MODELLING AND CONTROL OF A TWO-LINK FLEXIBLE MANIPULATOR USING FINITE ELEMENT MODAL ANALYSIS

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Specially dedicated to:

My kind parents for their patience, support, and motivation.

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ABSTRACT

This thesis focuses on Finite Element (FE) modeling and robust control of a two-link flexible manipulator based on a high resolution FE model and the system vibration modes. A new FE model is derived using Euler-Bernoulli beam elements, and the model is validated using commercial software Abaqus CAE. The frequency and time domain analysis reveal that the response of the FE model substantially varies with changing the number of elements, unless a high number of elements (100 elements in this work) is used. The gap between the complexity of the high order FE model capable of predicting dynamics of the multibody system, and suitability of the model for controller design is bridged by designing control schemes based on the reduced order models obtained using modal truncation/H_∞ techniques. Two reduced order multi-input multi-output modal control algorithms composed of a robust feedback controller along with a feed-forward compensator are designed. The first controller, Inversion-based Two Mode Controller (ITMC), is designed using a mixed-sensitivity H_∞ synthesis and a modal inversion-based compensator. The second controller, Shaping Two-Mode Controller (STMC), is designed with H_∞ loop-shaping using the modal characteristics of the system. Stability robustness against unmodelled dynamics due to the model reduction is shown using the small gain theorem. Performance of the feedback controllers are compared with Linear Quadratic Gaussian designs and are shown to have better tracking characteristics. Effectiveness of the control schemes is shown by simulation of rest-to-rest maneuver of the manipulator to a set of desired points in the joint space. The ITMC is shown to have more precise tracking performance, while STMC has higher control over vibration of the tip, at the expense of more tracking errors.

ABSTRAK

Tesis ini memfokuskan kepada pemodelan unsur terhingga (FE) dan kawalan tegap untuk pengolah fleksibel dua lengan berdasarkan model FE dengan resolusi tinggi dan mod getaran sistem. Model FE baru dihasilkan menggunakan unsur rasuk Euler-Bernoulli dan model ini disahkan menggunakan perisian komersial Abaqus CAE. Analisa domain frekuensi dan masa menunjukkan sambutan model FE sangat bergantung kepada bilangan unsur, melainkan bilangan unsur yang sangat tinggi digunakan (100 unsur dalam penyelidikan ini). Jurang di antara kerumitan model FE tertib tinggi yang diperlukan untuk meramal tingkah laku dinamik sistem berbilang jasad dan kesesuaian model untuk rekabentuk pengawal dihubungkan melalui rekabentuk sistem kawalan berasaskan model tertib kurang menggunakan kaedah pemangkasan mod/H_∞. Dua algoritma pengawal berbilang-masukan berbilang-keluaran tertib kurang telah direkabentuk terdiri daripada pengawal suapbalik tegap dan pemampas suap-hadapan. Pengawal pertama, ITMC, telah direkabentuk menggunakan kaedah kepekaan-bercampur H_∞ dan pemampas mod songsangan. Manakala pengawal kedua, STMC direkabentuk dengan pembentuk-gelung H_∞. menggunakan ciri-ciri mod sistem. Ketegapan sistem terhadap ciri-ciri dinamik yang tidak dimodelkan disebabkan oleh peringkasan model ditunjukkan menggunakan teorem gandaan kecil. Prestasi pengawal ini dibandingkan dengan rekabentuk Gaussian Kuasadua Lelurus dan telah menunjukkan hasil penjejakan yang lebih baik. Keberkesanan sistem-sistem pengawal ditunjukkan melalui simulasi pergerakan pengolah ke beberapa lokasi yang dikehendaki. ITMC telah menunjukkan prestasi penjejakan yang lebih tepat manakala STMC mempunyai prestasi kawalan getaran yang lebih baik dengan ralat penjejakan besar.

CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Flexible manipulators are commonly known as a class of robotic arms that are designed with long and slender links in order to reduce their weight. As a more technical definition, 'flexible manipulator' commonly addresses a manipulator that its first structural natural frequencies are exited severely in its operating speeds. This can happen due to either high acceleration motions, or low stiffness of the structure of the robot. Examples include space manipulators (Sabatini *et al.,* 2012), such as the shuttle remote manipulator Canadarm (Skaar and Ruoff, 1994), and high-speed industrial manipulators. The structural flexibility is caused by elastic deflections of the links and/or joints. When the elastic deflections of the links of a manipulator are considered, the robot is known as a Flexible Link Manipulator (FLM). In the theory of elasticity, a flexible link is an infinite dimensional continuous system. For controller design and simulation, generally, a finite-dimensional (spatially discrete) model of such systems is required.

A widely used method for discretization of the governing equations of the FLMs is the Assumed Mode Method (AMM). In AMM (Book *et al.*, 1975; Yu and Elbestawi, 1995), vibrational behavior of each link is assumed to be similar to the first vibration mode(s) of the link as a separate beam under some assumed Boundary Conditions (BC). The problem with the AMM, in particular in the case of multi-link manipulators, is that the vibration modes of a beam are very sensitive to the changes in the BCs (See for example Ata *et al.* (2012)). Therefore, describing the BCs of a moving link by classic BCs such as clamped, free, or carrying a mass/inertia can be a source of error. An alternative method that can provide a finite dimension model of a flexible multibody system is the method of Finite Elements (FE). The FE analysis has been used for open-loop or closed-loop simulation of the FLMs by many researchers. Tokhi *et al.* (2001) presented FE modelling of a single link flexible manipulator (SLFM).

Beside the modelling complexities, concurrent vibration and motion control of FLMs has been an interesting and active area in vibration control (Shaheed and Tokhi, 2013; Kumar, 2013; Yin *et al*. 2013). Various uncertainties and complexities of the system have been targeted in various studies. One of the most considered uncertainties in a flexible manipulator is the payload variation as such manipulators are normally expected to handle different payloads in remote fields (Sąsiadek, 2013).

In this work, an FE model is developed using the Euler-Bernoulli beam element and lumped mass model with arbitrary number of elements. The governing equations of motion are derived using the energy equations. Then an analysis of the FE model with different number of elements is presented to find out the necessary, sufficient, or optimum required mesh resolution (number of elements). It will be shown that independency of the model to the number of elements will be achieved only when the number of elements is chosen sufficiently high (here around 50 elements for each link). The high-order FE model is then verified using numerical measurements and commercial FE software. In order to prepare the model for a model based control algorithm, the FE model is approximated to a low-order system by employing modal decomposition and model reduction. In this manner, the resultant dynamic equations preserve the precision of a finely meshed FE model in low frequencies of interest or bandwidth of the system, while the order of the system is not too big for control algorithms. Based on the reduced model, a Multi-Input Multi-Output (MIMO) feedback control is designed that is shown to be robust against the uncertainty of truncated high frequency modes. Finally, a multi-stage rest-to-rest control algorithm based on the feedback controller and a feed-forward controller is simulated on the high-order system which drives the manipulator to desired postures through a smooth trajectory.

1.2 Problem Statement

The demand for reliable lightweight and high-speed manipulators, for space or industrial applications, has attracted many researchers to develop dynamic models and control methods for manipulators with elastic behavior. A planar Two-Link Flexible Manipulator (TLFM) is the most fundamental and, practically, common case of multi-link flexible-link manipulators. Flexible-link manipulators are continuous (infinite dimensional) systems, which need to be approximated as finite dimensional models. The everlasting demand for reducing the weight or increasing the speed of manipulators is associated with increasing the flexibility of the links of the robots. To fulfill this demand, continuous research is necessary in order to develop more accurate models (*e.g.* with more modes), and to design more advanced control algorithms, for TLFMs.

Due to natural complexities of flexible multibody systems, the dynamics and control of a TLFM is yet an open problem. The modeling methods developed up to now, generally sacrifice a degree of accuracy to get a low-order model that is manageable for controller design. In particular, in the FE models a low number of elements have been used for discretization of flexible links. If each link is modeled with one element, the FE and AMM will be essentially equivalent; except for the shape function that is 'presumed' for describing bending of the links. When multiple elements are used, the shape function approximation rules only inside the elements and the bending curve (*i.e.* the system vibration modes) can be measured based on position of the nodes. However, with multiple elements FE results in high-order models. The literature on FLMs show that AMM has been more of interest, because the assumed modes selected based on the BCs will be better approximations than the shape functions used in a single element.

To keep the advantages of a high-resolution FE model in model-based control design, a compromise between the order and resolution of the model is necessary. By using a model order reduction, the flexible links can be modeled with high number of elements. Then, to fully use the advantages of such elaborate modelling technique, employing advanced MIMO robust control techniques, as well as classic methods of maneuver control of manipulators is indispensable.

1.3 Objectives of the Research

This research aims at developing a high-order FE model of TLFMs and designing model-based control for maneuvering the system. The main objectives are as follows:

- To develop a multi-element FE model of a TLFM, free of the assumptions of component modes analysis and the floating frame of references. Validation of the model is performed with the commercial FE software Abaqus CAE.

- To measure the system vibration modes of the TLFM, and implement modal decomposition in order to reduce order of the model.

- To design a MIMO feedback controller for rest-to-rest maneuvers of the manipulator in free joint space. The controller needs to be robust to unmodelled dynamics resulted from the model order reduction.

1.4 Scope of the Research

The scope of this research comprises a theoretical study of reduced-order modeling and control of a TLFM, as well as simulation studies using MATLAB, as the main platform. Abaqus CAE is used for validation of the eigenproblem. In this work, some idealizations are adopted to focus on effects of flexibility of the links (flexibility of the joints is not considered in this research). Modeling of a two-link manipulator with small elastic deflections using Euler-Bernoulli beam element is considered. The motion and vibration of the manipulator are in the horizontal plane. The joints are actuated by external torque and are without gear box, friction, and joint flexibility. Damping is considered by adding modal damping terms.

The controller is designed based on the reduced-order model, and closed-loop simulation is performed using the high-order model as the plant. The feedback controller is designed using some MIMO H_∞ minimizing methods. To ensure robustness, firstly, the normalized coprime stability margin is considered in the controller design stage, and secondly, the small-gain theorem is checked for the specific uncertainty that is the unmodelled dynamics or the truncated modes in the model order reduction.

The overall control system is to drive the system to a typical set of point and stabilize the manipulator at the destination. The bandwidth is considered to be 0 to 50 Hz (vibrations of higher frequencies are ignorable). Performance of the controller will be compared with a Linear Quadratic Gaussian (LQG) design. In each stage of the rest-to-rest motion, the angular motion of the joints are supposed to be small (say less than 1 rad). For large motions, the controller can be equipped with adaptation algorithms, which is out of the scope of this research.

1.5 Research Methodology

In this section, an overview of the research methodology is presented. To introduce the readers who may not be familiar with the subjects, an overall road map is given. Figure 1.1 illustrates the flowchart of the project methodology. The research was started with reviewing the literature to figure out the research direction. Then, dynamic equations of the system are derived using analytical and numerical methods, and are simulated using MATLAB. The dynamic modelling starts with physical modelling and discretization of the links using arbitrary number of beam elements (FE discretization). The Lagrange's equations are used, then, to derive the dynamic equations. For convergence analysis, the time and frequency responses of the FE model are measured for the FE model with different mesh sizes. In parallel to the measurements, a model of the TLFM is made in Abaqus CAE. The measured model is compared and validated to the results of the Abaqus CAE. The verified model is named high-order FE model as any order reduction (including decreasing number of elements) has not been applied.

In the next step, the mode shapes of the system are measured and a model order reduction procedure based on the measured modes is employed. The reduced models prepared with different orders are verified with the original FE model. The model based control designs, then, will be based on the reduced order systems.

The control design starts with feedback control design and synthesis. The control design and simulation is performed in MATLAB. After verification of the robustness of the feedback loop, the control system is augmented with feed-forward compensators. Finally, the controllers are evaluated in rest-to-rest maneuvers to arbitrary points. The evaluation will be in terms of parameterization of trajectory error and vibration of the tip (end-effector).

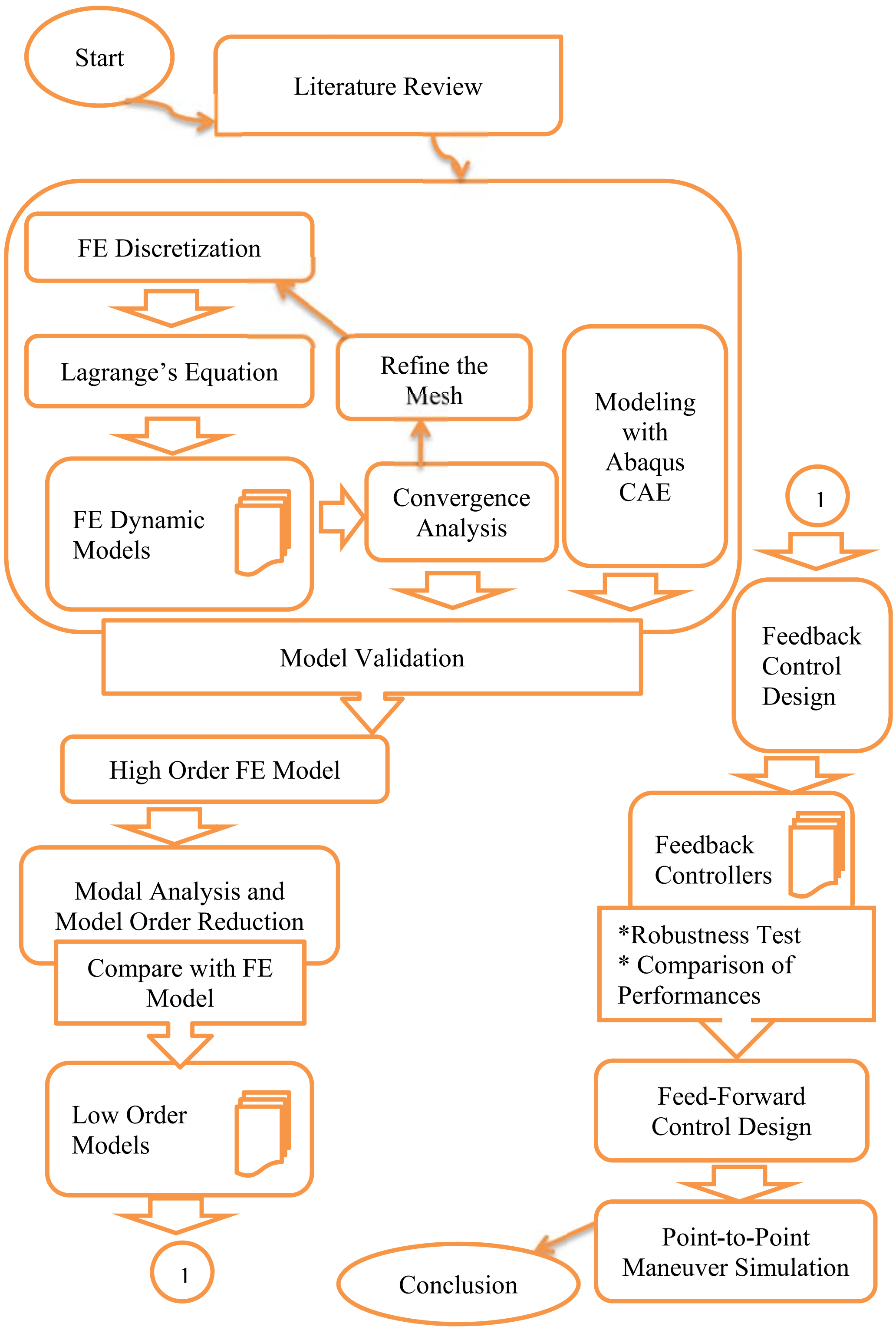

Figure 1.1 Flowchart of the research methodology

1.6 Thesis Contribution

A brief outline of the main contributions of this research is given in this section as follows:

1) An accurate FE model with a high resolution mesh of beam elements was developed for the TLFM. The number of required elements for discretization of the model was discussed based on convergence of frequency and time domain responses so that the FE model can predict the first modes monotonously when the number of elements is increased (considering system matrices of order lower than 200 to avoid measurement complexity of large scale matrices (Cullum, 2002))

2) The vibration modes of the system were measured. The modes were used for modal decomposition of the model, and measuring reduced-order model that perfectly matches with the high-order FE model in terms of the input-output characteristics.

3) Based on the modal decomposition, two modal control algorithms (named ITMC and STMC, in this work) were developed for multi-stage rest-to-rest maneuver of the manipulator. A method was proposed for reshaping the loop transfer matrix (loop-shaping control) for concurrent motion and vibration control of the flexible manipulator. Robustness of the feedback controlled system against unmodelled dynamics was shown using the small gain theorem. Performance of the controllers was evaluated by comparing time responses of the controlled system with the simulation results of some LQG control designs. The ITMC is suggested for the tasks requiring more precise tracking performance, and STMC for higher control on vibration at the expense of more tracking error.

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