Multifractal characterization of seismicity: the case of Carterbury region (New Zealand), 2000 -2018

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Abstract: *The Multifractal Detrended Fluctuation (MF-DF) algorithm is applied to measure the complexity of two time series, the inter-event hypocentral distance $\Delta\delta(t)$, and the inter-event time series $\Delta\tau(t)$ In particular, we apply this methodology to the seismic sequences produced in the Carterbury region during 18 years (2000-2018). Results indicate a clear multifractal behavior of $\Delta\delta(t)$ and $\Delta\tau(t)$. Moreover, an increses in the complexity is observed when a large event occurs . These results suggest that the MF-DF algorithm could be useful as a seismic precursor index.*

I. INTRODUCTION

The concept of multifractal modeling has been used intensively in various fields of science for characterizing measures with self- similarity. Since [1], numerous studies have characterized the self-similar (or scale-invariant) properties of a wide variety of natural phenomena by using the concepts of fractal geometry and fractal dimension [2].In the case of earthquake phenomena, several studies found that the temporal, epicentral and hypocentral, and also the energy distribution of earthquakes, have multifractal characteristics [3-6].

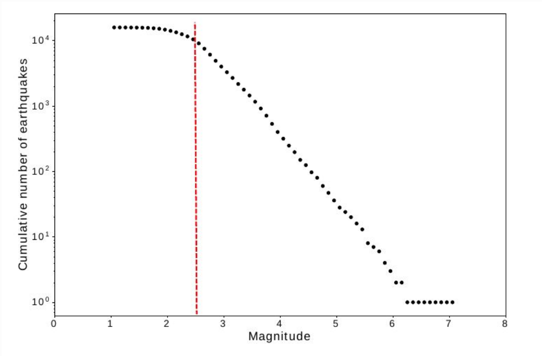
In this work the Multifractal Detrended Fluctuation (MF-DF) algorithm is applied to characterize multifractal scaling properties of two time series, the inter-event hypocentral distance $\Delta\delta(t)$, and the inter-event time series $\Delta\tau(t)$ from a particular seismic catalogue. These series give information of the seismic distribution in space and time.

II. SEISMIC CATALOGUE

New Zealand is located at the border of two major tectonic plates, the Australian and Pacific plates. The slowly driving movement of these plates breaks the Earth's crust into separated blocks producing fractures, also known as faults. One example is the Alpine Fault, which it is over 600 km long and is responsible for some of the largest earthquakes in New Zealand's history.

We analize in the Carterbury region, located in the South Island, close to Christchurch city. From 2000 to 2018 the seismic network registered 15889 events with magnitude larger than 1. However, we only consider the events with magnitudes larger than 2.5, because from this value a completeness in the seismic catalogue is assured to follow the Gutenberg-Richter relation [7].

Fig. 1. Frequency-Magnitude distribution of 15889 events registered in Carterbury (New Zealand) region from 2010 to 2018. Red line indicates the magnitude of completeness 2.5



III. METHODOLOGY

A. *Multifractal Detrended Fluctuation algorithm*

The multifractal properties of nonstationary series is analysed by means of the multifractal detrended fluctuation MF-DF, analysis [8-11]. After appling the MF-DF algorithm, one can compute the singularity spectrum $f(\alpha)$ as:

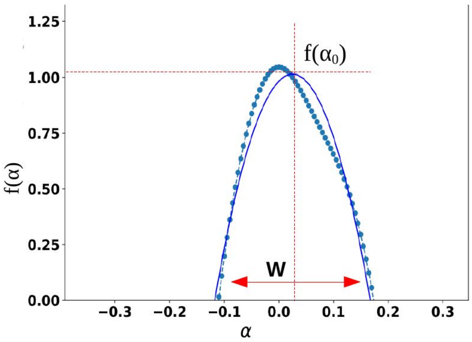
$$f(\alpha) = q\alpha - \tau(q) \quad (1)$$

where q is the q-th order fluctuation moment computed from the MF-DF algorithm, α is the singularity strength or Hölder exponent, and $\tau(q)$ is the global scaling exponent.

Fig. 2 shows an example of the singularity spectrum for an empirical time series. Eq. (1) can be well fitted by a polynomial of second order around the position α_0 which is Hölder exponent with maximum singularity spectrum, being

$$f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C \quad (2)$$

Fig. 2 Example of the singularity spectrum f(α) for an empirical time series (dotted line). Blue line is the fitting of a polynomial of second order.



We use the complexity index, CI, proposed in [12] to quantify the degree of complexity in a time series. This value is defined as,

$$CI(Z) = \{Z - \langle Z \rangle\} / \sigma(Z) \quad (3)$$

where $Z = z(\alpha_0) + z(B') + z(W)$, W is the spectral amplitude, $B' = -B/2A$ is the asymmetry, and $\langle Z \rangle$ and $\sigma(Z)$ are the mean and the standard deviation of the standardized Z respectively

B. Seismic data

We divided the total series using a moving window, in a sub-series of 1000 events length and shifted by 25 events each one. In total we consider 355 windows (or sub-series), and for each we compute their singularity spectrum and measure its multifractal behavior. Fig. 3 shows the length in time of each sub-series. The larger events are marked by dotted lines

III. RESULTS AND CONCLUSIONS

To summarize the results Fig. 4 shows the CI parameter because it is a combination of the other parameters (Eq. 3) and gives a measure of the multifractal behavior of $\Delta\delta(t)$ and $\Delta\tau(t)$ series. The results indicate evident multifractal behavior for the analyzed series $\Delta\delta(t)$ and $\Delta\tau(t)$. Moreover the maximum of the parameters captures a large event in the analyzed window. We interpreted that not only a large event produces a complexity increase in the seismicity but this could also be generated by seismic swarm. We explore the MF-DFA as a possible precursor index measure. However it is not obvious to define a precursor threshold value. To confirm this hypothesis would require to repeat this procedure in other series to obtain a wide perspective about the MF-DFA as a precursor tool.

Fig. 3 Evolution of each window length considering the same number of events per each one.

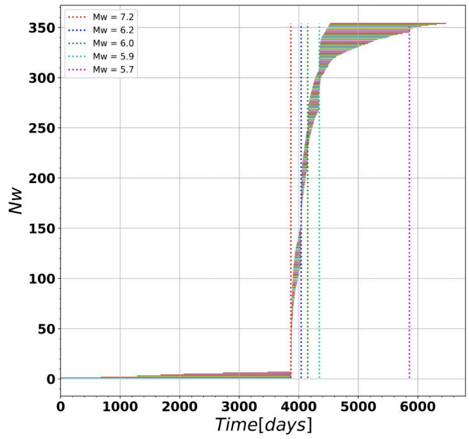
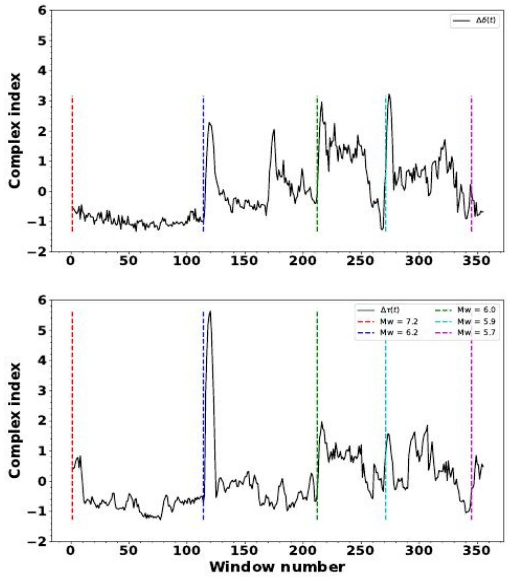


Fig. 4 Complexity Index as function of each window number. In dotted color lines the largest events are indicated.



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Marisol Monterrubio-Velasco received her PhD in computational physics by University Politécnica de Catalunya. She was postdoctoral researcher in the Geosciences center at Universidad Nacional Autónoma de México. At present she is postdoctoral researcher in the Barcelona Supercomputing Center Her research interests focus on computational physics, statistical analysis and numerical simulation applied to the Earthquakes.