Some remarks on the role of boundary layers in decaying 2D turbulence in containers with no-slip walls (*)

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Summary. — Direct numerical simulations of decaying two-dimensional (2D) turbulence inside a square container with no-slip boundaries have been carried out for Reynolds numbers up to 2000. The role of the boundary layers during the decay process has been illustrated with ensemble-averaged results for the power law behaviour of several characteristic properties of the coherent vortices which emerge during the decay of 2D turbulence. The evolution of the vortex density, the average vortex radius, the enstrophy and the vorticity extrema have been computed. An algebraic decay regime has been observed during the initial turbulent decay stage. The computed decay exponents disagree, however, with the exponents from the classical scaling theory for 2D decaying turbulence on an unbounded domain. This is attributed to the presence of no-slip boundaries. Additionally, the temporal evolution of the average boundary-layer thickness has been studied by computing the ensemble-averaged viscous stress and normal vorticity gradient near the no-slip boundaries. These computations reveal that $\delta(t) \simeq t^{0.4}$ and that the average boundary-layer thickness is proportional with $\mathrm{Re}^{-0.5}$.

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1. – Introduction

Numerical simulations of freely evolving 2D turbulence in containers with impermeable *no-slip* boundaries have revealed several interesting features such as the spontaneous spin-up of the fluid during the decay process [1], and the anomalous power

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law behaviour of the decay of the average number of vortices during the flow evolution [2]. The investigation of spontaneous spin-up of the fluid, which is characterised by a rapid increase of the absolute value of the total angular momentum of the flow due to interaction of the flow with the rigid no-slip boundaries, is based on several runs which have been carried out for $\mathrm{Re} = 1000$, 1500 and 2000. In each of the numerical simulations the initial condition consists of a flow field containing only a very small amount of net angular momentum. The large-scale Reynolds number in these simulations is based on the RMS velocity of the initial flow field and the half-width of the domain. The spontaneous spin-up of the fluid has been observed for a majority (approximately 75%) of the runs and the final decay regime is for these runs usually characterised by the formation of a strong monopolar vortex or a rotating tripolar structure. Since the angular momentum of unbounded viscous flows is conserved when the total circulation (as in bounded domains with no-slip walls) is zero, the spontaneous spin-up of the fluid is entirely due to the finiteness of the flow domain. Another interesting aspect is the temporal evolution of the number of vortices (vortex density), the evolution of the average vortex radius, the value of the vorticity extrema during the decay process, the enstrophy decay rate, etc. From a set of simulations with $1000 \leqslant \mathrm{Re} \leqslant 2000$ it appears that the temporal evolution differs nontrivially from the theoretically predicted evolution by Carnevale and coworkers [3, 4], which is valid for two-dimensional high Reynolds number flows on an infinite domain. It might be expected that this difference is due to the presence of no-slip boundaries, which serve, on the one hand, as a source of small-scale vorticity in the form of filaments or as a source of intermediate-scale vortices, but, on the other hand, vortices might be dissipated faster in the turbulent background flow when they are substantially weakened after strong vortex-wall interactions. Although the effect of the boundary layers on the vortex statistics is rather convincing, a more thorough comparison between runs with periodic and with no-slip boundary conditions should finally be carried out, preferably at higher Reynolds numbers, in order to support the present observations more firmly. Another interesting aspect, which has not been discussed so far, is the temporal evolution of the boundary layers during the initial turbulent decay stage, *i.e.* the creation of the boundary layers and the subsequent growth of the average boundary-layer thickness.

The organisation of this paper is as follows: the numerical scheme is recalled in sect. **2**, where also the initialisation procedure of the simulations is sketched. Results of numerical simulations of decaying 2D turbulence with no-slip boundary conditions and the vortex statistics data obtained with these computations are shortly summarised and compared to data from the literature in sect. **3**. In sect. **4** the growth of the boundary layers during the initial turbulent decay stage is discussed. Conclusions are presented in sect. **5**.

2. – Numerical scheme and initialisation procedure

We report here on results of numerical simulations of freely evolving 2D turbulence on a square domain with impermeable *no-slip* boundaries for Reynolds numbers up to 2000. Simulations of the full 2D Navier-Stokes equations are carried out with a 2D Chebyshev pseudospectral algorithm with a maximum of 289 Chebyshev modes in both directions [2, 5]. The flow domain D with boundary ∂D is a two-dimensional square cavity (in dimensionless form the square $[-1, 1] \times [-1, 1]$), Cartesian coordinates in a frame of reference are denoted by x and y, and the velocity field is denoted by $\mathbf{u} = (u, v)$. The equation governing the nondimensional (scalar) vorticity $\omega = \partial v/\partial x - \partial u/\partial y$ is obtained by taking the curl of the momentum equation. The following set of equations has to be solved numerically in D:

$$\frac{\partial \omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \frac{1}{\mathrm{Re}}\nabla^2\omega \,, \tag{1}$$

$$\nabla^2\mathbf{u} = \mathbf{k}\times\nabla\omega \,, \tag{2}$$

with the boundary condition $\mathbf{u} = \mathbf{0}$ and enforcing $\mathbf{k}\cdot\nabla\times\mathbf{u} = \omega$ on ∂D by an influence matrix method [5]. An initial condition, $\omega|_{t=0} = \mathbf{k}\cdot\nabla\times\mathbf{u}_i$, where $\mathbf{u}_i$ is the initial velocity field, is also supplemented. The Reynolds number is defined as $\mathrm{Re} = UW/\nu$, with U a characteristic velocity (based on the RMS velocity of the initial flow field), W the half-width of the box and ν the kinematic viscosity of the fluid. The time is made dimensionless with W/U, and $t \approx 1$ is comparable with an eddy turnover time. The time discretisation of the vorticity equation (1) is semi-implicit: it uses the explicit Adams-Bashforth scheme for the advection term and the implicit Crank-Nicolson procedure for the diffusive term. Both components of the velocity and the vorticity are expanded in a double truncated series of Chebyshev polynomials. All numerical calculations, except the evaluation of the nonlinear terms, are performed in spectral space, *i.e.* the Chebyshev coefficients are computed. Fast Fourier Transform methods are used to evaluate the nonlinear terms following the procedure designed by Orszag [6], where the padding technique has been used for de-aliasing.

The initial condition for the velocity field, denoted by $\mathbf{u}_i$, is obtained by a zero-mean Gaussian random realisation of the first 65×65 Chebyshev spectral coefficients of both u_i and v_i, and subsequently applying a smoothing procedure in order to enforce $\mathbf{u}_i = \mathbf{0}$ at the boundary of the domain. The variance σ_{nm} of the velocity spectrum of $\mathbf{u}_i$ is chosen as

$$\sigma^2_{nm} = \frac{n}{[1+(1/8n)^4]}\,\frac{m}{[1+(1/8m)^4]} \,, \tag{3}$$

with $0 \leqslant n, m \leqslant 64$, and $\sigma_{nm} \equiv 0$ for $n, m \geqslant 65$, and the resulting flow field is denoted by $\mathbf{U}(x, y)$. The smoothing function is chosen as $f(x) = [1 - \exp[-\beta(1-x^2)^2]]$, with $\beta = 100$. The initial velocity field is thus: $\mathbf{u}_i(x,y) = f(x)\,f(y)\,\mathbf{U}(x,y)$, where the flow field is normalised in order to enforce the L^2-norm of the velocity per unit surface of the initial flow field to be equal to unity. The kinetic energy E and the enstrophy Ω of the flow are defined as

$$E = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}[u^2(x, y) + v^2(x, y)]\,\mathrm{d}x\,\mathrm{d}y \,, \tag{4}$$

$$\Omega = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\omega^2(x, y)\,\mathrm{d}x\,\mathrm{d}y \,. \tag{5}$$

It should be emphasised that for all numerical runs the kinetic energy of the flow field drops from $E(t=0) = 2$ (see eq. (4)) to $E(t=0^+) \approx 1$ during the first time integration step, because the initial velocity field, with $\nabla \cdot \mathbf{u}_i \neq 0$, is then projected onto the subspace of divergence-free velocity fields. Values for the energy and the enstrophy during the simulations are normalised with their values obtained after the first time integration step. A more proper treatment for obtaining a divergence-free initial velocity field is discussed in a review paper by Gresho [7], but for the present simulations the procedure described above is sufficient; the details of the initial flow field are not important.

3. – Decaying 2D turbulence: general results

Simulations have been carried out for three values of the Reynolds number: $\mathrm{Re} = 1000$, 1500 and 2000. For each Reynolds number an ensemble of twelve simulations, initialised by different random flow fields which all have in common that the net angular momentum of these flow fields is approximately zero, have been used to compute ensemble-averaged values of several properties of the vortices. Additionally, these ensemble averages are the basis of the investigation of vortex-wall interactions and associated boundary-layer characteristics, and are used for determining the characteristic time scales of decaying 2D turbulence (including the spontaneous spin-up time scale). Present investigation requires so-called well-resolved simulations, especially when the flow in boundary layers has to be computed accurately. The qualification "well-resolved" for present simulations means that the smallest dynamically active scales are sufficiently resolved and that the decay process and boundary-layer dynamics becomes effectively independent of a further increase of the number of degrees of freedom (*i.e.* the number of Chebyshev expansion coefficients) used in the numerical simulations. In order to get a well-resolved simulation of the flow dynamics, it turned out that the minimum number of Chebyshev modes should be at least $N \approx 6\sqrt{\mathrm{Re}}$ [2].

The decay scenario of 2D turbulence is strongly modified by the presence of no-slip boundaries, and differs completely from the scenario observed for decaying 2D turbulence on a domain with periodic boundary conditions. This is nicely illustrated by a run with $\mathrm{Re} = 2000$ for which contour plots of the vorticity are shown in fig. 1. Self-organisation of the flow, due to an inverse energy cascade, is clearly visible by the increase of the average vortex radii (see, *e.g.*, the growth of the vortex in the top-left corner in figs. 1a-d). The formation of strong boundary layers as a consequence of vortex-wall interactions, and the results of the dynamical activity of the vorticity filaments, are clearly visible throughout the flow evolution. During the intermediate time scales ($t \geqslant 10$) boundary layers, together with the larger vortices, dominate the global flow evolution. Clearly, the observed flow behaviour is remarkably different from what is observed in simulations of freely evolving 2D turbulence with periodic boundary conditions, which have been reported abundantly in the literature (see, *e.g.*, refs. [8, 9]) and which are therefore not reproduced here.

Simulations with no-slip boundary conditions show thus that the boundary layers, due to shear nearby no-slip walls, play an important role in the decay of turbulence. The boundary layers serve as sources of small-scale vorticity in two different ways: either vorticity is continuously injected into the interior of the flow domain in the form of filamentary structures, or the boundary layer rolls up into a vorticity blob. In the latter case this vorticity blob usually pairs with the neighbouring (primary) vortex, thus forming a dipolar structure. Due to the process of boundary-layer activity, a rapid

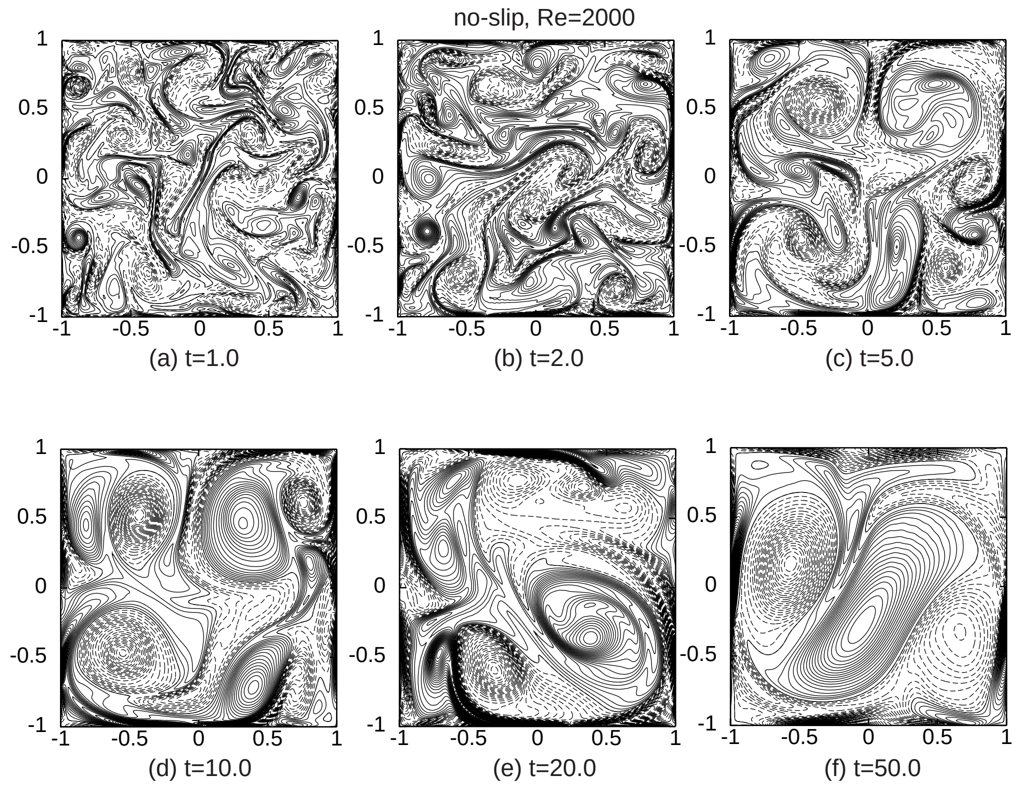

Fig. 1. – Vorticity contour plots of a simulation with no-slip boundary conditions, $\mathrm{Re} = 2000$. Dashed contours represent negative vorticity, and solid contours represent positive vorticity. The contour level increment is (a) 3, (b) 1.5, (c) 0.75, (d) 0.4, (e) 0.2, and (f) 0.1.

self-organisation of the flow towards one or two large vortices is inhibited in the early decay stage.

During the flow evolution of decaying 2D turbulence the sets of simulations with $\mathrm{Re} = 1000$, 1500 and 2000, respectively, show that three different stages can be distinguished [2]. The first one, which is relevant in present report on vortex statistics and average boundary-layer growth, is the *initial turbulent decay stage* ($0 \leqslant t \leqslant 0.2\sqrt{\mathrm{Re}}$). The average scale λ of the coherent structures, initially of the order of $\lambda = 0.01W$, grows rapidly due to merging of like-sign vortices and the formation of medium-sized dipoles (mainly by vortex-wall interactions). When the average scale of the vortices is increased to $\lambda \approx 0.4W$, vortex-wall interactions start to become a dominant feature during the further flow evolution. This is reflected in the non-monotonous decay of the enstrophy, which clearly indicates the production of vorticity in the boundary layers. The ratio of normalised enstrophy over kinetic energy, computed for the run shown in fig. 1, is plotted in fig. 2a. The non-monotonous decay of the enstrophy is clearly visible from this figure. The second stage ($0.2\sqrt{\mathrm{Re}} \leqslant t \leqslant 3\sqrt{\mathrm{Re}}$), which is also called the spontaneous spin-up stage, is characterised by strong vortex-wall interactions and the formation of coherent structures with sizes comparable with the container dimension (see fig. 1d-e). The majority of the runs

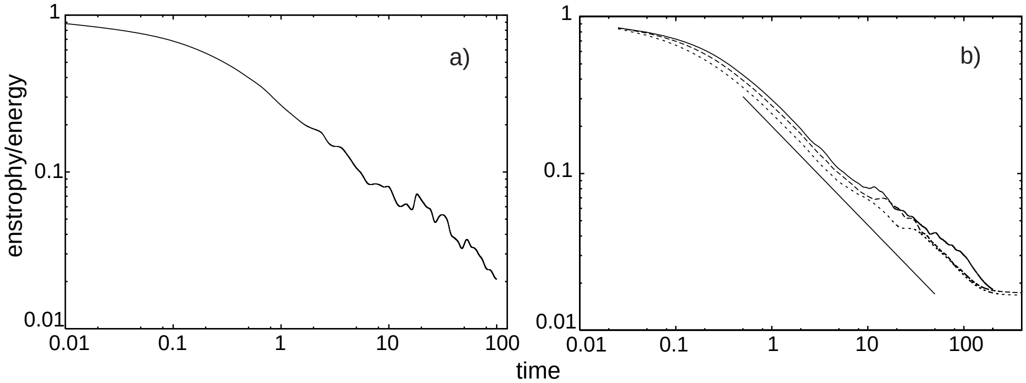

Fig. 2. – $\Omega(t)/E(t)$ for the run shown in fig. 1 (a), and the ensemble-averaged results for $\Omega(t)/E(t)$ for $\mathrm{Re} = 1000$, 1500 and 2000 (from left to right) (b). The straight line represents the decay exponent $\zeta = -0.63$.

shows a rapid growth of the (absolute value of the) angular momentum of the flow, reflecting the spontaneous spin-up, and the angular momentum decays afterwards only very slowly. The third stage ($t \geqslant 3\sqrt{\mathrm{Re}}$) shows a relaxation process to a more or less monopolar structure or a rotating tripole (see fig. 1f) that is situated in the centre of the container, subsequently followed by viscous relaxation. It is also possible to recognise a pure Stokes decay phase for $t \geqslant 6\sqrt{\mathrm{Re}}$ [10, 2].

The temporal evolution of decaying 2D turbulence in containers with no-slip walls during the initial decay stage ($0 \leqslant t \leqslant 0.2\sqrt{\mathrm{Re}}$) is characterised by merging of vortices, and consequently the number of vortices, denoted by $V(t)$, decreases. The precise role of the no-slip boundaries on the decrease of $V(t)$ is not known yet, because the boundary layer can act as a source of vorticity blobs, thus increasing the number of vortices, but the strength of vortices may also be decreased considerably after vortex-wall interactions both through the mechanisms of deformation and cross-diffusion. Such vortices are then much more susceptible to destruction by neighbouring vortices or by the background turbulent activity of the flow. One might not presume *a priori* that production and destruction of vortices due to vortex-wall interactions balance exactly, and a more detailed investigation of these processes is timely. For two-dimensional high-Reynolds number flows on an infinite domain the situation is not complicated by the presence of rigid walls, and the decrease of the vortex density is entirely due to merging processes only (and possibly by weak effects of viscosity). For this kind of flows, Carnevale *et al.* proposed a rather simple scaling theory for freely decaying 2D turbulence [3]. They assumed that both the kinetic energy of the flow and the vortex amplitude are conserved quantities during the flow evolution. Based on dimensional analysis, they found the following power law for the decay of the number of vortices: $V(t) \simeq t^{-\zeta}$. The decay exponent ζ is so far unknown, and has to be determined from numerical experiments. Both Navier-Stokes simulations (for decaying 2D turbulence with periodic boundary conditions) and modified point-vortex methods revealed that $\zeta \approx 0.70$-0.75 [3, 4]. The temporal evolution of several other properties such as the average vortex radius $a(t)$ and the enstrophy $\Omega(t)$ are: $a(t) \simeq t^{\zeta/4}$ and $\Omega(t) \simeq t^{-\zeta/2}$. Note that the enstrophy decay rate is considerably slower than predicted from the scaling theory by Batchelor [11]. In this latter theory, in contrast to the approach by Carnevale and coworkers, only the kinetic energy is assumed to be conserved and no conservation of the vorticity extremum is assumed. The power law for the enstrophy, as derived by Batchelor, is $\Omega(t) \simeq t^{-2}$, which represents indeed a much faster algebraic decay. Although the present simulations do not satisfy the conditions for freely decaying 2D turbulence in the infinite Reynolds number limit on an unbounded domain, it is still useful to compute the ensemble-averaged power law exponents based on the present simulations, and to compare them with experimental results.

For the *initial* turbulent decay stage the following power laws are found (note that both the enstrophy and the vortex amplitude had to be normalised in order to account for the decrease of kinetic energy of the flow):

$$V(t) \simeq t^{-0.90 \pm 0.03}, \tag{6}$$

$$a(t) \simeq t^{0.31 \pm 0.02}, \tag{7}$$

$$\omega_{\mathrm{ext}}(t) / \sqrt{E(t)} \simeq t^{-0.30 \pm 0.02}, \tag{8}$$

$$\Omega(t)/E(t) \simeq t^{-0.63 \pm 0.02}. \tag{9}$$

For a detailed account of the procedure to calculate the decay exponents and the calculation of the error margins, see ref. [2]. The decay exponents are found to be independent of the Reynolds number used in the present runs, thus indicating that Reynolds number dependent corrections of the decay exponents are small. A quantitative experimental study of freely decaying quasi-2D turbulence has recently been carried out by Cardoso *et al.* [12]. This investigation was aimed at measuring the power law exponents, and their results differ from the decay exponents computed with the present numerical simulations. They found the following power laws:

$$V(t) \simeq t^{-0.44 \pm 0.1}, \tag{10}$$

$$a(t) \simeq t^{0.22 \pm 0.03}, \tag{11}$$

$$\omega_{\mathrm{ext}}(t) / \sqrt{E(t)} \simeq t^{-0.22 \pm 0.06}, \tag{12}$$

$$\Omega(t)/E(t) \simeq t^{-0.44 \pm 0.06}. \tag{13}$$

Nevertheless, some similarities between the experimental and numerical results are found. For example, the decay exponent for $\omega_{\mathrm{ext}}(t)/\sqrt{E(t)}$ obtained from our simulations and those measured by Cardoso and coworkers do not differ much when the error margins are taken into account, and the ratio $\omega_{\mathrm{ext}}(t)/\sqrt{\Omega(t)}$, as computed from our numerical runs, is, like in Cardoso's experiments, approximately constant. It should be emphasised that this result is not trivial at all; the total enstrophy is a sum of ω^2 from vortices, boundary layers (containing much vorticity) and small-scale patches of vorticity from the turbulent background flow, while the vorticity extrema represent the coherent structures only.

During the *intermediate* decay stage ($0.2\sqrt{\mathrm{Re}} \leqslant t \leqslant 3\sqrt{\mathrm{Re}}$) different decay exponents are found in the present simulations for the vortex density and the

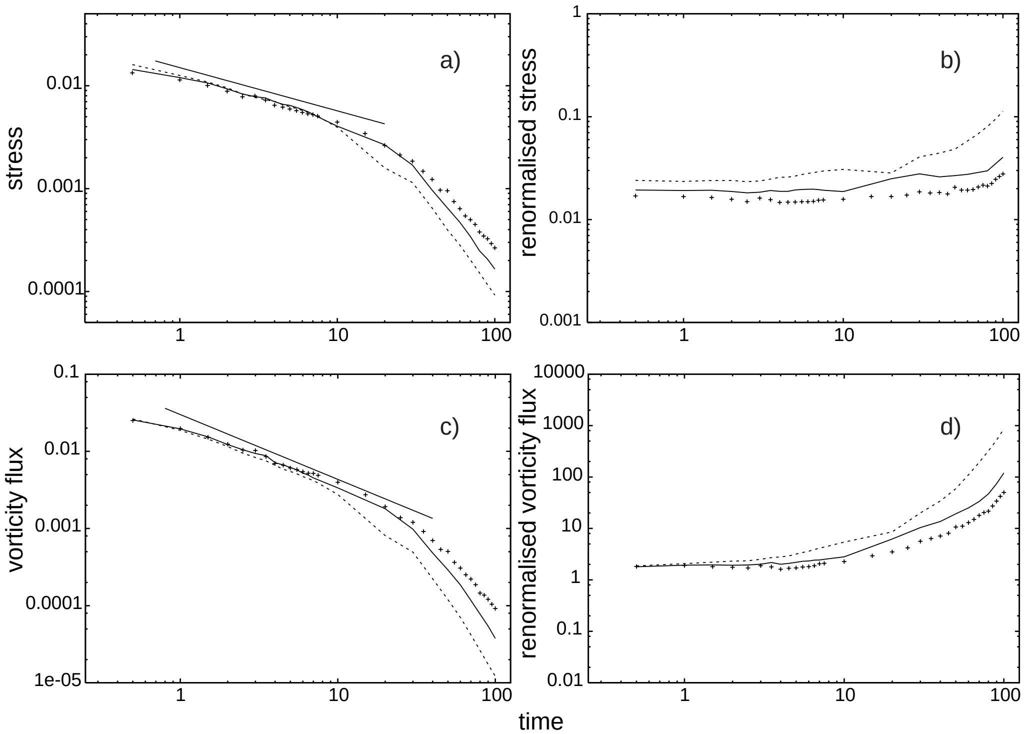

Fig. 3. – The ensemble-averaged viscous stress (a), the renormalised ensemble-averaged viscous stress (b), the ensemble-averaged value of the rescaled normal vorticity gradient (c) and the renormalised ensemble-averaged value of the rescaled normal vorticity gradient (d) plotted as a function of the dimensionless time for $Re = 1000$ (dotted), $Re = 1500$ (solid) and $Re = 2000$ (crosses). The straight lines represent the decay exponents $\zeta = -0.42$ (a) and $\zeta = -0.84$ (c).

renormalised vortex amplitude

$$V(t) \simeq t^{-0.7 \pm 0.1}, \tag{14}$$

$$\omega_{\text{ext}}(t)/\sqrt{E(t)} \simeq t^{-0.10 \pm 0.04}. \tag{15}$$

These results appear to be slightly more sensitive for variations of the Reynolds number, which might be expected because the kinetic energy of the flow has by then decreased considerably. The decay exponents in eqs. (14) and (15) agree reasonably well with some experimentally obtained values, which seem to be measured in a dynamically analogous decay regime as our results from the intermediate decay stage [13].

Finally, the power law behaviour of the renormalised enstrophy, computed in the intermediate decay regime, is the same as in the initial turbulent decay phase (see fig. 2b), although production of vorticity in the boundary layers is clearly observable.

4. – Scaling laws for the boundary layers

An essential difference between decaying 2D turbulence in containers with rigid no-slip walls and its counterpart with periodic boundary conditions is the presence of boundary layers in the former. In this section the growth of the boundary layers during the initial turbulent decay stage is discussed. It will be shown that the average boundary-layer thickness grows with a power law. Additionally, scaling behaviour of the vorticity and normal gradients of the vorticity at the boundaries has been observed.

The magnitude of the viscous stress (nondimensionalised by ϱU^2, with ϱ the fluid density and U a characteristic velocity) at the boundary of the domain with no-slip walls is computed as follows, where we have used that the normal viscous stress is absent at no-slip walls (since $\nabla \cdot \mathbf{u} = 0$),

$$S_{\mathrm{ns}} = \frac{1}{4}\sqrt{2}\,\mathrm{Re}^{-1}\left[\int_{\partial D}\left(\frac{\partial \mathbf{u}}{\partial n}\right)^2 \mathrm{d}s\right]^{1/2}, \tag{16}$$

with $\partial/\partial n$ denoting the normal derivatives, and $\mathrm{d}s$ the length of an infinitesimal element of the boundary ∂D. The numerical factor is due to normalisation by the square root of the dimensionless length of the boundary. Note that the average viscous stress is equivalent to the average magnitude of the vorticity on the boundary of the domain. Several interesting trends are observed for S_{ns}, especially during the turbulent initial decay stage. It appears that $S_{\mathrm{ns}}(t)$ is then approximately independent of the Reynolds number for $1000 \leqslant \mathrm{Re} \leqslant 2000$. It is clearly shown is clearly shown in fig. 3a that the ensemble averaged values for $S_{\mathrm{ns}}(t)$ collapse onto a single curve for the three values of the Reynolds number considered in this numerical investigation. Furthermore, the following decay rate has been found for $S_{\mathrm{ns}}(t)$ in this decay regime:

$$S_{\mathrm{ns}}(t) \simeq 0.012 t^{-0.42 \pm 0.02}. \tag{17}$$

Renormalisation of the average viscous stress by the total kinetic energy of the flow, *i.e.* computing $S_{\mathrm{ns}}(t)/E(t)$, yields another interesting result which is shown in fig. 3b. From this plot it is evident that the renormalised averaged viscous stress is approximately constant in the initial turbulent decay regime ($0 \leqslant t \leqslant 0.2\sqrt{\mathrm{Re}}$).

The ensemble-averaged value of the normal vorticity gradient near the no-slip boundaries can be quantified by computing

$$F_{\mathrm{ns}} = \frac{1}{4}\sqrt{2}\,\mathrm{Re}^{-1}\left[\int_{\partial D}\left(\frac{\partial \omega}{\partial n}\right)^2 \mathrm{d}s\right]^{1/2}. \tag{18}$$

The observed trend for the scaling behaviour of this quantity with respect to the Reynolds number of the flow is estimated to be $F_{\mathrm{ns}}(t) \simeq \sqrt{\mathrm{Re}}$. The decay rate of $F_{\mathrm{ns}}(t)/\sqrt{\mathrm{Re}}$ during the initial turbulent decay regime is (see fig. 3c)

$$\frac{F_{\mathrm{ns}}(t)}{\sqrt{\mathrm{Re}}} \simeq 0.024 t^{-0.84 \pm 0.03}. \tag{19}$$

Combination of the Reynolds number dependence of both S_{ns}, which gives an average value of the vorticity on the boundary, and F_{ns}, which gives the average normal gradient of the vorticity near the boundary, yields an expression for the average boundary-layer thickness in terms of the container size and the Reynolds number

$$\delta \simeq W(S_{\mathrm{ns}}/F_{\mathrm{ns}}) \simeq W/\sqrt{\mathrm{Re}}. \tag{20}$$

Combination of the power laws of $S_{\mathrm{ns}}(t)$ and $F_{\mathrm{ns}}(t)$ indicates that, during the initial turbulent decay stage, the boundary-layer thickness grows like $\delta(t) \simeq t^{0.4}$. Renormalisation of the ensemble-averaged normal vorticity gradient $F_{\mathrm{ns}}(t)/\sqrt{\mathrm{Re}}$ by, in this case, $E^2(t)$, leads in the initial turbulent decay regime to an approximately constant value of the renormalised vorticity gradient (see fig. 3d).

5. – Conclusion

Evidence for an important role of the boundary layers on the dynamics of decaying 2D turbulence has been presented with a set of direct numerical simulations. The role of the boundary layers near the no-slip walls during the decay process has been illustrated with ensemble-averaged results for the power-law behaviour of several characteristic properties of coherent vortices. The temporal evolution of the vortex density, the average vortex radius, the ratio of enstrophy over energy, and the extremum of vorticity (normalised by the square root of the energy) have been computed. The decay exponents of these quantities (see eqs. (6)-(9)), computed for the initial turbulent decay regime ($0 \leqslant t \leqslant 0.2\sqrt{\mathrm{Re}}$), disagree with the classical scaling theory for 2D decaying turbulence on an unbounded domain [3], which we attribute to the presence of no-slip boundaries. The decay exponents for $V(t)$, etc., appear to be independent of the Reynolds numbers considered in the present study, which supports the assumption that Reynolds number dependent corrections are, at least for $1000 \leqslant \mathrm{Re} \leqslant 2000$, small. The present results for the decay exponents do not agree satisfactorily with experimental data obtained by Cardoso *et al.* [12]. Unfortunately, it is so far not clear what causes the differences although a reason could be that the initial vortex size distribution is not the same for the experiment and the numerical runs. Decay exponents computed for the intermediate decay stage ($0.2\sqrt{\mathrm{Re}} \leqslant t \leqslant 3\sqrt{\mathrm{Re}}$), where the role of viscous effects is somewhat stronger, seem to compare rather well with experimental data by Hansen *et al.* [13]. Additionally, the temporal evolution of the boundary-layer thickness has been studied by computing the ensemble-averaged viscous stress and normal vorticity gradient near the boundaries. These computations reveal that $\delta(t) \simeq t^{0.4}$ and that the average boundary-layer thickness is proportional to $\mathrm{Re}^{-0.5}$.

Future research on vortex statistics and the role of boundary layers in decaying 2D turbulent flows in (rectangular) containers with no-slip walls is primarily aimed at three aspects. The first one concerns the power law behaviour and decay exponents in the case that the Reynolds number is increased by at least one order of magnitude. Additionally, a thorough comparison of the decay scenario of 2D turbulence in containers with periodic and with no-slip boundary conditions should be carried out. Such a comparison enables one to make definitive conclusions about the role of the boundary layers on the dynamics of decaying 2D turbulence. Finally, the effect of the initial distribution of vortex sizes should be studied in more detail. With such an investigation on is able to decide if different initial size distributions result in different decay exponents.

REFERENCES

[1] Clercx H. J. H., Maassen S. R. and van Heijst G. J. F., *Phys. Rev. Lett.*, **80** (1998) 5129.

[2] Clercx H. J. H., Maassen S. R. and van Heijst G. J. F., *Phys. Fluids*, **11** (1999) 611.

[3] Carnevale G. F., McWilliams J. C., Pomeau Y., Weiss J. B. and Young W. R., *Phys. Rev. Lett.*, **66** (1991) 2735.

[4] Weiss J. B. and McWilliams J. C. *Phys. Fluids A*, **5** (1993) 608.

[5] Clercx H. J. H., *J. Comput. Phys.*, **137** (1997) 186.

[6] Orszag S. A., *Phys. Fluids*, Suppl. II, **12** (1969) 250.

[7] Gresho P. M., *Annu. Rev. Fluid Mech.*, **23** (1991) 413.

[8] McWilliams J. C., *J. Fluid Mech.*, **146** (1984) 21.

[9] Matthaeus W. H., Stribling W. T., Martinez D., Oughton S. and Montgomery D., *Physica D*, **51** (1991) 531.

[10] Van de Konijnenberg J. A., Flór J. B. and van Heijst G. J. F., *Phys. Fluids*, **10** (1998) 595.

[11] Batchelor G. K., *Phys. Fluids*, Suppl. II, **12** (1969) 233.

[12] Cardoso O., Marteau D. and Tabeling P., *Phys. Rev. E*, **49** (1994) 454.

[13] Hansen A. E., Marteau D. and Tabeling P., *Phys. Rev. E*, **58** (1998) 7261.