THE RELATIONSHIP BETWEEN SMALL-GROUP DISCOURSE AND STUDENT-ENACTED LEVELS OF COGNITIVE DEMAND WHEN ENGAGING WITH MATHEMATICS TASKS AT DIFFERENT DEPTH OF KNOWLEDGE LEVELS

by

Kristy Litster

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Education

Approved:

Patricia S. Moyer-Packenham, Ph.D. Major Professor	Beth MacDonald, Ph.D. Committee Member
Jessica F. Shumway, Ph.D. Committee Member	Kerry Jordan, Ph.D. Committee Member
Colby Tofel-Grehl, Ph.D. Committee Member	Richard S. Inouye, Ph.D. Vice Provost for Graduate Studies

UTAH STATE UNIVERSITY
Logan, Utah

2019

ABSTRACT

The Relationship Between Small-Group Discourse and Student-Enacted Levels of Cognitive Demand when Engaging with Mathematics Tasks at Different Depth of Knowledge Levels

by

Kristy Litster, Doctor of Philosophy
Utah State University, 2019

Major Professor: Patricia Moyer-Packenham, Ph.D.
Department: Mathematics Education and Leadership

The purpose of this study was to examine the relationship between mathematics discourse and grade 5 students' enacted levels of cognitive demand (CD) when solving mathematics tasks at four different depth of knowledge (DOK) levels. To understand this relationship, this study used a quasi-experimental sequential mixed methods approach. A total of 97 students in four purposefully chosen grade 5 classrooms completed two mathematics task-sets, each spanning four DOK levels. Students worked individually on tasks in one task-set, followed by small-group Reflective Discourse of solutions and strategies. Students worked collaboratively with a small group on a second task-set, engaging in Exploratory Discourse of solutions and strategies.

The relationship between small-group discourse and student-enacted levels of CD were explored qualitatively using magnitude coding of written and video data to evaluate students' written and verbal evidence of student-enacted CD and discourse quality.

Within- and between-group relationships were analyzed quantitatively using percentage and frequency tables, graphical analyses, chi square tests, and difference in proportions tests. Finally, the relationship between types of discourse and student-enacted CD were analyzed qualitatively using descriptive pattern coding, open thematic coding, and structural coding to explain and explore quantitative results.

Results show that most students' written and verbal responses were high cognitive demand (HCD), regardless of the discourse type or intended level of the tasks. Results also found that although there were no significant differences in the overall quality of the mathematical discourse between the two different types of discourse, there were significant differences in quality when students engaged in different practices to organize the discussion or student responsibilities within the group. Additionally, results showed that while Exploratory Discourse tended to isolate typically struggling students, a supportive environment, such as the environment created by Reflective Discourse, helped to support typically struggling students in this study. Finally, this research reinforced the importance of dissonance in prompting students to engage with the mathematics tasks at higher levels of cognitive demand. These results show that teachers can use small-group discourse as an effective classroom practice to promote HCD in mathematics, regardless of the intended DOK of the task.

(184 pages)

PUBLIC ABSTRACT

The Relationship Between Small-Group Discourse and Student-Enacted Levels of Cognitive Demand when Engaging with Mathematics Tasks at Different Depth of Knowledge Levels

Kristy Litster

High cognitive demand (HCD) tasks can help students develop a deeper understanding of mathematics. Teachers need interventions that encourage students to engage in HCD activities. Small-group discourse provides HCD opportunities for students while solving mathematics problems. Discourse can take place after students solve problems individually (reflective) or in groups as students solve problems (exploratory). This study looks at the relationship between these two types of small-group discourse and student-enacted cognitive demand.

This study looks at how students engage with tasks that were designed at four different cognitive demand levels using Webb's depth of knowledge (DOK) framework. Ninety-seven grade 5 students from four different classrooms were grouped in small groups of two or three students to solve two sets of mathematics problems on operations with fractions and decimals. Each class engaged in Reflective Discourse after solving one set and engage in Exploratory Discourse while solving the other set. To help understand any order effects, half the classes used Reflective Discourse with Set 1 while the other half used Exploratory Discourse with Set 1. Then, they switched for Set 2, so that whoever used Reflective Discourse with Set 1 used Exploratory Discourse with Set 2 and vice versa.

The researcher analyzed whether there were patterns in levels of cognitive demand and quality of the discussion when students engaged in each type of discourse for math problems at four different levels. First, the researcher looked at any numerical differences between the intended cognitive demand of the problems and how students engaged with the problems using frequency tables, heat maps, and statistical analyses. Next, the researcher looked at differences in student actions and the way they talked about the math problems.

Findings showed that both Reflective and Exploratory Discourse can be used by teachers to promote high student-enacted levels of cognitive demand. Results also showed that a supportive environment, such as the environment created by Reflective Discourse, can help support typically struggling students. Finally, this research reinforced the importance of dissonance in prompting students to engage with the tasks at higher levels of cognitive demand.

ACKNOWLEDGMENTS

I would like to especially thank Dr. Patricia Moyer-Packenham for her time and mentorship throughout my entire doctoral experience and particularly in the design, implementation, and writing of this dissertation. I would like to thank my committee members, Drs. Beth MacDonald, Jessica F. Shumway, Colby Tofel-Grehl, and Kerry Jordan, for their support and assistance through-out the entire process. I would also like to thank Dr. Sarah Schwartz for her help and expertise in refining my tables and graphics for clarity and readability.

I give special thanks to my husband Ben and children Nicole, Matthew, Christopher, Jacob, and Jeremy, for their encouragement, moral support, and patience as I worked my way from my first doctoral course to this final document. I could not have done it without all of you.

Kristy Litster

CONTENTS

Page

ABSTRACT iii

PUBLIC ABSTRACT v

ACKNOWLEDGMENTS vii

LIST OF TABLES x

LIST OF FIGURES xiii

LIST OF VIDEO EXCERPTS xv

CHAPTER

1. INTRODUCTION 1

Background of the Problem 2
Problem Statement 3
Significance of the Study 5
Purpose of the Study and Research Questions 6
Summary of the Research Design 7
Scope of Study 7
Definition of Terms 9

2. LITERATURE REVIEW 11

Historical Overview of Cognitive Demand 11
Current Research Findings in Cognitive Demand 21
Conceptual Framework 24
Summary 29

3. METHODS 30

Research Design 30
Research Questions 31
Participants and Materials 31
Data Sources 34
Procedures and Data Collection 37
Data Analysis 42

Page

4. RESULTS 60

Research Question 1: Written CD vs Intended DOK 60
Research Question 2: Intended DOK vs. Verbal CD 69
Research Question 3: Reflective vs. Exploratory Discourse Contributions 78
Overarching Research Question 93

5. DISCUSSION 100

Relationships Between Discourse Type and Student-Enacted Levels of Cognitive Demand 101
Equitable Participation for Typically Struggling Students 103
Importance of Dissonance in Small-Group Discourse 104
Implications for Teachers 106
Limitations and Suggestions for Future Research 107
Conclusion 107

REFERENCES 109

APPENDICES 119

Appendix A: IRB Approval 120
Appendix B: Task-Sets 1 and 2 122
Appendix C: Coding Protocol 128
Appendix D: Written Cognitive Demand Quantitative Tables 137
Appendix E: Verbal Cognitive Demand Quantitative Tables 142
Appendix F: Mathematical Discourse Contributions Quantitative Tables 147

CURRICULUM VITAE 154

LIST OF TABLES

Table Page

3.1 Participation and Demographics by Class 33

3.2 Scope of the Written Data Sources 35

3.3 Scope of the Verbal Data Sources 36

3.4 Timeline of the Study........ 37

3.5 Mathematical Discourse Contributions Coding Rubric........ 44

4.1 Chi-Square Test of Independence for Written CD by Discourse Type 62

4.2 Chi-Square Test of Independence for Verbal CD by Discourse Type........ 71

4.3 Frequency of Time Spent Discussing Mathematics Tasks in Minutes........ 79

4.4 Instances of Nonmathematical Discourse........ 87

4.5 Number of Tasks Completed by DOK Level for each Organizational Practice........ 94

B.1 DOK Level for All Tasks in each Task-Set........ 123

C.1 Written Rubric with Sample Evidence 130

C.2 Sample Coding Verbal CD 131

C.3 Double Coder Procedures 134

D.1 Reflective Task-Set 1 Overall Percentage Written........ 138

D.2 Exploratory Task-Set 1 Overall Percentage Written 138

D.3 Reflective Task-Set 2 Overall Percentage Written........ 138

D.4 Exploratory Task-Set 2 Overall Percentage Written 139

D.5 Difference in Proportions Tests of Association for Intended DOK vs. Written CD........ 139

Table Page

D.6 Frequencies of Written CD Organized by Tasks and Discourse 140

D.7 Chi-Square Comparing Reflective and Exploratory Written CD by Task-Set 141

E.1 Reflective Task-Set 1 Overall Percentage Verbal 143

E.2 Exploratory Task-Set 1 Overall Percentage Verbal 143

E.3 Reflective Task-Set 2 Overall Percentage Verbal 143

E.4 Exploratory Task-Set 2 Overall Percentage Verbal 144

E.5 Difference in Proportions Tests of Association for Intended DOK vs. Verbal CD 144

E.6 Frequencies of Verbal CD Organized by Tasks and Discourse 145

E.7 Chi-Square Tests of Independence for Verbal CD 146

F.1 Frequency of Coded Discourse Contribution Levels, Organized by Tasks and Discourse 148

F.2 Sub Codes of Mathematical Discourse Contributions for Intended DOK Levels 149

F.3 Percentage Discourse Contribution Order by DOK Level 150

F.4 Percentage Discourse Contribution Priority by DOK Level 150

F.5 Percentage Discourse Contribution Together by DOK Level 150

F.6 Percentage Discourse Contribution Side-by-Side by DOK Level 151

F.7 Percentage Discourse Contribution Split Up by DOK Level 151

F.8 Percentage Discourse Contribution Take Over DOK Level 151

F.9 Percentage Written CD Order by DOK Level 152

F.10 Percentage Written CD Priority by DOK Level 152

Table Page

F.11 Percentage Written CD Together by DOK Level 152

F.12 Percentage Written CD Side-by-Side by DOK Level....................................... 153

F.13 Percentage Written CD Split Up by DOK Level 153

F.14 Percentage Written CD Take Over DOK Level 153

LIST OF FIGURES

Figure		Page
2.1	Memory encoding and retrieval	12
2.2	A comparison of three influential cognitive demand frameworks	17
2.3	Discourse and interpretation influence on cognitive demand	25
3.1	Relationships between data sources and phases of analysis	42
3.2	Percentage table comparing Intended DOK and Written CD	49
3.3	Sample heat map color scheme	50
3.4	Tint saturation values	51
4.1	Heat maps of Intended DOK vs Written CD for each crossover group	61
4.2	Samples of student work comparing reflective and exploratory responses	64
4.3	Samples of student work for four DOK2 tasks	66
4.4	Heat maps of intended DOK vs verbal CD for each crossover grouping	70
4.5	Reflective discourse level 4 written and verbal CD sample for Task-Set 2	74
4.6	Exploratory discourse Level 4 written and verbal CD sample for Task-Set 2	75
4.7	Discourse contributions for tasks at each DOK level	80
4.8	Categories of mathematical discourse contributions	84
4.9	Quality of mathematical discourse contributions and verbal CD based on organizational practices and intended DOK levels	95
4.10	Quality of written CD based on organizational practices and Intended DOK levels	98
B.1	Task-Set 1	124

Figure Page

B.2 Task-Set 2 126

C.1 DOK wheel coding rubric 129

C.2 Sample coded written CD 135

LIST OF VIDEO EXCERPTS

Page

[Video 410H2, 2:31-4:37] ... 72

[Video 280H2, 1:52-2:25] ... 73

[Video 310D3, 2:45-3:04] ... 74

[Video 270D4, 1:37-4:35] ... 75

[Video 370H1, 0.00-5:48] ... 85

[Video 490D1, 1:05-2:10] ... 86

[Video 460H2, 11:30-13:20] ... 89

[Video 460D2, 2:51-3:54] ... 91

[Video 210H1, 0:31-2:35] ... 92

CHAPTER 1

INTRODUCTION

High student-enacted levels of cognitive demand can promote deeper student understanding of mathematics properties and procedures, increase students' ability to solve related mathematics problems, and reinforce mathematics connections (Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996; Webb, 1999). Unfortunately, research shows that students' generally decrease their engagement with the mathematics of a task and lower their enacted levels of cognitive demand (Charalambous & Litke, 2018; Kessler, Stein, & Schunn, 2015; Otten, 2012). Mathematics discourse can increase student involvement in mathematics tasks (Brodie, 2011; Hufferd-Ackles, Fuson, & Sherin, 2004; National Council of Teachers of Mathematics [NCTM], 2014; Wouters, Van Nimwegen, Van Oostendorp, & Van Der Spek, 2013), which can increase opportunities for students to enact the tasks at a higher level of cognitive demand (Brodie, 2011; Charalambous & Litke, 2018). Small-group discourse further increases equitable access to participation for all students (Hung, 2015). This study will focus on two specific types of small group mathematics discourse (i.e., Reflective Discourse and Exploratory Discourse) and their relationship with the levels of cognitive demand (i.e., number and types of mental connections required to solve a specific task) enacted by students (called "student-enacted" in the remainder of this proposal) when engaging with mathematics tasks designed to engage students at four different levels of cognitive demand. In this study, different levels of cognitive demand are identified using Webb's (1999) Depth of Knowledge (DOK) model.

Background of the Problem

In 2017, only 40% of Grade 4 students and 34% of Grade 8 students in the U.S. were able to solve mathematics tasks at or above proficiency (Institute of Education Sciences [IES], 2018). Low student-enacted levels of cognitive demand when solving mathematics tasks contribute to these low proficiency scores. Results from the 2015 Trends in International Mathematics and Science Study (TIMSS) show that, although U.S. students ranked 9 of 46 countries in their ability to solve Depth of Knowledge (DOK) Level 1 tasks ("knowing"), their ranking dropped to 17 for DOK Level 2 tasks ("applying") and dropped to 22 for DOK Level 3 tasks ("reasoning"; IES, 2015). This shows that as the cognitive demand of the mathematics tasks increase, students' ability to solve the tasks at or above proficiency decreases, resulting in low overall proficiency.

In an effort to improve students' ability to solve more cognitive demanding mathematics tasks, research and practitioner articles recommend that teachers implement High Cognitive Demand (HCD) tasks (i.e., DOK 2-4; e.g., Keazer & Gerberry, 2017; Orrill, 2015; Schoenfeld, 2018; Tan, Ismail, & Abidin, 2018). Unfortunately, several factors impede the implementation of HCD tasks. First, research shows that conventional and standards-based mathematics textbooks contain mostly Low Cognitive Demand (LCD) tasks, which limits teachers' access to HCD tasks (Jones & Tarr, 2007; Moreno Alcázar, 2007; Porter, MacMaken, Hwang, & Yang, 2011; Tan et al., 2018). Second, many teacher actions and dispositions during implementation of HCD mathematics tasks generally decrease the intended levels of cognitive demand (Hong & Choi, 2016; Mdladla, 2017). Third, students' actions and dispositions also decrease their involvement in the mathematics task and lower the level of cognitive demand (Charalambous & Litke, 2018; Kessler et al, 2015; Otten, 2012). These results indicate that how students engage with mathematics tasks, and the practices teachers employ to increase this engagement, may be more important than the intended DOK levels of the tasks themselves.

Problem Statement

The Common Core State Standards for Mathematical Practice (SMP) explain that students who engage in high student-enacted levels of cognitive demand “continually ask themselves, ‘Does this make sense?’” [SMP 1] and “construct arguments using concrete referents” [SMP 3] (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). *Mathematical discourse*, a discussion between two or more students that focuses on specific mathematics properties or procedures, provides students with opportunities to engage in these two mathematical practices (NCTM, 2014) as they share and clarify mathematical ideas (Brodie, 2011; Hufferd-Ackles et al., 2004; Wouters et al., 2013).

Mathematics discourse can take place with the whole class or in small groups. Large group discourse can sometimes limit opportunities for every student to participate in the discourse, while small-group discourse increases equitable access to participation for all students by increasing their opportunities to talk (e.g., 1 of 3 vs. 1 of 30) and promotes collaborative dialogue as students can quickly respond to other students’ ideas without waiting for teacher permission (Charalambous & Litke, 2018; Coakley, 2018; Hung, 2015; Otten, 2012; Williams, 2010; Yackel, Cobb, & Wood, 1991). This is problematic, as much of the research in mathematics discourse focuses on whole class settings (e.g., Brodie, 2011; Erath, Prediger, Quasthoff, & Heller, 2018; Hufferd-Ackles et al., 2004; Wouters et al., 2013). Hence, there is need for more research on small-group discourse, such as the research in this study.

Research on mathematics discourse is divided on the timing of discourse - whether it should be after (reflective) or during (exploratory) student engagement with the mathematical tasks. Reflective Discourse takes place after students have had an opportunity to engage with the tasks. Walter (2018) explains that delaying discourse until after students have had sufficient time to process the mathematics or write down their own ideas can increase the quality of discussion and promote the inclusion of students who might otherwise be ignored. This increases opportunities for students to bring contrasting ideas to the discussion, which may potentially increase students' engagement in HCD activities such as justifying their own ideas or using their ideas as a counterargument. Rojas-Drummond and Mercer (2003) contest that waiting to engage in discourse until after the task is complete results in cumulative talk where students simply agree with one specific idea and do not elaborate on or discuss contrasting ideas. Instead, they recommend using Exploratory Discourse, which takes place during engagement with the task and allows students to jointly discuss relevant information in a timely manner. A joint discussion could also provide opportunities for students to justify their own ideas or use their ideas as a counterargument while strategizing about how to solve the tasks. However, there is little to no research specifically examining relationships between these two types of small-group mathematics discourse and student-enacted levels of cognitive demand. This indicates a critical need for the research proposed in this study on the relationship between reflective and exploratory small-group mathematics discourse and student-enacted levels of cognitive demand.

Significance of the Study

Research on student mathematical discourse shows that it can increase opportunities for student reasoning and engagement with mathematics tasks. Research on reflective and exploratory shows that each type of discourse is important. Reflective Discourse can promote equity by providing time for students to prepare their ideas, which provides the opportunity for all students to be prepared to share their thinking (not just students who think of an idea quickly). Exploratory Discourse can promote collaboration by providing opportunities for students to share their own ideas and build upon the ideas of other students. However, no study has looked simultaneously at both types of discourse in small group settings, settings that promotes both equity and collaboration due to group size. This study is significant because it provides a unique perspective on the timing of mathematics discourse. It contributes to current research by using a quantitative examination of how the affordances of the two types of discourse are related to student-enacted levels of cognitive demand and by using a qualitative examination of student behaviors that influence this relationship.

Research on HCD tasks shows increased opportunities for students to deepen their understanding of mathematics concepts and procedures. Although research identifies some teacher practices that can maintain high levels of cognitive demand, no study shows how student practices might influence levels of cognitive demand (e.g., Georgius, 2014; Morgan & Power, 2016; Williams, 2010). This study is significant because it will look at how the two types of discourse may support student understanding when engaging with Low Cognitive Demand (LCD) and HCD tasks. Knowledge gained from this study adds to the literature by highlighting specific student practices that support high student-enacted levels of cognitive demand when solving mathematics tasks. Teachers can potentially model these practices to encourage student use in mathematics classrooms.

Purpose of the Study and Research Questions

The purpose of this study was to examine the relationship between mathematics discourse and student-enacted levels of cognitive demand when students solve mathematics tasks at four different Depth of Knowledge levels. The overarching mixed methods research questions guiding this study was: What is the relationship between two types of small-group discourse and student-enacted levels of cognitive demand when engaging with mathematics tasks at four different intended Depth of Knowledge (DOK) levels? Evidence of student-enacted levels of cognitive demand (i.e., evidence of thinking or reasoning) can be identified from either written or verbal responses to the tasks. As such, there are three specific questions.

1. What are the differences within and between groups for intended DOK levels of the mathematical tasks and student-enacted levels of cognitive demand for written responses relating to the mathematical tasks?
2. What are the differences within and between groups for intended DOK levels of the mathematical tasks and student-enacted levels of cognitive demand for verbal responses relating to the mathematical tasks?
3. What are the differences in quantity and quality of mathematical discourse

> contributions during small-group Reflective Discourse (after students individually solve mathematical tasks at four DOK levels) and Exploratory Discourse (while students solve mathematical tasks as a group at four DOK levels)?

Summary of the Research Design

This study used a sequential mixed methods approach (Creswell & Plano Clark, 2011; Tashakkori & Teddlie, 2010) with a quasi-experimental crossover design (Shadish, Cook, & Campbell, 2002). Qualitative data, in the form of student written responses to mathematics tasks and video data of student verbal responses to the same tasks, was collected over the course of two weeks to answer the research questions.

Data analyses included within- and between-group analyses and focused on written evidence of student-enacted levels of cognitive demand, verbal evidence of student-enacted levels of cognitive demand, verbal evidence of quantity and quality of discourse, categorizing and interpreting evidence, and exploring relationships.

Scope of Study

There are three common models that categorize different levels of cognitive demand: Doyle's (1983) Categories of Academic Tasks, Smith and Stein's (1998) Mathematics Tasks Framework, and Webb's (1999) DOK. As explained further in the literature review, Webb's DOK model of cognitive demand provides a balanced model that clearly defines different levels of high and low cognitive demand mathematics tasks. Level 1 provides a clear outline of low cognitive demand tasks. Levels 2-4 provide a breakdown of HCD tasks into three distinct levels ranging from surface level connections to highly integrated levels of mathematical and contextual connections. This study uses Webb's DOK model because it is the only model that differentiates HCD into three distinct levels, which is critical to understanding the nuances among students' discourse at each level. In the remainder of this proposal, intended levels of cognitive demand for specific tasks was identified using the DOK levels identified during task design (e.g., intended DOK1).

Although cognitive demand research addresses all three phases of teaching (i.e., intended design, teacher implemented, and student enacted), the scope of this study is confined to only the student enacted portion. This allows for a specific focus on student reasoning and connections when engaging with tasks at different intended DOK levels. Evidences of student-enacted levels of cognitive demand (written and verbal) were coded using the DOK model. However, to avoid confusion between intended and enacted levels of cognitive demand, written and verbal evidence identifying student-enacted levels of cognitive demand is identified as written or verbal cognitive demand and the associated DOK level (e.g., written evidence of enacted DOK1 = written CD1) throughout the remainder of this proposal.

Additionally, this research focuses on Grade 5 students. By Grade 5, students should have the ability to verbalize reasoning related to their own strategies as well as other students' strategies and reasoning. Grade 5 students also have a variety of elementary mathematics classroom experiences from previous grade levels to draw upon during their engagement in mathematics discourse. This is important to this study in order to capture students' verbalization of possible internal reasoning and connections. Future research will be needed to expand these findings towards earlier or later grade levels.

Definition of Terms

Cognitive demand – The number and strength of the connections within and between mental networks, or schema, to solve a specific task (Webb, 1997).

Levels of cognitive demand – Cognitive demand is organized in levels based upon the number of schema connections anticipated to be involved in the thinking, reasoning, or processes elicited by a specific task (Stein & Lane, 1996). *Low Cognitive Demand* requires direct retrieval of facts and procedures from short- or long-term memory, which elicits the least number of schema connections to complete recall or procedural tasks. *High Cognitive Demand* requires two or more schema connections to make inferences or connections between mathematical ideas or context. *Models of cognitive demand*, such as Webb's Depth of Knowledge*,* organize differentiations between the thinking, reasoning or processes required for subset levels of low and high cognitive demand.

Phases of cognitive demand – There are three phases of cognitive demand: Intended, Implemented, and Enacted (Stein & Smith, 1998). The level of cognitive demand for a specific task can vary based upon the perceptions of the main instigator for each phase. The task designer is the main instigator for *Intended Cognitive Demand.* During this phase, the designer has a perception regarding the thinking, reasoning, or processes a task will elicit from students in order to complete the task. The teacher is the main instigator for *Implemented Cognitive Demand*. During this phase, the teacher interprets how to present and scaffold a designed task for their specific students. The way in which teachers implement the task within their instruction can alter (or maintain) the cognitive demand by changing (or maintaining) the thinking, reasoning, or processes elicited by the task. The student is the main instigator for *Enacted Cognitive Demand.* During this phase, the student interprets the task and enacts specific thinking, reasoning, or processes to complete it. These levels of thinking, reasoning or processes may mimic the hypothesized cognitive demand intended by the design or may change based on student perception and interpretation.

Mathematics discourse – Mathematics discourse is a discussion between two or more students to exchange ideas regarding how to complete a mathematics task (Hufferd-Ackles et al., 2004).

Quality of discourse is ranked using a scale based upon the depth of the mathematical content within student contributions to the mathematics discourse (Bishop, Hardison, & Przybyla-Kuchek, 2016; Cobb & Yackel, 1996; Durfee, 2018; Walter, 2018). On one end of the scale, *Low Quality* discourse contributions pertain to surface level conversations relating to basic recall of facts and organizing actions and responsibilities. On the other end of the scale, *High Quality* discourse contributions pertain to deeper levels of mathematical connections such as justifying strategies or extending results to relate to multiple sources.

Reflective discourse – Discourse that occurs after students have had time to process and/or write about (i.e., solve) the mathematics tasks (Walter, 2018).

Exploratory discourse – Discourse that occurs during the solution process for the mathematics tasks (Rojas-Drummond & Mercer, 2003).

CHAPTER 2

LITERATURE REVIEW

Two areas of literature influenced the design and implementation of this study: cognitive demand and mathematics discourse. First, this literature review briefly discusses a historical overview of cognitive demand and the relationship between levels of cognitive demand and student learning. Second, it summarizes current research findings related to cognitive demand (2008-2018), with an emphasis on student-enacted levels of cognitive demand. Finally, the conceptual framework discusses the relationship between two phases of cognitive demand (intended and enacted) and high-quality mathematics discourse.

Historical Overview of Cognitive Demand

Current research and practitioner models of cognitive demand have historical roots in the works of Carpenter and Fennema (1988), Greeno (1976), and Resnick and Ford (1981) who, in turn, based their work primarily on Cognitive Processing Theory. Cognitive Processing Theory is a learning theory that explains how students learn declarative knowledge, "knowledge 'about' things," or procedural knowledge, "knowledge about 'how to do' things" (Gagne, Yekovich & Yekovich, 1993, p. 91) and encode or retrieve this knowledge from short- and long-term memory. As seen in Figure 2.1, outside input is encoded to short-term memory and then stored in long-term memory as either declarative or procedural knowledge. Declarative knowledge can be retrieved from long-term memory and matched with procedural knowledge, which when retrieved,

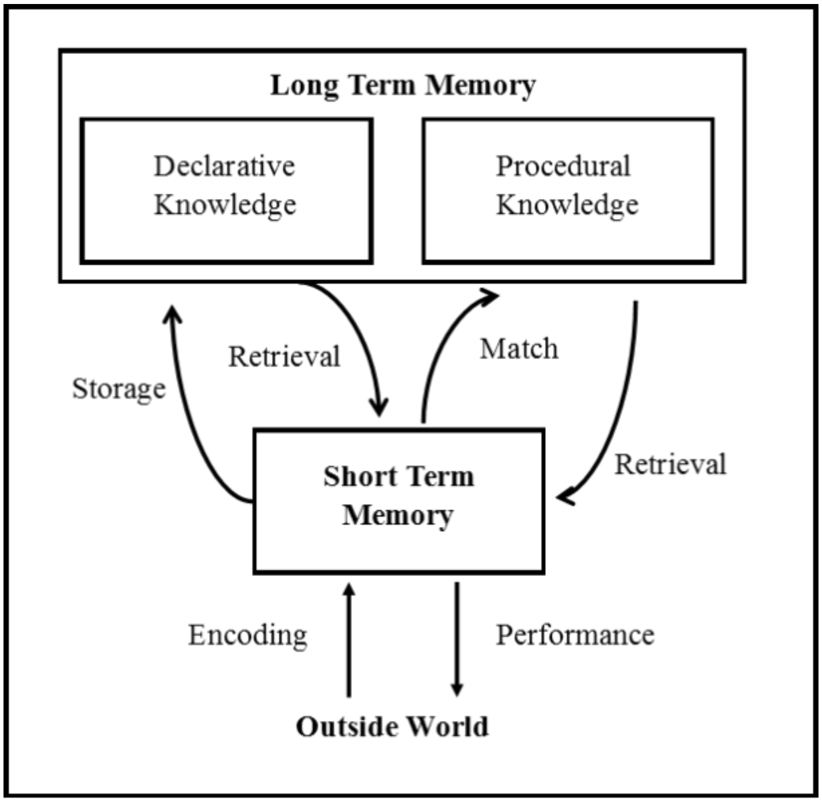


Figure 2.1. Memory encoding and retrieval (adapted from Gagne et al., 1993).

can influence students' physical performance (Anderson, 1982; Gagne et al., 1993). Sweller and Mayer (2015) explains that long-term memory provides the justification for the education process - when things are changed in long-term memory, they are learned.

A person's long-term memory is made of up a mental network of nerves that allows different senses to be stored and retrieved, such as images, sounds or even smells (Wittrock, 1992). Data structures, known as schemes, are built up in long-term memory to match associations between various declarative and procedural knowledge in an organized manner (Cowan, 2000). Schemes in long-term memory help facilitate inferences on what should be done, when it should be done, and how it should be done (Gagne et al., 1993; Mayer, 2006). Nussbaum (1999) explains that schemes allow a person to create a new series of neurons, create connections between schemes, or remove connections as old schemes are rejected for new ones. The number and strength of the connections within and between schemes related to a specific task are referred to as the cognitive demand of the task (Webb, 1997). Cognitive demand is sometimes confused with cognitive load; however, cognitive load focuses primarily on capacity of short-term memory, while cognitive demand focuses primarily on schemes activated in long-term memory (Grobman, 2015; van Merrienboer & Sweller, 2005). Low cognitive demand tasks many only activate one scheme to retrieve information; however, as the level of cognitive demand for a task increases, the number of schemes activated to complete the task also increases. The activation of multiple schemes can help students build a deeper and more flexible understanding and application of mathematics (Rittle-Johnson, Star, & Durkin, 2012).

Carpenter and Fennema (1988), Resnick and Ford (1981), and Greeno (1976) each proposed that student-enacted levels of cognitive demand directly relate to retention of what students learn (i.e., the ability to recall procedural or declarative knowledge) as well as the ability to apply that learning to novel problems or situations.

Resnick and Ford's (1981) work was based on the idea that "one of the fundamental assumptions of cognitive learning psychology is that new knowledge is in large part 'constructed' by the learner" (p. 249). She emphasized that the process of building new relationships for students is a key component of learning. Although this information about how students learn is interesting, many researchers wanted a more practical application of these theories. In order to use this information to facilitate research, Thomas Carpenter and Elizabeth Fennema developed a model for research and curriculum development. In this model, teachers' decisions affect classroom instruction, which in turn, affects students' cognitions and learning (Carpenter & Fennema, 1991).

Greeno (1976) focused his research relating to cognitive demand specifically on the tasks used in classroom instruction. He proposed that the goals of instruction could be inferred from the tasks that students perform. He also noted that researchers could analyze the cognitive structures that students would need to engage in to successfully complete mathematical tasks, which in turn, would help ensure that instruction promotes the most important goals. He explained that true cognition is situated within the circumstances of the learning.

One disadvantage of cognitive processing theory is that, although instructors and researchers can make inferences based on student actions and responses, learning takes place within the student's mind and is invisible. Based on the work of Carpenter and Fennema (1988), Resnick and Ford (1981), and Greeno (1976), Doyle (1983) developed a model to help identify and understand some of the processes and thinking that students would need to mentally engage with to be successful in completing different instructional tasks.

Categories of Academic Tasks Framework

Walter Doyle (1983) was one of the first researchers to create a framework to categorize four different levels of cognitive demand, which he called Categories of Academic Tasks. This is significant because it set the precedence of not only leveling cognitive demand, but inferring that there is a difference in student learning based on those levels. His framework was also influential in the design of Smith and Stein's (1998) Mathematics Tasks Framework, which is used in the majority of current mathematics research on cognitive demand.

Mathematics Tasks Framework

Stein et al. (1996) developed the first cognitive demand framework focused specifically on mathematics - the Mathematics Tasks Framework. They developed this framework to help stratify 144 mathematical tasks within their research study across four levels of cognitive demand: memorization, procedures without connections, procedures with connections, and doing mathematics. Stein et al. also adapted Doyle's definition of an academic task to apply specifically to mathematics. In their framework, a *mathematical task* relates to a specific mathematical idea. One lesson may have several tasks, with varying levels of cognitive demand that change as the foci changes to address different conceptual or procedural aspects of the lesson topic. This definition of a mathematical task was used to determine the grain size for the coding analysis in this study.

Using the Mathematics Task Framework, Stein and Lane (1996) reported a link between student gains on a performance assessment designed with cognitively demanding tasks and HCD instructional classroom tasks. In contrast, student performance was low for control groups that used low cognitive demand instructional classroom tasks or classrooms tasks that required little or no mathematics discussion. This was significant as it directly linked the intended and implemented cognitive demand of classroom tasks and student learning.

In 1998, Smith and Stein printed the complete Mathematics Tasks Framework in the practitioner journal *Mathematics Teaching in the Middle School*, along with a task sorting activity a few years later to allow teachers to practice identifying the intended cognitive demand of tasks (Smith, Stein, Arbaugh, Brown, & Mossgrove, 2004). Their goal was to bring their research directly to teachers as a framework to promote the practice of selecting HCD tasks for students. Current researchers who cite Doyle (1996) or Smith and Stein (1998) directly, often reproduce their models completely as a figure or appendix to work as a task analysis guide in their research (e.g., Jamieson, 2015; McCormick, 2016; Sherman, 2011). The early research on the Mathematics Task Framework influenced, in part, the development of Webb's Depth of Knowledge which is currently gaining practitioner attention.

Webb's Depth of Knowledge Framework

Webb (1997) developed a framework with four levels of cognitive demand to analyze the alignment between state mathematics standards and test items in standardized assessments and called the framework Depth of Knowledge (DOK). The four levels of the framework increase from low cognitive demand (DOK 1, Recall) to three different levels of HCD (DOK 2, Application; DOK 3, Strategic Thinking; and DOK 4, Extended Thinking). In 1999, Webb published a conceptual Depth of Knowledge framework for easy practitioner reference as well as training materials in 2005 (http://wat/wceruw.org/index.aspx). The DOK framework is being used by curriculum companies, such as Nextlesson Inc. to design instructional task-sets than span all four DOK levels for K-12 classrooms. Two Nextlesson task-sets were used in this study.

Comparison of Three Influential Cognitive Demand Frameworks

The three frameworks described above are not the only frameworks for organizing levels of cognitive demand. For example, Schoenfeld (2018) used the TRU Framework in analyzing cognitive demand, while Tan et al. (2018) used the Cognitive Dimensions of TIMSS 2015 Mathematics Framework. However, the three aforementioned models (Categories of Academic Tasks, Mathematics Tasks Framework, & Depth of Knowledge) are the most influential frameworks for organizing levels of cognitive demand in current classroom research and practice. Figure 2.2 compares the similarities among these three historical cognitive demand frameworks. For ease of

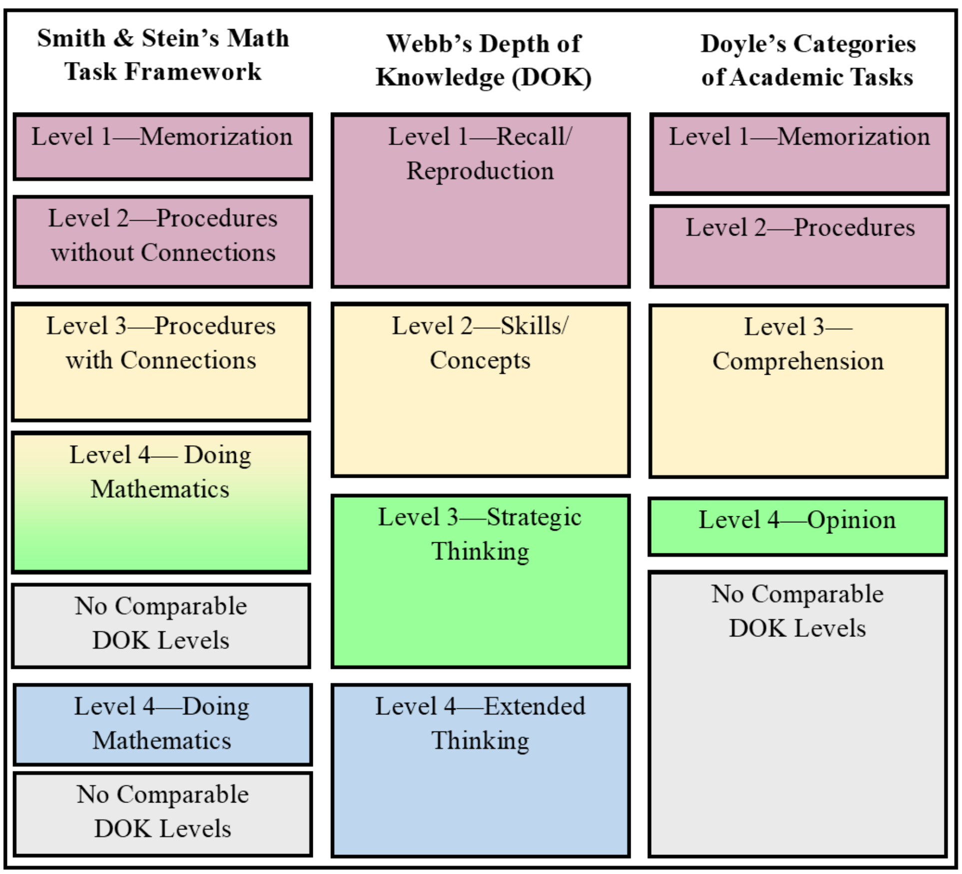


Figure 2.2. A comparison of three influential cognitive demand frameworks.

reference, all frameworks are compared to Webb's Depth of Knowledge, the framework used in this study, to better understand possible overlap or gaps.

Depth of Knowledge Level 1. As shown in Figure 2.2, Webb's (1999) first, and lowest level of cognitive demand, is recall/reproduction. Encoding and retrieval of declarative or procedural knowledge is the basis for Depth of Knowledge (DOK) level 1. Students passively retrieve basic declarative knowledge to recall memorized facts and perform basic sets of procedural behaviors. (DOK) Level 1 encompasses both level 1 and level 2 in each of the other two models. Smith and Stein (1998) define memorization tasks and simple procedural tasks as low cognitive demand tasks.

Depth of Knowledge Level 2. Webb's (1999) second cognitive demand level is skills/concepts. Where DOK Level 1 focuses on activating specific pieces of declarative or procedural knowledge, activation of schemes in long term memory are the basis of DOK Level 2. Schemes in long-term memory help facilitate student inference on what should be done, when it should be done, and how it should be done (Gagne et al., 1993; Mayer, 2006). At DOK Level 2, students encounter new situations or are asked to apply information or concepts to make decisions about how to approach a problem or activity beyond an automated response.

This response usually entails more than one step in the solution process. Students are still applying procedures; however, procedures are not applied mindlessly, nor without context. Students often apply learned procedures to new situations, such as word problems. For the task to truly require conceptual understanding, the student must have a variety of procedures to choose from and identify the correct one to accomplish their task.

Webb's Depth of Knowledge Level 2 aligns with Level 3 in the other two models and also encompasses part of Smith and Stein's (1998) highest level, doing mathematics, due to the inclusion of non-algorithmic thinking. Webb (1999) defines Depth of Knowledge Level 2 tasks as mid-level cognitive demand tasks; however, Smith and Stein (1998) designate tasks at this comparative level, as well as any higher-level tasks, as HCD demand. Therefore, for the purposes of this study, any tasks that are DOK Level 2 or higher are designated as HCD.

Depth of Knowledge Level 3. Webb's (1999) third cognitive demand level is strategic thinking. While DOK Level 2 tasks generally only required the activation of one scheme, DOK Level 3 tasks often required activation of multiple schemes to reason or think strategically about the task (Gagne et al., 1993). Strategic thinking requires reasoning, developing a plan, more than one possible solution, and requires students to justify their response in order to manipulate facts, procedures, and/or non-routine solutions.

For most DOK Level 3 tasks, there are usually multiple methods or strategies for solving the problem embedded within the task (Anderson & Schunn, 2000). The goal of DOK Level 3 tasks is "not to automate but to maintain conscious control of the procedure, because of the varying conditions under which strategies apply" in order to monitor outcomes and learn when various strategies apply (Gagne et al., 1993, p. 205). Developing a logical argument requires an explanation of these conscious strategies (Feldon, 2007).

DOK Level 3 tasks may also involve creating hypotheses, similar to Doyle's (1983) level 4 (Opinions), however Webb's level 3 continues beyond this by requiring generalization or justification of those opinions. These justifications relate more closely to Smith and Stein's (1998) level 4 activities of exploring and understanding relationships in mathematics; however, it should be noted that Smith and Stein's model never explicitly requires justification of understanding, which Webb (1997) emphasizes as a basic component of HCD tasks.

Depth of Knowledge Level 4. Webb's (1999) fourth, and highest level of cognitive demand, is extended thinking. Tasks at DOK Level 4 often "push the boundaries" of student schemes to promote far transfer across domains (Barnett & Cecci, 2002). Because DOK Level 4 and DOK Level 3 tasks both require the activation of multiple schemes, it can sometimes be hard to distinguish between the two levels. One distinguishing characteristic of DOK Level 4 tasks is they usually cross curriculum domains when activating multiple schema, while DOK Level 3 tasks can be confined to only the mathematics domain. A second distinguishing characteristic of DOK Level 4 tasks is that they usually require the use of multiple mathematics standards to complete the tasks, integrating different mathematical topics.

DOK Level 4 tasks require students to construct, create, analyze, and apply mathematical understanding across multiple disciplines or real-world contexts. Smith and Stein's Level 4 spans a broad range of activities as it also requires students to analyze tasks and requires self-monitoring and self-regulation. Although Depth of Knowledge Level 4 tasks often require longer periods of time to complete, time itself does not determine HCD (Webb, 1999, pp. 15-22).

The results of this comparison between cognitive demand models is significant to this study as it confirms Webb's DOK as a potential framework for use in mathematics classrooms. The DOK framework not only spans all levels and types of cognitive demand tasks, but it also offers unique perspectives and fills in gaps created by other models to clearly delineate among different levels when creating and evaluating tasks. Additionally, it provides three distinct levels differentiating different types of HCD tasks. This provides a lens in which to more accurately review results of student engagement with HCD tasks.

Current Research Findings in Cognitive Demand

Current research findings on cognitive demand fall within one of three phases of curriculum integration: intended (CD of task design), implemented (CD based on teacher presentation or actions), and enacted (CD enacted by student). Overall, research on task-intended levels of cognitive demand shows that the mathematics tasks in most commercial and teacher-designed curriculums are Low Cognitive Demand (LCD; e.g., Jones & Tarr, 2007; Kohar, Wardani, & Fachrudin, 2019; Moreno Alcázar, 2007; Porter et al., 2011; Tan et al., 2018). This is important to this study as it identifies the need for practices that can help increase student-enacted levels of cognitive demand for both high and low cognitive demand tasks.

Overall, research on teacher-implemented levels of cognitive demand shows that many teacher actions and dispositions, during the implementation of HCD mathematics tasks, generally decrease the intended levels of cognitive demand (e.g., Boston & Smith, 2009; Candela, 2016; Hong & Choi, 2016; McCormick, 2016; Mdladla, 2017; Roth, 2019; Son & Kim, 2015; Yanik & Serin, 2016). Store and College (2015) found that the launch of the task by the teacher was particularly important. The way that tasks are launched is critical in maintaining the cognitive demand of the tasks. The best launches give only minimal instructions that provide student expectations, without "funneling their thinking" (p. 1188). These results are important to this study as they outline the rationale for limiting teacher involvement in the study to a minimal launch of the mathematics tasks.

Overall, research on student-enacted levels of cognitive demand shows that the cognitive demand of intended and implemented curriculum is not correlated with the enacted curriculum. Most students enact mathematics tasks at low levels of cognitive demand, regardless of the intended or implemented levels, (e.g., Charalambous & Litke, 2018; Kessler et al, 2015; Otten, 2012).

Otten (2012) found that the nature of students' disposition towards HCD tasks influenced their involvement with the tasks and predicted levels of enacted cognitive demand. This outcome is supported by other research that found when students exhibit either indifferent dispositions towards mathematics tasks or exhibit a desire to quickly complete the tasks, they engage with the tasks in a manner that typically decreases their involvement with the mathematics tasks and lowers student-enacted levels of cognitive demand (e.g., Candela, 2016; Charalambous & Litke, 2018; Kessler et al, 2015; Morgan & Power, 2016; Ngware, Ciera, Musyoka, & Oketch, 2015; Otten, 2012 Sherman, 2011). For example, in Kessler et al.'s study, students interacted with online tasks in a tutor program. One student, Emma, skipped the introductory pages, which were designed to engage students in HCD tasks and reasoning which they would then use in the subsequent problems. Rather than reason through what each problem was asking, Emma duplicated previous procedures to answer questions through a series of trial and error. She also neglected to explain her final reasoning in her answers, though it was requested by the instructions for each task.

Emma's learning disposition (i.e., desire to "get it done") may have influenced her low student-enacted levels of cognitive demand, despite the high intended cognitive demand of the tutor program. When student involvement decreases, the intended elements of richness that facilitate HCD are lost (Charalambous & Litke, 2018). Conversely, students' dispositions can also increase the probability of engagement with HCD tasks. Gilbert (2016) found that when students had positive dispositions towards the context of the task, their motivation decreased task avoidance and negative emotions associated with performance.

Welder et al. (2015) found that the numbers used in mathematical tasks, regardless of the intended cognitive demand, affected student-enacted levels of cognitive demand for undergraduate students in their preservice teacher mathematics content course. For example, students were more likely to fall back on meaningless procedures (i.e., LCD) when working with fractions if the denominators were "friendly" towards decimals or common denominators (e.g., 1/10, 1/5). Students were more likely to engage in reasoning and justification (i.e., HCD) with nontraditional strategies when denominators were prime numbers.

In summary, current research indicates that how students engage with mathematics tasks, and practices to increase this engagement, may be more important than the original intended DOK level of the tasks. Student participation and interactions with mathematical tasks can play a determining role in the enacted cognitive demand. Research related to cognitive demand of enacted curriculum is of critical significance to teachers as they plan and implement HCD tasks during mathematics instruction. The limited research relating to enacted levels of cognitive demand focuses primarily on student interaction with technology or student surveys to identify their curricular experience. Research is needed that captures student thinking, either verbally or written, to capture a more holistic understanding of student engagement with tasks at different levels of cognitive demand. Additionally, research is needed on interventions that teachers can incorporate into their classrooms to promote student participation, which in turn, would support student learning opportunities.

Conceptual Framework

The conceptual framework in Figure 2.3 shows the potential relationships between intended/implemented levels of cognitive demand, student-enacted levels of cognitive demand, and mathematics discourse.

On the left side of Figure 2.3 is the intended or implemented DOK for a specific mathematics task (Webb, 1999). On the right are the high or low student-enacted CD for the same task (Stein et al, 1996; Webb, 1999). Students may enact multiple elements of high or low CD in response to a single task (e.g., using recall of facts to support counter-

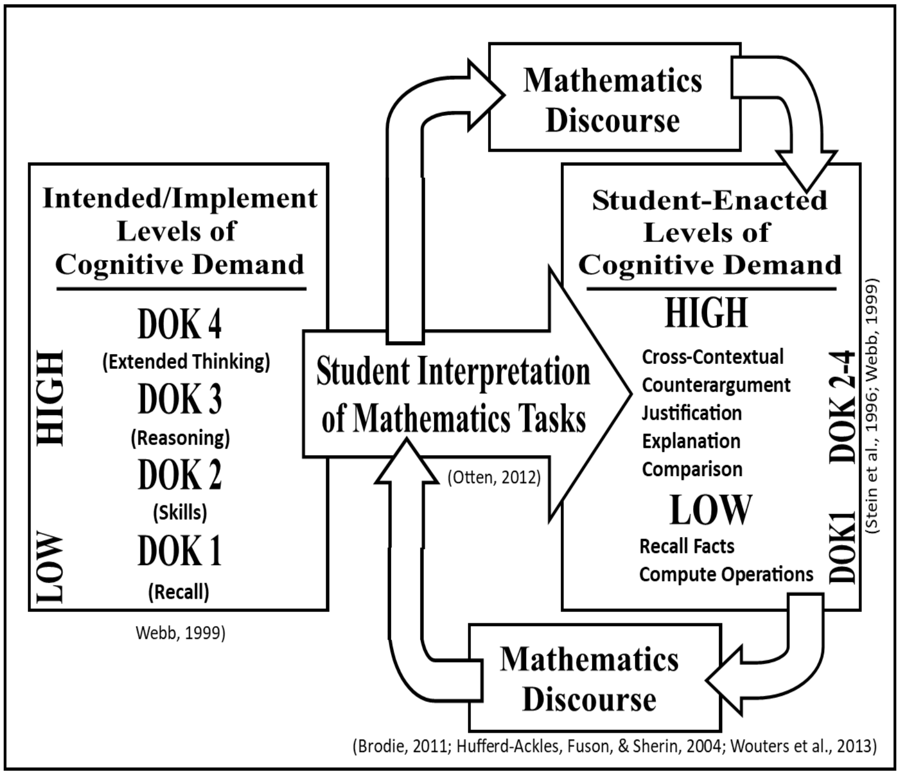


Figure 2.3. Discourse and interpretation influence on cognitive demand.

argument). The arrow in the middle indicates research findings that students' interpretation of the tasks primarily influences enacted CD (e.g., Otten, 2012).

An additional influence on student-enacted CD are potentially internal or external mathematics discourse in which the student engages. This relationship between cognitive demand and discourse is cyclical in that interpretation influences the discourse, which influences cognitive demand enactment, which can then influence a change in mathematics discourse, which in turn, can influence a student's interpretation of the mathematics task. Although discourse can potentially decrease student-enacted CD (e.g., cumulative talk, Rojas Drummond & Mercer, 2003), it has an even greater potential to increase student-enacted CD by eliciting student participation in evaluating and reasoning about mathematical properties or procedures (e.g., Charalambous & Litke, 2018; Falloon & Khoo, 2014; NCTM, 2014; Otten, 2012; Sfard, 2001; Williams, 2010). This study focuses on this relationship between discourse and cognitive demand.

Small Group Mathematics Discourse

When students work in small groups (i.e., 2-3 people), it provides participation opportunities for discourse of mathematical ideas and strategies. Learning is a social process that is facilitated through social discourse to lead to personal understanding (Bruner, 1983; Clements & Battista, 2009; Forman, 2003; Vygotsky, 1978). Mathematics discourse helps students to verbalize their understanding, which not only extends their thinking, but also allows them to assimilate the knowledge for a higher transfer of learning (Brodie, 2011; Hufferd-Ackles et al., 2004; Wouters et al., 2013).

During small-group discourse, students have the opportunity to share and clarify their ideas through constructive arguments, solidify mathematical language and ideas, and evaluate others' perspectives (NCTM, 2014). This collaboration of ideas and perspectives leads to a collective intelligence or socially shared meta-cognition (Bruce & Flynn, 2011; Iiskala, Vauras, Lehtinen, & Salonen, 2011; McGonigal, 2008). The co-construction of learning through explanations, redefinitions and reiterations promotes multiple strategies and deeper understanding of the mathematics (Hwang & Hu, 2013; Mueller, 2009; Wachira, Pourdavood, & Skitzki, 2013). Carpenter, Franke, and Levi (2003) explain that students who "justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding" (p. 6) of the mathematics. Anderson-Pence

(2014) found that discourse, when combined with HCD tasks, deepened students' connections which promoted learning. Additionally, the use of language in tasks that promotes HCD, can help increase students' mathematical fluency and accuracy (Grammer, Coffman, Sidney & Ornstein, 2016). Another benefit of discourse is that it can increase student interest in the context or task, which can help increase or maintain CD (Gilbert, 2016). In summary, discourse increases opportunities for students to reason through or justify procedures and concepts to make connections – elements of HCD (Georgius, 2014; Kalamar, 2018; van Leeuwen & Janssen, 2019; Williams, 2010).

As mentioned in the introduction, discourse can occur at different times during student engagement with mathematics tasks. Reflective Discourse occurs after students have had an opportunity to engage with the mathematics tasks independently, which allows students opportunities to reflect on their thinking and strategies (Cobb, Boufi, McClain, & Whitenack, 1997; Steffe, 1991; Walter, 2018). This reflection can be facilitated by the teacher in a whole-class setting or in small groups with students directing the conversation. Reflection on actions can facilitate opportunities for students to develop their mathematical understanding (Cobb et al., 1997; Silverman & Thompson, 2008; Steffe, 1991). Cobb et al. found in their study with young children, that "participation in this type of discourse *constitute conditions for the possibility of mathematical learning* [Italics in text]" (p. 264). When students reflect on their actions, "new, more advanced conceptions develop out of existing conceptions" (Silverman & Thompson, 2008, p. 506).

Research shows that delaying discourse allows students wait-time to process their ideas and bring a dynamic object with them to the discussion to facilitate discourse (Silverman & Thompson, 2008; Walter, 2018). Kalamar (2018) found that when students in her study were given time to prepare their response, they had more in-depth responses during discourse. When students have time to prepare their response, it also leads to more equitable opportunities for all students to participate in the discourse and share their ideas. In her paper synthesizing 20 years of research, Rowe (1986) noted that delaying discourse allows "previously 'invisible' people [to] become visible" (p. 45), which included both minority and typically struggling students. Kalamar found that by the end of her three-week intervention, 100% of students in the intervention class were participating during Reflective Discourse.

Exploratory Discourse occurs while students are engaging with the mathematics tasks. Joint discussion while engaging with tasks allows student discourse to occur in a timely manner, which allows students opportunities to build a collaborative understanding of the mathematics (Calcagni & Lago, 2018; Patterson, 2018; Rojas-Drummond & Mercer, 2003). Barron (2003) found that when groups worked together collaboratively using Exploratory Discourse, they had a meaningful conversation that allowed them to bring out a variety of thinking and ideas to solve the task. Hogan, Nastasi, and Pressley (1999) found that when students in their study were allowed to work in small groups (i.e., Exploratory Discourse) to solve problems, independent of the teacher, they were more likely to engage in high levels of reasoning (i.e., HCD) such as elaboration and justification of strategies and solutions. In summary, Reflective and Exploratory Discourse each provide students with different types of opportunities or resources to reason through mathematical concepts or procedures that may facilitate discourse containing elements of HCD for a deeper understanding of the mathematics.

Summary

This chapter outlined the importance of high student-enacted levels of cognitive demand to facilitate a deeper understanding of mathematics. Very little research has focused directly on how students enact mathematical tasks to understanding the role of student discourse in promoting high student-enacted levels of cognitive demand. The conceptual framework shows that small-group mathematics discourse can potentially target student perceptions and responses directly to increase student-enacted levels of cognitive demand. Research on students' mathematical discourse shows that it can increase opportunities for student reasoning and engagement with mathematics task. Reflective Discourse can promote equity, while Exploratory Discourse can promote collaboration. This study makes a significant contribution because it will provide a unique perspective and quantitative analysis on how the timing and affordances of two types of mathematics discourse (during the task or after the task) may relate to student-enacted levels of cognitive demand. It will also contribute to the literature by qualitatively examining student behaviors that influence this relationship. Knowledge gained from this study will add the literature by highlighting specific student practices that teachers can reinforce to promote high student-enacted levels of cognitive demand.

CHAPTER 3

METHODS

The purpose of this study was to examine relationship between mathematics discourse and student-enacted levels of cognitive demand when students solved mathematics tasks at all four DOK levels.

Research Design

This study used a sequential mixed methods approach (Creswell & Plano Clark, 2011; Tashakkori & Teddlie, 2010) with a quasi-experimental crossover design (Shadish et al., 2002). A crossover design allowed students to act as their own control group and control for class to class variations, which increased the reliability and validity of the study for generalized causal inferences.

Mixed methods provide a deductive lens to understand student-enacted levels of cognitive demand (written and verbal) and explore within- and between-group relationships between intended and enacted levels of cognitive demand when students engage in small-group mathematics discourse (Creamer, 2018). Mixed methods also provide an inductive lens to generate hypotheses around the relationships between mathematics discourse and student-enacted levels of cognitive demand (Creamer, 2018). The researcher collected qualitative data in the form of student written and verbal responses to mathematics tasks over the course of four weeks and then analyzed the data qualitatively and quantitatively in a sequential manner. Combining and building upon quantitative and qualitative results increased the internal and external reliability of research findings by evaluating the effectiveness of discourse and contextualizing the results (Creamer, 2018).

Research Questions

The overarching research question guiding this study was: What is the relationship between two types of small-group discourse and student-enacted levels of cognitive demand when engaging with mathematics tasks at four different intended Depth of Knowledge (DOK) levels? There were three specific questions.

1. What are the differences within and between groups for intended DOK levels of the mathematical tasks and student-enacted levels of cognitive demand for written responses relating to the mathematical tasks?
2. What are the differences within and between groups for intended DOK levels of the mathematical tasks and student-enacted levels of cognitive demand for verbal responses relating to the mathematical tasks?
3. What are the differences in quantity and quality of mathematical discourse contributions during small group Reflective Discourse (after students individually solve mathematical tasks at four DOK levels) and Exploratory Discourse (while students solve mathematical tasks as a group at four DOK levels)?

Participants and Materials

Participants

This study used a purposeful sample of four grade 5 public and charter school classrooms in the intermountain west. Utilizing four different classrooms allowed the researcher to control for class to class variations and compare class effects for within-and between-group variations which increases generalizability and validity of results.

Classrooms selected for inclusion in the study were limited to those where teachers identified that they had previously established classroom norms for group work. Cobb and Bauersfeld (1995) found that, in a typical school year that begins in the fall (September), small group activities required less teacher intervention by January, when the teacher began establishing classrooms norms in the fall of the year. This was important in this study as one assumption was that groups could work on the tasks with little or no teacher intervention. Previously established norms were defined as classroom norms for group work set up within the first month of the school year and reinforced throughout the year. The expectation of established norms for group work required for this study was that students worked collaboratively to answer the questions posed in the mathematical tasks, communicated their thinking and reasoning, and agreed on one answer to the task. During prior participation in cognitive demand professional development sessions with the researcher, teachers across multiple schools filled out a form expressing interest in future classroom research studies. Four grade 5 classrooms were identified for inclusion in this study from this self-nominated pool of candidates.

The teachers in each classroom in this study purposefully placed their students in groups of 2-3 that supported collaboration and communication. Every student ($N = 101$) in the four identified classrooms completed the informed consent form, with four students opting out of the study, for a total of 97 students grouped in 34 small groups participating in the study. Results from the demographic survey show that students in this study were primarily white (80%); one fourth of students identified as low socioeconomic status (SES); and one student per class had a math IEP. See Table 3.1 for participation and demographics by class.

Table 3.1

Participation and Demographics by Class

Demographic	Class 1	Class 2	Class 3	Class 4	Total
Class size	25	24	26	26	101
Participated	24	22	25	26	97
Opted out	1	2	1	0	4
Total groups	8	8	9	9	34
Group 2	0	2	2	1	5
Group 3	8	6	7	8	29
Gender					
Male	12	9	11	10	42
Female	12	13	14	16	55
Ethnicity					
White	21	19	20	20	80
Hispanic	2	3	2	4	11
Native American	0	0	2	2	4
Asian	1	0	1	0	2
Low SES	6	8	5	5	24
Math IEP	1	1	1	1	4

Materials

This study used pregenerated mathematics Task-Sets selected from Nextlesson Inc. resources (see Appendix B). These Task-Sets were chosen for several reasons: (a) Nextlesson Task-Sets are based on real-world problems, which increased opportunities for discourse at all ability levels. (b) Nextlesson Task-Sets are pre-coded using Webb's (1999) Depth of Knowledge (DOK) model with four levels of cognitive demand (1 = low cognitive demand, 2-4 = HCD). Three experts in cognitive demand double coded these task-sets to verify the validity of the codes before being published. (c) Nextlesson Task-Sets are organized in four levels to present a mixture of DOK Level 1-2 tasks in Level 1, then DOK Level 2-3 tasks in Level 2 and the Challenge (Level 3), and DOK Level 3-4 tasks in the Finale (Level 4). Each level can stand alone or be combined together to address the same real-world problem. This allowed groups to work at their own pace, with Level 4 containing multiple extended problems for fast finishers. (d) Nextlesson Task-Sets were created for every Common Core State Standard in Mathematics (CCSS-M), which allowed the researcher to choose two sets of tasks that related to key topics in Grade 5 mathematics standards.

The researcher chose task-sets that aligned with the following mathematics content: operations with fractions, operations with decimals, and volume (see Figures B.1 and B.2 in Appendix B). Task-Set 1 and Task-Set 2 each contain four DOK Level 1 tasks, four DOK Level 2 tasks, four DOK Level 3 tasks, and instructions to select one of five DOK Level 4 tasks, totaling 13 tasks in each task-set and 26 tasks across both task-sets (see Table B.1 in Appendix B). The researcher verified Nextlesson's previously coded intended DOK levels for each written and numbered task by blind coding both task-sets using a scale of 1 to 4, aligned with the DOK levels (e.g., DOK1 = 1) to validate the existing codes. The researcher's blind coding and Nextlesson coding had 100% agreement for DOK levels.

Data Sources

There were two main data sources in this study: students' written work relating to the mathematics tasks and video and audio recordings of students' interactions and discourse while engaging with the mathematics tasks.

Students' Written Work

The first data source was students' written work. The purpose of this data source was to collect written evidence of students' responses to the two task-sets when engaging in each discourse type. Students wrote their answers to Level 1-3 tasks directly on the task sheets for each Nextlesson Task-Set and used additional scratch paper to show their work as needed. Students wrote their outline to the Level 4 task using scratch paper or google documents/slides. Students' written responses provided clear information for the researcher's analysis of student-enacted DOK levels. The final scope of this data source can be found in Table 3.2.

Table 3.2

Scope of the Written Data Sources

Discourse type	Task sheets	Scratch paper	Electronic	Total
Reflective	554	224	9	787
Exploratory	197	229	21	447
Total	751	453	30	1,234

Note. The numbers reported in the table reflect the number of physical pages or electronic slides.

Video and Audio Recordings

The second data source was video and audio recordings of students' responses. The purpose of this data source was to collect physical and verbal evidence of students' responses to the two task-sets when engaging in each discourse type. The researcher recorded student interactions and discourse using one GoPro camera per small mathematics discourse group, with the student wearing the GoPro sitting in the middle of the group, and an audio recorder set on the desk as a back-up recorder. For example, Class 1 had 24 students in the class, grouped into eight groups of three, so there were eight GoPro Cameras and eight audio recorders used at the same time.

There are several advantages to using videos during task enactment. Roschelle (2000) explains that videos of student interactions allow for observations in a more natural environment (classroom), facilitates connections between student actions and words, and allows for repeated observation. A GoPro camera is an unobtrusive observation tool that captures relevant video and sound of student interactions with the task. Pilot projects related to this study using GoPro cameras found that the audio range of the GoPro allowed the researcher to clearly hear members of the group while minimizing noise from other groups in the room. The final scope of this data source can be found in Table 3.3.

Table 3.3

Scope of the Verbal Data Sources

Discourse type	Video recordings (in hours)	Audio recordings (in hours)	Total (in hours)
Reflective	20	40	60
Exploratory	40	40	80
Total	60	80	140

The researcher used the video recordings and audio files for all 34 student groups to examine students' verbal responses and code levels of productive discourse, as well as how students' discourse may have influenced or related to students' written answers on the task sheets or scratch paper.

Procedures and Data Collection

This study took place over the course of four weeks, with all four classes completing two task-sets during this time (see Table 3.4). The week before engaging with Task-Set 1, students and their parents filled out a consent form for participation and/or photograph and completed a demographic survey. Students who brought back the form (positive or negative consent) received a small nonfood prize to encourage them to return the forms. Two students (Class 2) who opted out of the study worked on traditional math instruction with a student teacher. The remaining two students who opted out of the study were absent on the days that the class completed the task-sets.

Table 3.4

Timeline of the Study

Week	Group A (Class 1 & 2)	Group B (Class 3 & 4)
Week1	Collect consent and survey forms	
Week2	Task-Set 1 (13 Tasks) Reflective Discourse Individual (35 min)→Group (35 min)	Collect consent and survey forms
Week3	Task-Set 2 (13 Tasks) Exploratory Discourse Group (70 min)	Task-Set 1 (13 Tasks) Exploratory Discourse Group (70 min)
Week4		Task-Set 2 (13 Tasks) Reflective Discourse Individual (35 min)→Group (35 min)

As seen in Table 3.4, all four classes completed Task-Set 1 first and Task-Set 2 second. However, the students in Group A completed Task-Set 1 using Reflective Discourse procedures (discussed in detail below), while the students in Group B completed Task-Set 1 using Exploratory Discourse procedures (discussed in detail below). Based on the crossover design, Group A completed Task-Set 2 using Exploratory Discourse procedures while Group B completed Task-Set 2 using Reflective Discourse procedures. This allowed for an analysis of within- and between-group effects. Each class completed Task-Set 1 during one 70-minute block, and then completed Task-Set 2 during one 70-minute block, for a total of 140 minutes of engagement with the two task-sets.

Reflective Discourse Procedures

During Reflective Discourse procedures, the researcher provided each group with a numbered GoPro camera, a matching numbered digital recorder, and a matching numbered folder containing enough task sheets for each student to have their own copy of the task sheets as well as a piece of scratch paper. The group leader (wearing GoPro) gave each student their own Level 1 task-sheet. The researcher explained that students were working independently to complete the tasks and to make their best guess if they were unsure of an answer. The researcher encouraged students to show their work, either in the space provided or on scratch paper, and informed them that they would be getting into groups to talk through their ideas about Level 1 in 10 minutes. The researcher provided students with two pens (black and colored) and instructed them to use the black pen when working independently and the colored pen when making any changes while working as a group. After each student had a copy of Level 1 on their desk, the researcher introduced the real-world problem posed in the Task-Set to the whole class and addressed any questions relating to the real-world context, which included defining new terms (e.g., gross, net, retail) and reminding students of classroom norms for individual work, group work, and care of the digital equipment (< 5 minutes).

Students worked individually for 10 minutes to complete all of the tasks in Level 1 of the Task-Set. At the end of 10 minutes, the researcher instructed students to “talk with your group about your answers and how you got your answers.” Students then worked with their small group for 10 minutes to discuss their results and strategies. Although the group leaders wore the GoPro during the entire 70 minutes, they turned the video recording off during individual work time and on during group work time. The audio recorder continued to record during both individual and group work time. After the group discussion, the researcher asked group leaders to distribute a Level 2 task sheet to each student and repeated the Level 1 procedures. After discussing the Level 2 task sheet, the group leader distributed Level 3-4 task sheet(s) and students worked independently for 15 minutes to complete all tasks in Level 3, choose one Level 4 choice, and start an outline for their Level 4 choice. Following this independent work time, students worked with their small group for 15 minutes to discuss their results and strategies.

The times provided for individual work time were based on pilot projects showing that students spent approximately 10 minutes to complete Level 1 or Level 2 tasks and 5-10 minutes to complete Level 3 tasks or create an outline for a Level 4 task. Due to the high variation in student completion times for Level 3 and 4 tasks, these two levels were combined during reflective discourse to allow students that completed Level 3 quickly to move on to a Level 4 task while still providing students who took longer on Level 3 tasks to have time to engage with a Level 4 task. The researcher provided students with 35 minutes for individual reflection and computation and 35 minutes for group Reflective Discourse, for a total of 70 minutes. See Table 4.3 in Chapter 4 for average time spent verbally discussing the mathematics tasks for each task-set and discourse type.

Both the teacher and researcher walked around the classroom while students worked on the task-sets to encourage students to stay on-task and to answer any questions. When asked specific questions about procedures for the tasks, the teacher/researcher responded by asking, “What does it ask you to do?” or directed students to re-read the instructions. If asked to confirm an answer, the teacher/researcher responded by asking, “What do you think?” if the student was working independently or directed them to talk to their group if working together. The teacher/researcher answered any organizational questions (e.g., materials they could use, which tasks to complete, time, bathroom, camera issues, etc.). The researcher collected all task sheets and scratch paper at the end of each class session, and then scanned and uploaded the digital copies to a secure electronic folder by the end of each day. Groups that used an electronic device to record their Level 4 responses using a google doc or google slide shared their electronic responses with the researcher using google drive. The researcher downloaded all GoPro video data, audio recordings, and electronic responses to a secure electronic folder by the end of each day.

Exploratory Discourse Procedures

During Exploratory Discourse procedures, the researcher provided each group with a numbered GoPro camera, a matching numbered digital recorder, and a matching numbered folder containing one copy of the task-set (stapled together) and three pieces of scratch paper. The researcher also provided different colored pens for each student in a group to differentiate between work completed by different students on the task-set and scratch paper.

The researcher explained that students were working in groups to complete the tasks in the Task-Set and asked the group leader (wearing the GoPro) to share the task-set with their group. After each group had a copy of the task-set, where everyone in the group could see it, the researcher introduced the real-world problem posed in the Task-Set to the whole class and addressed any questions relating to the real-world context, which included defining new terms (e.g., gross, net, retail) and reminding students of classroom norms for group work and care of the digital equipment (< 5 minutes).

Students worked as a group and engaged in Exploratory Discourse for 70 minutes to complete all tasks in the Task-Set. The group leader turned on the GoPro video and the researcher turned on the audio recorders, both of which remained on during the entire 70 minutes. See Table 4.3 in Chapter 4 for average time spent verbally discussing the mathematics tasks for each task-set and discourse type.

The teacher and researcher consistently implemented the Reflective Discourse procedures. The researcher collected all task sheets and scratch paper at the end of each session, and then scanned and uploaded the digital copies to a secure electronic folder by the end of each day. Groups that used an electronic device to record their Level 4 responses using a google doc or google slide shared their electronic responses with the researcher using google drive. The researcher downloaded all GoPro video data, audio recordings, and electronic responses to a secure electronic folder by the end of each day.

Data Analysis

The researcher conducted the data analysis in three distinct phases. Figure 3.1 provides an overview of the relationships between data sources and phases of the analysis. The phases of the analysis are explained in the following sections. As seen in Figure 3.1, data were magnitude coded in Phase 1 to assist in quantitative analyses related to each research sub question in Phase 2, and then revisited to provide explanatory insights regarding quantitative results in Phase 3 to explain and explore quantitative results and answer the overarching research question.

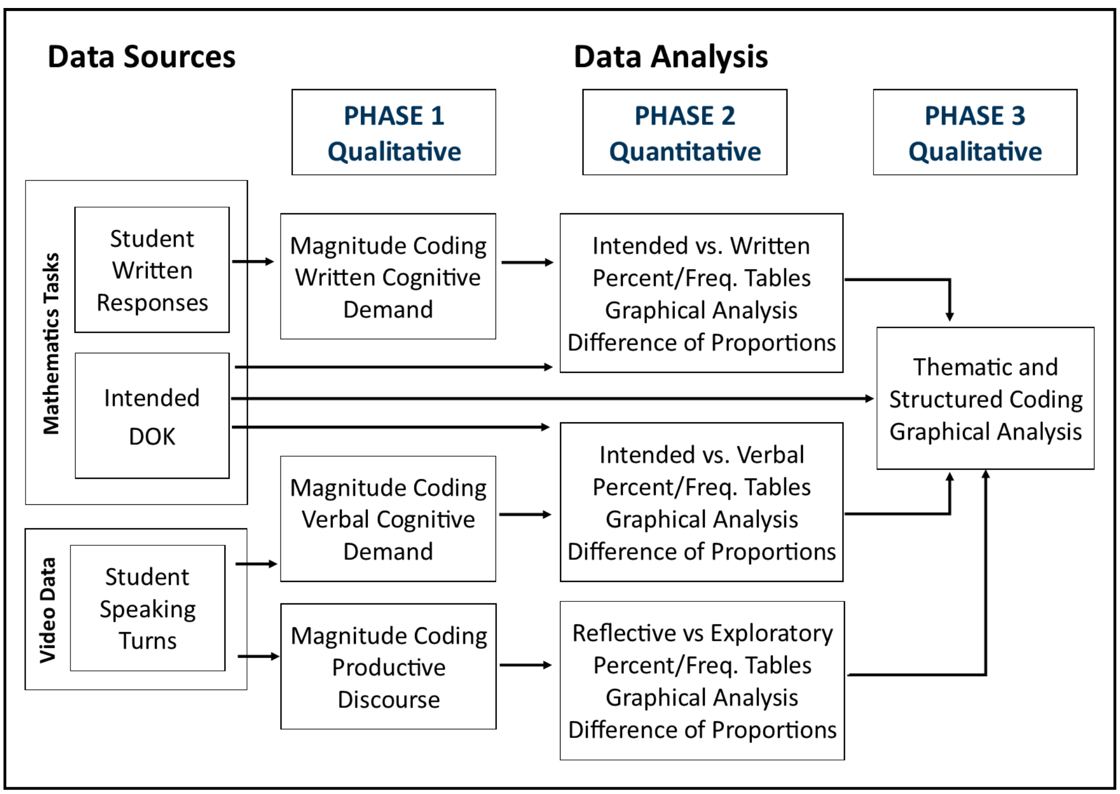


Figure 3.1. Relationships between data sources and phases of analysis.

Phase 1: Qualitative Magnitude Coding of Video and Written Work

The researcher paid two undergraduate researchers to transcribe 50% of the video/audio data (30 of 60 hours) to assist in timely completion of the data analysis. The researcher verified 10% of the paid transcriptions and transcribed the remaining video/audio data. The researcher used Microsoft Excel to organize video/audio transcriptions. The researcher used qualitative magnitude coding (Saldaña, 2015) to identify and "quantitize" (Saldaña, 2015, p. 86) the enacted levels of cognitive demand in students' verbal and written responses to the mathematics tasks. The researcher used Microsoft Excel to organize coded data electronically.

Written cognitive demand levels. The researcher coded all written task responses using Webb's (1999) Depth of Knowledge (DOK) coding scale (see Appendix C, Figure C.1 and Table C.1), relating to the four DOK levels (e.g., DOK1=1). Responses showing a recall of short facts or use of standard computational procedures were coded as "1." Responses showing an application of skills or concepts such as comparisons, meaningful algorithms (Brownell, 1945), or estimations were coded as "2." Reponses that explained solution steps were also coded as "2." Responses that justified a hypothesis or solution by citing evidence were coded as "3." Responses showing revision processes were also be coded as "3." Responses that synthesized or cited multiple contexts or sources of information to analyze, prove, or design a solution were coded as "4." Figure C.2 provides a sample of one completed task-set and the associate Written CD codes. The researcher used Written CD codes during Phase 2 quantitative analyses for RQ1 (Figure 3.1).

Verbal cognitive demand levels and quality of mathematics discourse. Next, the researcher magnitude coded videos of students' verbal responses relating to the mathematics tasks for DOK levels as well as quality of the mathematics discourse using the Mathematical Discourse Contributions Coding Rubric (Table 3.5). The grain size for codable student responses was determined using Stein et al.'s (1996) definition of a mathematical task. Using this definition, all sequential verbal responses relating to the same conceptual or procedural aspect of the task were coded as a single response. Any foci change within sequential responses by the same student was coded as a new response. Student verbal responses following another student's response was coded as a new response, regardless of the response's relation to previous responses by this student or other students.

Table 3.5

Mathematical Discourse Contributions Coding Rubric

Code	Category	Sub code descriptions
0	None	0a Student did not talk during the mathematical task 0b Responses were off-topic 0c Responses organized actions or responsibilities related to problem
1	Minimal DOK 1	1a Perform routine calculations 1b Recall facts or state answer 1c Identify problem or component of problem 1d Reiterate prior ideas, solutions, or indicate agreement
2	Considerable DOK 2	2a Share new solution strategy (similar efficiency/sophistication) 2b Share more efficient or accurate solution strategy 2c Share more sophisticated solution strategy
3	Substantive DOK 3	3a Generalizations regarding solutions or problems 3b Justification of a solution's accuracy or efficiency 3c Counterargument against a solution's accuracy or efficiency
4	Extended DOK 4	4a Connect or apply solutions to multiple disciplines or contexts 4b Design a new mathematical model to inform the solution strategy 4c Analyze or synthesize information from multiple sources

The researcher adapted the framework in Table 3.5 from three research design and discussion coding frameworks that evaluated teacher-led discourse in a whole class setting (Bishop et al., 2016; Cobb & Yackel, 1996; Durfee, 2018), to address student-led problem-solving discourse and associated levels of enacted cognitive demand for different comment types. In Table 3.5, responses in the "none" category do not directly relate to solving the mathematics tasks. This code indicates that either the student did not participate, their responses were distractors from the mathematics, or their responses organized student actions (Bishop et al., 2016). In the remaining categories, student responses facilitate problem solving and extend mathematical understanding by introducing, justifying, or applying possible solutions that increase either efficiency or accuracy (Cobb & Yackel, 1996; Durfee, 2018). This rubric was tested by the researcher in a pilot project.

Responses in the "minimal" category relate to recalled facts, ideas, or mathematical procedures (Bishop et al., 2016) each of which requires low levels of cognitive demand to complete (Webb, 1999). As such, this category is associated with DOK Level 1. Responses in the "considerable" category illustrate children's attempts to apply and share prior learning to new situations, which is associated with DOK Level 2 tasks (Webb, 1999). Responses in the "substantive" category relate to DOK Level 3 as students are "providing justifications, making generalizations, or participating in mathematical argumentation" (Durfee, 2018, p. 19; Webb, 1999). Responses in the "extended" category go beyond the context of the current task to bridge or connect ideas across multiple disciplines, contexts, or sources, which is associated with DOK Level 4 tasks (Webb, 1999). These categories and sub-codes provided insight into enacted levels of cognitive demand as students worked through the mathematical tasks.

Verbal cognitive demand levels. To quantitize verbal cognitive demand levels of student responses, the researcher used a 0 to 4 score, with numbers matching the associated DOK levels. The researcher coded videos for DOK levels by determining which category code best represented each student's response. For example, off-topic responses were coded as "0" as they did not relate to any mathematics. A response relating to recall of memorized facts or procedures was coded as "1." A response reflecting an application of skills or concepts was coded as "2." A response reflecting strategic, relational thinking was coded as "3." A response reflecting extended thinking beyond the current context was coded as "4." Specific examples for each category code are listed in Table 3.5.

The researcher coded each discussion relating to specific tasks for the highest verbal cognitive demand level. For example, a group discussion relating to Task 1 in Task-Set 1 that included coded responses ranging from 0-3 had a highest verbal cognitive demand level of 3. The researcher used codes for highest verbal cognitive demand levels during Phase 2 quantitative analyses relating to RQ2 (Figure 3.1).

Mathematical discourse contributions. General categories of mathematical discourse contributions align with coding for the five verbal levels of cognitive demand (None = 0; Minimal = 1, Considerable = 2, Substantive = 3, Extended = 4). In addition to general codes for productive discourse, the researcher coded for specific types of discourse contributions using categorical sub-codes (a-d) within each overall code (see

Table 3.5). The researcher coded videos to determine the specific type of productivity for each codable student response. For example, a response sharing the first strategy or similar strategy to previously shared strategies were coded as 2a, a more efficient strategy was coded as 2b, and a more sophisticated strategy was coded as 2c. The number “2” relates to the general category “considerable” with the letters relating to the specific types of responses within that category. Appendix Table C.2 provides a sample of one group’s Reflective Discourse and the associate Verbal CD codes and Discourse Contribution categorical sub-codes. The researcher used codes and sub-codes for categories of discourse contributions during Phase 2 quantitative analyses relating to RQ3 (Figure 3.1).

Reliability of magnitude coding levels. Twelve percent of the written task sheets and video data were double coded to ensure reliability of coding (Creswell & Plano Clark, 2011). Double coders were two doctoral students at the same university as the researcher, and were trained by the researcher for approximately 45 minutes using excerpts from Webb’s (2005) publicly available DOK level coding training materials (http://wat/wceruw.org/index.aspx; Appendix C, Figure C.1), a Written CD sample evidence coding table (Appendix C, Table C.1), and the Mathematical Discourse Contributions Coding Rubric (Table 3.5). See Appendix C, Table C.3 for detailed double coder procedures.

The researcher used Krippendorff’s alpha test (Hayes & Krippendorff, 2007; Krippendorff, 2013) to estimate inter-coder reliability using the KALPHA SPSS syntax. This quantitative analysis was appropriate for several reasons. First, it can control for any number of raters, which was appropriate for this study as it utilized 3 coders. Second, it can be used with all data types, which was appropriate as cognitive demand codes are both ordinal and categorical. Third, it used 10,000 bootstrapping samples to identify the distribution rather than assume an approximation. Results show a relatively high inter-coder reliability for written ($\alpha = 0.8381$) as well as verbal ($\alpha = 0.9212$) cognitive demand.

Phase 2: Quantitative Analyses

The researcher used results from Phase 1 analyses in Phase 2 quantitative analyses related to frequencies of codes, graphical analyses of these frequencies, chi square tests of independence, and difference in proportions tests of associations (Figure 3.1). Phase 2 analyses are organized by research question and are detailed in the following sections.

Research Question 1 analysis. Research Question 1 compared written and intended levels of cognitive demand within and between groups. To answer this question, the researcher created tables that compare the intended cognitive demand levels for tasks in each Task-Set (DOK 1-4) and percentages of the written levels of cognitive demand (CD 1-4) for all tasks at the same DOK level. Rows and columns within the table matrices were serialized by level of cognitive demand to order results (Wilkinson & Friendly, 2009; see Figure 3.2).

As shown in Figure 3.2, these tables provide a measure of student-enacted levels of cognitive demand on, above, or below the intended DOK of the tasks. One table was created for each of the four unique groupings within the crossover design (i.e., Group A Reflective Discourse with Task-Set 1, Group B Reflective Discourse with Task-Set 2, Group A Exploratory Discourse with Task-Set 2, Group B Exploratory Discourse with Task-Set 1) to facilitate within- and across-group analyses (i.e., within and across

Intended DOK	Incomplete Tasks	Written Cognitive Demand Percentages and Frequencies			
	0	CD 1	CD 2	CD 3	CD 4
DOK 1	I	Same	+1	+2	+3
DOK 2	I	-1	Same	+1	+2
DOK 3	I	-2	-1	Same	+1
DOK 4	I	-3	-2	-1	Same

Figure 3.2. Percentage table comparing Intended DOK and Written CD.

discourse types; within and across Task-Sets).

The researcher ran difference in proportions tests of association to test statistical differences between written and intended levels of cognitive demand for each of the four unique 4x5 grids (i.e., Figure 3.2) within the crossover design using Jamovi Software (see Appendix D, Tables D.1-D.4). The difference in proportions tests were based upon the null hypothesis that there is no relationship between intended and enacted levels of cognitive demand. The categorical nature of cognitive demand (e.g., the difference between level 1 and 2 cannot be assumed to be the same as the difference between level 3 and 4), as well as the restrictions of the data set (e.g., DOK Level 1 tasks cannot be enacted at a lower level), prohibited the use of parametric and non-parametric tests, such as an ANOVA or a Mann-Whitney U test. A difference in proportions tests is appropriate as it is designed to test differences in categorical data, such as the different levels of cognitive demand in the data set for this study, as well as 5x4 grids containing frequencies less than 5. These results provided a probability of within-group associations for written and intended levels of cognitive demand occurring due to chance.

Graphical representations in the form of color heat maps overlaying the frequency tables using conditional formatting assisted in these analyses. Color heat maps differentiating quantitative results using hue and tint can reveal structural and hierarchical patterns to assist in quantitative comparisons (Kelleher & Wagener, 2011; Wilkinson & Friendly, 2009). A divergent color scheme is used to contrast student-enacted levels of cognitive demand on (yellow), above (blue), and below (red) intended DOK levels (Kelleher & Wagener, 2011). A neutral gray depicts incomplete tasks (see Figure 3.3).

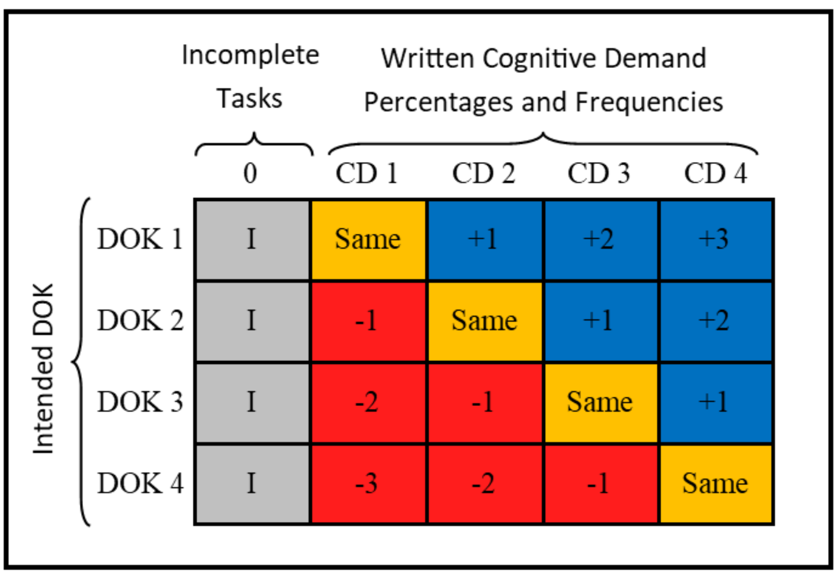


Figure 3.3. Sample heat map color scheme.

Tint saturation is used to represent percentage values within each cell, relative to each row (same intended DOK level). Light tints represented low values and dark tints represented high values (Kelleher & Wagener, 2011; see Figure 3.4).

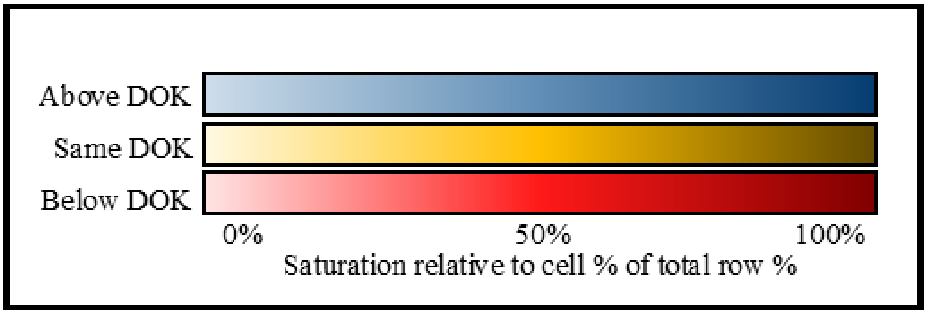


Figure 3.4. Tint saturation values.

The researcher conducted Chi-square Tests of Independence to compare the Written CD for each discourse type (i.e., Reflective & Exploratory). Written CD was dichotomously coded as either ‘less than’ the intended DOK or ‘greater than/equal to’ the intended DOK. Chi-square Tests of Independence were appropriate as each value in the 4x4 Chi-square grids was expected to be 5 or greater (Cohen, 2013) and the categorical nature of Written CD prohibited the use of parametric and non-parametric tests such as an ANOVA or Mann-Whitney U test. While the heat maps provide a visual representation of all of the data in the study related to Written CD, the results from the Chi-square Test of Independence was used to provide a focused between-discourse comparison between the Written CD that was greater than or equal to the intended DOK levels and the Written CD that was less than the intended DOK levels.

The researcher organized additional comparisons of frequencies of written levels of cognitive demand into a table using four different units of analysis: (a) overall composite score, (b) all tasks related to the same Task-Set (1 or 2), (c) all tasks of the same intended DOK level, and (d) individual tasks. Results for each unit of analysis were separated by type of discourse (reflective and exploratory) and also combined for an overall composite score of both types of discourse (see Appendix D, Table D.6). These results provided a measure of variation across tasks and types of discourse.

Research Question 2 analysis. Research Question 2 compared verbal and intended levels of cognitive demand within and between groups. To answer this question, the researcher created tables that compare the intended cognitive demand levels for tasks in each Task-Set (DOK 1-4) and percentages of the highest verbal level of cognitive demand for all tasks at the same DOK level (similar to Figure 3.2).

The researcher created one table for each of the four unique groupings within the crossover design (i.e., Group A Reflective Discourse with Task-Set 1, Group B Reflective Discourse with Task-Set 2, Group A Exploratory Discourse with Task-Set 2, Group B Exploratory Discourse with Task-Set 1) to facilitate within- and between-group analyses (i.e., within and across discourse types; within and across Task-Sets). The researcher created color heat maps using the same procedures outlined for Written CD (similar to Figure 3.3).

The researcher conducted difference in proportions tests of association for each 4x5 grid of intended DOK and verbal CD frequencies for each of the four unique groupings, using the same procedures outlined for Written CD (see Appendix E, Tables E.1-E.4). These results provided a probability of within-group (i.e., within each of the four 4x5 grids) associations for verbal and intended levels of cognitive demand.

The researcher conducted chi-square tests of independence to compare the Verbal CD for each discourse type (i.e., Reflective & Exploratory) using the same procedures outlined for Written CD. These results provided a between-group (i.e., between discourse types) comparison of Verbal CD.

The researcher organized additional comparisons of frequencies of highest verbal levels of cognitive demand into a table using four different units of analysis, similar to the Written CD table (see Appendix E, Table E.6). These results provided a measure of variation of Verbal CD across tasks and types of discourse.

Research Question 3 analysis. Research Question 3 focused on differences in quantity and quality of mathematical discourse contributions between Reflective and Exploratory Discourse. Only mathematical discourse contributions in the 'none' category coded as sub-code 0c (i.e., responses organized actions or responsibilities) related directly to the mathematics tasks. As such, categorical codes for sub-code 0c are referred to as 'Organizational' to distinguish these types of discourse contributions from other discourse contributions in the 'none' category that did not directly relate to the mathematics tasks.

Quantity of mathematical discourse contributions. The researcher created a frequency table comparing the difference in average time spent verbally discussing the mathematics tasks (i.e., sub-codes 0c-4c) for each discourse type (i.e., reflective and exploratory), organized by a) whether groups were on-task the entire time (i.e., discussing the tasks or working silently to solve problems or write answers the entire 70 minutes) or off-task at least once during the task-set (i.e., talking about other topics beyond the scope of the task-set, such as their love life); b) task-set (i.e., Task-Set 1 and Task-Set 2); and c) overall average time spent verbally discussing the mathematics tasks. These results provided a measure of differences in quantity of time engaged in mathematical discourse for each discourse type.

The researcher calculated overall percentages of mathematical discourse contributions (i.e., codes 0c-4c) for the four intended DOK levels (i.e., DOK1-4) and each discourse type (i.e., reflective & exploratory). These results provided a measure of quantity of discussion spent on low cognitive demand tasks (DOK1) and each of the three levels of HCD tasks (DOK2-4) for each discourse type.

Quality of mathematical discourse contributions. The researcher conducted chi-square tests of independence to compare the categories of mathematical discourse contributions for each discourse type (i.e., Reflective and Exploratory). The researcher dichotomously coded mathematical discourse contributions as either 'organizational to minimal' or 'considerable to extended' Chi-square Tests of Independence were appropriate as each value in the 4x4 Chi-square grids was 5 or greater (Cohen, 2013) and the categorical nature of the mathematical discourse contributions prohibited the use of parametric and non-parametric tests such as an ANOVA or Mann-Whitney U test. These results provided a general between-group comparison of quality of mathematical discourse contributions for each discourse type.

The researcher calculated overall percentages of mathematical discourse contributions in five different categories (i.e., Organizational, Minimal, Considerable, Substantive, Extended) for each discourse type and presented the results in a bar graph. This bar graph provided a more specific visual between-group comparison of quality of mathematical discourse contributions for each discourse type.

The researcher organized comparisons of frequencies of mathematical discourse contributions into a table for each discourse type (reflective and exploratory) using four different units of analysis: (a) overall composite score, (b) all tasks related to the same Task-Set (1 or 2), (c) all tasks of the same intended DOK level, and (d) individual tasks (see Appendix F, Table F.1). These results provided a measure of variation of mathematical discourse contribution categories across tasks and types of discourse.

The researcher organized additional comparisons of frequencies of mathematical discourse contribution sub-codes for tasks at each intended DOK level (DOK1-4) into a table by discourse type (reflective and exploratory; see Appendix F, Table F.2). These results provided a measure of variation of quality of mathematical discourse contribution sub-codes across tasks of different intended DOK levels and types of discourse.

Phase 3 Qualitative Analysis

In Phase 3, the researcher revisited students' written and verbal responses using qualitative pattern, thematic and structured coding (Saldaña, 2015) to explain and explore quantitative results.

Explanatory pattern coding. The researcher used qualitative pattern coding to help explain the quantitative results. First, the researcher revisited students' written or verbal responses relating to unexpected high or low percentages and frequencies within written and verbal tables and heat maps to identify patterns that might explain these unexpected values. Next, the researcher revisited students' written and verbal responses relating to unexpected differences between written and verbal quantitative results to identify patterns that might help explain these differences. Finally, the researcher revisited students' mathematical discourse contribution sub-codes relating to each discourse type (reflective and exploratory) to identify patterns that might help explain why each discourse type was higher in alternating general categories.

Exploratory thematic and structured coding. The researcher identified three student groups with high percentages of substantive and extended mathematics discourse contributions as well as three groups with low percentages of substantive and extended mathematics discourse. Using thematic coding, the researcher explored similarities and differences within- and between- each set of groups (high and low) to identify themes relating to the quality of mathematics discourse. Group organizational practices emerged as one theme that may have influenced the quality of mathematics discourse.

Further exploration of organizational practices within task-sets completed using Reflective Discourse procedures identified order of reflection as a possible influence on quality for this type of discourse. Three categories for the order in which students reflected on the task were originally identified: (a) chronological order, (b) reverse chronological order, and (c) prioritized order in which students jumped around to different questions they struggled with during independent work time. However, the researcher removed reverse chronological order as students who started by talking about the last task they worked on either jumped to the first task and completed the rest of the tasks in chronological order and revisiting the last task or proceeded to address the remaining tasks in a prioritized order.

Further exploration of organization practices within task-sets completed using Exploratory Discourse procedures identified the division of workload as a possible influence on quality for this type of discourse. Five categories for the division of workload were originally identified: (a) working together to solve a single task, (b) working side-by-side to individually solve the same task at the same time, (c) splitting up similar tasks so each student was working on a different task at the same time, (d) alternating which student completed similar tasks so that only one person was working on a task at a time, and (e) one student took over a portion of the task-set to complete a task or set of tasks without input from the rest of the group.

The researcher refined these categories for tasks originally coded in the fourth category (alternating tasks). Tasks in which students collaboratively defined a strategy to solve a set of similar tasks and then alternated which student completed the calculation or wrote an answer while other students watched silently to check their work were grouped with the third category (splitting up similar tasks) as groups in this category also collaboratively defined their strategy before splitting up the tasks. Tasks in which students alternated which student was completing the task and the students who were not actively solving the task were engaged in off-task behaviors or discussions were grouped with the fifth category (take-over) as groups in this category also worked completely independent of other members of their group.

Following the identification and refinement of organizational practices, the researcher structurally coded all tasks for the primary organizational practice used to complete the task. The researcher coded by task as most groups employed different organizational practices for different tasks within a single task-set. Usually, groups completed a single task using only one organizational practice. However, occasionally a group would start a task using one organizational practice, engage in off-tasks behaviors or discussions, and then return to the task using a different organization practice. In these rare cases, the researcher coded the organizational practice with the most mathematical discourse contributions as the primary organizational practice used for that task.

The researcher created a frequency table comparing the numbers of tasks at each DOK level using each of the six organizational practices. These results provided a numerical comparison of the quantity of tasks completed using each type of organizational practice.

The researcher created percentage tables comparing intended DOK levels and mathematical discourse contributions for each of the six organizational practices (see Appendix F, Tables F.3-F1.4). The researcher compiled results from each of these tables within two heat maps (verbal CD and written CD) to better visualize the similarities and differences in student-enacted levels of CD based on intended DOK level and organizational practice. Contrasting colors indicate the primary enacted discourse category (i.e., red = minimal, yellow = considerable, blue = substantive, and green = extended) within each organizational practice and intended DOK level. The size of the circle indicates the percentage of discourse for the displayed primary enacted discourse category. For example, the highest quality of mathematical discourse contributions for the 59 DOK3 tasks discussed using Reflective Discourse and a chronological order organizational practice was minimal for 30.51% of the tasks, considerable for 6.78% of the tasks, substantive for 45.76% of the tasks, and extended for 16.95% of the tasks. The primary enacted discourse category DOK3 tasks using this organizational practice (i.e., highest percent) was substantive at 45.76%, which is indicated in the heat map by the color blue. The percentage of substantive discourse is indicated by a diameter that is

45.76% (.4 in) of a full-sized circle (.88 in) and the numerical representation of this percentage rounded to the nearest whole located in the center of the circle. These results provide visual comparisons of the relationship between the two discourse types and student-enacted CD across all four DOK levels to answer the overarching researching question.

CHAPTER 4

RESULTS

The results in this chapter are organized by the research questions in this study. Quantitative results for each of the three major research questions are presented first with explanatory qualitative data embedded throughout to help contextualize quantitative findings. Exploratory qualitative results relating to the overarching research question are presented at the end of the chapter.

Research Question 1: Written CD vs Intended DOK

Research Question 1 (RQ1) compared differences in student-enacted levels of cognitive demand (CD) based on students' written responses and the intended DOK levels of the mathematical tasks within and between groups. The quantitative results relating to students' written responses to the tasks show that students enacted 63-88% of the tasks at or above the intended DOK levels (Appendix D, Tables D.1-D.4). Results from the difference in proportions χ^2 test of associations show significant *p* values (see Appendix D, Table D.5), indicating that the enacted levels of CD for written tasks within each of the four unique groups within the crossover design (i.e., Group A Reflective Discourse with Task-Set 1, Group B Reflective Discourse with Task-Set 2, Group A Exploratory Discourse with Task-Set 2, Group B Exploratory Discourse with Task-Set 1) were not likely due to chance.

The heat maps in Figure 4.1 illustrate three different comparisons of intended and written evidence of student-enacted levels of CD for tasks at each DOK level. First,

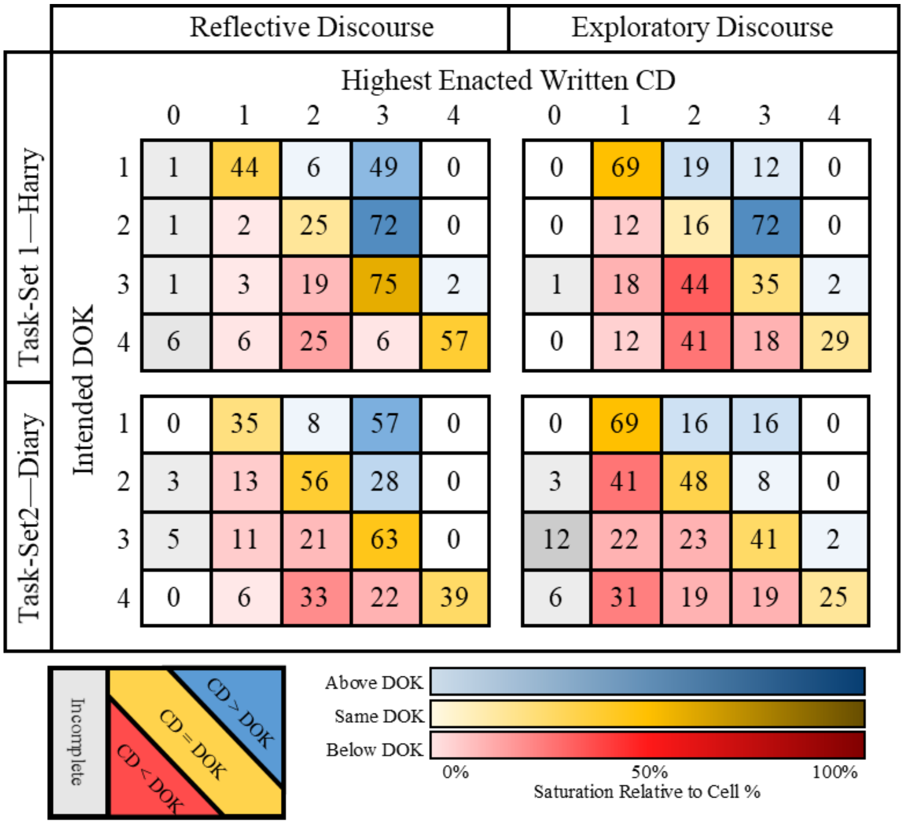


Figure 4.1. Heat maps of Intended DOK vs Written CD for each crossover group.

Figure 4.1 shows a comparison between reflective (left) and exploratory (right) discourse types. Numbers within the cells indicate the percentage of tasks at each written CD level relative to the total number of tasks at the same DOK level (same row). Next, Figure 4.1 shows a comparison between Task-Set 1 – Harry (top) and Task-Set 2 – Diary (bottom). Finally, Figure 4.1 shows a comparison of tasks at each DOK level that were incomplete during the study between each of the four groups. Zero indicates the percentage of tasks at each DOK level that were incomplete.

As seen in Figure 4.1, overall, most of the tasks were coded at (Yellow) or above (Blue) the intended DOK, regardless of the discourse type or task-set. Reflective Discourse (left side) had a higher percentage of tasks with written CD at or above the intended DOK than Exploratory Discourse (right side). Additionally, Task-Set 1-Harry (top row) had a higher percentage of tasks with written CD coded at or above the intended DOK, particularly for DOK2 Tasks, than Task-Set 2-Diary (bottom row). Finally, more tasks were incomplete for Task-Set 1 when students engaged in Reflective Discourse and more tasks were incomplete for Task-Set 2 when students engaged in Exploratory Discourse. Quantitative and qualitative findings relating to these three relationships are elaborated below.

Comparison of Written CD by Discourse Type

The first comparison of intended DOK and written CD showed that Reflective Discourse had a higher percentage of tasks with written CD at or above the intended DOK than Exploratory Discourse. Results from the chi-square test of independence, comparing written CD of tasks completed using Reflective and Exploratory Discourse, support these findings (Table 4.1).

Table 4.1 shows that 368 of the 442 (83%) tasks completed using Reflective Discourse had written CD at or above the intended DOK levels and only 74 tasks (17%)

Table 4.1

Chi-Square Test of Independence for Written CD by Discourse Type

	Written outcome				
	CD < DOK		CD ≥ DOK		
Discourse type	Count	%	Count	%	Total
Reflective	74	16.7	368	83.3	442
Exploratory	140	32.6	289	67.4	429
Total	214	24.6	357	75.4	871

Note. $\chi^2 = 29.667$, $p < .001$.

were below the intended DOK level. In contrast, 140 of the 429 (33%) tasks completed using Exploratory Discourse resulted in students' written responses being below the intended DOK level, and only 289 tasks (67%) resulted in students' written responses being at or above the intended DOK level (see Appendix Table D.7 for breakdown by Task-Set). This result may indicate that Reflective Discourse may have prompted students' written work to be at higher DOK levels.

Qualitative findings comparing students' written responses for DOK1-3 tasks, when those responses were consistently above or below the intended DOK levels, show that during Reflective Discourse, students were more likely to go back and revise or justify their previous answers. Figure 4.2 shows examples of student work from four different groups (for DOK1 task H4 and DOK3 task D3c) for both Reflective and Exploratory Discourse. In these examples, the written work from students after engaging in Reflective Discourse is on the left and written work from students after engaging in Exploratory Discourse is on the right. During Reflective Discourse, students used a black pen for their independent work and a colored pen for any revisions made during group work; during Exploratory Discourse, each student used a different colored pen.

On the left side of Figure 4.2, shows one student's original written response for Task H4 while the student was working independently (as noted by black pen). In this example, the student's response consisted of five black 'X's below the number line. This response is similar to the final written group response by a group engaged in Exploratory Discourse on the right side of Figure 4.2, which is CD1 level as it shows no work beyond the answer. However, during the Reflective Discourse, the student on the left for H4

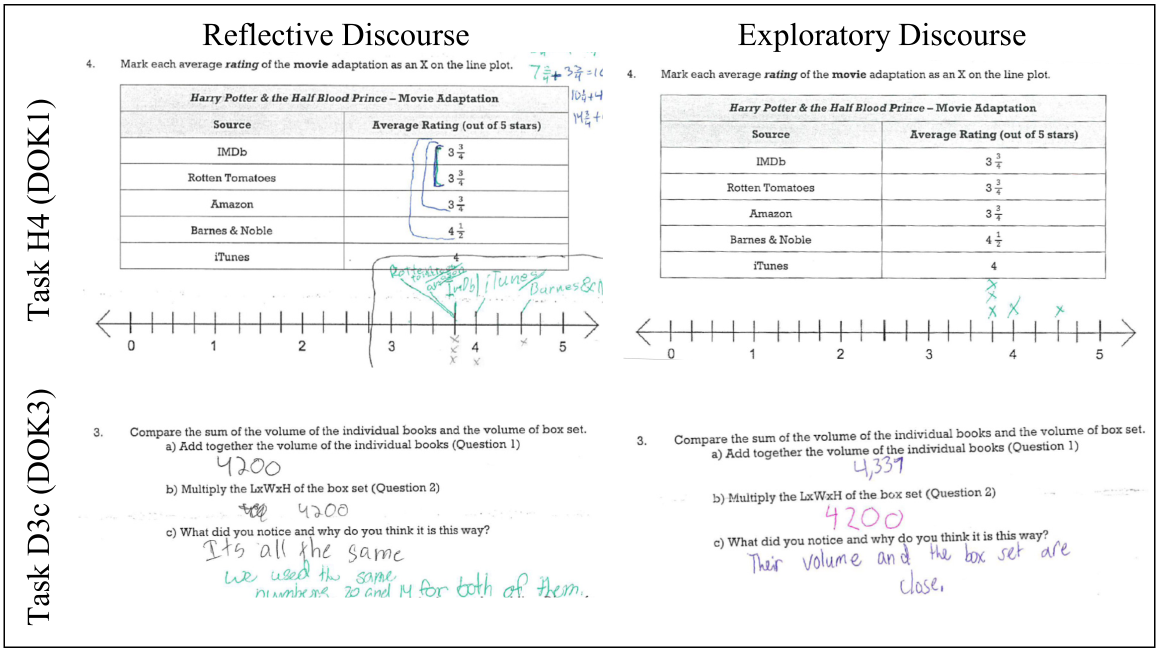


Figure 4.2. Samples of student work comparing reflective and exploratory responses.

changed her response (as noted by the green pen; the blue pen shows changes related to task H8) that justified the placement of each X by labeling them with the names of each source. These changes increased the student's original written CD level from CD1 to CD3. This same pattern is also apparent for a different students' responses to Task D3c (Figure 4.2, bottom row). The changes during Reflective Discourse (noted by green pen) increased the original written CD for one student to provide an explanation for why both answers were the same. On the right side, there are no visible revisions to the final response for either group, which was common in the written work for groups who engaged in Exploratory Discourse. This will be discussed further in the next section relating to verbal CD results.

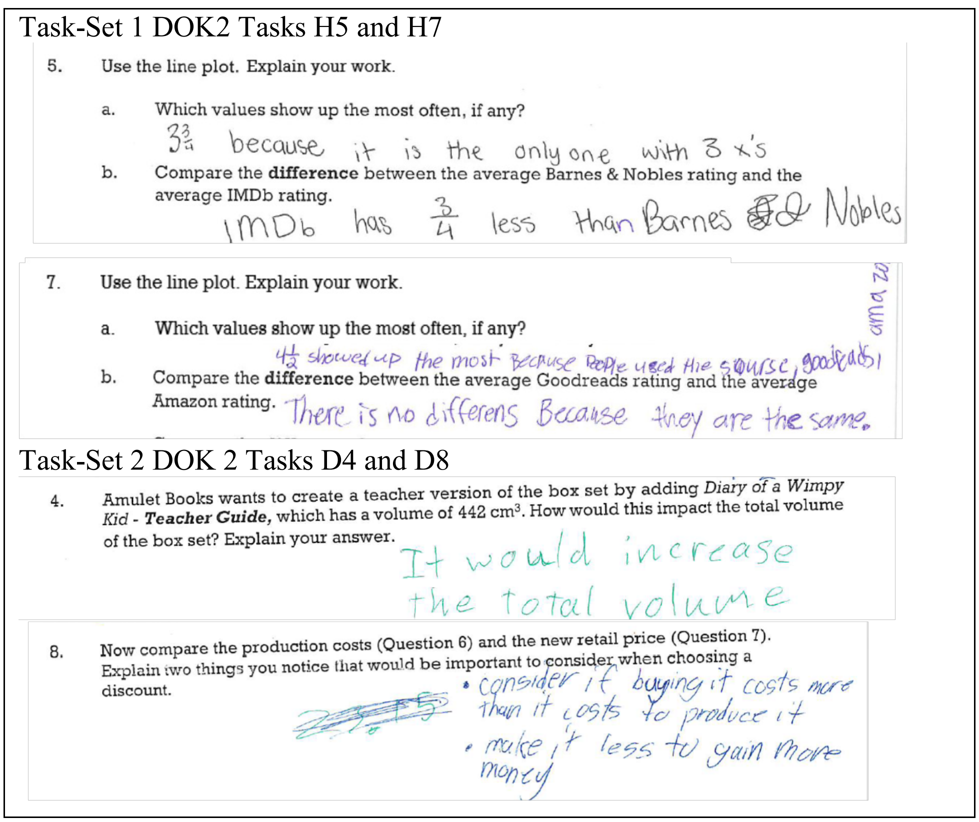
Comparison of Written CD by Task-Set

The second comparison of intended DOK and written CD showed that Task-Set 1 had a higher percentage of students' written CD coded at or above the intended DOK, particularly for DOK 2 Tasks, than Task-Set 2. Although this could be due to order effects of completing a second task-set one week after the first, qualitative results seem to indicate that this is more likely due to the theme of each task-set or the organization of the DOK2 tasks in each set.

The theme for Task-Set 1 revolved around the Harry Potter series, while the theme for Task-Set 2 revolved around the Diary of a Wimpy Kid series. Although all students in this study seemed familiar with the basic premises of these two series, many students may have had more personal experience with the Harry Potter series than the Diary of the Wimpy Kid series. For example, three times as many groups had students verbally share that they had seen or watched the Harry Potter series than the Diary of a Wimpy Kid series. Additionally, students who mentioned their experiences with the Harry Potter series were more likely to note their expertise with the series (e.g., "I can provide information because I have read and watched;" "He's read the book series like 8 times."). In contrast, students who mentioned their experiences with the Diary of a Wimpy Kid series were more likely to note their lack of expertise with the series (e.g., "I've read 'em, but I don't remember the summary;" "I've only seen the first movie so it's probably not the same."). The increased personal experience with the theme for Task-Set 1 may have increased motivation towards the task-set or provided students with additional schema, either of which could have increased engagement with the tasks at a higher level of CD.

A closer look at the DOK 2 tasks shows that two tasks from Task-Set 1 and two tasks from Task-Set 2 had very little variation in enacted levels of cognitive demand and may also help explain differences in written CD between the two task-sets. The written work for tasks H5 and H7 (Task-Set 1) were enacted at CD3 for almost every student group, whereas tasks D4 and D8 (Task-Set 2) were enacted at CD2 for almost every student group (see Appendix D.5). Figure 4.3 provides samples of student work, from four different groups of students, for these four tasks. The sample for Task H5 is from a group who engaged in Reflective Discourse; the samples for Tasks H7, D4, and D8 are from groups who engaged in Exploratory Discourse. All four DOK2 tasks in Figure 4.3 are comparative tasks that ask students to explain their response.

Figure 4.3 shows several justifications in students' responses for the H5 and H7


Task-Set 1 DOK2 Tasks H5 and H7

5. Use the line plot. Explain your work.

a. Which values show up the most often, if any?

$3\frac{3}{4}$ because it is the only one with 3 x's

b. Compare the **difference** between the average Barnes & Nobles rating and the average IMDb rating.

IMDb has $\frac{3}{4}$ less than Barnes & Nobles

7. Use the line plot. Explain your work.

a. Which values show up the most often, if any?

$4\frac{1}{2}$ showed up the most because people used the sourse, goodreads

b. Compare the **difference** between the average Goodreads rating and the average Amazon rating.

There is no differens Because they are the same.

Task-Set 2 DOK 2 Tasks D4 and D8

4. Amulet Books wants to create a teacher version of the box set by adding *Diary of a Wimpy Kid - **Teacher Guide**,* which has a volume of 442 cm³. How would this impact the total volume of the box set? Explain your answer.

It would increase the total volume

8. Now compare the production costs (Question 6) and the new retail price (Question 7). Explain two things you notice that would be important to consider when choosing a discount.

• consider if buying it costs more than it costs to produce it
• make it less to gain more money

Figure 4.3. Samples of student work for four DOK2 tasks.

tasks. For example, when one group wrote, "...because they are the same," for task H7b, they justified their response that there was no difference between the average Goodreads rating and the average Amazon rating. Most students, regardless of discourse type, justified their answers for H5a and H7a and justified or explained their responses for H5b and H7b, similar to the responses for these tasks in Figure 4.3. One possible explanation is that these tasks in Task-Set 1 required students to compare information contained within a single source (Line Plot, Figure 4.2) and a justification may have been an easy way to explain how they found their answer.

Figure 4.3 shows examples of general comparisons in students' responses for the D4 and D8 tasks. For example, when one group wrote, "It would increase the total volume," for task D4, they made a general comparison that adding the teacher's guide would increase the total volume of the box set. In contrast to the single source for both DOK2 task comparisons from Task-Set 1, both DOK2 tasks from Task-Set 2 required students to make comparisons between information from multiple tasks. For example, task D8 requires students to make comparisons between their responses to tasks D6 and D7. This may have prompted students to make general comparisons or statements similar to those in Figure 4.2, D4 and D8, regardless of discourse type.

Comparison of Incomplete Tasks by Group

The final comparison of intended DOK and written CD showed that more tasks were left incomplete for Task-Set 1 when students engaged in Reflective Discourse and more tasks were left incomplete for Task-Set 2 when students engaged in Exploratory Discourse. This may be due in part to class effects, as the two classes that completed

Task-Set 1, engaging in Reflective Discourse, are the same two classes that completed Task-Set 2 engaging in Exploratory Discourse.

Timing may have also played a role in the percentage of tasks that were incomplete during the study. For example, two groups engaged in Exploratory Discourse spent a lot of time on Task-Set 2 calculations for Level 1 questions due to a misconception (e.g., calculated 442 x 442 x 442 instead of 4200 + 442). This meant that they only completed tasks D1-D8 before they were out of time, with one group jumping from D8 to DF during the last 5 minutes (e.g., "We're almost out of time. This one is funner so let's jump to this."). This accounts for the high incompletion percentage for Task-Set 2 DOK3 and DOK4 tasks. The other incomplete tasks during Exploratory Discourse for Task-Set 1 stemmed from a group skipping questions (i.e., "We can't agree, so let's just move on."; "I don't know, let's skip to the next one and then come back."). Three groups accounted for the incomplete percentages for exploratory work. In contrast, the structured timing allotted for the Reflective Discourse (i.e., 10 min individual and 10 min group for Level 1 and 2 tasks, respectively; 15 min individual and 15 min group for combined Level 3 and 4 tasks) required students to move to the next level after a set period of time. This made it less likely that students would run out of time before reaching the later tasks in the set. However, it did make it more likely that students would run out of time before the last task in each level. This occurred once for students in nine different groups across seven different tasks and both task-sets, indicating that it was not the same groups nor the same tasks that accounted for the incomplete percentages during Reflective Discourse.

In summary, a majority of students' written responses were coded as HCD (CD2-CD4) while engaging in both Reflective and Exploratory Discourse. However, it is important to note that more of students' written responses were coded at or above the intended DOK level when students engaged in Reflective Discourse than when they engaged in Exploratory Discourse. One possible interpretation for this result may be the increased number of written revisions that students made during their participation in Reflective Discourse.

Research Question 2: Intended DOK vs. Verbal CD

Research Question 2 (RQ2) compared differences in student-enacted levels of cognitive demand (CD) based on students' verbal responses and the intended Depth of Knowledge (DOK) levels of the mathematical tasks within and between groups. The quantitative results relating to students' verbal responses show that 54-81% of tasks were enacted at or above the intended DOK levels. Specifically, students enacted 54-62% of reflective tasks and 63-81% of exploratory tasks at or above the intended DOK level, based on their verbalizations of the tasks (see Appendix E, Tables E.1-E.4). Results from the difference in proportions χ^2 test of associations for each group show significant *p*-values (see Appendix Table E.5), indicating that students enacted verbal CD levels within each of the four groups within the crossover design were not likely due to chance.

The heat maps in Figure 4.4 illustrate three different comparisons of intended and verbal evidence of student-enacted levels of CD for tasks at each DOK level, similar to Figure 4.1: (a) a comparison between reflective (left) and exploratory (right) discourse

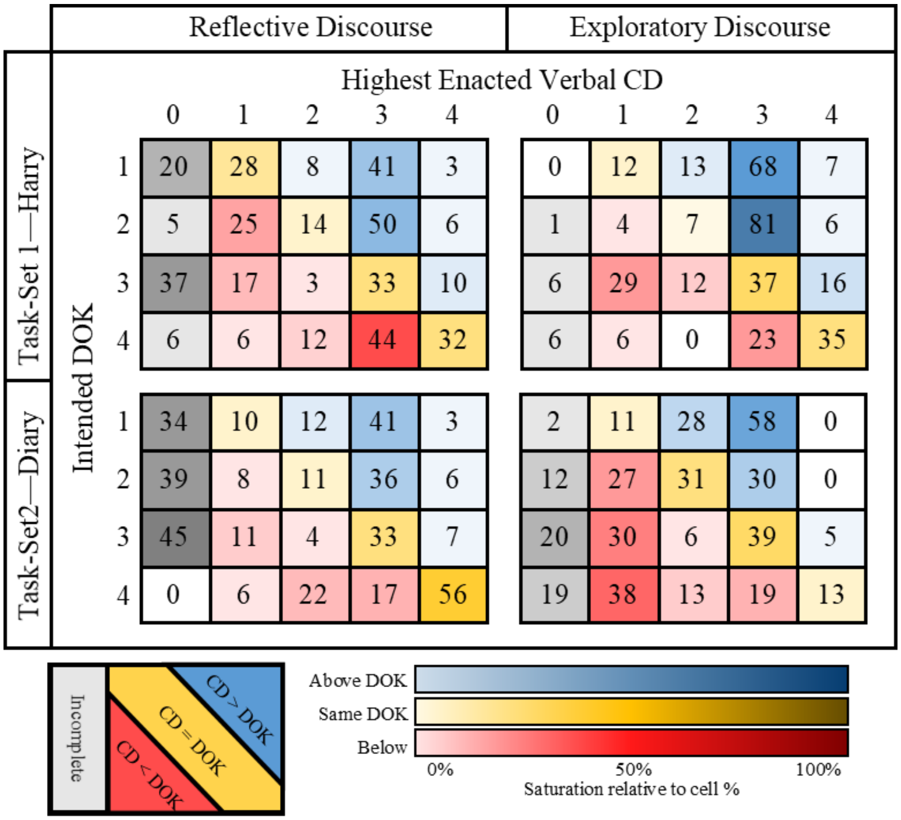


Figure 4.4. Heat maps of intended DOK vs verbal CD for each crossover grouping.

types; (b) a comparison between Task-Set 1 – Harry (top) and Task-Set 2 – Diary (bottom); and (c) a comparison of tasks at each DOK level that were not discussed during the study between each of the four groups.

As seen in Figure 4.4, overall, most of the tasks were coded at (yellow) or above (blue) the intended DOK, regardless of the discourse type or task-set. Exploratory Discourse (right side) had a higher percentage of tasks with verbal CD at or above the intended DOK than Reflective Discourse (left side), with the exception of Task-Set 2 DOK4 tasks. Additionally, Task-Set 1-Harry (top row) had a higher percentage of tasks with verbal CD coded at or above the intended DOK, particularly for DOK2 Tasks, than

Task-Set 2-Diary (bottom row). Finally, a higher percentage of tasks were not discussed during Reflective Discourse than during Exploratory Discourse. Quantitative and qualitative findings relating to these three relationships are elaborated below.

Comparison of Verbal CD by Discourse Type

The first comparison of intended DOK and verbal CD showed that Exploratory Discourse has a higher percentage of tasks where students' verbal responses were at or above the intended DOK levels than Reflective Discourse, with the exception of Task-Set 2 DOK4 tasks. Results from chi-square tests of independence comparing verbal CD of tasks completed using Reflective and Exploratory Discourse support these finding (Table 4.2).

Table 4.2.

Chi-Square Test of Independence for Verbal CD by Discourse Type

	Verbal outcome				
	CD < DOK		CD ≥ DOK		
Discourse type	Count	%	Count	%	Total
Reflective	190	43.	252	57.0	442
Exploratory	118	27.5	311	82.5	429
Total	308	35.4	563	64.6	871

Note. $\chi^2 = 22.825$, $p < .001$.

Table 4.2 shows that 311 of the 429 (72%) tasks completed using Exploratory Discourse were enacted by students with a verbal CD level at or above the intended DOK level and only 118 tasks (28%) were below the intended DOK. In contrast, 190 of the 442 (43%) tasks completed using Reflective Discourse were enacted by students with a verbal

CD level below the intended DOK level and only 252 tasks (57%) at or above the intended DOK (see Appendix Table E.7 for breakdown by Task-Set).

The results for students' enacted levels of verbal CD do not align with the results for students' written levels of CD. Qualitative results show that when students engaged in Exploratory Discourse, they were more likely to justify their ideas verbally, but not write the justifications on the paper, whereas when students engaged in Reflective Discourse, they were more likely to write down their justifications based on their reflections from the group discussion, but not necessarily talk about those justifications during the Reflective Discourse. The following excerpts from students' verbal responses help to illustrate this idea. Student names are pseudonyms for all excerpts. The first excerpt is from a group solving the H5b task (see Figure 4.2) while students were engaging in Exploratory Discourse.

Alice: So we have to minus it.

Bruce: Yeah, cause it says 'difference' and difference means subtract. So 4 ½ minus 3 ¾ cause Barnes and Nobles is 4 ½ and IMBd is 3 ¾.

Carrie: Write 4 ½ first cause Barnes and Nobles is bigger.

Bruce: [subtracting silently on paper] So ¾, what else should I write or should I only write ¾?

Alice: And then why did you subtract that.

Bruce: No, it doesn't ask that.

Alice: Yeah, cause it doesn't ask how.

Carrie: [writes ¾ on the paper]

[Video 410H2, 2:31-4:37]

In this excerpt, we can see several examples of high verbal CD as the students share solution strategies and justify their reasoning for those strategies. For example, when Bruce says, “Yeah, cause it says ‘difference’ and difference means subtract,” this shows that he his justifying the subtraction strategy based on key words. However, the final written CD for this group of students for the task was coded as low because there was no evidence of the justifications from the conversation in the group’s simple written response “3/4.” In contrast, the next excerpt is from a group of three students reflecting on the same task using Reflective Discourse.

Danika: Okay, next one. So what did you get for B?

Erik: That [pointing to the written response “4 ½ > 3 ¾ by ¾”]

Frank: Yeah, I did that [pointing to the written response “¾”]

Erik: What did you get on B Danika?

Danika: [pointing to written response “The difference is ¾ cause I subtracted.”]

[Video 280H2, 1:52-2:25]

In this excerpt, we can see several examples of high written CD for comparisons and explanations in the written responses, however the verbal CD focuses on organization for sharing responses and identifying answers which were coded as low verbal CD. For example, when Erik and Danika simply point to their written responses, this shows that they are only identifying their solutions, but not explaining or justifying their solutions or strategies verbally to each other. This pattern was repeated by multiple groups for both task-sets and tasks at all four DOK levels. For example, Figures 4.5 and Figures 4.6 provide samples of students’ written and verbal responses for a Level 4 task (DOK4) from Task-Set 2 where the written and verbal CD do not match. Figure 4.5 shows the first three google slides created by one student, Hank, in response to a Task-

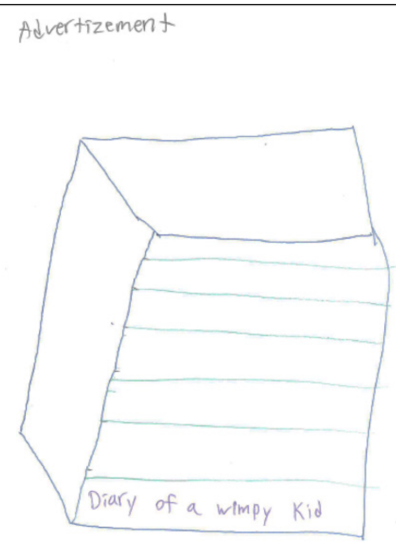
Greg: I did this one [points to #2 and shows picture]
Hank: I did presentation [scrolls through slides on Chromebook]
Isaac: [points to written summary under #4]
Greg: Ok. Diary. Wimpy Kid. Awesomeness. Done.
[Video 310D3, 2:45-3:04]

Figure 4.5. Reflective discourse level 4 written and verbal CD sample for Task-Set 2.

Set 2 Level 4 task, before engaging in Reflective Discourse with his group (top) as well as an excerpt of the small-group Reflective Discourse relating to Hank and his Level 4 task response (bottom).

As seen in Figure 4.5, the high written CD sample does not align with the low verbal CD in the excerpt from the students in this group discussing the task. This excerpt shows that even though Hank's written response had elements of high CD such as synthesizing information for multiple sources using technology, he did not verbally share his reasoning or strategies during the 20 second reflection with his group. Instead when Hank says, "I did presentation," his verbal response only identifies the medium he used to complete the task. This pattern of low verbal CD for DOK4 tasks was more common with groups engaging in Reflective Discourse where everyone in the group chose a different Level 4 task (e.g., "We all got different answers, so...") than with groups engaging in Reflective Discourse where students all chose the same Level 4 task and compared the differences in their written responses. Students engaging in Reflective Discourse were more likely to choose the same Level 4 task as other people in their group for Task-Set 2, which may explain the higher verbal CD for Reflective Discourse Task-Set 2 DOK4 tasks.

Figure 4.6 shows the collaborate hand-drawn written response to a Task-Set 2 Level 4 task from a group engaging in Exploratory Discourse (top) as well as an excerpt of students in this group discussing the task (bottom).



Janet:	I think that number 2 would be the best because it would get more people because we would be able to do a billboard and more people would see it and want to buy it.
Karen:	Like if someone sees a McDonald's hamburger and it looks really good, they will get one because they know where it is. So if we put stuff on the billboard it would make sense because more people would see it.
Laney:	Yea, but if the McDonalds burger looked that good, it doesn't mean it could be good cause this [pointing to task-set] could only have bar graphs to show if it's good or not.
Karen:	But we could put line graphs on the billboard.
Janet:	So let's do an advertisement [writes "Advertizement" on paper]
Laney:	So should we draw a picture of the box set?
Karen:	The box set? Yeah! I think the outline should be blue.
Janet:	They have all colors: blue, brown, orange, but mostly blue. Let's get something to look at [finds a box set for Beyonders and a copy of a Diary of a Wimpy kid book to look at]
Laney:	So you draw the box set and we will write the titles on the spines and figure out something after that.

[Video 270D4, 1:37-4:35]

Figure 4.6. Exploratory discourse Level 4 written and verbal CD sample for Task-Set 2.

On the right side of Figure 4.6, the low written CD sample does not align with the high verbal CD in the excerpt from the students in this group discussing the task. In this excerpt we can see several examples of high verbal CD as the students synthesize information from multiple sources to clarify and justify their ideas. For example, when Laney says, “Yea, but if the McDonalds burger looked that good, it doesn’t mean it could be good cause this [pointing to task] could only have bar graphs to show if it’s good or not,” this shows that Laney is providing a counterargument for Karen’s idea by synthesizing information from her multiple experiences with McDonalds, the current mathematics task-set, and using bar graphs to represent and interpret data. The class session ended shortly after this excerpt, which may explain why the coded written response only showed the low written CD outline of a drawing for a box set. If the students had more time, the written results for DOK4 tasks may have more closely reflected the verbal responses.

Comparison of Verbal CD by Task-Set

The second comparison of intended DOK and verbal CD showed that Task-Set 1 (Harry) had a higher percentage of students’ verbalizations enacted at or above the intended DOK levels, particularly for DOK 2 Tasks, than Task-Set 2 (Diary). These results are similar to the results for RQ1 which showed that students’ written work on Task-Set 1 was at a higher level. These results imply that students increased personal experiences with the Harry Potter series as well as the comparisons for Task-Set 1 DOK2 tasks focusing on one source may have influenced students’ verbal CD at higher levels of CD for Task-Set 1 than for Task-Set 2.

Comparison of Tasks not Discussed by Group

The final comparison of intended DOK and verbal CD showed that a higher percentage of tasks were not discussed during Reflective Discourse than during Exploratory Discourse. This means that students engaging in Reflective Discourse had more tasks where not a single student talked about their strategies or solutions.

Qualitative results show that this may be due to differences in how time was organized by students and their groups for Reflective and Exploratory Discourse. For example, during Reflective Discourse, the researcher provided 10 minutes for students to discuss Tasks H1-3. One group that had no differences in their responses for Tasks H1, H2, or H3 were able to discuss all three tasks in 5 minutes and then spent the remaining 5 minutes justifying and checking their responses. In contrast, a different group where each student had a different response for H1, spent the entire 10 minutes discussing their solutions and strategies for H1 and never discussed H2 or H3, even though they each had a response written for these tasks.

During Exploratory Discourse, students organized the time they allotted to discuss each task. This allowed them to spend more time on tasks where they disagreed on the solutions or strategies and less time on tasks where they agreed on the solutions or strategies, touching on most of the tasks at least once. Most tasks that were not discussed during Exploratory Discourse were completed independently by one member of the group or, similar to the written CD results, were left incomplete because the group ran out of time.

In summary, a majority of the verbal discussion contained elements of HCD

(CD2-CD4) during both Reflective and Exploratory Discourse. The highest levels of students' verbal responses (i.e., those coded at or above the intended DOK levels) occurred more often during Exploratory Discourse than Reflective Discourse. This contrasted written results, possibly due to groups engaging in Reflective Discourse that did not verbally elaborate on their written responses and groups engaging in Exploratory Discourse that did not write everything they verbally discussed. The next section looks deeper at these differences between student discussion practices for Reflective and Exploratory Discourse.

Research Question 3: Reflective vs. Exploratory Discourse Contributions

Research Question 3 (RQ3) focused on differences in the quantity and quality of mathematical discourse contributions during small group Reflective Discourse (after students individually solved the mathematical tasks at four DOK levels) and Exploratory Discourse (while students solved the mathematical tasks as a group at four DOK levels). Quantity was defined in two ways: a) actual amount of time engaged in verbal discussions of the mathematics tasks, and b) number of codable mathematics discourse contributions. Quality was defined by the categories in the Mathematical Discourse Contribution Rubric (Table 3.3) which include: Organizational and Minimal (low quality), Considerable (sharing strategies), Substantive (justifications, generalizations), and Extended (multiple sources).

The results for this section are organized first by differences in the quantity of time and number of discourse contributions for each discourse type and second by the quality of the mathematical discourse contributions for each discourse type.

Quantity of Time and Discourse Contributions for Each Discourse Type

Results relating to the quantity of time spent discussing the mathematics tasks show that there was an average difference of less than one minute in the time that groups spent discussing the mathematics tasks across the discourse types (i.e., reflective and exploratory). Table 4.3 illustrates the time differences for groups that were on-task the entire time as well as groups that were off-task at least once during the task-set. The Ns in the table indicate the number of groups who were on- or off-task for Reflective and Exploratory Discourse, respectively.

Table 4.3

Frequency of Time Spent Discussing Mathematics Tasks in Minutes

	Reflective discourse		Exploratory discourse		
Heading	Mean	Min-max	Mean	Min-max	Difference (Ref – Col)
On-Task (N = 16, 17)	14.75	11.10-17.80	15.41	11.67-20.10	-0.66
Task-Set 1 (H)	14.65	13.06-17.80	15.81	11.67-20.10	-1.16
Task-Set 2 (D)	14.81	11.10-17.23	14.47	13.08-18.43	0.34
Off-Task (N = 18, 17)	8.83	3.05-13.15	8.45	2.70-12.60	0.38
Task-Set 1 (H)	7.75	3.05-11.48	9.65	8.80-11.12	-1.90
Task-Set 2 (D)	9.78	4.32-13.15	7.90	2.70-12.60	1.88
Overall (N = 34)	11.17	03.05-17.80	12.03	02.70-20.10	-0.86

As seen in Table 4.3, groups of students who were on-task during the entire task-set spent an average of 15 minutes actively engaging in group mathematical discourse and the remaining 55 minutes engaging in individual reflection, computation, or writing –

regardless of discourse type. Groups of students who were off-task at least once during the task-set spent an average of 8 or 9 minutes engaged in group mathematical discourse. Overall, students spent an average of 11-12 minutes engaging in mathematical discourse. This indicated that, despite the amount of time provided for discussion, students following Reflective or the Exploratory Discourse procedures, were engaging in similar amounts of time for verbal discussions and individual reflections or computations.

Results for the quantity of discourse also indicate that, when students engaged in Reflective Discourse, they spent more time discussing HCD tasks (DOK2-4). In contrast, when students engaged in Exploratory Discourse, they spent more time discussing low cognitive demand tasks (DOK1). Figure 4.7 provides a visualization of the percentages of discussion spent on tasks at each DOK level (see Appendix F, Table F.1 for specific tasks).

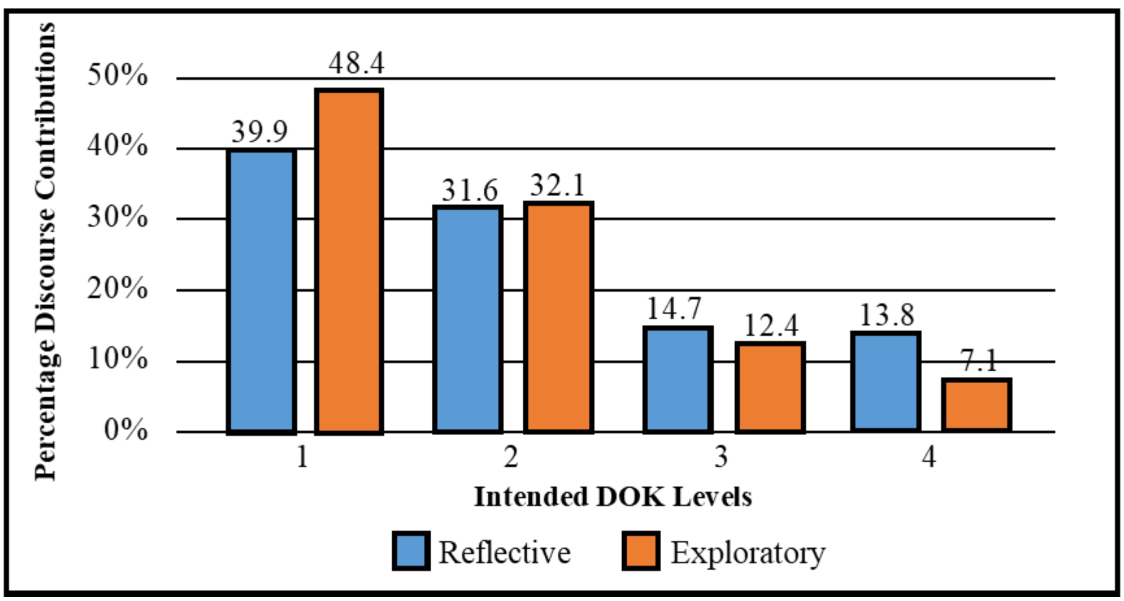


Figure 4.7. Discourse contributions for tasks at each DOK level.

As seen in Figure 4.7, about 48% of the discourse contributions during Exploratory Discourse were focused on DOK1 tasks, whereas only about 40% of the discourse contributions during Reflective Discourse were focused on those same tasks. Additionally, the percentage of discussion contributions when students were engaged in DOK4 tasks during Reflective Discourse was almost double the percentage of discussion contributions when students were engaged in Exploratory Discourse.

Qualitative results show that one possible reason for the distribution in Figure 4.7 could be attributed to how the researcher allotted the amount of time students engaged with different levels within the task-sets during each discourse type. For example, DOK3 and DOK4 tasks were located primarily in Level 3 and Level 4. During Reflective Discourse procedures, the researcher allotted 15 minutes (approximately 1/3 of discussion time) for students to work individually on Level 3 and Level 4 tasks and then 15 minutes more for groups to discuss their results and strategies. This may have positively influenced the percentage of time students engaged with tasks at DOK3 and DOK4 levels.

During Exploratory Discourse procedures, the researcher allotted 70 minutes for Level 1-4 tasks, and allowed students to choose how to allot time for individual work and group discussions for each level. Although most groups started Level 3 tasks with about 30 of their 70 minutes remaining (similar to reflective groups), several groups started Level 3 tasks with 15 minutes or less remaining to discuss and work on the tasks, which could explain low percentages of the discussion time focusing on tasks at these levels.

The structured time during Reflective Discourse provided a scaffold for students to encourage the discussion of tasks at each DOK level. This scaffold had positive benefits in prompting students to discuss results and strategies for tasks at each DOK level. However, it also created conflicts for some groups who either completed their reflective discourse early (and then proceeded to off-topic discussions) or other groups that wanted extra time to reflect on the tasks, but were forced to move on to the next set before they had completed their discussion (e.g., high percentage of tasks not discussed during reflective discourse). In contrast, during exploratory discourse, the removal of the time scaffold allowed students the discretion to move to the next task or spend extra time discussing a particular task. However, it may have limited the time for some groups to discuss the higher DOK level tasks (e.g., the two groups that ran out of time).

Additionally, most groups engaged in Exploratory Discourse had at least one group member comment on how long they had been working on the tasks about 25-40 minutes into the task-set (e.g., "Guys, the camera says we have been working on this for 34 minutes! Hey, [talking to another group] how much time does your camera say?"). A few groups engaging in Exploratory Discourse revisited the time again about 10-15 minutes before the end of the class session (e.g., "How much time is left?") and then either responding with a frantic desire to work harder (e.g., "We need to hurry!") or a fatigued lack of initiative (e.g., "This is long...can we just be done?"). In contrast, no groups engaged in Reflective Discourse asked about the time or commented on the amount of time displayed on the camera. The verbal reminders by the researcher (e.g., "You have 10 minutes to work on your own;" "You have 10 minutes to talk with your group."), may have eliminated the need for students to comment on the time.

Additionally, the alternating time between individual and group work for groups engaged in Reflective Discourse may have provided students with a break or change of pace which decreased the fatigue experienced by groups engaging in repetitive actions during Exploratory Discourse.

A second possible reason for the distributions of time for DOK1 tasks is that many groups engaging in Exploratory Discourse spent time discussing strategies to solve the DOK1 tasks, whereas groups engaging in Reflective Discourse had already chosen their own strategies during their individual work time before they engaged in Reflective Discourse with their groups. Results relating to the types of discourse contributions, such as strategy, are discussed more in the next section relating to the quality of discourse.

Quality of Mathematical Discourse Contributions

A quantitative chi-square analysis of the mathematics discourse contributions showed no significant differences between the quality of mathematical discourse contributions, based on whether the discussion was reflective or exploratory ($\chi^2 = .832$, $p = .362$). Although there were no significant differences, there were two subtle differences between Reflective and Exploratory Discourse patterns among the groups in the quality of students' mathematical discourse contributions relating to students' on-task and off-task discourse practices.

Quality was defined by the categories in the Mathematical Discourse Contribution Rubric (Table 3.3). Organizational and Minimal categories are considered low quality, as they pertain to surface level basic recall of facts and organizing responsibilities and actions (Walter, 2018). Quality increases as the categories progress to Considerable (sharing strategies), then Substantive (justifications, generalizations), with the Extended (multiple sources) category as the highest quality. Each progressive category aligns with a deeper level of mathematical connections (Bishop et al., 2016; Durfee, 2018).

Mathematical discourse practices. Students' mathematical contributions coded as 0c-4c directly related to the mathematical tasks. Figure 4.8 shows a visual representation comparing percentages of student contributions for each category of mathematical discourse by discourse type (see Appendix F, F.2 for specific sub-codes within each category).

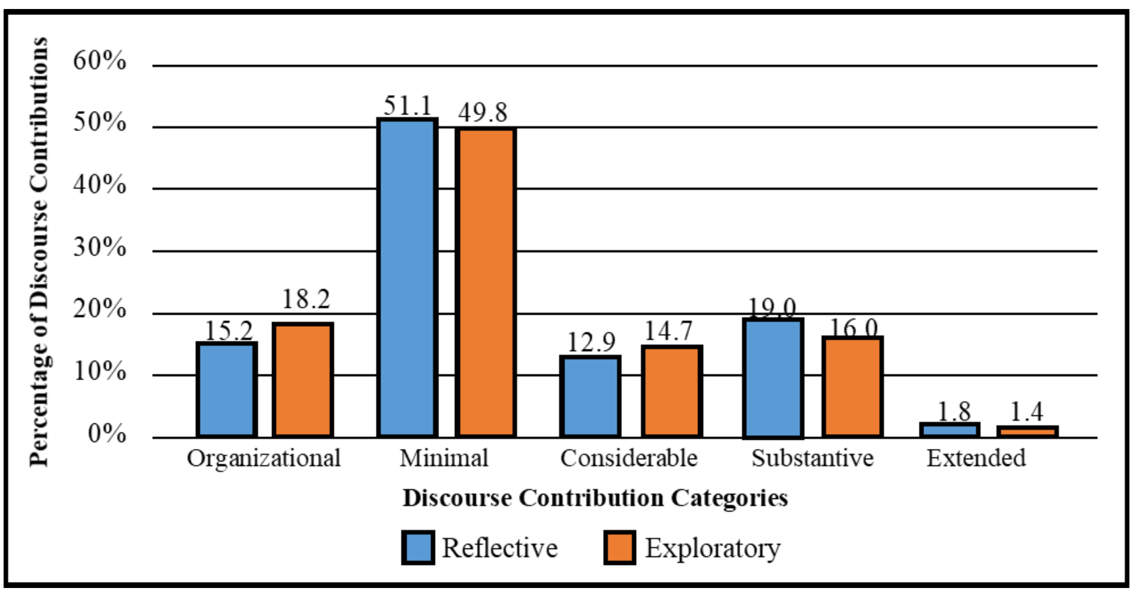


Figure 4.8. Categories of mathematical discourse contributions.

As seen in Figure 4.8, students' discourse contributions during Exploratory Discourse were slightly higher for organizational and considerable categories, whereas students' discourse contributions during Reflective Discourse were slightly higher for minimal and substantive categories. A qualitative examination of subcodes within each of these categories indicates that students engaged in Exploratory Discourse were more likely to complete tasks by (a) sharing strategies (considerable) to complete tasks, then (b) organizing how to complete the tasks (organization), and finally (c) discussing the calculations or facts related to the task (minimal). Most students only engaged in counterarguments or justifications if one strategy was unknown to a member of the group or if there was a contradiction between calculations for different members of the group. The following excerpt provides an example of this practice for task H1. The first letter of the coded category for each response is written to the left of each response.

[O] **Mary:** [Reading Instructions] So what are we going to do first?

[M] **Nora:** First we need to figure out how many they sold, so... total sold price… so they sold, so how

[C] **Mary**: So July tenth through august first they made 35.32 dollars.

[C] **Nora:** That's millions of tickets.... I'm pretty sure we are supposed to times or divide these

[C] **Mary:** So each ticket is 7 dollars, they sold that many tickets, how much money did they make? Divide?

[C] **Nora:** Multiply. So 35.32 times 7 or just 3532 times 7.

[O] **Nora:** I'll do this one. [calculating]

[M] **Mary:** 24,724! That's huge!

[M] **Nora:** Then we move the decimal guy in here.

[O] **Mary:** You write the answer. [Nora writes 247.24]

[M] **Nora:** Okay first one, okay now we got to figure out August 7-December

[C] **Mary:** So divide it. 7 divided by 37

[O] **Mary:** I'll do this one [calculating]

[M] **Mary:** So 4.31.

[O] **Nora:** Write that in the box. [Mary write 4.31 on task-sheet]

[S] **Nora:** I think if it's on this side you divide, if it's on this side you multiply

[O] **Mary:** Okay, then let's do that. [Video 370H1, 0.00-5:48]

In this excerpt, with 17 discourse contributions, 6 were organizational [O], 5 were minimal [M], 5 were considerable [C], and 1 was substantive [S]. This shows that the focus of this discussion, like many exploratory discussions without contradictory student answers, had the largest contributions in the organizational, minimal, and considerable categories.

In contrast, students engaged in Reflective Discourse were more likely to share their answers as statements (minimal) unless there was a contradiction of answers between different members of the group. At that point, students engaged in counterarguments or justifications of their answers (substantive). Most students only shared new or more efficient strategies (considerable) during Reflective Discourse if the group determined that most or none of the answers shared were correct. The following excerpt provides an example of this practice for task D1. The first letter of the coded category for each response is written to the left of each response.

[M]	**Olsen:**	Okay, so for the first one, I got 662
[M]	**Peter:**	672. You wrote 672.
[M]	**Olsen:**	Oh yeah, 672 and for the next one . . .516
[S]	**Quincy:**	616 dude cause this one's gotta be bigger than this one.
[O]	**Olsen:**	I am going to solve it again, just in case.
[C]	**Peter:**	I checked it two ways, subtraction and multiplication.
[S]	**Peter:**	I learned that each time you add the point 1 to this, it adds 28
[M]	**Olsen:**	Oh, Oops, I forgot the 1. [Revising answer]
[M]	**Peter:**	The next one was 532
[M]	**Olsen:**	The next was 616 after that
[M]	**Peter:**	and then 644
[M]	**Olsen:**	yeah, 644 and then 588
[M]	**Quincy:**	and then 532
[M]	**Olsen:**	yea 532. [Video 490D1, 1:05-2:10]

In this excerpt, with 14 discourse contributions, 1 was organizational [O], 10 were minimal [M], 1 was considerable [C], and 2 were substantive [S]. This shows that the focus of this discussion, like many reflective discussions was minimal confirmations, except when there was a contradiction in answers, which prompted considerable or substantive responses.

Nonmathematical discourse practices. The researcher coded two non-mathematical discourse sub codes: Code 0a: Student was silent during the task (silent); and Code 0b: Student's verbal contributions were not related to the mathematics task (off-topic). Table 4.4 shows a frequency count for these two codes for Reflective and Exploratory Discourse.

Table 4.4

Instances of Nonmathematical Discourse

Discourse	Silent (0a) No. of instances	Off-Topic (0b) No. of instances
Reflective	56	91
Exploratory	78	130

As seen in Table 4.4, students were more likely to remain silent during the discourse for a task or contribute an off-topic comment during Exploratory Discourse than during Reflective Discourse. However, it should be noted that during Reflective Discourse procedures, although students had half the time allotted where they could potentially remain silent or contribute an off-topic comment, the number of reflective instances is more than half the number of exploratory instances. This may be due in part to groups that completed their discussion before the end of the allotted time during reflective discourse and then engaged in off-topic discussions until prompted to move to the next set of tasks.

Although some silent students were not engaged with the task (e.g., playing with paperclips in their desk), most students who were silent appeared to be engaged in active listening (e.g., looking at other students in group and writing on paper in response to other students' discourse contributions). A closer look at the instances where a student remained silent showed that all 56 of the Reflective Discourse instances were spread across 29 different students and 27 of the 78 exploratory instances were spread across 12 different students (about 1-2 silent tasks per student). Many of these silent students gave non-verbal indications of agreement to other students' comments. In contrast, the remaining 51 exploratory instances were spread across only eight students (about 6-8 silent tasks per student). A qualitative examination of these eight students seemed to indicate that they typically struggled with mathematics, based on self-identification during the discourse (e.g., "I'm usually wrong.") or listed with a math IEP on the survey.

Qualitative results looking at these eight students, as well as two more students whose comments identified them as potentially typically struggling with the mathematics (e.g., "I just don't really get how to do math"), found that overall, Exploratory Discourse did not seem to support these students. This was evident in three common behavioral patterns for the struggling students during Exploratory Discourse. First, similar to the results from Table 4.4., the struggling students often said little to nothing during the entire task-set and were often ignored by their group. For example, the only phrase one student with an IEP, Penny, said while completing a task-set using Exploratory Discourse was, “You are going too fast.” After this comment, her group told her the answers and what to write on her paper after they solved each task. Other struggling students often went without speaking for three or four tasks in a row.

Second, the struggling students’ ideas were often devalued by other members of their group. The following excerpt provides one example of this behavioral pattern. Shawn is the typically struggling student in this excerpt from the H5 task. In earlier tasks leading up to this point, Shawn usually took longer than the other two group members to complete his calculations. Shawn’s calculations also frequently contained errors.

> Ryan: So Amazon is 3 3/4, minus iTunes, which is 4.
>
> [Ryan and Tara start to subtract the fractions]
>
> Shawn: It’s 1/4
>
> Tara: You have to show your work
>
> Shawn: I just know it
>
> Tara: Then how did you do it in your head? Cause you can’t minus that. That would equal 1 ¾ so how did you do that in your head?
>
> Shawn: It’s just like...[pointing to line plot].
>
> Tara: [interrupting] If you can’t say it then you need to do it.
>
> Shawn: Maybe if you do 4 minus 3 and you get 1...and then minus ¾
>
> Tara: You can’t do that or you would get negative. You have to change this to improper fraction and that makes 4/1 and this [points to 3¾] makes 9/4 and you have to make a common denominator so this is 16/4 and then you subtract so it’s 7/4 and then you have to change it to a mixed number so the answer is 1¾.
>
> Shawn: Yours is probably correct. [Video 460H2, 11:30-13:20]

In this example, we can see that even though Shawn had the correct answer and what appeared to be a valid strategy to get to the answer, Tara’s interruption did not give him the time to fully work out his reasoning to support his answer. In the end, Shawn conceded that Tara was probably correct, even though her calculations were incorrect. This pattern of concession was similar among other typically struggling students whose divergent answers or strategies were not valued or explored (e.g., "Okay, you're the smart one, so what's the answer;" or "You are probably right cause you are usually right."). Two groups that ignored divergent ideas from typically struggling students actually caused themselves more work. These two groups were the same groups that did not complete the entire task-set while engaging in Exploratory Discourse.

There was one exception to these patterns for Valerie, a student with an IEP. Valerie was placed with a group who was very patient with her and never took over her work. For example, if Valerie said, "I need help," her group would ask, "Which part," or "Would you like to try...[specific strategy from class]." Additionally, Valerie's group would use questioning techniques such as "Do you remember how [teacher's name] taught us last week?" or "Okay, so what do you need next to be able to do that?" at least three times before offering a specific suggestion for the next step. This may have provided Valerie with the support to ask questions, request more time to complete a task, or even offer the occasional strategy to complete a task.

In contrast, groups engaged in Reflective Discourse were more likely to support students that typically struggled with mathematics. One reason Reflective Discourse may have supported typically struggling students is that it provided students time to think through their own strategies. For example, in this except from task D7, Shawn had a chance to try out his strategy before comparing answers with Ryan and Tara.

Tara: So on this one I multiplied and ½ times 60 is 30.

Shawn: I divided.

Tara: No, so you have to multiply. You change this to an improper fraction so 60 is on top of 1 and then 60 times 1 is 60 and...

Shawn: [interrupts] But it's the same answer.

Ryan: Me too. So 1/3 of 60 is like 20.

Tara: Yep and 1/4 times 60 is 40.

Shawn: It's 15. Cause 40 + 40 + 40 + 40 is more than 60.

Ryan: So 1/5 of 60 is 12.

Shawn: And 60 divided by 10 is 6.

Tara: So 1/7 times 60 is...49?

Shawn: [Shows Tara his division] so it rounds to 8.

Tara: Ok [revises her answer]. [Video 460D2, 2:51-3:54]

In this excerpt, Shawn is a lot more confident in his answer and in his strategy than he was in the exploratory excerpt. During Reflective Discourse, Shawn was able to show Tara that his strategy produced the same answer as hers on the first few calculations. This may have helped Tara to accept his strategies and answers on the later calculations that did not match her answers.

When student groups reflected on the tasks in a prioritized order, the typically struggling students were more likely to ask for help. For example, Penny, our IEP student that never talked during the tasks set where her group engaged in Exploratory Discourse, contributed 19 different times during the task-set where her group engaged in Reflective Discourse. Most of these contributions confirmed shared answers (e.g., "Yeah, I got that too"); however, occasionally she would ask her group for help or add ideas to the conversation. An example of this is found in the excerpt below.

Wendy: What did you get down here [pointing to H2] cause I didn't finish.

Penny: I am pretty slow writer and thinker so I did not make it down there. But show me what you did.

Wendy: I did .5 million times 12.99. [points to work for 500,000 times 12.99 that is half finished]

Penny: It says round. $12.99 is like $13 so can you just do 13 times 5?

Xander: So I did that and I got 65.

Wendy: So on this one [points to Q3], I am pretty sure this is the movie [points to Q1] and this is the books [points to Q2]

Penny: Paperpacks means books

Wendy: So yea, so up here is 247.24 million and the highest one here was just 65 million so I think down here people saw the movie more.

Penny: Yea, I think they saw the movie more.

Xander: The price was bigger than $7 and the price was bigger but it is a smaller number.

Penny: Yea and I think people don't really like reading any more. [Video 210H1, 0:31-2:35]

In this excerpt, we can see that, not only is Penny asking for help, her group is willing to explain their answers rather than just asking her to copy them. Additionally, Penny was able to add ideas and strategies to the discussion. One factor that may help explain the increased willingness to ask for help is that many groups engaged in Reflective Discourse had one student (not typically struggling) who started the discussion by asking about a task they struggled with. For example, in this excerpt, Wendy moved the discussion directly to Task H2 because she struggled to complete her very large (and unnecessary) calculation. Seeing other students struggle may have helped students who typically struggle feel more comfortable asking for help. It also may have helped the other students in the group feel more patient with their explanations because they could point to their work during their explanation or compare their work to the struggling student's work to find where their calculations diverged.

In summary, solving tasks together while engaging in Exploratory Discourse had the highest quality of mathematical discourse contributions. However, the long period of time allotted for Exploratory Discourse may have increased student fatigue and resulted in less time spent on HCD tasks. Exploratory Discourse was also more likely to exclude students who typically struggle with mathematics. Reflective Discourse typically had a high quality of mathematical discourse contributions. Reflective Discourse was also more likely to include students who typically struggle with mathematics. However, students were more likely to confirm each other's answers (low-quality discourse) unless there was dissonance between students' answers or strategies.

Overarching Research Question

The overarching research question sought to understand the relationship between the two types of small-group discourse (reflective and exploratory) and student-enacted levels of CD when engaging with mathematics tasks at the four different intended DOK levels.

Qualitative results comparing groups with unusually high or low student-enacted levels of CD revealed different organizational practices that may have contributed to the quality of mathematical discourse contributions and student-enacted CD. Two organizational practices identified for students engaged in Reflective Discourse were: (a) "order" - chronological order (started with the first task and continued sequentially) and (b) "priority" - prioritized order (students jumped around to different tasks they struggled with during independent work time). Four organizational practices identified for students engaged in Exploratory Discourse were: (a) “together”—working together to solve a single task, (b) “side-by-side”—working side-by-side to individually solve the same task at the same time, (c) “split up”—splitting up similar tasks so that different students worked on different tasks, usually at the same time, and (d) “take over”—one student took over a portion of the task-set to complete a task or set of tasks without input from the rest of the group. Table 4.5 shows the number of tasks at each DOK level completed using theses organizational practices.

Table 4.5

Number of Tasks Completed by DOK Level for each Organizational Practice

	Reflective				Exploratory							
	Order		Priority		Together		Side by side		Split up		Take over	
DOK level	Count	%	Count	%	Count	%	Count	%	Count	%	Count	%
DOK1	70	71	28	29	47	40	43	36	20	17	9	7
DOK2	78	75	26	25	57	51	20	18	14	12	21	19
DOK3	59	76	19	24	71	68	7	7	7	7	20	18
DOK4	23	75	8	25	21	72	0	0	0	0	8	8
TOTAL	230	74	81	26	196	54	70	19	41	11	58	16

As seen in Table 4.5, most groups engaged in Reflective Discourse reflected on tasks in sequential order, while most groups engaged in Exploratory Discourse either worked together on the tasks or worked on the same task side-by-side. Additionally, groups either worked together on DOK4 tasks or allowed one student to complete the tasks on their own.

The first synthesis shows the results of the analysis of students’ verbalizations.

The heat map in Figure 4.9 provides a visual representation which combines the highest level of verbal CD and quality of discourse for tasks in each of these organizational groups and DOK levels. Color indicates the primary category of the highest number of discourse contributions (category with the most codes). Size indicates the percentage of discourse codes at the primary category relative to all discourse codes for the indicated DOK level. For example, the large blue circle with the number 84 under Reflective Discourse indicates that 84% of DOK3 tasks coded in the Priority category had at least one Substantive (CD3) discourse contribution as the highest level of discourse contributions. The other 16% is not displayed here, but is distributed among the other

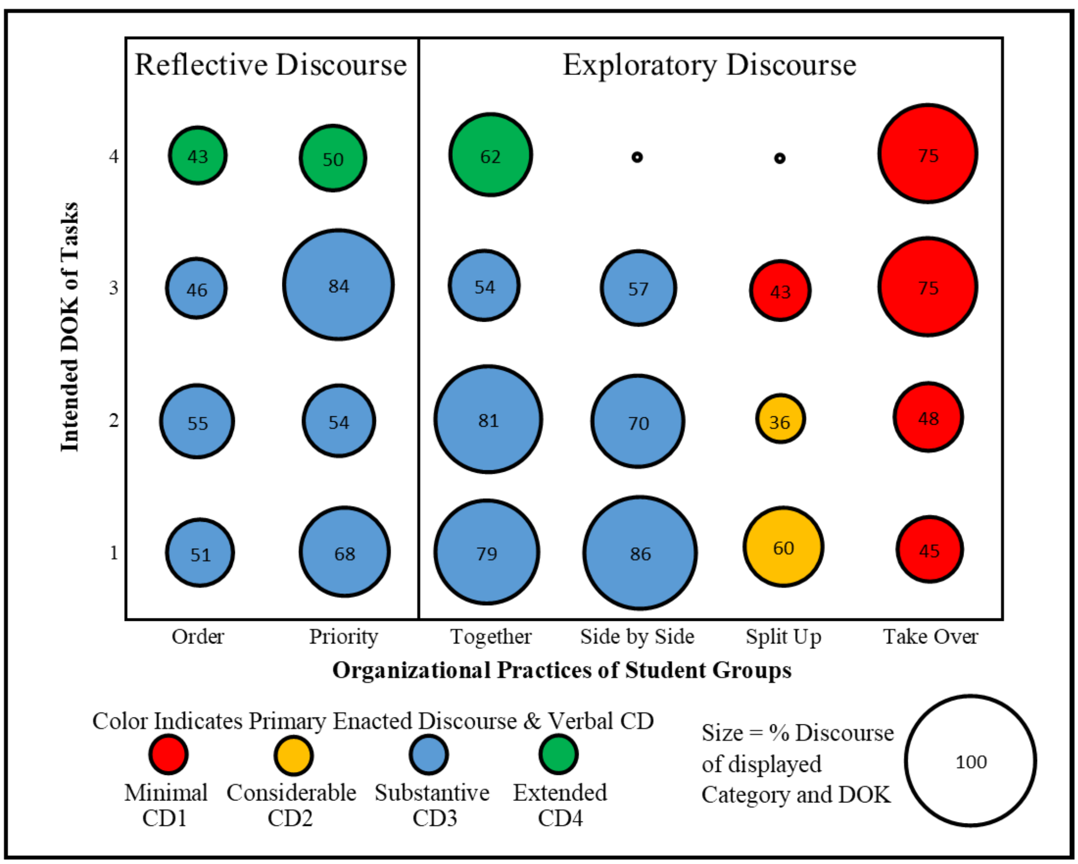


Figure 4.9. Quality of mathematical discourse contributions and verbal CD based on organizational practices and intended DOK levels.

three categories (see Appendix F, Tables F.3-F.8 for percentages of highest discourse contributions for each category and DOK level.). The small red circle with the number 43 under Exploratory Discourse indicates that 43% of DOK3 Tasks in the split-up category did not have any discourse contributions higher than Minimal (CD1). The other 57% of DOK3 Tasks in this category were distributed among the other three categories, with no categories higher than 42%.

As seen in Figure 4.9, Reflective Discourse tasks discussed in a prioritized order had a higher quality of discourse than tasks discussed in chronological order, with the primary discourse categories being substantive (verbal CD3) or extended (verbal CD4). Exploratory Discourse tasks that were discussed together or worked side-by-side had the highest quality of discourse. This seems to indicate that organizational practices that focused on collaboration, such working together to solve problems, had a higher quality of discourse and students' verbalizations were at a higher level of CD, regardless of the intended DOK level. Exploratory tasks that were split up between members of the group had lower qualities of discourse, especially with regards to DOK3 tasks. Tasks in which one student took over had the lowest quality of discourse, with 45-75% of these tasks discussed at minimal levels, regardless of the intended DOK level. This seems to indicate that organizational practices that did not focus on collaboration had a lower quality of discourse.

Figure 4.9 also shows that different discourse types seemed to be more effective for tasks at different intended DOK levels. For example, students working together during Exploratory Discourse were more likely to engage in substantive discourse (e.g., verbal CD3) for DOK1 and DOK2 tasks than students engaging in Reflective Discourse. In contrast, students engaging in prioritized Reflective Discourse for DOK3 tasks were more likely to engage in substantive discourse (e.g., verbal CD3) than students engaging in Exploratory Discourse. Both practices supported extended discourse (verbal CD4) for DOK4 tasks, with 62% of groups working together during Exploratory Discourse engaging in extended discourse. None of the student groups completed DOK4 tasks using Side-by-Side or Split-Up organizational practices, as indicated by the small dots.

The next synthesis shows the results of the analysis of students' written work. The heat map in Figure 4.10 provides a visual representation of the written CD for each of these organizational groups and DOK level. As the organizational groups are task-based and not group based, the tasks in Figure 4.10 are limited to the same tasks that are represented in Figure 4.9 for quality and verbal CD. (See Appendix F, Tables F.9-F.14 for percentages of written CD for each organizational practice and DOK level.)

As seen in Figure 4.10, students' final written CD levels for tasks completed after engaging in Reflective Discourse were primarily above the intended level for DOK1-2 tasks and primarily at the intended level for DOK3-4 tasks. These results are similar to the verbal CD in Figure 4.8 for Reflective Discourse. Students' final written CD levels for tasks completed collaboratively (i.e., together & side by side), while engaging in Exploratory Discourse, were primarily above the intended level for DOK2 tasks and at the intended level for DOK1, 3, and 4 tasks. Students' final written CD levels for tasks completed separately (i.e., split up and take over), while engaging in Exploratory Discourse varied, with a high percentage of tasks coded primarily as low cognitive

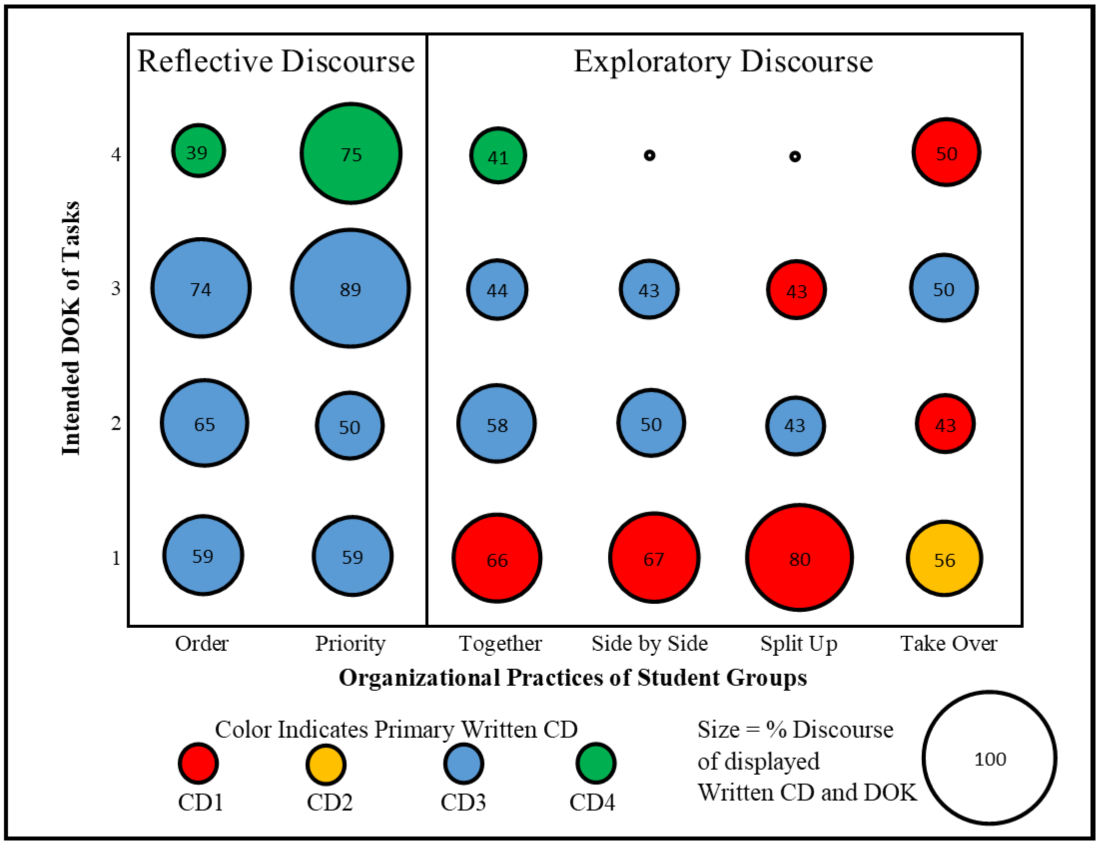


Figure 4.10. Quality of written CD based on organizational practices and Intended DOK levels.

demand (LCD). These results contrast students' verbal CD levels in Figure 4.9 for Exploratory Discourse. This seems to indicate that students' written CD for Reflective Discourse was primarily higher than students' written CD for Exploratory Discourse. This also shows that students' written CD may potentially be a better representation of students' verbal CD levels for Reflective Discourse than for Exploratory Discourse.

In summary, students exhibited verbal evidence of HCD for tasks at all four DOK levels during both Reflective and Exploratory Discourse; however, only Reflective Discourse had similar written evidence of HCD for these same tasks. Groups of students who organized the completion of tasks separately (i.e., split up & take over) during

Exploratory Discourse primarily showed verbal and written evidence of low cognitive demand for tasks at all four DOK levels. Additionally, low cognitive demand tasks (DOK1) and the lowest level of HCD tasks (DOK2) had the highest quality of discourse during Exploratory Discourse procedures, while HCD tasks (DOK3) had the highest quality of discourse during Reflective Discourse procedures.

CHAPTER 5

DISCUSSION

The purpose of this study was to explore the relationship between two types of small-group discourse (reflective and exploratory) and student-enacted levels of cognitive demand (CD) (written and verbal) when engaging with mathematics tasks at the four different intended Depth of Knowledge (DOK) levels. Results showed that most written and verbal responses were at high levels of cognitive demand, regardless of discourse type. Additionally, results showed that students' organizational practices to solve and discuss the tasks in each task-set that focused on students working collaboratively (i.e., prioritized, together, side-by-side) had the highest quality of discourse.

Results relating to students' written and verbal responses to tasks at each of the four DOK levels showed differences in student-enacted levels of cognitive demand for each discourse type. Reflective Discourse generally had higher written responses and lower verbal responses, possibly due to increased student revisions of written responses after engaging in a discussion of the tasks with their group. Exploratory Discourse generally had higher verbal responses and lower written responses, possibly due to increased opportunities for students to elaborate on their strategies and solutions before engaging with independent calculations.

Results showed that the verbal cognitive demand for DOK1 and DOK2 tasks were consistently higher when students engaged in Exploratory Discourse, whereas the verbal cognitive demand for DOK3 and DOK4 tasks were consistently higher when students engaged in Reflective Discourse. Results also seemed to indicate that groups of students that may not have been able to structure their time efficiently were more likely to run out of time to: a) complete all written tasks during Exploratory Discourse, or b) verbally discuss all tasks during Reflective Discourse.

Results relating to the quality and quantity of discourse contributions showed that, while there were no overall differences in the quantity of time spent verbally discussing the tasks or the quality of discourse contributions between each discourse type, there were differences between discourse types for students that typically struggle with mathematics. In this study, Exploratory Discourse did not support struggling students; however, Reflective Discourse did support struggling students.

Based on the main results in this study, three overall themes emerged: a) relationships between discourse type and student-enacted levels of cognitive demand; b) equitable participation for typically struggling students; and c) the importance of dissonance in small-group discourse. Each of these themes are discussed below.

Relationships Between Discourse Type and Student-Enacted Levels of Cognitive Demand

The first theme related to the relationship between small-group discourse type and student-enacted levels of cognitive demand. Results showed that most written and verbal responses were HCD, regardless of discourse type. This contrasts previous research that found that students' enacted levels of CD were more likely to be Low Cognitive Demand (LCD) regardless of the intended DOK level (Charalambous & Litke, 2018; Kessler et al., 2015; Otten, 2012). Although the reasons for this discrepancy between prior research and the current study may be explained by the high-quality tasks used in this study or the existence of classroom norms for collaborative work in the classes in this study, the differences are most likely explained by engagement of students in small-group discourse during this study. This supports prior research that both Reflective and Exploratory Discourse can engage students in reasoning and justification of strategies and solutions. (Georgius, 2014; Hogan et al., 1999; Kalamar, 2018; Rojas-Drummond & Mercer, 2003). As students verbalized their ideas during discourse, this may have provided them with opportunities to use multiple schemes (i.e., high CD) in order to understand and explain the ideas they automatically "just knew." For example, in an earlier exploratory discourse prompt, Shawn proposed an answer of "1/4." However, when Tara asked him how he knew, the subsequent discourse prompted him to think about how he knew the answer was ¼ as he contemplated the relationship between his proposed answer, the number line, and computation algorithms.

Results across both discourse types confirmed Rojas-Drummond and Mercer's (2003) concerns that students engaging in Reflective Discourse were more likely to engage in cumulative talk than students engaging in Exploratory Discourse. However, the results extend these findings to show that cumulative talk generally occurred during Reflective Discourse for lower cognitively demanding tasks (DOK1 and 2) with exploratory talk showing higher levels of verbal CD for these tasks. In this study, results showed that higher cognitively demanding tasks that require reasoning (DOK3 and 4) had higher verbal and written CD during Reflective Discourse procedures than Exploratory Discourse procedures. This aligns with research showing that HCD tasks may require more wait time in order to allow students to process their ideas and reasoning (Ingram & Elliott, 2016; Walter, 2018). Similar to Kalamar's (2018) findings, delaying discourse until after students had time to think about the mathematics, may have "enable[d] them to produce stronger answers to be shared out with their peers" (p. 69).

Equitable Participation for Typically Struggling Students

The second theme among the results related to equitable participation for typically struggling students. Results showed that overall, Reflective Discourse was more likely to support typically struggling students than Exploratory Discourse. Qualitative results showed that the ideas of many typically struggling students were ignored or rejected during Exploratory Discourse. Barron (2003) explains that this may be due to a competitive rather than collaborative relationship within the groups engaged in Exploratory Discourse. Barron hypothesized that, "When schooling values student identities that are based on being smarter than others, the extent to which students are willing to engage mutually in intellectual discourse may be compromised" (p. 350). In these situations, the ideas of typically struggling students are seen as less credible (Coplan, Hughes, Bosacki, & Rose-Krasnor, 2011). This was evidenced in this study as groups more likely to ignore or reject student ideas were also more likely to justify the rejection with a need to complete tasks quickly (e.g., "No, my way is faster") or correctly (e.g., "This is the way [teacher] explained it multiple times"). Unfortunately, when typically struggling students' ideas are ignored or rejected, it can reinforce labels such as "smart" or "dumb" and make these students feel isolated from their peers (Bishop, 2012;

Yang, Chen, & Wang, 2015). This was also evidenced in this study, as students whose ideas were ignored or rejected were less likely to speak up during later tasks.

Research also shows that a friendly or supportive environment can break the cycle for non-participation by typically struggling students (Lin et al., 2015). This was evidence in this study for typically struggling students such as Valerie and Penny. In supportive environment where students were willing to listen to each other's strategies and solutions, typically struggling students were more confident and likely to share their ideas. Results from this study show that Reflective Discourse is more likely than Exploratory Discourse to support typically struggling students by creating a more friendly or supportive environment. For example, during Reflective Discourse, many students who did not typically struggle with the mathematics started the discourse by asking for help on a problem they struggled with. This may have created a friendly and supportive environment where students could see that it was okay to struggle. The delay in discourse until after students had time to think through their solutions and strategies also helped students to be prepared to share and justify their strategies and solutions. This supports prior research that Reflective Discourse can provide more equitable student participation and voice (e.g., Hung, 2015; Kalamar, 2018; Walter, 2018).

Importance of Dissonance in Small-Group Discourse

The third and final theme among the results was the importance of dissonance in small-group discourse. Overall, results show that the presence of dissonance seemed to result in higher levels of student-enacted CD for both discourse types. For example, when students engaged in either Reflective or Exploratory Discourse suggested a different strategy to solve the same task, the dissonance created by the lack of consensus usually prompted students to justify their strategy or provide a counterargument against another student's strategy. Dissonance created by differences in solutions for students in groups engaged in Reflective Discourse often prompted students to revise their answers. Additionally, these revisions typically raised the written CD from the original response.

Dissonance in strategies or solutions during reflected discourse also seemed to prompt students to disengage from cumulative talk as they justified their own strategies and solutions or used their written responses to provide a counterargument against other students' ideas. In a few cases, the discussion that arose in response to the dissonance in strategies and solutions seemed to allow students to identify misconceptions in all of their strategies (e.g., "Oh, I guess we all got it wrong.") and prompt students to work together to identify and implement a more efficient and accurate strategy. This supports Hogan et al.'s (1999) research which found that groups with a high quality of discourse shared ideas that served to "articulate and clarify what they did not know" rather than focusing on organizational or conformational ideas (p. 424). Hogan et al. also found that groups engaged in Exploratory Discourse presented "provocative ideas" (p. 424) that provided dissonance increase the quality of the discussion to focus on deeper conceptual understanding. This was evidenced in this study as students were more likely to prompt someone to explain or justify their strategy during Exploratory Discourse when the strategy was radically different than their own (e.g., "What? Why did you do that?).

Implications for Teachers

There are several implications for teachers, based on the results of this study. Results indicate that teachers can use both reflective and exploratory small-group discourse as effective classroom practices to help increase student-enacted levels of cognitive demand to promote a deeper understanding of the mathematics. Teachers can potentially use Reflective Discourse to facilitate high quality discourse among students with different experiences or solution strategies to promote higher written student-enacted levels of cognitive demand, particularly with HCD tasks. Teachers can potentially use Exploratory Discourse to facilitate high quality discourse between students with different experiences or solution strategies to promote higher verbal student-enacted levels of cognitive demand, particularly with HCD tasks.

These results indicate that teachers can support students that typically struggle with the mathematics by providing them with time to think through their own ideas before reflecting on tasks with their group or class. Teachers can also potentially help students learn to structure their time in order to engage with the tasks more efficiently to solve or discuss mathematics tasks. Additionally, teachers can potentially support all students by helping their class build a friendly and supportive environment where students can work together collaboratively, rather than competitively, to solve different mathematics tasks and increase the quality of the discourse and student-enacted levels of cognitive demand.

Limitations and Suggestions for Future Research

This study is limited in its generalizability, specifically relating to grade level, mathematical content, and length of task. Due to the scope of this study, the generalizability of this study does not extend beyond Grade 5 students and tasks relating to operations with fractions and decimals. Operations with fractions and decimals can be difficult mathematical topics and may have increased or decreased the range of student strategies or levels of dissonance. Additional research is needed to expand these findings to other grade levels or mathematical domains.

This study involved 26 unique tasks completed in two 70-minute sessions. The extended length of time for students to work with minimal teacher support limits the generalizability of this study towards single tasks completed during smaller chunks of time or tasks completed with additional teacher support. More research is needed to expand these findings to shorter discourse sessions. More research is also needed to identify teacher actions that could potentially support or increase group organizational practices during small-group discourse that support high student-enacted levels of cognitive demand.

Conclusion

This research showed that both Reflective and Exploratory Discourse can be used by teachers to promote high student-enacted levels of cognitive demand. Results also showed that a supportive environment, such as the environment created by Reflective Discourse, can help support typically struggling students. Finally, this research reinforced the importance of dissonance in prompting students to engage with the tasks at higher levels of cognitive demand.

REFERENCES

Anderson, J. R. (1982). Acquisition of cognitive skill. *Psychological Review, 89*, 369-406.

Anderson, J. R., & Schunn, C. D. (2000). Implications of the ACT-R learning theory: No magic bullets. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 5, pp. 1-27). Mahwah, NJ: Erlbaum.

Anderson-Pence, K. L. (2014) *Examining the impact of different virtual manipulative types on the nature of students' small-group discussions: An exploratory mixed-methods case study of techno-mathematical discourse* (Doctoral dissertation). Utah State University, Logan, UT. Retrieved from https://digitalcommons.usu.edu/etd/2176

Barnett, S. M., & Ceci, S. J. (2002). When and where do we apply what we learn? A taxonomy for far transfer. *Psychological Bulletin, 138*(4), 612-637.

Barron, B. (2003). When smart groups fail. *The Journal of the Learning Sciences, 12*(3), 307-359.

Bishop, J. P. (2012). "She's always been the smart one. I've always been the dumb one": Identities in the mathematics classroom. *Journal for Research in Mathematics Education, 43*(1), 34-74.

Bishop, J. P., Hardison, H., & Przybyla-Kuchek, J. (2016). Profiles of responsiveness in middle grades mathematics classrooms. *Psychology of Mathematics & Education of North America, 8*, 1173-1180.

Boston, M. D., & Smith, S. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education*, *40*(2), 119-156.

Brodie, K. (2011). Working with learners' mathematical thinking: Towards a language of description for changing pedagogy. *Teaching and Teacher Education, 27*(1), 174-186.

Brownell, W. A. (1945). When is arithmetic meaningful? *The Journal of Educational Research, 38*(7), 481-498.

Bruce, C. D., & Flynn, T. (2011). Which is greater: One half or two fourths? An examination of how two Grade 1 students negotiate meaning. *Canadian Journal of Science, Mathematics and Technology Education, 11*(4), 309-327.

Bruner, J. (1983). Play, thought, and language. *Peabody Journal of Education*, *60*(3), 60-69.

Calcagni, E., & Lago, L. (2018). The three domains for dialogue: A framework for analyzing dialogic approaches to teaching and learning. *Learning, Culture and Social Interaction, 18,* 1-12.

Candela, A. G. (2016). Mathematics teachers' perspectives on factors affecting the implementation of high cognitive demand tasks. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 315-322). Tucson, AZ: The University of Arizona.

Carpenter, T. P., & Fennema, E. (1988). Research and cognitively guided instruction. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 2-19). Madison: University of Wisconsin, National Center for Research in Mathematical Sciences Education

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school.* Portsmouth, NH: Heinemann.

Charalambous, C. Y., & Litke, E. (2018). Studying instructional quality by using a content-specific lens: the case of the Mathematical Quality of Instruction framework. *ZDM*, 1-16.

Clements, D. H., & Battista, M. T. (2009). Constructivist learning and teaching. *Putting Research into Practice in the Elementary Grades: Readings from Journals of the NCTM,* 6-11.

Coakley, M. (2018). *Using strategic discourse for building understanding in elementary mathematics: What do teachers and students think?* (Doctoral dissertation). University of Massachusetts. Amherst, MA. Retrieved from https://scholarworks. umass.edu/dissertations_2/1333

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: Interaction in classroom cultures.* Hillsdale, NJ: Psychology Press.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education, 28*(3), 258-277.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist 31*, 175-190.

Cohen, B. H. (2013). *Explaining psychological statistics* (4th ed.). Hoboken, NJ: Wiley.

Coplan, R. J., Hughes, K., Bosacki, S., & Rose-Krasnor, L. (2011). Is silence golden? Elementary school teachers' strategies and beliefs regarding hypothetical shy/quiet and exuberant/talkative children. *Journal of Educational Psychology, 103*(4), 939-951.

Cowan, N. (2000). The magical number 4 in short-term memory: A reconsideration of mental storage capacity. *Behavioral and Brain Sciences, 24*, 87-114.

Creamer, E. G. (2018). *An introduction to fully integrated mixed methods research.* Thousand Oaks, CA: Sage.

Creswell, J. W., & Plano Clark, V. L. (2011). *Designing and conducting mixed methods research* (2nd ed.). Thousand Oaks, CA: Sage.

Doyle, W. (1983). Academic work. *Review of Educational Research*, *53*(2), 159-199.

Durfee, M. V. (2018). *An exploratory case study of how high-performance team training develops sociomathematical norms and differing levels of math-talk* (Doctoral dissertation). Utah State University, Logan, UT. Retrieved from https://digitalcommons.usu.edu/etd/7357

Erath, K., Prediger, S., Quasthoff, U., & Heller, V. (2018). Discourse competence as important part of academic language proficiency in mathematics classrooms: The case of explaining to learn and learning to explain. *Educational Studies in Mathematics, 99*(2), 161-179.

Falloon, G., & Khoo, E. (2014). Exploring young students' talk in iPad-supported collaborative learning environments. *Computers & Education, 77*, 13-28.

Feldon, D. F. (2007). Implications of research on expertise for curriculum and pedagogy. *Educational Psychology Review, 19*(2), 91-110.

Forman, E. A. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 333-352). Reston, VA: National Council of Teachers of Mathematics.

Gagne, E. D., Yekovich, C. W., & Yekovich, F. R. (1993). *The cognitive psychology of school learning.* New York, NY: Harper Collins.

Georgius, K. (2014). *Planning and enacting mathematical tasks of high cognitive demand in the primary classroom* (Doctoral dissertation). The University of Nebraska-Lincoln. Retrieved from http://digitalcommons.unl.edu/teachlearnstudent/41.

Gilbert, M. C. (2016). Relating aspects of motivation to facets of mathematical competence varying in cognitive demand. *The Journal of Educational Research, 109*(6), 647-657. doi:10.1080/00220671.2015.1020912

Grammer, J. K., Coffman, J. L., Sidney, P., & Ornstein, P. A. (2016). Linking teacher instruction and student achievement in mathematics: The role of teacher language. *Journal of Cognition and Development, 17*(3), 468-485.

Greeno, J. G. (1976). Cognitive objectives of instruction: Theory of knowledge for solving problems and answering questions. In D. Klahr (Ed.), *Cognition and instruction* (pp. 123-159). Hillsdale, NJ: Erlbaum.

Grobman, K. (2015, September 4). *What is the difference (in mathematics education) between cognitive load and cognitive demand?* [Msg 1]. Message posted to https://www.researchgate.net/post/What_is_the_difference_in_mathematics_education_between_cognitive_load_and_cognitive_demand

Hayes, A. F., & Krippendorff, K. (2007). Answering the call for a standard reliability measure for coding data. *Communication Methods and Measures, 1*(1), 77-89.

Hogan, K., Nastasi, B. K., & Pressley, M. (1999). Discourse patterns and collaborative scientific reasoning in peer and teacher-guided discussions. *Cognition and Instruction, 17*(4), 379-432.

Hong, D. S., & Choi, K. M. (2016). Challenges of maintaining cognitive demand during limit lessons. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (p. 1408). Tucson, AZ: The University of Arizona.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, 35(2), 81-116.

Hung, M. (2015). Talking circles promote equitable discourse. *Mathematics Teacher, 109*(4), 256-260.

Hwang, W. Y., & Hu, S. S. (2013). Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving. *Computers & Education, 62*, 308-319.

Iiskala, T., Vauras, M., Lehtinen, E., & Salonen, P. (2011). Socially shared metacognition of dyads of pupils in collaborative mathematical problem-solving processes. *Learning and Instruction, 21*(3), 379-393.

Ingram, J., & Elliott, V. (2016). A critical analysis of the role of wait time in classroom interactions and the effects on student and teacher interactional behaviours. *Cambridge Journal of Education, 46*(1), 1-17. https://doi.org/10.1080/0305764X. 2015.1009365

Institute of Education Sciences [IES]. (2015). *Trends in international mathematics and science study (TIMSS).* Retrieved from http://nces.ed.gov/timss/timss2015.

Institute of Education Sciences [IES]. (2018). *Condition of education 2018.* Retrieved from http://nces.ed.gov/pubs2018/2018144.pdf

Jamieson, T. S. (2015). *Changes in elementary mathematics teachers' understanding of cognitive demand: When adapting, creating, an using mathematical performance tasks* (Doctoral dissertation). George Mason University, George Mason, VA. Retrieved from ProQuest Digital Dissertations. (UMI 10031804)

Jones, D. L., & Tarr, J. E. (2007). An examination of the levels of cognitive demand required by probability task in middle grades mathematics textbooks. *Statistics Education Research Journal, 6*(2), 4-27.

Kalamar, C. (2018). *Questioning techniques that increase student engagement during the mathematics lesson* (Master's thesis). Moravian College, Bethlehem, PA. Retrieved from https://www.moravian.edu/static/education/2018/thesis/Kelsey.pdf

Keazer, L., & Gerberry, C. (2017). Supporting common core sense making. *Teaching Children Mathematics, 23*(8), 484-490.

Kelleher, C., & Wagener, T. (2011). Ten guidelines for effective data visualization in scientific publications. *Environmental Modelling & Software, 26*(6), 822-827.

Kessler, A. M., Stein, M. K., & Schunn, C. D. (2015). Cognitive demand of model tracing tutor tasks: Conceptualizing and predicting how deeply students engage. *Technology, Knowledge, and Learning, 20,* 317-337. doi:10.1007/s10758-015-9248-6

Kohar, A. W., Wardani, A. K., & Fachrudin, A. D. (2019). Profiling context-based mathematics tasks developed by novice PISA-like task designers. *Journal of Physics: Conference Series, 1200*(1), Retrieved from https://iopscience.iop.org/article/10.1088/1742-6596/1200/1/012014

Krippendorff, K. (2013). *Content analysis: An introduction to its methodology* (3rd ed.). Thousand Oaks, CA: Sage, Lin, T. J., Anderson, R. C., Jadallah, M., Nguyen-Jahiel, K., Kim, I. H., Kuo, L. J., ... Li, Y. (2015). Social influences on children's development of relational thinking during small-group discussions. *Contemporary Educational Psychology, 41*, 83-97.

Mayer, R. E. (Ed.). (2006). *The Cambridge handbook of multimedia learning*. New York, NY: Cambridge University Press.

McCormick, M. (2016). Exploring the cognitive demand and features of problem solving tasks in primary mathematics classrooms. In B. White, M. Chinnappan, & S. Trenholm (Eds.), *Opening up mathematics education research*: Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia (pp. 455-462). Adelaide, Australia: Mathematics Education Research Group of Australasia.

McGonigal, J. (2008). Why I love bees: A case study in collective intelligence gaming. In K. Salen (Ed.) *The ecology of games: Connecting youth, games, and learning*, (pp. 199-228). Cambridge, MA: MIT Press.

Mdladla, E. P. (2017). *Tasks used in mathematics classrooms* (Doctoral dissertation). University of the Witwatersrand, Johannesburg, South Africa. Retrieved from ProQuest Digital Dissertations. (UMI 0217853V)

Moreno Alcázar, M. T. (2007). *Alignment of cognitive demand: Peruvian national assessment, mandated curriculum, teaching and textbook in second grade math* (Doctoral dissertation). University of Delaware, Newark, DE. Retrieved from ProQuest Digital Dissertations. (UMI 3267160)

Morgan, M., & Power, R. (2016). Facilitating classroom discourse: A project designed to help preservice teachers implement high-cognitive demand tasks. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (p. 955). Tucson, AZ: The University of Arizona.

Mueller, M. F. (2009). The co-construction of arguments by middle-school students. The *Journal of Mathematical Behavior, 28*(2-3), 138-149.

National Governors Associate Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to action: Ensuring mathematical success for all*. Reston, VA: Author.

Ngware, M. W., Ciera, J., Musyoka, P. K, & Oketch, M. (2015). Quality of teaching mathematics and learning achievement gains: Evidence from primary schools in Kenya. *Educational Studies in Mathematics, 89*, 111-131. doi:10.1007/s10649-015-9594-2

Nussbaum, E. M. (1999, February). *Understanding cognitive views of learning*. Paper presented at University of Nevada, Las Vegas, NV.

Orrill, C. H. (2015, May 11). *Tasks, questions, and practices*. [Teaching Children Mathematics online blog]. Retrieved from https://www.nctm.org/Publications/Teaching-Children-Mathematics/Blog/Tasks-Questions-and-Practices/

Otten, S. (2012). *Participation and cognitive demand: linking the enacted curriculum and student learning in middle school algebra* (Doctoral dissertation). Michigan State University, East Lansing, MI. Retrieved from ProQuest Digital Dissertations. (UMI 3525714)

Patterson, E. W. (2018). Exploratory talk in the early years: analyzing exploratory talk in collaborative group activities involving younger learners. *Education 3-13, 46*(3), 264-276.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new US intended curriculum. *Educational Researcher, 40*(3), 103-116.

Resnick, L., & Ford, W. (1981). *The psychology of mathematics for instruction.* Hillsdale, NJ: Erlbaum.

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2012). Developing procedural flexibility: Are novices prepared to learn from comparing procedures? *British Journal of Educational Psychology, 82*(3), 436-455.

Rojas-Drummond, S., & Mercer, N. (2003). Scaffolding the development of effective collaboration and learning. *International Journal of Educational Research, 39*(1-2), 99-111.

Roschelle, J. (2000). Choosing and using video equipment for data collection. In A. Kelly & R. Lesh (Eds.), *Research design in mathematics & science education* (pp. 709-731). Amsterdam, The Netherlands: Kleuwer.

Roth, J. A. (2019). *Making the struggle productive: Conceptualizing the role and impact of the mathematics teacher in episodes of productive struggle.* (Doctoral Dissertation). Kennesaw State University, Kennesaw, GA. Retrieved from https://digitalcommons.kennesaw.edu/seceddoc_etd

Rowe, M. B. (1986). Wait time: Slowing down may be a way of speeding up! *Journal of Teacher Education, 37*(1), 43-50. https://doi.org/10.1177/002248718603700110

Saldaña, J. (2015). *The coding manual for qualitative researchers*. Thousand Oaks, CA: Sage.

Schoenfeld, A. H. (2018). Video analyses for research and professional development: the teaching for robust understanding (TRU) framework. *ZDM*, 50(3), 491-506.

Sfard, A. (2001). There is more to discourse than meets the ears: Learning from mathematical communication things that we have not known before. *Educational Studies in Mathematics, 46*(1/3), 13-57.

Shadish, W., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference.* Boston, MA: Houghton Mifflin.

Sherman, M. (2011). *An examination of the role of technological tools in relation to the cognitive demand of mathematical tasks in secondary classrooms* (Doctoral dissertation). University of Delaware, Newark, DE. Retrieved from ProQuest Digital Dissertations. (UMI 3485770)

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education, 11*(6), 499-511.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*, 344-350.

Smith, M. S., Stein, M. K., Arbaugh, F., Brown, C. A., & Mossgrove, J. (2004). Characterizing the cognitive demand of mathematical tasks: A sorting activity. *Professional development guidebook for perspectives on the teaching of mathematics: Companion to the sixty-sith yearbook* (pp. 45-72). Reston, VA: National Council of Teachers of Mathematics.

Son, J., & Kim, O. (2015). Teachers' selection and enactment of mathematical problems from textbooks. *Mathematics Education Research Journal*, *27*(4), 491-518. doi: 10.1007/s13394-015-0148-9

Steffe, L. (1991). The learning paradox: A plausible example. In L. Steffe (Ed.), *Epistemological foundations of mathematical experience* (pp. 26-44). New York, NY: Springer-Verlag.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50-80.

Store, J. C., & College, A. (2015). Maintaining cognitive demands of algebraic reasoning tasks. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield, & H. Dominguez (Eds.), *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (p. 1188). East Lansing, MI: Michigan State University

Sweller, J., & Mayer, R. (2015, November 1). *Virtual seminar - professor Richard Mayer talking with professor John Sweller.* Retrieved from https://www.youtube.com/watch?v=T88InIudNk0

Tan, K. J., Ismail, Z., & Abidin, M. (2018). A comparative analysis on cognitive domain for the Malaysian primary four textbook series. *Eurasia Journal of Mathematics, Science and Technology Education, 14*(4), 1273-1286.

Tashakkori, A., & Teddlie, C. (Eds.). (2010). *Sage handbook of mixed methods in social & behavioral research* (2nd ed.). Los Angeles, CA: Sage.

van Leeuwen, A., & Janssen, J. (2019). A systematic review of teacher guidance during collaborative learning in primary and secondary education. *Educational Research Review, 27,* 71-89.

van Merrienboer, J. J. G., & Sweller, J. (2005). Cognitive load theory and complex learning: Recent developments and future directions. *Educational Psychology Review, 17,* 147-177.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological functions*. Cambridge, MA: Harvard University Press.

Wachira, P., Pourdavood, R. G., & Skitzki, R. (2013). Mathematics teacher's role in promoting classroom discourse. *International Journal for Mathematics Teaching and Learning, 13*(1), 1-40.

Walter, H. A. (2018). Beyond turn and talk: Creating discourse. *Teaching Children Mathematics, 25*(3), 180-185.

Webb, N. L. (1997). *Criteria for alignment of expectations and assessments in mathematics and science education* (Council of Chief State School Officers and National Institute for Science Education Research Monograph No. 6). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

Webb, N. L. (1999). *Alignment of Science and Mathematics Standards and Assessments in Four States* (Research Monograph No. 18). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

Welder, R. M., Tobias, J. M., Feldman, Z., Hillen, A., Olanoff, D., & Thanheiser, E. (2015). Modifying children's tasks into cognitively demanding tasks for preservice elementary teachers. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield, & H. Dominguez (Eds.), *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 984-987). East Lansing, MI: Michigan State University

Wilkinson, L., & Friendly, M. (2009). The history of the cluster heat map. *The American Statistician, 63*(2), 179-184.

Williams, L. (2010). *Building connections between sociomathematical norms and cognitive demand to improve the quality of whole class mathematics conversations* (Doctoral dissertation). University of Wisconsin-Milwaukee, WI. Retrieved from ProQuest Digital Dissertations. (UMI 3437973)

Wittrock, M. C. (1992). Generative learning processes of the brain. *Educational Psychologist, 27*(4), 531-541.

Wouters, P., Van Nimwegen, C., Van Oostendorp, H., & Van Der Spek, E. D. (2013). A meta-analysis of the cognitive and motivational effects of serious games. *Journal of Educational Psychology, 105*(2), 249-265.

Yackel, E., Cobb, P., & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education*, *22*(5)*,* 390-408.

Yang, F., Chen, X., & Wang, L. (2015). Shyness-sensitivity and social, school, and psychological adjustment in urban Chinese children: A four-wave longitudinal study. *Child Development, 86*(6), 1848-1864.

Yanik, H. B., & Serin, G. (2016). Two fifth grade teachers' use of real-world situations in science and mathematics lessons. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 89*(1), 28-37. doi:10.1080/00098655.2015.1130014

APPENDICES

Appendix A

IRB Approval

Subject: Approval letter from USU IRB
Date: Wednesday, February 13, 2019 at 4:10:14 PM Mountain Standard Time
From: noreply@usu.edu
To: Patricia Moyer-Packenham, Kristy Litster

Institutional Review Board

USU Assurance: FWA#00003308

USU RGS Logo

Expedite #6 & #7

AAHRP seal

Letter of Approval

FROM:	Melanie Domenech Rodriguez, IRB Chair Nicole Vouvalis, IRB Administrator
	Melanie Signature / Nicole Signatur
To:	Patricia Moyer-Packenham, Kristy Litster
Date:	February 13, 2019
Protocol #:	9516
Title:	The Relationship Between Small Group Discourse And Student-Enacted Levels Of Cognitive Demar When Engaging With Mathematics Tasks At Different Depth Of Knowledge Levels
Risk:	Minimal risk

Your proposal has been reviewed by the Institutional Review Board and is approved under expedite procedure #6 & # (based on the Department of Health and Human Services (DHHS) regulations for the protection of human research subjects, 45 CFR Part 46, as amended to include provisions of the Federal Policy for the Protection of Human Subjects November 9, 1998):

> #6: Collection of data from voice, video, digital, or image recordings made for research purposes.
> #7: Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

This approval applies only to the proposal currently on file for the period of one year. If your study extends beyond th approval period, you must contact this office to request an annual review of this research. Any change affecting huma subjects must be approved by the Board prior to implementation. Injuries or any unanticipated problems involving ris to subjects or to others must be reported immediately to the Chair of the Institutional Review Board.

Prior to involving human subjects, properly executed informed consent must be obtained from each subject or from a authorized representative, and documentation of informed consent must be kept on file for at least three years after the project ends. Each subject must be furnished with a copy of the informed consent document for their personal records.

Appendix B

Task-Sets 1 and 2

Table B.1

DOK Level for All Tasks in each Task-Set

DOK Level	Task-Set 1 (Harry)	Task-Set 2 (Diary)
1	H1, H2, H4, H6	D1, D5, D6, D7
2	H3, H5, H7, H8	D2, D4, D8, D11
3	H9, HC1, HC2, HC3	D3, D9, D10, D12
4	HF	DF

Books on Screen Harry Potter & the Half Blood Prince

Name:

You are a ***movie analyst***. You are evaluating films that are ***adaptations*** of popular children's books.

Your Challenge: Do people prefer the book or the movie adaptation?

LEVEL 1

To begin your analysis, you look at data from the year that the movie was released in theaters. *Harry Potter & the Half Blood Prince* was released in 2009. The average ticket price in 2009 was $7.

1. Divide or multiply to calculate the estimated ***ticket sales*** and the ***box office gross*** for the weeks that the movie was showing in theaters.
Round your answers to the nearest hundredth as necessary.

Dates	Estimated Ticket Sales (millions)	Box Office Gross ($ millions)
July 10th - August 6th	35.32	
August 7th - September 3rd		30.17
September 4th - October 1st		4.97
October 2nd - 29th	0.19	
October 30th - November 26th		0.49
November 27th - December 17th	0.1	

2. Calculate the ***revenue*** for the books sold in the same year.
Round your answers to the nearest hundredth as necessary.

	Total Sold (millions)	Price	Revenue (millions)
Paperback	0.5	$12.99	

3. Interpret the data.
   a. Did more people see the movie or buy the book? How do you know?

   b. Did the movie or book bring in more money? How do you know?

NextLesson

Updated September 2014

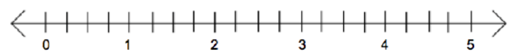
Books on Screen Harry Potter & the Half Blood Prince

LEVEL 2

As well as judging their success by the number of people who saw the movie or read the book, you also want to analyze what people actually thought of *Harry Potter & the Half Blood Prince*.
You find several ***sources*** of reviews.

4. Mark each average ***rating*** of the **movie** adaptation as an X on the line plot.

***Harry Potter & the Half Blood Prince* – Movie Adaptation**	
Source	**Average Rating (out of 5 stars)**
IMDb	$3\frac{3}{4}$
Rotten Tomatoes	$3\frac{3}{4}$
Amazon	$3\frac{3}{4}$
Barnes & Noble	$4\frac{1}{2}$
iTunes	**4**



5. Use the line plot. Explain your work.
   a. Which values show up the most often, if any?
   b. Compare the **difference** between the average Barnes & Nobles rating and the average IMDb rating.
   c. Compare the **difference** between the average Amazon rating and the average iTunes rating.

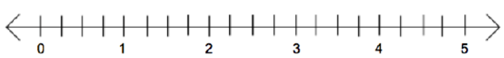
NextLesson

Updated September 2014

Books on Screen Harry Potter & the Half Blood Prince

6. Mark each average rating of the **book** as an X on the line plot.

***Harry Potter & the Half Blood Prince* – Book**	
Source	**Average Rating (out of 5 stars)**
Goodreads	$4\frac{1}{2}$
Amazon	$4\frac{1}{4}$
Barnes & Noble	$4\frac{1}{2}$
iTunes	$3\frac{1}{2}$
Target	5



7. Use the line plot. Explain your work.
   a. Which values show up the most often, if any?
   b. Compare the **difference** between the average Goodreads rating and the average Amazon rating.
   c. Compare the **difference** between the average Target rating and the average Barnes & Noble rating.

8. Now compare the ratings.
   a. Use all the movie ratings to calculate the movie's total rating out of 25 stars.
   b. Use all the book ratings to calculate the book's total rating out of 25 stars.
   c. Compare the two new 25-star ratings. Calculate the difference between the two ratings.

9. Based on all of the ***data***, do people prefer the original book or the movie adaptation? Justify and explain your reasoning.

NextLesson

Updated September 2014

Books on Screen Harry Potter & the Half Blood Prince

Challenge

Finally, you examine the **number** of people who have provided a rating of *Harry Potter & the Half Blood Prince* on each of the websites.

Movie Adaptation		**Book**	
Source	**Raters**	**Source**	**Raters**
IMDb	247,993	Goodreads	1,200,990
Rotten Tomatoes	1,552,692	Amazon	4,543
Amazon	1,365	Barnes & Noble	4,015
Barnes & Noble	170	iTunes	28
iTunes	492	Target	1

1. Do you see any relationships between the number of people who provided a rating and the average rating that the source has posted? ***Cite evidence*** from the data to explain your thinking.

2. How does this additional data affect your opinion of people's preferences for the book or the movie adaptation? ***Justify*** your reasoning.

3. What other data would you want to gather to help you assess whether people prefer the book or the movie? Justify why you think it would help you prove your findings.

NextLesson

Updated September 2014

(*Figure continues*)

Books on Screen Harry Potter & the Half Blood Prince

Finale

You could give students one of the following ideas or have them choose themselves.

1. Create an infographic to present to movie producers that shows why they should consider producing film adaptations of popular children's books. Think and write like a mathematician using the language, data, and examples from this task.

2. Write a blog entry for a movie website explaining whether the movie adaptation of this book was a success. Include data to justify your reasons. Give your opinion on making movie adaptations of popular books. Does it help box office sales or book sales do you think?

3. You have been asked to speak at Career Day at a local elementary school. Prepare a speech to explain how you use mathematics as a movie analyst. In your speech, also persuade students to both read books and see their movie adaptations. Include data and examples on *Harry Potter & the Half Blood Prince* to encourage students to write book and movie reviews.

4. Conduct research to gather data about other popular books that have been made into movies. Create a multi-media presentation to convince authors to agree to sign movie deals for their books. Cite evidence from the data to defend your position.

5. As a movie analyst evaluating adaptations of popular children's books, you like to actually take on the role of a reader and a viewer. Read *Harry Potter & the Half Blood Prince* and watch the movie adaptation. Write a review that compares and contrasts the two. Provide a rating out of five stars for each.

NextLesson

Updated September 2014

Figure B.1. Task-Set 1.

BOX SET BARGAINS Diary of a Wimpy Kid

Name: ______________________

You are the ***Manager*** of the ***publishing design*** team for Amulet Books. You are designing a single ***box set*** to pack and ***ship*** box sets of the Diary of a Wimpy Kid series in ***hardcover***.

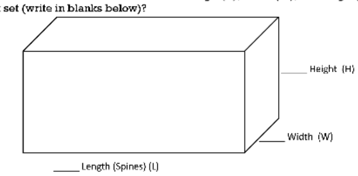
Your Challenge: How would you design and package your box set?

LEVEL 1

1. Calculate the volume of each book. Round your answers to the nearest whole number.

Title of Book	Dimensions of Book (cm) (H x W x L)	Volume
Diary of a Wimpy Kid	20 x 14 x 2.4	
Rodrick Rules	20 x 14 x 2.2	
The Last Straw	20 x 14 x 1.9	
Dog Days	20 x 14 x 2.2	
The Ugly Truth	20 x 14 x 2.3	
Cabin Fever	20 x 14 x 2.1	
The Third Wheel	20 x 14 x 1.9	

2. Draw in the 7 books. What would be the height(H), Width(W), and length(L) of the combined box set (write in blanks below)?



BOX SET BARGAINS Diary of a Wimpy Kid

3. Compare the sum of the volume of the individual books and the volume of box set.
   a) Add together the volume of the individual books (Question 1)
   b) Multiply the LxWxH of the box set (Question 2)
   c) What did you notice and why do you think it is this way?

4. Amulet Books wants to create a teacher version of the box set by adding *Diary of a Wimpy Kid* - ***Teacher Guide,*** which has a volume of 442 cm^3. How would this impact the total volume of the box set? Explain your answer.

5. Stores prefer to buy multiple sets at a time. What would the volume be for a crate of 12 student box sets (without teacher guide)?

BOX SET BARGAINS Diary of a Wimpy Kid

LEVEL 2

Next you need to assess how to price your box set at a ***discount*** from buying all of the books separately.

Title of Book	*Production Cost*	*Purchase Price*
Diary of a Wimpy Kid	$2.63	$7.88
Rodrick Rules	$2.63	$7.90
The Last Straw	$3.21	$9.63
Dog Days	$3.30	$9.90
The Ugly Truth	$2.64	$7.93
Cabin Fever	$2.64	$7.93
The Third Wheel	$3.10	$9.31

6. What is the cost for **seller** to produce (production cost) and buyer to **purchase** (purchase price) for all of the books in the series individually (as opposed to in a box set)?
   a. How much would it cost to **produce** one set of all of the books in the series?
   b. How much would it cost to **purchase** one set of all of the books in the series?

7. Fill in the table below to think about different discounts you could offer:
   a. Round the total retail price (Question 6b) to the nearest dollar to fill in the first column.
   b. Multiply retail price by fractional discounts to calculate each discount in dollars. Round your answer to the nearest dollar.
   c. Use your rounded discount amount to calculate the new retail price of the box set after subtracting the discount from the retail price.

Retail Price from Question 6b (rounded to the nearest dollar)	Fractional Discount	Discount Amount (rounded to the nearest dollar)	Discounted Price
	$\frac{1}{2}$		
	$\frac{1}{3}$		
	$\frac{1}{4}$		
	$\frac{1}{5}$		
	$\frac{1}{6}$		
	$\frac{1}{7}$		

8. Now compare the production costs (Question 6) and the new retail price (Question 7). Explain two things you notice that would be important to consider when choosing a discount.

9. Which discount would be the best discount to offer? Justify why this is the best discount for either you, the buyer, or both.

(*Figure continues*)

BOX SET BARGAINS Diary of a Wimpy Kid

Challenge

Once you have your box sets produced and priced you will need to think about how you will ship them to stores around the country.

10. Choose discounts they will get based on whether they make single orders (1 box set), multiple orders (2-9 box sets), or bulk orders (10 or more box sets)

Order Type	Fractional Discount (Question 7)	Price for 1 Box Set with this discount (Question 7c)
Single Order (1)		
Multiple Order (2-9)		
Bulk Order (10-100)		

Justify your reasoning for your discount decision for each order type.

11. On the back side is a draft order form. Use this to answer the following questions:
    a. Calculate the cost of a Single Order to include shipping.
    b. Calculate the cost of any two Multiple Orders to include shipping.
    c. Calculate the cost of any two Bulk Orders to include shipping
    d. Compare your answers. What do you notice about the total cost in the different discount groups?

You would like to finalize your order form.

12. Review your draft order form (back side). What changes, if any, would you want to make to the order form, These could include layout, discounts offered, multiple order grouping, etc. Justify your reasoning for making those changes.

NextLesson

Updated September 2014

BOX SET BARGAINS Diary of a Wimpy Kid

Number of Box Sets	Cost of All Box Sets in this order (at Discount)	Shipping Cost	Total Cost
SINGLE ORDERS – Cost of 1 Box Set at Discount Rate (Question 10): _____			
1		$6.00	
MULTIPLE ORDERS – Cost of 1 Box Set at Discount Rate (Question 10): _____			
2		$16.00	
3		$24.00	
4		$32.00	
5		$40.00	
6		$48.00	
7		$56.00	
8		$64.00	
9		$72.00	
BULK ORDERS – Cost of 1 Box Set at Discount Rate (Question 10):			
10		$120.00	
20		$240.00	
30		$360.00	
40		$480.00	
50		$600.00	
100		$1,200.00	

NextLesson

Updated September 2014

BOX SET BARGAINS Diary of a Wimpy Kid

Finale

You could give students one of the following ideas or have them choose themselves.

1. Design a presentation for the board of directors of Amulet Books defending your pricing and shipping options. Use at least one graphic organizer (i.e, tables, line graph, bar graph, etc.) to illustrate your point. Consider any problems or concerns the board might voice.

2. Create a promotional advertisement for your box set. Determine your audience as single buyers or bulk buyers. Be sure to include pricing and/or discounts.

3. Amulet Books is considering creating box sets of books based on common genres, such as putting 4 different fantasy authors together in one set. Write an email to the production team to explain why this is or is not a good investment. Include an example of a genre box set that could be created.

4. Amazon.com would like a summary of your box set to include on its online preview. Write a short summary of each book in the series, but don't give away the ending! Include 2-3 sentences to conclude on why buyers should buy the box set rather than the books individually.

5. Design a cover for your new box set in ***net form*** and label the dimensions for each part of the net (include and opening for books). Also include instructions for the printer for the size of the paper to be printed on, how much paper should be cut away to allow overlap or folding around the opening. The design should incorporate themes from all the books.

NextLesson

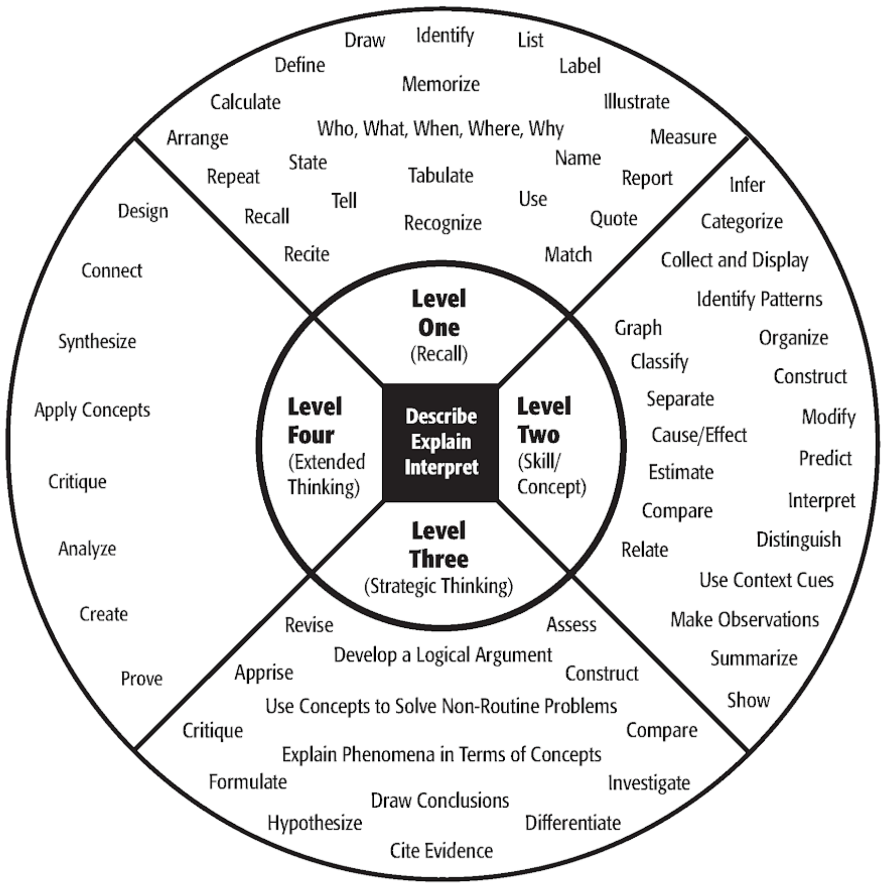
Updated September 2014

Figure B.2. Task-Set 2.

Appendix C

Coding Protocol

Depth of Knowledge (DOK) Levels



Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting. Conduct basic mathematical calculations. Label locations on a map. Represent in words or diagrams a scientific concept or relationship. Perform routine procedures like measuring length or using punctuation marks correctly. Describe the features of a place or people.	Identify and summarize the major events in a narrative. Use context cues to identify the meaning of unfamiliar words. Solve routine multiple-step problems. Describe the cause/effect of a particular event. Identify patterns in events or behavior. Formulate a routine problem given data and conditions. Organize, represent and interpret data.	Support ideas with details and examples. Use voice appropriate to the purpose and audience. Identify research questions and design investigations for a scientific problem. Develop a scientific model for a complex situation. Determine the author's purpose and describe how it affects the interpretation of a reading selection. Apply a concept in other contexts.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions. Apply mathematical model to illuminate a problem or situation. Analyze and synthesize information from multiple sources. Describe and illustrate how common themes are found across texts from different cultures. Design a mathematical model to inform and solve a practical or abstract situation.

Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>.

Figure C.1. DOK wheel coding rubric.

Table C.1

Written Rubric with Sample Evidence

Code	Evidence
1	-Calculates with Algorithm / Routine Procedures -Showed Not Work (Possible Recall) -Stated Fact /Make Statement (Tell, Label, Repeat) -Writes “I copied” on the worksheet -Nonsense Answer (Draws a chicken for a calculation problem)
2	-Explains Answer (but does not Justify; I think . . . I see . . .) -Made Comparison -Used Estimation (not rounding procedure) -Summarized Information -Identified Patterns -Organized Data
3	-Justified placement (with word or arrows) -Justified Answer (With words or arrows) -Revised Answer (Illustrates strategic thinking) -Cite Evidence to Support Answer (from One Source)
4	-Synthesize information from multiple sources -Crossed multiple domains to solve or justify solution (e.g., Pulled resources from outside the task-set to prove, synthesize or create something)

Table C.2

Sample Coding Verbal CD

Task	Student	Student Response	Code	Sub Code
D1	153	So just find out the volume?	2	2a
D1	152	Yeah	1	1d
D1	153	Then let's split it up. One person do number one, one person do number two	0	0c
D1	152	It's not even. There are 7 and we have three people.	0	0c
D1	153	I can do an extra and let's get our own scratch paper.	0	0c
D1	152	Okay, I'll do the first one. You can do this one.	0	0c
D1	153	She will do the Last Straw. Then you will do Dog Days. I will do The Ugly Truth She can do Cabin Fever and I can do the third Wheel	0	0c
D1		[Solves assigned independently.]	1	1b
D1	152	What did you get for the last one?	1	1d
D1	151	I don't know, it wasn't one I was assigned.	0	0c
D1	152	[solves problem] Ok, write that one down, it is 560 and 588	1	1a
D2	153	Okay, so basically, someone will add all the decimals together. So I think those two will be the same but the decimals won't so we need to add all the decimals	1	1c
D2	152	Yeah	1	1d
D2		[Starts calculating]	1	1a
D2	152	I got 13	1	1c
D2	152	I forgot to add two whole so that would be 15.	1	1c
D2	153	Height is 14 . . . wait So would the books be like [swipe sideways] or that [swipe up and down].	2	2a
D2	152	I think it would be [swipe up and down]	1	1d
D2	153	Okay, so length would be added together. Width would be 14 and height would be 20.	1	1c
D2	151		0	0a
D3	152	[reads questions]	0	0c
D3	153	So we just add all the volumes together? I think we should just add some and then add them together so we don't have to keep adding.	2	2b
D3	152	Okay [starts adding]	1	1a
D3	153	She will do the Last Straw and Cabin Fever and you will add those two together.	0	0c
D3	152	So 560 and 588	1	1d
D3	153	And I can do Mine	0	0c
D3	152	Do you want to do the third wheel?	1	1c
D3	153	Do you want to?	0	0c
D3	152	I don't care	0	0c
D3	153	Ok, I'll do it.	0	0c

Task	Student	Student Response	Code	Sub Code
D3	152	Ok, I got mine 1,344	1	1c
D3	153	4,337	1	1c
D3	152	Ok [writes on sheet] Okay, multiply 15	1	1a
D3	153	It is close to rounding by 16	1	1c
D3	152	Who wants to multiply them?	0	0c
D3	153	We multiply all of them?	1	1d
D3	152	Yea	1	1d
D3		[all multiplying]	1	1a
D3	153	I got 4200	1	1d
D3	152	42? [writes answer and reads question]	1	1d
D3	153	These two are really close	1	1c
D3	152	Ok, so their volume are really close	1	1d
D3	153	Their volume and box set are close. Wait a minute, wouldn't those two be the same because if you added all of the volume they would be the same.	3	3b
D3	152	So did we not do this one right?	1	1d
D3	153	Oh, because of the .49 it would be.	3	3b
D3	152	Oh, so they are the same. But wait, what should we write?	1	1d
D3	153	The volume and box set are close.	1	1d
D4	152	[reads question]	0	0c
D4	153	And you have to multiply it.	2	2a
D4	152	Wow, just wow	0	0b
D4		[multiply numbers]	1	1a
D6	153	So that retail goes there and production goes there.	1	1c
D6	152	So I am going to go back here and do this [flips paper over and starts adding]	1	1a
D6	153	So [reads question] so we would add all of those together. So you add those together [starts solving]	2	2a
D6	152	So I think I got this wrong.	1	1d
D6	153	At least we have an answer. I got 20.15 [is handed paper and writes answer]	1	1c
D6	153	So then we add all the retail prices together.	1	1c
D6		[start calculating]	1	1a
D7	153	60:48 [writes answer, turns paper over, reads question] so 60	1	1a
D7	153	You round them so it would be like 8, 9, 10	2	2a
D7	152	Yea, 8, 8 and 9	1	1d
D7	153	That is weird, there is 1, 2, 3, 4, 5, 6, 7, [books] but there is only 1, 2, 3, 4, 5, 6 [discounts]. There is 6 columns and 7 books.	3	3a
D7	152	60 goes in each of these boxes.	1	1c
D7	153	Oh, that makes more sense. 30?	1	1c
D7	152	Yea, that would be 20	1	1d
D7	153	Yea and that would be 15. So that would be a half and you would subtract this from that so it would be this is how much is off and this is selling.	3	3b

Task	Student	Student Response	Code	Sub Code
D7		[calculates problems]	1	1a
D7	153	Weird, 7 doesn't go into 60. [continues calculating] Just round it to 9?	1	1c
D7	151		0	0a
D8	153	Now you add all that?	1	1c
D8	152	Yea, and subtract from discount.	1	1d
D8		[calculate sum]	1	1a
D8	153	So you wouldn't want it to be too high or too low [writes answer]	1	1c
D9	153	So maybe 45	1	1c
D9	152	Yes	1	1a
D9	153	Cause I think it's a good amount off and you don't lose money	3	3b
D9	152	Yeah [writes answer]	1	1d
D11	153	So the discount would be minus $15. But would we change the discount rate for more orders? I think we should make it lower so people would want to buy more orders so I would make it 20% and then a little bit lower. No you would want it to be a little bit higher, so 33% off and then half off.	3	3b
D11	153	[starts filling in columns]	1	1a
D10	153	[reads questions and copies from back]	1	1b
DF		[reads choices silently to themselves]	1	1b
DF	152	Which one would you want to do?	0	0c
DF	153	I would design a cover.	0	0c
DF	152	Yea.	0	0c
DF		[both start drawing on the same paper, adding details while discussing drama class for last 3 minutes of the task-set]	1	1c

Table C.3

Double Coder Procedures

Time	Procedures
45 min	Review DOK Wheel and 4 DOK Levels Review Written Coding Rubric and Compare to DOK Wheel Co-Code Written Harry Task. Double Coder coders first identified the DOK level and explained their reasoning for the identified level. Next, the researcher confirmed the coding or provide justification for an alternative DOK level. Type codes in google sheet double coding form. Review Verbal Coding Rubric Co-Code Verbal Harry Task. (Same as Written). (3%)
1-2 hrs	Double Coder codes Harry and Diary Tasks for 1 Group (3%)
15 min	Researcher and Double Coder Meet to Compare Codes, discuss any discrepancies and answer and questions.
2-3 hrs	Double Coder codes Harry and Diary Tasks for 2 Groups (6%)
15 min	Research compiles SPSS files with original codes from Researcher and Two Double Coders. Runs KALPHA Reliability test.

Books on Screen Harry Potter & the Half Blood Prince

Name: [redacted] 470

You are a *movie analyst*. You are evaluating films that are *adaptations* of popular children's books.

Your Challenge: Do people prefer the book or the movie adaptation?

REAL LIFE · REAL WORLD CERTIFIED · REAL DATA

LEVEL 1

To begin your analysis, you look at data from the year that the movie was released in theaters. *Harry Potter & the Half Blood Prince* was released in 2009. The average ticket price in 2009 was $7.

DOK 1 - Work (Scrach Paper - Used Algorithm)

1. Divide or multiply to calculate the estimated *ticket sales* and the *box office gross* for the weeks that the movie was showing in theaters. *Round your answers to the nearest hundredth as necessary.*

Dates	Estimated Ticket Sales (millions)	Box Office Gross ($ millions)
July 10th - August 6th	35.32	247.24
August 7th - September 3rd	4.31	30.17
September 4th - October 1st	0.71	4.97
October 2nd - 29th	0.19	1.33
October 30th - November 26th	0.07	0.49
Novermber 27th - December 17th	0.1	0.7

2. Calculate the *revenue* for the books sold in the same year. *Round your answers to the nearest hundredth as necessary.*

DOK 1 - Shows no Work

	Total Sold (millions)	Price	Revenue (millions)
Paperback	0.5	$12.99	6.500

3. Interpret the data.
   a. Did more people see the movie or buy the book? How do you know?

DOK 3 - Used Comparison to Justify Answer

There is already more in one number then the books so more people bought tickets

   b. Did the movie or book bring in more money? How do you know?

The first amount of money (247.24 million) is more than the Revenue (6.5m)

NextLesson Updated September 2014 © NextLesson 2014

Books on Screen Harry Potter & the Half Blood Prince

LEVEL 2

As well as judging their success by the number of people who saw the movie or read the book, you also want to analyze what people actually thought of *Harry Potter & the Half Blood Prince*. You find several *sources* of reviews.

4. Mark each average *rating* of the *movie* adaptation as an X on the line plot.

Harry Potter & the Half Blood Prince – Movie Adaptation	
Source	Average Rating (out of 5 stars)
IMDb	$3\frac{3}{4}$
Rotten Tomatoes	$3\frac{3}{4}$
Amazon	$3\frac{3}{4}$
Barnes & Noble	$4\frac{1}{2}$
iTunes	4

DOK 2 - Organized Data (Labeled Number Line)

0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1 $1\frac{1}{4}$ $1\frac{2}{4}$ $1\frac{3}{4}$ 2 $2\frac{1}{4}$ $2\frac{2}{4}$ $2\frac{3}{4}$ 3 $3\frac{1}{4}$ $3\frac{2}{4}$ $3\frac{3}{4}$ 4 $4\frac{1}{4}$ $4\frac{2}{4}$ $4\frac{3}{4}$ 5

DOK 3 - Justified answer with "Because"

5. Use the line plot. Explain your work.
   a. Which values show up the most often, if any?

   the $3\frac{3}{4}$ shows up the most because it had 3 out of 5

   b. Compare the **difference** between the average Barnes & Noble rating and the average IMDb rating. $\frac{3}{4}$

   ~~IMDb showed up more and barnes & Nobles has 1 rating~~

   c. Compare the **difference** between the average Amazon rating and the average iTunes rating. $\frac{1}{4}$

NextLesson Updated September 2014 © NextLesson 2014

Books on Screen Harry Potter & the Half Blood Prince 470

6. Mark each average rating of the **book** as an X on the line plot.

Harry Potter & the Half Blood Prince – Book	
Source	Average Rating (out of 5 stars)
Goodreads	$4\frac{1}{2}$
Amazon	$4\frac{1}{4}$
Barnes & Noble	$4\frac{1}{2}$
iTunes	$3\frac{1}{2}$
Target	5

DOK 2 - Organized Data (Labeled Number Line)

0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1 $1\frac{1}{4}$ $1\frac{2}{4}$ $1\frac{3}{4}$ 2 $2\frac{1}{4}$ $2\frac{2}{4}$ $2\frac{3}{4}$ 3 $3\frac{1}{4}$ $3\frac{2}{4}$ $3\frac{3}{4}$ 4 $4\frac{1}{4}$ $4\frac{2}{4}$ $4\frac{3}{4}$ 5

7. Use the line plot. Explain your work.

DOK 3 - Justified answer with "Because"

   a. Which values show up the most often, if any? $4\frac{1}{2}$ Because there are two $4\frac{2}{4}$ and 1 of each other

   b. Compare the **difference** between the average Goodreads rating and the average Amazon rating. $4\frac{1}{2} = 4\frac{2}{4}$ $4\frac{2}{4} - 4\frac{1}{4} = \frac{1}{4}$

   c. Compare the **difference** between the average Target rating and the average Barnes & Noble rating. $\frac{2}{4}$

8. Now compare the ratings.

DOK 1 - Work (Scrach Paper - Used Algorithm)

   a. Use all the movie ratings to calculate the movie's total rating out of 25 stars. $19\frac{3}{4}$

   b. Use all the book ratings to calculate the book's total rating out of 25 stars. $21\frac{3}{4}$

   c. Compare the two new 25-star ratings. Calculate the difference between the two ratings. $21\frac{3}{4} - 19\frac{3}{4}$ = 2

DOK 3 - Justifies answer with "because" and Comparison

9. Based on all of the **data**, do people prefer the original book or the movie adaptation? Justify and explain your reasoning.

more people liked the book because the rating was $21\frac{3}{4}$ for the book and for the movie the rating was $19\frac{3}{4}$.

NextLesson Updated September 2014 © NextLesson 2014

Books on Screen Harry Potter & the Half Blood Prince

Challenge

Finally, you examine the **number** of people who have provided a rating of *Harry Potter & the Half Blood Prince* on each of the websites.

Movie Adaptation		Book	
Source	Raters	Source	Raters
IMDb	247,993	Goodreads	1,200,990
Rotten Tomatoes	1,552,692	Amazon	4,543
Amazon	1,365	Barnes & Noble	4,015
Barnes & Noble	170	iTunes	28
iTunes	492	Target	1

1. Do you see any relationships between the number of people who provided a rating and the average rating that the source has posted? *Cite evidence* from the data to explain your thinking.

   Amazon, Barnes and Noble, iTunes also rated both the book and the movie

DOK 2 - Explains Answers without Extending

2. How does this additional data affect your opinion of people's preferences for the book or the movie adaptation? *Justify* your reasoning.

DOK 2 - Explains Answer without Extending

   It makes me dislike the movie more. It is an opinion it cannot be justified.

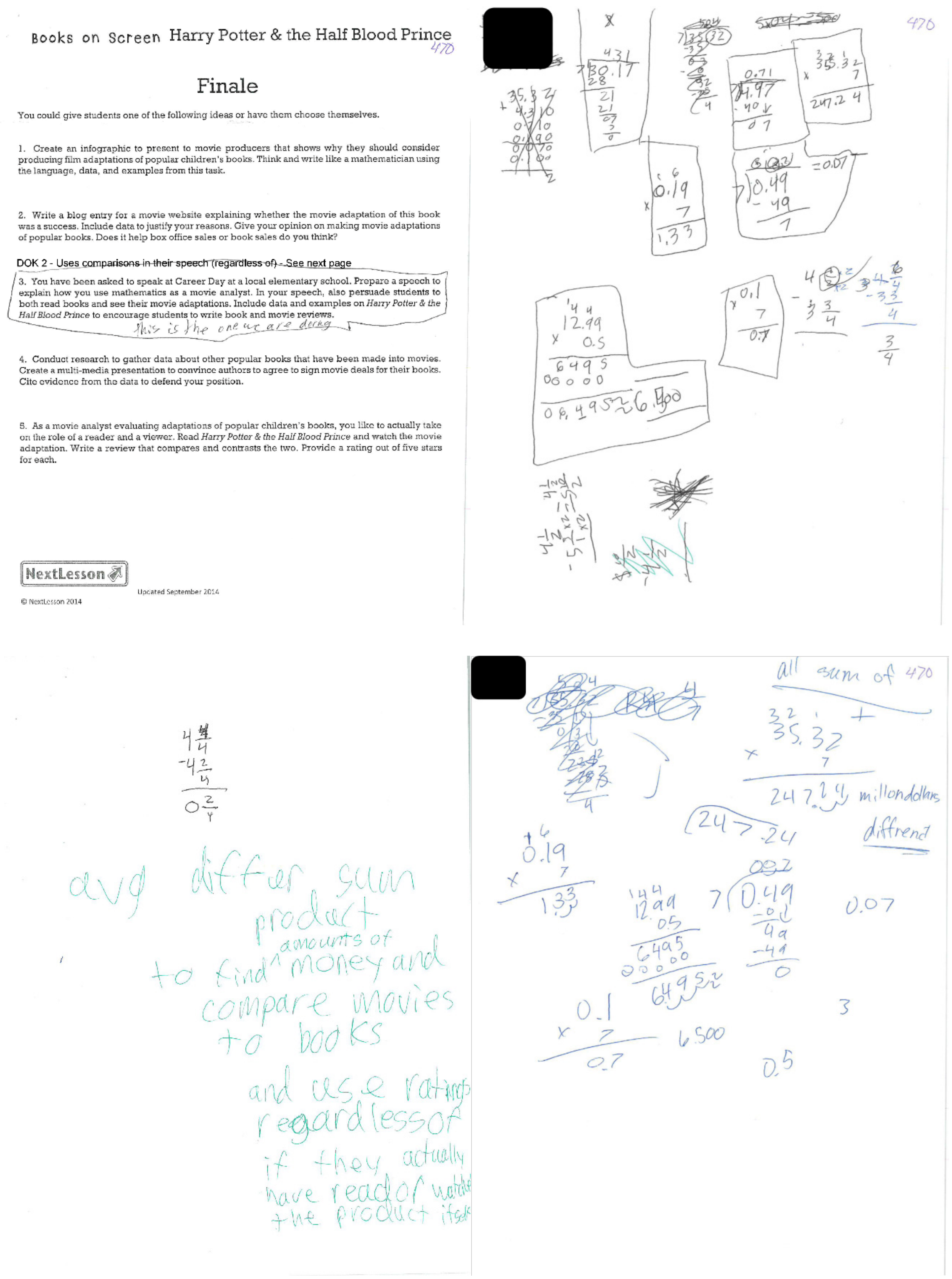
3. What other data would you want to gather to help you assess whether people prefer the book or the movie? Justify why you think it would help you prove your findings.

   I perfer the book but I asked people and two people perfered the book two peopl perfured the movie

DOK 4 - Synthesize information from outside sources

NextLesson Updated September 2014 © NextLesson 2014

(*Figure continues*)


Books on Screen Harry Potter & the Half Blood Prince

Finale

You could give students one of the following ideas or have them choose themselves.

1. Create an infographic to present to movie producers that shows why they should consider producing film adaptations of popular children's books. Think and write like a mathematician using the language, data, and examples from this task.

2. Write a blog entry for a movie website explaining whether the movie adaptation of this book was a success. Include data to justify your reasons. Give your opinion on making movie adaptations of popular books. Does it help box office sales or book sales do you think?

DOK 2 - Uses ~~comparisons in their speech (regardless of)~~ - See next page

3. You have been asked to speak at Career Day at a local elementary school. Prepare a speech to explain how you use mathematics as a movie analyst. In your speech, also persuade students to both read books and see their movie adaptations. Include data and examples on *Harry Potter & the Half Blood Prince* to encourage students to write book and movie reviews.

4. Conduct research to gather data about other popular books that have been made into movies. Create a multi-media presentation to convince authors to agree to sign movie deals for their books. Cite evidence from the data to defend your position.

5. As a movie analyst evaluating adaptations of popular children's books, you like to actually take on the role of a reader and a viewer. Read *Harry Potter & the Half Blood Prince* and watch the movie adaptation. Write a review that compares and contrasts the two. Provide a rating out of five stars for each.

NextLesson

Updated September 2014

© NextLesson 2014

Figure C.2. Sample coded written CD.

Appendix D

Written Cognitive Demand Quantitative Tables

Table D.1

Reflective Task-Set 1 Overall Percentage Written

	Written DOK					
Intended DOK	0	1	2	3	4	Total
1	0.48%	13.46%	1.92%	14.90%	0.00%	30.77%
2	0.48%	0.48%	7.69%	22.12%	0.00%	30.77%
3	0.48%	0.96%	5.77%	23.08%	0.48%	30.77%
4	0.48%	0.48%	1.92%	0.48%	4.33%	7.69%
Total	1.92%	15.38%	17.31%	60.58%	4.81%	100.00%

Table D.2

Exploratory Task-Set 1 Overall Percentage Written

	Written DOK					
Intended DOK	0	1	2	3	4	Total
1	0.00%	21.27%	5.88%	3.62%	0.00%	30.77%
2	0.00%	3.62%	4.98%	22.17%	0.00%	30.77%
3	0.45%	5.43%	13.57%	10.86%	0.45%	30.77%
4	0.00%	0.90%	3.17%	1.36%	2.26%	7.69%
Total	0.45%	31.22%	27.60%	38.01%	2.71%	100.00%

Table D.3

Reflective Task-Set 2 Overall Percentage Written

	Written DOK					
Intended DOK	0	1	2	3	4	Total
1	0.00%	10.68%	2.56%	17.52%	0.00%	30.77%
2	0.85%	3.85%	17.52%	8.55%	0.00%	30.77%
3	1.71%	3.42%	6.41%	19.23%	0.00%	30.77%
4	0.00%	0.43%	2.56%	1.71%	2.99%	7.69%
Total	2.56%	18.38%	29.06%	47.01%	2.99%	100.00%

Table D.4

Exploratory Task-Set 2 Overall Percentage Written

	Written DOK					
Intended DOK	0	1	2	3	4	Total
1	0.00%	21.15%	4.81%	4.81%	0.00%	30.77%
2	0.96%	12.50%	14.90%	2.40%	0.00%	30.77%
3	3.85%	6.73%	7.21%	12.50%	0.48%	30.77%
4	0.48%	2.40%	1.44%	1.44%	1.92%	7.69%
Total	5.29%	42.79%	28.37%	21.15%	2.40%	100.00%

Table D.5

Difference in Proportions Tests of Association for Intended DOK vs. Written CD

Task-set and discourse type	χ^2	df	*p*
Task-Set 1 Reflective	166.0	12	< 0.001
Task-Set 2 Reflective	149.0	12	< 0.001
Task-Set 1 Exploratory	144.0	12	< 0.001
Task-Set 2 Exploratory	96.4	12	< 0.001

Table D.6

Frequencies of Written CD Organized by Tasks and Discourse

	Reflective				Exploratory				Overall			
Tasks	1	2	3	4	1	2	3	4	1	2	3	4
Overall	75	104	236	17	158	120	128	11	233	224	364	28
Task-Set 1 (H)	32	36	126	10	69	61	84	6	101	97	210	16
Task-Set 2 (D)	43	68	110	7	89	59	44	5	132	127	154	12
DOK 1 Tasks	53	10	72	0	91	23	18	0	144	33	90	0
DOK 2 Tasks	10	57	66	0	34	42	54	0	44	99	120	0
DOK 3 Tasks	10	27	93	1	26	45	50	2	36	72	143	3
DOK 4 Tasks	2	10	5	16	7	10	6	9	9	20	11	25
H1 (DOK 1)	10	1	5	0	11	1	5	0	21	2	10	0
H2 (DOK 1)	9	1	5	0	13	3	1	0	22	4	6	0
H3 (DOK 2)	0	4	11	0	0	1	16	0	0	5	27	0
H4 (DOK 1)	3	1	12	0	13	4	0	0	16	5	12	0
H5 (DOK 2)	0	2	14	0	1	2	14	0	1	4	28	0
H6 (DOK 1)	6	1	9	0	10	5	2	0	16	6	11	0
H7 (DOK 2)	0	1	15	0	1	1	15	0	1	2	30	0
H8 (DOK 2)	1	9	6	0	6	7	4	0	7	16	10	0
H9 (DOK 3)	2	0	13	0	4	1	12	0	6	1	25	0
H10 (DOK 3)	0	4	12	0	1	15	1	0	1	19	13	0
H11 (DOK 3)	0	2	13	1	1	7	9	0	1	9	22	1
H12 (DOK 3)	0	6	10	0	6	7	2	1	6	13	12	1
HF (DOK 4)	1	4	1	9	2	7	3	5	3	11	4	14
D1 (DOK 1)	4	0	14	0	8	2	6	0	12	2	20	0
D2 (DOK 2)	1	5	12	0	7	7	2	0	8	12	14	0
D3 (DOK 3)	2	4	11	0	4	5	5	0	6	9	16	0
D4 (DOK 2)	0	18	0	0	3	13	0	0	3	31	0	0
D5 (DOK 1)	9	4	5	0	14	2	0	0	23	6	5	0
D6 (DOK 1)	9	2	7	0	11	5	0	0	20	7	7	0
D7 (DOK 1)	3	0	15	0	11	1	4	0	14	1	19	0
D8 (DOK 2)	4	9	3	0	6	8	2	0	10	17	5	0
D9 (DOK 3)	1	4	11	0	1	2	11	0	2	6	22	0
D10 (DOK 3)	3	1	14	0	4	3	7	0	7	4	21	0
D11 (DOK 2)	4	9	5	0	10	3	1	0	14	12	6	0
D12 (DOK 3)	2	6	9	0	5	5	3	1	7	11	12	1
DF (DOK 4)	1	6	4	7	5	3	3	4	6	9	7	11

Table D.7

Chi-Square Comparing Reflective and Exploratory Written CD by Task-Set

	Written Outcome			Chi Square	
Task-set and discourse type	CD < DOK	CD ≥ DOK	Total	χ^2	*p*
Task-Set 1 (H)				17.864	<0.001
Reflective	25	183	208		
Exploratory	63	158	221		
Total	88	341	429		
Task-Set 2 (D)				13.969	<0.001
Reflective	49	185	234		
Exploratory	77	131	208		
Total	126	316	442		
Both Task-Sets				29.667	<0.001
Reflective	74	368	442		
Exploratory	140	289	429		
Total	214	357	871		

Appendix E

Verbal Cognitive Demand Quantitative Tables

Table E.1

Reflective Task-Set 1 Overall Percentage Verbal

	Verbal DOK					
Intended DOK	0	1	2	3	4	Total
1	6.25%	8.65%	2.40%	12.50%	0.96%	30.77%
2	1.44%	7.69%	4.33%	15.38%	1.92%	30.77%
3	11.54%	5.29%	0.96%	10.10%	2.88%	30.77%
4	0.48%	0.48%	0.96%	3.37%	2.40%	7.69%
Total	19.71%	22.12%	8.65%	41.35%	8.17%	100.00%

Table E.2

Exploratory Task-Set 1 Overall Percentage Verbal

	Verbal DOK					
Intended DOK	0	1	2	3	4	Total
1	0.00%	3.62%	4.07%	20.81%	2.26%	30.77%
2	0.45%	1.36%	2.26%	24.89%	1.81%	30.77%
3	1.81%	9.05%	3.62%	11.31%	4.98%	30.77%
4	0.45%	0.45%	0.00%	1.81%	4.98%	7.69%
Total	2.71%	14.48%	9.95%	58.82%	14.03%	100.00%

Table E.3

Reflective Task-Set 2 Overall Percentage Verbal

	Verbal DOK					
Intended DOK	0	1	2	3	4	Total
1	10.26%	2.99%	3.85%	12.39%	0.85%	30.34%
2	11.97%	2.56%	3.42%	11.11%	1.71%	30.77%
3	14.10%	3.42%	1.28%	10.26%	2.14%	31.20%
4	0.00%	0.43%	1.71%	1.28%	4.27%	7.69%
Total	36.32%	9.40%	10.26%	35.04%	8.97%	100.00%

Table E.4

Exploratory Task-Set 2 Overall Percentage Verbal

	Verbal DOK					
Intended DOK	0	1	2	3	4	Total
1	0.48%	3.37%	8.65%	17.79%	0.48%	30.77%
2	3.85%	8.17%	9.62%	9.13%	0.00%	30.77%
3	6.25%	9.13%	1.92%	12.02%	1.44%	30.77%
4	1.44%	2.88%	0.96%	1.44%	0.96%	7.69%
Total	12.02%	23.56%	21.15%	40.38%	2.88%	100.00%

Table E.5

Difference in Proportions Tests of Association for Intended DOK vs. Verbal CD

Task-set and discourse type	χ^2	df	p
Task-Set 1 Reflective	42.8	12	< 0.001
Task-Set 2 Reflective	64.4	12	< 0.001
Task-Set 1 Exploratory	77.4	12	< 0.001
Task-Set 2 Exploratory	45.4	12	< 0.001

Table E.6

Frequencies of Verbal CD Organized by Tasks and Discourse

	Reflective				Exploratory				Overall			
Tasks	1	2	3	4	1	2	3	4	1	2	3	4
Overall	68	42	168	34	81	66	214	37	149	108	382	75
Task-Set 1 (H)	46	18	86	17	35	22	130	31	78	40	216	48
Task-Set 2 (D)	22	24	82	21	49	44	84	6	71	68	166	27
DOK 1 Tasks	25	14	55	4	15	27	83	6	40	41	138	10
DOK 2 Tasks	22	17	58	8	20	25	74	4	42	42	132	12
DOK 3 Tasks	19	5	45	11	39	12	50	14	58	17	95	25
DOK 4 Tasks	2	6	10	15	7	2	7	13	9	8	17	28
H1 (DOK 1)	4	1	11	0	0	0	14	3	4	1	25	3
H2 (DOK 1)	2	3	10	1	0	3	14	0	2	6	24	1
H3 (DOK 2)	1	1	8	4	0	0	13	4	1	1	21	8
H4 (DOK 1)	7	1	3	0	2	3	11	1	9	4	14	1
H5 (DOK 2)	2	1	13	0	0	4	13	0	2	5	26	0
H6 (DOK 1)	5	0	2	1	6	3	7	1	11	3	9	2
H7 (DOK 2)	10	1	5	0	2	1	14	0	12	2	19	0
H8 (DOK 2)	3	6	6	0	1	0	15	0	4	6	21	0
H9 (DOK 3)	4	0	5	1	3	2	8	2	7	2	13	3
H10 (DOK 3)	4	1	5	2	6	3	3	5	10	4	8	7
H11 (DOK 3)	1	0	5	3	7	0	7	3	8	0	12	6
H12 (DOK 3)	2	1	6	0	4	3	7	1	6	4	13	1
HF (DOK 4)	1	2	7	5	1	0	4	11	2	2	11	16
D1 (DOK 1)	2	2	12	0	1	1	14	0	3	3	26	0
D2 (DOK 2)	1	1	14	1	3	4	9	0	4	5	23	1
D3 (DOK 3)	1	2	9	0	2	3	8	0	3	5	17	0
D4 (DOK 2)	1	0	0	0	4	7	1	0	5	7	1	0
D5 (DOK 1)	0	0	0	0	5	7	2	1	5	7	2	1
D6 (DOK 1)	4	2	7	0	1	8	7	0	5	10	14	0
D7 (DOK 1)	1	5	10	2	0	2	14	0	1	7	24	2
D8 (DOK 2)	1	6	4	1	5	6	4	0	6	12	8	1
D9 (DOK 3)	0	0	5	3	2	0	11	1	2	0	16	4
D10 (DOK 3)	4	1	9	1	6	1	4	2	10	2	13	3
D11 (DOK 2)	3	1	8	2	5	3	5	0	8	4	13	2
D12 (DOK 3)	3	0	1	1	9	0	2	0	12	0	3	1
DF (DOK 4)	1	4	3	10	6	2	3	2	7	6	6	12

Table E.7

Chi-Square Tests of Independence for Verbal CD

	Verbal Outcome			Chi Square	
Task-set and discourse type	CD < DOK	CD ≥ DOK	Total	χ^2	*p*
Task-Set 1 (H)				19.933	<0.001
Reflective	80	128	208		
Exploratory	42	179	221		
Total	122	307	429		
Task-Set 2 (D)				4.953	0.026
Reflective	110	124	234		
Exploratory	76	132	208		
Total	188	256	442		
Both Task-Sets				22.825	<0.001
Reflective	190	252	442		
Exploratory	118	311	429		
Total	308	563	871		

Appendix F

Mathematical Discourse Contributions Quantitative Tables

Table F.1

Frequency of Coded Discourse Contribution Levels, Organized by Tasks and Discourse

	Reflective					Exploratory				
Tasks	0	1	2	3	4	0	1	2	3	4
Overall	511	1723	436	642	61	1066	2920	862	939	81
Task-Set 1 (H)	218	709	101	242	26	697	1870	557	665	69
Task-Set 2 (D)	293	1014	335	400	35	369	1050	305	274	12
DOK 1 Tasks	201	718	161	255	12	450	1491	449	441	9
DOK 2 Tasks	150	558	141	205	11	364	896	289	326	11
DOK 3 Tasks	84	223	51	123	14	178	351	66	107	24
DOK 4 Tasks	76	224	83	59	24	74	182	58	65	37
H1 (DOK 1)	46	165	19	34	0	149	580	181	174	6
H2 (DOK 1)	18	50	15	24	1	43	103	48	36	0
H3 (DOK 2)	38	69	17	54	10	68	164	53	80	10
H4 (DOK 1)	9	38	0	3	0	36	72	21	28	1
H5 (DOK 2)	18	87	7	35	0	94	210	74	73	0
H6 (DOK 1)	5	15	1	8	1	26	40	4	19	1
H7 (DOK 2)	12	47	2	7	0	56	134	29	37	0
H8 (DOK 2)	15	70	10	11	0	65	214	60	101	0
H9 (DOK 3)	8	21	1	8	1	21	42	7	14	4
H10 (DOK 3)	10	28	1	8	2	32	85	13	15	10
H11 (DOK 3)	5	9	0	12	4	32	57	6	18	3
H12 (DOK 3)	3	12	2	9	0	13	34	11	11	1
HF (DOK 4)	31	98	26	28	7	62	135	50	59	33
D1 (DOK 1)	44	153	41	82	0	114	402	116	126	0
D2 (DOK 2)	50	168	60	62	1	42	93	52	24	0
D3 (DOK 3)	38	76	34	29	0	48	84	22	20	0
D4 (DOK 2)	1	1	0	0	0	5	9	2	0	0
D5 (DOK 1)	0	0	0	0	0	2	8	1	2	0
D6 (DOK 1)	14	64	14	29	0	40	106	43	15	1
D7 (DOK 1)	53	222	64	71	5	42	190	40	42	0
D8 (DOK 2)	19	48	26	11	1	17	37	14	6	0
D9 (DOK 3)	5	27	3	15	4	7	25	1	23	1
D10 (DOK 3)	13	42	5	35	1	10	15	4	5	5
D11 (DOK 2)	8	74	25	29	4	17	37	7	5	1
D12 (DOK 3)	0	9	0	2	0	15	7	0	1	0
DF (DOK 4)	48	130	63	35	19	10	37	3	5	4

Table F.2

Sub Codes of Mathematical Discourse Contributions for Intended DOK Levels

	Reflective				Exploratory			
Tasks	1	2	3	4	1	2	3	4
Organization	201	150	84	76	450	364	178	74
	14.9%	14.1%	17.0%	16.3%	15.8%	19.3%	24.5%	17.7%
Action	201	150	84	76	450	364	178	74
	14.9%	14.0%	16.9%	16.3%	15.8%	19.3%	24.5%	17.7%
Minimal	718	558	223	224	1491	896	351	182
	53.3%	52.3%	45.0%	48.0%	52.5%	47.5%	48.3%	43.7%
Calculations	112	54	31	2	358	196	28	0
	8.3%	5.0%	6.2%	0.4%	12.6%	10.3%	3.8%	0%
Recall Facts	51	49	15	40	199	171	80	34
	3.7%	4.6%	3.0%	8.5%	7.0%	9.0%	11.0%	8.1%
State Problem	377	328	127	145	664	392	196	126
	27.9%	30.8%	25.6%	31.1%	23.3%	20.7%	27.0%	30.2%
Agreement	178	127	50	37	270	137	47	22
	13.21%	11.92%	10.10%	7.94%	9.51%	7.26%	6.47%	5.29%
Considerable	161	141	51	83	449	289	66	58
	11.9%	13.2%	10.3%	17.8%	15.8%	15.3%	9.0%	13.9%
New Strategy	144	132	47	81	406	266	60	52
	10.6%	12.3%	9.4%	17.3%	14.3%	14.1%	8.2%	12.5%
Efficient Strategy	17	9	4	2	43	23	6	6
	1.2%	0.8%	0.8%	0.4%	1.5%	1.2%	0.7%	1.4%
Substantive	255	205	123	59	441	326	107	65
	18.9%	19.2%	24.8%	12.6%	15.5%	17.2%	14.7%	15.6%
Generalize	21	3	2	3	23	6	2	1
	1.5%	0.2%	0.4%	0.6%	0.8%	0.3%	0.2%	0.2%
Justify	98	126	75	34	183	166	62	31
	7.2%	11.8%	15.1%	7.3%	6.4%	8.8%	8.5%	7.4%
Counter	136	76	46	22	235	154	43	33
	10.1%	7.1%	9.2%	4.7%	8.2%	8.1%	5.9%	7.9%
Extended	12	11	14	24	9	11	24	37
	0.8%	1.0%	2.8%	5.1%	0.3%	0.5%	3.3%	8.8%
Connect Contexts	11	9	3	12	6	8	12	5
	0.8%	0.8%	0.6%	2.5%	0.2%	0.4%	1.6%	1.2%
Multiple Sources	1	2	11	12	3	3	12	32
	0.1%	0.2%	2.2%	2.5%	0.1%	0.1%	1.6%	7.6%

Note. Percentage out of 100 for each column.

Table F.3

Percentage Discourse Contribution Order by DOK Level

	Mathematical Discourse Contributions			
Intended DOK	Minimal	Considerable	Substantive	Extended
DOK1	30.00%	15.71%	51.43%	2.86%
DOK2	21.79%	17.95%	55.13%	5.13%
DOK3	30.51%	6.78%	45.76%	16.95%
DOK4	4.35%	21.74%	30.43%	43.48%

Table F.4

Percentage Discourse Contribution Priority by DOK Level

	Mathematical Discourse Contributions			
Intended DOK	Minimal	Considerable	Substantive	Extended
DOK1	14.29%	10.71%	67.86%	7.14%
DOK2	19.23%	11.54%	53.85%	15.38%
DOK3	5.26%	5.26%	84.21%	5.26%
DOK4	0.00%	12.50%	37.50%	50.00%

Table F.5

Percentage Discourse Contribution Together by DOK Level

	Mathematical Discourse Contributions			
Intended DOK	Minimal	Considerable	Substantive	Extended
DOK1	8.51%	6.38%	78.72%	6.38%
DOK2	5.26%	7.02%	80.70%	7.02%
DOK3	18.31%	8.45%	53.52%	19.72%
DOK4	4.76%	4.76%	28.57%	61.90%

Table F.6

Percentage Discourse Contribution Side-by-Side by DOK Level

	Mathematical Discourse Contributions			
Intended DOK	Minimal	Considerable	Substantive	Extended
DOK1	4.65%	4.65%	86.05%	4.65%
DOK2	5.00%	25.00%	70.00%	0.00%
DOK3	28.57%	14.29%	57.14%	0.00%
DOK4	0.00%	0.00%	0.00%	0.00%

Table F.7

Percentage Discourse Contribution Split Up by DOK Level

	Mathematical Discourse Contributions			
Intended DOK	Minimal	Considerable	Substantive	Extended
DOK1	5.00%	60.00%	35.00%	0.00%
DOK2	28.57%	35.71%	35.71%	0.00%
DOK3	42.86%	14.29%	42.86%	0.00%
DOK4	0.00%	0.00%	0.00%	0.00%

Table F.8

Percentage Discourse Contribution Take Over DOK Level

	Mathematical Discourse Contributions			
Intended DOK	Minimal	Considerable	Substantive	Extended
DOK1	44.44%	44.44%	11.11%	0.00%
DOK2	47.62%	23.81%	28.57%	0.00%
DOK3	75.00%	15.00%	10.00%	0.00%
DOK4	75.00%	12.50%	12.50%	0.00%

Table F.9

Percentage Written CD Order by DOK Level

	Written Cognitive Demand			
Intended DOK	CD1	CD2	CD3	CD4
DOK1	30.00%	15.71%	51.43%	2.86%
DOK2	21.79%	17.95%	55.13%	5.13%
DOK3	30.51%	6.78%	45.76%	16.95%
DOK4	4.35%	21.74%	30.43%	43.48%

Table F.10

Percentage Written CD Priority by DOK Level

	Written Cognitive Demand			
Intended DOK	CD1	CD1	CD1	CD1
DOK1	14.29%	10.71%	67.86%	7.14%
DOK2	19.23%	11.54%	53.85%	15.38%
DOK3	5.26%	5.26%	84.21%	5.26%
DOK4	0.00%	12.50%	37.50%	50.00%

Table F.11

Percentage Written CD Together by DOK Level

	Written Cognitive Demand			
Intended DOK	CD1	CD1	CD1	CD1
DOK1	8.51%	6.38%	78.72%	6.38%
DOK2	5.26%	7.02%	80.70%	7.02%
DOK3	18.31%	8.45%	53.52%	19.72%
DOK4	4.55%	4.55%	27.27%	63.64%

Table F.12

Percentage Written CD Side-by-Side by DOK Level

	Written Cognitive Demand			
Intended DOK	CD1	CD1	CD1	CD1
DOK1	4.65%	4.65%	86.05%	4.65%
DOK2	5.00%	25.00%	70.00%	0.00%
DOK3	28.57%	14.29%	57.14%	0.00%
DOK4	0.00%	0.00%	0.00%	0.00%

Table F.13

Percentage Written CD Split Up by DOK Level

	Written Cognitive Demand			
Intended DOK	CD1	CD1	CD1	CD1
DOK1	5.00%	60.00%	35.00%	0.00%
DOK2	28.57%	35.71%	35.71%	0.00%
DOK3	42.86%	14.29%	42.86%	0.00%
DOK4	0.00%	0.00%	0.00%	0.00%

Table F.14

Percentage Written CD Take Over DOK Level

	Written Cognitive Demand			
Intended DOK	CD1	CD1	CD1	CD1
DOK1	44.44%	44.44%	11.11%	0.00%
DOK2	47.62%	23.81%	28.57%	0.00%
DOK3	75.00%	15.00%	10.00%	0.00%
DOK4	75.00%	12.50%	12.50%	0.00%

CURRICULUM VITAE

KRISTY L. LITSTER

Business Address:
Utah State University
College of Education & Human Services
2805 Old Main Hill
Logan, UT 84321
Kristy.litster@usu.edu

Home Address:
27 Aggie Village Apt B
Logan, UT 84341
801-400-2354
Chemile@hotmail.com

EDUCATION

Ph.D. Expected Summer, 2019
Education, Utah State University
Specialization: Curriculum and Instruction,
Concentration: Mathematics Education and Leadership

M.E. August 2010
Masters of Education, Southern Utah University

B.S. August 2003
Elementary Education, Brigham Young University
Level 2 Elementary Teaching Certificate (1-8), Utah
English as a Second Language Endorsement, Southern Utah University
Mathematics Endorsement, Utah State University

EMPLOYMENT HISTORY

UTAH STATE UNIVERSITY

Adjunct Profession (May 2019-August 2019)
Utah State University, College of Education & Human Services, School of Teacher Education and Leadership, Logan, Utah
Teaching responsibilities include teaching online Masters level courses for in-service teachers towards an endorsement.

Graduate Research Assistant (August 2015- May 2019)
Utah State University, College of Education & Human Services, School of Teacher Education and Leadership, Logan, Utah
Research responsibilities include assisting professors on various research projects in

mathematics education such as coding data, writing literature reviews, conducting research interviews, finding emergent themes and participating in group papers and presentations.

Graduate Teaching Assistant (August 2015- May 2019)
Utah State University, College of Education & Human Services, School of Teacher Education and Leadership, Logan, Utah
Teaching assistant responsibilities include teaching face-to-face Mathematics Methods for Elementary Teachers and mentoring pre-service teachers in practicum setting as well as teaching online Masters level courses for in-service teachers towards an endorsement.

UTAH COUNTY PUBLIC AND PRIVATE SCHOOLS

Elementary School Teacher, Grade 5, all subjects (2007-2015)
Wilson Elementary, Payson, Utah
Responsibilities included planning, designing and teaching curricula in all content areas. Worked with a Professional Development Committee (PLC) to design data driven instructional assessments and interventions in a Response to Intervention (RtI) model. I led committees, clubs, and activities such as, student council, school newspaper, Hope of America, science club, recorder club, and knitting club. I served as the DART school representative, district science fair judge, and school communications manager.

Music Specialist, Grades K-6, music and drama (2004-2007)
Meadow Elementary, Lehi, Utah
Responsibilities included planning, designing, and teaching curricula in music and drama as well as one-on-one tutoring in mathematics. I led the committee for Triple R Night, organized drama and music programs for each grade level including, but not limited to, the 4^{th} Grade Utah Centennial Celebration, 6^{th} Grade Mother Goose Musical, and 3-6^{th} Recorder concerts. I also designed and maintained the school website.

AWARDS & PROFESSIONAL RECOGNITION

Hillcrest Elementary Volunteer of the Year ($110): Volunteer of the Year award for service relating to the Hillcrest Elementary Science Fair and Teacher Mathematics Professional Development. Awarded by the Logan City Board of Education, Logan, Utah. (April 23, 2019)

Outstanding Service Commendation: Outstanding Service Commendation for service to the Special Interest Group for Research in Mathematics Education, American Education Research Association in designing and implementing collaborative networking opportunities. AERA, Toronto, ON, Canada (April 6, 2019).

Outstanding Paper Award: Outstanding Paper Award for "How the Balance of Gaming and Mathematics Elements Effects Student Learning in Digital Math Games," SITE, Las

Vegas, NV (March 20, 2019).

Graduate Student Researcher of the Year. Graduate Student Researcher of the Year for the School of Teacher Education and Leadership, in the College of Education and Human Services (2018-2019)

Scholarship ($20,400): Graduate Research and Teaching Assistantship, Utah State University, Logan, UT (2018-2019)

Graduate Enhancement Award ($4,000): Award for using knowledge gained through educational opportunities to contribute to Utah State University campus, to local and national communities, and to the professional field to foster lasting change, Utah State University, Logan, UT (March 29, 2018)

Outstanding Paper Award: Outstanding Paper Award for "The Role of Design Features in the Affordances of Digital Math Games," SITE, Washington D.C. (March 27, 2018)

Scholarship ($20,400): Graduate Research and Teaching Assistantship, Utah State University, Logan, UT (2017-2018)

Scholarship ($17,000): Graduate Research and Teaching Assistantship, Utah State University, Logan, UT (2016-2017)

Scholarship ($17,000): Graduate Research and Teaching Assistantship, Utah State University, Logan, UT (2015-2016)

RESEARCH

Research Interests:

- Webb's Depth of Knowledge (Levels of Cognitive Demand)
- Pre-Service and In-service teacher professional development in teaching mathematics.
- The effects of authentic or real-world applications of mathematics on student learning.

Research Projects:

Discourse and Cognitive Demand Dissertation Project (2018-Present). Utah State University (with PI Dr. Patricia Moyer-Packenham) in partial fulfillment for the requirements of a Doctor of Philosophy in Education. My role: Design study to include questions, tasks, implementation procedures, and analysis procedure; conduct literature review; complete IRB; solicit district, school, teacher, and student participation; conduct study in four classrooms, solicit transcribers and transcribe data; quantitatively and qualitatively analyze data using frequency tables, heat maps, difference in proportions tests, as well as open and structured coding; solicit double coders and check for inter-rater reliability; and write up results and discussion.

Early Count Project (2018-Present). Utah State University (with PI Dr. Beth MacDonald) as part of an independent study course, TEAL 7900. My role: Upload all transcripts to NVivo software and code 50% of transcripts for emerging themes. Qualitative coding of longitudinal data for 6 students' strategies. Quantitative analysis of TEMA-3 scores, task variation and student solution strategies. Co-author collaborative presentations and publications.

Affordances of Virtual Manipulatives Grades 3-6 (2016 - Present). Utah State University (with PI Dr. Patricia Moyer-Packenham and the Virtual Manipulatives Research Group). My role: design Experiment 3 (select 3 mathematics iPad apps, design pre/post assessments, design semi-structured interview); organize interview schedules between interviewers and participants for over 200 participants across four experiments; design and distribute study brochure; track demographic information; develop iPad-based interview protocols; conduct iPad-based interviews with over 60 participants; collect and code data; qualitatively and quantitatively analyze data; author and co-author collaborative presentations and publications.

Elementary Pre-Service Teachers' Beliefs as Related to their Intended Pedagogy (2016-present). Utah State University (with PI Dr. Beth MacDonald). My role: design pre-service teacher belief survey; disseminate survey; qualitatively and quantitatively analyze data; co-author collaborative presentations and publications.

Supporting kindergarten, first grade, and second grade students' ability to utilize mental reversibility when solving computational problems (2015-Present). Utah State University (with PI Dr. Beth MacDonald). My role: Code video observations of participation strategies and find emerging themes and data analysis. Quantitative analysis of task variation and student solution strategies, participate in qualitative analysis for strategies across tasks. Author and co-author collaborative presentations and publications.

Affordances of Virtual Manipulatives Grades PreK-2 (2015-2019). Utah State University (with PI Dr. Patricia Moyer-Packenham and the Virtual Manipulatives Research Group). My role: qualitatively analyze data of 100 participants; author and co-author collaborative presentations and publications.

Effects of Dual Language Immersion on Early Numerical Cognition (2017-2018). Utah State University (with PI Dr. Kerry Jordan and Emily Speed as lead graduate researcher) as part of an independent research course, TEAL 7910. My role: meet with principals and district leaders to obtain project consent, collect data at local elementary schools that offer dual language immersion programs, collect completed consent forms from schools.

PUBLICATIONS

Journal Articles

Litster, K., Lommatsch, C. W., Moyer-Packenham, P. S., Novak, J., Ashby, M. J., Roxburgh, A., & Bullock, P. (Revise and Resubmit) Attitude, app use, and affordances: A mediation model on learning transfer from digital math games. *Journal of Research in Mathematics Education.*

Litster, K., MacDonald, B. L., & Shumway, J. F., (In Press). Experiencing active mathematics learning: Meeting the expectations for teaching and learning in elementary math classrooms. *Math Enthusiast.*

Litster, K., Moyer-Packenham, P. S., Reeder, R. (2019). Base-10 blocks: A study of iPad virtual manipulative affordances across primary-grade levels. *Mathematics Education Research Journal. () 1-17. https://doi.org/10.1007/s13394-019-00257-2.*

Moyer-Packenham, P. S., Lommatsch, C. W., **Litster, K**., Ashby, J., Bullock, E. K., Roxburgh, A. L., Shumway, J. F., Speed, E., Covington, B., Hartmann, C., Clarke-Midura, J., Skaria, J., Westenskow, A., MacDonald, B., Symanzik, J., & Jordan, K. (2019). How design features in digital math games support learning and mathematics connections. *Computers in Human Behavior, 91*(2019), 316-332. doi: 10.1016/j.chb.2018.09.036.

Litster, K., Reeder, R., Di Stefano, M., & MacDonald, B. L. (2018). Turn it around: Culturally and Linguistically Responsive Teaching. *Utah Mathematics Teacher, 11*(Fall-Winter 2018-2019), 57-63.

Di Stefano, M., **Litster, K.**, & MacDonald, B. L. (2017). Mathematics intervention supporting Allen, an English Learner: A case study. *Education Sciences, 7*(2), pp. 1-24. DOI:10.3390/educsci7020057

Monograph

Moyer-Packenham, P. S., **Litster, K.**, Bullock, E. P. & Shumway, J. F., (2018) Using video analysis to explain how virtual manipulative app alignment affects children's mathematics performance and efficiency. In L. Ball, P. Drijvers, S. Ladel, H. Siller, M. Tachack, & C. Vale (Eds.) *Uses of Technology in Primary and Secondary Matheamtics Education* (pp. 9-34). Springer, Cham.

Book

Brunson, S, **Jones, K.** & Wilson, T. (1999). Service Learning: What it is and Why I am Doing It? In, Harline, P. & Johnson, L. (Eds.), *800 N 200 E, Addressing Honors Intensive Writing,* (pp. 195-199). Provo, UT: Wasatch Lithography.

Conference Proceedings

Litster, K., & Moyer-Packenham, P.S. (2019). How the Balance of Gaming and Mathematics Elements Effects Student Learning in Digital Math Games. In K. Graziano (Ed.), *Proceedings of the Society for Information Technology and Teacher Education (SITE) International Conference* (pp. 1834-1843). Las Vegas, Nevada, United States: Association for the Advancement of Computing in Education (AACE).
OUTSTANDING PAPER AWARD

Litster, K., Moyer-Packenham, P.S., Ashby, M. J., Roxburgh, A., & Kozlowski, J. (2019). Digital Math Games: Importance of Strategy and Perseverance on Elementary Children's Learning Opportunities. In K. Graziano (Ed.), *Proceedings of the Society for Information Technology and Teacher Education (SITE) International Conference* (pp. 1828-1833). Las Vegas, Nevada, United States: Association for the Advancement of Computing in Education (AACE).

Moyer-Packenham, **Litster, K.**, P.S., Roxburgh, A., Kozlowski, J. & Ashby, M. J., (2019). Relationships between Mathematical Language, Representation Connections, and Learning Outcomes in Digital Games. In K. Graziano (Ed.), *Proceedings of the Society for Information Technology and Teacher Education (SITE) International Conference* (pp. 1872-1880). Las Vegas, Nevada, United States: Association for the Advancement of Computing in Education (AACE).

Moyer-Packenham, Ashby, M. J., **Litster, K.**, P.S., Roxburgh, A., & Kozlowski, J. (2019). How Design Features Promote Children's Awareness of Affordances in Digital Math Games. In K. Graziano (Ed.), *Proceedings of the Society for Information Technology and Teacher Education (SITE) International Conference* (pp. 1863-1871). Las Vegas, Nevada, United States: Association for the Advancement of Computing in Education (AACE).

Moyer-Packenham, P. S., & **Litster, K.** (2018, October). How teachers can enhance mathematics learning with technology-infused experiences. *Program book of the 3rd Sriwijaya University Learning and Education International Conference (3rd SULE-IC 2018)* (p. 7). *Sriwijaya University, Palembang, Indonesia.*

Litster, K., Moyer-Packenham, P. S., & Reeder, R. (2018, March). Affordances of Simultaneous Linking Features in a Base-10 Blocks Mathematics App for Young Children. In E. Langran & J. Borup (Eds.), *Proceedings of the Society for Information Technology and Teacher Education (SITE) International Conference* (pp. 761-767), Waynesville, NC: Association for the Advancement of Computing in Education (AACE).

Moyer-Packenham, P. S., **Litster, K.**, Lommatsch, C., Ashby, M. J., & Roxburgh, A. (2018, March). Mediators of learning in game-based mathematics apps. In E. Langran & J. Borup (Eds.), *Proceedings of the Society for Information Technology and Teacher*

Education (SITE) International Conference (pp. 454-464), Waynesville, NC: Association for the Advancement of Computing in Education (AACE).

Moyer-Packenham, P. S., Lommatsch, C., **Litster, K.**, Ashby, M. J., & Roxburgh, A. (2018, March). The role of design features in the affordances of digital math games. In E. Langran & J. Borup (Eds.), *Proceedings of the Society for Information Technology and Teacher Education (SITE) International Conference* (pp. 465-473), Waynesville, NC: Association for the Advancement of Computing in Education (AACE).
OUTSTANDING PAPER AWARD

MacDonald, B., Ashby, J., & **Litster, K.** (2016). Preliminary Findings of First Grade Students' Development of Reversibility. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (p. 205). Tucson, AZ: The University of Arizona.

Unpublished Manuscripts

Litster, K., Moyer-Packenham, P. S., Lommatsch, C. M., Ashby, J., & Roxburgh, A., Bullock, E. P., Shumway, J. F., Speed, E., Covington, B., Harmann, C., Clarke-Midura, J., Skaria, J., Westenskow, A., Macdonald, B., Symanzik, J., & Jordan, K. (Under Review). *How Children's Affect, Mathematical Connections, and Strategies Influence Learning with Digital Math Games.*

Litster, K. (Under Review) *Money makes sense: Understanding the standard algorithm for long division.*

DiStefano, M., **Litster, K.**, MacDonald, B. L., & Ashby, M. J. (Formatting for Re-submission). *Culturally Sustaining Mathematics Education: A Conceptual Framework for Enhancing ELs STEM Education.*

MacDonald, B.L., Hunt, J., Roxburgh, A., & **Litster, K.** (Formatting for Re-submission). *Diego's Use of Doubles and Near Doubles when Subitizing and Counting: A Case Study*

MacDonald, B. L**., Litster, K.**, & Ashby, M. J., (Formatting for Journal Submission). *Low-Achieving Students' Reversibility Development and Early Mathematics Achievement.*

Litster, K., Moyer-Packenham, P. S., & Feldon, D. (Formatting for Journal Submission) *Elementary mathematics apps: Balancing gaming and mathematics affordances for student learning.*

Manuscripts in Preparation

Moyer-Packenham, P. S., Rocburgh, A. L., **Litster, K.**, & Kozlowski, J. S. *How the representational connections that students make in digital games impact their mathematics learning.*

MacDonald, B. L**.**, Ashby, M. J., **Litster, K.**, Di Stefano, M., & Maahs-Fladung, C. *Mental reversibility tasks in early elementary: Interventions for struggling students.*

Roxburgh, A., Moyer-Packenham, **Litster, K.**, P.S., & Bullock, E. *How Design Features, Such as Feedback in Digital Math Games, Promote Children's Understanding.*

UNIVERSITY TEACHING

Utah State University, Logan, Utah (2015-2019)
College of Education and Human Services

Courses Taught – Utah State University

EDUC 4060 - Teaching Mathematics & Practicum Level III *(Fall 2015, Spring 2016, Fall 2016)*
Undergraduate Course. Relevant mathematics instruction in the elementary and middle-level curriculum; methods of instruction, evaluation, remediation, and enrichment. A field experience practicum is required.

- Face-To-Face Course
- Practicum In-school Supervision

EDUC 4062 - Teaching Elementary School Mathematics II: Number, Operations, and Algebraic Reasoning *(Spring 2018, Spring 2019)*
Undergraduate Course. Development of pedagogical content knowledge in number, operations, and algebraic reasoning for teaching grades preschool to grade 6. Methods for designing and implementing mathematics instruction, assessment, remediation, and intervention will be applied in a field-based placement.

- Face-To-Face Course (*Spring, 2018)*
- Mixture of Broadcast(IVC) and online course.(*Spring, 2019; 32 students in 11 different distance sites throughout Utah)*
- Practicum In-school Supervision (All Sections)

TEAL 6300- Special Topics: Elementary Mathematics Teaching Academy (*Spring 2017, Summer 2017, Summer 2018, Fall 2018, Summer 2019)*
Graduate Personalized Field-based program focusing upon characteristics of effective teaching methodologies, teaching performance, curriculum decision

making, value guidelines, and the characteristics of the learner. (3, 6, or 9 credits)

TEAL 6521/TEAL 5560 - Mathematics for Teaching K-8: Numbers and Operations
This course, for K-8 teachers, will cover the content of Number and Operations to develop comprehensive understanding of our number system and relate its structure to computation, arithmetic, algebra, and problem solving. Online Course

- TEAL 5560 – Undergraduate Course (*Fall, 2018*)
- TEAL 6521 – Graduate Course (*Spring 2017, Summer 2017, Summer 2018, Fall 2018, Summer 2019)*

TEAL 6522- Mathematics for Teaching K-8: Rational Numbers and Proportional Reasoning (*Spring 2017, Summer 2017, Summer 2018, Fall 2018, Summer 2019)*
Graduate course. Designed for K-8 teachers to explore the content of Rational Numbers and Proportional Reasoning. Online course.

TEAL 6523/TEPD 6523 - Mathematics for Teaching K-8: Algebraic Reasoning (*Spring 2017, Summer 2017, Fall 2017, Summer 2018, Fall 2018, Summer 2019)*
Graduate course. To provide practicing teachers a deeper understanding of algebraic expressions, equations, functions, real numbers, and instructional strategies to facilitate the instruction of this content for elementary students.

- Mixture Online and Face-to-Face Professional Development (*Fall, 2017)*
- Online Course (*All other sections)*

TEAL 6524– Mathematics for Teaching K-8: Geometry and Measurement (*Summer 2017, Summer 2018, Fall 2018, Summer 2019)*
Graduate course. Part of the Elementary Mathematics Endorsement (Level 1) Series. To provide practicing teachers a deeper understanding of the geometry and measurement context that exists in the state core and instructional strategies to facilitate the instruction of this content. Online Course

TEAL 6525/CETE 6525 - Mathematics for Teaching K-8: Data Analysis and Problem Solving (*Spring 2017, Summer 2017, Summer 2018, Fall 2018, Summer 2019)*
Graduate course. Part of the Elementary Mathematics Endorsement (Level 1) Series. This course will provide practicing teachers a deeper understanding of probability and data representation and analysis. Online Course

TEAL 6551- Mathematics for Teaching K-8: Assessment and Intervention (*Spring 2017, Summer 2017, Fall 2018, Summer 2019)*
Graduate course. Part of the Elementary Mathematics Endorsement (Level 1) Series. To provide practicing teachers a deeper understanding of the various types of assessment and their appropriate use for guiding instruction, intervention and evaluation of student learning. Online Course

Courses Adapted Collaboratively– Utah State University

Redesign EDUC 4060 into two courses: EDUC 4061 & 4062 (see above). My role in this adaptation was to attend initial meetings to determine topics covered in each of the two new courses and identify potential textbooks. For EDUC 4061 (online), review modules, powerpoints, and quizzes for accuracy and clarity. For EDUC 4062, identify specific sub-topics, design progressions, identify readings, revise tasks and assignments, create homework and quiz questions, create slides and resources.

Revise TEAL 6525 (See above). My role in this online course was to meet collaboratively to discuss proposed changes and design of the revised course as well as publications and resources. Review modules and quiz questions for clarity and accuracy. Compose discussion board/free response quiz questions.

GRANTS FUNDED

International Travel Grant, School of Teacher Education and Leadership (TEAL). ($400). International travel for presentation at annual American Educational Research Associate (AERA). (2019) Toronto, ON Canada.

International Travel Grant, Research and Graduate Studies. ($400). International travel for presentation at annual American Educational Research Associate (AERA). (2019) Toronto, ON, Canada.

Travel Grant, Research in Mathematics Education (AERA SIG-RME). ($350) Presentation, business meeting, and mentoring sessions at annual American Educational Research Associate (AERA). (2019) Toronto, ON, Canada.

Travel Grant, School of Teacher Education and Leadership (TEAL). ($300). Presentation at Annual American Educational Research Association (AERA). (2018) New York City, NY.

Travel Grant, Research and Graduate Studies. ($300). Presentation at Annual American Educational Research Association (AERA). (2018) New York City, NY.

Travel Grant, Research in Mathematics Education (AERA SIG-RME). ($350). Presentation and business meeting at Annual American Educational Research Association (AERA). (2018) New York City, NY.

Travel Grant, School of Teacher Education and Leadership (TEAL). ($300). Presentation at 21st Annual Association of Mathematics Teacher Educators (AMTE). (2017) Orlando, FL.

Travel Grant, Research and Graduate Studies. ($300). Presentation at 21st Annual Association of Mathematics Teacher Educators (AMTE). (2017) Orlando, FL. Utah State University.

Tuition Grant, College of Education and Human Services ($50). Tuition Award, Spring 2017.

Tuition Grant, College of Education and Human Services ($50). Tuition Award, Fall 2016.

Help Keep the Music. ($185). Donors Choose. Donation included private donations and matched funds from George S. and Dolores Doré Eccles Foundation and Wells Fargo Utah. December 21, 2010. The purpose of this grant was to provide music scores and personal recorder musical instruments for 24 fifth grade student in a low-SES, title one elementary school.

Recorder Music Program. ($304). Donors Choose. Donation included private donations, funds from George S. and Dolores Doré Eccles Foundation, and Wells Fargo Utah. January 4, 2010. The purpose of this grant was to provide advanced classroom recorders and personal recorder musical instruments for 30 fifth grade student in a low-SES, title one elementary school.

Recorder Music Program. ($254). Donors Choose. Donation included private donations, George S. and Dolores Doré Eccles Foundation, and Wells Fargo Utah. December 30, 2008. The purpose of this grant was to provide music stands and personal recorder musical instruments for 28 fifth grade student in a low-SES, title one elementary school.

GRANTS SUBMITTED

(*Not Funded*)

Principal Investigator: Patricia Moyer-Packenham ($1.4 million). *The GAME Project: Exploring Digital Games for Mathematics Learning.* (2017). U.S. Department of Education, Institute of Education Sciences (US DOE IES). Project Goal: investigate game-based math apps and their relation to student math learning outcomes for students in Grades 3-5. My role: conducted literature reviews for the proposal, developed figures for the narrative and appendices.

PRESENTATIONS

International and National Presentations

Litster, K., Moyer-Packenham, P.S., Ashby, M. J., & Bullock, E. P. (April 2019). Attitude, App Use, and Affordances: Mediators of Learning from Digital Math Games. *American Educational Research Association (AERA) Annual Meeting, Toronto, Canada.*

Litster, K., Moyer-Packenham, P. S., Ashby, M. J., Bullock, E. P. & Clarke-Midura, J. E. (April 2019). Relationship between Children's Affect, Mathematical Connections, Strategies and Learning with Digital Math Games. *American Educational Research Association (AERA) Annual Meeting, Toronto, Canada.,*

Moyer-Packenham, P. S., Ashby, M. J., **Litster, K.** Bullock, E. P., Shumway, J. F., & Clarke-Midura, J. E. (April, 2019). Design Features that Promote Children's Awareness of the Affordances in Digital Math Games. *American Educational Research Association (AERA) Annual Meeting, Toronto, Canada.*

Litster, K., Moyer-Packenham, P.S., Ashby, M. J., & Roxburgh, A. (Anticipated, April 2019). Digital Math Games: Affect, Vocabulary, and Strategy Influences on Learning. *NCTM Annual Research Conference,* San Diego, California.

MacDonald, B. & **Litster, K.** (Anticipated, April 2019). Benchmark Computation Strategies: The Importance of Fives, Tens, & Doubles. *NCTM Annual Research Conference,* San Diego, California.

Litster, K., Moyer-Packenham, P.S., Ashby, M. J., Roxburgh, A., & Kozlowski, J. (March 2019). Digital Math Games: Importance of Strategy and Perseverance on Elementary Children's Learning Opportunities. *30th annual conference of the Society for Information Technology and Teacher Education (SITE), Las Vegas, Nevada.*

Litster, K., & Moyer-Packenham, P.S. (March 2019). How the Balance of Gaming and Mathematics Elements Effects Student Learning in Digital Math Games. *30th annual conference of the Society for Information Technology and Teacher Education (SITE), Las Vegas, Nevada.*

Moyer-Packenham, **Litster, K.**, P.S., Roxburgh, A., Kozlowski, J. & Ashby, M. J., (March 2019). Relationships between Mathematical Language, Representation Connections, and Learning Outcomes in Digital Games. *30th annual conference of the Society for Information Technology and Teacher Education (SITE), Las Vegas, Nevada.*

Moyer-Packenham, Ashby, M. J., **Litster, K.**, P.S., Roxburgh, A., & Kozlowski, J. (March 2019). How Design Features Promote Children's Awareness of Affordances in Digital Math Games. *30th annual conference of the Society for Information Technology and Teacher Education (SITE), Las Vegas, Nevada.*

Litster, K. & Moyer-Packenham, P. S. (2018, April). Elementary Mathematics Apps: Balancing Gaming and Mathematics Affordances for Student Learning. *Research Presentation at American Educational Research Association (AERA) Annual Meeting, New York City, New York.*

Moyer-Packenham, P. S., Lommatsch, C., **Litster, K.,** Ashby, M. J., Bullock, E. P.,

Shumway, J. F., & MacDonald, B. (2018, April). Affordances of Digital Games for Mathematics Learning in Grades 3-6. *Research Presentation at American Educational Research Association (AERA) Annual Meeting, New York City, New York.*

Lommatsch, C., Moyer-Packenham, P. S., & **Litster, K.** (2018, April). Differences in children's affordance awarenesss and access between novice and experienced learners. *Research Presentation at American Educational Research Association (AERA) Annual Meeting, New York City, New York.*

MacDonald, B.L., Ashby, M.J., Maahs-Fladung, C., **Litster, K.**, & Di Stefano, M. (2018, April). Relationships between low-achieving students' reversibility development and early mathematics achievement. *Research Presentation at American Educational Research Association (AERA) Annual Meeting, New York City, New York.*

Litster, K., Moyer-Packenham, P. S., & Reeder, R. (March, 2018). Affordances of Simultaneous Linking Features in a Base-10 Blocks Mathematics App for Young Children. *29th annual conference of the Society for Information Technology and Teacher Education (SITE), Washington D.C.*

Moyer-Packenham, P. S., **Litster, K.,** Lommatsch, C., Ashby, M. J., & Roxburgh, A. (March, 2018). Mediators of Learning in Game-Based Mathematics Apps. *29th annual conference of the Society for Information Technology and Teacher Education (SITE), Washington D.C.*

Moyer-Packenham, P. S., Lommatsch, C., **Litster, K.,** Ashby, M. J., & Roxburgh, A. (March, 2018). The Role of Design Features in the Affordances of Digital Math Games. *29th annual conference of the Society for Information Technology and Teacher Education (SITE), Washington D.C.*

DiStefano, M., **Litster, K.,** & MacDonald, B. L. (2017, August) The Interdependence of Language and Math: K-2 ELs Solving Inversion and Compensation Tasks. *1st Annual Build Math Minds Virtual Summit, International Webinar.*

Litster, K. & Watts, C. (2017, April). Virtual Cookies: Free Virtual Resources to Increase Participation, Discussion, and Collaboration. *NCTM Annual Meeting and Exposition,* San Antonio, TX.

Di Stefano, M., **Litster, K.**, & MacDonald, B.L. (2017, April) Language Effects in K-2 ESL Students Receiving Mathematics Intervention Support. *NCTM Annual Meeting and Exposition,* San Antonio, TX.

MacDonald, B.L., Ashby, M.J., & **Litster, K.** (2017, April). Early Elementary Algebraic Reasoning Development for Students Receiving Intervention Support. *NCTM Annual Meeting and Exposition,* San Antonio, TX.

MacDonald, B. L., **Litster, K.**, & Ashby, M. J. (2017, February). Measuring elementary preservice teachers' beliefs as related to their pedagogy, *21st Annual Association of Mathematics Teacher Educators (AMTE) Conference*, Orlando, FL.

MacDonald, B., Ashby, J., & **Litster, K.** (2016, November). Preliminary Findings of First Grade Students' Development of Reversibility. Poster Session. *PMENA-38,* Tucson, Arizona.

State & Regional Presentations

Litster, K., and Ashby, M. J. (October, 2018). Identifying and building depth of knowledge in mathematics standards and objectives, *Utah Council of Teachers of Mathematics (UCTM)*, Draper, Utah.

Litster, K. and Sawyer, L. (October, 2018). Students' early number strategies to guide educators' instruction, *Utah Council of Teachers of Mathematics (UCTM)*, Draper, Utah.

MacDonald, B. L., Maahs-Fladung, C., **Litster, K.**, & Ashby, M. J. (2017, March). Measuring elementary preservice teachers' beliefs as related to their pedagogy, *The 17th Annual UAMTE Conference*, Provo. Utah.

Litster, K. (2016, March). Bridging the gap between beginner and expert in training. Poster Session. *8th Annual SOTE Conference on Scholarship of Teaching and Engagement,* Orem, Utah.

Litster, K. & Watts, C. (2016, March). Virtual cookies do not taste the same as physical ones. Poster Session. *8th Annual SOTE Conference on Scholarship of Teaching and Engagement,* Orem, Utah.

Local Presentations

Litster, K., MacDonald, B. L., & Roxburgh, A. (2018, August).Virtual Cookies: Online Digital Resources and Strategies to Enhance In-Class and Distance Learning Experiences and Promote an Active Learning Environment. *Together We Teach Conference,* Utah State University, Logan, Utah.

Litster, K. (2017, April). Preliminary findings on the role of app design on student success and learning. Oral Presentation, *SRS Student Research Symposium,* Utah State University, Logan, Utah.

Litster, K. (2017, April). Preliminary findings on the role of app design on student success and learning. Poster Presentation, *SRS Student Research Symposium,* Utah State University, Logan, Utah.

LEADERSHIP & SERVICE

AERA SIG-RME Graduate Vice President (2017-2019) . Participate in board meetings to support development of AERA Annual Meeting Program. Plan and Facilitate graduate programs for AERA annual meeting and SIG-RME business meetings. Select incoming graduate rep. Design and disseminate announcements for upcoming programs and networking opportunities.

Hillcrest Elementary Science Fair Chair, Logan School District, Logan, Utah (2017-2019) Organize permission slips, student project packets, after school program, judging forms, advertising, science fair layout, and awards. Design and distribute advertising flyers, banners, and displays. Run before/after school program to help students prepare their projects. Solicit judges for over 100 individual and group projects and compile judging score cards. Present awards.

Hillcrest Elementary Professional Development, Logan School District, Logan, Utah (2018-2019). Provide professional development on cognitive demand. Evaluating tasks and standards, design tasks, assessments, and goals at different levels of Webb's DOK. Promoting critical thinking and reasoning when problem solving.

Debate Judge, University of Utah, Salt Lake City, Utah (2018). John R. Park Debate Society High School Beehive Bonanza Tournament judge. Judge teams of high school debate students on current issues and literature; Judging categories: dramatic interpretation, Original Oratory, & Extraneous.

Guest Lecturer, Utah State University, Logan, Utah (2018). GRA Professional Development Seminar (for Department of Mathematics and Statistics) (September, 2018). *Online Digital Resources and Strategies to Enhance In-Class and Distance Learning Experiences and Promote and Active Learning Environment.*

Presider, Society for Information Technology & Teacher Education International Conference, Washington D.C. (March, 2018). Facilitate presentations at the SITE conference. Introduce speakers and monitor time to facilitate presentations, questions, and transitions from one speaker to the next.

Guest Lecturer, Utah State University, Logan, Utah (2018). MATH 2010 Algebraic Thinking & Number Sense for Elementary Education School Teachers (for Jean Culbertson) (February, 2018). *Properties of Addition and Subtraction across Multiple Base Systems*

Student Council Vice President of Graduate Studies, Emma Eccles Jones College of Education and Human Services, Utah State University (2016-2017). Liaison between the graduate and undergraduate students within the college. Work with the council in outreach and service activities designed to support students and faculty in research and career development.

Graduate Student Senate Chair, Emma Eccles Jones- College of Education and Human Services (2016-2017). Liaison between the graduate students in the College of Education and the graduate student senator for the university. Work with the council to identify and address graduate student needs.

Reviewer (2016-Present)
Review articles and books for Teaching Children Mathematics.
Review articles for Education Sciences
Review proposals for the 2016 PMENA-38 Conference.

Mathematics Classroom Aid (2016-Present)
Work with local elementary teachers to improve mathematics teaching practices. Work one-on-one and with small groups of students to reduce misconceptions and increase conceptual understanding of mathematics topics.

Reading Tutor, Americorp, Logan, Utah (2016-Present). Work with elementary students to increase reading confidence, vocabulary, and fluency.

Judge, Utah Odyssey of the Mind Tournament, *Park City, Utah* (2016), *Ogden, Utah* (2017). Judge teams of kindergarten through college students from around the state of Utah on problem solving to choose teams who will continue to the world competition. Teams were judged on critical thinking, creativity, and teamwork in solving open ended problems.

Invited Presenter, Utah State University, Logan, Utah (2016). ELED 4060 Teaching Mathematics and Practicum Level III (For Dr. Jessica Shumway) (May, 2016) *Bridging the Gap Between Beginner and Expert in Training.*

Judge, Nebo School District, Science Fair, Spanish Fork, Utah (2012-2015). Judged elementary students' performance in designing, executing, and documenting a science project to choose students who would continue to the regional competition. Students were evaluated on their adherence to the Scientific method, originality, and understanding of applications for their projects.

PROFESSIONAL AFFILIATIONS

Association of Mathematics Teacher Educators (Since 2016)
National Council of Teachers of Mathematics (Since 2015)
American Educational Research Association (Since 2015)
Nebo Education Association (2007-2015)
Utah Education Association (2003-2015)