


Observation of an Excited B_c^+ State

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Using pp collision data corresponding to an integrated luminosity of 8.5 fb^{-1} recorded by the LHCb experiment at center-of-mass energies of $\sqrt{s} = 7, 8,$ and 13 TeV , the observation of an excited B_c^+ state in the $B_c^+ \pi^+ \pi^-$ invariant-mass spectrum is reported. The observed peak has a mass of $6841.2 \pm 0.6(\text{stat}) \pm 0.1(\text{syst}) \pm 0.8(B_c^+) \text{ MeV}/c^2$, where the last uncertainty is due to the limited knowledge of the B_c^+ mass. It is consistent with expectations of the $B_c^*(2^3S_1)^+$ state reconstructed without the low-energy photon from the $B_c^*(1^3S_1)^+ \rightarrow B_c^+ \gamma$ decay following $B_c^*(2^3S_1)^+ \rightarrow B_c^*(1^3S_1)^+ \pi^+ \pi^-$. A second state is seen with a global (local) statistical significance of 2.2σ (3.2σ) and a mass of $6872.1 \pm 1.3(\text{stat}) \pm 0.1(\text{syst}) \pm 0.8(B_c^+) \text{ MeV}/c^2$, and is consistent with the $B_c(2^1S_0)^+$ state. These mass measurements are the most precise to date.

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The B_c meson family is unique in the standard model as its states are formed from two heavy quarks of different flavors. The spectrum of masses of B_c mesons can reveal information on heavy-quark dynamics and improve the understanding of the strong interaction. Specifically, it provides tests of nonrelativistic quark-potential models [1–13], which have been successfully applied to quarkonium, since the B_c family shares properties with both the charmonium and bottomonium systems. The B_c family is predicted to have a rich spectroscopy by various potential models [1–13] and lattice quantum chromodynamics [12]. However, the B_c mesons are much less explored compared to quarkonia due to the small production rate, since their predominant production mechanism requires the production of both $c\bar{c}$ and $b\bar{b}$ pairs. The ground state meson B_c^+ was first observed by the CDF experiment [14] at the Tevatron collider. Knowledge of the properties of the B_c^+ meson has been greatly advanced by the LHCb experiment with the measurement of the B_c^+ mass, lifetime, and production rate [15–20], and the discovery and precise measurement of the branching fractions of several new decay channels [16,21–30]. Charge conjugation is implied throughout this Letter.

Excited B_c^+ states that lie below the threshold for decay into a beauty and charm meson pair are expected to have decay widths smaller than a few hundred keV [3,4]. Depending on its mass, an excited B_c^+ resonance may

undergo either cascade radiative or pionic decays to the B_c^+ state, which decays weakly. The second S -wave B_c state occurs as either a pseudoscalar (0^-) or a vector (1^-) spin state, i.e., the singlet $B_c(2^1S_0)^+$ or the triplet $B_c^*(2^3S_1)^+$. The $B_c(2^1S_0)^+$ and $B_c^*(2^3S_1)^+$ states are denoted as $B_c(2S)^+$ and $B_c^*(2S)^+$, respectively. The $B_c(2S)^+$ state decays directly to $B_c^+ \pi^+ \pi^-$, while the $B_c^*(2S)^+$ state decays to $B_c^*(1^3S_1)^+ \pi^+ \pi^-$, followed by the $B_c^*(1^3S_1)^+ \rightarrow B_c^+ \gamma$ electromagnetic transition. The low-energy photon produced in this decay is not considered in this analysis, since the reconstruction efficiency for such photons is too low to be useful with the current data sample. The $B_c^*(1^3S_1)^+$ state is denoted as B_c^{*+} hereafter. The transitions among the $B_c^{(*)}(2S)^+$ and $B_c^{(*)+}$ states are illustrated in Fig. 1. Decays of both $B_c^{(*)}(2S)^+$ states produce a narrow peak in the $B_c^+ \pi^+ \pi^-$ invariant-mass spectrum [31,32]; however, the $B_c^*(2S)^+$ state peaks at $M(B_c^{(*)}(2S)^+)_{\text{rec}} = M(B_c^{(*)}(2S)^+) - \Delta M(B_c^{*+})$ due to the missing photon, where $\Delta M(B_c^{*+})$ is the mass difference between the intermediate state B_c^{*+} and

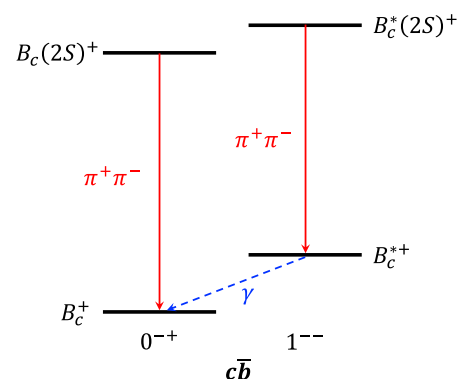


FIG. 1. Transitions among the $B_c^{(*)}(2S)^+$ and $B_c^{(*)+}$ states.

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the B_c^+ meson. Since the B_c^{*+} state has not been observed yet, the quantity $\Delta M(B_c^{*+})$ is unknown and the value of $M(B_c^*(2S)^+)$ cannot be determined with this technique at the moment. Taking into account the unreconstructed photon, the mass difference between the two peaks in the $B_c^+\pi^+\pi^-$ mass distribution originating from the two $B_c^{(*)}(2S)^+$ states, $M(B_c(2S)^+) - M(B_c^*(2S)^+)_{\text{rec}}$, is predicted to be in the range 11 to 53 MeV/ c^2 [1–13]. The production cross section of the $B_c^*(2S)^+$ state is predicted to be twice as large as that of the $B_c(2S)^+$ state [8,31,33,34], while the branching fractions of the decays $B_c(2S)^+ \rightarrow B_c^+\pi^+\pi^-$ and $B_c^*(2S)^+ \rightarrow B_c^+\pi^+\pi^-$ are expected to be similar [8,34].

With the large samples of B_c^+ mesons produced at the Large Hadron Collider, the ATLAS Collaboration first reported the observation of a signal in the $B_c^+\pi^+\pi^-$ mass distribution peaking at a value of $6842 \pm 4(\text{stat}) \pm 5(\text{syst})$ MeV/ c^2 using pp collision data at $\sqrt{s} = 7$ and 8 TeV corresponding to a luminosity of 24 fb $^{-1}$ [35]. Because of large mass resolution and low signal yield, no determination could be made as to whether the observed peak was either the $B_c(2S)^+$, the $B_c^*(2S)^+$ state, or a combination of the two states. The LHCb experiment also performed a search for excited B_c^+ states in the $B_c^+\pi^+\pi^-$ mass distribution using pp collision data at center-of-mass energy of $\sqrt{s} = 8$ TeV, corresponding to an integrated luminosity of 2 fb $^{-1}$. No evidence of any signal was found [36]. Recently, the CMS Collaboration reported the observation of the $B_c(2S)^+$ and $B_c^*(2S)^+$ states [37], in which the mass of the $B_c(2S)^+$ state and the mass difference between the two peaks were measured to be $6871.0 \pm 1.2(\text{stat}) \pm 0.8(\text{syst}) \pm 0.8(B_c^+)$ and $29.0 \pm 1.5(\text{stat}) \pm 0.7(\text{syst})$ MeV/ c^2 , respectively. The third uncertainty is due to the limited knowledge of the B_c^+ mass.

This Letter presents an updated search for excited B_c mesons in the $B_c^+\pi^+\pi^-$ mass distribution. The analysis makes use of run 1 and run 2 data collected by the LHCb experiment from 2011 to 2018 at center-of-mass energies of $\sqrt{s} = 7, 8,$ and 13 TeV, corresponding to integrated luminosities of about 1.0, 2.0, and 5.5 fb $^{-1}$, respectively.

The LHCb detector [38,39] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing b and/or c quarks. The detector elements that are particularly relevant to this analysis are a silicon-strip vertex detector surrounding the pp interaction region that allows c and b hadrons to be identified from their characteristically long flight distance, a tracking system that provides a measurement of the momentum p of charged particles with a relative uncertainty that varies from 0.5% at low momentum to 1.0% at 200 GeV/ c , and two ring-imaging Cherenkov detectors that are able to discriminate between different species of charged hadrons. The minimum distance of a track to a primary vertex (PV), the impact parameter (IP), is

measured with a resolution of $(15 + 29/p_T)$ μm , where p_T is the component of the momentum transverse to the beam, in GeV/ c . The online event selection is performed by a trigger, which consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction. At the hardware stage, events are required to have at least one muon with high transverse momentum p_T or a hadron with high transverse energy. At the software stage, two muon tracks or three charged tracks are required to have high p_T and to form a secondary vertex with a significant displacement from the interaction point. The momentum scale in data is calibrated using the J/ψ and B^+ mesons [40] with well-known masses.

Simulated samples are used to model the signal behavior. In the simulation, pp collisions are generated using PYTHIA 6 [41] with a specific LHCb configuration [42]. The generator BCVEGPy [33] is used to simulate the production of B_c^+ mesons. Decays of unstable particles are described by EVTGEN [43], in which final-state radiation is generated using PHOTOS [44]. The interaction of the generated particles with the detector, and its response, are implemented using the GEANT4 toolkit [45] as described in Ref. [46].

To form the $B_c^{(*)}(2S)^+$ candidates, first the intermediate B_c^+ state is reconstructed from the $B_c^+ \rightarrow J/\psi\pi^+$ decay. The J/ψ candidates are reconstructed with a pair of oppositely charged particles identified as muons. The muons are required to have $p_T > 550$ MeV/ c and good track-fit quality. They are required to form a common decay vertex with an invariant mass in the range [3040, 3140] MeV/ c^2 , corresponding to approximately 6 times the J/ψ mass resolution. The J/ψ candidate is combined with a charged pion to form the B_c^+ candidate. Each particle is associated with the PV that has the smallest value of χ_{IP}^2 , where χ_{IP}^2 is defined as the difference in the vertex fit χ^2 of a given PV reconstructed with and without the particle under consideration. The pion must have $p_T > 1000$ MeV/ c , good track-fit quality, and be inconsistent with originating from any PV. The B_c^+ candidate is required to have a good-quality vertex, a trajectory consistent with coming from its associated PV, and a decay time larger than 0.2 ps.

To further suppress background, a boosted decision tree (BDT) [47,48] classifier is used, as done in the B_c^+ production measurement [20]. The input variables of the BDT classifier are taken to be the p_T of each muon, the J/ψ meson and the charged pion, the decay length, decay time and vertex fit χ^2 of the B_c^+ meson, and the χ_{IP}^2 of the muons, the pion, the J/ψ meson, and the B_c^+ meson with respect to the associated PV. The BDT classifier is trained using signal candidates from simulation and background candidates from the upper sideband of the $J/\psi\pi^+$ mass distribution in data, corresponding to the range [6370, 6600] MeV/ c^2 . The BDT threshold is chosen to maximize $S/\sqrt{S+B}$, where S and B are the expected yields of signal and background in the range $M(J/\psi\pi^+) \in [6251, 6301]$ MeV/ c^2 , respectively.

This mass window corresponds to around 4 times the resolution of $M(J/\psi\pi^+)$. To improve the signal-to-background ratio in the $B_c^{(*)}(2S)^+$ search, the transverse momentum of the B_c^+ meson is required to be larger than $10 \text{ GeV}/c$.

An unbinned maximum-likelihood fit is performed to the $M(J/\psi\pi^+)$ distribution. To improve the mass resolution, the mass $M(J/\psi\pi^+)$ is calculated by constraining the J/ψ mass to its known value [49] and the B_c^+ meson to originate from the associated PV [50]. The signal component is described by a Gaussian function with asymmetric power-law tails [51]. The parameters of the tails are determined from the simulation, while the mean and width of the Gaussian function are left free in the fit. The combinatorial background is modeled with an exponential function. The contamination from the Cabibbo-suppressed decay $B_c^+ \rightarrow J/\psi K^+$, with the kaon misidentified as a pion, is modeled by a Gaussian function with asymmetric power-law tails. The parameters of this Gaussian function are fixed according to the simulation, except that the mean is constrained relative to that of the $B_c^+ \rightarrow J/\psi\pi^+$ signal. The invariant-mass distribution of the $J/\psi\pi^+$ candidates is shown in Fig. 2. The B_c^+ signal yield is 3785 ± 73 . The fitted B_c^+ mass and mass resolution are 6273.7 ± 0.3 and $15.1 \pm 0.3 \text{ MeV}/c^2$, respectively.

To reconstruct the $B_c^{(*)}(2S)^+$ candidates, B_c^+ candidates with $M(J/\psi\pi^+) \in [6200, 6320] \text{ MeV}/c^2$ are combined with a pair of oppositely charged particles identified as pions. These pion candidates are required to originate from the PV, and each have $p_T > 300 \text{ MeV}/c$, $p > 1500 \text{ MeV}/c$, and a good track-fit quality. The $B_c^{(*)}(2S)^+$ candidate is required to have a good vertex-fit quality. To improve the mass resolution, a fit [50] is performed in which the J/ψ and B_c^+ masses are constrained to their known values [49] and the daughters of the $B_c^{(*)}(2S)^+$ meson are required to point to the associated PV. The χ^2 per number of degrees of freedom of this fit must be smaller than 9. The value of $M(B_c^+\pi^+\pi^-) - M(B_c^+) - M(\pi^+\pi^-)$ is required to be smaller

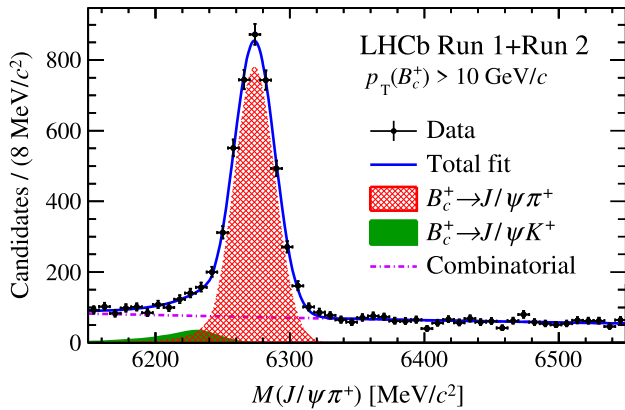


FIG. 2. Invariant-mass distribution of the selected B_c^+ candidates. The fit results are overlaid.

than $200 \text{ MeV}/c^2$. To ensure that the selection does not produce any artificial peaks in the $M(B_c^+\pi^+\pi^-)$ spectrum, the same requirements are applied to a same-sign sample, constructed from $B_c^+\pi^+\pi^+$ or $B_c^+\pi^-\pi^-$ combinations. The efficiency of the selections is found to change smoothly with the invariant mass $M(B_c^+\pi\pi)$ and no peaks are seen in the same-sign sample.

The $M(B_c^+\pi^+\pi^-)$ distribution in the data sample after all the selections are applied is shown in Fig. 3, with those of the same-sign sample and a sample drawn from the B_c^+ sidebands ($M(J/\psi\pi^+) \in [6150, 6200] \cup [6320, 6550] \text{ MeV}/c^2$) superimposed for comparison. The same-sign and B_c^+ mass sideband distributions are scaled to the opposite-sign distribution in the sideband region, $M(B_c^+\pi^+\pi^-) \in [6735, 6825] \cup [6895, 6975] \text{ MeV}/c^2$. The $M(B_c^+\pi^+\pi^-)$ distribution presents an obvious peak at approximately $6840 \text{ MeV}/c^2$, and a less significant structure at about $6870 \text{ MeV}/c^2$.

The masses and yields of the $B_c^{(*)}(2S)^+$ peaks are determined using an unbinned maximum-likelihood fit to the distribution of the mass difference $\Delta M \equiv M(B_c^+\pi^+\pi^-) - M(B_c^+)$ to eliminate the dependence on the reconstructed B_c^+ mass. Here the mass $M(B_c^+\pi^+\pi^-)$ is calculated with no constraint on the B_c^+ mass, but only constraining the J/ψ mass to its known value [49] and requiring the $B_c^{(*)}(2S)^+$ meson to come from the associated PV [50]. Each $B_c^{(*)}(2S)^+$ peak is modeled by a Gaussian function with asymmetric power-law tails [51]. The tail parameters are fixed to the values determined from simulation, while the Gaussian mean and width are treated as free parameters. The combinatorial background is described by a second-order polynomial function.

The fit to the ΔM distribution is shown in Fig. 4, and the results are summarized in Table I. The $B_c^{(*)}(2S)^+$ signal yield is determined to be $51 \pm 10(\text{stat})$, corresponding to a local statistical significance of 6.8σ . The significance is evaluated with a likelihood-based test, in which the

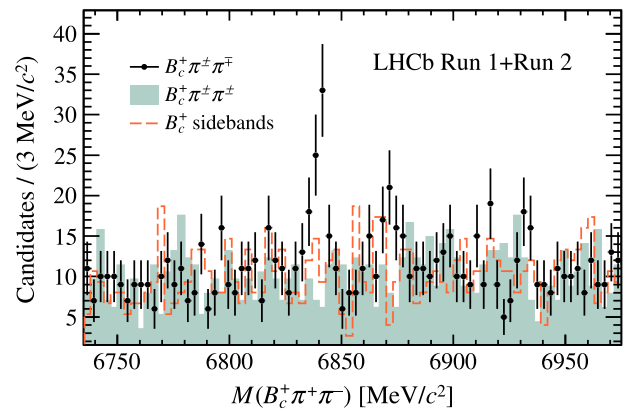


FIG. 3. Invariant-mass $M(B_c^+\pi^+\pi^-)$ distributions for the data and same-sign samples with the distribution of the B_c^+ mass sidebands overlaid.

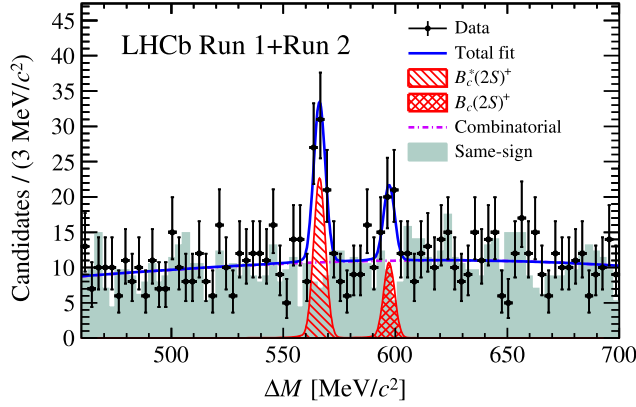


FIG. 4. Distribution of $\Delta M = M(B_c^+ \pi^+ \pi^-) - M(B_c^+)$ with the fit results overlaid. The same-sign distribution has been normalized to the data in the $B_c^{(*)}(2S)^+$ sideband region.

likelihood distribution of the background-only hypothesis is obtained using pseudoexperiments [52]. The yield of the $B_c(2S)^+$ state is $24 \pm 9(\text{stat})$ with a local statistical significance of 3.2σ . The Gaussian widths of the two peaks are consistent with the expectation of negligible resonance widths. The mass difference between the two peaks is measured to be $31.1 \pm 1.4(\text{stat}) \text{ MeV}/c^2$. Taking the known B_c^+ mass, $M(B_c^+) = 6274.9 \pm 0.8 \text{ MeV}/c^2$ [53], the quantities $M(B_c^{*}(2S)^+)_{\text{rec}}$ and $M(B_c(2S)^+)$ are determined to be $6841.1 \pm 0.6(\text{stat}) \pm 0.8(B_c^+) \text{ MeV}/c^2$ and $6872.1 \pm 1.3(\text{stat}) \pm 0.8(B_c^+) \text{ MeV}/c^2$, respectively. The second uncertainty is due to the limited knowledge of the B_c^+ mass. After considering the look-elsewhere effect in the predicted mass regions [54], $M(B_c^+ \pi^+ \pi^-) \in [6790, 6895]$ for the $B_c^{*}(2S)^+$ state, and $M(B_c^+ \pi^+ \pi^-) \in [6845, 6895] \text{ MeV}/c^2$ for the $B_c(2S)^+$ state [1–10,55], the global statistical significances of the two states are determined to be 6.3σ and 2.2σ , respectively.

Several sources of systematic uncertainty on the determination of the mass difference ΔM are studied. The dominant contribution is from the uncertainty on the momentum scale, which is due to imperfections in the description of the magnetic field and the imperfect alignment of the subdetectors. The uncertainty of the momentum calibration is estimated using other particles, such as K_S^0 and Υ mesons, and leads to an uncertainty of $0.12 \text{ MeV}/c^2$

TABLE I. Results of the fit to the ΔM distribution. Uncertainties are statistical only.

	$B_c^{*}(2S)^+$	$B_c(2S)^+$
Signal yield	51 ± 10	24 ± 9
Peak ΔM value (MeV/c^2)	566.2 ± 0.6	597.2 ± 1.3
Resolution (MeV/c^2)	2.6 ± 0.5	2.5 ± 1.0
Local significance	6.8σ	3.2σ
Global significance	6.3σ	2.2σ

on the ΔM measurements. The unreconstructed photon emitted in the $B_c^{*}(2S)^+$ decay chain could be an additional source of systematic uncertainty. Studies on simulated events show that the missing photon introduces a small bias, and a correction of $+0.08 \text{ MeV}/c^2$, with negligible uncertainty, is applied to the fitted value of the $B_c^{*}(2S)^+$ mass peak. All other systematic uncertainties are negligible and are briefly described as follows. The effects of the imperfect modeling of the signal and background components are estimated by using alternative models. The alternative model for the signal peaks uses Hypatia functions [56], while for the background the alternative model consists of a sum of two threshold functions, each of the form $(\Delta M - m_t)^p \times e^{-C(\Delta M - m_t)}$, where p and C are free parameters, and m_t represents the threshold, which is taken to be $2m_{\pi^\pm}$. The changes in ΔM obtained with the alternative models are found to be negligible. The effect of final-state radiation is also studied with simulated events and the associated uncertainty on the fitted mass values is found to be negligible. The total systematic uncertainty on ΔM for both the $B_c(2S)^+$ and $B_c^{*}(2S)^+$ states of $0.12 \text{ MeV}/c^2$ is fully correlated, and therefore cancels in the mass difference of the two peaks.

In conclusion, using pp collision data collected by the LHCb experiment at center-of-mass energies of $\sqrt{s} = 7, 8,$ and 13 TeV , corresponding to an integrated luminosity of 8.5 fb^{-1} , a peaking structure consistent with the $B_c^{*}(2S)^+$ state is observed in the $B_c^+ \pi^+ \pi^-$ mass spectrum with a global (local) statistical significance of 6.3σ (6.8σ). The mass associated with the $B_c^{*}(2S)^+$ state, for which the low-energy photon in the intermediate decay $B_c^{*+} \rightarrow B_c^+ \gamma$ is not reconstructed, is measured to be

$$6841.2 \pm 0.6(\text{stat}) \pm 0.1(\text{syst}) \pm 0.8(B_c^+) \text{ MeV}/c^2, \quad (1)$$

where the last uncertainty is due to the limited knowledge of the B_c^+ mass. It is equal to $M(B_c^{*}(2S)^+)_{\text{rec}} = M(B_c^{*}(2S)^+) - (M(B_c^{*+}) - M(B_c^+))$. The mass difference between the $B_c^{*}(2S)^+$ and B_c^{*+} state is determined to be $566.3 \pm 0.6(\text{stat}) \pm 0.1(\text{syst}) \text{ MeV}/c^2$. The data also show a hint for a second structure consistent with the $B_c(2S)^+$ state with a global (local) statistical significance of 2.2σ (3.2σ). Assuming this peak is due to the $B_c(2S)^+$ state, its mass is measured to be

$$6872.1 \pm 1.3(\text{stat}) \pm 0.1(\text{syst}) \pm 0.8(B_c^+) \text{ MeV}/c^2. \quad (2)$$

The mass difference between the $B_c(2S)^+$ and B_c^+ state is $597.2 \pm 1.3(\text{stat}) \pm 0.1(\text{syst}) \text{ MeV}/c^2$. The mass difference of the two $B_c^{(*)}(2S)^+$ peaks is determined to be

$$31.0 \pm 1.4(\text{stat}) \pm 0.0(\text{syst}) \text{ MeV}/c^2, \quad (3)$$

in which both the uncertainty from the B_c^+ mass and the systematic uncertainty cancel. The mass measurements

are the most precise to date, and are consistent with the results from the CMS Collaboration [37]. They are also within the range of the theoretical predictions [1–13].

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[1] S. S. Gershtein *et al.*, *Yad. Fiz.* **48**, 515 (1988) [*Sov. J. Nucl. Phys.* **48**, 327 (1988)].
 [2] Y.-Q. Chen and Y.-P. Kuang, *Phys. Rev. D* **46**, 1165 (1992); **47**, 350(E) (1993).
 [3] E. J. Eichten and C. Quigg, *Phys. Rev. D* **49**, 5845 (1994).
 [4] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. V. Tkabladze, *Phys. Rev. D* **51**, 3613 (1995).
 [5] S. N. Gupta and J. M. Johnson, *Phys. Rev. D* **53**, 312 (1996).
 [6] L. P. Fulcher, *Phys. Rev. D* **60**, 074006 (1999).
 [7] D. Ebert, R. N. Faustov, and V. O. Galkin, *Phys. Rev. D* **67**, 014027 (2003).
 [8] S. Godfrey, *Phys. Rev. D* **70**, 054017 (2004).
 [9] K.-W. Wei and X.-H. Guo, *Phys. Rev. D* **81**, 076005 (2010).
 [10] A. Abd El-Hady, J. R. Spence, and J. P. Vary, *Phys. Rev. D* **71**, 034006 (2005).
 [11] J. Zeng, J. W. Van Orden, and W. Roberts, *Phys. Rev. D* **52**, 5229 (1995).

[12] C. T. H. Davies, K. Hornbostel, G. P. Lepage, A. J. Lidsey, J. Shigemitsu, and J. Sloan, *Phys. Lett. B* **382**, 131 (1996).
 [13] R. J. Dowdall, C. T. H. Davies, T. C. Hammant, and R. R. Horgan, *Phys. Rev. D* **86**, 094510 (2012).
 [14] F. Abe *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **81** (1998) 2432; *Phys. Rev. D* **58**, 112004 (1998).
 [15] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **109**, 232001 (2012).
 [16] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **87**, 112012 (2013).
 [17] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **113**, 152003 (2014).
 [18] R. Aaij *et al.* (LHCb Collaboration), *Eur. Phys. J. C* **74**, 2839 (2014).
 [19] R. Aaij *et al.* (LHCb Collaboration), *Phys. Lett. B* **742**, 29 (2015).
 [20] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **114**, 132001 (2015).
 [21] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **108**, 251802 (2012).
 [22] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **87**, 071103(R) (2013).
 [23] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **09** (2013) 075.
 [24] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **111**, 181801 (2013).
 [25] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **11** (2013) 094.
 [26] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **05** (2014) 148.
 [27] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **92**, 072007 (2015).
 [28] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **09** (2016) 153.
 [29] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **95**, 032005 (2017).
 [30] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **118**, 111803 (2017).
 [31] Y.-N. Gao, J.-B. He, P. Robbe, M.-H. Schune, and Z.-W. Yang, *Chin. Phys. Lett.* **27**, 061302 (2010).
 [32] A. Berezhnoy and A. Likhoded, *Proc. Sci.*, QFTHEP2013 (2013) 051.
 [33] C.-H. Chang, X.-Y. Wang, and X.-G. Wu, *Comput. Phys. Commun.* **197**, 335 (2015).
 [34] I. P. Gouz *et al.*, *Yad. Fiz.* **67**, 1581 (2004) [*Phys. At. Nucl.* **67**, 1559 (2004)].
 [35] G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. Lett.* **113**, 212004 (2014).
 [36] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **01** (2018) 138.
 [37] A. M. Sirunyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **122**, 132001 (2019).
 [38] A. A. Alves, Jr. *et al.* (LHCb Collaboration), *J. Instrum.* **3**, S08005 (2008).
 [39] R. Aaij *et al.* (LHCb Collaboration), *Int. J. Mod. Phys. A* **30**, 1530022 (2015).
 [40] R. Aaij *et al.* (LHCb Collaboration), *Phys. Lett. B* **708**, 241 (2012).
 [41] T. Sjöstrand, S. Mrenna, and P. Skands, *J. High Energy Phys.* **05** (2006) 026.

- [42] I. Belyaev *et al.*, *J. Phys. Conf. Ser.* **331**, 032047 (2011).
 [43] D. J. Lange, *Nucl. Instrum. Methods Phys. Res., Sect. A* **462**, 152 (2001).
 [44] P. Golonka and Z. Was, *Eur. Phys. J. C* **45**, 97 (2006).
 [45] J. Allison *et al.* (Geant4 Collaboration), *IEEE Trans. Nucl. Sci.* **53**, 270 (2006).
 [46] M. Clemencic, G. Corti, S. Easo, C. R. Jones, S. Miglioranza, M. Pappagallo, and P. Robbe, *J. Phys. Conf. Ser.* **331**, 032023 (2011).
 [47] L. Breiman, J. H. Friedman, R. A. Olshen, and C. J. Stone, *Classification and regression trees*, *Wadsworth International Group, Belmont, California, USA, 1984*.
 [48] Y. Freund and R. E. Schapire, *J. Comput. Syst. Sci.* **55**, 119 (1997).
 [49] C. Patrignani *et al.* (Particle Data Group), *Chin. Phys. C* **40**, 100001 (2016).
 [50] W. D. Hulsbergen, *Nucl. Instrum. Methods Phys. Res., Sect. A* **552**, 566 (2005).
 [51] T. Skwarnicki, A study of the radiative cascade transitions between the Upsilon-prime and Upsilon resonances, Ph.D. thesis, Institute of Nuclear Physics, Krakow, 1986, DESY-F31-86-02, <http://inspirehep.net/record/230779/>.
 [52] G. Cowan, K. Cranmer, E. Gross, and O. Vitells, *Eur. Phys. J. C* **71**, 1554 (2011); **73**, 2501(E) (2013).
 [53] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018).
 [54] E. Gross and O. Vitells, *Eur. Phys. J. C* **70**, 525 (2010).
 [55] A. K. Rai and P. C. Vinodkumar, *Pramana* **66**, 953 (2006).
 [56] D. Martínez Santos and F. Dupertuis, *Nucl. Instrum. Methods Phys. Res., Sect. A* **764**, 150 (2014).

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