ABSTRACT

Title of dissertation: ESSAYS ON CENTRALIZED MARKET ALLOCATIONS

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A long-standing policy concern in many countries is the difficulty of filling medical positions in rural areas. In Colombia, the Ministry of Health requires newly-graduated health professionals to work in a rural or marginalized urban area for a year in order to receive professional certification. The decentralized mechanism used until 2013 to allocate graduates to slots was one that health professionals could manipulate to avoid an assignment. In 2014, a single-offer centralized mechanism that cannot be manipulated to avoid an assignment, based on Gale and Shapley's deferred acceptance algorithm, was adopted. Following a revealed preference approach, I estimate health professionals' hospital preferences using the 2014 data. Using these estimates and the fact that under the decentralized mechanism health professionals were able to avoid positions that fall below their acceptance threshold, I obtain the average marginal utility a health professional would require to accept a position by simulating the outcome had the decentralized mechanism still been in use. Then, I simulate the outcome of the centralized mechanism in the absence of

the requirement that students accept the assignment determined by the mechanism. I find that, given the choice, about 30% of physicians would be left unassigned, implying that it is important for the policy's success that assignments be mandatory. I review many algorithms that have been discussed in the literature and find some that result in significant welfare gains. Finally, I show that, in this setting, there is no evidence that manipulable mechanisms can yield a higher welfare gains.

ESSAYS IN CENTRALIZED MARKET ALLOCATIONS

by

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Preface

This dissertation consists of three chapters. The main topic throughout this dissertation is the lessons that can be drawn from the implementation of a centralized clearinghouse to allocate health professionals to rural areas in Colombia. Chapter 1 explores the theoretical motivations of the clearinghouse. Paula Jaramillo and Çağatay Kayı are coathors of that chapter. Chapter 2 is an empirical evaluation of the implemented clearinghouse. Chapter 3 discusses the lessons that can be drawn from the clearinghouse implemented for those used in the US that allocate students to public schools. Chapter 4 discusses the aftermath of the allocation and concludes. All errors are mine. The views expressed here are those from the author and do not reflect the views of the Colombian Ministry of Health. I next summarize each chapter.

In chapter 1, we study many–to–one matching problems where doctors have strict preferences over hospitals, hospitals have strict preferences over sets of doctors, and the law assigns priorities to hospitals over sets of doctors. A mechanism assigns a matching between doctors and hospitals. We are interested in mechanisms that respect priority, are efficient and immune to strategic behavior, and minimize unfilled positions. We show that the mechanisms that have been used recently in Colombia do not satisfy any of these desirable properties. We consider three mechanisms proposed in the literature: the deferred acceptance mechanism, the immediate acceptance mechanism (so-called Boston mechanism), and the top trading cycle mechanism. We explore different versions of *efficiency*, namely *doctor–*

efficiency, *efficiency*, and *priority constrained–efficient*. We find that all these mechanisms are *priority constrained–efficient* and they *minimize unfilled positions*. As expected, the deferred acceptance mechanism and the top trading cycle mechanism are *strategy–proof* but the immediate acceptance mechanism is not. We show that the deferred acceptance mechanism always assigns a matching that is *priority constrained–efficient* and *respects priority* although it is neither *doctor–efficient* nor *efficient*. On the other hand, the top trading cycles mechanism and the immediate acceptance mechanism are *doctor–efficient*, but do not *respect priority*.

In chapter 2, I evaluate the centralized clearinghouse implemented through the mechanism motivated in the previous chapter as a policy tool. A long-standing policy concern in many countries is the difficulty of filling medical positions in rural areas. In Colombia, the Ministry of Health requires newly-graduated health professionals to work in a rural or marginalized urban area for a year in order to receive professional certification. The decentralized mechanism used until 2013 to allocate graduates to slots was one that health professionals could manipulate to avoid an assignment. In 2014, a single-offer centralized mechanism that cannot be manipulated to avoid an assignment, based on Gale and Shapley's deferred acceptance algorithm, was adopted. Following a revealed preference approach, I estimate health professionals' hospital preferences using the 2014 data. Using these estimates and the fact that under the decentralized mechanism health professionals were able to avoid positions that fall below their acceptance threshold, I obtain the average marginal utility a health professional would require to accept a position by simulating the outcome had the decentralized mechanism still been in use. Then, I simulate the outcome of the centralized mechanism in the absence of the requirement that students accept the assignment determined by the mechanism. I find that, given the choice, about 30% of physicians would be left unassigned, implying that it is important for the policy's success that assignments be mandatory. One feature of the centralized mechanism is that, in the case of multiple individuals having the same priority for a particular position, the tie is broken randomly. I show that breaking the ties in favor of those who listed a specific hospital as preferred can yield welfare gains of up to 12%. Finally, I show that moving from the random lottery to a merit-based tie-break, based on the results of the examination that health professionals take at the completion of their studies, raises inequality concerns.

In chapter 3, I revisit a long standing debate in the way how students are allocated to public/charter schools in many districts in the US. In particular within the school choice literature, the Immediate Acceptance and Deferred Acceptance (DA) mechanisms are defined as different algorithms. I develop a framework in which these mechanisms differ only in the priority structure. When "rank-based priority relations" are added on top of the DA mechanism's priorities the Immediate Acceptance mechanism is obtained. This framework naturally suggests a new mechanism; the one obtained by adding rank priorities below DA's priorities. Using the utilities of each professional at each hospital estimated in the previous chapter, I calculate the aggregate welfare for each mechanism. I find no evidence of a possible welfare increase when moving to a manipulable algorithm.

Acknowledgments

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I owe my deepest thanks to my wife, my mother, and my father who have always supported me. I have been able to pull through the road that ends with this dissertation because of them. Words cannot express the gratitude I owe them.

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Special thanks to my former roommate Mateo Uribe who has always been of great support. I would also want to thank Prateik Dalmia, Juan Echenique, Cody Tuttle, Matt Staiger, Alejandro Graziano, Xinlu Yao, Rodrigo Heresi, and Fernando Saltiel. It is impossible to remember all, and I apologize to those I've inadvertently left out.

Finally, this work would have not been possible without Paula Jaramillo and Çağatay Kayı. They introduced me to the marvels of the matching literature and gave me the wonderful opportunity of developing the software that conducts the allocation of health professionals to rural areas. They have become like family to me. This dissertation outlines that we must be proud of our work. It was also fundamental the contribution of the Ministry of Health in Colombia. In particular, I would like to thank Diego Restrepo and the Department of Human Talent at the Ministry of Health for their continuous support.

Ad maiorem Dei gloriam!

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List of Abbreviations

CSS	Compulsory Social Service
DA	Deferred Acceptance
IA	Immediate Acceptance
TTC	Top Trading Cycles
SOSM	Student Optimal Stable Match

Chapter 1: Matching Problems with Priorities and Preferences: Compulsory Social Service Allocation in Colombia

This chapter[1] studies two-sided many–to–one matching markets where doctors have strict preferences over hospitals, hospitals have strict preferences over sets of doctors, and the law assigns priorities to hospitals over doctors.

There are many real–life two-sided many–to–one matching markets. Two important instances are the national resident matching program and student assignment system to public schools. The former is an entry-level matching market for hospitals and medical school graduates in the United States where doctors have strict preferences over hospitals and hospitals have strict preferences over sets of doctors. The latter is the public school match where students have strict preferences over schools and the law assigns priorities to schools over students.

Many real–life matching markets employ centralized mechanisms to match students to hospitals and the only information that the matchmaker asks from each participating agent is the preference over the other side of the market. In particular, we assume that the agents' quotas (i.e., the number of available positions) are commonly known by the agents. In practice, agents are only allowed to submit ordered

[1]This chapter is co-authored with Paula Jaramillo and Çağatay Kayı

lists of individual partners. Throughout the chapter we focus on mechanisms that only demand ordered lists of potential individual partners.

An assignment between doctors and hospitals is called a matching. A mechanism assigns a matching to each problem. Next, we describe desirable properties of mechanisms. First, we are interested in mechanisms that *respect priority*, i.e., for each problem, the mechanism assigns a matching such that there is no doctor–hospital pair such that they are not mates in the matching but the doctor would prefer to be matched to the hospital and the hospital has unfilled positions or is matched to another doctor who has a lower priority for the hospital. Second, we are interested in a set of properties related to Pareto-efficiency. A mechanism is *doctor–efficient* if for each problem, the mechanism assigns a matching such that there is no other matching such that each doctor finds it at least as desirable and at least one doctor prefers. A mechanism is *efficient* if for each problem, the mechanism assigns a matching such that there is no other matching such that each agent finds it at least as desirable and at least one agent prefers. A mechanism is *priority constrained–efficient* if for each problem, the mechanism assigns a matching such that there is no other matching that respects priority such that each agent finds it at least as desirable and at least one agent prefers. The third property is *strategy–proofness*, i.e., no doctor should benefit from misrepresenting his preferences. The last property is *minimizing unfilled positions*, i.e., for each problem, the mechanism assigns a matching such that there is no doctor–hospital pair such that the hospital has unfilled positions but the doctor is not matched to any hospital.

1.1 Compulsory Social Service Allocation in Colombia

The compulsory social service allocation of medical doctors in Colombia was created to tackle the problems of inequality in allocation of doctors to rural areas in Colombia. The allocation is decided and implemented by the Ministry of Health.

The Colombian law and the regulations of the Ministry of Health impose a priority relation over doctors. Some priorities are being an indigenous, being a "raizal" (native of Archipelago of San Andrés, Providencia, and Santa Catalina), being pregnant, being a mother or a father with small children, being a doctor with impairments or disabilities, being a doctor who needs a special treatment. The Colombian law has two classes of priorities. First, a doctor who is "raizal" has a priority at the hospitals in the Archipelago of San Andrés, Providencia, and Santa Catalina. Second, a doctor who is indigenous has a priority at the hospitals in the indigenous regions. The regulations of the Ministry of Health have also two classes of priorities. First, a doctor who is a mother with small children has a priority at any hospital. Second, a doctor who has impairments or disabilities, or needs a special medical treatment. Also, the Colombian law take precedence over the regulations of the Ministry of Health. For example, a doctor who is a "raizal" has a priority over a doctor who is a mother with small children at the hospitals in the Archipelago of San Andrés, Providencia, and Santa Catalina. Similar precedence takes place for indigenous doctors. However, a doctor who is a mother with small children, or who has impairments or disabilities, or who needs a special medical treatment has priority over other doctor at any hospital.

The mechanism has been used to allocate doctors since 2012 is a lottery for each state (departamento) of Colombia. Each doctor enrolls in a state and a lottery assigns the enrolled doctors to a hospital in that state. The doctors that are not assigned to any hospital are exempt from the compulsory social service. In 2013, the ministry changed the mechanism by putting enrollment limits for each state. In each state, the number of enrolled doctors cannot be more than the double of the total available positions in that state. It was a first-come first-serve basis and the doctors who could not enroll to their favourite state have to choose another state to enroll in. Once again, after the enrollment, at each state, a lottery assigns the enrolled doctors to a hospital in that state. The doctors that are not assigned to any hospital are exempt from the compulsory social service.

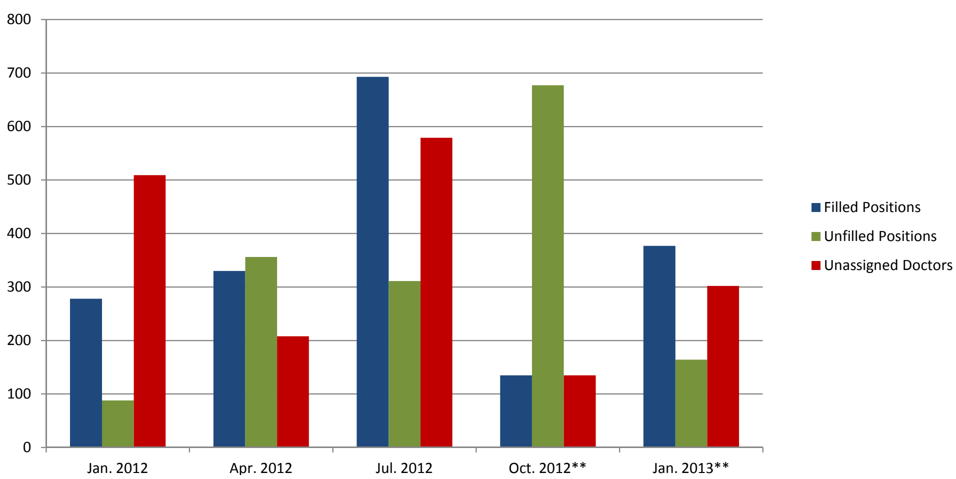


Figure 1.1: Filled positions, unfilled positions, and unassigned doctors in the mechanisms used by the Ministry of Health in 2012 and 2013.

These mechanisms created many problems. First, although the number of positions are smaller than the number of doctors, there are always unfilled positions (Figure 1.1). Second, the doctors do not have incentives to reveal their preferences truthfully and they coordinate to be exempt from the social service (Figure 1.2). Third, some doctors reject the allocation assigned by the mechanism since they cannot live in some states because of health problems or being mothers.

1.1.1 Overview of the Results

To analyze compulsory social service allocation of medical doctors in Colombia, we study a combination of many–to–one matching problem and school choice problem where each doctor can work for at most one hospital and each hospital can hire at most the number of positions it offers. Each doctor has a strict preference over hospitals and being exempt, each hospital has strict preferences over sets of doctors and being unmatched, and the Colombian law or the Ministry of Health assigns priorities to hospitals over doctors.

For the compulsory social service allocation of medical doctors in Colombia, we consider Gale and Shapley (1962) deferred acceptance algorithm incorporating the preferences of the doctors and the priorities dictated by the Colombian law or the Ministry of Health. This mechanism solves three aforementioned problems. The incorporation of priorities allows doctors with disabilities, doctors who are mothers with small children, or doctors with indigenous origins to have a priority over other doctors in the hospitals to which they apply. The doctors have incentives to reveal their true preferences. Also, if the number of positions are smaller than the number of doctors, there are no unfilled positions. The mechanism minimizes the number of unfilled positions.

We also consider the immediate acceptance mechanism (so-called Boston mechanism) and the top trading cycle mechanism. We find that all these mechanisms are *priority constrained–efficient* and they *minimize unfilled positions*. We show that the deferred acceptance mechanism always assigns a matching that is *priority constrained–efficient* and *respects priority* although it is neither *doctor–efficient* nor *efficient*. On the other hand, the top trading cycles mechanism and the immediate acceptance mechanism are *doctor–efficient*, but do not *respect priority*. As expected, the deferred acceptance mechanism and the top trading cycle mechanism are *strategy–proof* but the immediate acceptance mechanism is not.

The remainder of the chapter is organized as follows. In Section 1.2, we describe the model. Section 1.3, we present our results. The results in this chapter advocate the use of the Gale and Shapley's deferred acceptance mechanism in the setting mentioned above.

1.2 Model

There are two finite and disjoint sets of agents: a set of **doctors** $\boldsymbol{D}$ and a set of **hospitals** $\boldsymbol{H}$. Let $I = D \cup H$ be the set of agents. We denote a generic doctor, hospital, and agent by d, h, and i, respectively. For each hospital h, there is an integer **quota** $\boldsymbol{q_h} \geq 1$ that represents the number of positions it offers. Each doctor d can work for at most one hospital and each hospital h can hire at most q_h doctors. Let $q = (q_h)_{h \in H}$. For each $i \in I$, the set of potential partners of agent i is denoted by $\boldsymbol{N_i}$. If $i \in D$, $N_i = H$ and if $i \in H$, $N_i = D$.

Each doctor d has a complete, transitive, and strict **preference relation $\boldsymbol{P_d}$** over the hospitals and the prospect of "being exempt", which is denoted by $\emptyset$. For $h, h' \in H \cup \{\emptyset\}$, we write $h\, P_d\, h'$ if doctor d prefers h to h' ($h \neq h'$), and $h\, R_d\, h'$ if d finds h at least as good as h', i.e., $h\, P_d\, h'$ or $h = h'$. Note that for each doctor, each hospital is acceptable but a doctor may prefer being exempt to working for a hospital.[2] Let $\boldsymbol{P_D} = (P_d)_{d \in D}$ be the preference profile for the doctors.

Let $h \in H$. A subset of doctors $D' \subseteq D$ is **feasible** for hospital h if $|D'| \leq q_h$. Let $\mathcal{F}(D, q_h) = \{D' \subseteq D : |D'| \leq q_h\}$ denote the collection of feasible subsets of doctors for hospital h. The element $\emptyset \in \mathcal{F}(D, q_h)$ denotes "being unmatched". Each hospital h has a complete, transitive, and strict **preference relation $\boldsymbol{\mathcal{P}_h}$** over $\mathcal{F}(D, q_h)$. Also, for each hospital, each feasible subset of doctors is "acceptable", i.e., for each $D' \in \mathcal{F}(D, q_h) \backslash \{\emptyset\}$, $D'\, \mathcal{P}_h\, \emptyset$. For $D', D'' \in \mathcal{F}(D, q_h)$ we write $D'\, \mathcal{P}_h\, D''$ if hospital h prefers D' to D'' ($D' \neq D''$), and $D'\, \mathcal{R}_h\, D''$ if hospital h finds D' at least as good as D'', i.e., $D'\, \mathcal{P}_h\, D''$ or $D' = D''$. Let $\boldsymbol{\mathcal{P}_H} = (\mathcal{P}_h)_{h \in H}$ be the preference profile for the hospitals.

Let $\boldsymbol{P_h}$ be the restriction of preference relation $\mathcal{P}_h$ to $\{\{d\} | d \in D\} \cup \{\emptyset\}$, i.e., individual doctors in D and being unmatched. For each $d, d' \in D \cup \{\emptyset\}$, we write $d\, P_h\, d'$ if $d\, \mathcal{P}_h\, d'$ and $d\, R_h\, d'$ if $d\, \mathcal{R}_h\, d'$.[3] Since each doctor is acceptable, for each $d \in D$, we have $d\, P_h\, \emptyset$. Let $\boldsymbol{P_H} = (P_h)_{h \in H}$.

[2] In the literature, each doctor d has a preference relation over the hospitals and the prospect of "being unmatched" (or some outside option), which is denoted by d and if $h \in H$ such that $h\, P_d\, d$, then h is an acceptable hospital for doctor d. If the hospital is not acceptable, then the doctor may reject the assignment and wait to apply to the mechanism again in the future. In the extreme case, the doctor does not get the medical license to practice medicine. Since the consequences might be dire, we assume that a doctor finds each hospital acceptable.

[3] With some abuse of notation, we often write x for a singleton $\{x\}$.

We assume that for each hospital $h \in H$, $\mathcal{P}_h$ is **responsive**[4], or more precisely, a responsive extension of P_h, such that **(r1)** as long as a hospital's quota is not reached, it prefers to fill a position with a doctor rather than leaving it unfilled and **(r2)** a hospital if faced with two sets of potential doctors that differ only in one doctor, it prefers the set of doctors containing the more preferred doctor, i.e., for each $D' \in \mathcal{F}(D, q_h)$,

(r1) if $d \in D \backslash D'$ and $|D'| < q_h$, then $(D' \cup d)\mathcal{P}_h D'$ and

(r2) if $d \in D \backslash D'$ and $d' \in D'$, then $((D' \backslash d') \cup d) \; \mathcal{P}_h \; D'$ if and only if $d \, P_h \, d'$.

Each hospital $h \in H$, has a complete and transitive **priority relation** $\boldsymbol{\succeq_h}$ over D (not necessarily strict). The priority relation of each hospital is given by the law, hence the priority surpasses preference. Hence, the preferences are used to break ties in priorities. If $d \succ_h d'$, then d has a higher priority than d' at h. Let $\boldsymbol{\succeq_H} = (\succeq_h)_{h \in H}$ be the priority profile.

A **problem** is given by $(P_D, \mathcal{P}_H, \succeq_H, q)$. Let $\mathbb{P}$ be a set of problems. Let $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$ be a problem. A **matching** is a function μ on the set $D \cup H$ such that (1) each doctor is either matched to exactly one hospital or exempt, i.e., for each $d \in D$, either $\mu(d) \in H$ or $\mu(d) = \emptyset$; (2) each hospital is matched to a feasible set of doctors, i.e., for each $h \in H$, $\mu(h) \in \mathcal{F}(D, q_h)$; and (3) a doctor is matched to a hospital if and only if the hospital is matched to the doctor, i.e., for each $d \in D$ and $h \in H$, $\mu(d) = h$ if and only if $d \in \mu(h)$. Let μ be a matching and $i, j \in I$. If $j \in \mu(i)$, then we say that i and j are matched to one another and that they are mates in μ. The set $\mu(i)$ is agent i's match. Let $\mathcal{M}\,(P_D, \mathcal{P}_H, \succeq_H, q)$ be the

[4] Roth (1985) and Roth and Sotomayor (1992) for a discussion about this assumption.

set of matchings for the market $(P_D, \mathcal{P}_H, \succeq_H, q)$.

A matching *respects priority* if there is no doctor–hospital pair such that they are not mates in a matching but the doctor would prefer to be matched to the hospital and the hospital has unfilled positions or another doctor who has a lower priority than the doctor at the the hospital is matched to the hospital. Formally, a matching μ **respects priority** if there are no $d \in D$ and $h \in H$ such that $[h\ P_d\ \varphi_d(P_D, \mathcal{P}_H, \succeq_H, q)]$ and $[|\varphi_h(P_D, \mathcal{P}_H, \succeq_H, q)| < q_h$ or there is $d' \in \varphi_h(P_D, \mathcal{P}_H, \succeq_H, q)$ such that $d \succ_h d']$. Let $\mathcal{R}(P_D, \mathcal{P}_H, \succeq_H, q)$ be the set of matchings that respect priority for the market $(P_D, \mathcal{P}_H, \succeq_H, q)$.

A matching is *doctor–efficient* if there is no other matching such that each doctor finds it at least as desirable and at least one doctor prefers. A matching is *efficient* if there is no other matching such that each agent finds it at least as desirable and at least one agent prefers. A matching is *priority constrained–efficient* if there is no other matching that respects priority such that each agent finds it at least as desirable and at least one agent prefers. Formally, a matching μ is **doctor–efficient** if there is no matching μ' such that for each $d \in D$, $\mu'(d)\ R_d\ \mu(d)$ and there is $d \in D$, $\mu'(d)\ P_d\ \mu(d)$. A matching μ is **efficient** if there is no matching μ' such that for each $i \in D$, $\mu'(i)\ R_i\ \mu(i)$ and there is $i \in I$, $\mu'(i)\ P_i\ \mu(i)$. A matching μ is **priority constrained–efficient** if there is no matching $\mu' \in \mathcal{R}(P_D, \mathcal{P}_H, \succeq_H, q)$ that respects priority such that for each $i \in D$, $\mu'(i)\ R_i\ \mu(i)$ and there is $i \in I$, $\mu'(i)\ P_i\ \mu(i)$. Let $\mathcal{DE}(P_D, \mathcal{P}_H, \succeq_H, q)$, $\mathcal{E}(P_D, \mathcal{P}_H, \succeq_H, q)$, and $\mathcal{PCE}(P_D, \mathcal{P}_H, \succeq_H, q)$ be the set of doctor–efficient, efficient, and priority constrained–efficient matchings for the market $(P_D, \mathcal{P}_H, \succeq_H, q)$, respectively.

Many real-life centralized matching markets employ mechanisms that only ask for the preferences over individual partners. Since the social service is compulsory, the doctors cannot report whether they would like to be exempt. Hence, for each doctor $d \in D$, the mechanism only focuses on the restriction of preference relation P_d on H. We also assume that quotas are commonly known by the hospitals.[5] Throughout the paper, we focus on this class of mechanisms. A **mechanism** $\boldsymbol{\varphi}$ assigns a matching to each problem. We often denote agent i's match $\varphi(P)(i)$ by $\varphi_i(P)$.

Next, we describe desirable properties of mechanisms. The first set of properties is related to Pareto-efficiency. A mechanism is *doctor–efficient* if for each problem, the mechanism assigns a matching such that there is no other matching such that each doctor finds it at least as desirable and at least one doctor prefers. A mechanism is *efficient* if for each problem, the mechanism assigns a matching such that there is no other matching such that each agent finds it at least as desirable and at least one agent prefers. A mechanism is *priority constrained–efficient* if for each problem, the mechanism assigns a matching such that there is no other matching that respects priority such that each agent finds it at least as desirable and at least one agent prefers. Formally, a mechanism φ is **doctor–efficient** if for each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, we have $\varphi(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathcal{DE}(P_D, \mathcal{P}_H, \succeq_H, q)$. A mechanism φ is **efficient** if for each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, we have $\varphi(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathcal{E}(P_D, \mathcal{P}_H, \succeq_H, q)$. A mechanism φ is **priority constrained–efficient** if for each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, we have $\varphi(P_D, \mathcal{P}_H, \succeq_H, q) \in$

[5]Contrary to Sönmez (1997), we assume that quotas cannot be manipulated.

$\mathcal{PCE}\,(P_D, \mathcal{P}_H, \succeq_H, q)$.

The second property is *respecting priority*, i.e., for each problem, the mechanism assigns a matching that respects priority. Formally, a mechanism φ **respects priority** if for each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, we have $\varphi\,(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathcal{R}\,(P_D, \mathcal{P}_H, \succeq_H, q)$.

The third property is *strategy–proofness*, i.e., no doctor should benefit from misrepresenting his preferences. Formally, a mechanism φ is **strategy–proof** if for each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, each $d \in D$, and each P'_d, we have $\varphi(P_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)(d)\ R_d\ \varphi(P'_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)(d)$.

The last property is *minimizing unfilled positions*, i.e., for each problem, the mechanism assigns a matching such that there is no doctor–hospital pair such that the hospital has unfilled positions but the doctor is not matched to any hospital. Formally, a mechanism φ **minimizes unfilled positions** if for each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, there are no $d \in D$ and $h \in H$ such that $|\varphi_h(P_D, \mathcal{P}_H, \succeq_H, q)| < q_h$ and $\varphi_d(P_D, \mathcal{P}_H, \succeq_H, q) \notin H$.

Next, we define different mechanisms that are employed in many real-life problems. We start with the mechanisms that have been used in Colombia by the Ministry of Health. To better understand these mechanisms, we make two changes: First, we describe the mechanisms in terms of hospitals rather than in terms of states of Colombia (departamentos). This allows to compare effectively these mechanisms to other well-known mechanisms in the literature. Second, we incorporate the priorities into the mechanism. This allows us to show that incorporating the priorities to the current mechanism is not sufficient to satisfy *respecting priorities.*

The first one is the mechanism that the ministry was using during 2012.

Algorithm 1. [Ministry of Health - 2012, M^{2012}]

Step 1: Each doctor enrolls in a hospital.

Step 2: At each hospital, a lottery is used to assign doctors to the hospital. The doctors that are not assigned to any hospital are exempt from the compulsory social service.

For each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, let $M^{2012}(P_D, \mathcal{P}_H, \succeq_H, q)$ be the matching resulting from Algorithm 1.

The above mechanism was modified in January 2013 by the Ministry of Health.

Algorithm 2. [Ministry of Health - 2013, M^{2013}]

Step 1: Each doctor enrolls in an hospital. However, in each hospital, the number of enrolled doctors cannot be more than the double of the total available positions in that hospital.

Step 2: At each hospital, a lottery is used to assign doctors without priority to the hospital. The doctors that are not assigned to any hospital are exempt from the compulsory social service.

For each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, let $M^{2013}(P_D, \mathcal{P}_H, \succeq_H, q)$ be the matching resulting from Algorithm 2.

Next mechanism is the Deferred Acceptance with priority ($DA^{\succeq}$) which is based on Gale and Shapley (1962) deferred acceptance algorithm.[6]

Algorithm 3. [Deferred Acceptance with priority, $DA^{\succeq}$]

Step 1: Each doctor proposes to his preferred hospital. Each hospital tentatively assigns the positions to its proposers one at a time following the priority relation. If two doctors are indifferent in terms of priority relation, then the strict preference relation of the hospital is used for the tentative assignment. All remaining proposers, if any, are rejected.

$\vdots$

Step k: Each doctor that is rejected in Step $k-1$ proposes to the next hospital in his preference relation. Each hospital considers the doctors that were tentatively assigned a position in Step $k-1$ together with its new proposers. The hospital tentatively assigns the positions to these doctors one at a time following the priority relation. If two doctors are indifferent in terms of priority relation, then the strict preference relation of the hospital is used for the tentative assignment. All remaining proposers are rejected. A doctor that is rejected by all hospitals is exempt from the compulsory social service.

The algorithm terminates when each doctor is assigned to a hospital or exempt from the compulsory social service. For each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, let $DA^{\succeq}(P_D, \mathcal{P}_H, \succeq_H, q)$ be the matching resulting from Algorithm 3.

The next mechanism is Immediate Acceptance (IA) which also known as the

[6] We refer to Roth (2008) for an account on the history and applications of the deferred acceptance algorithm.

Boston Mechanism (Abdulkadiroğlu and Sönmez, 2003).

Algorithm 4. [Immediate Acceptance, IA]

Step 1: Each doctor proposes to his preferred hospital. Each hospital assigns the positions to its proposers one at a time following the priority relation. If two doctors have the same priority relation, then the strict preference relation of the hospital is used for the tentative assignment. All remaining proposers, if any, are rejected.

⋮

Step k: Each doctor that is rejected in Step $k-1$ proposes to the next hospital in his preference relation. Each hospital assigns its remaining unfilled positions to its proposers one at a time following the priority relation. If two doctors are indifferent in terms of priority relation, then the strict preference relation of the hospital is used for the assignment. All remaining proposers, if any, are rejected. A doctor that is rejected by all hospitals is exempt from the compulsory social service.

The algorithm terminates when each doctor is assigned to a hospital or exempt from the compulsory social service. For each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, let $IA(P_D, \mathcal{P}_H, \succeq_H, q)$ be the matching resulting from Algorithm 4.

Finally, we introduce the Top Trading Cycle mechanism which is introduced by Shapley and Scarf (1974). Abdulkadiroğlu and Sönmez (2003) adapt this mechanism for the problems with priorities. We extend this mechanism to the problems with preferences and priorities.

Algorithm 5. [Top Trading Cycle, TTC]

Step 1: Assign a counter for each hospital which keeps track of how many unfilled positions there are. Initially, set the counter of a hospital equal to its quota. Each doctor points to his preferred hospital. Each hospital points to the doctor who has the highest priority (If two doctors are indifferent in terms of priority relation, then the strict preference relation of the hospital is used for the assignment.) Since the number of doctors and hospitals are finite, there is at least one cycle. (A *cycle* is an ordered list of distinct hospitals and distinct doctors $(d_1, h_1, d_2, ..., d_k, h_k)$ where d_1 points to h_1, h_1 points to d_2,...,d_k points to h_k, and h_k points to d_1.) Moreover, each hospital can be part of at most one cycle. Every doctor in a cycle is assigned a position at the hospital he points to and is removed. The counter of each hospital in a cycle is reduced by one, and if it is reduced to zero, the hospital is removed. Counters of other hospitals stay put.

⋮

Step k: Each remaining doctor points to the preferred hospital among the remaining ones, and each remaining hospital points to the doctor with the highest priority among the remaining ones.(If two doctors are indifferent in terms of priority relation, then the strict preference relation of the hospital is used for the assignment.) There is a cycle. Every doctor in a cycle is assigned a position at the hospital he points to and is removed. The counter of each hospital in a cycle is reduced by one, and if it is reduced to zero, the hospital is removed. Counters of other hospitals stay put. A doctor that is never a part of a cycle is exempt from the compulsory social service.

The algorithm terminates when each doctor is assigned to a hospital or exempt from the compulsory social service. For each problem $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, let $TTC(P_D, \mathcal{P}_H, \succeq_H, q)$ be the matching resulting from Algorithm 5.

1.3 Results

First, we present our results about the set of matchings. Figure 1.3 summarizes the relationship between set of matchings.

Theorem 1.1. *The set of* doctor–efficient *matchings is a subset of the set of* efficient *matchings, i.e., for each problem* $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, $\mathcal{PCE}(P_D, \mathcal{P}_H, \succeq_H, q) \subseteq \mathcal{E}(P_D, \mathcal{P}_H, \succeq_H, q)$. *However, the converse is not true.*

Proof. First, we show that any *doctor–efficient* matching is *efficient*. Let $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$ be a problem and $\mu \in \mathcal{DE}(P_D, \mathcal{P}_H, \succeq_H, q)$ be a *doctor–efficient* matching. Assume on the contrary that μ is not *efficient*. Then, there is $\mu' \in \mathcal{M}(P_D, \mathcal{P}_H, \succeq_H, q)$ such that for each $i \in I$, $\mu'(i)\ R_i\ \mu(i)$ and there is $i \in I$, $\mu'(i)\ P_i\ \mu(i)$. In particular, for each $d \in D$, $\mu'(d)\ R_d\ \mu(d)$. However, there is $h \in H$ such that $\mu'(h) \neq \mu(h)$. Then, there is $d \in D$ such that $\mu'(d) \neq \mu(d)$. Since the preference relation is strict, then $\mu'(d)\ P_d\ \mu(d)$ which is contradiction to the fact that μ is a *doctor–efficient* matching. Hence, $\mu \in \mathcal{E}(P_D, \mathcal{P}_H, \succeq_H, q)$. However, the converse is not true. To see this, consider a problem $(P_D, \mathcal{P}_H, \succeq_H, q)$ with 2 doctors, 2 hospitals, and preferences over individual partners and priorities given by the columns in Table 1.1. Each hospital h has quota $q_h = 1$.

Doctors		Hospitals			
P_{d_1}	P_{d_2}	$\succ_{h_1}$	$\succ_{h_2}$	P_{h_1}	P_{h_2}
h_1	h_2	d_1	d_2	d_2	d_1
h_2	h_1	d_2	d_1	d_1	d_2

Table 1.1: Preferences P and priorities $\succ$ in Theorem 1.1.

Note that $\mu = \begin{pmatrix} h_1 & h_2 \\ d_2 & d_1 \end{pmatrix}$ is *efficient* but not *doctor-efficient*. □

Theorem 1.2. *The set of* efficient *matchings is a subset of the set of* priority constrained–efficient *matchings, i.e., for each problem* $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, $\mathcal{E}(P_D, \mathcal{P}_H, \succeq_H, q) \subseteq \mathcal{PCE}(P_D, \mathcal{P}_H, \succeq_H, q)$. *However, the converse is not true.*

Proof. First, we show that any *efficient* matching is *priority constrained–efficient*. Let $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$ be a problem and $\mu \in \mathcal{E}(P_D, \mathcal{P}_H, \succeq_H, q)$ be a *efficient* matching. Assume on the contrary that μ is not *priority constrained–efficient*. Then, there is $\mu' \in \mathcal{R}(P_D, \mathcal{P}_H, \succeq_H, q)$ such that for each $i \in I$, $\mu'(i)\ R_i\ \mu(i)$ and there is $i \in I$, $\mu'(i)\ P_i\ \mu(i)$. Since $\mu' \in \mathcal{R}(P_D, \mathcal{P}_H, \succeq_H, q) \subseteq \mathcal{M}(P_D, \mathcal{P}_H, \succeq_H, q)$ such that for each $i \in I$, $\mu'(i)\ R_i\ \mu(i)$ and there is $i \in I$, $\mu'(i)\ P_i\ \mu(i)$, it is a contradiction to the fact that μ is a *efficient* matching. Hence, $\mu \in \mathcal{PCE}(P_D, \mathcal{P}_H, \succeq_H, q)$.

However, the converse is not true. To see this, consider a problem $(P_D, \mathcal{P}_H, \succeq_H, q)$ with 3 doctors, 3 hospitals, and preferences over individual partners and priorities given by the columns in Table 1.2. Each hospital h has quota $q_h = 1$.

Doctors			Hospitals					
P_{d_1}	P_{d_2}	P_{d_3}	$\succ_{h_1}$	$\succ_{h_2}$	$\succ_{h_3}$	P_{h_1}	P_{h_2}	P_{h_3}
h_2	h_1	h_1	d_1	d_2	d_2	d_2	d_1	d_3
h_1	h_2	h_2	d_3	d_1	d_1	d_3	d_2	.
h_3	h_3	h_3	d_2	d_3	d_3	d_1	d_3	.

Table 1.2: Preferences P and priorities $\succ$ in Theorem 1.2.

Consider the following subset of matchings $\{\mu^1, \mu^2, \mu^3, \mu^4, \mu^5, \mu^6\} \subset \mathcal{M}(P_D, \mathcal{P}_H, \succeq_H, q)$ where

$$\mu^1 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_2 & d_1 & d_3 \end{pmatrix}, \quad \mu^2 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_3 & d_1 & d_2 \end{pmatrix}, \quad \mu^3 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_3 & d_2 & d_1 \end{pmatrix},$$

$$\mu^4 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_2 & d_3 & d_1 \end{pmatrix}, \quad \mu^5 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_1 & d_3 & d_2 \end{pmatrix}, \quad \mu^6 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_1 & d_2 & d_3 \end{pmatrix}.$$

Note that $\mu^6 \in \mathcal{PCE}\left(P_D, \mathcal{P}_H, \succeq_H, q\right)$ but $\mu^6 \notin \mathcal{E}\left(P_D, \mathcal{P}_H, \succeq_H, q\right)$. □

Theorem 1.3. *The intersection of the set of* doctor–efficient *matchings and the set of matchings that respect priority might be empty, i.e., there is a problem* $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, $\mathcal{DE}\left(P_D, \mathcal{P}_H, \succeq_H, q\right) \cap \mathcal{R}\left(P_D, \mathcal{P}_H, \succeq_H, q\right) = \emptyset$. *The intersection of the set of* efficient *matchings and the set of matchings that respect priority might be empty, i.e., there is a problem* $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, $\mathcal{E}\left(P_D, \mathcal{P}_H, \succeq_H, q\right) \cap \mathcal{R}\left(P_D, \mathcal{P}_H, \succeq_H, q\right) = \emptyset$

Proof. To show that the intersection of the set of *doctor–efficient* matchings and the set of matchings that respect priority might be empty, consider the problem in Theorem 1.2. It is easy to verify that μ^1, μ^2, μ^3, and μ^4 are *doctor–efficient* but do not *respect priority*. Similarly, to show that the intersection of the set of *efficient* matchings and the set of matchings that respect priority might be empty, consider the problem in Theorem 1.2. It is easy to verify that μ^1, μ^2, μ^3, and μ^4 are *efficient* but do not *respect priority*. $\square$

Theorem 1.4. *The intersection of the set of* priority constrained-efficient *matchings and the set of matchings that respect priority is not empty, i.e., for each problem* $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$, $\mathcal{PCE}(P_D, \mathcal{P}_H, \succeq_H, q) \cap \mathcal{R}(P_D, \mathcal{P}_H, \succeq_H, q) \neq \emptyset$. *Moreover, the matching resulting from the deferred acceptance algorithm with priority is always in the intersection, i.e.,* $DA^{\succeq}(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathcal{PCE}(P_D, \mathcal{P}_H, \succeq_H, q) \cap \mathcal{R}(P_D, \mathcal{P}_H, \succeq_H, q)$. *Also, the set of matchings that respect priority is not always a subset of the set of* priority constrained-efficiently–efficient *matchings, i.e., there is a problem* $(P_D, \mathcal{P}_H, \succeq_H, q) \in \mathbb{P}$ *such that* $\mathcal{R}(P_D, \mathcal{P}_H, \succeq_H, q) \nsubseteq \mathcal{PCE}(P_D, \mathcal{P}_H, \succeq_H, q)$.

Proof. First, we show that the matching resulting from the deferred acceptance algorithm with priority *respects priority*. Suppose it is not the case. Then, there are $(P_D, \mathcal{P}_H, \succeq_H, q)$, $d \in D$, and $h \in H$ such that $\left[h \; P_d \; DA^{\succeq}_d(P_D, \mathcal{P}_H, \succeq_H, q)\right]$ and $\left[|DA^{\succeq}_h(P_D, \mathcal{P}_H, \succeq_H, q)| < q_h\right.$ or there is $d' \in DA^{\succeq}_h(P_D, \mathcal{P}_H, \succeq_H, q)$ such that $\left.d \succ_h d'\right]$. There are two cases:

- $|DA^{\succeq}_h(P_D, \mathcal{P}_H, \succeq_H, q)| < q_h$. Since $h \; P_d \; DA^{\succeq}_d(P_D, \mathcal{P}_H, \succeq_H, q)$, d has been tentatively assigned to h before being assigned to $DA^{\succeq}_h(P_D, \mathcal{P}_H, \succeq_H, q)$. Since $|DA^{\succeq}_h(P_D, \mathcal{P}_H, \succeq_H, q)| < q_h$, d should never been rejected by h at any step of the algorithm.

- There is $d' \in DA_h^{\succeq}(P_D, \mathcal{P}_H, \succeq_H, q)$ such that $d \succ_h d'$. Since $h\ P_d\ DA_d^{\succeq}(P_D, \mathcal{P}_H, \succeq_H, q)$, d has been tentatively assigned to h before being assigned to $DA_h^{\succeq}(P_D, \mathcal{P}_H, \succeq_H, q)$. Since there is $d' \in DA_h^{\succeq}(P_D, \mathcal{P}_H, \succeq_H, q)$ such that $d \succ_h d'$, d should never been rejected by h at any step of the algorithm before d'.

Next, we show that the matching resulting from the deferred acceptance algorithm with priority is *priority constrained–efficient*. Assume on the contrary that $\mu = DA_h^{\succeq}(P_D, \mathcal{P}_H, \succeq_H, q)$ is not *priority constrained–efficient*. Then, there is $\mu' \in \mathcal{R}(P_D, \mathcal{P}_H, \succeq_H, q)$ such that for each $i \in I$, $\mu'(i)\ R_i\ \mu(i)$ and there is $i \in I$, $\mu'(i)\ P_i\ \mu(i)$.

In particular, for each $d \in D$, $\mu'(d)\ R_d\ \mu(d)$. However, there is $h \in H$ such that $\mu'(h) \neq \mu(h)$. Then, there is $d \in D$ such that $\mu'(d) \neq \mu(d)$. This contradicts to the fact that the matching resulting from the deferred acceptance algorithm with priority weakly Pareto–dominates any matching that respects priority since the set of matchings that respect priority is a lattice (Roth, 2008). Hence, it is *priority constrained–efficient*.

In the problem in Theorem 1.2, the matching $\mu^6 = DA_d^{\succeq}(P_D, \mathcal{P}_H, \succeq_H, q)$ which is the matching resulting from the deferred acceptance algorithm with priority is *priority constrained–efficient* and *respects priority*.

To see that the set of matchings that respect priority is not always a subset of the set of *priority constrained-efficient* matchings, consider a problem $(P_D, \mathcal{P}_H, \succeq_H, q)$ with 3 doctors, 3 hospitals, and preferences over individual partners and priorities given by the columns in Table 1.3. Each hospital h has quota $q_h = 1$.

Doctors			Hospitals					
P_{d_1}	P_{d_2}	P_{d_3}	$\succ_{h_1}$	$\succ_{h_2}$	$\succ_{h_3}$	P_{h_1}	P_{h_2}	P_{h_3}
h_1	h_2	h_3	d_2	d_3	d_1	d_1	d_2	d_3
h_2	h_3	h_1	d_3	d_1	d_2	d_3	d_1	d_2
h_3	h_1	h_2	d_1	d_2	d_3	d_2	d_3	d_1

Table 1.3: Preferences P and priorities $\succ$ in Theorem 1.4.

Consider the following subset of matchings $\{\mu^1, \mu^2, \mu^3, \mu^4, \mu^5, \mu^6\} \subset \mathcal{M}(P_D, \mathcal{P}_H, \succeq_H, q)$ where

$$\mu^1 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_2 & d_3 & d_1 \end{pmatrix}, \quad \mu^2 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_1 & d_2 & d_3 \end{pmatrix}, \quad \mu^3 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_3 & d_2 & d_1 \end{pmatrix}$$

$$\mu^4 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_1 & d_3 & d_2 \end{pmatrix}, \quad \mu^5 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_2 & d_1 & d_3 \end{pmatrix}, \quad \mu^6 = \begin{pmatrix} h_1 & h_2 & h_3 \\ d_3 & d_1 & d_2 \end{pmatrix}.$$

It is to verify that μ^1 and μ^6 respect priority but they are not *priority constrained–efficient* since each agent is better–off at μ^2 which respects priority.

□

Next, we present our results regarding the properties of the mechanisms.

Theorem 1.5. *The mechanisms M^{2012} and M^{2013} are not priority constrained–efficient, strategy–proof, and neither* respect priority *nor minimize unfilled positions.*

Proof. Consider a problem $(P_D, \mathcal{P}_H, \succeq_H, q)$ with 2 doctors, 2 hospitals, and preferences over individual partners and priorities given by the columns in Table 1.4.

Each hospital h has quota $q_h = 1$.

Doctors		Hospitals		
P_{d_1}	P_{d_2}	$\succ_h$	P_{h_1}	P_{h_2}
$\boxed{h_1}$	h_1	d_1	$\boxed{d_1}$	d_1
h_2	h_2	d_2	d_2	d_2
$\emptyset$	$\boxed{\emptyset}$		$\emptyset$	$\boxed{\emptyset}$

Table 1.4: Preferences P and priorities $\succ$ in Theorem 1.5.

Suppose that the matching recommended by M^{2012} and M^{2013} is:

$$\mu = \begin{pmatrix} h_1 & h_2 & \emptyset \\ \\ d_1 & \emptyset & d_2 \end{pmatrix}$$

which is the boxed matching in Table 1.4.

The mechanisms M^{2012} and M^{2013} are not *priority constrained–efficient* since there is a matching $\mu' = \begin{pmatrix} h_1 & h_2 \\ \\ d_1 & d_2 \end{pmatrix}$ which *respects priority* such that each agent finds it at least as desirable and at least one agent prefers. Hence, the mechanisms M^{2012} and M^{2013} are not neither *doctor–efficient* nor *efficient*.

The mechanisms M^{2012} and M^{2013} do not *respect priorities* since $d_2 \in D$ prefers to be matched to h_2 to not to be assigned and h_2 has unfilled position.

The mechanisms M^{2012} and M^{2013} do not *minimize unfilled positions*, since h_2 is not assigned to any doctor, i.e., $|\varphi_{h_2}(P_D, \succeq_H, \mathcal{P}_H, q)| = 0 < q_h = 1$ and d_2 is not assigned to any hospital.

The mechanisms M^{2012} and M^{2013} are not *strategy–proof.* Suppose that d_2 reports $P'_{d_2} = h_2, h_1$. In this case, suppose that the matching recommended by

M^{2012} and M^{2013} is $\mu' = \begin{pmatrix} h_1 & h_2 \\ d_1 & d_2 \end{pmatrix}$. Note that $\mu'(d_2)\ P_{d_2}\ \mu(d_2)$. Hence, P'_{d_2} is a profitable manipulation for d_2 in violation of *strategy–proofness*. □

Theorem 1.6. *The mechanisms IA and TTC do not* respect priorities*.*

Proof. To see this, consider the problem in Theorem 1.2. It is easy to verify that the matching $\mu^2 = IA(P_D, \mathcal{P}_H, \succeq_H, q)$ does not *respect priority*. Similarly, the matching $\mu^1 = TTC(P_D, \mathcal{P}_H, \succeq_H, q)$ does not *respect priority*.

□

Theorem 1.7. *The mechanisms IA and TTC are doctor–efficient. Hence, they are efficient and priority constrained–efficient.*

Proof. Abdulkadiroğlu and Sönmez (2003) showed that the mechanisms IA and TTC are *doctor–efficient*. □

Theorem 1.8. *The mechanism $DA^{\succeq}$ is strategy–proof.*

Proof. Suppose that it is not the case. Then, there are $(P_D, \mathcal{P}_H, \succeq_H, q)$, $d \in D$ and P'_d such that $DA^{\succeq}_d(P'_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)\ P_d\ DA^{\succeq}_d(P_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)$. There are two cases:

- $DA^{\succeq}_d(P'_d, P_{-d}, \mathcal{P}_H, \succeq_H, q) = h \in H$. Note that at $(P_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)$, d has been tentatively assigned to h and has been rejected. Since P_H is as before, h would reject d as well in this case.

- $DA^{\succeq}_d(P'_d, P_{-d}, \mathcal{P}_H, \succeq_H, q) = \emptyset$. Note that $|D| > \sum q_i$ and at $(P_d, P_{-d}, \mathcal{P}_H, \succeq_H$

$, q)$ d has been rejected by each hospital. Since (P_{-d}, P_H) is as before, d would be rejected by each hospital as well. Therefore, $DA_d^{\succeq}(P_d, P_{-d}, \mathcal{P}_H, \succeq_H, q) = \emptyset$.

$\square$

Theorem 1.9. *The mechanism TTC is strategy–proof.*

Proof. Suppose that it is not the case. Then, there are $(P_D, \mathcal{P}_H, \succeq_H, q)$, $d \in D$ and P'_d such that $TTC_d(P'_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)\ P_d\ TTC_d(P_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)$. There are two cases:

- $TTC_d(P'_d, P_{-d}, \mathcal{P}_H, \succeq_H, q) = h \in H$. Note that at $(P_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)$, d points to h and d is not a part of a cycle. Since (P_{-d}, P_H) is as before, h never closes a cycle that d participates.

- $TTC_d(P'_d, P_{-d}, \mathcal{P}_H, \succeq_H, q) = \emptyset$. Note that $|D| > \sum q_i$ and at $(P_d, P_{-d}, \mathcal{P}_H, \succeq_H, q)$, d is not a part of any cycle. Since (P_{-d}, P_H) is as before, the rest of the agents point as they were pointing and d cannot be a part of any cycle. Therefore, $TTC_d(P_d, P_{-d}, \mathcal{P}_H, \succeq_H, q) = \emptyset$.

$\square$

Theorem 1.10. *The mechanism IA is not strategy–proof.*

Proof. To see this, consider the problem in Theorem 1.2. The matching $\mu^2 = IA(P_D, \mathcal{P}_H, \succeq_H, q)$ is the matching resulting from the immediate acceptance algorithm. Suppose that d_2 reports $P'_{d_2} = h_2, h_1, h_3$. In this case, the matching resulting from the immediate acceptance algorithm is $\mu^3 = IA(P'_{d_2}, P_{D\setminus\{d_2\}}, \mathcal{P}_H, \succeq_H, q)$ Note that $\mu^3(d_2)\ P_{d_2}\ \mu^2(d_2)$. Therefore, P'_{d_2} is a profitable manipulation for d_2 in violation of *strategy–proofness.*

□

Theorem 1.11. *The mechanisms* $DA^{\succeq}$, IA, *and* TCC *minimize unfilled positions.*

Proof. Suppose it is not the case. Then, there are $(P_D, \mathcal{P}_H, \succeq_H, q)$, $h \in H$ such that $|\varphi_h(P_D, \mathcal{P}_H, \succeq_h, q)| < q_h$, and $d \in D$ such that $\varphi_d(P_D, \mathcal{P}_H, \succeq_H, q) = \emptyset$.

- $DA^{\succeq}$: Since $\varphi_d(P_D, \mathcal{P}_H, \succeq_H, q) = \emptyset$, d is rejected by any hospital. However, h would not reject d if there is unfilled position.

- IA: Since $\varphi_d(P_D, \mathcal{P}_H, \succeq_H, q) = \emptyset$, d is rejected by any hospital. However, h would not reject d if there is unfilled position.

- TTC: Since $\varphi_d(P_D, \mathcal{P}_H, \succeq_H, q) = \emptyset$, d points to each hospital and never be a part of cycle. However, in the last step of the algorithm, h would point d and d would point to h forming a cycle.

□

Table 1.5 summarizes our results.

Property	M^{2012}	M^{2013}	$DA^{\succeq}$	IA	TTC
Doctor–Efficiency	−	−	−	+	+
Efficiency	−	−	−	+	+
Priority constrained–efficiency	−	−	+	+	+
Respecting priority	−	−	+	−	−
Strategy–proofness	−	−	+	−	+
Minimizing unfilled positions	−	−	+	+	+

Table 1.5: Summary of the results. The mechanism corresponding to a column satisfies (does not satisfy) the property corresponding to a row if the associated cell contains a + (−).

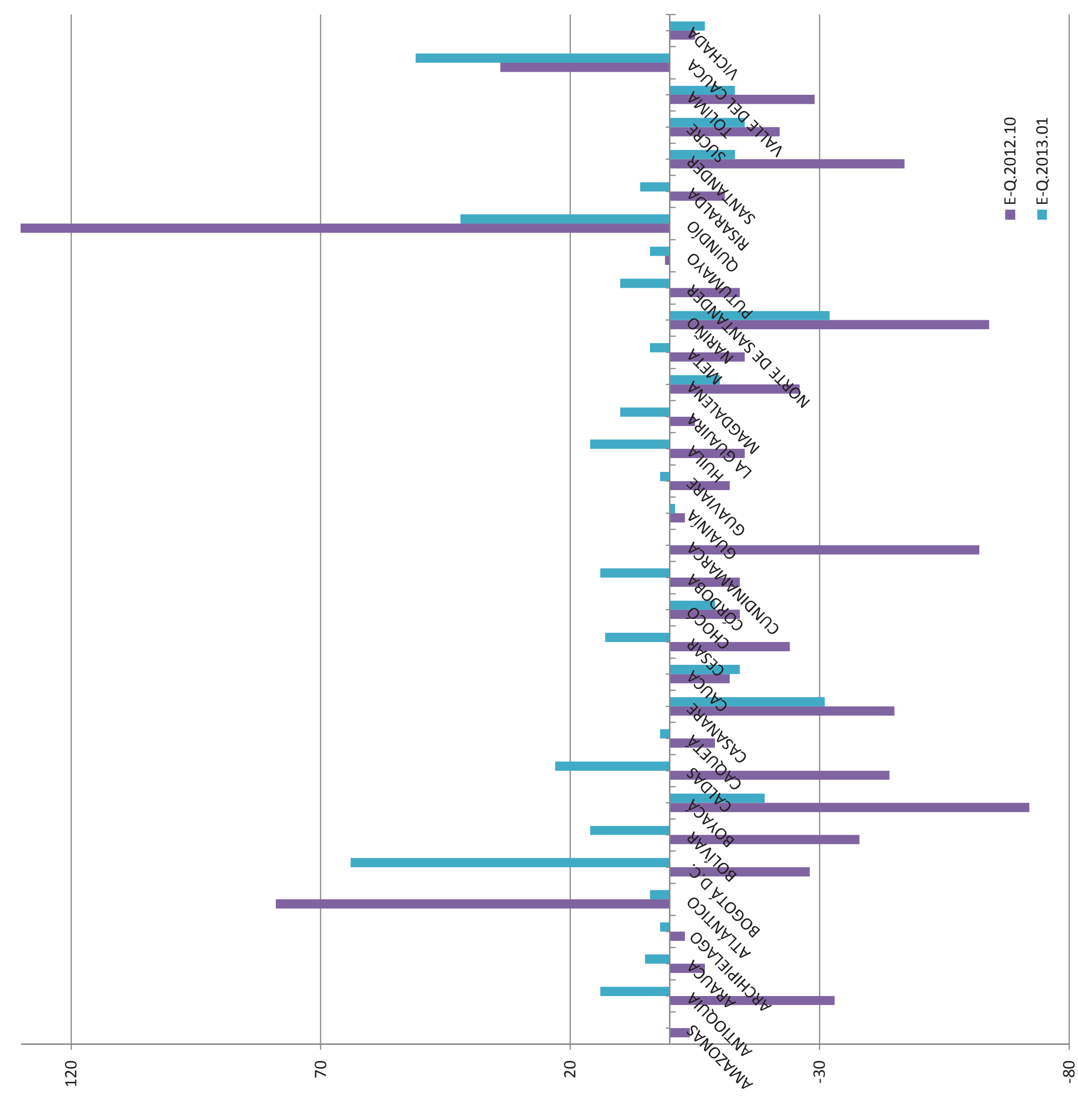


Figure 1.2: The difference between the number of enrolled doctors and the positions in each state of Colombia in October, 2012 and January, 2013.

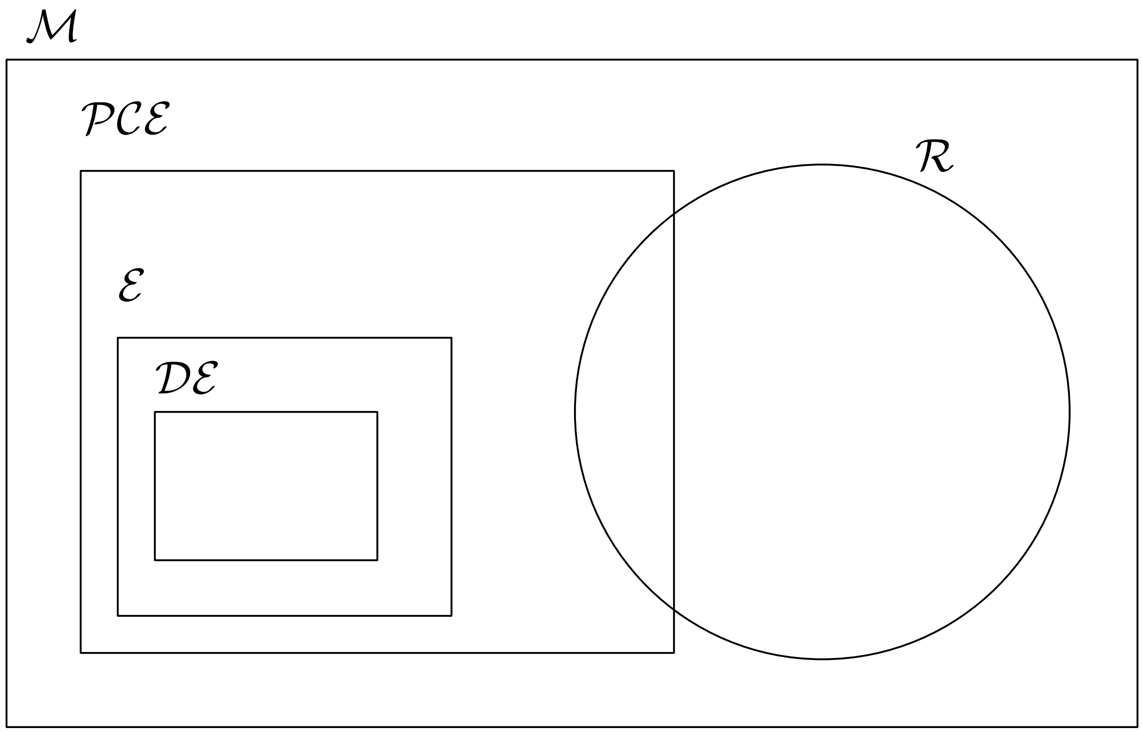


Figure 1.3: The relationships of the set of matchings.

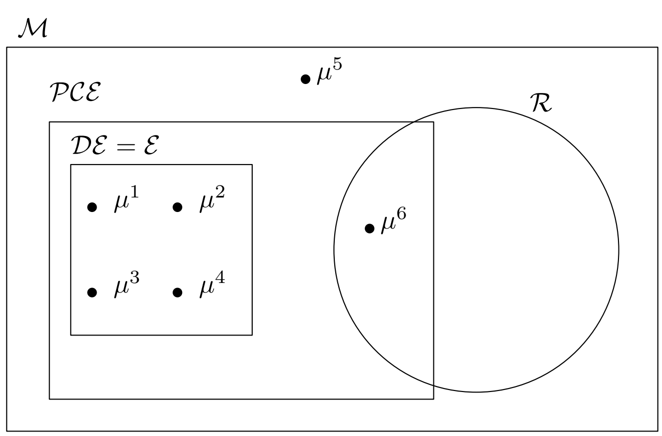


Figure 1.4: The relationships of the set of matchings for the problem in Theorem 1.2.

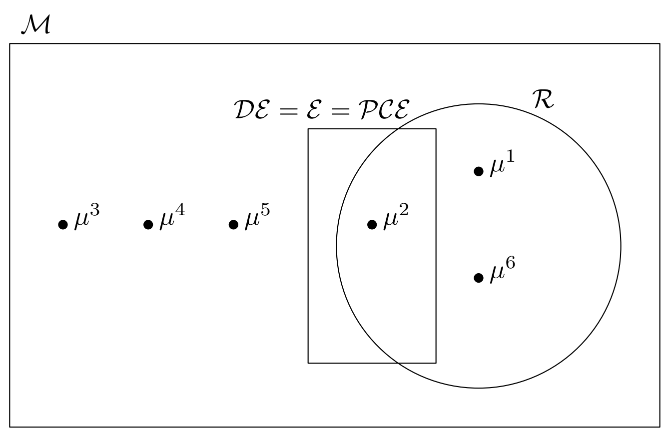


Figure 1.5: The relationships of the set of matchings in Theorem 1.4.

Chapter 2: Overcoming the Rural Hospital Theorem: Compulsory Social Service Allocation in Colombia

2.1 Introduction

The quality of health care systems depends highly on the availability of health professionals. However, rural communities tend to suffer from a lower availability of these professionals. The small population and scale makes the loss of a single health professional likely to have far-reaching consequences (Burrows et al., 2012). Rural areas lack health professionals primarily because health professionals try to avoid them. The allocation of medical residents in the US achieved through the National Residency Matching Program (NRMP) has shown the following pattern: "A number of hospitals, particularly Rural Hospitals, fail each year to fill as many positions as they have available, and find that a high percentage of the positions they do fill are filled by foreign medical school graduates" (Roth, 1986). In the health professionals market, a *stable* allocation is one in which there does not exist a pair of hospital and health professionals not mutually assigned and that they prefer each other. The so called "*Rural Hospital Theorem*" states that for any stable allocation the unfilled positions will be the same. Hence the solution to the vacancy of rural hospital positions cannot be found within this type of allocations. Therefore Roth (1986) concludes that "this maldistribution seems unlikely to be changed by any system that does not involve some element of *compulsion*, or some change in the relative numbers of available positions".

In Colombia, the Ministry of Health has tackled this problem through a program called Compulsory Social Service[1], hereafter CSS. The health professions that participate are Medical Lab Science, Nursing, Medicine, and Dentistry. The CSS requires health professionals to work at a hospital they assign them to for one year, 70% of which are located in rural areas, but some are located in marginalized urban ones. The compulsion element comes in through the fact that professionals in these fields are required to be professionally certified by the Ministry of Health; this happens only after participating in the CSS.

There are other entry level clearinghouses that have been studied in the literature. Examples of such are the National Residency Matching Program which allocates medical school students in the US to residency training and the allocation of teachers done by the Ministry of Education in Turkey. However, the compulsory characteristics makes the CSS different than any other clearinghouse studied in the literature. The previous chapter details the theoretical motivations underpinning the question on how to adapt the previously used mechanisms in the literature to this setting. In this chapter, I evaluate the implemented mechanism.

Since October 2014, the CSS allocation of health professionals has been determined using a single-offer centralized mechanism based on Gale and Shapley's

[1] The name in Spanish is *Servicio Social Obligatorio*.

(1962) deferred acceptance algorithm (DA). Health professionals report their five preferred positions and also rank all of the states where they might be assigned. Based on that information, the ministry creates a ranking of all hospitals for each participant, and that ranking is used in the assignment process. Additionally, the Ministry of Health assigns a (coarse) priority order to each hospital. [2] The allocation is determined using the deferred acceptance algorithm, henceforth DA, with the rankings and priorities[3] as inputs.

To study and evaluate the performance of three dimensions of the program, this chapter uses empirical estimates of the preferences. The first dimension I evaluate is the compulsory element, or the fact that participation is compulsory rather than based on individual rationality. Second, I estimate the allocative efficiency of the health professionals in the centralized mechanism. Third, I measure the consequences of moving from a single random lottery to a merit-based tie-break.

Colombia's CSS mechanism has one of the strongest compulsory elements of any centralized mechanism that has been studied in the literature. I show that compared to the previous decentralized mechanism, the centralized mechanism when adopted results in significant welfare gains. The first counterfactual answers the empirical question of whether, given the welfare gains, the program still needs the compulsory characteristic to successfully allocate health professionals. To this end I refer to the fact that in October 2012, the allocation was determined by a decentralized system of lotteries that was regularly manipulated by physicians to avoid being

[2] In this program hospitals do not report preferences of health professionals. Hence, the hospitals are not strategic agents in this setting, making this allocation problem one-sided. In the literature this is known as *school choice* (Abdulkadiroğlu and Sönmez, 2003).

[3] A single random tie-break is used to break ties in priorities.

assigned. I simulate the outcome of the centralized mechanism in the absence of the requirement that students accept the assignment determined by the mechanism. I find that if physicians were given the opportunity of not ranking all hospitals, about 30% of the them would be left unassigned, implying that for the policy to succeed, assignments must be mandatory. This contrasts with the 5% rejection rate observed in the actual allocations.

The second counterfactuals are designed to test the allocative efficiency of the implemented mechanism. Improving the allocative efficiency, besides improving the welfare of health professionals, would also reduce the rejection rate. Since this mechanism uses the DA algorithm to determine the allocations, it is known to *respect priorities* and to be *strategy proof*. The first of these characteristic establishes that whenever there is position that a health professional prefers relative to the one she is assigned, that position must be assigned to someone with at least the same priority. The latter property establishes that no health professional benefits from misrepresenting his preferences. These properties, although desirable, constrain the possible allocations and (possibly) entail efficiency costs. In these counterfactuals I measure the cost in efficiency of these properties. In order to measure the cost of the *respecting priorities* property, I compare the allocative efficiency of the allocations determined by the implemented mechanism with the *Top Trading Cycles*. This is due to the fact that the former is known to always yield *doctor-efficient* allocations while also being *strategy proof*. I find small welfare gains implying that *respecting priorities* does not come at a major cost. This is due to the fact that the preferences are significantly heterogeneous.

To measure the cost of the *strategy proof* property, I evaluate the allocative efficiency of mechanisms that, although not *strategy proof*, are known to be difficult to manipulate. When determining the allocation, the implemented mechanism treats equally a reported (one of the most preferred ones) and an implied preference through the ranking of states (potentially one of the least desired). I show that breaking ties in priorities in favor of reported hospitals could induce welfare gains of $8\% - 12\%$. This welfare increase results in a reduction of the rejection rates to levels of $2\% - 5\%$.

Another proposed mechanism is one which the allocation is determined in two rounds, using the reported preferences in the first and the implied preferences, i.e. those implied from state reports, on the second. This results in welfare gains of up to 13% while also having no rejections in some cases. This mechanism is neither *strategy proof* nor *respects priorities*. However it respects the priorities of the reported preferred hospitals and is strategy proof in the relative ranking of the reported preferences.

In the aforementioned mechanism treating differently the reported preferences than the not reported ones resulted in significant welfare gains. I study next what the optimal number of hospitals to report is, under the scenario where health professionals report truthfully. I find that in some cases the number is significantly low while on others it is a high number. Most of the environments where DA is used a truncation of possible reports is also at hand. I propose changing the termination rule of the deferred acceptance algorithm in order to reduce the cost of congestion while also allocating all slots.

Overall, I show that the cost of *strategy proofness* has first order consequences on the aggregate welfare of the allocation. Therefore, in this setting fine-tuning the algorithm has significant welfare consequences. This contrasts with other centralized mechanisms settings in which it has been observed that the fine-tuning of the algorithm is of second order (Abdulkadiroğlu et al., 2017).

The last counterfactual studies the consequences of moving to a merit-based tie-break. When designing the current program, the ministry decided not to use the results of the end of study nation-wide exam (known as SaberPro). They did this because the goal of the mechanism is to tackle the inequality of the allocation of health professionals; if they used the results, then the better health professionals would be allocated to the most desired positions. Conversely, not using the results would improve the odds that very good professionals would be allocated to remote positions. The allocation of physicians was not congested—that is, there were more positions than participants. In this case, the quality of allocated professionals would be a zero sum game in which the more desired positions would have a higher chance of going to good physicians. This would generate professions that are strongly congested, by means of giving the professionals with lower scores a higher probability of not being assigned, an increase in the mean quality of the set of allocated health professionals. Nevertheless, because of regional priorities, a small fraction of the hospitals are made worse off.

If statements about welfare are to be accurate, preference estimates must be accurate, too. Since the allocation mechanism is based on DA, the stated preferences are treated as the true preferences. This is due to the fact that DA is known to be strategy-proof. Had the health professionals not been restricted in the number of hospitals they report directly it is dominant for them to report their true preferences (as shown in Chapter 1). This is true even if their objective was not to be assigned at all. Since professionals report five (5) hospitals, they might misreport their true preferences in order to improve their chances of getting into a reported position. I do not observe evidence of this strategic behavior being systematic. Since their ranking will include all positions, there are many positions at which health professionals can be allocated regardless of their report. In particular, were one to be strategic with the report, the last position reported is the one that results in the highest incentive to do so. This would result in having a higher probability of being assigned to the last position reported than to the second to last.

Overall, this chapter evaluates a program designed to solve the rural hospital theorem problem of the health professionals market. This theorem happens to be framed in the context of rural hospitals but is applicable to all markets where stability is expected in the outcome. Therefore the lessons learned from the evaluation of the program at hand are valuable for other markets in which vacant positions are seen as market failures.

2.1.1 Related Literature

This chapter makes five significant contributions to the literature. First, it contributes to the public policy literature that examines the inequality of the allocation of health professionals to rural and urban positions. Several current policies tackle this problem. For example, in the US, foreign health professionals are waived the two year residency requirement if they work in a Health Professional Shortage Area (Burrows et al., 2012). In the case of Japan, Kamada and Kojima (2015) propose a centralized mechanism that uses hard regional caps to allocate a higher share of medical interns to that country's rural regions. Agarwal (2017) discusses how monetary incentives would modify the allocation of residents in the US. The approach taken by the Ministry of Health is different in that it uses a centralized mechanism and makes participation in the allocation compulsory.

Second, it contributes to the empirical study of preferences and allocations in the case of health professionals. To the best of my knowledge Agarwal (2015) is the only scholar who has carried out this type of analysis, which he applies to family medicine residents in the US. Agarwal's estimates are obtained from the analysis of observed matches. This chapter, in contrast, uses reported preferences. The estimates of the preferences show that the health professionals' preferences are significantly heterogeneous.

Third, it contributes to the empirical literature on centralized mechanisms, particularly those that use DA to determine the allocation. In the context of school choice, Abdulkadiroğlu et al. (2017) estimate parents' preferences for high schools in New York and compares the allocation efficiency of centralized and decentralized mechanisms. In the context of College Admissions, Luflade (2017) studies the value of information in the performance of the sequential use of truncated DA to allocate college positions in Tunesia. For the case of health professionals in Colombia, this chapter shows how, on top of using a centralized mechanism, a compulsory feature is central to successfully allocating health professionals to rural areas.

Fourth, it contributes to several influential theoretical results in the large literature of mechanism design. This is done by comparing several algorithms that have been proposed previously in terms of allocation efficiency. Erdil and Ergin (2008) propose an algorithm that eliminates the welfare reducing cycles that the coarse priorities may induce. I find small welfare gains from moving to that algorithm. There is a family of algorithms that can be described as adding rank priorities to the priorities determined by the Ministry of Health, which is described in detail in Chapter 3. In this family we can find the Immediate Acceptance (Kojima and Ünver, 2014) and the New Haven algorithms (Kapor et al., 2017). Within this family, I compare the efficiency of giving a priority to reported hospitals above any other priority has substantial welfare gains but at a cost of a having a significant number of the original priorities not being respected. I show that breaking the ties in favor of the health professionals who report a hospital as preferred is also within this family. However, this mechanism respects priorities while having welfare gains that range from $5\% - 9\%$.

These results can be seen as a contribution to the literature that measures the costs (gauged in terms of the welfare of health professionals) of the properties of the allocations that results from DA. Roth (2008) and Abdulkadiroğlu et al. (2017) do this in the case of the high school slots allocation in New York City. In sharp contrast to their findings, I find that because the preferences are completed (due to the fact that the rankings are complete), the fine tuning of the algorithm used to determine the allocation is a first-order concern.

Finally, it also contributes to the literature on merit/effort-based priorities in centralized allocations. This is exemplified by the case of the cadet branch-of-choice in the US Military Academy (Sönmez and Switzer, 2013). I show that under certain conditions merit based tie-break can entail welfare gains under certain conditions yet raise inequality concerns under others.

This chapter is organized as follows. In the next section I discuss the institutional details and the October 2014 allocation data. In Section 3, I detail the estimation and the results of a random coefficients model of the preferences of health professionals. Section 4 evaluates the performance of the program in the absence of the compulsory feature. In section 5, I describe the theoretical properties of centralized mechanisms and evaluate the allocative efficiency (in terms of health professional welfare) that would have occurred if other allocation mechanisms had been used. In section 6, I evaluate the consequences of moving from a random tie-break to a merit-based one. In section 7, I discuss the assumption that reported preferences are the true preferences and present evidence that support this is the case. Section 8 concludes.

2.2 Institutional Background

The Compulsory Social Service, hereafter CSS, of health professionals in Colombia was created to tackle the problems of inequality in the allocation of these professionals to rural areas and marginalized urban areas in Colombia. It started in 1949 for physicians, 1951 for dentists, 1971 for medical lab scientists, and 1971 for nurses. CSS lasts one year.

The inequality in the allocation of health professionals is not exclusive to Colombia and neither is the compulsory social service solution. Currently Bolivia, Costa Rica, Ecuador, Honduras, Nicaragua, Mexico[4], Peru, and El Salvador also require, at least for physicians, compulsory social service. All of these programs emphasize that the *social service* is a contribution to society. The programs differ primarily in whether the service must be undertaken before graduation (as part of their studies) or after graduation (as a first professional work experience).

The CSS in Colombia can be seen as mechanism to overcome the Rural Hospital Theorem effects of the country's health professionals market. In the matching literature the Rural Hospital Theorem predicts that under any stable allocation the set of unmatched hospital positions is the same. The name of this theorem reflects the fact that rural hospitals tend to have the highest rate of unmatched or vacant positions. When designing the centralized allocation mechanism, for the Ministry of Health it was a central objective that the mechanism *minimized the number of unfilled positions.*

A centralized mechanism was needed because prior to July 2014, the allocation mechanism was a system of state-level (departamento) lotteries marred by serious incentive and efficiency flaws. Specifically, the mechanism generally did not minimize the number of unfilled positions. In the July 2012 allocation, for example, almost 600 physicians were not allocated even though 300 positions were left unfilled. The

[4] In 1936, Mexico was the first country who implemented a compulsory social service for physicians. Today this program is known as *Servicio Social en Medicina*

main purpose of the mechanism, to allocate health professionals to rural areas, was not being accomplished.

If a health professional is not assigned, she is exempted from the CSS and can ask to be professionally certified. The incentive flaws occurred because health professionals, and specially physicians, were gaming the mechanism to avoid being allocated. Since October 2014, the allocation in Colombia has been decided and implemented by the Ministry of Health, which uses a centralized mechanism. This mechanism, whose design was guided and implemented by economists, implemented a version of Gale and Shapley (1962) deferred acceptance algorithm. Chapter 1 discuss in detail the motivations needed to implement Gale and Shapley's (1962) deferred acceptance algorithm as well as the incentive flaws of the previous mechanisms.

Professionals can apply for three types of positions in the CSS. First are positions in rural areas (at least 70% of the positions). Second are positions in marginal urban areas (at most 25% of the positions). Third, are research positions (at most 5%). The first two types can only be obtained through the centralized mechanism. Investigation positions are assigned directly by the entities that conduct the investigations, and professionals do not participate in the centralized allocation. To determine the allocation, the Ministry of Health imposes a priority order that needs to be satisfied by every hospital. Consequently, hospitals in this setting are not strategic agents. Thus, the allocation is a one-sided problem known in the literature as *school choice* (Abdulkadiroğlu and Sönmez, 2003).

Health professionals are given three types of priorities that define the priority order under which they are accepted at each hospital. In the first type (which corresponds to Colombian law), a professional who is indigenous is given priority at hospitals located in indigenous regions. Similarly, a professional who is "raizal" (a native of the Archipelago of San Andres, Providencia, and Santa Catalina) is given priority at hospitals in the Archipelago. In the second type the priorities follow from Health Ministry regulations. Specifically, priorities are given to professionals who 1) are mothers with small children, 2) have impairments or disabilities, or 3) need special medical treatment. The third priority type takes into account hospital's regional preferences: hospital prefer professionals who graduated from their state or who were born in their state. Within this system Colombian law takes precedence over the Ministry's regulations and the latter take precedence over hospital preferences. Hence, the priority structure is as follows:

In summary, the priority structure is the following:

- Being indigenous,
- Being a "raizal" (native of Archipelago of San Andrés, Providencia, and Santa Catalina),
- Being pregnant,
- Being a mother or a father who has small children,
- Being a professional who has impairments or disabilities,
- Being a professional who needs special treatment,
- Graduating from a University from the same state as the hospital,
- Being born in the same state as the hospital.

Under this system the priority order would work as follows. Assuming that health professionals will qualify for no more than one of the Ministry-regulation priorities, pregnant women will have the top priority at all hospitals that do not grant priorities given by the Colombian law. In any given state, pregnant women who were born in and who graduated in that state will be placed first, while those who graduated in but were not born in that state will be placed next. Finally, those who were born in that state but did not graduate in it will be placed third in the priority order.

In all of the clearinghouses previously studied in the literature, having a higher priority is good news for each participant. In this case it is not necessarily the case since having a very high priority results in a high chance of getting assigned and if a health professional wants to avoid being allocated at all she will have no chance of doing so. Hence, as there are more participants than positions, having a higher priority has a higher chance of meaning bad news for the health professionals. Also, notice that to hire a pregnant professional might not be on the best interest of the hospitals. As such, some of the priorities go counter to the interest of the hospitals, another difference with what has been studied in the literature.

Health professions who participate in the CSS belong to one of four professions[5] Medical Lab Scientist[6], Medicine, Nursing, and Dentistry. To be professionally certified, professionals in these fields need a valid professional ID card that is authorized by the Ministry of Health. The social service is compulsory because to

[5]This health professions are all undergraduate studies.

[6]In Colombia this profession is known as *Bacteriology and Clinical Lab* and professionals in this area are usually referred to as simply *bacteriologists*.

become certified, health professionals must participate in the CSS.

The Ministry allocates the positions on a quarterly basis. As mentioned earlier, professionals who are not assigned can ask to be professionally certified immediately. If a professional rejects her allocation, she is not allowed to participate in the centralized allocation during the next two allocations (i.e., she will need to wait 9 months before participating again) and because she is not certified, she cannot work in her profession during that period. A 9 month penalty is a high penalty considering that it is for a 12 month position. However, it has been observed that about $3-6\%$ of the health professionals reject the position they are assigned to.

The mechanism incorporates the *compulsory* characteristic by not allowing the professionals to report an outside option in addition to the penalty for not accepting a position. This is due to the fact that if there is an equal number of professionals and positions, allowing professionals to report an outside option could result in positions being left unfilled. As mentioned earlier, one of the objectives of the mechanism is to minimize the unfilled positions. Hence, the mechanism is also compulsory in the respect that a professional cannot simply accept an outside option and pay a penalty for it.

A health professional can choose any position that currently is unassigned or that will become available before the next allocation. However, because Colombia has a large number of hospitals, professionals are required to report up to five preferred hospitals. They also are asked to rank the states in which they would like to fulfill their CSS obligation. Thus, their ranking of hospitals is constructed as follows: preferred hospitals are placed at the top; then all hospitals in each reported state (excluding those reported as preferred) are randomly allocated in the professional's ranking following the state's stated order.

Ties in the ranking of Professionals (Hospitals) in the Hospital's (Professional's) rankings (priorities) are broken using a single tie-break. Using these rankings and priorities the allocation is determined using the *Deferred Acceptance*-Algorithm, described in Chapter 1.

The mechanism rests on the assumption that no professional is willing to incur the penalty of not being able to work in his or her profession for 9 months. This a substantial penalty if one takes into account that the assignments are due for one year. As shown in Chapter 1, the restriction on preferences under this assumption does not deteriorate the mechanism's strategy-proofness because the weakly dominant strategy is for each professional to report their true restricted preferences (i.e., report their true preferences with the restriction that the outside option is placed last). This is true even in the limiting case in which no positions are acceptable for a health professional. This is due to the fact that, as shown below, since the allocation is determined using DA, she needs to be rejected by all of the hospitals at some point in the algorithm.

2.2.1 October 2014 Allocation

In October 2014, a total of:

- 194 Medical Lab Scientists participated for 83 positions in 81 hospitals.
- 708 Dentists participated for 109 positions in 97 hospitals.

- 386 Physicians participated for 1025 positions in 544 hospitals.

- 828 Nurses participated for 194 positions in 148 hospitals.

Note that the ratio of professionals to available positions varies by profession. In the case of physicians, for example, there are more than 2 positions available per physician (a case of very low congestion), whereas in the other professions the opposite is true. Dentists, for instance, face the most congested allocation, with over 7 participants per position. The reason why there are so many positions available for physicians is the mechanism used before the centralized one was being manipulated to avoid allocation.

I obtained secure access to administrative data for all health professionals that participated in the first allocation that used the centralized mechanism. For each health professional, I have data on the hospital's reported ranking (up to five). I have information on the state where the professional was born, the state where he or she graduated, and her or his gender. Although I have no data on professionals' exact city of birth, most of Colombia's biggest cities (where 75% of the country's population resides) are state capitals, and most universities where a health professional degree can be obtained are located in these cities. Therefore, I use the capital of the state as a proxy for location. From this I construct key variables, such as the distance from the hospital to the capital of the state of graduation or origin.

The webpage interface used by the Ministry of Health forces professionals to report five positions. However, the interface also allows participants to report the

	Med-Lab	Nurses	Med-Docs	Dentists
Female (%)	80	91	58	76
Reports St. (%)	92	89	94	91
Repeats Reported Pos.(%)	6	8	8	7
Diff. St. of Grad (%)	43	25	34	34
Dummy Priority (%)	6	13	10	9
Total Num. of Prof. with Priority	12	104	39	64
Num. Priority Raizal	0	0	2	2
Num. Priority Indigenous	2	11	2	6
Num. Priority Pregnant	7	50	25	31
Num. Priority Mother	1	19	6	9
Num. Priority Father	1	2	3	9
Num. Priority Handicap	1	12	4	9
Num. Priority Safety	0	0	0	1

Table 2.1: Professionals Characteristics by Profession

same position more than once. Indeed, about 7% of participants report a preferred position more than once, presumably in an attempt to game the system. But this choice does not give participants an extra chance to receive their preference, nor does it allow them to have a full five choices. Moreover, when reporting the ranking of states, these by default were ranked in alphabetical order. Around 8% of the participants decided not to modify the displayed ranking. I observe that the different professions differ mainly in terms of the share of participants who are female. The profession with the most females is Nursing (91%), whereas Medicine has the lowest share (58%).

In the case of non-regional priorities, the one most observed is being pregnant. I observe that Nursing has the highest share of professionals who have non-regional priorities (13%), while among Medical Lab scientists this share is only 6%. Hence,

	Med-Lab	Nurses	Med-Docs	Dentists
Rural (%)	71	78	85	86
Assigned (%)	73	75	79	87
Dist. Capital (km)	89	66	70	77
Compensatorio (%)	20	20	12	14
Coca (%)	25	32	19	26
Quota	1.02	1.21	1.88	1.13

Table 2.2: Hospital Characteristics by Profession

the priority structure for all of the professions is quite coarse because most participants will only have a regional priority. Moreover, participants who were both born and raised in the same state are given precedence in that state over those who were not. Because of the manner in which the priority structure was developed, subjects who have any of the aforementioned priorities will compete among themselves for all of the positions. Then the subjects who do not have any priority will compete among themselves for the remaining positions, as seen in Table 2.1.

With regards to hospital characteristics, I observe that about 80% of positions were occupied at the time of the allocation–that is, they will become available during the next three months. In all cases, a significant number of the positions are located in state-capital cities (18%). I also observe that positions on average are located 79km from their capital. Moreover, around 25% are located in counties where coca was produced in 2013. Additionally, 16% of the positions offered a *compensatorio* day, or an additional paid rest day mandated by Colombian law for employees who work four consecutive weekends, as seen in Table 2.2.

Hospitals need authorization from the Ministry of Health to open a CSS position. When they do so, hospitals offer for that position a wage that is set in reference to the Colombian Monthly Minimum Wage. There is no negotiation over wages: the Colombian Monthly Minimum Wage is set on a yearly basis and wages are automatically updated so that there is no need to reapply for the position (an administrative process that can take several months). Moreover, when a professional applies to the CSS he or she observes a list of available positions and the wage offered for each. In other words, during each quarterly allocation, wages are exogenously determined. A histogram of the wages is shown in Figure 2.1. In 2014, the Colombian Minimum Monthly Wage $616,000 COP, approximately $300 USD. As is evident from the histogram, there is a significant dispersion on the wages offered. Most of the hospitals are public state institutions and as such the wages belong to the state budget.

Because professionals choose both the hospital where they will work and the county where they will live, the characteristics of the counties in which hospitals are located play an important role in hospital choice . The Municipal Panel Data CEDE[7] provides demographic information about the counties in which the hospitals are located. I find that both geographic location and the Unsatisfied Basic Necessities Index, hereafter UBN, which is a measure of poverty in each county, affect hospital choice.

2.3 Estimating Health Professional's Preferences

The Ministry of Health publishes on a webpage information about available positions, and it includes both an email address and a telephone number where

[7]See Acevedo et al. (2014) for a description of the Municipal Panel Data CEDE.

Figure 2.1: Histogram of wages offered by hospitals (in Colombian Minimum Monthly Wage)

questions can be answered[8].

Health professionals' preferences exhibit consistent regularities. They prefer hospitals close to their current location, higher wages, and hospitals in wealthier counties (as measured by the UBN-index), as shown in Table 2.3. Overall more than 90% of health professionals report at a least 3 hospitals. Table 2.3 shows the regularity of health professionals' preferences.

Figure 2.2 shows a histogram of the distance to the first reported hospital from the capital of the state where they were born[9]. From it we can observe that despite

[8]For a detailed version of the instructions (in Spanish) go to `https://tramites.minsalud.gov.co/tramitesservicios/DefaultSSO.aspx`-Accessed May 2018

[9]In the Appendix a similar histogram is shown for the distance to the first reported hospital from the capital of the state of graduation

	Avg.	1st	2nd	3rd	4th	5th
Med-Lab ranking choice (%)		100	96	96	96	91
Mean Dist. Graduation (km)	446	202	224	264	299	310
Mean Dist. from St. of Origin (km)	476	181	222	245	287	302
UBN index	49	40	46	48	44	44
Wage (MMW)	3.2	3.29	3.33	3.24	3.22	3.22
Same State of Graduation (%)	4	47	39	24	18	8
Same State of Origin (%)	4	56	44	33	20	17
Nurses ranking choice (%)		100	97	90	76	55
Mean Dist. Graduation (km)	488	123	158	172	195	238
Mean Dist. from St. of Origin (km)	493	132	164	178	203	252
UBN index	42	27	30	31	32	33
Wage (MMW)	3.0	3.20	3.12	3.09	3.13	3.11
Same State of Graduation (%)	4	66	56	46	41	34
Same State of Origin (%)	4	64	53	46	39	31
Med-Docs ranking choice (%)		100	95	87	72	54
Mean Dist. Graduation (km)	405	106	106	120	146	161
Mean Dist. from St. of Origin (km)	428	128	139	153	160	188
UBN index	41	23	24	25	24	25
Wage (MMW)	4.49	4.73	4.73	4.68	4.84	4.86
Same State of Graduation (%)	4	70	67	63	55	51
Same State of Origin (%)	4	65	59	54	52	47
Dentists ranking choice (%)		100	97	96	93	72
Mean Dist. Graduation (km)	428	170	183	237	254	285
Mean Dist. from St. of Origin (km)	443	165	182	241	250	284
UBN index	44	34	35	36	37	36
Wage (MMW)	3.6	3.82	3.76	3.82	3.82	3.80
Same State of Graduation (%)	4	51	40	25	20	20
Same State of Origin (%)	4	58	44	30	24	21

Table 2.3: Preferences regularities of Health professionals by Ranking

having a high average, more than $120km$ for all health professions, the preference for distance is highly skewed and most of the health professionals prefer to be close to the capital of the state where they were born. Overall we observe that the preference for location, in the sense of being either close to the capital city of the state they were born or the city where they graduated from, is stronger than the preference for wage. Moreover, from Table 2.3 we can observe that the reported hospitals wage is not decreasing while it increases strongly in distance. Nevertheless, health professionals tend to prefer hospitals that pay wages higher than the average.

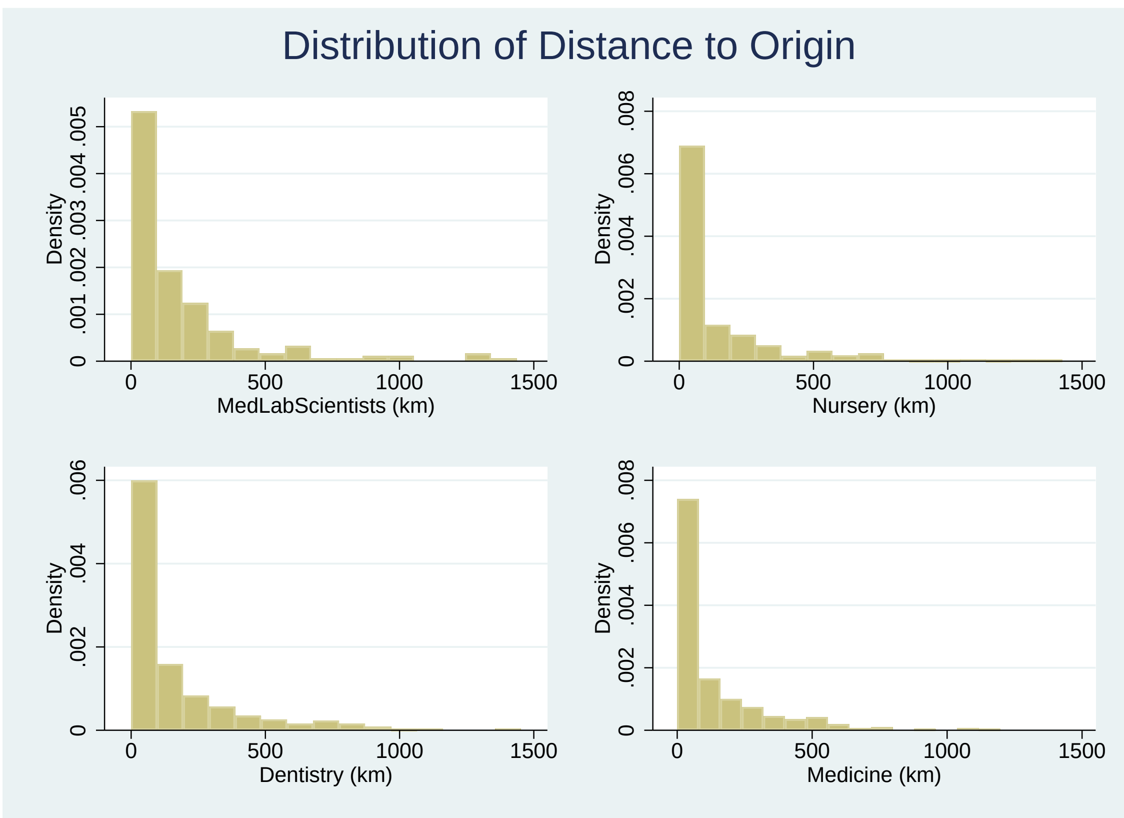


Figure 2.2: Travel distance from hospital to capital of state of origin city (km)

2.3.1 A Random Coefficients Model of Preferences

To model preferences for rural hospitals I use a standard random utility framework. Denote with U_{ij} the utility of health professional i from performing the CSS at hospital j. I assume that health professionals choose the hospitals that maximize their utility. Their utility is assumed to depend on the wage offered by the hospital W_j, the distance to their place of graduation D_{ij}, and a set of characteristics of the county where the hospital is located C_j, resulting in the following utility:

$$U_{ij} = W_j + \beta_i^D D_{ij} + \beta_i^H H_j + \beta_i^C C_j + \epsilon_{ij} \tag{2.1}$$

Where β_i^D, β_i^H, and β_i^M represent the weight that health professional i places on the distance to the town of origin[10], the characteristics of the hospital, and the characteristics of the county where a given hospital is located, respectively. In the estimation, these variables are interacted with the gender of the health professionals. Thus, I allow different preference distributions and correlations on the weights by gender.

I use the same weight on wage for all health professionals in order to find utilities in a common numeraire, namely nominal wage. In particular, the wage coefficient is constrained to be equal to 1. Therefore, all else being equal, since all health professionals are likely to like higher wages, treating the wage as numeraire is a scale normalization. To estimate their different weights for distance to the town of origin, and hospital's town characteristics, I use the reported choices of the

[10] This distance was calculated using the *geocodeopen* command in Stata (Anderson, 2013).

health professionals and a random coefficients model (McFadden and Train (2000), Train (2009)). Hence, this coefficients allowed to be random in order to allow for heterogeneity of preference in the choice behavior.

In the data set most health professionals report several hospitals. Each hospital ranked by each health professional constitutes an observation. The first observation for each health professional indicates when the first ranked alternative was chosen among all alternatives. The second(nth) observation identities the case in which the second alternative was chosen when the first (previous n-1) chosen alternative(s) was (are) removed. Removing the previously chosen alternatives is important because it creates variation in the choice set. This variation allows a better prediction of both the agents' utilities and the ranking beyond the reported choices. This is so because when a health professional makes multiple choices that share a common attribute, I can infer that the individual has a strong preference for that attribute because independence in the additive error terms across choices makes observing such a pattern very unlikely.

I assume that ϵ_{ij} in equation (2.1) is a distributed i.i.d. extreme value and captures the idiosyncratic tastes for hospitals. The vector of coefficients β follows a multivariate normal mixing distribution and therefore allows for preference heterogeneity by allowing random coefficients. A dummy variable for each state is included in the county's characteristics. The distribution of the weights of these variables is constrained to have a zero mean. I include these variables to allow for a correlation between hospitals within the same state, which, in turn, allows for a rich variety of distribution patterns. This is analogous to a nested logit in which states define mutually exclusive nests.

The main assumption in this estimation is that conditional on observed hospital and county characteristics, unobserved idiosyncratic tastes are independent on wages. A violation to this assumption may occur if, given the controls, health professionals prefer hospital offering higher wages. It is unlikely in this case since, as shown in Table 2.3, preferences regularities show that the preferences for location are stronger than those for wage. Also, the wages are published by the Ministry of Health and there is no possibility of negotiation regarding the wage. Therefore, the wage is orthogonal to the health professionals' characteristics.

Estimation was carried out using a hierarchical Bayes procedure. Table 2.4 presents the results of the random coefficients estimation of health professional preferences for rural hospitals for a selection of the variables. The distribution of the weights of the coefficients are consistent with expectations. They show that health professionals prefer higher wages, being closer to their state of graduation, and wealthier counties. In all cases there is significant heterogeneity among professionals. Moreover, the estimates show that female health professionals have a stronger preference for staying close to the capital of the state in which they were born. In the appendix the complete table of the means and standard deviation of the weights is shown. There it can be observed that the coefficients of the standard deviations of the dummy variables of states show that there is a significant heterogeneity in preferences for states. This indicates that the decision by the Ministry of Health to ask for a ranking of states produced a relatively good description of the rankings.

2.3.2 Measuring Welfare with the Estimates

The estimated model provides the posteriors of the weights for each health professional. With these posteriors I estimate the utility of each health professional at each hospital. However, these estimated utilities need not result in the same ordinal order implied by the reported preferences. The reported preferences then introduce restrictions in the unobserved terms ϵ_{ij}.

Accurately incorporating the restrictions implied by the reported preferences plays a central role in the welfare analysis that I develop below. Previous studies, such as Abdulkadiroğlu et al. (2017), reveal that higher-ranked alternatives tend to have the highest unobserved terms. Hence the unobserved role plays an important role in explaining the observed preferences. In the counterfactuals I use reported preferred and not reported hospitals utilities. Therefore, it is important to incorporate the fact that reported hospitals will have a high unobserved term and that the not reported ones will have a low one. This is done in order for the welfare analysis not to be driven by the unobserved terms.

In this research I estimate unobserved terms by means of placing restrictions on the bounds of each ϵ_{ij}. The bounds are set in order to maximize the probability of observing the reported ranking. Denote with $\hat{U_{dj}}$ the estimated utility of health professional d at hospital j. The constraints imposed by the reported preferences cause a hospital ranked in the j-th position among N alternatives to satisfy:

$$\hat{U}_{dj+1} + \epsilon_{dj+1} > \hat{U}_{dj} + \epsilon_{dj} > \hat{U}_{dj-1} + \epsilon_{dj-1} \tag{2.2}$$

		Med-Lab	Nursing	Med-Docs	Dentists
Means					
Wage					
		1	1	1	1
Travel Distance Origin					
	F	-1.413	-1.863	-2.455	-1.444
	M	-1.192	-1.417	-1.798	-1.413
UBN-Index					
	F	-0.056	-0.114	-0.141	-0.083
	M	-0.023	-0.108	-0.118	-0.109
Coca					
	F	-1.419	-0.644	-1.095	-1.124
	M	-0.147	-2.013	-0.997	-0.338
Living Place					
	F	0.866	0.256	0.577	0.587
	M	0.163	0.882	0.179	0.541
Weekends Shift					
	F	-2.296	-0.555	-0.176	-1.231
	M	-0.001	-0.382	-0.114	-0.031
Std. Deviation					
Travel Distance Origin					
	F	1.506	1.427	2.430	1.244
	M	1.875	2.202	1.542	1.455
Travel Distance Graduation					
	F	1.265	2.068	1.862	1.131
	M	1.358	2.157	2.736	0.237
UBN-Index					
	F	0.237	0.667	0.178	0.261
	M	0.214	0.603	0.191	0.470
Coca					
	F	1.630	1.102	1.940	1.252
	M	0.568	2.291	0.875	1.012
Living Place					
	F	1.111	0.663	0.491	1.016
	M	0.585	1.254	0.418	0.942
Weekends Shift					
	F	2.276	1.028	0.414	1.297
	M	0.037	0.779	0.329	0.202

Table 2.4: Preference Estimates of Health Professionals

Therefore, the ranking constraints the possible values the error term ϵ_{dj} can have. Let $\underline{\epsilon}_j(\bar{\epsilon}_j)$ denote the lower (upper) bound of ϵ_j. The lower (upper) limit of ϵ_j will be bounded by the upper(lower) limit of ϵ_{j+1} (ϵ_{j-1}) as implied by the following restriction:

$$\hat{U}_{dj+1} + \underline{\epsilon}_{dj+1} = \hat{U}_{dj} + \bar{\epsilon}_{dj} \tag{2.3}$$

With N alternatives ranked, there are $2(N-1)$ variables because the first (last) ranked alternative has no restrictions on the upper (lower) bound. Moreover, there are $N-1$ restrictions like the one mentioned above. Thus, the optimization problem in order to find the maximum probability of observing the ranking is:

$$\max_{(\underline{\epsilon}_{di}, \bar{\epsilon}_{dj})} \log Prob(\epsilon \geq \underline{\epsilon}_1) + \sum_{k=2}^{N-1} \log Prob(\bar{\epsilon}_k \geq \epsilon \geq \underline{\epsilon}_k) + \log Prob(\epsilon < \bar{\epsilon}_N) \tag{2.4}$$

After having estimated the boundaries for each unobserved term (i.e., the restrictions implied by the ranking order of the alternatives), I estimate the expected value of each term and add them to the expected utilities. These are the final estimates that will be denoted $E\left[u_{ij}|r_i, H_j, C_j, \beta_i\right]$ (the expected utility of professional i at hospital j given his reports r_i), and they are the estimates that will be used to estimate the counterfactuals.

Denote with $E\left[u_{ij}|r_i, H_j, C_j, \beta_i\right]$ the expected utility of professional i at hospital j given his reports r_i, the hospital's characteristics H_j, the county's characteristics C_j, and the estimated coefficients of the professional β_i. Also denote with $\mu : \mathcal{P} \rightarrow \mathcal{H} \cup \mathcal{P}$ a matching, such that each professional is assigned either to a single hospital or to itself. Define the estimated average professional welfare as:

$$\bar{W}(\mu) = \frac{1}{|\mathcal{P}|} \sum_{i \in \mathcal{P}} E\left[u_{ij} | r_i, H_j, C_j, \beta_i\right]$$

.

The difference between average professional welfare between two matchings, μ and μ', is: $\bar{W}(\mu) - \bar{W}(\mu')$.

2.4 Individual Rationality vs Compulsion

In this section, I test whether the efficiency improvement is high enough to make the compulsory feature unnecessary. In other words, I test what would happen if the allocation was based on the individual rationality that the single-offer centralized allocation mechanisms are built upon. I do this by exploiting the allocation mechanism changes carried out from 2012 to 2014.

In section 2, I characterized the single-offer centralized mechanism implemented since October 2014. Prior to October 2012, a decentralized system was used to determine the allocation. Each health professional enrolled in a state, and among the enrolled health professionals, each state conducted a lottery to determine the allocation. If in a given state there were more health professionals than participants, the unassigned professionals were exempt from the CSS. In the final allocation that used this decentralized mechanism, 135 (out of 380) of the physicians enrolled in a state that had only 2 positions available, causing 133 of them to be exempt.

During the previous two allocations, others applied to the same state with the goal of not being assigned. In the April (June) 2012 allocation, a total of 74 (225) physicians applied for 6 (7) positions. Because this strategy was used only in the allocation of physicians, I focus on that profession. Excluding the state just mentioned, 35 states participated in the allocation, and professionals were exempt from only 3 of them. Forty-two (42) health professionals were exempted, 39 of whom came from one state: Valle del Cauca. However, Valle del Cauca had a significant number of positions available (52) and many graduates in the 2014 allocation. Thus, I assume that these graduates were not attempting to avoid the allocation.

In 2013, the Ministry of Health established enrollment limits in each state. The number of enrolled health professionals could not double the number of positions. This constraint reduced significantly the number of unfilled positions. However, the fact that there were more exempted physicians than unfilled positions motivated the ministry to design the centralized mechanism. Due to the presence of this constraint, I use the 2012 data rather than the 2013 data to find the marginal utility for physicians of accepting a position.

A similar number of physicians participated in the October 2012 and the October 2014 allocations: 382 in the former and 386 in the latter. Because I do not observe detailed data on participants in the 2012 allocation, I examine the 2014 participants. I assume that the characteristics of the two cohorts are significantly similar. To make the 2014 allocation more closely resemble the October 2012 situation, I randomly choose the participating positions in the 2014 allocation, when 213 more positions were available, in order to have the same number of positions in each state.

I simulate the outcome that would have occurred if the decentralized mechanism used in 2012 had still been in use in October 2014. I model the decision of physicians of enrolling to their preferred state (in order to work for one year) or to avoid being allocated[11] To this end I use information about the rankings of states submitted by physicians. For those physicians who did not modify the default ranking of states, and who, therefore, reported them in alphabetical order, I use their state of origin as the preferred state. I assume that had the decentralized mechanism still being in place and that in the lottery physicians would have enrolled in their reported preferred state. I do not find significant differences when I include the state of graduation.

I estimate the average marginal utility a physician would require in order not to avoid being allocated. I do so by making, in equilibrium, the number of physicians that decide to avoid being allocated the same as the ones that applied to the state that had 2 positions in 2012. I estimate this marginal utility to be 1.91 Colombian Monthly Minimum Wages. Notice that I assume this marginal utility remained the same in 2012 and 2014. Given that the percentage of the PIB spent on health in Colombia between 2012 and 2014 increased from 4.47% to 4.64%, the aforementioned value is most likely to have increased.

[11] I use the simplifying assumption that when a health professional applies to the state with 2 positions with a probability of 1 she is not assigned.

I then simulate the outcome of the October 2014 allocation if there had been voluntary participation–that is, if the physicians had been able to apply only for positions that yielded them a utility higher than the aforementioned acceptance threshold. I find that about 115 (30%) of the physicians would have been unassigned. Notice that the number of physicians that would be left unassigned under the centralized mechanism (115) is similar to the number of physicians that strategized to avoid being allocated (133). This is so despite the fact that the average utility of the assigned physician went from 2.95 Colombian Monthly Minimum Wages to 6.02 Colombian Monthly Minimum Wages. In other words, for the policy to be successful assignments need to be mandatory.

Notice that this counterfactual has been carried out in an allocation that has low congestion, when about 3 positions were available for each physician. Allocations conducted in January and July tend to be much more congested. A higher congestion is likely to cause a lower utility per allocated physician. Hence, the compulsory feature is important for the program's success.

Another way to motivate physicians to participate in the hospital allocation is to increase wages. Figure 2.3 shows the number of physicians who would be allocated if the wage was increased optimally for the vacant positions. The term "optimally" refers to the fact that filling the first position comes at a lower cost than filling the second one. The results show that an average increase of 600 USD (2 Colombian Minimum Monthly Wages) would cause 35 more physicians to be allocated. This would entail having about 82 physicians not being allocated while having many options to choose from. Therefore, the alternative to making compulsory the

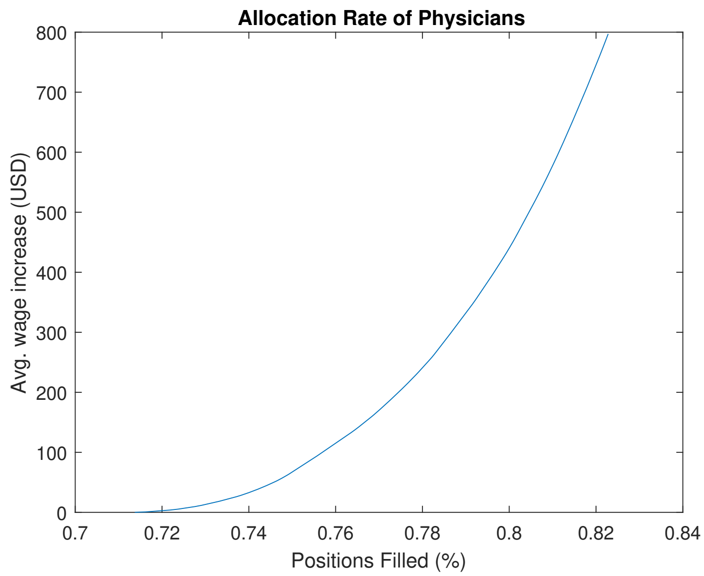


Figure 2.3: Increase of Physicians Allocated with Wage

participation of raising optimally the wages implies a very high cost.

Increasing the wage is also a competing way to motivate physicians to be allocated to the positions in the participating hospitals. Figure 2.3 shows the number of physicians that would be allocated if the wage was increased optimally for the vacant positions. Optimally refers to the fact that filling the first position comes at a lower cost than filling the second one. The results show that an average increase of 600USD (2 Colombian Minimum Monthly Wages of the time) would result in 35 more physicians being allocated – resulting in an allocation rate change of 10%. This would entail having about 82 physicians not being allocated while having many options to choose from. Therefore, the alternative to making compulsory the participation of raising optimally the wages implies a very high cost.

2.5 Comparing Alternative Mechanisms

The CSS's design, as a centralized mechanism, was built on the assumption that the cost of rejecting a position was so high, that no health professional would do so. However, in practice it has been observed that, depending on the profession, about $4\% - 6\%$ of the positions are rejected. Hence, improving the welfare of the health professionals will also have the purpose of reducing the aforementioned rate of rejections. I incorporate the fact that positions are rejected on the welfare estimates by calculating the outside value of rejection in order to have, on average, the same rejection rates in each profession as observed.

In this section, I will compare the welfare in terms of efficiency for the health professionals and estimated rejection rate of different algorithms. Then I will discuss the welfare implications of changing the total number of hospitals in the ranking of health professionals. The properties and trade-offs in matching were already described in Chapter 1.

In the physians case, the fact that there are almost 3 positions per each professional makes the allocative analysis futile. This is due to the fact that a high share of health professionals is getting what they want. Therefore, in this section I will focus on the efficiency analysis of the other professions, i.e. the ones that were significantly congested. For those professions, in contrast to the results reported by Abdulkadiroğlu et al. (2017) and Abdulkadiroğlu et al. (2009) that conclude for their settings that any algorithm modification has a second order relevance when compared to DA, I observe that in this case there are significant welfare gains from the fine-tuning of the mechanism.

The following example illustrates a case in which having one more professional than available positions precipitates significant welfare costs for the allocated health professionals:

Suppose there are 4 health professionals, denoted with $hp_{i \in 1-4}$, and three hospitals with one positions each, which are denoted with $H_{i \in 1-3}$ with the following preferences and priorities:

P_{hp_1}	P_{hp_2}	P_{hp_3}	P_{hp_4}	Pr_{H_1}	Pr_{H_2}	Pr_{H_3}
H_2	H_3	H_2	H_2	hp_3	hp_2	hp_1
H_1	H_1	H_3	H_1	hp_4	hp_4	hp_4
H_3	H_2	H_1	H_3	hp_2	hp_3	hp_3
				hp_1	hp_1	hp_2

The allocation determined by using DA is: $(hp_1, H_3)(hp_2, H_2)(hp_3, H_1)$ and leaving hp_4 unassigned. However, had hp_4 not been present, the allocation would have been: $(hp_1, H_1)(hp_2, H_3)(hp_3, H_2)$. This allocation is strictly preferred by every allocated health professional. Therefore, the costs of congestion can be substantial.

2.5.1 Competing Mechanisms

I compare the welfare output of different algorithms, all of which satisfy the minimizing unfilled positions property. The results derive from simulating the tie-breaks and determining the allocation with them 100 times. The implemented mechanism is based on the DA algorithm, which is known to be strategy-proof and efficient.

If a priority structure is strict, then it would Pareto-dominate any other allocation that respects priorities; however, this expectation does not necessarily hold in this case because the priority structure is significantly coarse (Erdil and Ergin, 2008). A policy maker that is willing to sacrifice some strategy-proofness for efficiency, would use the SOSM[12]. This algorithm improves the allocation upon the one resulting from DA by removing the welfare-decreasing cycles induced by use of the tie-break. I find that the SOSM yields few welfare gains for all professions. This is so because I use a single-tie break rather than multiple ones. The Appendix describes other mechanisms under both single and multiple tie-break scenarios.

The second mechanism I compare to the implemented mechanism uses the TTC algorithm to determine allocation. This approach makes each allocation efficient and maintains the strategy-proofness property. Each subject who has the highest priority at a hospital is endowed with a seat and is allowed to trade positions. The trade does not take priorities into account, and, consequently, priority violations are allowed in the resulting allocations. For example, two health professionals with the highest priorities at a hospital may exchange their positions at that hospital even thou they have the lowest priority in the hospital they are receiving. Indeed, we observe that violations do occur and they involve on average 13-27 professionals. Comparing these two mechanisms reveals that in this scenario with strategy-proof mechanisms, due to the aforementioned heterogeneity of preferences,

[12]SOSM stands for the *Student Optimal Stable Match*. In this setting this algorithm produces a *Health Professional Optimal Respecting Priorities Match*.

there is no trade-off between efficiency and respecting priorities (Abdulkadiroğlu and Sönmez, 2003). The welfare gains achieved from moving to this mechanisms are not significant. However, the rejection rate is reduced by in 1 unit.

In a unique characteristic of the implemented mechanism, preferences are completed for each health professional using the reported ranking of states. As noted below in a discussion of welfare analysis, this fact generates significant competition between the health professionals for positions. Furthermore, imposing the strategy-proofness condition with completed preferences comes at a first order cost in welfare. The next mechanisms proposed are designed to limit competition for positions between health professionals in order to obtain significant welfare gains. They belong to a family of mechanisms that uses ranked-priorities, i.e. a priority given to a health professional because of where she ranked the hospital. The proposed mechanisms yield higher welfare than the TTC and the first one respects priorities. We conclude that in this case the strategy-proofness property is much more costly in terms of efficiency than the respecting priorities property.

The third proposed mechanism is to break ties in favor of the professionals who report a hospital. This modifies how the mechanism handles ties in priority, and it meets the respect priorities criterion. Regarding incentives to report the truth, a professional might want to misreport his preferences in order to get an exemption, yet this action could induce a strategic behavior equivalent to the one that occurs when truncated lists are reported Haeringer and Klijn (2009). A position undesired by all professionals cannot be avoided. The welfare gains from moving to this mechanism range from $6\% - 8\%$. Notice that this mechanism respects all priorities. As of the rejection rate, it is predicted to be reduced to 2%.

The final proposed mechanism uses two rounds. In the first round, the set of allocations uses only the reported hospitals. In the second round, the remaining hospitals use the state rankings. This mechanism presents professionals with the highest incentives to manipulate: if a professional avoids being matched in the first round she is very unlikely to be matched in the second. This mechanism is a version of the *Parallel DA* mechanism used in China to determine college admissions (Chen and Kesten, 2017). The welfare gains achieved by moving to this mechanism can be as high as 55%. Again, this gain carries the cost that occurs when the priorities of many health professionals are not respected (as is the case of Nursing with around 101). Table 2.5 summarizes the results when these mechanisms are compared.

In contrast to the results reported by Abdulkadiroğlu et al. (2017) and Abdulkadiroğlu et al. (2009) who conclude that for their settings any algorithm modification has a second-order relevance compared to DA, I find that when the mechanism is fine-tuned, significant welfare gains are achieved.

2.5.2 How many hospitals to report?

In the alternative mechanisms discussed in the previous section, allowing reported preferences to have an edge produces significant welfare gains. In this section I explore what would happen if a truncated DA was used–a condition that is consistent with the majority of school choice applications, whose reports need to be truncated [13]. In this application professionals were asked to report up to five hospitals. The number was not decided using a technical motivation.

In the next set of counterfactuals, the change in welfare is conditional on the number of hospitals that professionals are allowed to report using DA with truncated reports. In all of these cases I assume that professionals report truthfully–that is, when allowed to report N hospitals, they report their preferred ones. Therefore, when they are allowed to report five (5) or less, I use their reported hospitals. When professionals report more than the reported number, I use those that have the highest estimated welfare. Figure 2.4 summarizes the results. Interestingly, due to the fact that the physicians allocation was not congested, their utility increases with the number of positions. For the other professions, the number is four or five.

There is a trade-off in hand with the number of positions reported. On the one hand, the higher the number of hospitals reported the higher the share of positions allocated. On the other, the higher the competition between health professionals for these positions and hence the lower the welfare of the allocated health professionals. Due to the preference heterogeneity and the congestion of some markets, the maximum is achieved at a low number of positions for the very congested markets (Nursing and Dentistry) while achieving it at a high number for the other ones.

In most of the environments where DA is used, there is a truncation of possible reports. To reduce the cost of congestion while allocating all positions, I propose the following modification of the termination rule of the deferred acceptance algo-

[13] Rumania and Boston are the only cases I know of in which the number of options that can be reported is unrestricted. In the former, the mechanism employed uses a serial dictatorship to make the allocation

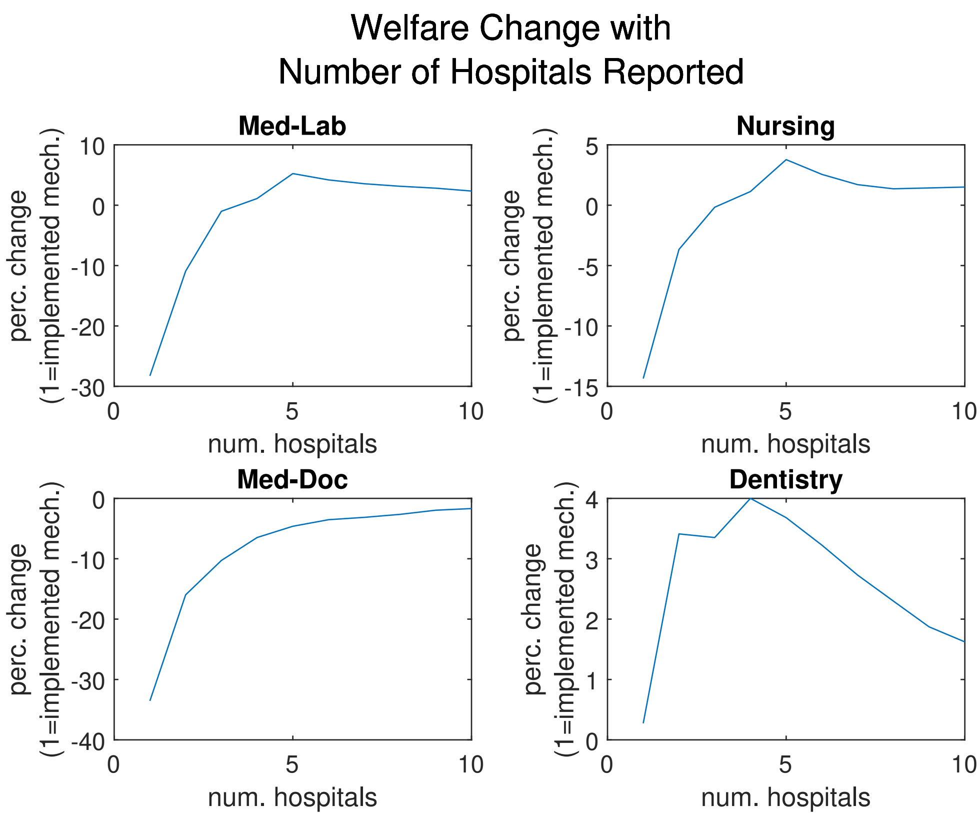


Figure 2.4: Average Change in Utility with Number of Hospitals Reported

rithm: The algorithm terminates when there are no new proposals, when all the positions are enrolled, or when all rejected students have exhausted their ranking. This mechanism is not strategy proof: a professional may want to modify his reports in order to precipitate an early termination of the algorithm. However, it would be difficult to manipulate because the odds of inducing early termination are roughly one over the number of professionals. This modification is advantageous because under it exempt professionals would not need to "apply" to and be rejected by each hospital. That said, the mechanism does not respect priorities; it could end a health professional's exemptions before she is given the chance to a apply to hospitals in which she has a high priority. The modification of the termination rule would result in welfare increases of $8-9\%$ and a significant drop in rejection rates.

Another mechanism can also be applied to this problem: add the reported hospital priority and modify the termination rule of the DA algorithm. Under this and the previous mechanism, the same manipulation incentives would be present, yet welfare would increase $9-12\%$ and rejection rates would drop significantly.

2.6 Merit-based tie-break

The SaberPro is an end-of-study nationwide exam. To graduate all students in all fields of study in the country must take the test, which is used to decide admissions into Colombian graduate studies. In this section I analyze what would happen if the exam results were used to break ties within the coarse priority structure. In a procedure known as merit-based tie-breaking, this process is standard procedure

	Welfare Gains vs DA (%)	Num. of Prof. with Not Respected Priorities	Num. of Allo. Prof. with Not Respected Priorities	% of Pos. Rejected
Med-Lab				
Implemented Mechanism	0	0	0	5
TTC	5	14	0.2	4
DA-Stopping	8	9	0.3	1
Report TB	9	0	0	3
DA Stopping+Report TB	12	10	1	0
Sequential DA	13	13	0.1	1
SOSM	0	0	0	4
Resp. Prio. Allo.	2	3	0	4
Nursing				
Implemented Mechanism	0	0	0	4
TTC	0	14	0	3
DA-Stopping	8	88	0	3
Report TB	5	0	0	4
DA Stopping+Report TB	9	89	0	3
Sequential DA	8	112	0	0
SOSM	0	0	0	4
Resp. Prio. Allo.	0	2	0	4
Med-Doc				
Implemented Mechanism	0	0	0	4
TTC	0	26	26	4
DA-Stopping	0	0	0	4
Report TB	1	0	0	4
DA Stopping+Report TB	1	0	0	4
Sequential DA	1	9	9	4
SOSM	0	0	0	4
Resp. Prio. Allo.	0	0	0	4
Dentistry				
Implemented Mechanism	0	0	0	3
TTC	2	11	0.13	2
DA-Stopping	9	64	0	1
Report TB	9	0	0	1
DA Stopping+Report TB	12	64	0	1
Sequential DA	12	65	0	0
SOSM	0	0	0	3
Resp. Prio. Allo.	2	4	0	2

Table 2.5: Comparison Between Different Algorithms (Complete Preferences)

	Med-Lab	Nurses	Med-Docs	Dentists
Worse	10	34	85	11
	(1.47)	(2.83)	(3.23)	(2.69)
Same	11	28	213	31
	(5,96)	(5.78)	(7.36)	(6.71)
Better	60	86	74	54
	(3.4)	(3.04)	(6.45)	(3.52)

Table 2.6: Merit-based vs Random Tie-break

in several school choice settings.

I recovered the results for 189 (out of 196) Medical Lab Scientists, 713 (out of 828) Nurses, 337(out of 386) physicians, and 661 (out of 708) Dentists. The tests usually consist of two sections: a generic section that is the same for all fields; and an advanced section specific to each field. Unfortunately, the advanced sections specific to particular fields were modified many times prior to the October 2014 allocation. Consequently, to break the ties in the priority structure, I use as the means the average in the generic portion of the test.

As in the previous sections, I run 100 simulations of the allocation using the random tie-break, and I compare it to the allocation produced by the merit-based approach. Table 2.6 shows the results. In the case of the allocation of physicians, because there are more positions than physicians, the quality of these positions is a zero sum game. Hence, the more desired positions have a higher chance of receiving good physicians, which generates inequality within the country in the quality of the allocated professionals.professionals. This outcome is contrary to the objective of the program, which is to give rural communities the chance to acquire good health professionals.

When the merit-based approach is applied to the other professions, congestion leads to a significant increase in the average quality of allocated health professionals, from which most hospitals benefit. However, in some states, regional priorities cause health professionals to be redistributed, and this negatively affects some hospitals. With regards to the welfare of health professionals, I do not observe a significant change int he average welfare when there is a merit-based tie break.

2.7 Robustness Check

The accuracy of the statements about welfare made in the previous sections depend naturally on the accuracy of the preference estimates. These estimates are based on the assumption that the reported preferences are the true preferences. Beyond the fact that the allocation is determined using the DA algorithm, there are several reasons to make this assumption.

As noted above, a professional cannot manipulate the allocation to achieve an exemption. A health professionals' strongest motivation to report strategically would be to over-report positions in their state of origin or graduation because doing so would give them a higher priority. This, in turn, would produce a higher probability of allocation. Two regularities show that this strategy would not be effective. First, as reported in Table 3, states that give a higher priority experience a decrease in their ranking position. Indeed, Haeringer and Klijn (2009) show that in the truncated DA there are no incentives to misreport preferences within the

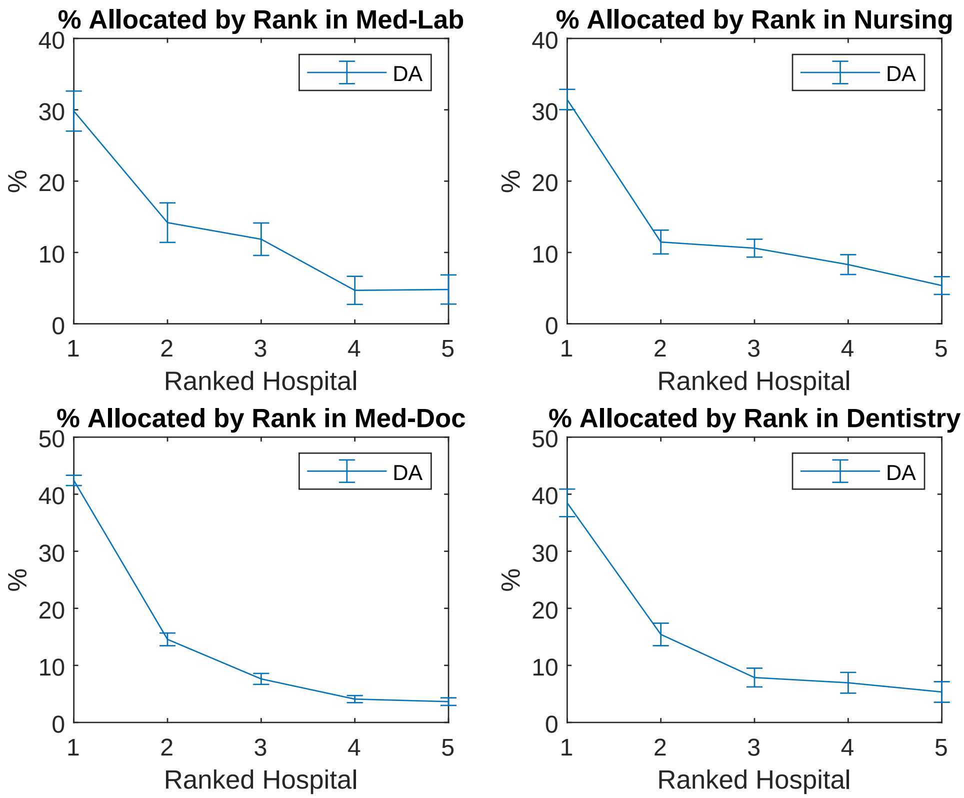


Figure 2.5: Percentage of Health Professionals Allocated by Rank

reported preferences. The same result holds in the environment that I examine here. In other words, health professionals have no incentive to misreport preferences. If the last positions reported had a higher probability of being from a priority-giving state, this would constitute evidence of strategic behavior, but I have found no such evidence. In fact, I observe that the probability of being allocated to the last reported hospital is lower than the probability of being allocated to the second-to-last reported hospital, as shown in Figure 2.5.

Second, the greater the congestion in a allocation, the greater the incentives to report a priority giving state. This should produce a higher weight in the travel distance coefficient. However, as Table 2.4 shows, the biggest average weight of travel costs is observed in the least congested allocation (that of physicians).

Chapter 3: School Choice with Rank Priorities

3.1 Introduction

Market design is known as the field of designing practical allocation mechanisms. It has been successfully applied to a number of cases. The literature is divided into two main topics: matching and auctions. The main difference between them arises from them is that they are thought to take opposite sides in the cardinal/ordinal utility spectrum (Abdulkadiroğlu and Ausubel, 2016).

In this chapter, I revisit one of the most studied problems on the matching side of market design: school choice. In doing so, I assume that agents have cardinal utilities instead of the traditional ordinal ones. Within this problem a long-lasting debate concerns use of the *Immediate Acceptance* mechanism (also known as *Boston*) in several school districts in the US to allocate students to schools (Kesten and Ünver, 2015). Use of *Immediate Acceptance* is problematic because, as Abdulkadiroğlu and Sönmez (2003) have demonstrated, it has several undesirable properties.

I use the allocation of health professionals in Colombia studied in the previous chapter to conduct counterfactuals. In particular I use the estimates of the preferences of medical lab scientists that participated in the October 2014 allocation. As I showed in the previous chapter, these estimates allow to give the preferences a

	v_1^j	v_2^j	v_3^j
$j = s_1$	0.8	0.8	0.6
$j = s_2$	0.2	0.2	0.4
$j = s_3$	0	0	0

Table 3.1: Abdulkadiroğlu et al. (2011) example

cardinal utility and this allows to unveil the intensity of preferences. While there are other papers that follow this approach,[1] I am unaware of other studies that compute a data supported Nash equilibrium of the *IA* mechanism allocation. This enables the comparison between mechanisms such as *DA* and *IA*.

The research most closely related theoretically to this one is Abdulkadiroğlu et al. (2011). They too, take a cardinal utility approach to studying the school choice problem and show, in a highly specialized setting, that when the priorities used by schools are completely coarse and the preferences of students completely correlated, the *Immediate Acceptance* mechanism may outperform the *DA* mechanism in terms of ex-ante *Pareto efficiency.* I reproduce their Example 1 to illustrate the efficiency gains that manipulable mechanisms may entail. Suppose three students, $1, 2, 3$ are assigned to three schools, s_1, s_2, s_3, each with one seat. Assume further that students have von-Neumann Morgenstern utilities and the same priority at each school. Let v_i^j be the utility of student i at school j. Table 3.1 shows the information of this example.

Note that student 3 benefits by reporting $(s_2 \succ s_1 \succ s_3)$ instead of the truthful $(s_1 \succ s_2 \succ s_3)$ when the mechanism used is IA, since this guarantees him a seat

[1] see for example (Abdulkadiroğlu et al., 2017) and Hastings et al. (2009).

at s_2. Note also that the expected welfare for each student under DA is 0.33 due to the fact that they have the same probability of being assigned at each school. However, once student 3 misreports his preferences, the expected welfare for each student under IA is 0.4 in the Nash equilibrium.

I show that the ex-ante efficiency gains in situations similar to the one depicted by the example above are not exclusive to the IA mechanism. Using the axiomatization of the IA mechanism provided by Kojima and Ünver (2014) I show, as a corollary to that theorem, that the IA mechanism is isomorphic to a DA when ranked priorities are placed on top of the priorities otherwise used in DA. I define the priorities traditionally used in DA as *exogenous*. This framework naturally suggests a new mechanism, namely the one that arises when ranked priorities are located below the exogenous priorities. I show that because ranked priorities are located above the exogenous priorities, the IA mechanism fails to respect priorities. The new proposed mechanism allows priorities to be violated but only as a result of untruthful reporting. I define a new property this mechanism satisfies, a weaker version of respecting priorities, that I call *claim free*. Therefore, this new mechanism is defined as *Claim Free Boston* mechanism. Recently, it has been shown by Kapor et al. (2017) that this mechanism has been used in New Haven to allocate students to schools.

Going one step further I define a family of mechanisms that satisfy the *claim free* property. This family of mechanisms depend on how strict or coarse ranked priorities are. In one end, when the ranked priorities added are completely coarse in the sense that ranking a school in any position gives the same ranked priority, results in the usual DA. When the opposite is true, i.e. when ranking a school higher gives strictly higher ranked priorities, the *Claim Free Boston* is obtained.

I conduct a counterfactual analysis using the estimates of the Medical Lab Science professional utilities at the *Compulsory Social Service*. I compare the aggregate welfare resulting from using the *IA*,*DA*, and the family of *claim-free* mechanisms. I find no evidence that moving to manipulable mechanisms results in higher aggregate welfare of the medical lab scientists.

This chapter is divided as follows. Section 2 defines and motivates the new family of mechanisms that satisfies the *claim free* property. Section 3 describes the *Compulsory Social Service* and the Medical Lab Science cohort used for the results. Section 4 outlines the details of the counterfactual analysis and discusses the results.

3.2 Claim Free Boston

In the *school choice* problem, the priority structure traditionally is very coarse because it usually involves only two student priorities: having a sibling in the school or living in the neighborhood served by the school. I define these as the exogenous priorities of the problem. The priorities are complemented with a random tie-break to determine the allocation in any either IA, DA, or TTC algorithms.

Kojima and Ünver (2014) provide an axiomatization of the *IA* mechanism. They show in their Theorem 1 that the characteristics that define the IA mechanism are favoring higher ranks and satisfying consistency, resource monotonicity, and rank-respecting invariance. I develop next a corollary of that theorem that relates

IA and DA. Suppose the priority structure of a school choice problem had rank priorities on top of the exogenous priorities. In this case, the priority structure would be:

Rank 1: Students who ranked the school first.

Rank 2: Students who ranked the school second.

⋮

Rank k: Students who ranked the school in the k-th position.

Exo: Exogenously given priorities.

R: Random tie-break.

With this definition in mind I can state and prove the following corollary to the axiomatization of the IA mechanism.

corollary 3.1. *Given the same random tie-break, the allocation using IA Algorithm is the same as the one with DA with the above priority structure.*

Proof. (Induction) The allocation of both algorithms is the same in the first step. If a student is assigned in the first step under DA, then his or her final assignment will be that school because in the following steps only students who have ranked the school lower will apply. The same holds for professionals who were first assigned to a hospital at any step of the algorithm. □

This formulation explains why the *IA* mechanism does not *respect priorities*. When *rank priorities* are given a higher status than the *exogenous* ones, with whom the *respecting priorities* property is defined, *exogenous* priorities become vulnerable.

This framework naturally suggests a new mechanism- the one generated by adding *rank priorities* below the *exogenous* ones. I define this new mechanism as *Claim Free Boston*. Adding these rank priorities gives the priority structure of the schools some endogeneity and, therefore, it comes at the cost of losing *strategy proofness*.

In the *IA* mechanism a student risks "losing" any exogenous priorities whenever he or she does not ranking a school first. Under the *Claim Free Boston* this does not happens, and, thus, as the name suggests, the mechanism is *claim-free*. Additionally, the chance that any subject will deviate from reporting truthfully should be lower than in the IA mechanism. Finally, in contrast to the *TTC*, students under the Claim Free Boston are not allowed to trade their priorities, and this comes at a cost: the mechanism is still not being ordinal-*Pareto efficient*.

Notice that in the example provided in the introduction the *Claim Free Boston* mechanism accomplishes the same cardinal utility as IA. The gains from this mechanism arise when a student who deviates from her or his true preferences earns a higher probability of being allocated to a least preferred school, which increases the chance that others will get into one of their preferred schools. Hence, the gains from this mechanism occur in the cardinal utility.

3.2.1 Deferred Acceptance with Rank Priorities

The *Claim Free Boston* mechanism defined in the previous section has a very particular structure in its ranked priorities: it gives a higher priority to a higher ranked alternative. However, there are many possibilities for ranked priorities. For example one can give the highest ranked priority if an alternative is ranked first or second while otherwise giving no priority. I define a rank priority structure as a function $\pi : \mathbb{Z}_{++} \to \mathbb{Z}_{++}$ that is interpreted as ranking an alternative in position $k \in \mathbb{Z}_{++}$ results in a ranked priority of $\pi(k)$ for that alternative. Ranking an alternative in position $k \in \mathbb{Z}_{++}$ yields a higher ranked priority to ranking it in position $w \in \mathbb{Z}_{++}$ if $\pi(k) < \pi(w)$. The ranked priority structure of both the IA and the *claim free Boston* mechanisms is $\pi(k) = k$.

I define two properties over ranked priorities functions: *monotonicity* and *unit increments*. *Monotonicity* is defined as the fact that for any r, $r' \in \mathbb{Z}_{++}$, if $r < r'$ then $\pi(r) \leq \pi(r')$. This property implies that ranking an alternative higher should always weakly increase its ranked priority. Not having this property may result in the undesirable case of an individual ranking an unfeasible school for the sole purpose of earning a higher priority in a lower ranked alternative. *Unit increments* property is defined as the fact that if $r \in \pi^{-1}$ and $r \neq 1$ then $r - 1 \in \pi^{-1}$. This property implies that giving the *k-th* priority to a ranked position only makes sense if some other position has the $(k-1)$-th priority.

In the *Compulsory Social Service*, health professionals are allowed to report 5 rural hospitals. In these cases, 16 *monotonic* and *unit increment* ranked priority structures are possible. In the next section I analyze the allocations that result from adding each of these possible *ranked priorities* below the *exogenous* priorities. I denote the mechanism that results from adding the ranked priority structure $\pi(1) = 1$, $\pi(2) = W$,$\pi(3) = X$,$\pi(4) = Y$, and $\pi(5) = Z$ below the exogenous priorities as

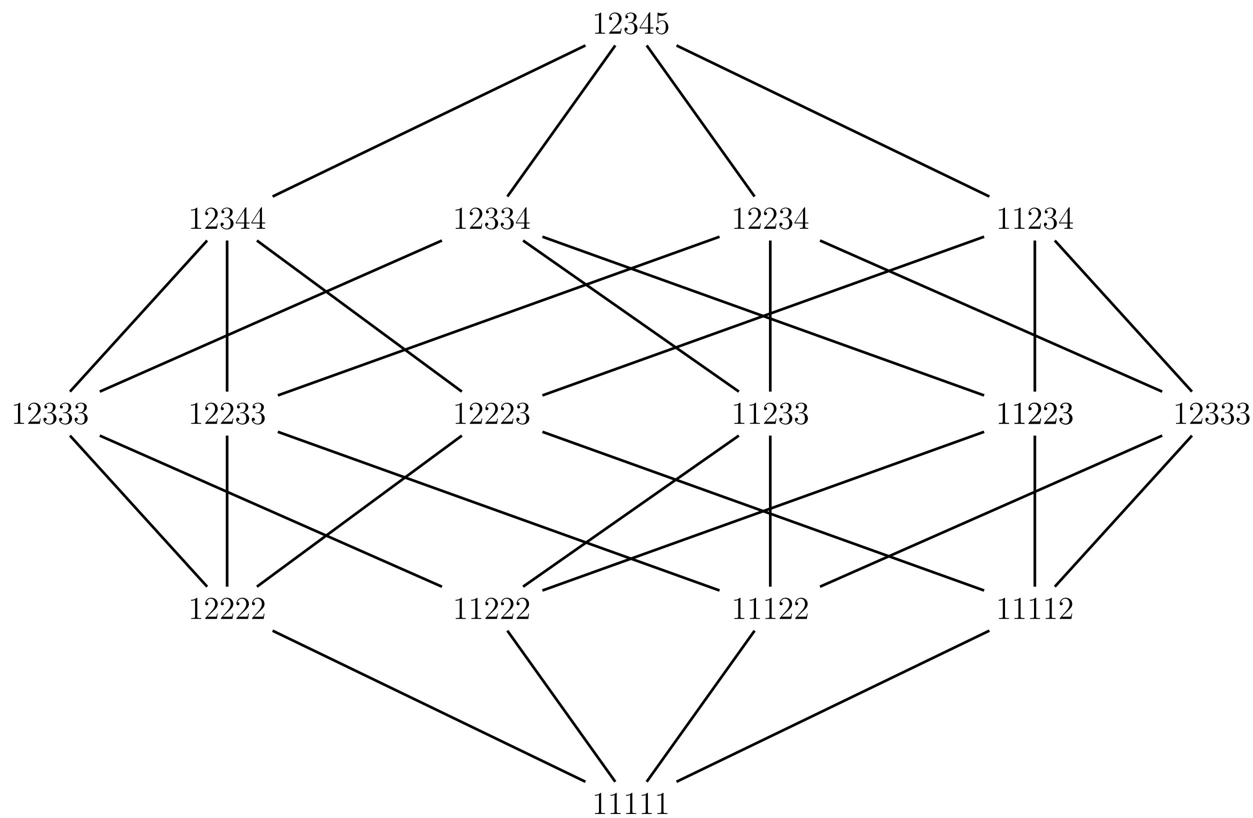


Figure 3.1: Complete Partial Ordering of Rank Priorities

$DA + RP(1WXYZ)$. In this case, the *claim free Boston* mechanism is $DA + RP(12345)$.

The *monotonicity* and *unit increment* property induces a *complete partial order* to the family of possible ranked priority structures. The top (bottom) element of the order will be the *claim free Boston* (DA) mechanism. Figure 3.1 shows what CPO of priority structures look like when it is possible to report up to five choices.

3.3 Comparing Different Allocation Mechanisms

The Compulsory Social Service allocation of health professionals in Colombia was created to tackle the problem of inequality in the allocation of health professionals to rural areas in Colombia. Since October 2014, the allocation has been determined by the Ministry of Health using a version of Gale and Shapley (1962) deferred acceptance.

The health professions that participate in the Compulsory Social Service are Medical Lab Science, Medicine, Nursing, and Dentistry. In this chapter I focus on the allocation of Medical Lab Scientists. I make this selection because this allocation had the smallest number of participants ($n = 182$). Recall that when dealing with manipulable mechanisms, I find Nash Equilibiums in this setting. This a computationally intense task. On top of the number of participants, each one has a strategy set of more than 30,000 alternatives. Therefore, the allocation of Medical Lab Scientists provides us with an interesting setting while having a small number of players, which makes the aforementioned exercise feasible to compute.

The following types of priorities are used to define the priority structure of hospitals in the CSS. First are the priorities that follow from regulations of the Health Ministry. These are given to professionals who either are mothers with small children or who have impairments, disabilities, or are in need of special medical treatment. During the period of study, only 6% of the Medical Lab Scientists had these priorities. They included 7 pregnant women, 1 mother or father with small children, and 2 professionals with special treatment needs. The second and inferior type prioritizes location. Hospitals had to give priority to professionals who graduated from their state (departamento) and to professionals who were born in their state (departamento). The remaining 94% of medical lab scientists had these priorities. Therefore, two professionals who were born and graduated from the same state will tie in priority at all hospitals. Since these ties are broken randomly, the allocation has a high degree of randomness.

The types of priorities used to define the *priority structure* of hospitals in the CSS is the following. First type of priorities were the ones that follow from regulations of the Health Ministry. These priorities are given by being to a professional who is a mother with small children or having impairments, disabilities, or needs of a special medical treatment. Only 6% of the Medical Lab Scientists had any of these priorities, in particular there were 7 pregnant women, 1 mother of father with small children, and 2 professionals with needs for special treatments participated. The second and inferior type of priorities location priorities. Hospitals had to give priority to professionals who graduated from their state (departamento) and to professionals who were born in their state (departamento). These were the only priorities the other 94% of the medical lab scientists had. This results many health professionals tying in priority at the any given hospital, i.e. the priority structure was significantly coarse. As such, the allocation had a high degree of randomness since ties in priorities were broken using a single tie-break.

I use the estimates found in Chapter 2, which employ a random coefficients model that allows me to capture the heterogeneity of preferences among medical lab scientists. The preferences have a vertical component (the wage offered) and several horizontal ones (e.g., location). I

nd that the horizontal component outweighs the vertical one and as such the preferences are significantly heterogeneous. Moreover, the estimates are consistent with expectations. The means of the coefficients show that bacteriologists prefer higher wages and being closer to their town of origin. They also prefer wealthier and safer municipalities and municipalities that have a higher number of health institutions. Regarding the variance of the coefficients, I find that only the coefficients on wage, travel distance, and staying within the same state are heterogeneous among the professionals. Moreover, all three of them are positively correlated.

Finally, I define the family of $DA + RP(1XYWZ)(Boston + RP(1XYWZ))$ mechanism as those that arise from adding $RP(1XYWZ)$ below (above) the weak priority structure of the hospitals and determining the allocation with DA over those priorities and the lists of professionals. I am limiting to five the slots that can be given a ranked priority because in the implemented mechanism medical lab scientists are allowed to report only five options. Notice that in this case, because the ranking of hospitals is completed for each professional, the matchings that result from the DA and $DA + RP(11111)$ $(Boston + RP(11111))$ are the same. This is so because a professional who ranked a hospital cannot "lose" the position with one who did not.

I conduct simulations of the allocations for *DA*, *IA*, and the families of mechanisms of *DA+Rank Priorities* and *Boston+Rank Priorities* discussed above. For each mechanism, I simulate the allocations using 20 draws of the random tie-breaking and analyze each of them under two treatments. In the first simulation, I assume that the medical lab scientists behave naively and, consequently, they report their preferences truthfully at all mechanisms. This is the *truthful* reporting treatment. In the second simulation, I find a *full information* Nash equilibrium with *limited consideration*. In this case *full information* means that the agents are able to calculate exactly the probability of being assigned to any given hospital by reporting it at any position. This implies that they know the realization of the 20 draws of the tie-break and that each one has an equal probability of being used[2]. The same draws are used for all mechanisms. *Limited consideration* comes from the fact that I assume that each medical lab scientist is only aware of its eight preferred alternatives. This constraint limits the strategic space of each agent and makes the computation of a *Nash* equilibrium feasible.

Despite the fact that the choice literature has points out a number of situations and the consequences for welfare analysis of boundedly-rational individuals, I have no evidence that this is the case for the particular setting examined in this research. Medical lab scientists face a crucial decision and the number of alternatives (81) is small relative to the importance of the decision at hand. Therefore, the aforementioned restriction is included only because it eases the computational burden and not because of any resemblance with reality or behavioral assumptions.

This is a very complex environment in which the strategy space of each agent is huge and, as a result, the number of Nash equilibrium is very large. The claim here is that *a subset* of the possible Nash equilibriums is found for each case, while it is acknowledged that there are many others. However, in each case the Nash equilibrium is calculated the same way, using iterative best response. Medical lab scientists initially report their true preferences and then are asked, one at a time in a fixed order, their best response given the others reports. To calculate the best

[2]Increasing the number of simulations weakens the constraint of having them know the results of the tie-breaks

	v_1^j	v_2^j	v_3^j
$j = s_1$	0.8	0.6	0.6
$j = s_2$	0.2	0.4	0.4
$j = s_3$	0	0	0

Table 3.2: Coordination Problem

responses, I use the probability of allocation at a hospital (conditional on the position reported) and the preference estimates obtained using the random coefficients model.

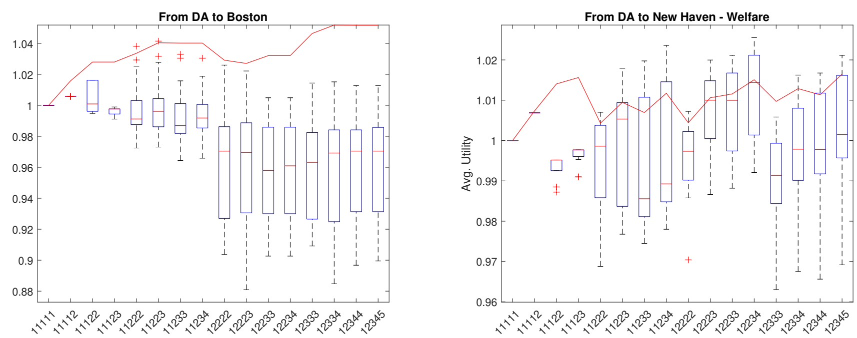
The iterative best response is done one at a time because the Nash equilibrium might entail two participants with the same utilities who report different preferences. An example of such a case is the following modification of the earlier example. Suppose three students, $1, 2, 3$ are assigned to three schools, s_1, s_2, s_3, each of which has one seat. Assume further that students have von-Neumann Morgenstern utilities and the same priority at each school. Let v_i^j be the utility of student i at school j. Table 3.2 shows the information of this example.

Note that in this case, both students 2 and 3 benefit from reporting $(s_2 \succ s_1 \succ s_3)$ instead of the truthful $(s_1 \succ s_2 \succ s_3)$ under IA only if the other student reports truthfully. This example has two Nash Equilibriums: one where only student 2 misrepresents his preferences and one where student 3 does.

Therefore, when the previous example is played statically, a coordination problem arises from students 2 and 3. Finding the Nash Equilibrium through iterative best responses, one participant at the time, solves the aforementioned problem. In the previous example, the participant (among students 2 and 3) who is asked to best response will optimally choose to misrepresent his preferences.

3.4 Results and discussion

Once the final allocations are determined for each mechanism under both the *truthful* and *Nash* treatments, the aggregate welfare is calculated in each case. I normalize the aggregate welfare of the *DA* truthful reporting to 1, so that the results are interpreted as percentage gain relative to the mechanism implemented- namely *DA*. Figure 3.2 shows the results of the computation of the aggregate welfare average for each mechanism under both treatments.



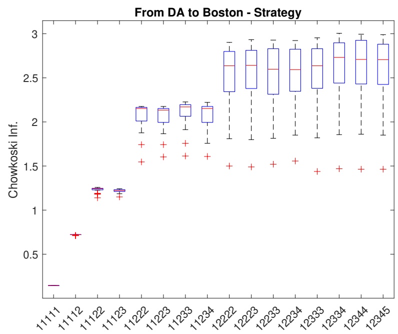
(a) Aggregate Welfare - From DA to Immediate Acceptance

(b) Aggregate Welfare - From DA to New Haven

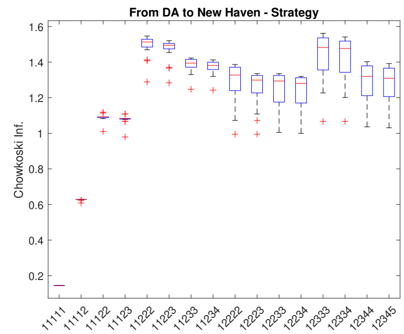
Figure 3.2: Aggregate Welfare

Examining which mechanism the Ministry of Health should use, I conclude that significant improvements can be made by changing the pure-DA mechanism. There is no evidence that significant welfare gains are achieved by moving towards manipulable mechanisms.

To measure the manipulability of the different mechanisms, I compare the Chowkoski Information difference between the true preferences and the ones reported in the Nash Equilibrium setting. The Chowkoski Information measures the number of pairwise switches between the true and the Nash Equilibrium preferences. For example, if the true preferences are $H_1 \succ H_2 \succ H_3$ and the reported one is $H_1 \succ H_3 \succ H_2$, then the Chowkoski Measure between the two is 1. Figure 3.3 shows the results of the computation of the manipulability measures of each mechanism under both treatments.



(a) Manipulability - From DA to IA



(b) Manipulability - From DA to New Haven

Figure 3.3: Manipulability

We can easily observe that a significant amount of strategizing is happening in the most manipulable mechanisms. This result is important because it rejects a possible explanation of the lack of welfare gains: that not much strategizing is occuring. In this scenario, it is the case that when all participants are strategizing, non-strategic behavior results in a *running to keep in the same place.*

Chapter 4: Aftermath of the Centralized Clearinghouse Implementation and Conclusions

4.1 Aftermath of the Centralized Clearinghouse Implementation

The centralized clearinghouse used to determine the allocation of health professionals for the Compulsory Social Service program was implemented in October 2014. Since then it has been used to determine the allocations. Before it was implemented, a system of decentralized lotteries marred with incentive flaws was used to determine the allocation. The key purpose of the program, taking health professionals to under-served areas was not being fulfilled. Evidence of this was the vacancy rate of positions for physicians of about 50%. This was specially troublesome since there were more physicians participating in the program than positions.

After the implementation of the centralized clearinghouse as a mean to determine the allocation, the vacancy rates have decreased by an order of magnitude. Currently the vacancy rate of positions if of 5% and it is because of health professionals rejecting their allocated hospital. Therefore, the Colombia transitioned from having about 1450 newly graduate professionals in rural areas in 2012 to 2750 in 2015- a significant increment in the professionals labor force for under-served areas.

Barriers to Healthcare Acces - Colombia			
	2014	**2015**	**2016**
Urban	5.4	5.9	4.0
Rural	9.9	**8.5**	**5.9**

Table 4.1: Evolution of the Barriers to Healthcare Access Index in Colombia

Since 2011, Colombia measures the poverty levels with a poverty multidimensional index. One particular index is the *barriers to healthcare access*. Table 4.1 shows the evolution of the index mentioned above. Colombian rural population, the main target of the program, has now a much higher access to healthcare services as shown in the fact that the above index went from 9.9% in 2014 to 5.9% in 2016. Colombia is know close to achieving the World Bank's objective of having 97% of the population with healthcare coverage.

Among the 15 indexes that are recorded in the multidimensional poverty index, *Barriers to Healthcare Acces* was the one that contributed the most in reducing poverty in 2016. This is infact a surprise given that the health indexes are the best performing indicators in the multidimensional poverty index.

4.2 Conclusions

This dissertation evaluates the Compulsory Social Service (CSS), a program developed by Colombia's Ministry of Health to tackle inequality in the allocation of the country's health professionals. This is a long-standing policy concern in many countries because of the difficulty of filling medical positions in rural areas. A salient characteristic of this program is that it is compulsory: health professionals in the areas of Medical-Lab Science, Dentistry, Nursing, and Medicine must participate in this allocation to become professionally certified. This characteristic makes the studied clearinghouse different than any other studied in the literature despite entry level clearinghouse have been extensively used in other settings.

Since October 2014, the allocation has been determined by a single-offer centralized mechanism that uses the Deferred Acceptance algorithm to determine the allocation. Chapter 1 discusses the theoretical motivations that pointed towards the use of that particular algorithm among many others that have been proposed in the literature. The main idea in that chapter is how to adapt the mechanisms that have been used in other settings to an environment that is compulsory in nature.

I then, in chapter 2, use the data on the reported preferences in the first allocation, which was conducted in October 2014, to estimate the preferences of each health professional for every hospital. I estimate a random coefficients model of the preferences. These preferences allow for a correlation in the coefficients, and they are specifically modeled to allow for a correlation for hospitals within each state. I find the health professionals' preferences for hospitals to be significantly heterogeneous.

Referring to these estimates, I show that moving from the previously-used decentralized system of lotteries to the current centralized mechanism has produced significant welfare gains for physicians. From the fact that under the decentralized mechanism health professionals were able to avoid positions that fell below their acceptance threshold, and simulating the outcome had the decentralized mechanism still been in use, I obtain the average marginal utility a health professional would require to accept a position. I then simulate the outcome of the centralized mechanism in the absence of the requirement that students accept the assignment determined by the mechanism. I find that, given the choice, about 30% of physicians would reject their hospital assignment, which implies that for the policy to be successful, assignments must be mandatory.

Then I study the allocations' efficiency from the viewpoint of health professionals. Allocations determined through use of the DA algorithm are known to be strategy proof and respect priorities. However, these characteristics constrain possible allocations and, therefore, (possibly) they entail welfare costs. I show that the respect priorities component is not very costly in terms of average welfare. Thus, I show that under several mechanisms all priorities are respected. I also show that the cost of *strategy-proofness* is of the first order. Slight deviations can result in welfare gains of up to 12%. This reflects the fact that the preferences of health professionals include all possible hospitals.

I also show that in case of the physicians market, using a random tie-break rather than a merit-based one was a good policy decision. The former would have allocated the best physicians to the most desirable positions, leaving those located further away with worse physicians (as measured by the results of their end-of study-exams). Doing a merit-based tie-break would benefit most of the other health professions, which face significant congestion (i.e., their ratios of health professionals to positions are very high). Behind this fact is the intuition that the less attractive health professionals would have a greater chance of not being assigned.

Finally, in chapter 3, show there is no evidence that manipulable mechanisms, like the one used in Boston and the one currently used in New Haven to allocate students to schools, in the setting of the Compulsory Social Service will result efficiency gains. This is further evidence that when preferences are significantly heterogeneous, the allocations when determined using the Deferred Acceptance algorithm achieve a high efficiency.

There are other alternatives that I plan to study in the future. One of the things that makes this setting interesting is the fact that the allocation is centralized but the wages are determined in a decentralized way. In the future I plan to study how the allocation would change if the wages are determined in a centralized way, i.e. they are determined by the Ministry of Health. This opens the possibility of capturing the intensity of preferences and thus determine the allocation in a more efficient way. In particular I will explore what the Vickrey-Clarkes-Groove mechanism outcome is in the setting at hand. This will not only be interesting in the allocative nature of the allocation but will also help outline how wages can be set optimally.

Appendix A: Appendix

A.1 Distribution of Distance from Preferred Hospital to Graduation City

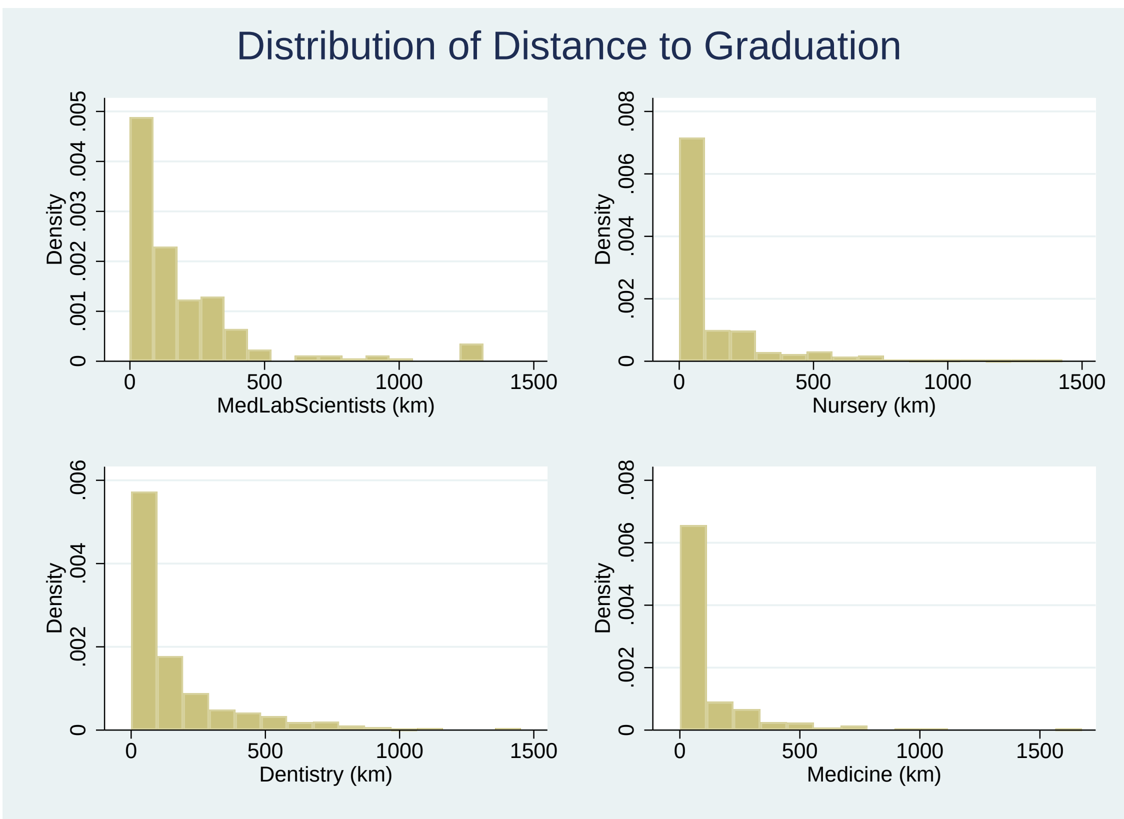


Figure A.1: Travel distance from hospital to graduation (km)

The understanding of nonlinear processes in optical fibers is crucial towards extending the capabilities of modern optical communication systems based on wavelength division multiplexing (WDM), where each communication channel is represented by a unique wavelength. One of the nonlinear processes that limits the information carrying capacity of a WDM system is four-wave mixing (FWM), which causes cross-talk between neighboring channels. This places a lower limit on the wavelength separation between adjacent channels and an upper limit on the input power in each channel. In this study, we describe a process by which the evolution of FWM processes in an optical fiber can be used to estimate the inhomogeneities in the fiber core material, in particular the fluctuations in the linear refractive index of the fiber core.

A.2 Single vs Multiple Tie-Break

I use different algorithms proposed in the literature and compare their efficiency, as measured by average welfare, with the one resulting from the implemented mechanism. In order to make our results comparable with the results in the *school choice* literature (e.g. Abdulkadiroğlu et al. (2009), He et al. (2012), and Calsamiglia et al. (2014)) I compare the efficiency of the allocation as if the implemented algorithm used only the reported preferred hospitals. I will refer to it as the *DA* allocation.

It is important outline that the implemented algorithm with these characteristics is *strategy proof* and *respects priorities*. Also, the priority structure of the hospitals is so coarse that the mechanism has a significant randomness, i.e. it relies a lot on the single tie-breaks. Define a matching $\mu = \mu'$ iff $\mu(i) = \mu'(i)$ for every $i \in \mathcal{P}$. I conducted 1'000,000 simulations of the *DA* allocation and found all resulting matchings being different.

The first algorithm I compare *DA* with is the *Top Trading Cycles* or *TTC* proposed initially by Shapley and Scarf (1974) and in the *school choice* setting by Abdulkadiroğlu and Sönmez (2003). This algorithm is *strategy proof* and *efficient*. Hence, comparing *DA* with *TTC* yields a measure of how costly it is in terms of efficiency the *respecting priorities* property. I find that the welfare actually decreases for the allocation of physicians and nurses. For the other two professions the gains from TTC are slightly higher.

The next algorithm I compare *DA* with is the *Student(Professional) Optimal Stable Match* or *SOSM* proposed by Erdil and Ergin (2008). The single tie break used can generate welfare decreasing cycles. Comparing this algorithm with *DA* gives a measure of how much welfare is lost on average due to this welfare reducing cycles. I find that there are small welfare gains when moving from *DA* to *SOSM* in every profession there are some potential gains from using this mechanism.

Another proposed modification in the literature is moving from a single tie-break to a multiple tie-break, i.e. instead of having the same tie-break for all hospitals allowing each hospital to have their own. Ashlagi et al. (2015) show that when comparing the allocations resulting from *DA* under single and multiple tie-breaking rules, the former has more subjects allocated at the firsts positions and very lasts positions while the latter tends to allocate more concentrated on the middle. Our results coincide with their results (as shown in table A.1). In the particular case I are studying I find that moving from the single tie-break to the multiple one induces

Avg Welfare		Med-Lab	Nursery	Med-Doc	Dentists
Single Tie-Break					
	DA STB	3.53	4.39	5.43	4.38
	MLDA STB	3.96	4.85	5.56	5.14
	SOSM STB	3.92	4.47	5.44	4.48
	TTC STB	3.96	4.50	5.38	4.60
	MLTTC STB	4.21	4.91	5.59	5.28
Multiple Tie-Break					
	DA STB	3.19	4.22	5.09	4.27
	MLDA MTB	3.65	4.72	5.12	5.20
	SOSM MTB	3.78	4.55	5.84	4.61
	TTC MTB	3.94	4.56	5.42	4.62
	MLTTC MTB	4.32	4.98	5.56	5.22

Table A.1: Comparing Different Algorithms

a welfare loss for all professions.

I compare all of the aforementioned mechanisms under the single and multiple tie-breaking rules. As shown by Pathak and Sethuraman (2011) I find no difference in the *TTC*'s welfare between the two. For the other mechanisms I consistently find that the single tie-break is superior in welfare than the multiple tie-break.

Finally, I modify the compare the *DA* with the *Augmented Choice Deferred Acceptance* (or *ACDA*) proposed by (Abdulkadiroğlu et al., 2011). This algorithm ideally should ask the professionals at which positions they would like to have an additional advantage in the tie-break. They show how this mechanism is *strategy proof*. I first calculate the probability of being assigned at each hospital for every professional under *DA*. This is done through a computation of 10,000 simulations. Then the additional advantage in the tie-break is given to each professional in their most likely hospital. I define this algorithm as *Most Likely Deferred Acceptance*. I find this algorithm produces a welfare gain of in cases relative to the Deferred

Acceptance. However, the welfare gains relative to the TTC are ambiguous. Interestingly, the resulting allocation first order stochastically dominates the allocation and results in an increase of almost 2% in the allocated professionals. Interestingly, following the same principle with the *TTC* I find once again that it procedure, i.e. giving a higher to the priority in the tie-break to the most likely alternative, results in similar welfare gains and a first order stochastic improvement.

A.3 Complete Table of Estimates

		Med-Lab	Nursing	Med-Docs	Dentists
Means	Gender				
Wage					
		1	1	1	1
Travel Distance Origin					
	F	-1.413	-1.863	-2.455	-1.444
	M	-1.192	-1.417	-1.798	-1.413
Travel Distance Graduation					
	F	-0.532	-1.626	-1.656	-0.694
	M	-0.708	-3.024	-1.511	-0.028
UBN-Index					
	F	-0.056	-0.114	-0.141	-0.083
	M	-0.023	-0.108	-0.118	-0.109
Coca					
	F	-1.419	-0.644	-1.095	-1.124
	M	-0.147	-2.013	-0.997	-0.338
Living Place					
	F	0.866	0.256	0.577	0.587
	M	0.163	0.882	0.179	0.541
Weekends Shift					
	F	-2.296	-0.555	-0.176	-1.231
	M	-0.001	-0.382	-0.114	-0.031
Tot. Pop.$< 100,000$					
				-0.047	
Tot. Num. Positions					
				0.061	
Std. Deviation					
Travel Distance Origin					
	F	1.506	1.427	2.430)	1.244
	M	1.875	2.202	1.542	1.455
Travel Distance Graduation					
	F	1.265	2.068	1.862	1.131
	M	1.358	2.157	2.736	0.237
UBN-Index					
	F	0.237	0.667	0.178	0.261
	M	0.214	0.603	0.191	0.470
Coca					
	F	1.630	1.102	1.940	1.252
	M	0.568	2.291	0.875	1.012
Living Place					
	F	1.111	0.663	0.491	1.016
	M	0.585	1.254	0.418	0.942

	Med-Lab	Nursing	Med-Docs	Dentists
Weekends Shift				
F	2.276	1.028	0.414	1.297
M	0.037	0.779	0.329	0.202
Tot. Pop. $< 100,000$				
			0.183	
Tot. Num. Positions				
			0.114	
Antioquia	3.613	3.180	2.443	2.250
Atlantico	2.599	2.313	0.993	
Bogota	4.127	2.504	2.415	3.531
Bolivar	2.659	1.958	5.473	2.377
Boyaca	3.145	1.550	1.011	2.631
Caldas	2.022	2.482	1.739	2.266
Caqueta	2.383		1.571	1.995
Cauca		1.924	1.912	2.026
Cesar	2.563	1.948	1.678	1.575
Cordoba	3.510	1.875	0.568	1.949
Cundinamarca	2.543	2.005	1.300	2.041
Choco	2.569	1.937	0.833	
Huila	2.606	1.721	1.280	2.237
La Guajira	2.047	1.808	2.607	1.703
Magdalena	2.233	1.872	1.110	3.749
Meta		1.926	0.948	1.676
Narino	2.419	3.917	1.725	2.189
N. De Santander	2.705	2.160	0.775	1.707
Quindio			2.781	
Risaralda			3.013	1.904
Santander	3.890	2.537	1.253	2.292
Sucre	3.008	2.355	2.797	
Tolima	2.074	1.707	0.499	1.675
Valle del Cauca	3.758	3.163	4.418	1.965
Arauca	3.227	2.341	2.395	1.826
Casanare	2.597	1.774	0.776	1.515
Putumayo	2.346	2.579	2.415	2.369
San Andres			5.377	
Amazonas	2.527	1.939	4.450	2.610
Guainia		2.705	1.706	2.587
Guaviare		2.112	0.768	
Vaupes			0.718	

Table A.2: Preference Estimates of Health Professionals

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