# A Pragmatic Gaze on Stochastic Resonance Based Variability Tolerant Memristance Enhancement

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***Abstract*****—**Stochastic Resonance (SR) is a nonlinear system specific phenomenon, which was demonstrated to lead to system unexpected (counter-intuitive) performance improvements under certain noise conditions. Memristor, on the other hand, is a enhancement manifests as an increase of $R_{MAX}/R_{MIN}$ ratio, facilitated by noise presence, that provides higher discreteness of the ON/OFF states.
Thus, in this paper, memristor SR effects are explored, assuming various memristor models, and SR-based memristance range enhancement, tolerant to device-to- device variability, is demonstrated. Our experiments reveal that SR can induce significant $R_{MAX}/R_{MIN}$ ratio increase under up to 60% variability, getting as high as 3.4 for 29 dBm noise power.

## I. Introduction

The Memristor is a theoretically conceived two-terminal electrical element which total resistance varies according to the applied signal history, postulated in the seminal work of Leon Chua [1] that recently reached the spotlight of emergent nanoelectronic devices as a connection between an actual de- vice and Chua's theory [2]. After its hardware implementation, although many computing applications of memristor have been proposed [3], its most adequate current use is as memory element in Resistive Random Access Memories (ReRAM) [4]. Noteworthy, the mass production of non-volatile memristor memories has to tackle a number of problems towards the scaling of the memory capabilities. One of the most significant problems is the variability of memristor devices' charac- teristics, caused during the nano-fabrication processes, that produces inconsistencies in the *SET* and *RESET* process between the devices [5], [6].

Aiming to alleviate the negative effects of memristor vari- ability, device fabrication level [7] techniques as well as error correction methods [8] have been proposed. However, device variability immunity has been also enabled by variability tolerant methods and architectures [9]–[11]. Following this direction, in this work, inspired by two different approaches from the literature, a noise-related phenomenon, known as Stochastic Resonance (SR), is exploited to enhance of the memristance range ($R_{MAX}/R_{MIN}$ ) while using accurate memristor models, and its performance is demonstrated in the presence device-to-device variability. The memristance ratio, reaching 3.4 for up to 60% variability of switching rate and *SET* and, *RESET* and 29 *dBm* noise power.

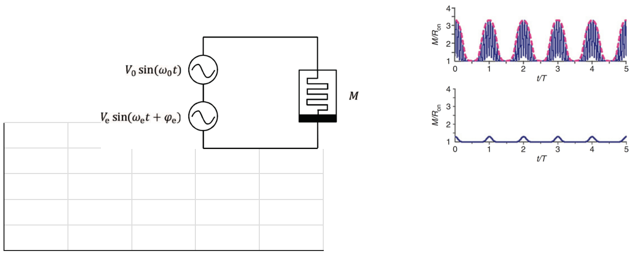
## II. SR-based Memristance Enhancement

Stochastic Resonance (SR) is a nonlinear system specific phenomenon that exploits noise to increase the performance of a nonlinear system for a specific range of noise power [12]– [14], extensively noticed in a wide range of systems, from biological systems to electronics [15]–[18].

Having in mind that memristor is naturally a nonlinear element, SR can effectively affect it. Thus, SR as a way to improve the performance of memristive systems has been proposed in previous works with two different approaches. The first one utilises a full spectrum noisy signal [19], known as White or Gaussian noise, while the second, in a more circuit- level effective way, introduces single-frequency noisy signal [20]. Either approach presents advantages and disadvantages, as the white noise approach proposes a more power-efficient method but less efficient to control the power of noise in circuit-level, whilst the other provides an easy way for circuit designers but the signal-to-noise ratio (SNR) of the input signal is not sufficiently high (<1) for actual power-efficient applications.

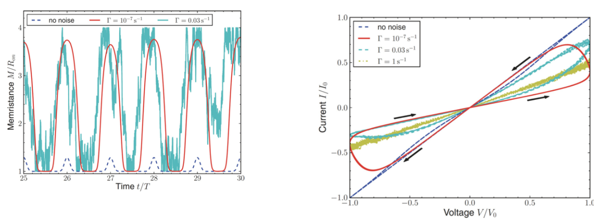
### A. Single-Frequency Noise

Starting from the single-frequency approach, Tanaka *et al.* [20] proposed a circuit configuration of two in-series sinu- soidal voltage sources, one for the driving signal ($V_0 sin(\omega_0 t)$) and one for the high-frequency signal of noise ($V_e sin(\omega_e t + \varphi_e) = pV_0 sin(q\omega_0 t + \varphi_e)$) (Fig. 1(a)). In this approach, the driving signal is characterised by a low-amplitude and low-frequency sinusoidal voltage, while noise signal is more than 10 times larger in both amplitude and frequency. Fig. 1(b) presents time evolution of memristance, normalised by mini- mum value ($M/R_{ON}$), when only the driving signal is applied on the memristor (bottom), and when the combination of driving and noise signals is applied (top). The latter exhibits a wider range of memristance values always controlled by the driving signal's phase. The optimal conditions for the




(a) (b)

Fig. 1. (a) Tanaka's *et al.* [20] proposed circuit configuration and (b) results for memory enhancement (top) with and (bottom) without high-frequency input signal. (Adopted from [20])




(a) (b)

Fig. 2. (a) Memristance time evolution and (b) pinched hysteresis loop of HP's $TiO_2$ memristor linear drift model [2] under small input signal and various noise scenarios, as presented in [19]. (Adopted from [19]) achievement of the maximum memristance enhancement using this configuration are further studied in [21].

### B. White Noise

Following the basic principles of SR, Stotland and DiVentra presented how the introduction of a white noise signal to the driving input signal can affect the memristance value range in a positive way [19]. Utilising the nonlinear dynamics of the memristor, the introduced noise is able to improve the quality of memristor's hysteresis loop by achieving a higher $R_{MAX}/R_{MIN}$ ratio under the same driving input signal. In Fig. 2(a), three examples with different values of noise power are presented, illustrating that for a specific amount of noise power the $R_{MAX}/R_{MIN}$ ratio can be improved. However, after a certain point, the inserted noise can result to the degradation of system's performance as the $R_{MAX}/R_{MIN}$ ratio is mostly related to the noise signal. The aforementioned effect of noise is further presented in I-V domain by Fig. 2(b).

### C. Understanding SR on linear HP memristor model

In both of the above approaches, memristor's behaviour is described by the original HP's $TiO_2$ memristor model [2], which maps the total memristor's resistance (Memristance, $M$) to two in-series variable resistors corresponding to the resistance of the undoped ($R_{OFF}$) and the doped ($R_{ON}$) areas of the device, with $R_{ON} << R_{OFF}$. The total resistance of the device is calculated based on the width of each area, according to the following expression:

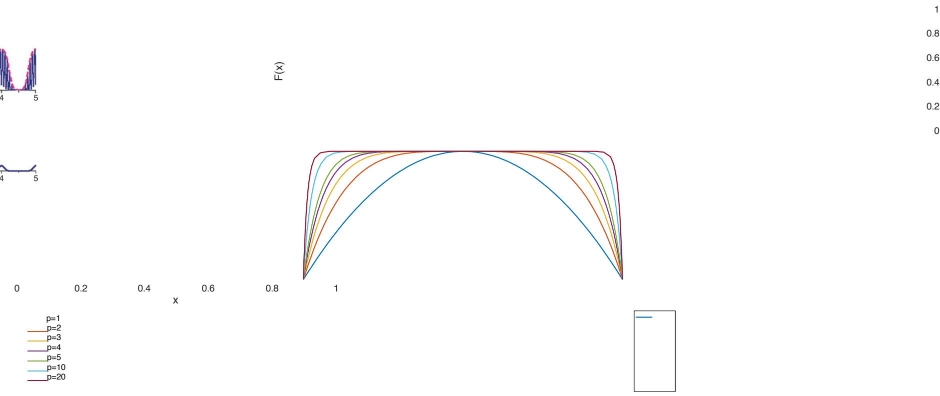



Fig. 3. Joglekar's window function [22] for various values of parameter $P$.

where $\mu_V$ is the average ion mobility. Given that the changing rate of memristance is governed by a linear expression, the requested nonlinearity for the emergence of SR-related behavior is missing.

Towards the deeper investigation of how memristance enhancement is achieved and of its relationship with SR, the above missing point is further explored. Specifically, although Eq. (2) is a linear expression, memristor is a physical system and its state values have to be bounded. So, during the use of memristor model in [19], [20], the handling of the boundary conditions was achieved by applying the Joglekar's window function $F_{window}(x, P) = 1 - (2x - 1)^{2P}$ [22] (Fig. 3). In specific, they are multiplying Eq. (2) by the window function $F_{window}(\frac{w(t)}{D}, 1)$, which fences memristor's state ($0 \le w(t)/D \le 1$) by suppressing, in nonlinear manner, its changing rate when close to the boundaries, even makes it zero when exactly at a boundary value independent of the applied signal.

The above realisation provided a useful tool on the deeper understanding of how memristance enhancement was achieved before. More particularly, setting the initial memristor state near the boundaries, the driving signal alone is incapable of changing the state sufficiently away from the boundary and as a result it hardly affects the memristance. On the other hand, the inserted noise supports the driving signal on moving the state away from the boundaries and, therefore, the window function is lesser suppressing the state's changing rate, giving the driving signal the capability to achieve higher $R_{MAX}/R_{MIN}$ ratio.

## III. Single-Frequency Noise Study

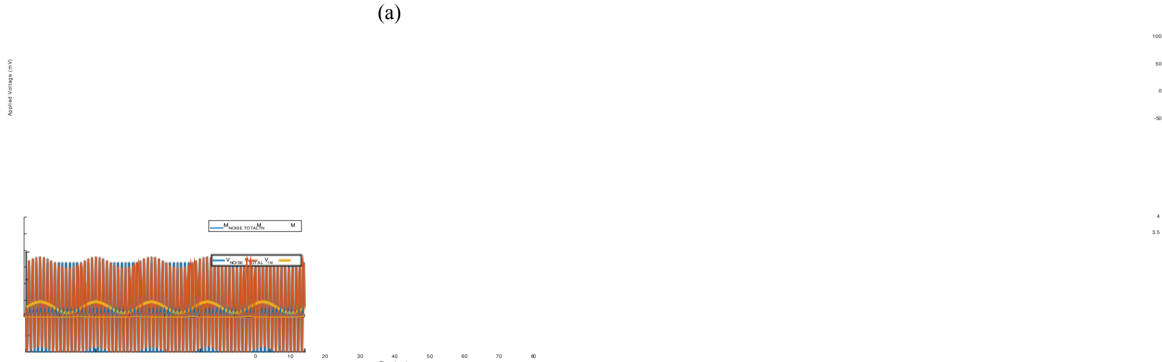
Delving deeper into the single-frequency memristance en- hancement approach, after exploring the mechanisms of Tanaka's *et al.* examples, the applicability of this approach in a wider range of memristor models is examined. Although HP's original $TiO_2$ memristor model is the most widely used it constitutes the first attempt to model the behaviour of the actual device, while, since now, other memristor models have been proposed.

[23] in HP Labs, and the Tungsten Oxide ($WO_x$), proposed by Chang *et al.* [24], memristor models have been adopted

where $w(t)$ is the width of the doped area and $D$ the thickness of the thin semiconductor film. In this model, the changing rate of $w(t)$ depends on the current $i(t)$ that passes through the device and constitutes the state equation of system by the following linear expression:

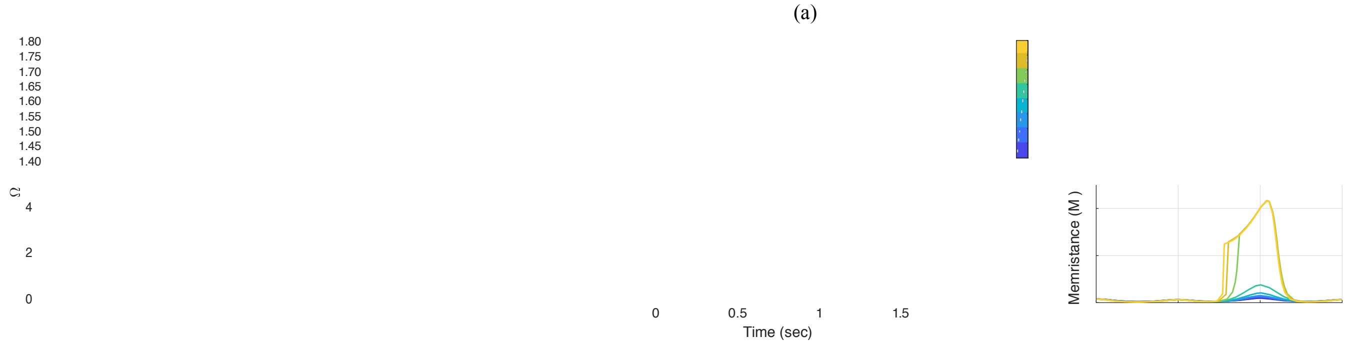
to explore the validity of single-frequency memristance en- hancement approach with more updated realistic models.

(a)



(b)


(a)

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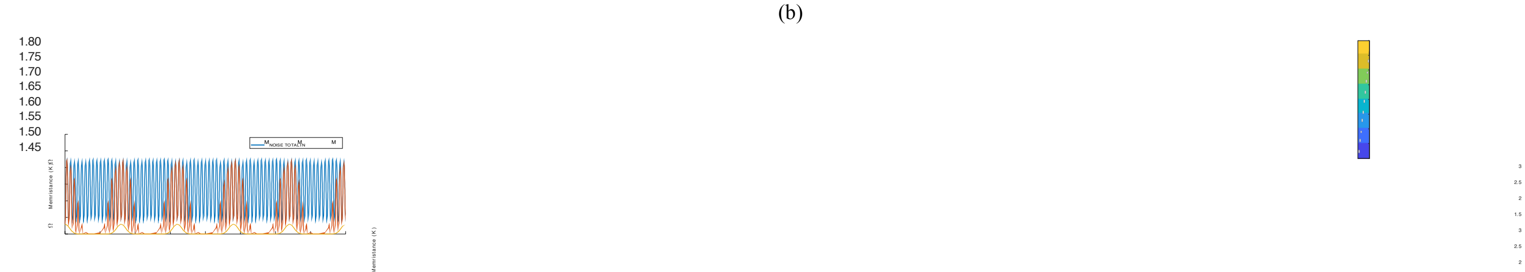
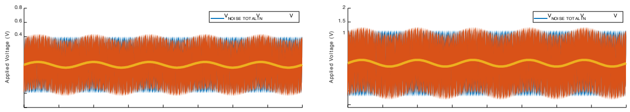
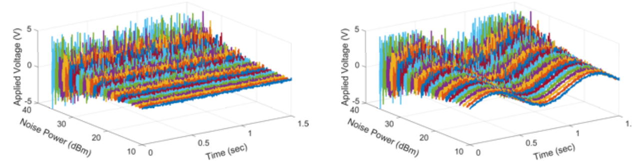


Fig. 5. $WO_x$ memristor model's response to sinusoidal applied voltage signals with different amplitudes $V_{peak}$.

(c)

(d)







0 10 20 30 40 50 60 70 80
Time (sec)

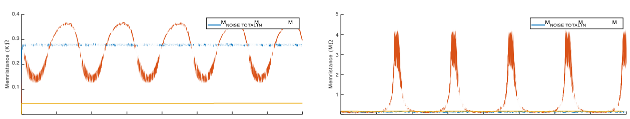
(e)

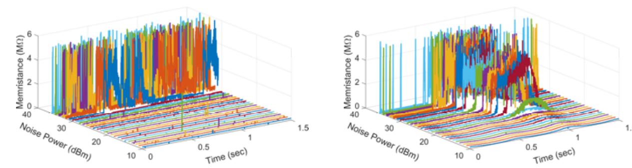
0 10 20 30 40 50 60 70 80
Time (sec)

(f)

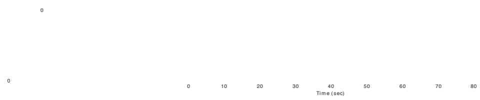
(a)

(b)



(g)



(h)

Fig. 4. (a), (b), (e), (f) Applied voltages for driving (yellow), noise (blue) and combined (orange) signals. (c), (d), (g), (h) Memristance time evolution according to the corresponding applied signals (a), (b), (e), (f), using Tanaka's example [20], modified example with $TiO_2$ model, example with $TaO_x$ model and example with $WO_x$ model, respectively.

model is considered a stable and accurate memristor model that provides a robust SPICE-compatible netlist.

In specific, in order to achieve memristance enhancement with different memristor models, the applied voltage has to be modified according to each model's requirements and constrains. Thus, firstly, the original example of [20] was reproduced and in Figs. 4(a), (c), the applied signal and its effect to the memristance for the cases of only noise, driving and combined signals are illustrated. Moreover, using the knowledge acquired previously, during the study of SR on HP's model, the initial state of memristor is crucial to the emergence of memristance enhancement, so a second example with $TiO_2$ model for initial conditions ($x_0 = \frac{w_0}{D} = 0.9997$ $M_0$ $1.001K\Omega$) close to the boundary value $x = 1$, corresponding to $R_{ON}$ state, is further presented in Figs. 4(b), (d). As result of our modification, the effect of noise alone on the memristance is negligible compared to the previous example; however, the combination of driving and noise signal is able to achieve memristance enhancement. Additionally, for the modified example, the amplitude ratio between the driving signal and the noise ($p$) is decreased from

(c) (d)

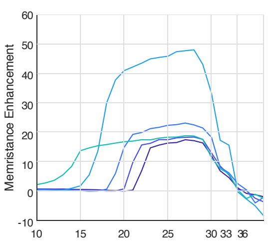
Fig. 6. (a) Only-noise and (b) combined signals for various noise power levels. (c), (d) $WO_x$ memristor model's response to the applied voltages signals of (a) and (b), respectively.

the noise signal is increased radically to reduce its effect on the memristance, so from $q_{Tanaka} = \frac{\omega_e}{\omega_0} = 15$ it is here $q = 200$. Furthermore, in each case the value of amplitude ratio was fixed at $p = 10$. Nevertheless, the amplitude of driving signal is adjusted according to model's characteristics, so $V_0 = 40mV$ and $V_0 = 120mV$ for $TaO_x$ and $WO_x$ examples, respectively.

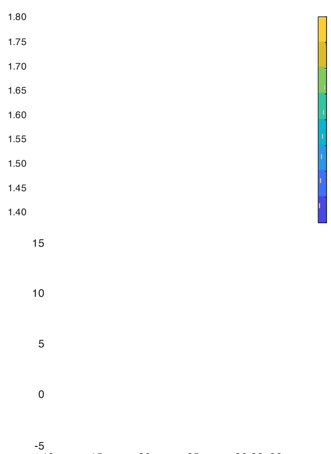
## IV. White Noise and Variability

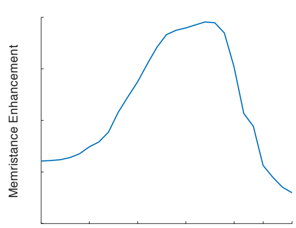
Aiming to study the use of white noise on memristance enhancement for future application on actual devices, for the rest of the paper only the $WO_x$ memristor model is taken under consideration, as provides realistic results without the convergence issues that emerge during the simulations of $TaO_x$ model under the highly fluctuating voltages of the input signal.

ing signal can aid at the switching of memristor's state accord- ing to the latter signal, corresponding to higher $R_{MAX}/R_{MIN}$ ratio. So, in Figs. 6(a), (c), the effect of noise alone on a $WO_x$ memristor is presented for a wide range of levels of noise's power, while in Figs. 6(b), (d), the aforementioned noise signals are combined with a specific driving signal ($V_{peak} = 1.45V$).

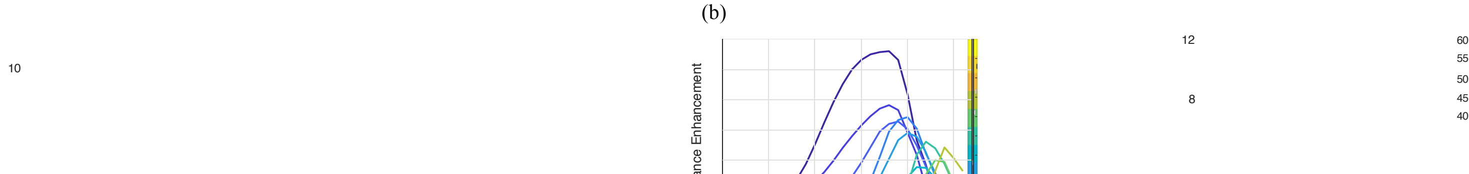



(a)






(b)



6 4 2 -2

35 30 25 20 15 10 5

0

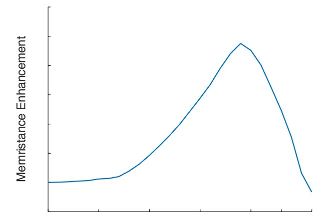
10 15 20 25 30 35

Noise Power (dBm)

(a)

4 3.5 3 2.5 2 1.5 1 0.5

10 15 20 25 30 33 36




Noise Power (dBm)

(b)

Fig. 7. (a) Average values of memristance enhancement using white noise for different $V_{peak}$ values in regard to the noise power. (b) Total average memristance enhancement for all $V_{peak}$ values over the noise power.

Different from previous work on $TiO_x$ model which reveal a memristance response following the input signal sinusoidal behavior [19], the dynamics of $WO_x$ model provides a more complicated memristance vs applied signals evolution. Thus, the memeristance enhancement evaluation in this case cannot be simply calculated as the on-resonant frequency response of combined input and noise signal over the corresponding only- noise value. So, in a way to evaluate the effect of noise on memristor's switching for the $WO_x$ model, the area of the memristance ($A$) in the Memristance-Time graph was calcu- lated, as a way to measure the duration of memristance being in $R_{MAX}$ state, for each of the aforementioned cases, and the following expression is utilised to evaluate the memristance enhancement:

$$\text{Memristance Enhancement} = \frac{A_{signal-and-noise} - A_{only-noise}}{A_{no-noise}}. \quad (3)$$

The main idea of the above calculation is to show how much the memristance range improved over the corresponding no- noise case, but also subtracting the effect of the noise alone. The average results over 50 simulations of each $V_{peak}$ value and noise power level are presented in Fig. 7(a), where is shown that increasing the $V_{peak}$ values, the enhancement is lower as driving signal achieves higher memristance value alone, even the switching of the device and the enhancement by noise is disappearing ($V_{peak} > 1.65V$). Additionally, the general average memristance enhancement of all simulations over the noise power in Fig. 7(b) depicts the SR-like region that noise utterly improves the memristance range.

*Variability addressed by Memristance Enhancement*

In spite of the fact that memristance enhancement can effectively increase the range of memristance values, the exploration of its suitability on the handling of memristors' variability is evident. Given that individual memristor's be- havior inside a memristor crossbar array is varying due to the nanofabrication processes, a way to tackle with the diversion of memristor switching is necessary. Thus, at this point we performed a study on the effect of noise on memristance for a wide range of memristor devices with variability character- istics introduced on model's state equation, as shown in the following modified form of [24]'s expression:

$$\frac{dw}{dt} = (\lambda + \delta_\lambda)[\exp(\eta_1 + \delta_{\eta_1})V) - \exp(\eta_2 + \delta_{\eta_2})V)], \quad (4)$$

Fig. 8. (a) Average value of memristance enhancement using white noise for different $Var$ values in regard to the noise power. (b) Total average memristance enhancement for all $Var$ values over the noise power.

where $\lambda$, $\eta_1$, and $\eta_2$ are fitting parameters of the original model [24], corresponding to the changing rate, the $SET$ and the $RESET$ threshold, respectively. So, $\delta_\lambda$, $\delta_{\eta_1}$, and $\delta_{\eta_2}$ are randomly selected fluctuations of the corresponding entities.

During our variability tests, 30 different sets of parameters $\delta_\lambda$, $\delta_{\eta_1}$, $\delta_{\eta_2}$ were selected by using Gaussian distribution around variability $Var = \{5, 10, ..., 60\}$ % of the original values of $\{\lambda, \eta_1, \eta_2\}$ and tested for the cases of: (a) only driving signal, (b) only noise, and (c) the combination of them, for 10 to 36 $dBm$ noise power.

The memristance enhancement results for the aforemen- tioned simulations are depicted in Fig. 8. In specific, the average results for each noise power level per variability percentage $Var$ are presented in Fig. 8(a), which indicates that the memristance enhancement is diminishing (and requires higher noise levels) for high variability sets. The general average memristance enhancement of all simulations over the noise power is illustrated in Fig. 8(b), with a maximum average enhancement of 3.4× for 29 $dBm$ noise power.

## V. Conclusions

In this paper, the study of the positive effect of noise, that accompanies a driving voltage applied to a memristor, is performed. Initially, a single-frequency approach of mem- ristance enhancement was adopted and simulations assum- ing accurate $TaO_x$ and $WO_x$ memristor models have been held, demonstrating the potential applicability of this circuit- friendly approach on actual devices. Furthermore, memristance enhancement based on white noise experiments have carried on using the robust $WO_x$ model in the presence ofdevice- to-device variability. Our experiments indicated that white noise based SR meristance enhancement requires higher noise levels for high device variability but it can deliver up to 3.4 $R_{MAX}/R_{MIN}$ ratio improvement in the presence of up to 60% device variability, which makes it a strong candidate for practical implementations. As future work, the effect of noise against variability issues in resistive memory cells will be studied, aiming to the enhancement of the margin between the resistive states.


## ACKNOWLEDGMENT

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