuation utility of choosing a seller k in a state e, and will depend on both prices and the expected value to be a potential buyer in the states that can be reached in the next period. The seller's cost and transition probabilities, and therefore values and prices, will depend on ρ and δ. We can denote this system of equations $F(\mathbf{V}^*, \mathbf{p}^*; \delta, \rho) = 0$.

A.3 Homotopy Algorithm: Overview

The idea of the homotopy is to trace out an equilibrium correspondance as one of the parameters of interest (δ or ρ) is changed, holding the other fixed. First, starting at a (unique) equilibrium where $\delta = 0$, a numerical algorithm traces a path where δ, and the vectors $V^B(\mathbf{e})$, $V^S(\mathbf{e})$ and $p(\mathbf{e})$ are changed together so that the equations F continue to hold, by solving a system of differential equations. We call this the "δ-homotopy". The path is continued until (hopefully) δ=1. Multiple equilibria for given values of δ can be found if the path turns back upon itself with respect to δ. Of course, the algorithm does not necessarily identify the solutiuons at our gridpoints, so an additional procedure is used to find the these solutions. The equilibria found on the δ homotopies can then be used as the starting points for ρ-homotopies, where ρ is changed and δ is held fixed. This procedure can collect more equilibria as the new path may provide a different solution to the equations at a particular gridpoint than the δ-homotopy did. In principle, this criss-crossing procedure can be continued using any new equilibria that are found at each iteration. The results in the paper are based on running a complete set of δ-homotopies and a set of ρ-homotopies from all of the equilibria at a sparser set of gridpoints (details below). Some experimentation indicates that many of the additional solutions that subsequent additional runs might find would not satisfy the criteria, described below, that we use to identify solutions as being sufficiently different from each other and sufficiently close to the same gridpoint to count as representing distinct equilibria. However, we are working to show that this is the case in a systematic way. In any event, this is not an issue for higher values of b^p where we find no multiplicity in any of the δ or ρ homotopy runs.

A.4 Homotopy Procedure Details

Step 1: Finding Equilibria for $\delta = 0$. The first step is to find an equilibrium (i.e., a solution to the 2,265 equations) for $\delta = 0$ for each value of ρ on the grid. There will be a unique Markov Perfect equilibrium for $\delta = 0$, as, in this case, movements through the state space are unidirectional, so that the state will eventually end up in the state (M, M) where no more learning is possible.[1]

We solve for an equilibrium using the Levenberg-Marquardt algorithm implemented using `fsolve` in MATLAB, where we supply analytic graduents for each equation. For $\rho = 0.02$, we use the solution when $\rho = 0.01$ as the starting values. To ensure that the solutions are precise we use a tolerance of 10^{-7} for the sum of squared values of each equation, and a relative tolerance of 10^{-14} for the variables that we are solving for.

Step 2: δ-Homotopies. Using the notation of BDKS, we explore the correspondance

$$F^{-1}(\rho) = \{(\mathbf{V}^*, \mathbf{p}^*, \delta) | F(\mathbf{V}^*, \mathbf{p}^*; \delta, \rho) = \mathbf{0}, \quad \delta \in [0, 1]\},$$

The homotopy approach introduces an additional parameter s. Denoting $\mathbf{x} = (\mathbf{V}^*, \mathbf{p}^*)$, $F(\mathbf{x}(s), \delta(s), \rho) = \mathbf{0}$ can be implicitly differentiated to find how $\mathbf{x}$ and

[1] BDKS discuss this result for $b^p = 0$. It will also hold for any higher value of b^p. The key to this result is that, as well as unidirectional movements through the state space, there will be a unique equilibrium in each state for given future values. This follows from the fact that the game within each period is sequential: sellers first choose prices and then the buyer decides which firm to choose from. Given future values, the buyer is choosing between the firms based on their prices and its private information logit preference shocks, and, excepting the case of a probability zero tie, it will have a unique optimal choice. From the perspective of the sellers, they face a buyer with logit choice probabilities, so that the results of Caplin et al. (1991) will imply uniqueness.

δ must change for the equations still to hold as s changes.

$$\frac{\partial F(\mathbf{x}(s),\delta(s),\rho)}{\partial \mathbf{x}}\mathbf{x}'(s)+\frac{\partial F(\mathbf{x}(s),\delta(s),\rho)}{\partial \delta}\delta'(s)=\mathbf{0}$$

where $\frac{\partial F(\mathbf{x}(s),\delta(s),\rho)}{\partial \mathbf{x}}$ is a (2,265 x 2,265) matrix, $\mathbf{x}'(s)$ and $\frac{\partial F(\mathbf{x}(s),\delta(s),\rho)}{\partial \delta}$ are both (2,265 x 1) vectors and $\delta'(s)$ is a scalar. The solution to these differential equations will have the following form, where $y_i'(s)$ is the derivative of the ith element of $\mathbf{y}(s)=(\mathbf{x}(s),\delta(s))$,

$$y_i'(s)=(-1)^{i+1}\det\left(\left(\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}\right)_{-i}\right)$$

where $_{-i}$ means that the ith column is removed from the (2,266 x 2,266) $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$.

To implement the path-following procedure the routine FIXPNS from HOMPACK90 is used, with the ADIFOR 2.0D automatic differentiation package used to evaluate the sparse Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ and STEPNS used to find the next point on the path.[2,3]

When allowed to run the FIXPNS routine will return solutions at values of δ that are not equal to the gridpoints. Therefore we adjust the code so that after *each* step, the algorithm checks whether a gridpoint has been passed and, if so, it called the routine ROOTNX to calculate the equilibrium at the gridpoint, using information on the solutions at either side.

[2]STEPNS is a predictor-corrector algorithm where hermetic cubic interpolation is used to guess the next point, and an iterative procedure is then used to return to the path.

[3]For details of the HOMPACK subroutines, please consult manual of the algorithm at `https://users.wpi.edu/~walker/Papers/hompack90,ACM-TOMS_23,1997,514-549.pdf`.

The time taken to run a homotopy is usually between one hour and seven hours, when it is run on UMD's BSWIFT cluster (a moderately sized cluster for the School of Behavioral and Social Sciences).

Step 3: Verifying Solutions. The numerical nature of the procedure, and the possiblity that a routine such as ROOTNX will end prematurely, means that the returned solutions may not be exactly at the gridpoints. This can matter because we would not want to count two solutions as multiple equilibria if they are only different because one of them is calculated using a slightly different value of δ. We therefore delete solutions where the Euclidean distance between the gridpoint and the parameters is more than 10^{-6}.

To illustrate the problem, Table A.1 provides an example of two solutions that we find close to the gridpoint ($\delta = 0.194, \rho = 0.87$). The first two columns give the solutions returned by the homotopy procedure, and some elements of the price vectors differ by more than 0.01 (fourth column). The third column shows what happens when we use `fsolve` in MATLAB to solve for an equilibrium at the gridpoint using the vector in the second column (the column that is further away from the gridpoint) as a starting value. As can be seen from the fifth column, the differences are very small, but still close to the values in the second column, suggesting that we should not consider this outcome to be a different equilibrium.

Figure A.1 shows the distribution of the Euclidean distances between the gridpoint and the solution (ρ, δ) when $b^p = 0.025$. As can be seen, our rule eliminates a small number of solutions on this basis.

	Solution 1	**Solution 2**	**'Improved Solution 2'**	**Difference Between Soln 2 & Soln 1**	**Difference Between Impr. Soln 2 & Soln 1**
Exact value ρ Exact value δ Distance to grid	**0.870000** **0.193999** 3.23E-08	**0.870106** **0.194000** 1.06E-04			
Elements of P					
1	2.128583	2.141516	2.128585	0.012933	1.665E-06
2	7.041244	7.045969	7.041244	0.004725	1.106E-07
3	10.762308	10.762449	10.762308	0.000142	4.713E-08
4	11.257930	11.258070	11.257930	0.000140	9.208E-09
...	...	...	...	...	...
899	5.468426	5.472013	5.468426	0.003587	1.290E-07
900	2.739311	2.746190	2.739311	0.006878	4.498E-07
Elements of V^S					
1	3.951730	3.953952	3.951730	0.002222	6.835E-08
2	2.903783	2.906150	2.903783	0.002367	3.079E-08
3	2.989675	2.992032	2.989675	0.002357	1.882E-08
...	...	...	...	...	...
899	16.771803	16.768819	16.771803	0.002984	8.608E-08
900	11.861553	11.862831	11.861552	0.001278	5.235E-07
Elements of V^B					
1	46.171971	46.170929	46.171971	0.001042	5.778E-08
2	45.968869	45.968181	45.968869	0.000688	1.534E-08
3	46.257907	46.256799	46.257907	0.001108	5.821E-08
...	...	...	...	...	...
464	46.638726	46.637410	46.638726	0.001315	7.362E-09
465	46.755808	46.754386	46.755808	0.001422	1.805E-08

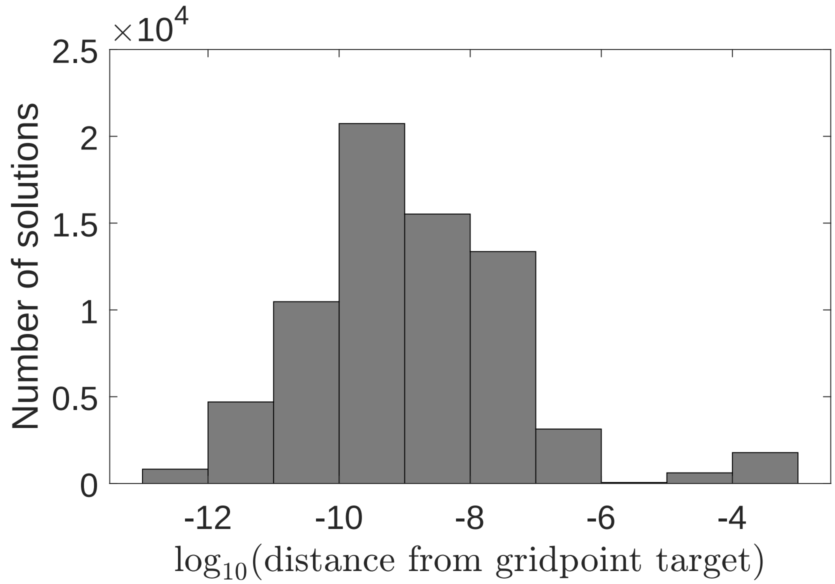


Figure A.1: Distribution of Euclidean Distances Between the (ρ, δ) Values Returned By the Algorithm and our Gridpoints for the δ-Homotopies and $b^p = 0.025$

A second numerical issue is whether the returned values of $\mathbf{x}$ are numerically close enough to solving the equations to be counted as equilibria. The criteria that we use is that solutions where the objective funciton (i.e., the sum of squared deviations across the equations) of $\mathbf{F}(\mathbf{x})$ are more than 10^{-10} are rejected. Figure A.2 shows the distribution of the value of the objective function for the solutions that fail our restriction. As can be seen clearly, the vast majority of rejected solutions represent outcomes that are not especially close to our cutoff, and so are not plausibly equilibria.

Step 4: Dealing with Failed Homotopies. As noted by BDKS (p. 467), the homotopies may stop if they reach a point where the Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ has less than full rank. We find that this happens for around 40% of homotopy runs. While this sounds like a major "problem" for the procedure, we do not believe that this having

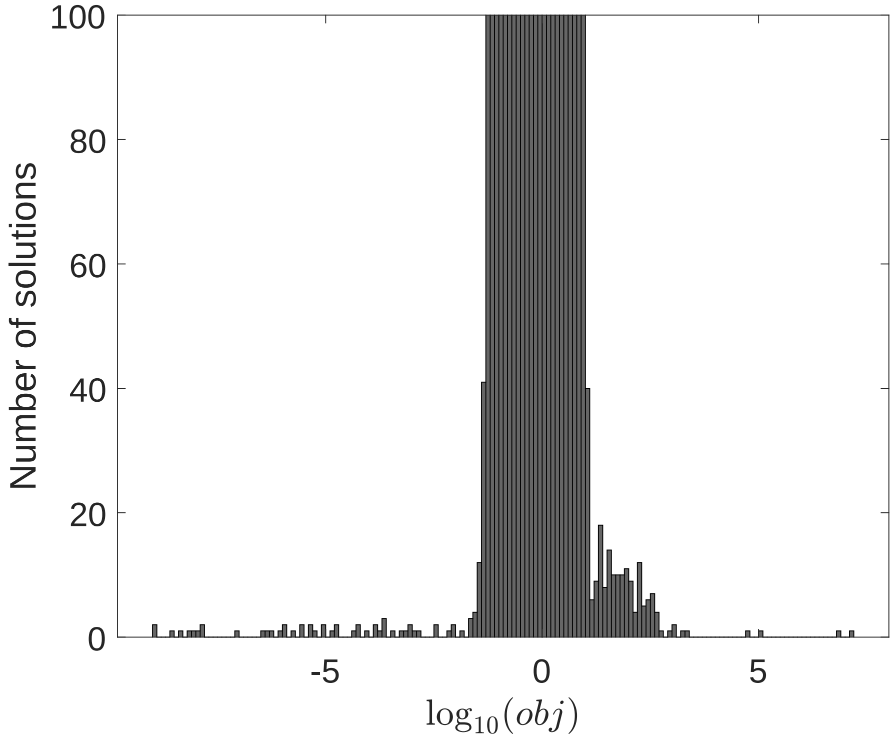


Figure A.2: Objective Functions Values for Solutions Rejected as Equilibria for the δ-Homotopies and $b^p = 0.025$

a major effect on our results because it usually happens outside the range of δ where either we or BDKS identify multiplicity (i.e., $\delta > 0.2$), and when we use ad-hoc procedures to try to restart the homotopy we are rarely able to find additional solutions that count as different equilibria even when we are able to do so.[4]

Step 5: Enumerating Equilibria. Once we have collected the solutions at each of the (ρ, δ) gridpoints we need to identify which solutions represent distinct equilibria, taking into account that small differences may arise because of numerical errors (or differences in the (ρ, δ) gridpoints that are within our tolerances.For this paper, we use the rule that solutions count as different equilibria if at least some elements of the price vector differ by more than 0.001.

Step 6: ρ-Homotopies. With a set of equilibria from the δ homotopies in hand, we can perform the second element of the criss-crossing procedure. In principle one would use all of the equilibria found in the first round (when $\delta > 0$) as starting points. However, the richness of our grid means that this is computationally prohibitive. We therefore use as starting points the step 2 equilibria found at the grid points $(\delta, \rho) \in \{0.001, 0.002, \ldots, 0.2, 0.25, 0.3, 0.35, \ldots, 1\} \times \{0.01, 0.11, 0.21, \ldots, 0.91\}$, where $\times$ denotes Cartesian product.[5]

As an example of how the different homotopies identify equilibria, Figures A.3

[4]Note that because one cannot show that the problem defined by the equations must be regular it is theoretically possible that the paths should stop at certain points. Therefore it is not necessarily surprising that we are unable to restart them.

[5]This choice reflects the fact that, like BDKS, we only find evidence of multiplicity for $\delta < 0.2$. The same Fortran subroutines as round 1 are used to run the ρ-homotopies. Experimentation indicates that we would not find significantly more equilibria if we used a richer grid, but we are doing some additional analysis to confirm this statement for all values of b^p. Steps 4 and 5 are applied to the results of the ρ-homotopies in the same way as they were for the δ-homotopies.

and Figures A.4 plot the number of new equilibria identified at each gridpoint for $b^p = 0.025$ in the δ- and the ρ-homotopy rounds.

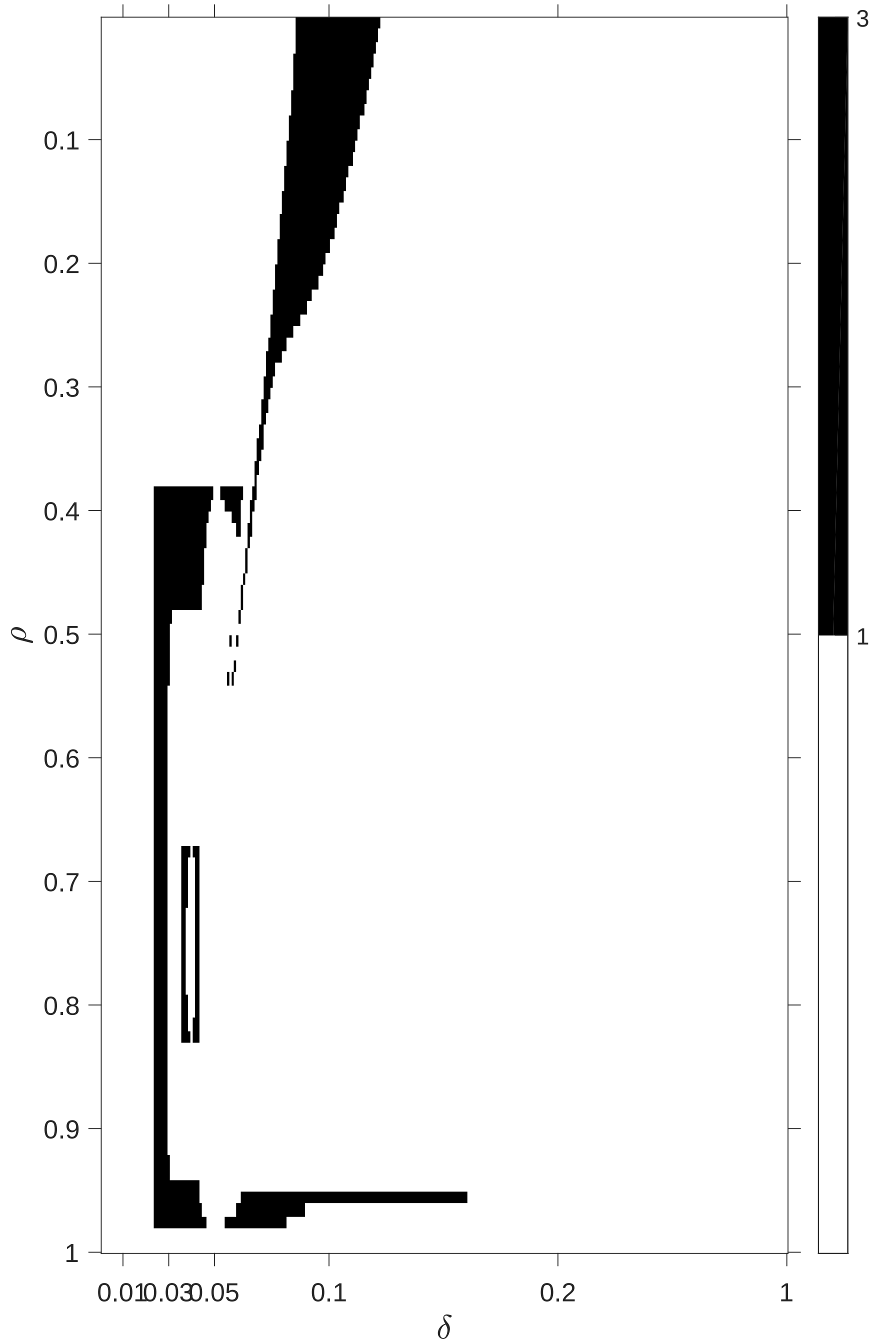


Figure A.3: Number of Equilibria Identified Using the δ-Homotopies for $b^p = 0.025$

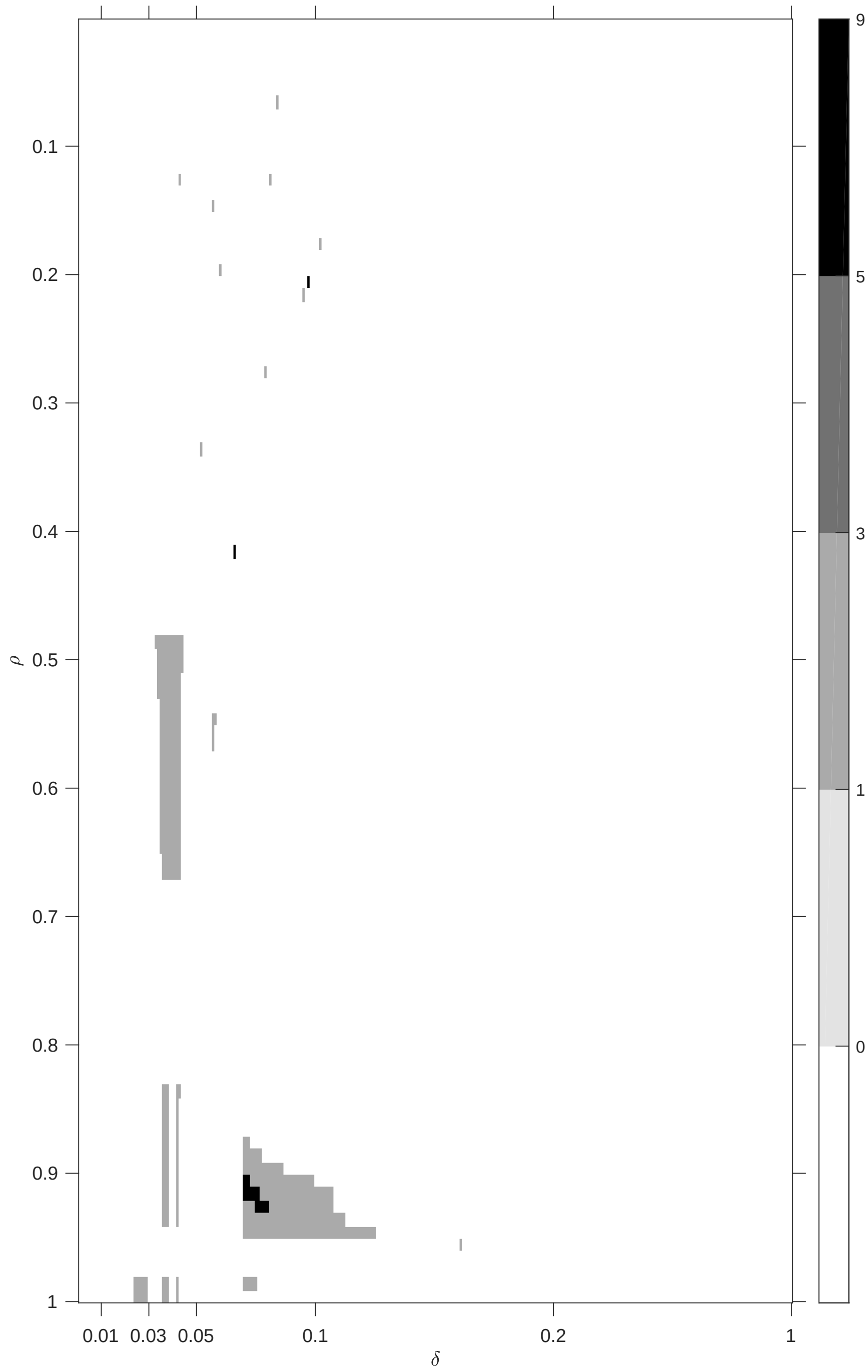


Figure A.4: Number of Additional Equilibria Identified Using the ρ-Homotopies for $b^p = 0.025$

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